Functional RG: from weak to strong coupling in the 2D Hubbard model

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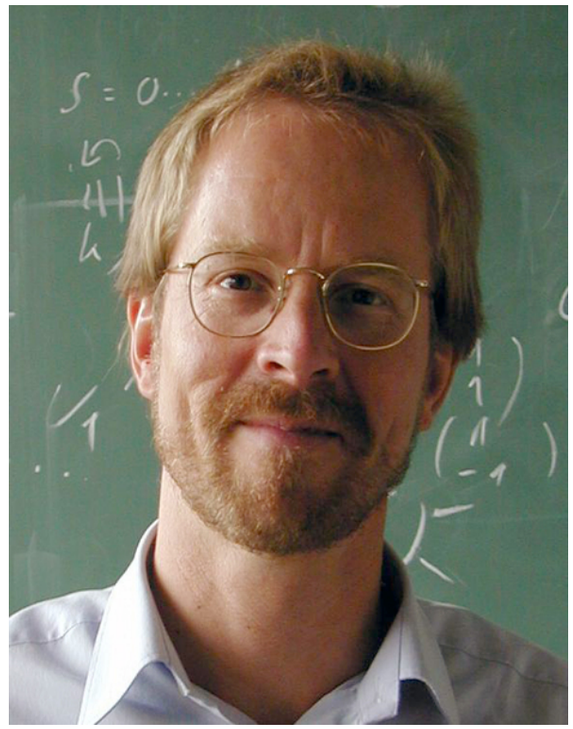
Collaborators:

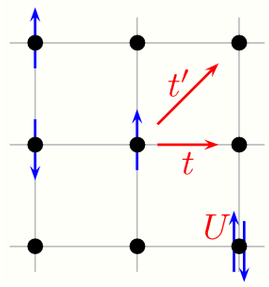
Ciro Taranto



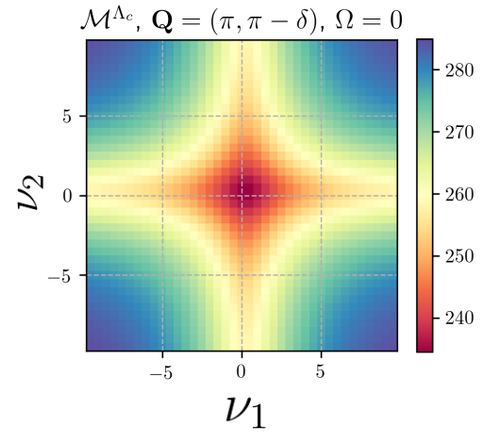
Walter Metzner

MAX-PLANCK-GESELLSCHAFT

Outline

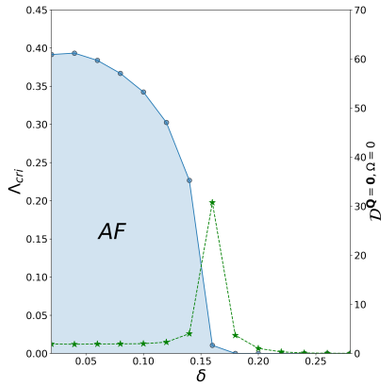


fRG: weak-to-intermediate coupling



- ➡ Frequency dependent interaction
- ➡ Charge divergence
- ➡ Self energy and pseudogap

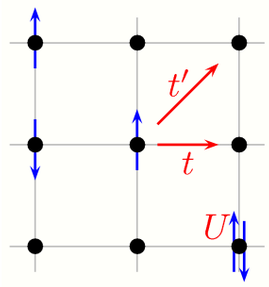
DMF²RG: strong coupling by starting from DMFT solution



- ➡ Local DMFT vertex affects non-local susceptibility
- ➡ *d*-wave pairing fluctuations

Conclusions and outlook

Outline



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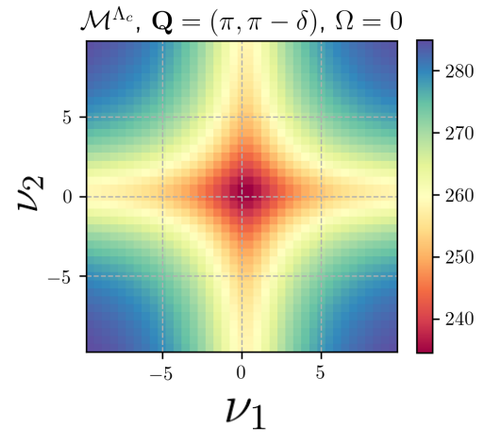

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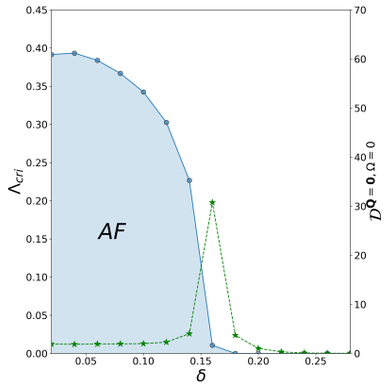
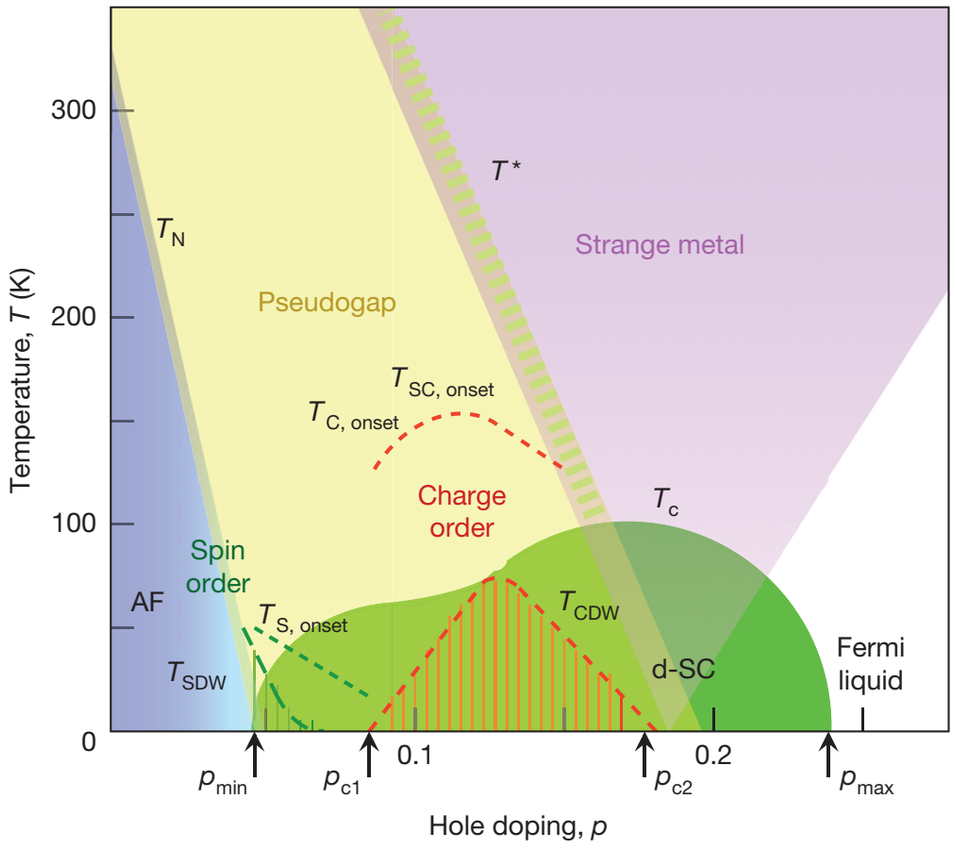


Conclusions and outlook

Motivation: CuO_2 high temperature superconductors

Phase diagram



Keimer et al., Nature (2015)

Known phases:

- ➡ Antiferromagnetism in undoped case
- ➡ *d*-wave superconductivity for sufficient doping

Interest due to:

- ➡ Competition of instabilities
- ➡ Strong correlations
- ➡ Quantum criticality

Prototype: 2D Hubbard model

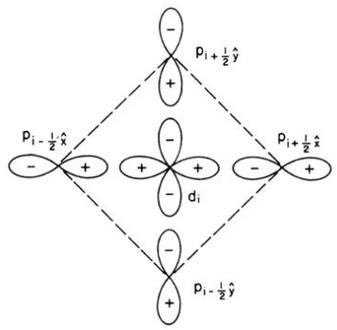
Effective model for HTSC

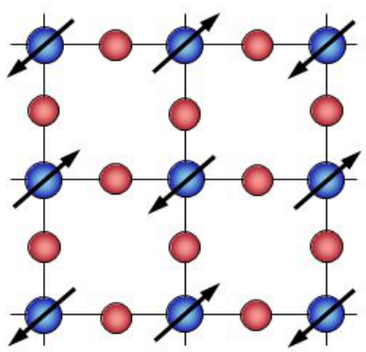
Anderson (1987)

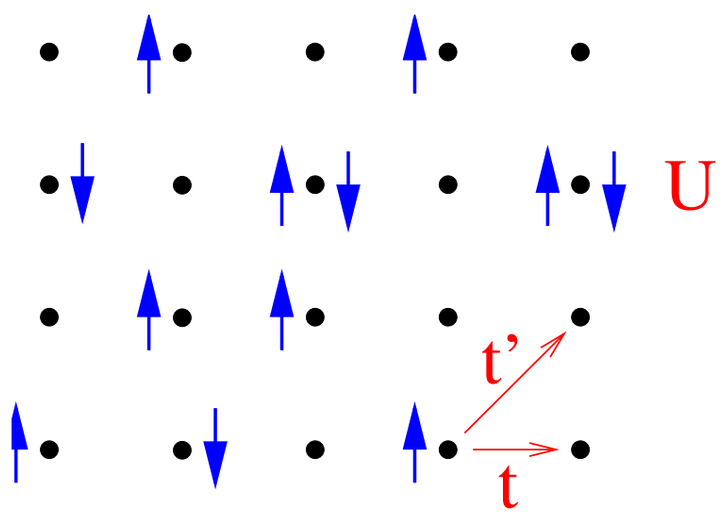
Zhang and Rice, PRB (1988)

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

CuO_2 plane





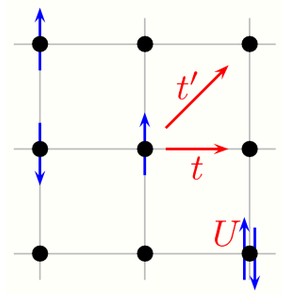


➡ Low energy model: Cu and O hopping effectively included in t-t' kinetic energy

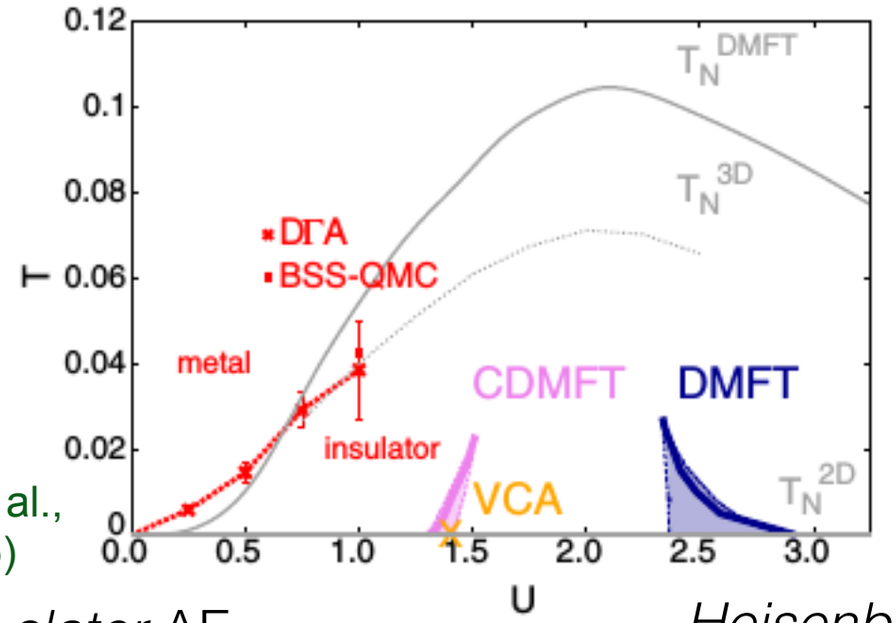
➡ Lack of charge instability observed in cuprates

2D Hubbard model

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$



$n = 1 \quad t' = 0$



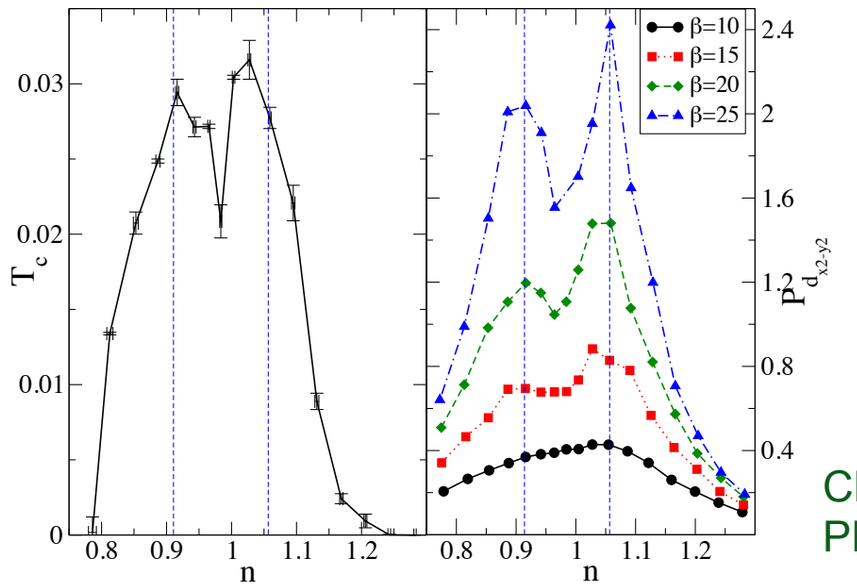
Schäfer et al., PRL (2015)

slater AF

Heisenberg AF

N_C=8 DCA

$n < 1 \quad t' \neq 0$



Chen et al., PRL (2015)

- SC d-wave phase in the limit $U \to 0$

Raghu, Kivelson and Scalapino, PRB (2010)

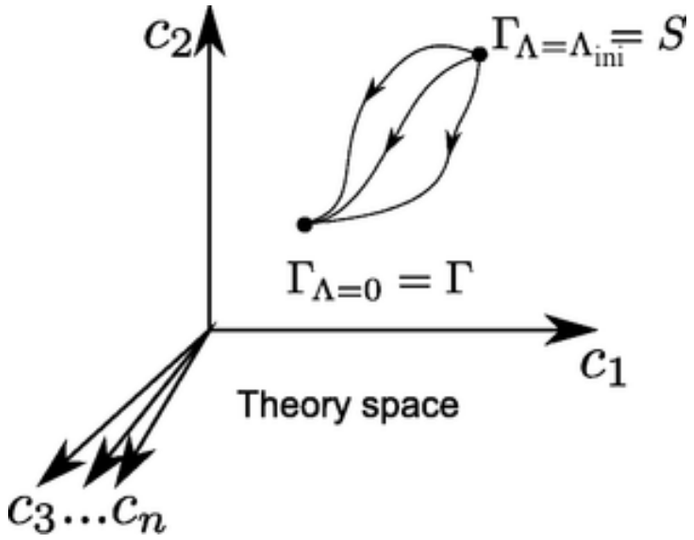
$$T_c \sim W \;\; \exp\left\{-\alpha_2 (t/U)^2 - \alpha_1 (t/U) - \alpha_0\right\} \times \left[1 + \mathcal{O}(U/t)\right]$$

Functional renormalization group (fRG)

Metzner et al. RMP (2012)

RG idea:

➡ Successive rescaling of the effective interaction



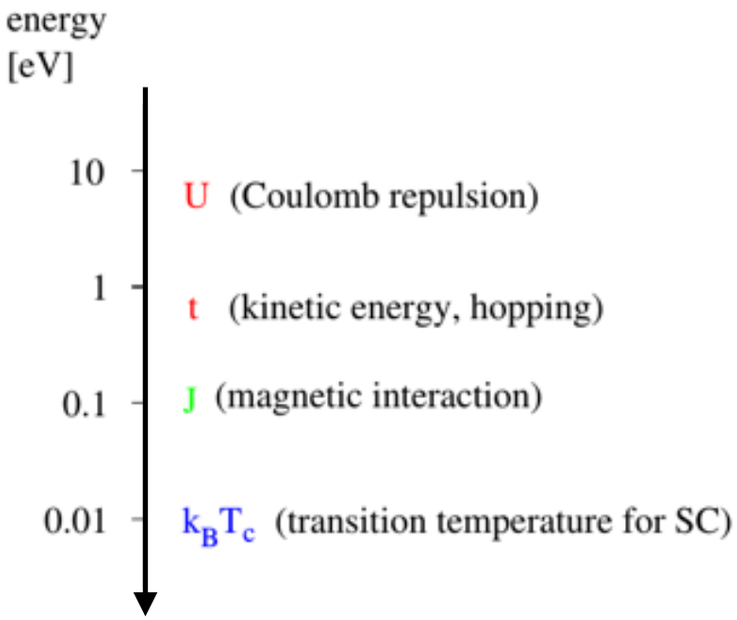
- Integrate degrees of freedom following hierarchy of energy scales
- Exact flow from *bare* to the effective action
- Unbiased channel competition (cuprate physics)

Scale dependent propagator:

$$G_0(\mathbf{k}, i\omega) \Rightarrow G_0^{\Lambda}(\mathbf{k}, i\omega)$$

Equivalence between multi-loop fRG and parquet approx.

Kugler and von Delft, arXiv:1703.06505



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Hierarchy of flow equations

Metzner et al. RMP (2012)

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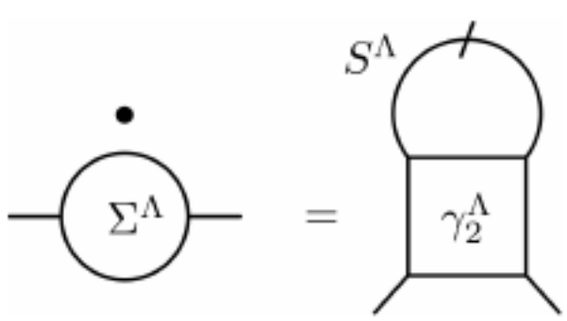
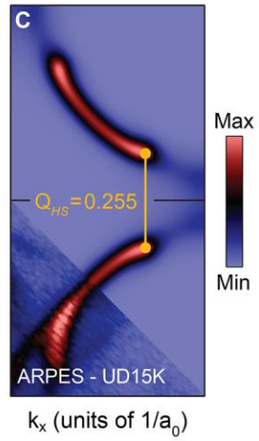


$$S^\Lambda = -G_\Lambda^2 \frac{d}{d\Lambda} G_{0,\Lambda}^{-1}$$



ARPES

Comin et al., Science (2013)

Metzner et al. RMP (2012)

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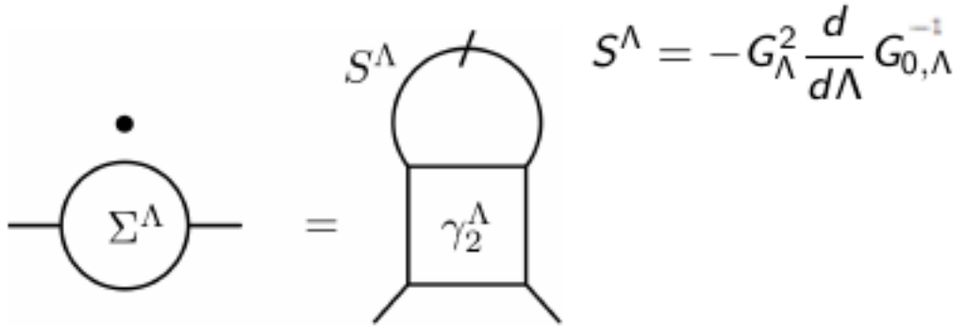

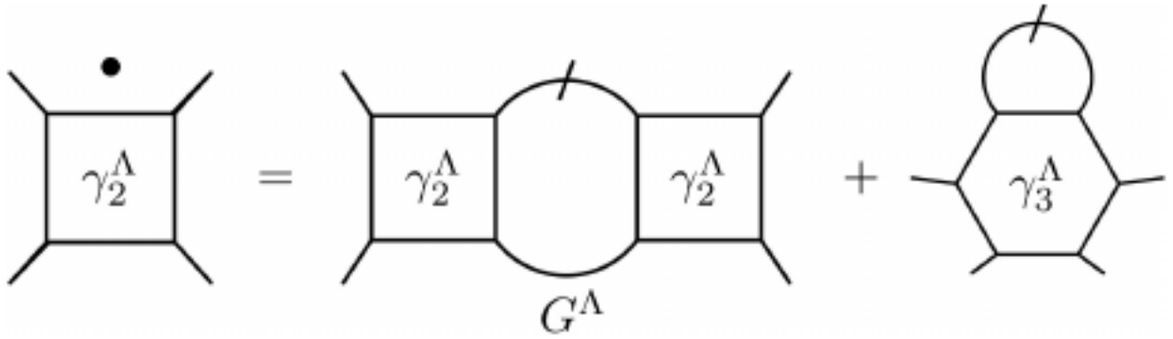

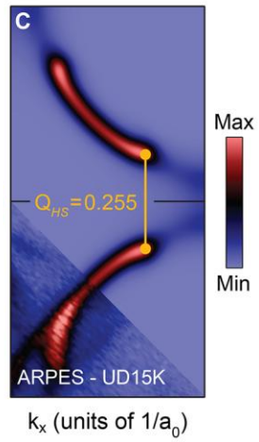

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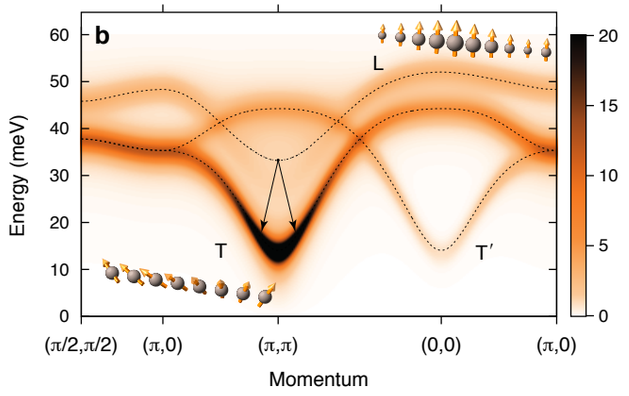

spin-wave

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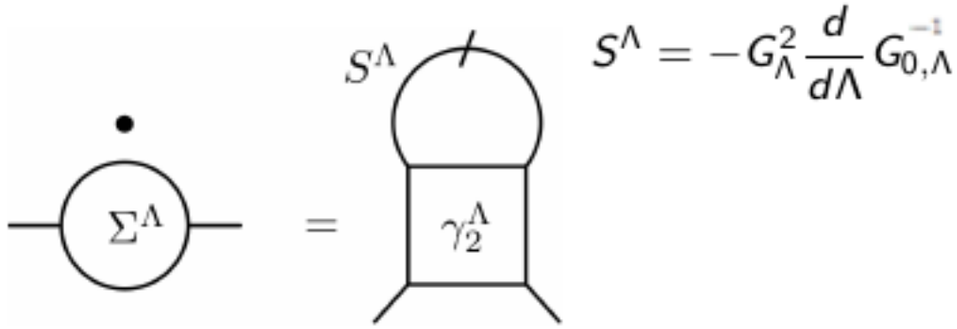
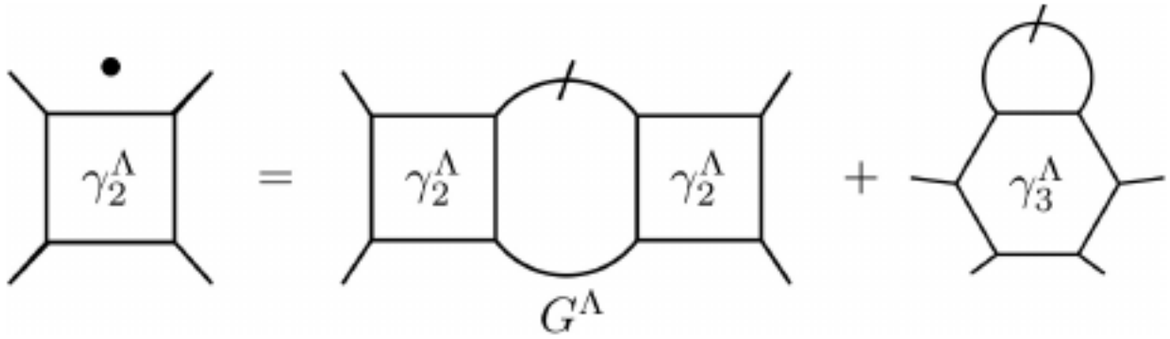
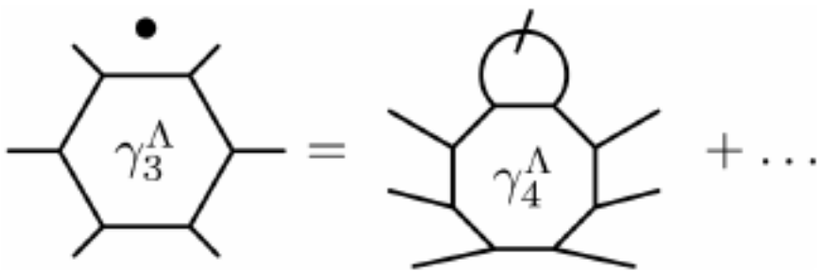
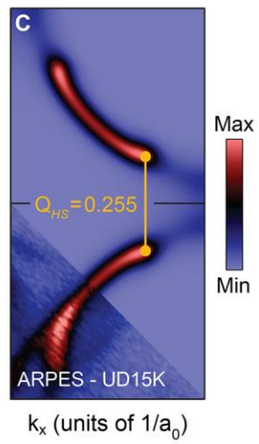






Metzner et al. RMP (2012)



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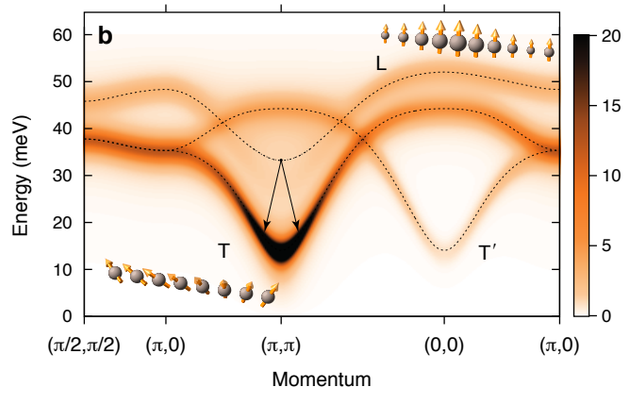

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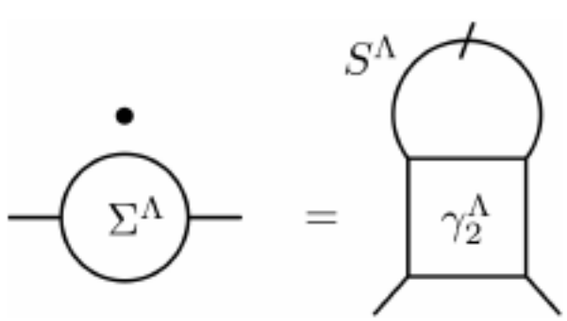

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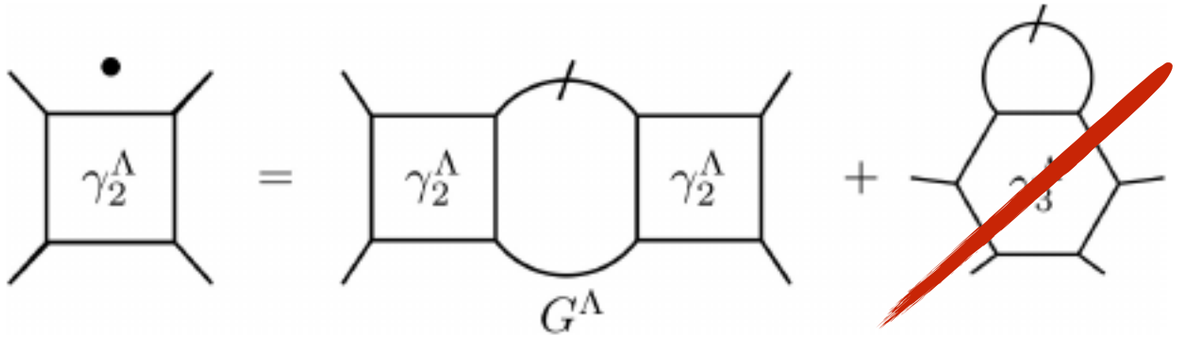

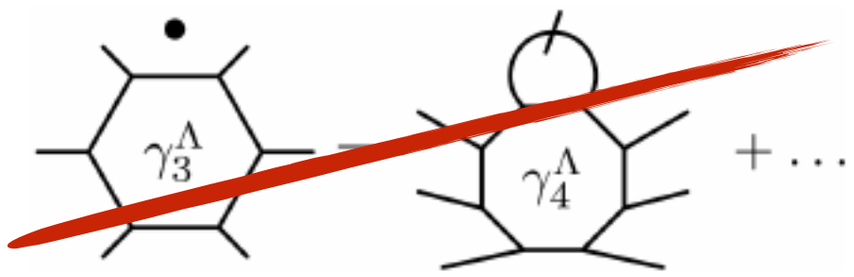

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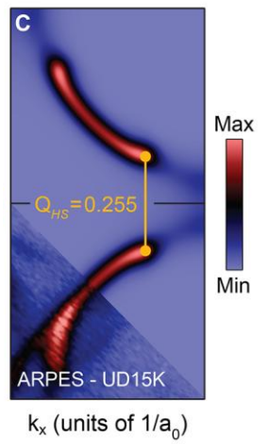

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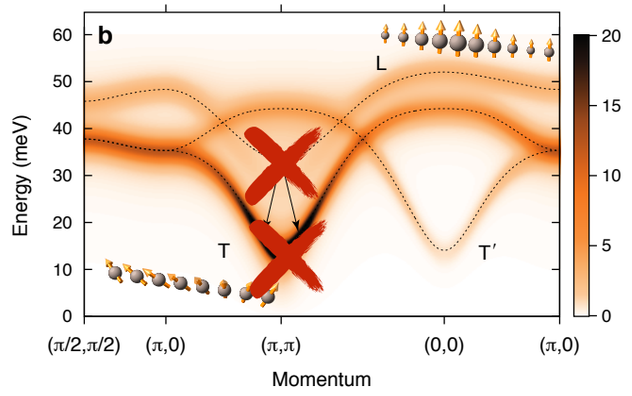


spin-wave

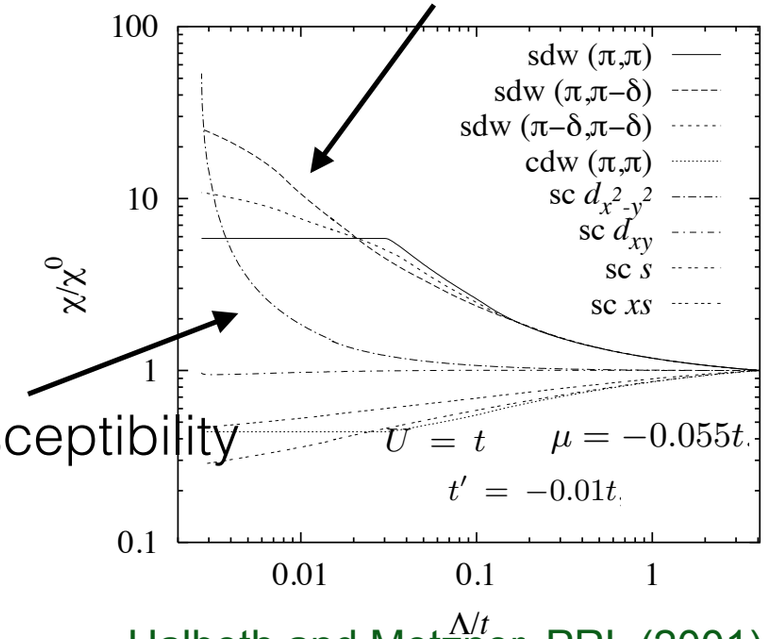
Jain et al., Nature (2013)

One loop: improper inclusion of soft and amplitude modes

Salmhofer et al., PTP (2004)

2D Hubbard model seen by fRG

Spin susceptibility



d-wave susceptibility

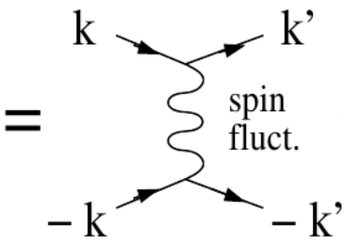
Halboth and Metzner, PRL (2001)

Spin fluctuation driven superconductivity

Gap equation:

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}}$$

$V_{kk'}$ =

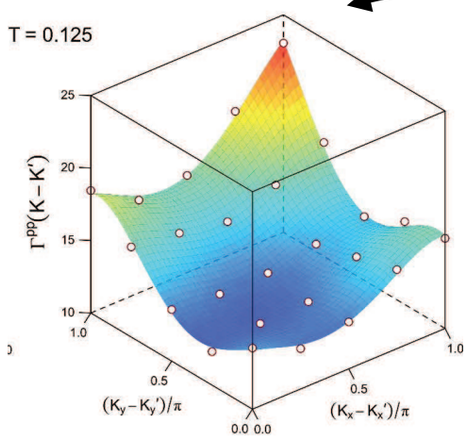


d-wave solution for $V_{k,k'} > 0$

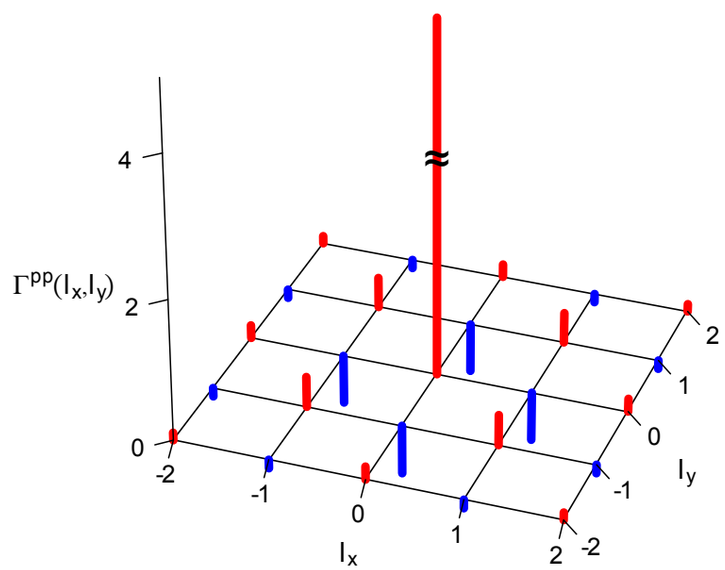
Effective interaction in PP channel

Scalapino, RMP (2012)

$k - k' \sim (\pi, \pi).$



real space



Frequency space in fRG

DV et al., arXiv: 1708.03539

Full frequency dependent interaction:

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = U\ -\phi_{sc}^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
$$+\frac{1}{2}M^{\nu_1,\nu_2,\nu_1+\nu_2-\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3) - \frac{1}{2}C^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3)$$

Based on Husemann and Salmhofer PRB (2009)

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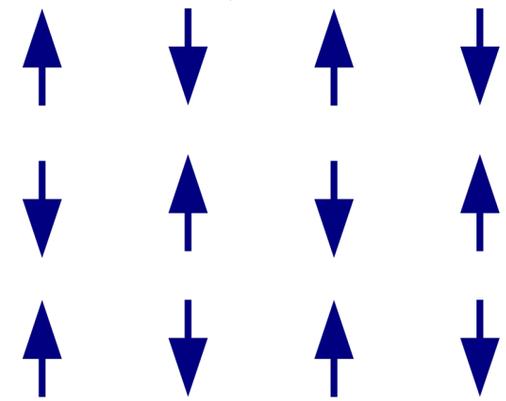

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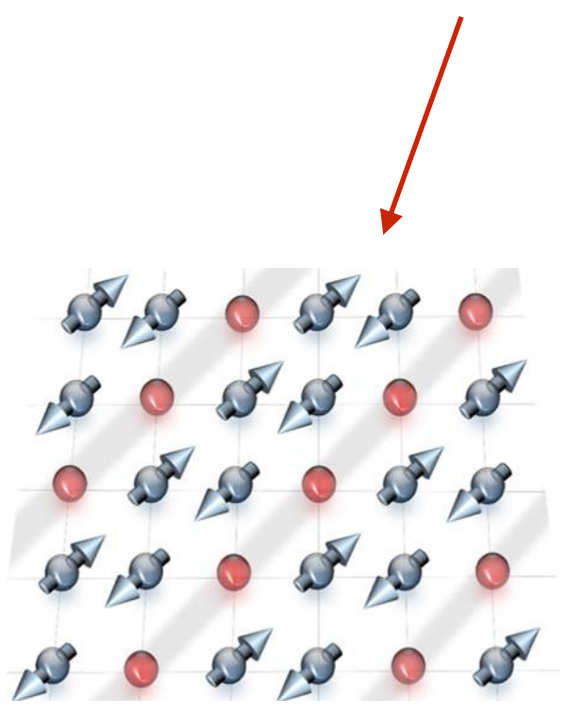

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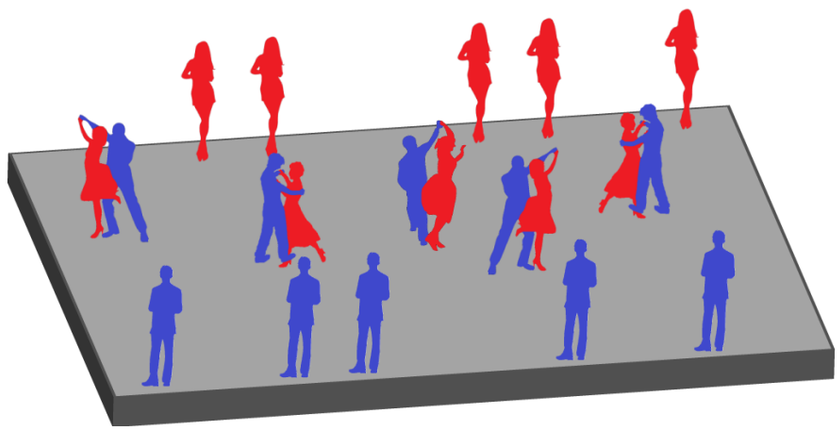
$$\phi_{SC}^{\nu_1,\nu_2,\nu_3}(\boldsymbol{Q},\boldsymbol{k}_1,\boldsymbol{k}_3) = S^{\nu_1,\nu_2,\nu_3}(\boldsymbol{Q}) + d_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}_1} d_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}_3} D^{\nu_1,\nu_2,\nu_3}(\boldsymbol{Q})$$



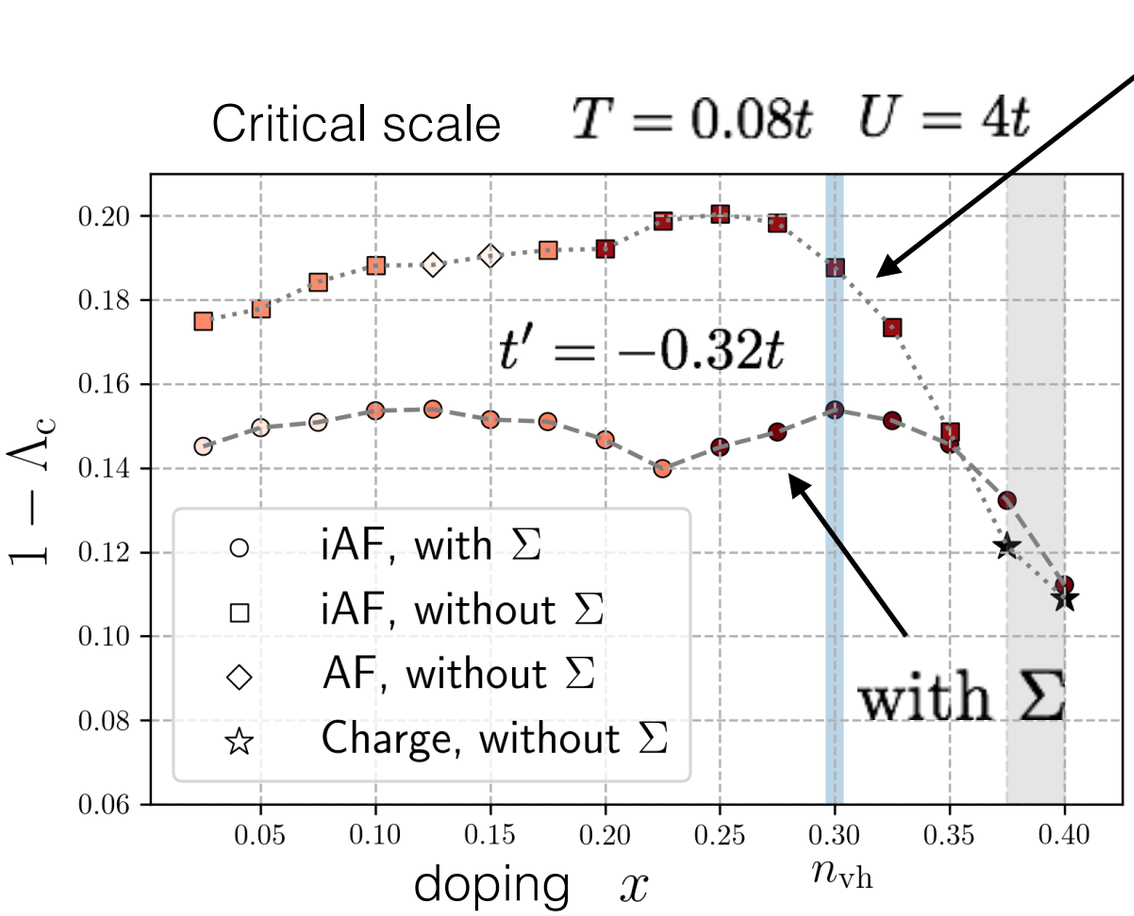
Based on Husemann and Salmhofer PRB (2009)

Interaction flow and phase diagram

Interaction scheme: (Honerkamp et al., PRB 2004)

$$G_0^{\Lambda}(k) = \Lambda G_0(k) = \frac{\Lambda}{i\nu + \mu^{\Lambda} - \varepsilon_{\mathbf{k}}}$$

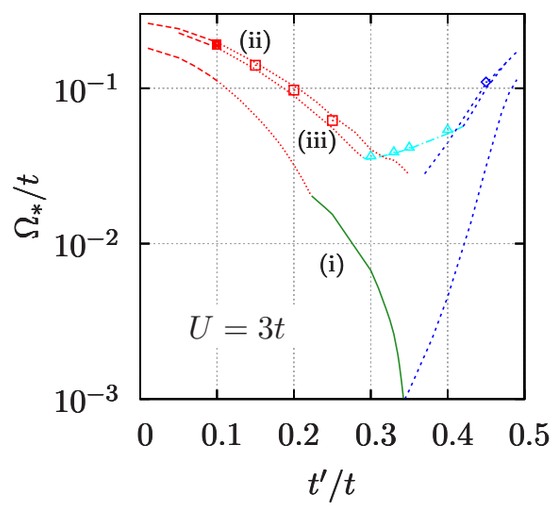
Critical scale interpreted as critical coupling: $U_c = \Lambda_c^2 U$



- ➡ Charge divergence at finite exchange frequency
- ➡ Self energy feedback “cures” charge divergence

DV et al., arXiv: 1708.03539

Consistent with Husemann et al. PRB 2012



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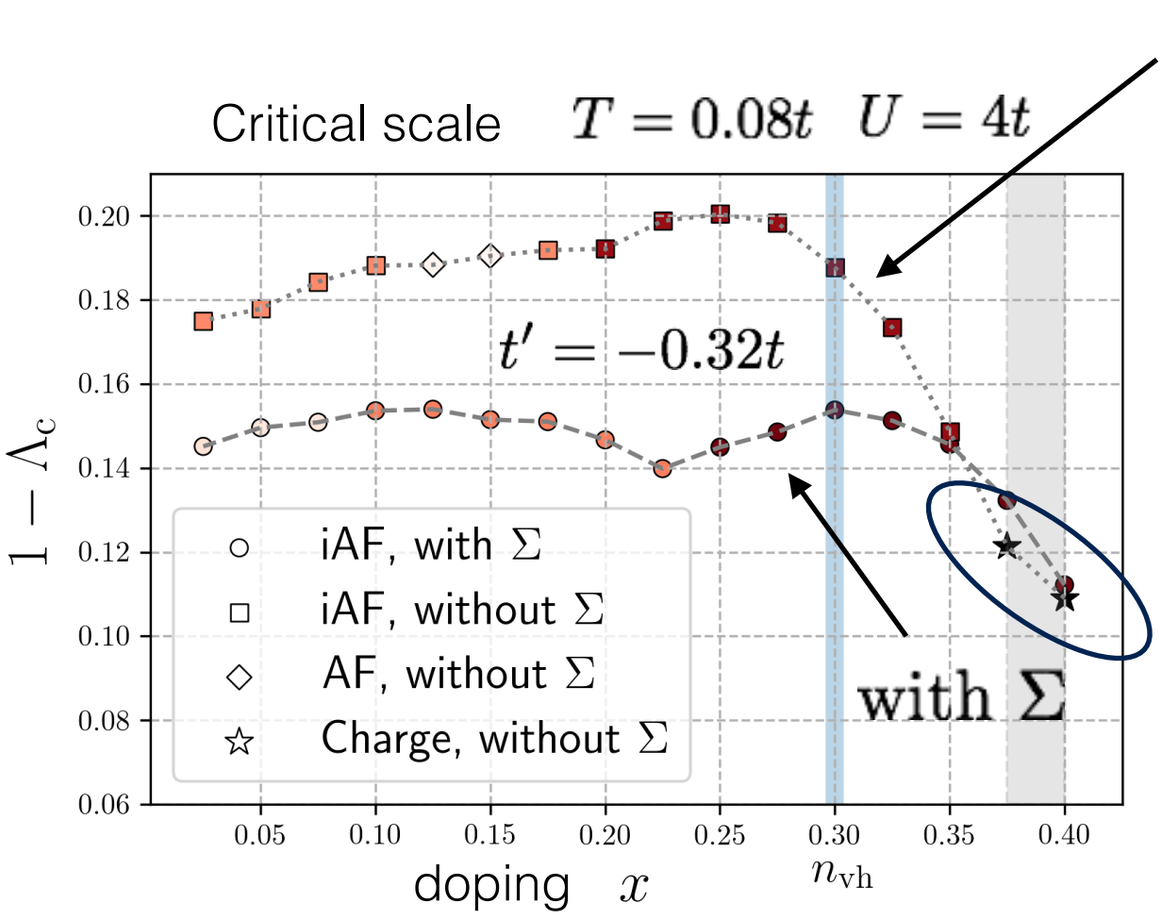

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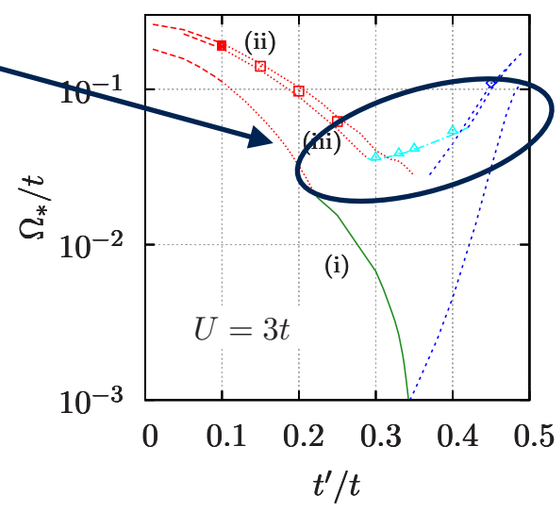
- ➡ Self energy feedback “cures” charge divergence

Consistent with Husemann et al. PRB 2012

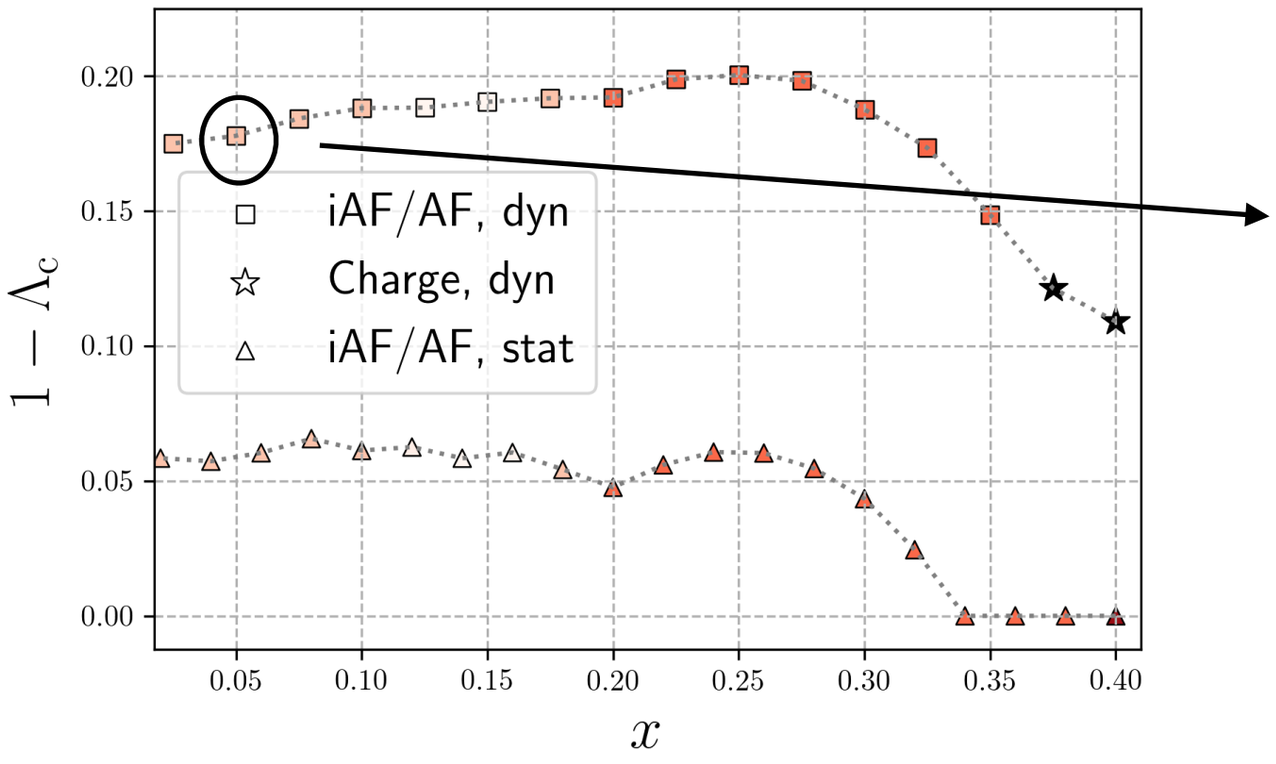


DV et al., arXiv: 1708.03539

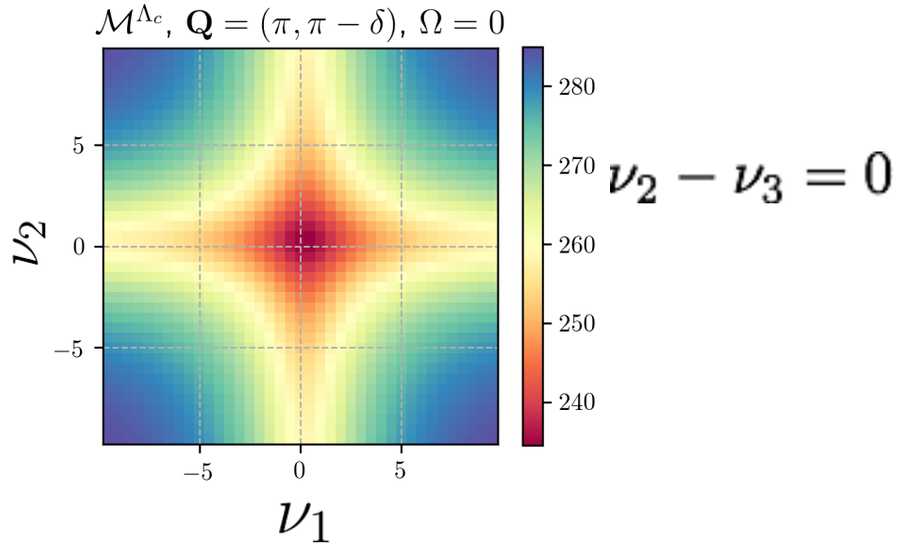
Interaction flow and phase diagram

DV et al., arXiv: 1708.03539

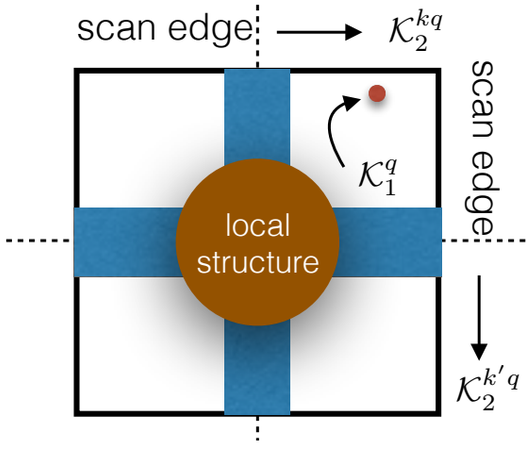
Dynamic vs static approximation



Magnetic channel



Consistent with Wentzell et al. (2016)



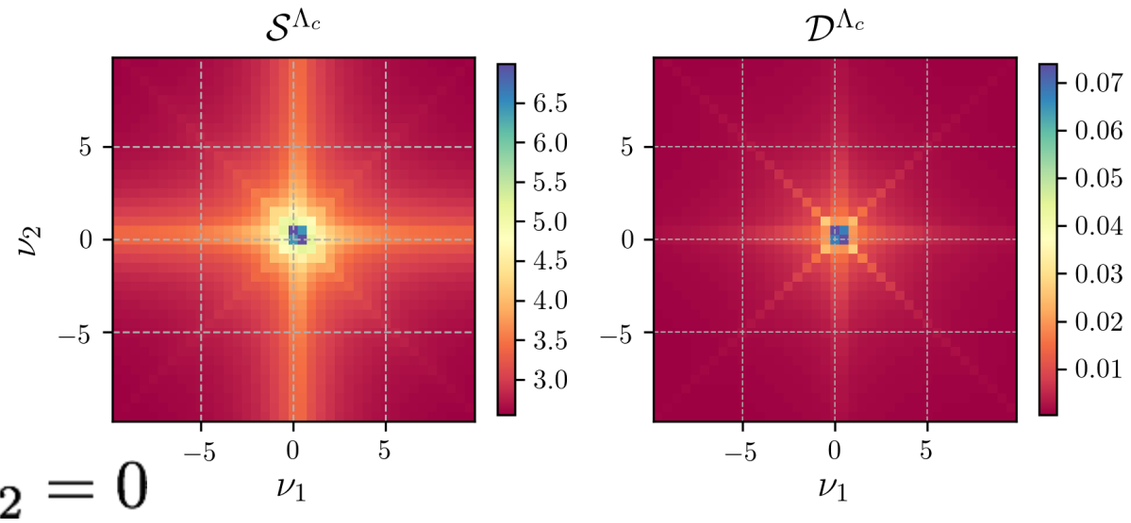
- Frequency dependence enhances $1 - \Lambda_c$
- The static approximation overestimates the effect of channel competition on the magnetic one

Effect of frequency dependence on PP d-wave channel

DV et al., arXiv: 1708.03539

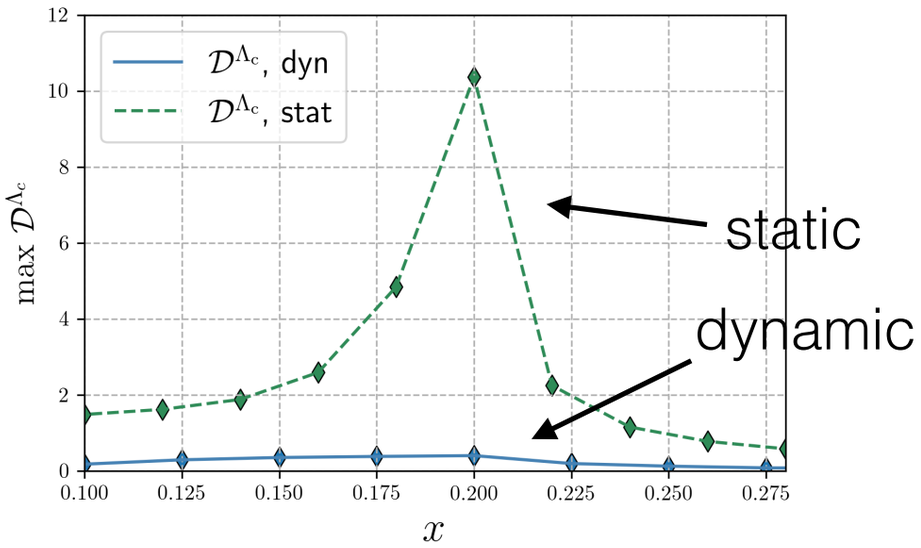
Pairing channels

doping = 0.4



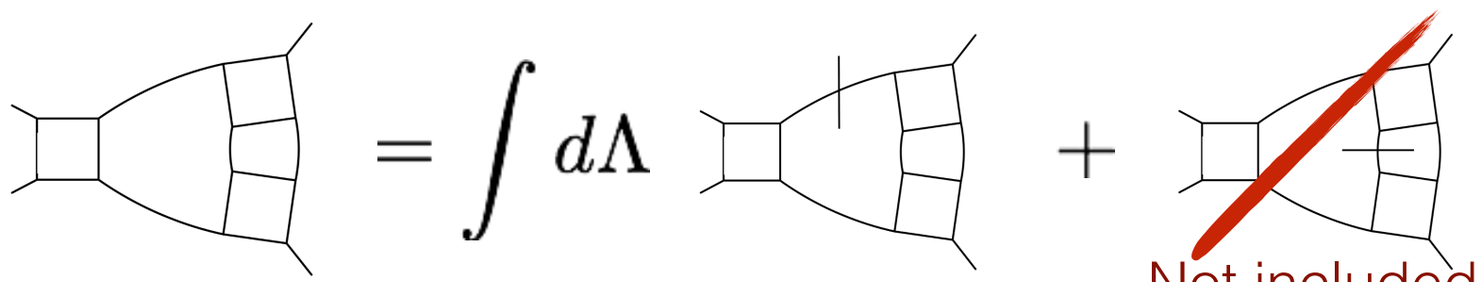
$$\nu_1 + \nu_2 = 0$$

d-wave: dynamic vs static



d-wave channel affected by two factors:

- Localized frequency structure Wentzell et al., 2016
- Interaction flow underestimates *d*-wave pairing

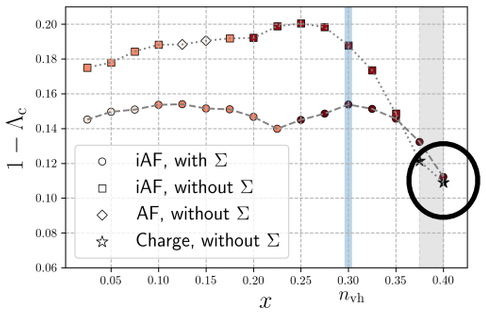


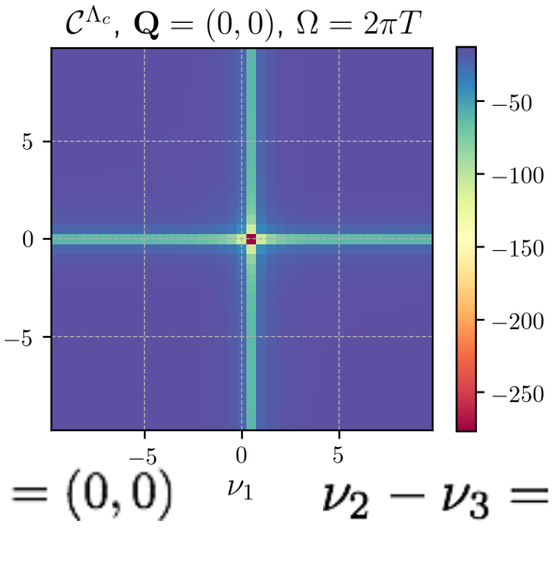
Taranto, PhD Thesis

Not included in one-loop

Charge divergence

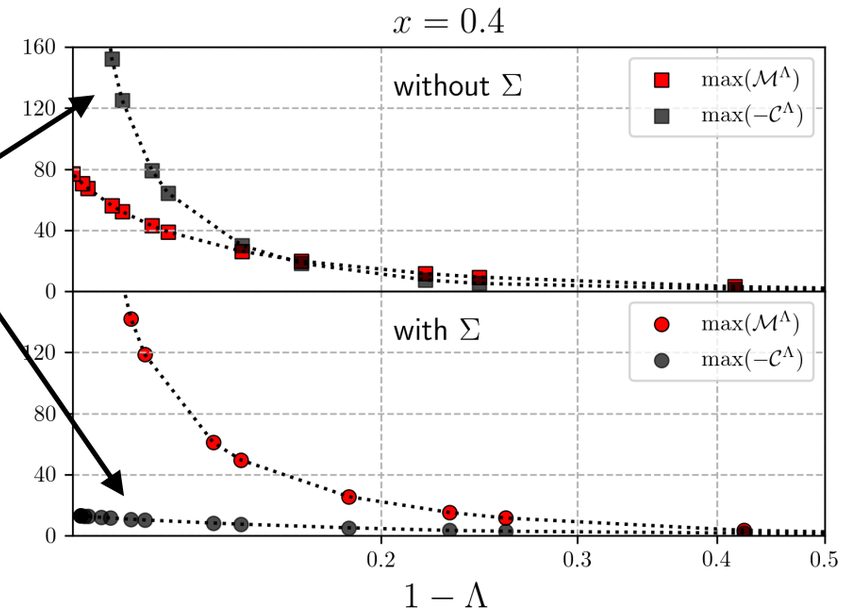
DV et al., arXiv: 1708.03539





Self energy suppresses the divergence

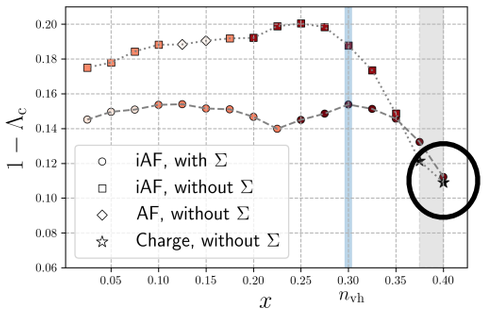
abs Charge

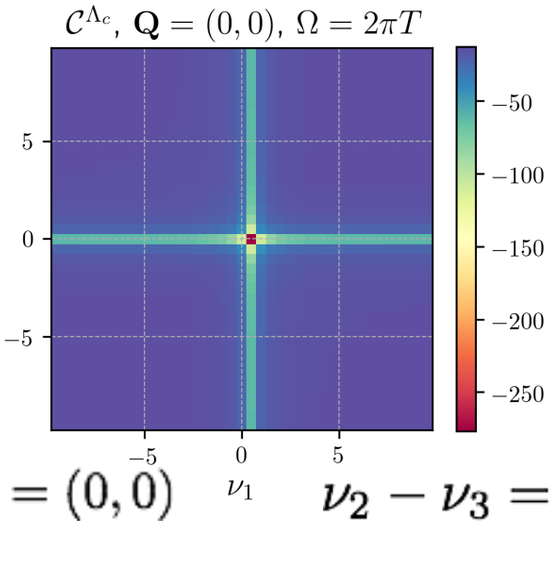


What is the origin of the divergence?

Charge divergence

DV et al., arXiv: 1708.03539





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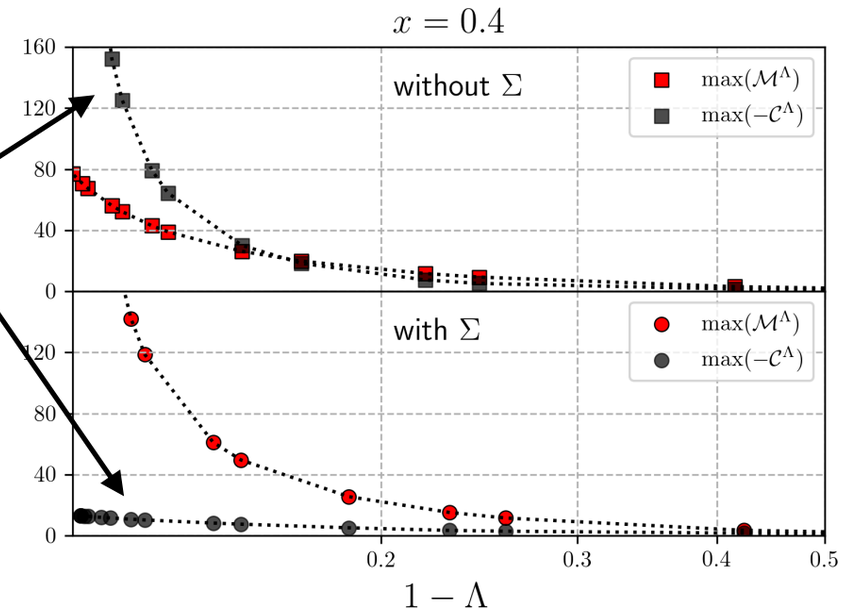


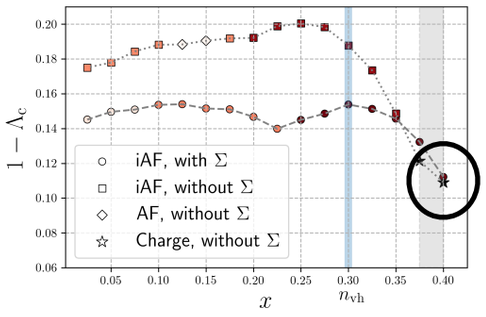
What is the origin of the divergence?

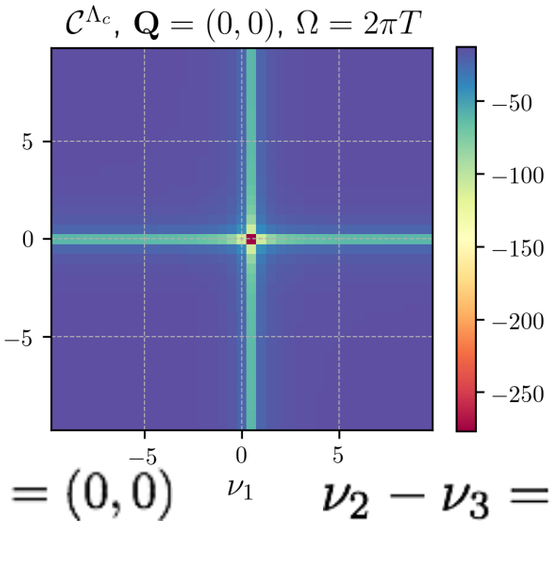
$$U^{\rm eff}_{\mathbf{Q},\Omega}=\frac{U}{1-U\Pi_{\mathbf{Q},\Omega}} \quad = \quad \bullet + \bigcirc\!\!\bullet + \bullet\!\!\bigcirc\!\!\bullet + \bullet\!\!\bigcirc\!\!\bigcirc\!\!\bullet + \cdots$$

Charge divergence

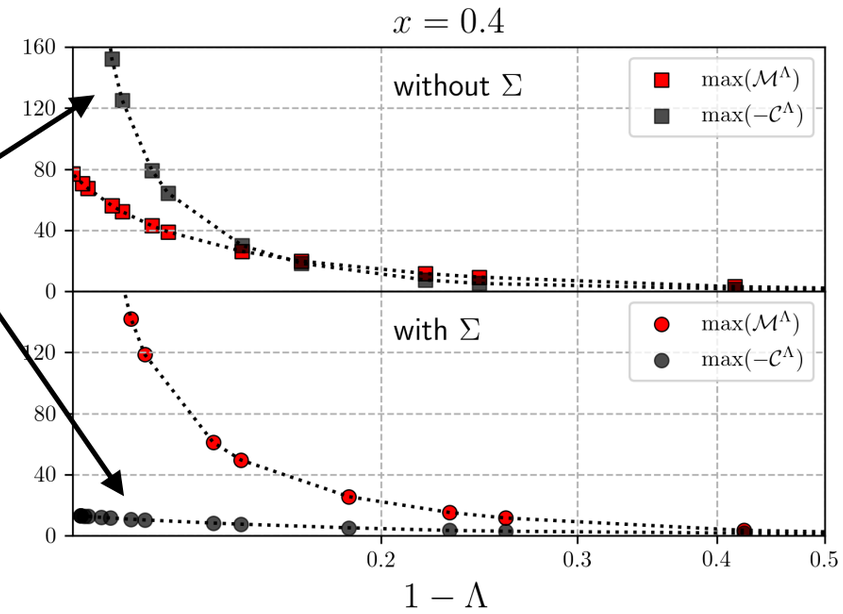
DV et al., arXiv: 1708.03539

Self energy suppresses the divergence





abs Charge



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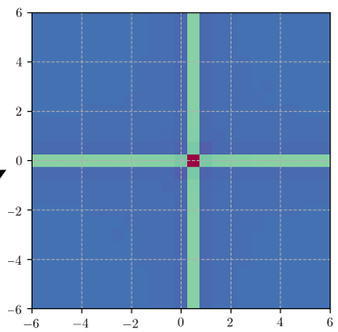
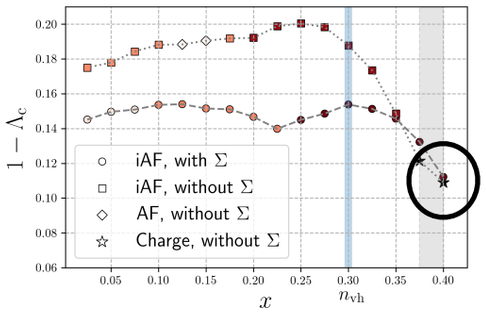
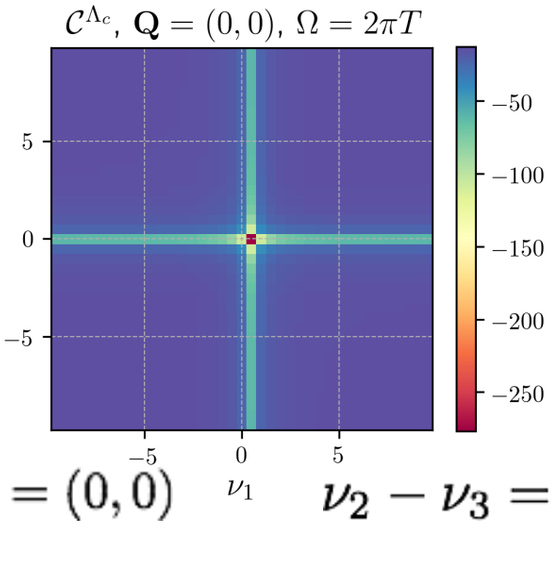
$$\tilde{\mathcal{C}}_{\mathbf{Q},\Omega}(\nu_1,\nu_3) = \quad + \quad + \quad + \cdots$$



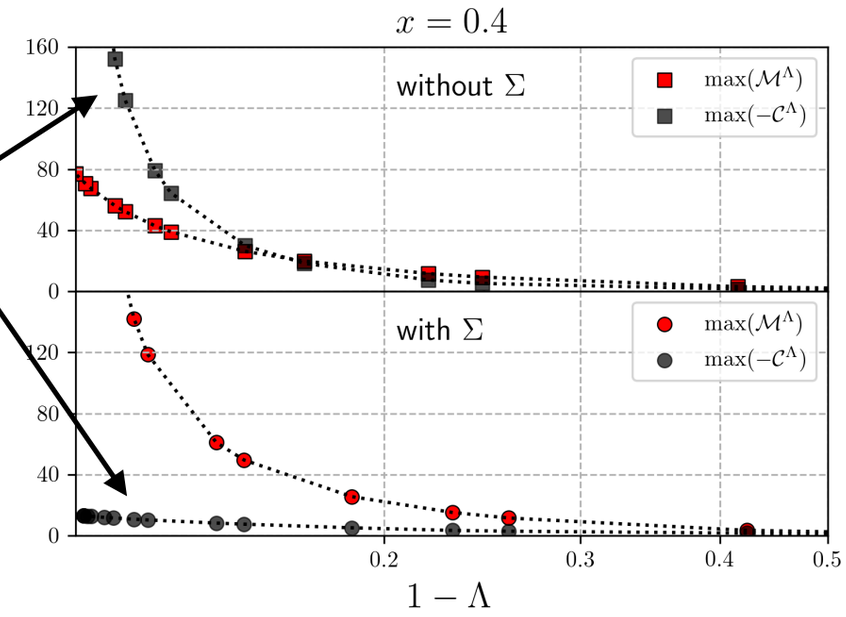
Charge divergence

DV et al., arXiv: 1708.03539





Self energy suppresses the divergence



abs Charge

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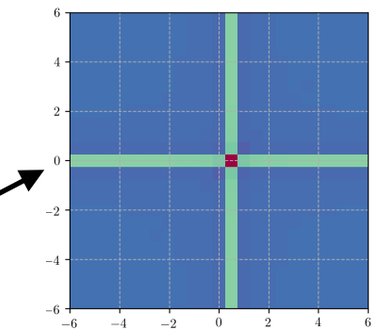
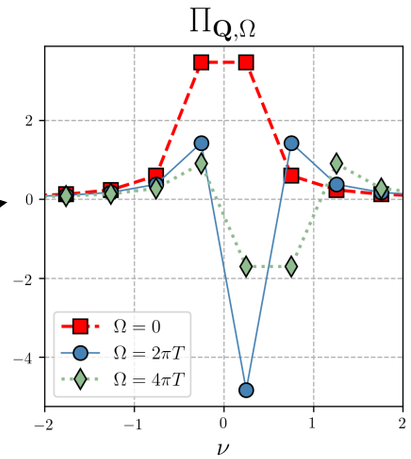
$$\tilde{\mathcal{C}}_{\mathbf{Q},\Omega}(\nu_1,\nu_3) = \quad + \quad + \quad + \cdots$$

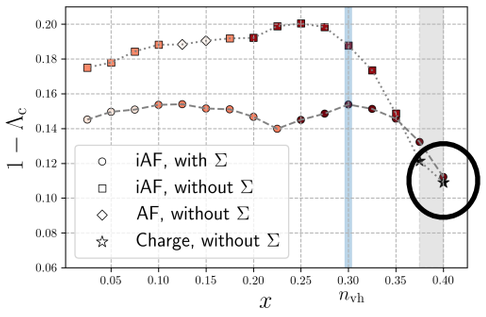
$$\Pi_{\mathbf{Q},\Omega}(\nu) = -\int_{\mathbf{p}} G_0(\mathbf{p},\nu)G_0(\mathbf{p}+\mathbf{Q},\nu+\Omega).$$

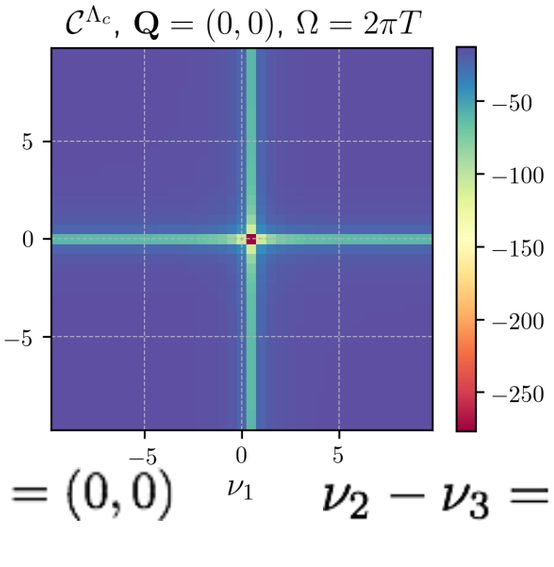




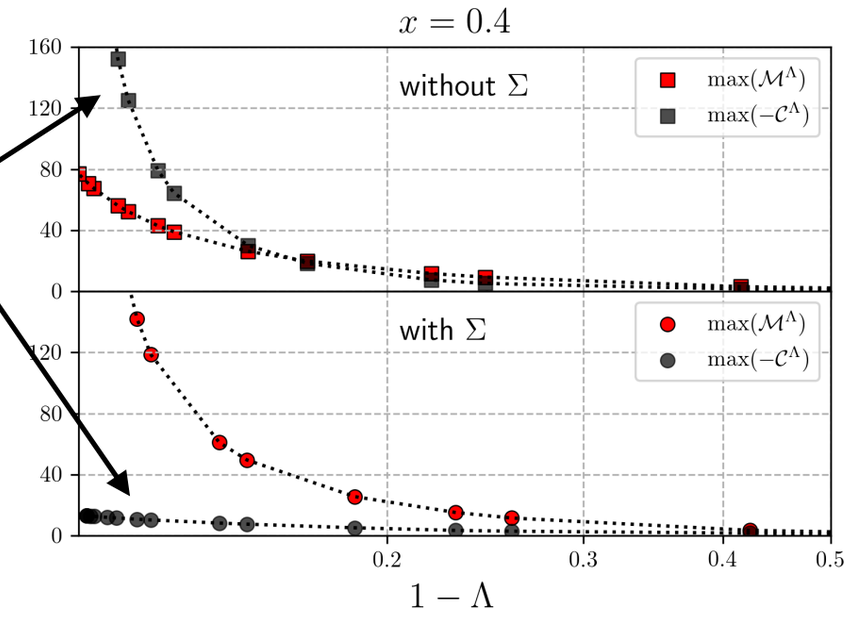
Charge divergence

DV et al., arXiv: 1708.03539





Self energy suppresses the divergence



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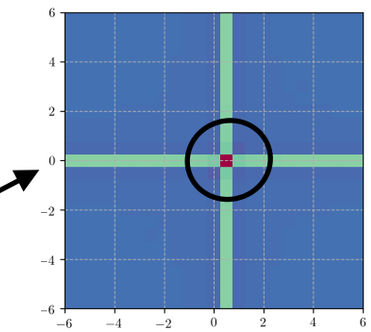
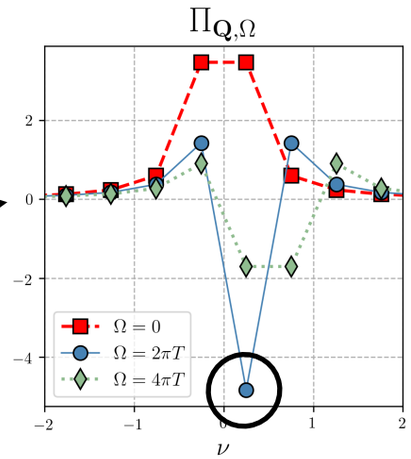
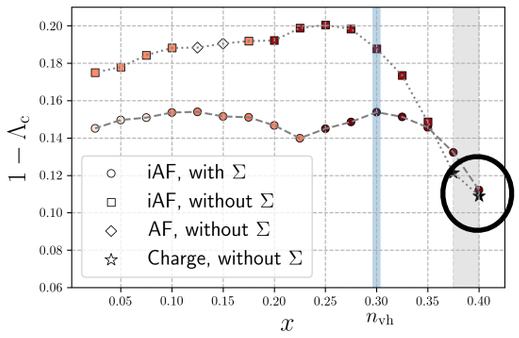


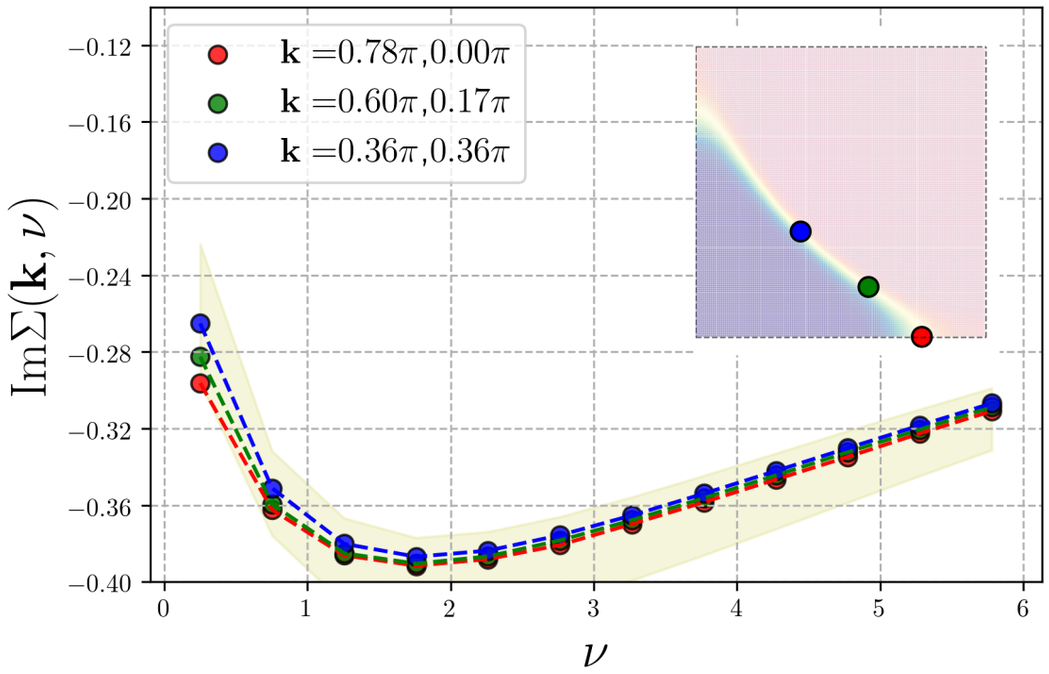




Self energy

DV et al., arXiv: 1708.03539

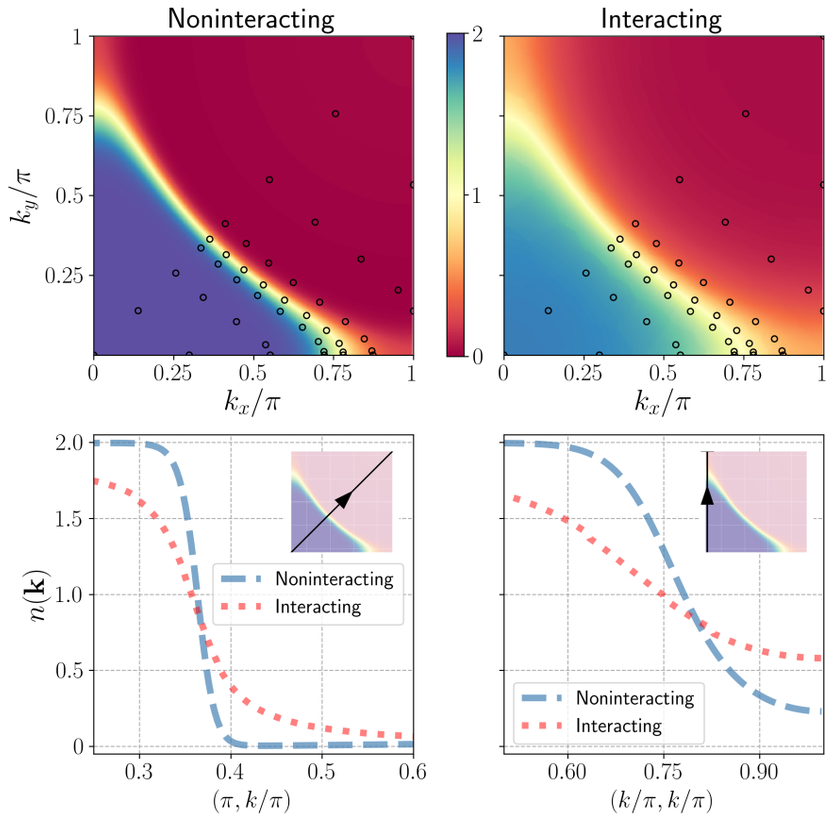
Self energy in Matsubara axis



- Almost local along the FS
- Fermi liquid behaviour even close to iAF instability

Momentum distribution

$$n^{\Lambda}(\mathbf{k}) = 2T\sum_{\nu}\frac{e^{i\nu 0^+}}{i\nu-\varepsilon_{\mathbf{k}}+\mu^{\Lambda}-\Lambda\Sigma^{\Lambda}(\mathbf{k},\nu)}.$$



- More broadening in antinodal direction

Hartree-Fock self energy

Wu, Georges and Ferrero, PRB 2017

Diagrammatics:

Close to magnetic instability $\mathbf{Q} \simeq (\pi, \pi)$

$$\Sigma(\mathbf{k}, i\omega_n) \propto \sum_{\mathbf{q}} \sum_{p} \frac{1}{i\omega + i\nu_p + \mu - \epsilon_{\mathbf{k}+\mathbf{q}}} \times \frac{1}{(\mathbf{q}-\mathbf{Q})^2 + \xi^{-2}\nu_p/\omega_{\mathrm{sp}} + \xi^{-2}}.$$

Ornstein-Zernike spin susceptibility: Dare' et al., PRB 1996

Fock self energy with strong AF fluctuations:

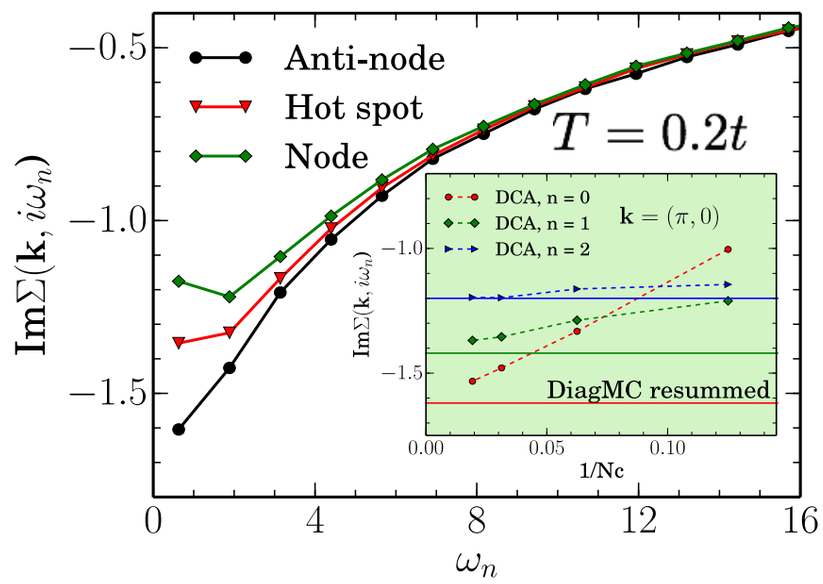
$\xi >> a$

$$\Sigma(\mathbf{k}, \omega) = \frac{\Delta^2}{\omega + \mu - \epsilon_{\mathbf{k}+\mathbf{Q}} + i0^+},$$

➡ Pseudogap phase

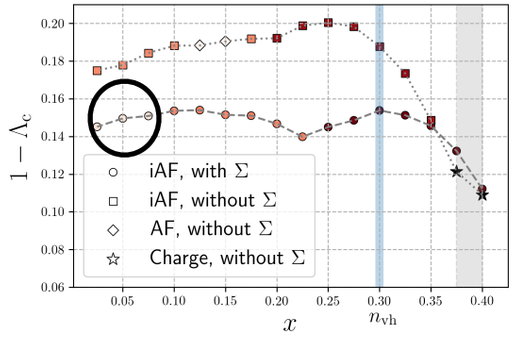
DiagQMC

$n = 0.96$

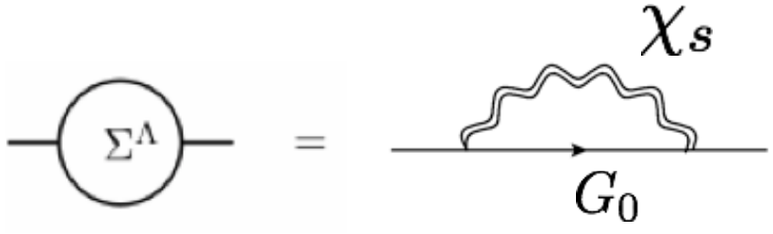


Wu, Ferrero, Georges and Kozik, PRB 2017

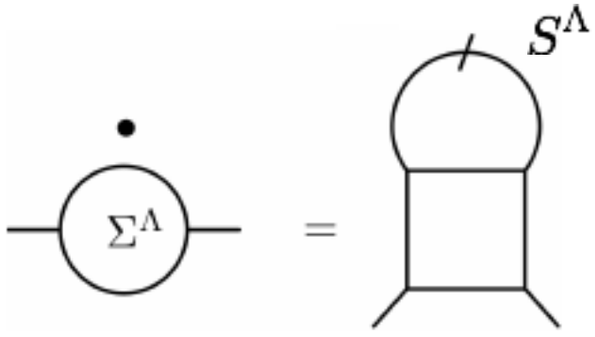
Self energy: Fock vs fRG



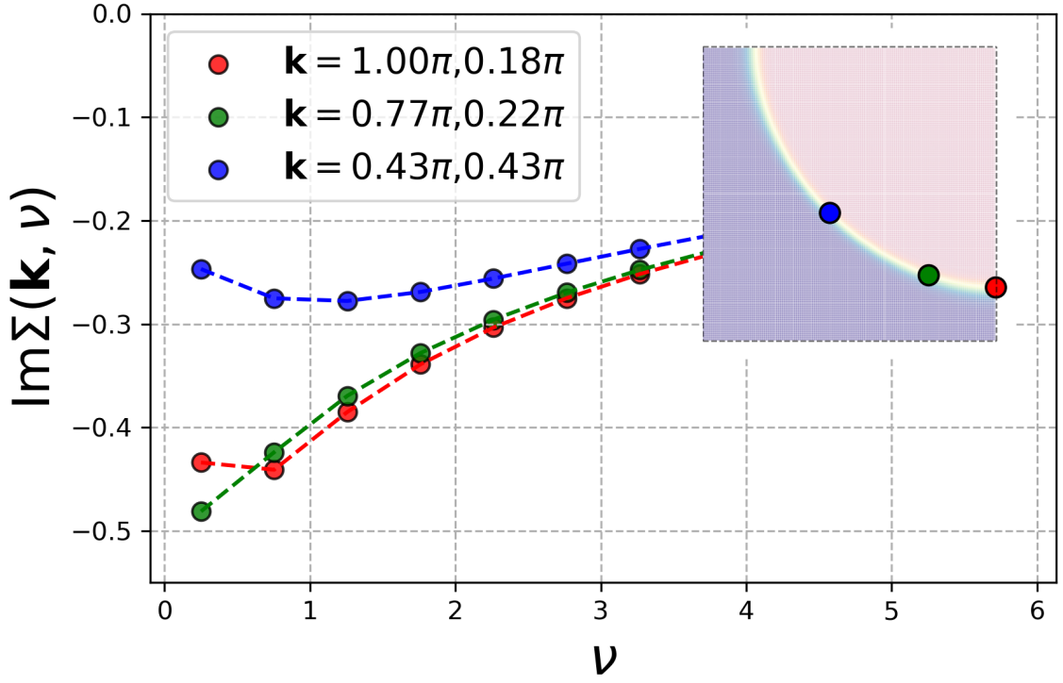
Fock self energy

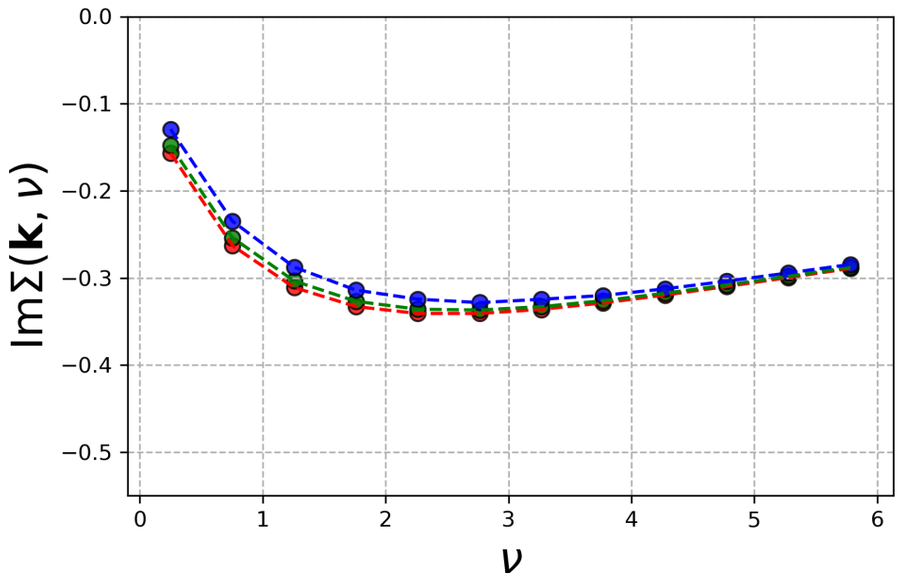


fRG self energy



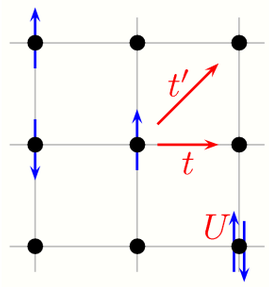
➡ Exact only when the vertex is exact



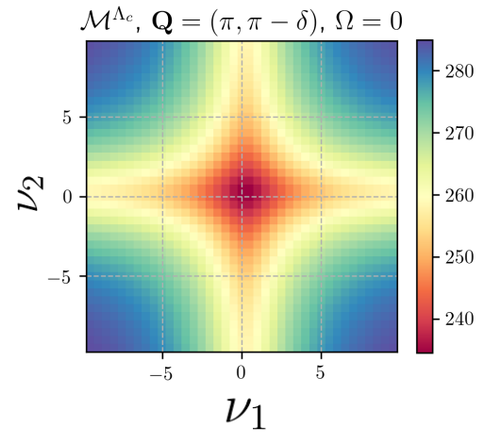


➡ Non Fermi-liquid at the hot-spots

Outline

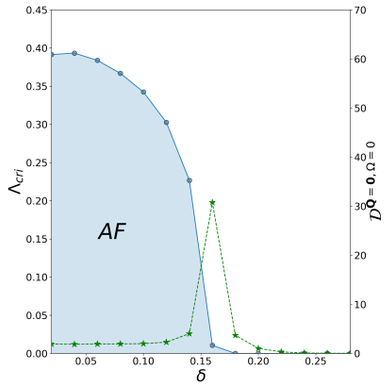


fRG: weak-to-intermediate coupling



- ➡ Frequency dependent interaction
- ➡ Charge divergence
- ➡ Self energy and pseudogap

DMF²RG: strong coupling by starting from DMFT solution



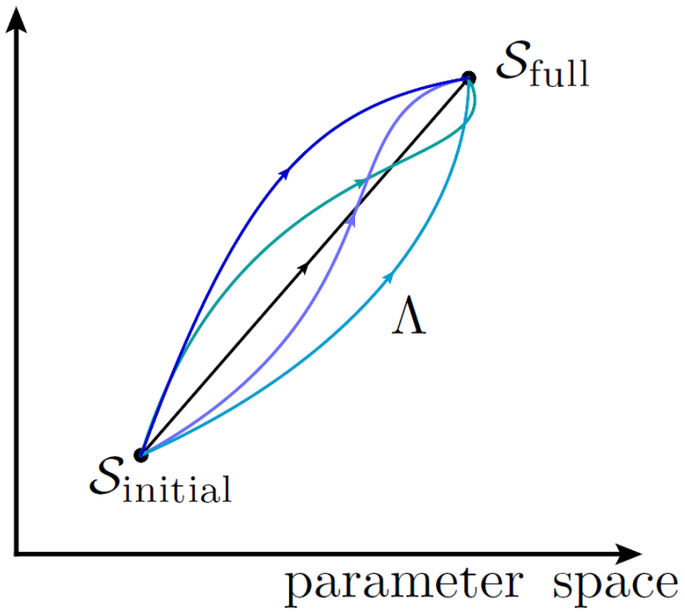
- ➡ Local DMFT vertex affects non-local susceptibility
- ➡ *d*-wave pairing fluctuations

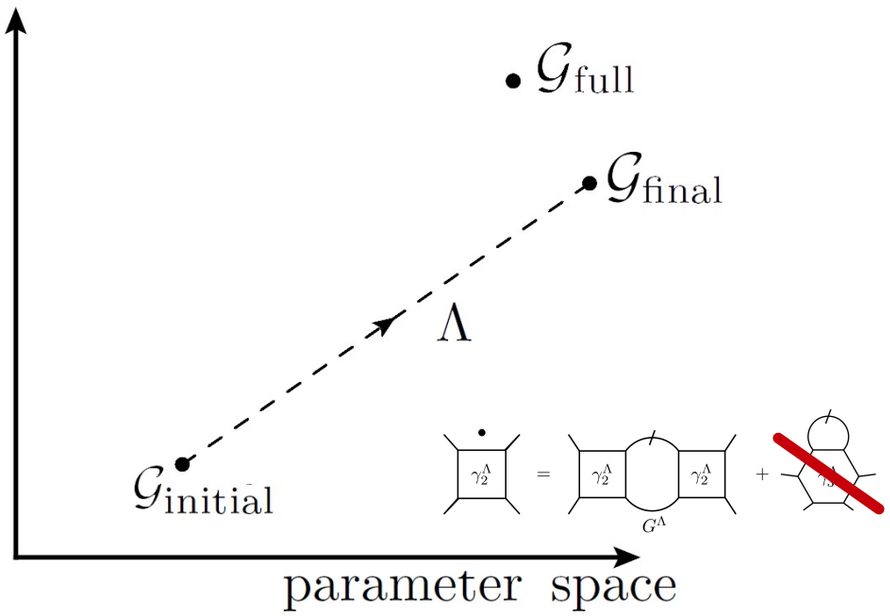
Conclusions and outlook

DMF²RG: flow for the strong coupling

Wentzel et al., PRB 2015

Conventional fRG





- Start from bare action: no fluctuations from the beginning
- Truncation: error accumulated during the flow

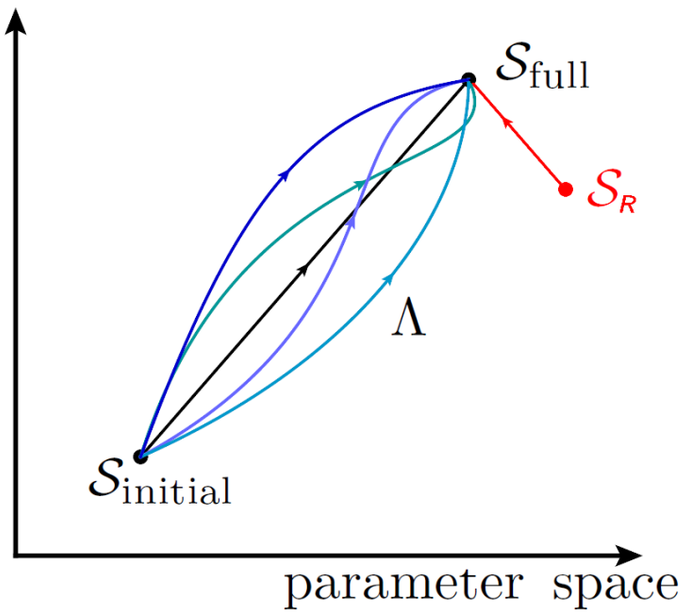
C. Taranto et al., PRL (2014)

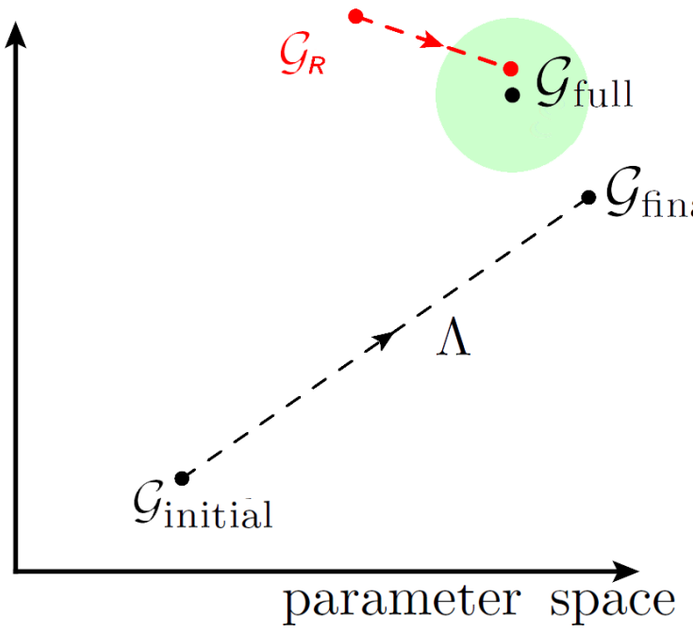
Metzner et al. RMP (2012)

DMF²RG: flow for the strong coupling

Wentzel et al., PRB 2015

Extension to strong coupling





$$G_0^{\Lambda_{ini}} = G_{0,R} \neq 0$$

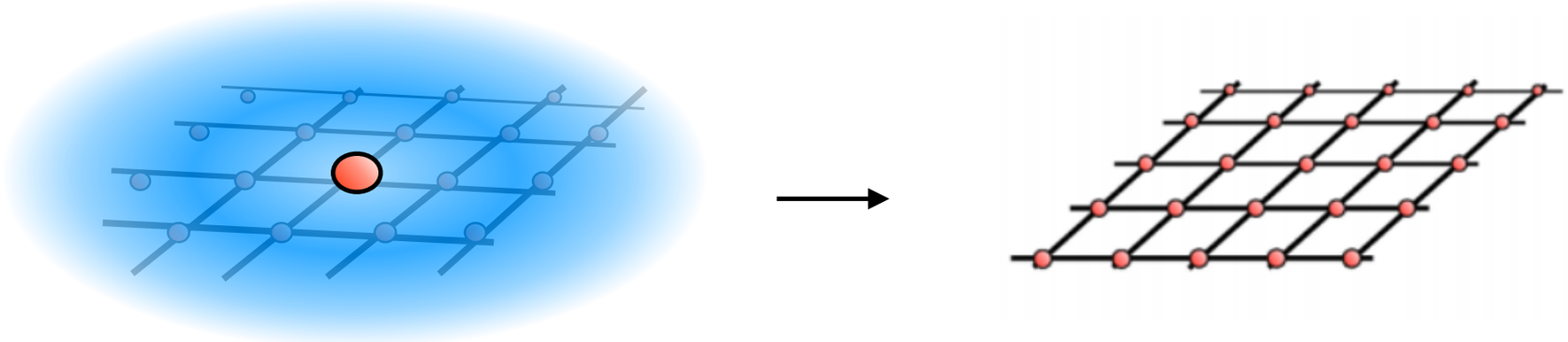
- Inclusion of correlation from the beginning

- Reduce truncation error by starting ‘closer’ to final action

C. Taranto et al., PRL (2014)

Georges et al. RMP (1996)

DMF²RG: start from DMFT solution

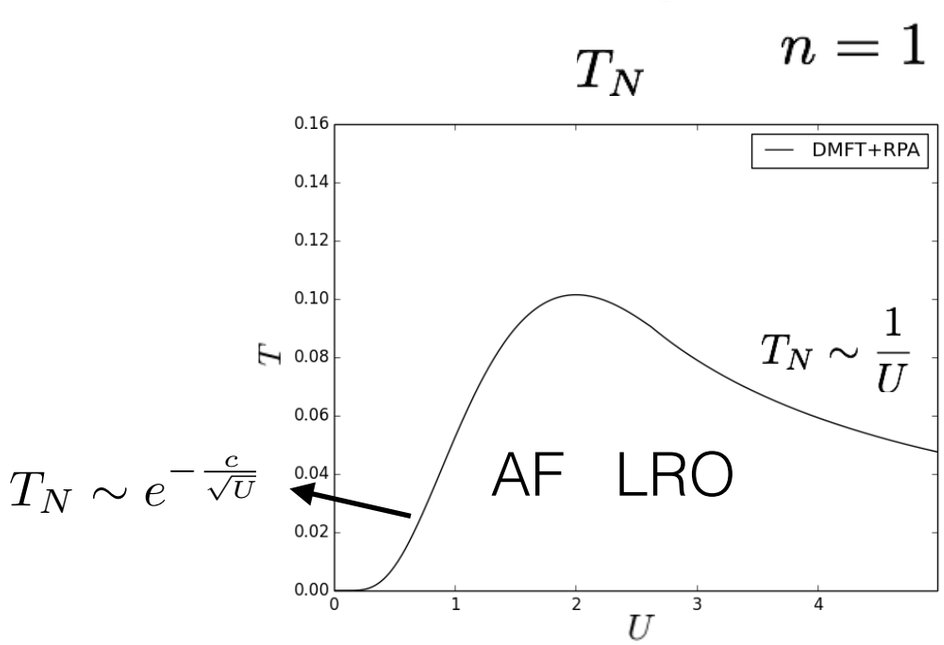


- Only local fluctuations
- Exact in infinite dimensions

➡ Use converged DMFT solution as initial condition for fRG

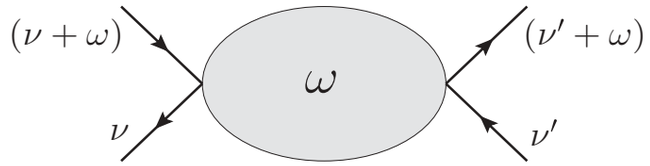
➡ DMFT capture strong coupling physics with local fluctuations

DMFT Phase diagram with SSB

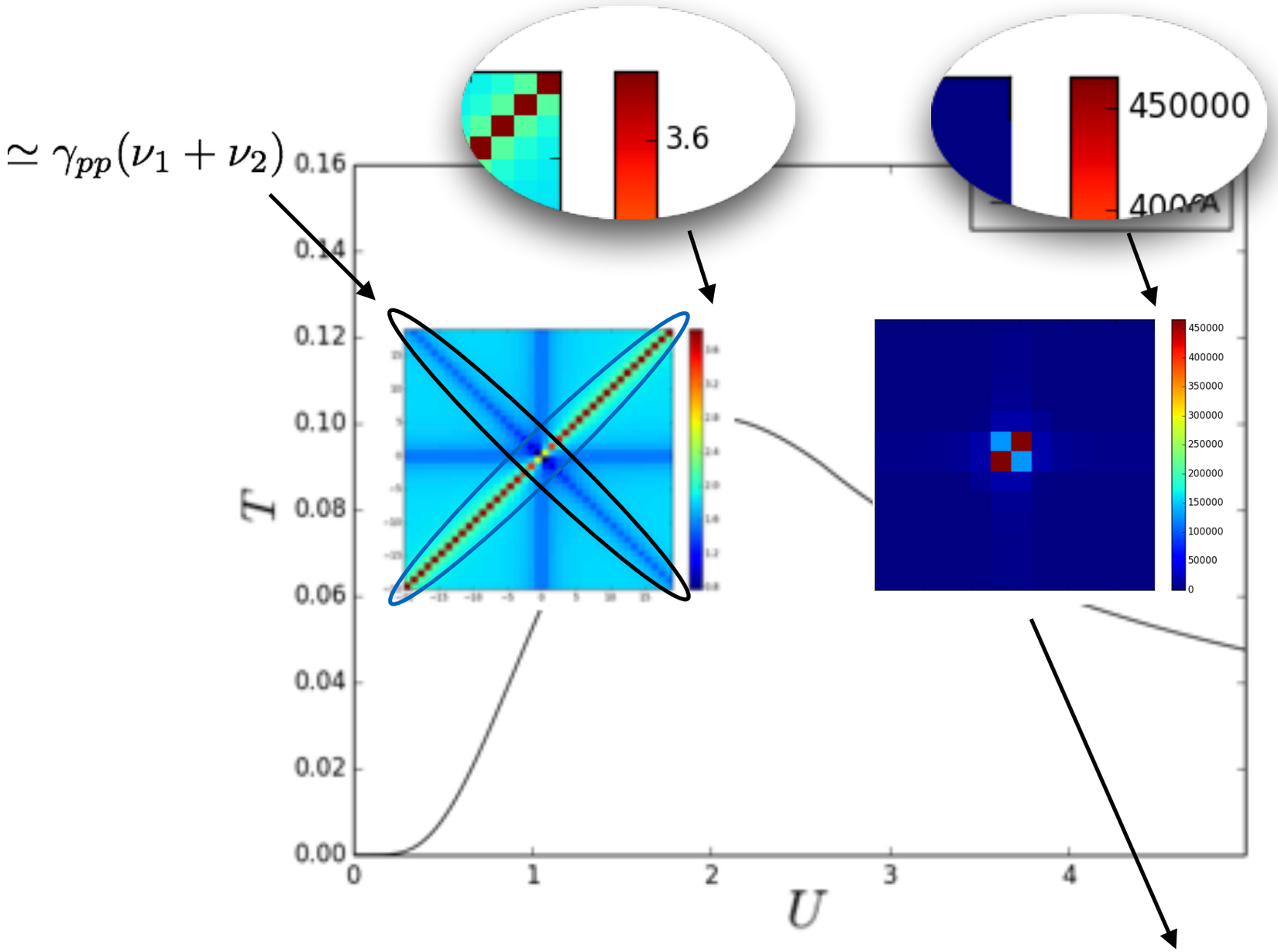


Metzner and Vollhardt PRL (1989) Georges and Kotliar PRB (1991)

DMF²RG: DMFT vertex $F^{\omega,\nu,\nu'}_{DMFT}$



Rohringer et al., PRB (2012)



$n = 1 \qquad t' = 0$

At strong coupling:

$$\chi_{loc} \sim \beta$$

$$F_{\omega=0,\nu=\nu'=\frac{\pi}{\beta}} \sim \beta^5 U^6$$

➡ Localized structure due to local moment

DMF²RG: strong coupling flow

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F^{\nu_1,\nu_2,\nu_3}_{\mathrm{DMFT}} - \phi^{\nu_1,\nu_2,\nu_3}_{SC}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
$$+\frac{1}{2}M^{\nu_1,\nu_2,\nu_1+\nu_2-\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3) - \frac{1}{2}C^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3)$$

DMF²RG: strong coupling flow $n = 1 \quad t' = 0$

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F^{\nu_1,\nu_2,\nu_3}_{\mathrm{DMFT}} - \phi^{\nu_1,\nu_2,\nu_3}_{SC}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
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Single channel (PH_{cr}) DMF²RG equivalent to *ladder*-DMFT

DMF²RG: strong coupling flow $n = 1 \quad t' = 0$

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F^{\nu_1,\nu_2,\nu_3}_{\mathrm{DMFT}} - \phi^{\nu_1,\nu_2,\nu_3}_{SC}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
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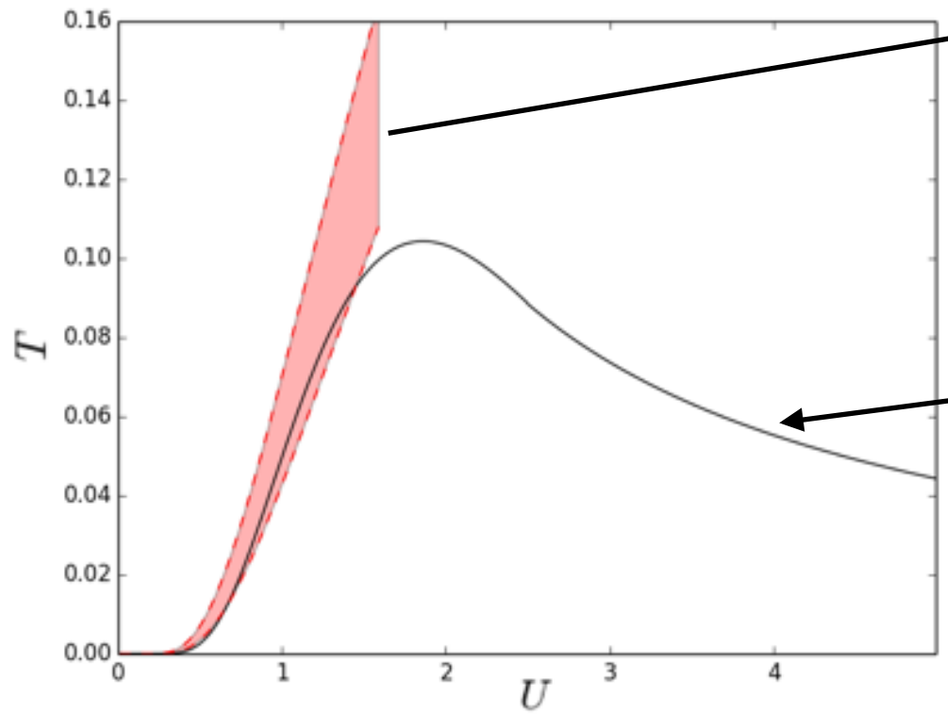
Single channel (PH_{cr}) DMF²RG equivalent to *ladder*-DMFT



boson propagator

$$\phi^{\omega,\nu,\nu'}(\boldsymbol{q}) \simeq \phi^{\omega}(\boldsymbol{q})$$

Full frequency dependence

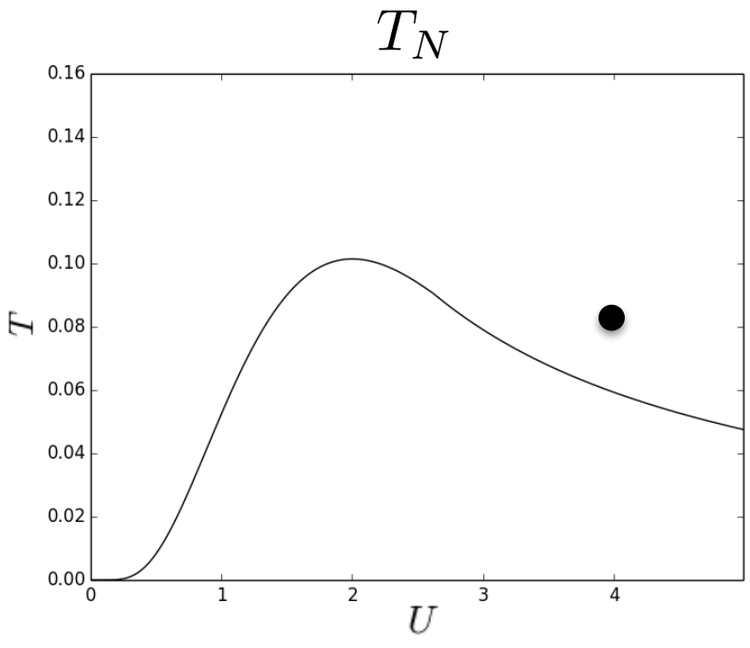
➡ Capture the Heisenberg coupling

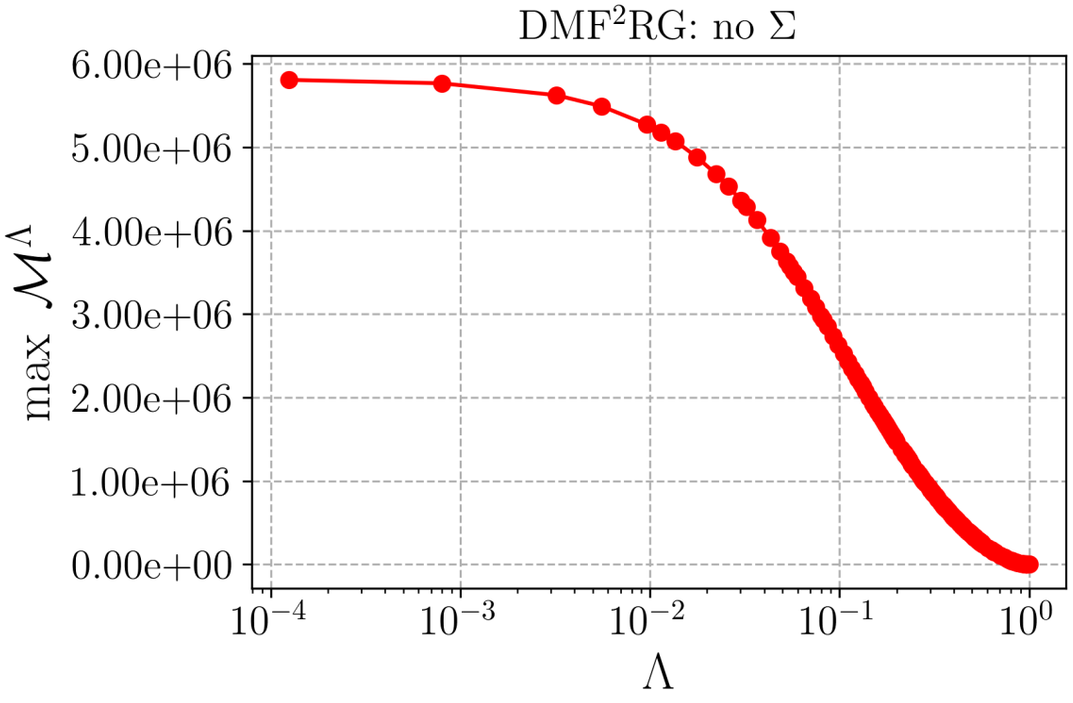
$$J \sim \frac{t^2}{U}$$

➡ Recovery of DMFT Néel temperature only with full frequency dependence

DMF²RG: strong coupling flow $n = 1 \quad t' = 0$

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F^{\nu_1,\nu_2,\nu_3}_{\mathrm{DMFT}} - \phi^{\nu_1,\nu_2,\nu_3}_{SC}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
$$+\frac{1}{2}M^{\nu_1,\nu_2,\nu_1+\nu_2-\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3) - \frac{1}{2}C^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_2-\boldsymbol{k}_3)$$





➡ Strong but localised vertex structure

$$\mathbf{max}M \sim 10^6$$

DMF²RG: strong coupling flow $n = 1$ $t' = 0$

$$V^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F^{\nu_1,\nu_2,\nu_3}_{\mathrm{DMFT}} - \phi^{\nu_1,\nu_2,\nu_3}_{SC}(\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + M^{\nu_1,\nu_2,\nu_3}(\boldsymbol{k}_3-\boldsymbol{k}_1)$$
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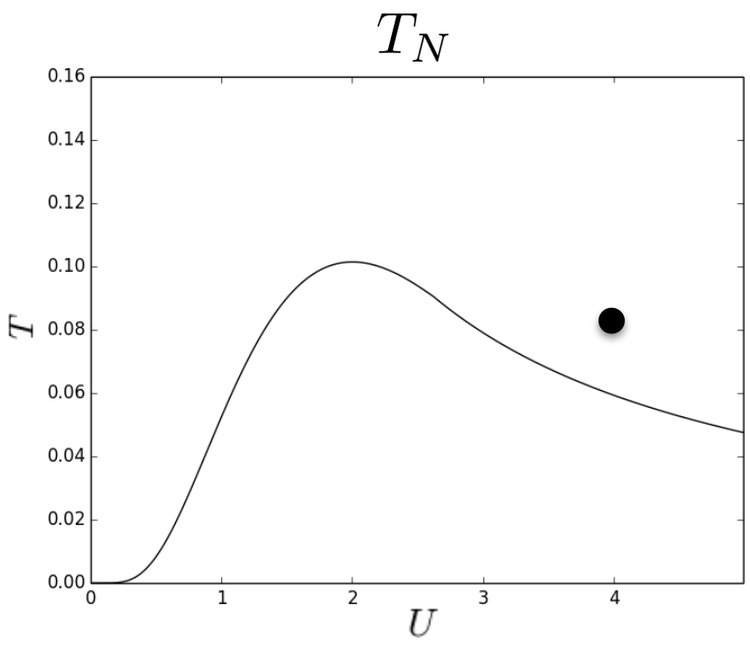
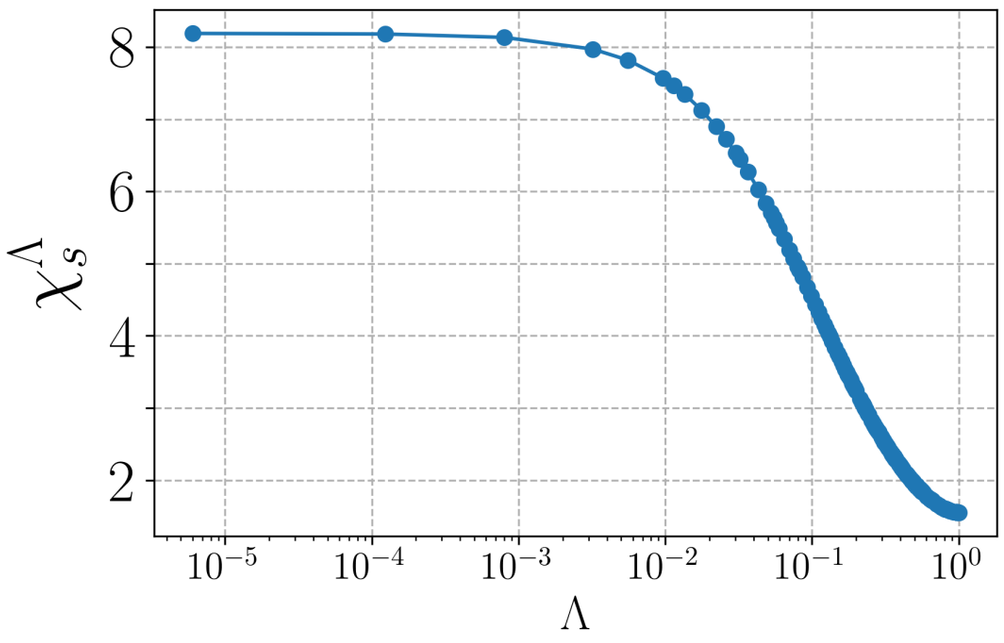


Spin susceptibility



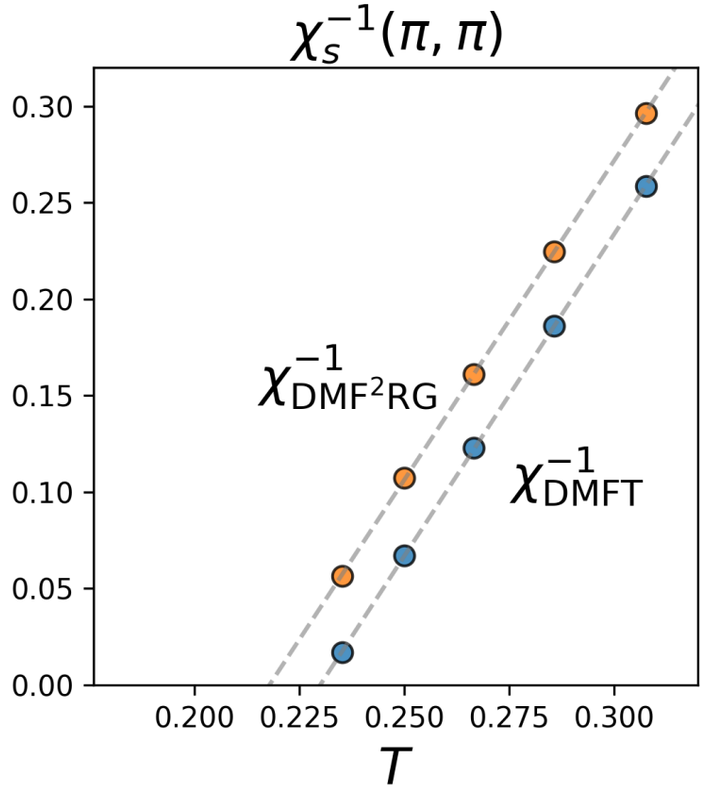
➡ Strong but localised vertex structure

$$\mathbf{max}M \sim 10^6$$

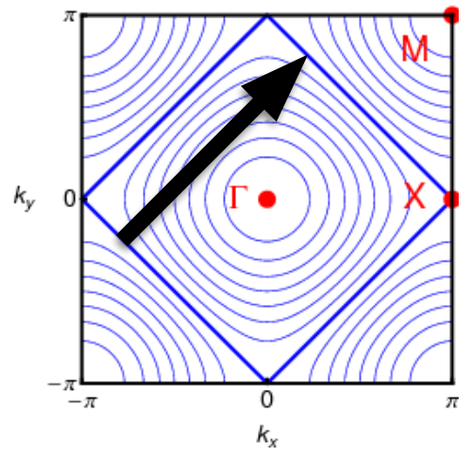
DMF²RG: Néel temperature at half-filling

Spin susceptibility

$$n = 1 \qquad t' = 0 \qquad U = 4t$$



Perfect nesting:



Brillouin zone

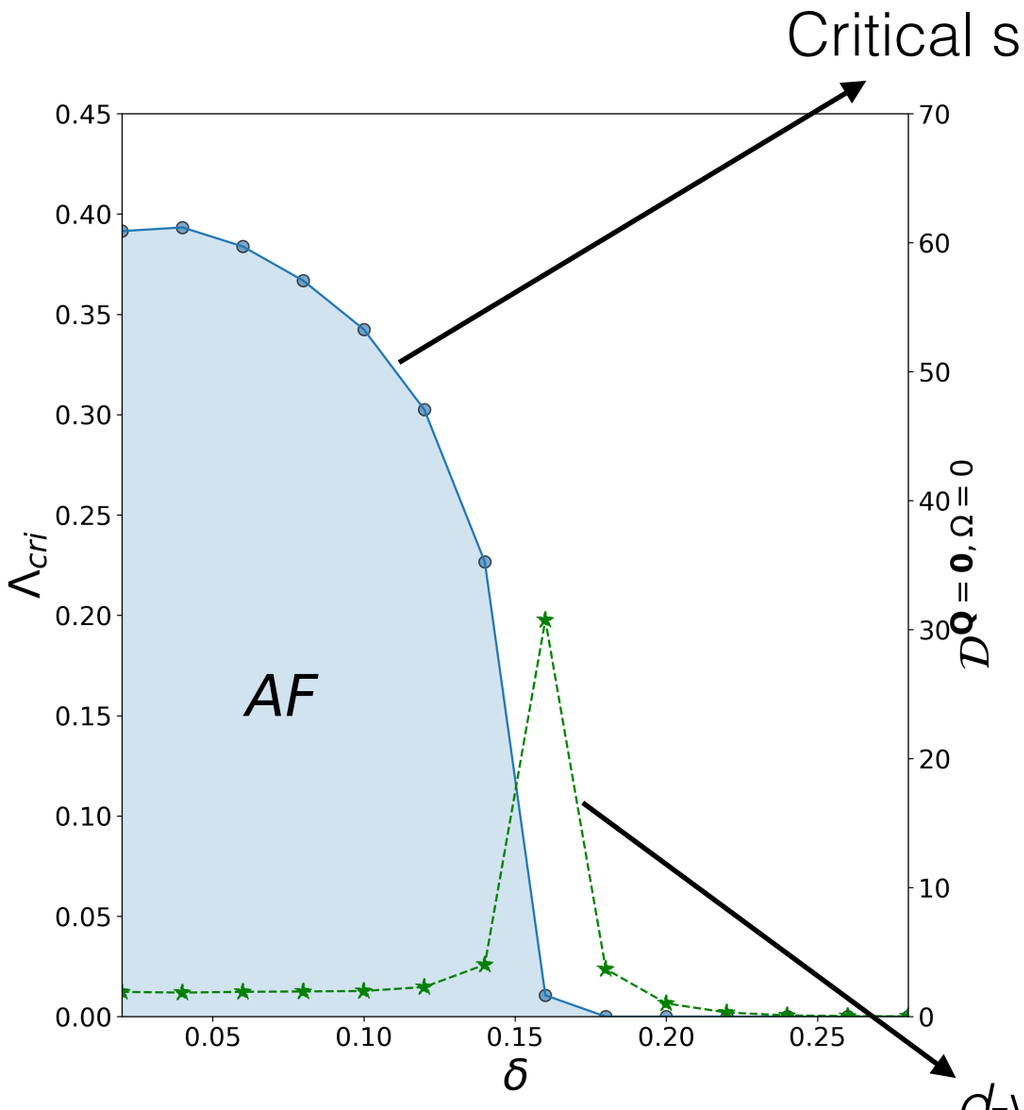
- Mean field critical exponent
- *Non-local* fluctuations slightly reduce the Néel temperature
- Result stable over a large range of coupling strength

DMF²RG: away from half-filling

$$T = 0.08t$$

Strong coupling regime and doped region

$$U = 8t \qquad t' = -0.2t$$



➡ No *d*-wave instability at this temperature

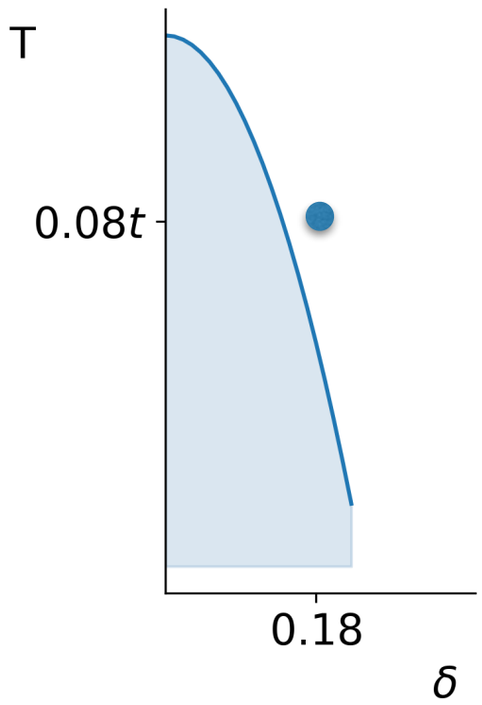
➡ Incommensurate antiferromagnetism in doped case

➡ Emergency of large *d*-wave pairing fluctuations at border of AF onset

d-wave pairing fluctuations

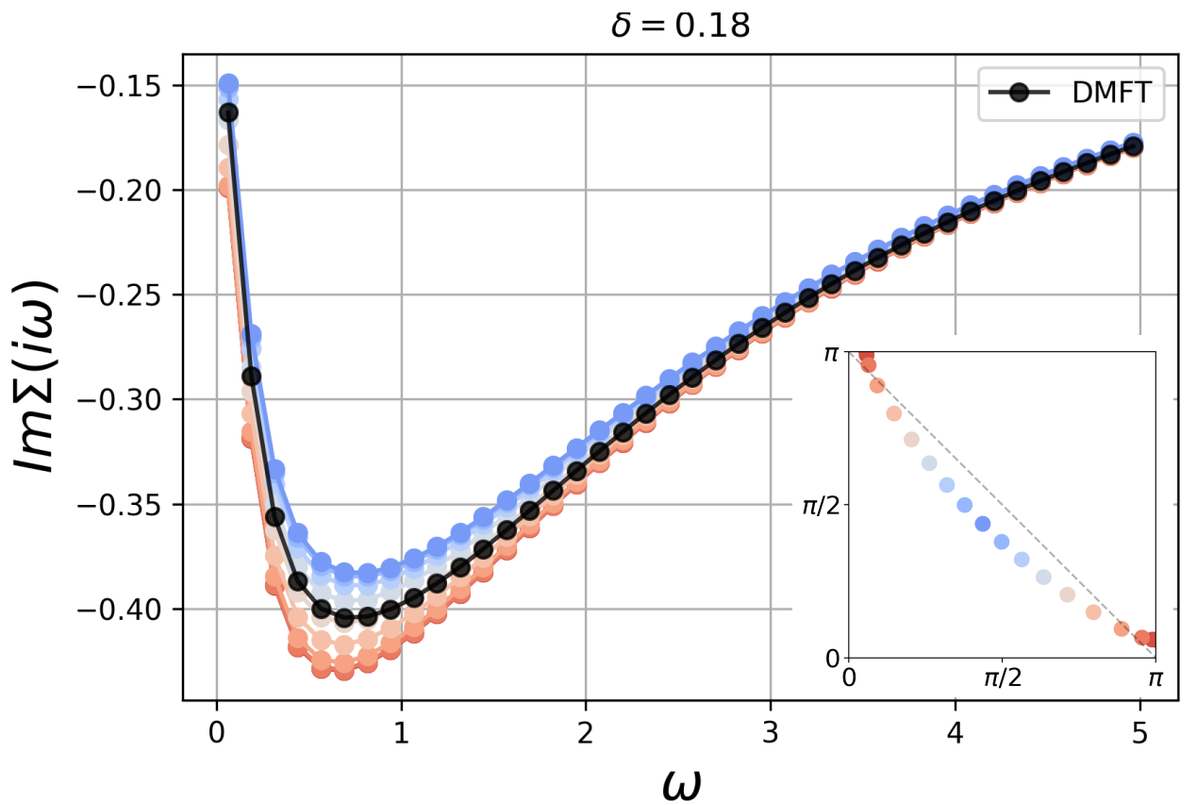
DMF²RG: away from half-filling

Strong coupling regime and doped region $U = 8t \quad t' = -0.2t$



Results:

- ➡ Incommensurate antiferromagnetism in undoped case
- ➡ Local and Fermi-liquid self-energy as in weak-to-intermediate coupling fRG



No indications for pseudogap:
Lack of method or model?

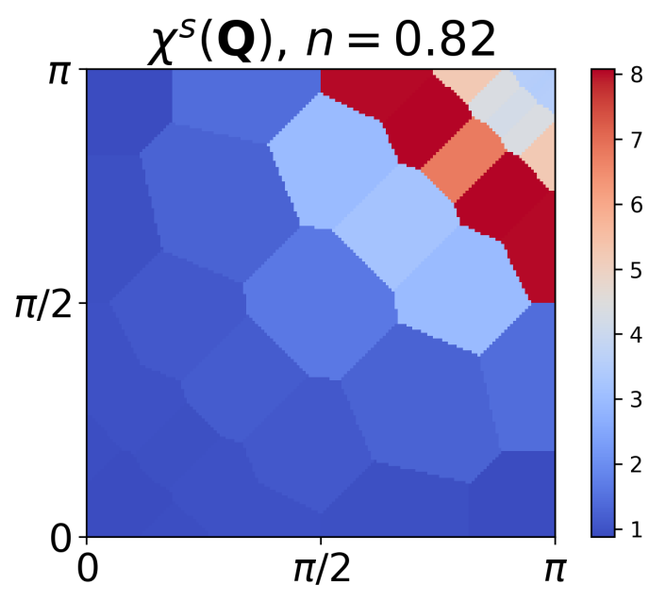
DMF²RG: away from half-filling

$$T = 0.08t$$

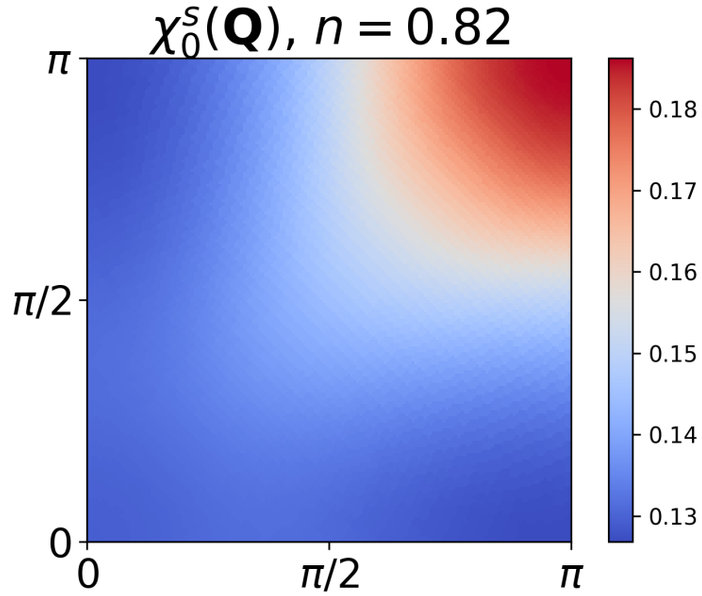
Strong coupling regime and doped region

$$U = 8t \qquad t' = -0.2t$$

Spin susceptibility



DMFT bubble



Where the incommensurate peak comes from?

$$\chi_0^s = -T \sum_\nu \int_{\boldsymbol{k}} G_{\boldsymbol{k},\omega} G_{\boldsymbol{Q}+\boldsymbol{k},\omega+\nu}$$

$$\text{with } \Sigma = \Sigma_{\text{DMFT}}$$

Three possibilities:

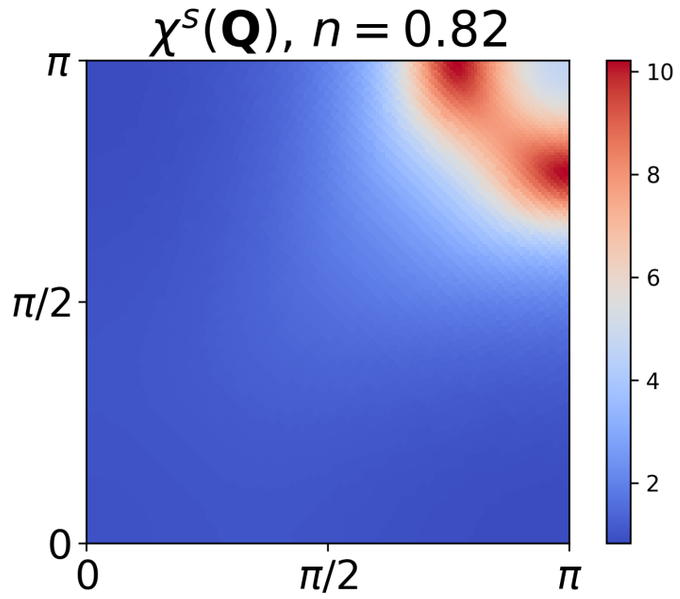
★ From the *non-locality* of the self-energy?

★ From the channel competition?

✓ From the DMFT vertex?

ladder-DMFT: away from half-filling $T = 0.08t$

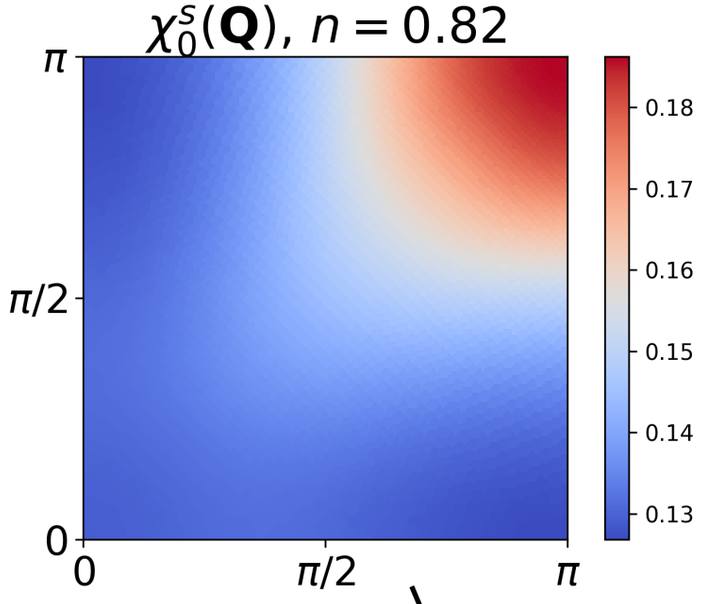
Strong coupling regime and doped region $U = 8t \quad t' = -0.2t$



$$\chi_s^{\boldsymbol{q}} = \chi_0^{\boldsymbol{q}} - \chi_0^{\boldsymbol{q}} \Gamma_{\mathrm{irr}} \chi_s^{\boldsymbol{q}}$$

Matrix multiplication in frequency space

DMFT bubble



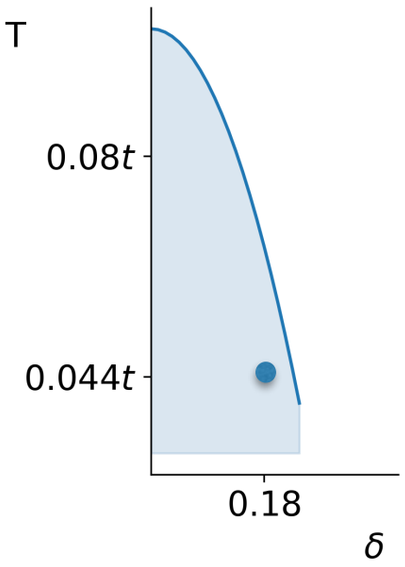
Frequency summed bubble

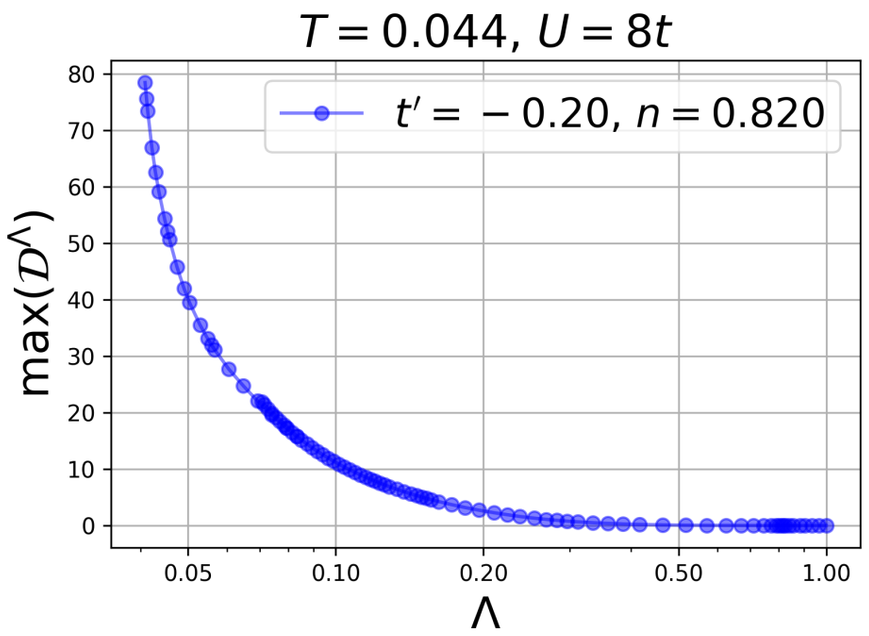
Local vertex affects *non-local* spin susceptibility

DMF²RG: *d*-wave pairing fluctuations

Lowering the temperature:

$$U = 8t \qquad t' = -0.2t$$

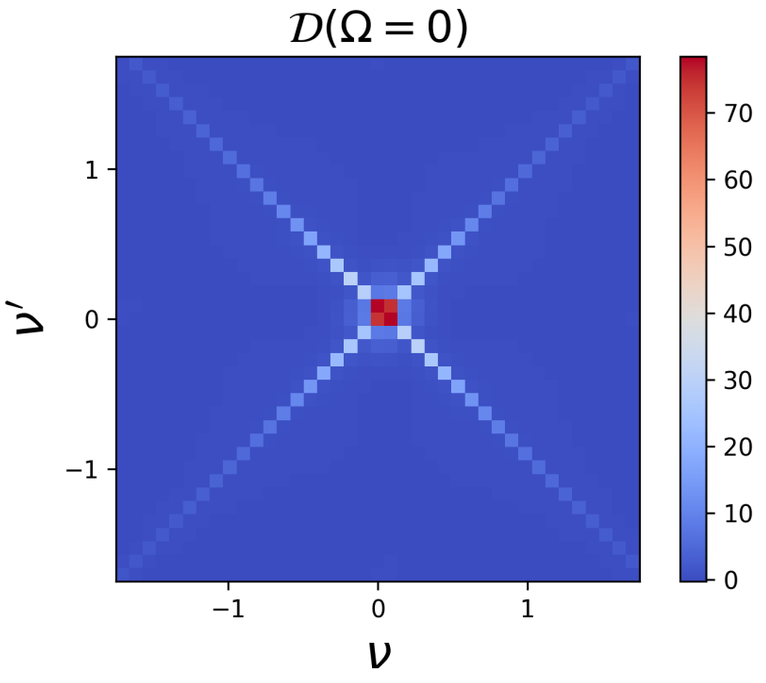




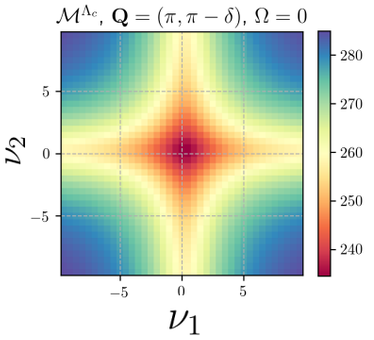
➡ Strong *d*-wave pairing fluctuations in the iAF phase

➡ Localised frequency structure as in fRG

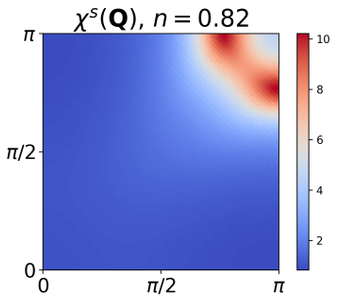
➡ Precursor of *d*-wave instability at lower T_C?



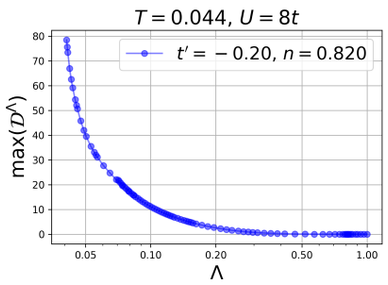
Conclusion



- Important frequency dependence in the intermediate coupling
- Presence of charge divergence, explained by RPA-like formulae and suppressed by self energy feedback
- Flow to the strong coupling by starting from DMFT solution
- *Local* vertex affects *non-local susceptibility*



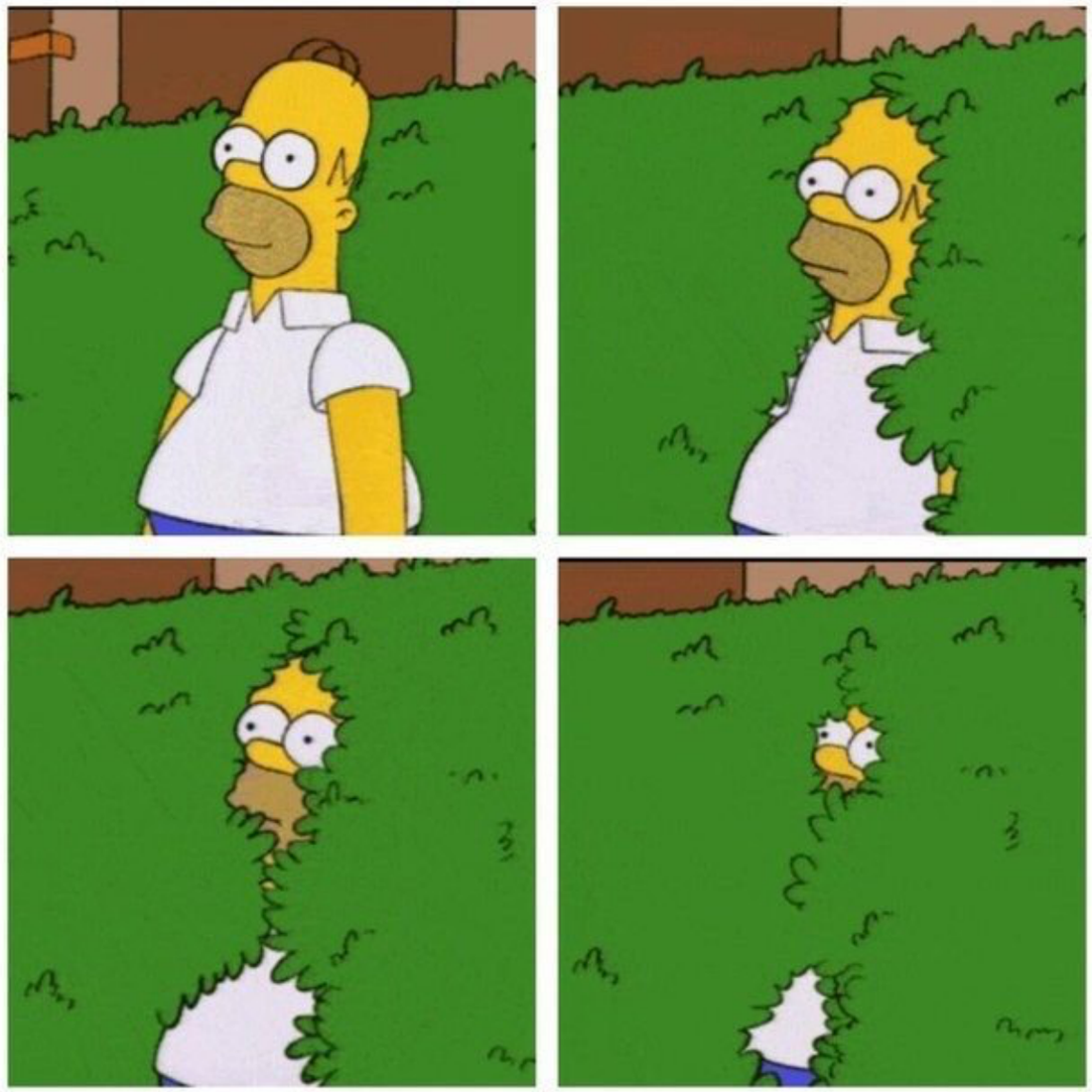
- Strong *d*-wave pairing fluctuations at Temperature studied



Outlook

- Lowering the Temperature to enhance the interplay between AF and SC

Questions?



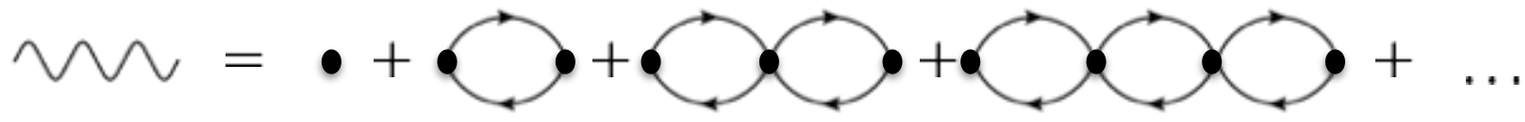
Thank you for your attention

DV et al., arXiv: 1708.03539

Charge divergence: perpendicular ladder

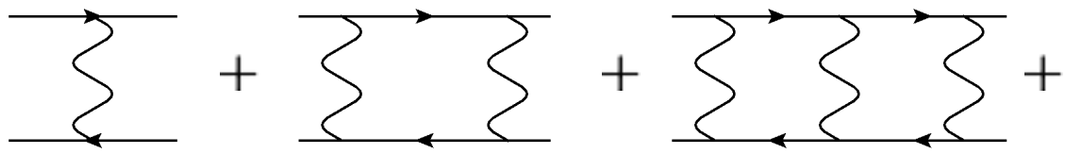
RPA-like magnetic fluctuations

$$U^{\rm eff}_{\mathbf{Q},\Omega} = \frac{U}{1 - U\Pi_{\mathbf{Q},\Omega}} =$$



RPA-like charge fluctuations (generated by magnons)

$$\tilde{\mathcal{C}}_{\mathbf{Q},\Omega}(\nu_1,\nu_3) = U^{\rm eff}_{\nu_1-\nu_3}\left[\delta_{\nu_1,\nu_3} + U^{\rm eff}_{\nu_1-\nu_3}\Pi_{\mathbf{Q},\Omega}(\nu_1)\right]^{-1} =$$



- Bosonic frequency $\Omega = \nu_2 - \nu_3$ enters only in the (not summed) bubble

Frequency dependent bubble

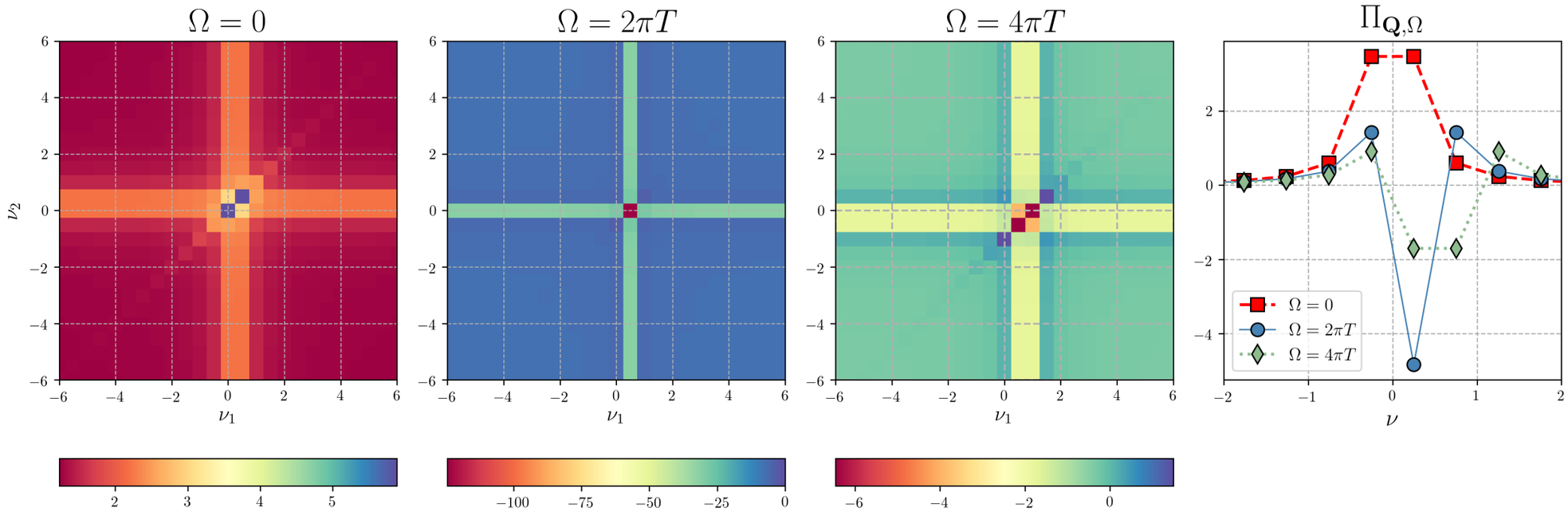
$$\Pi_{\mathbf{Q},\Omega}(\nu) = -\int_{\mathbf{p}} G_0(\mathbf{p},\nu)G_0(\mathbf{p}+\mathbf{Q},\nu+\Omega).$$

Similar to FLEX approx.

DV et al., arXiv: 1708.03539

Charge divergence: perpendicular ladder

$$\tilde{\mathcal{C}}_{\mathbf{Q},\Omega}(\nu_1,\nu_3) = U^{\text{eff}}_{\nu_1-\nu_3}\left[\delta_{\nu_1,\nu_3} + U^{\text{eff}}_{\nu_1-\nu_3}\Pi_{\mathbf{Q},\Omega}(\nu_1)\right]^{-1} \qquad \Pi_{\mathbf{Q},\Omega}(\nu) = -\int_{\mathbf{p}} G_0(\mathbf{p},\nu)G_0(\mathbf{p}+\mathbf{Q},\nu+\Omega).$$



- Same structure appearing in full fRG
- Divergent when lowering the temperature
- Qualitative but not quantitative picture

Property of the bubble:

$$T\sum_{\nu}\Pi_{\mathbf{Q}\to(0,0),\Omega}(\nu) = C\delta_{\Omega,0},$$

fRG: the self energy feedback cures the divergence