Disordered fermions in two dimensions: is Anderson insulating phase the only possibility?

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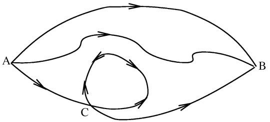
Outline

- Basic concepts on the Anderson localization
- QFT approach: derivation of the non linear σ-model and symmetry classifications
- (Anti)-localization effects for all symmetry classes
- Combined effects of disorder and interactions: not-universal behaviors and enhacement of critical temperatures.

Anderson localization

In the presence of strong enough disorder in D>2 or for any amount of disorder in D$\leq$ 2 a metal can turn into an insulator.

Interference effect ($\lambda_{DB} \simeq \ell$) $\Rightarrow$ localization of the wavefunctions



The probability to find the particle at point C is:

$$|a_1|^2 + |a_2|^2 + 2Re(a_1 a_2^*) = 4|a_1|$$

$\Rightarrow$ enhancement of probability to find a particle at C

$\Rightarrow$ reduction of probability to find it at B (conductivity $\searrow$)

Probability of self-intersection

$$\frac{\delta\sigma}{\sigma} \sim -\int_{\tau}^{\tau_\varphi} \frac{v\lambda^{d-1}dt}{(Dt)^{d/2}} \quad \Rightarrow \begin{cases} \delta\sigma \propto -(\frac{1}{\ell} - \frac{1}{L_\varphi}), & d = 3 \\ \delta\sigma \propto -\log(\frac{L_\varphi}{\ell}), & d = 2 \\ \delta\sigma \propto -(L_\varphi - \ell), & d = 1 \end{cases}$$

with $L_\varphi = \sqrt{D\tau_\varphi}$ and $\tau_\varphi \sim T^{-1}$

Scaling theory of localization

(Thouless, Phy.Rep. (1974); Abrahams, Anderson, Licciardello and Ramakrishnan PRL (1979); Gor'kov, Larkin, and Khmel'nitskii, JETP (1979))

Thouless idea: sample $(2L)^d$ made of cubes L^d

$\Rightarrow$ an eigenstate for $(2L)^d$ is a mixture of e.s. of L^d depending on overlap integrals and energy differences (as in perturbation theory)

- energy differences $\sim$ level spacing $\delta W = (\nu_0 L^d)^{-1}$
- overlap $\sim$ bandwidth δE (if localized e.s. δE exp. small, otherwise $\sim \hbar D/L^2$)

One parameter: $\frac{\delta E}{\delta W}$ related to the conductance G (units of $e^2/\hbar$)

- small disorder: $G(L) = \sigma L^{d-2}$
- strong disorder: $G(L) \sim \exp(-L/\xi)$

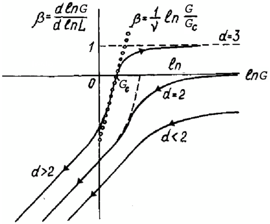
Scaling theory of localization

- strong disorder : $\mathrm{G}(L) \sim exp(-L/\xi)$

$$\beta(\mathrm{G}) = \frac{d \log G}{d \log L} = \log \frac{\mathrm{G}}{\mathrm{G_c}}.$$

- small disorder : $G(L) \sim \sigma\, L^{d-2}$, expanding in $1/G$

$$\beta(\mathrm{G}) = (d-2) - \frac{a}{\mathrm{G}}$$



$$\Rightarrow\ \sigma(L) - \sigma_0 \propto -\begin{cases} (\frac{1}{\ell} - \frac{1}{L}) & \text{d=3 (metal)} \\ \log(\frac{L}{\ell}) & \text{d=2 (insulator)} \\ (L-\ell) & \text{d=1 (insulator)} \end{cases}$$

Diagrammatics

Hamiltonian with some random potential

$$H = H_0 + V$$

Disorder variance $\overline{V(r)V(r')} = w_0 \delta_{rr'} =$

Bare Green function $\mathcal{G}_0 =$ 

In Born approximation, $\Sigma =$  $= i/2\tau$ (τ mean free time)

Green function

$$\mathcal{G}^{\pm}(E,p) = (E - H_0(p) \pm i/2\tau)$$

Kubo formula for conductivity (paramagnetic part)

$$\sigma(\omega) = \frac{e^2}{2\pi} \int d\varepsilon \frac{\partial n_\varepsilon}{\partial \varepsilon} Tr \left[\hat{v}\, \mathcal{G}^+_{\varepsilon+\omega} \hat{v} \left(\mathcal{G}^+_\varepsilon - \mathcal{G}^-_\varepsilon \right) \right] \simeq \frac{e^2 \nu v_F^2}{d} \frac{\tau}{1 + i\omega\tau}$$

$\sigma_0 = \sigma(0) = \frac{e^2 \nu v_F^2 \tau}{d}$ (Drude conductivity)

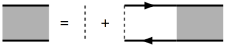
Diagrammatics

The dc electrical conductivity can be written in terms of current-current or density-density correlation functions

$$\sigma = i \lim_{\omega \to 0} \frac{1}{\omega} K^{ij}(\mathbf{0}, \omega)\delta_{ij} = i \lim_{\omega \to 0} \lim_{\mathbf{q} \to 0} \frac{\omega}{\mathbf{q}^2} K^{00}(\mathbf{q}, \omega)$$

Ladder summation (diffuson)

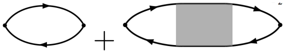
$\mathcal{D}(\mathbf{q}, \omega) =$



$$= \frac{1}{2\pi\nu\tau^2} \frac{1}{D\mathbf{q}^2 - i\omega}$$

with $D = v_F \ell / d = v_F^2 \tau / d$ (diffusion coefficient)

$K^{00}(\mathbf{q}, \omega) =$

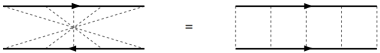


$$= -e^2\nu \frac{D\mathbf{q}^2}{D\mathbf{q}^2 - i\omega}$$

from which $\sigma = \sigma_0 = e^2 \nu D$.

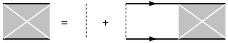
Diagrammatics: Weak Localization

Inclusion of crossing diagrams



Ladder summation in the particle-particle channel: cooperon

$$\mathcal{C}(\mathbf{q},\omega) = $$



$$= \frac{1}{2\pi\nu\tau^2}\frac{1}{D\mathbf{q}^2 - i\omega}$$

Since now $\mathbf{q} = \mathbf{p} + \mathbf{p}'$, the contribution to the current-current correlator

$$\delta K^{ii}(\mathbf{0},\omega) = $$

$$= \frac{i\omega\sigma_0}{\nu\pi}\int d\mathbf{q}\frac{1}{D\mathbf{q}^2 - i\omega}$$

The correction to the dc conductivity is

$$\delta\sigma = -\frac{\sigma_0}{\nu\pi}\int d\mathbf{q}\frac{1}{D\mathbf{q}^2 - i\omega} \propto -\begin{cases} \left(\frac{1}{\ell} - \frac{1}{L}\right) & d = 3 \\ \log\left(\frac{L}{\ell}\right) & d = 2 \\ (L - \ell) & d = 1 \end{cases}$$

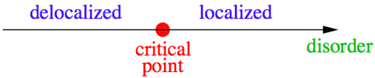
Anderson insulator

- 1D - 2D: Weak localization is IR-divergent in 1D and 2D: $\delta\sigma \sim \sigma_0$ at a scale

$$\xi \sim \pi\nu D, \quad \text{for 1D}$$

$$\xi \sim \ell \exp\left(\pi^2\nu D\right), \quad \text{for 2D}$$

- 3D: Localization only above a critical value of the disorder



localization lenght at criticality

$$\xi \sim \left(\sigma_0 - \sigma_c\right)^{-\nu}$$

In the localized phase $\mathcal{D}(\mathbf{q},\omega) = \mathcal{C}(\mathbf{q},\omega)$ becomes massive

$$\mathcal{D}(\mathbf{r},\omega) \sim \exp\left(-r/\xi\right)$$

Field theory approach: non-linear σ-model

(Wegner, ZPB (1979); Efetov, Larkin, Khmel'nitsky, JETP (1980))

- Write $\mathcal{G}^{\pm}$ in terms of Grassmann variables with action

$$S = \int \bar{\Psi}(E - H_0 - V \pm i\omega)\Psi$$

- Average over disorder V by replica method

$$S_{eff} = \int \bar{\Psi}(E - H_0 \pm i\,\omega)\Psi + w_0 \int (\bar{\Psi}\Psi)^2$$

- Hubbard Stratonovich transformation (auxiliary field Q)
- Integrating over fermionic fields $\Rightarrow S(Q)$
- Saddle point: $\frac{\delta S}{\delta Q} = 0 \Rightarrow Q_{sp}$
- Fluctuations around saddle point
- Gradient expansion $\Rightarrow$ N.L.σ M.

Hubbard Stratonovich transformation

Integration over disorder $\Rightarrow$ a quartic term in the action

$$e^{-S_{eff}} = e^{-(S_0+S_{imp})}$$

By Hubbard-Stratonovich decoupling,

$$e^{-S_{imp}} = e^{w_0 \int (\overline{\Psi}\Psi)^2} = \int dQ\, e^{\int \frac{1}{2w_0} Tr[QQ^\dagger] - iTr[\overline{\Psi} Q \Psi]}$$

For bipartite lattices the auxiliary field is not hermitian
$Q_j = Q_{0j} + i(-1)^j Q_{3j}$ (smooth and staggered components)

Integrating over Ψ

$$S(Q) = \sum \frac{1}{2w_0} Tr\left[Q^\dagger Q\right] - \frac{1}{2} Tr \ln\left(-H + iQ\right)$$

Saddle point: $\frac{\delta S}{\delta Q} = 0 \longrightarrow Q_{sp} = \Sigma \propto \tau^{-1}$

the self-energy at the Born level, in the diagrammatics!

Transverse modes and symmetry classification

Quantum fluctuations around Q_{sp} that leave H invariant

$$Q = U^{-1} Q_{sp} U$$

$U \in$ G and $[U, Q_{sp}] \neq 0$

If H subgroup of G such that $h \in$ H, $[h, Q_{sp}] = 0 \Rightarrow U \in$ G/H (Coset)

Hamiltonian Class	**RMT**	$\mathcal{T}$	**SU(2)**	**NLσ-model manifolds**
Wigner-Dyson classes				
A	GUE	$-$	$\pm$	U(2n)/U(n)×U(n)
AI	GOE	$+$	$+$	Sp(4n)/Sp(2n)×Sp(2n)
AII	GSE	$+$	$-$	O(2n)/O(n)×O(n)
Chiral classes				
AIII	chGUE	$-$	$\pm$	U(n)
BDI	chGOE	$+$	$+$	U(4n)/Sp(2n)
CII	chGSE	$+$	$-$	U(n)/O(n)
Bogoliubov-de Gennes				
C		$-$	$+$	Sp(2n)/U(2n)
CI		$+$	$+$	Sp(2n)
D		$-$	$-$	O(2n)/U(n)
DIII		$+$	$-$	O(n)

Non linear σ-model

From the real part of $S(Q)$

$$
\begin{aligned}
&Tr \ln(-H + iQ) + Tr \ln\left(-H - iQ^\dagger\right) = \\
&= -Tr \ln\left(H^2 + Q_{sp}^2\right) - Tr \ln(1 + G_0 U)\,,
\end{aligned}
$$

where $G_0 = \left(H^2 + Q_{sp}^2\right)^{-1}$ and

$$
U_{RR'} = iQ_R^\dagger H_{RR'} - iH_{RR'} Q_{R'} \simeq -\vec{J} \cdot \vec{\nabla} Q
$$

the current operator appears

$$
J = -iH_{RR'}(R - R')
$$

Expanding in $U_{RR'}$, the second term reads

$$
Tr(G_0 U G_0 U) \simeq (JG_0 JG_0)\; Tr\left(\partial Q^\dagger \partial Q\right)
$$

the factor $(JG_0 JG_0)$ is the Kubo formula for the conductivity!

Effective action (NLSM)

The final effective action in long wavelength limit

$$S[Q] = \frac{\pi}{8}\sigma \int dR \, Tr\left(\nabla Q \nabla Q^{\dagger}\right) - 4\nu \, Tr(\hat{\omega} Q)$$

the bare $\sigma = e^2 \nu D$ is the Drude conductivity!

Quantum corrections from Renormalization Group (RG) procedure:

Gaussian propagators = *diffuson* and *cooperon* in diagrammatics

$$< QQ >= \frac{1}{2\pi\sigma} \int \frac{d^2 q}{4\pi^2} \frac{1}{\mathbf{q}^2 - i\omega} \equiv g \log(s)$$

where the effective coupling constant which controls the perturbative expansion is given by $g = \frac{1}{2\pi^2\sigma}$ (the resistivity)

$$\beta(g) = \frac{d\,g}{d\log s} \quad \text{(s energy scaling factor)}$$

g is the running coupling constant.

RG of NLSMs (Wigner-Dyson classes) in $(2+\epsilon)d$

Beta-functions by ϵ-expansion

- Class A (unitary symmetry class, broken $\mathcal{T}$)
  $\beta(g) = -\epsilon g + g^3/2 + 3g^5/8 + O(g^7)$
- Class AI (ortogonal symmetry class, preserved $\mathcal{T}$ and SU(2))
  $\beta(g) = -\epsilon g + g^2 + 3\zeta(3)g^5/4 + O(g^6)$
- Class AII (simplettic symmetry class, preserved $\mathcal{T}$, no SU(2))
  $\beta(g) = -\epsilon g - g^2 + 3\zeta(3)g^5/4 + O(g^6)$

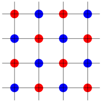
($+g^2 \Rightarrow$ weak localization, $-g^2 \Rightarrow$ weak anti-localization)

Anderson transitions ($\beta(g_c) = 0$)

- 3D ($\epsilon = 1$). Example: class AI
  critical point: $g_c = \epsilon - 3\zeta(3)\epsilon^4/4 + O(\epsilon^5)$
  localization lenght exponent: $\nu = -1/\beta'(g_c) = \simeq 1.7$
  (in good agreement with numerics, $\nu \simeq 1.57$)
- 2D for class AII
  critical point: $g_c = (4/3\zeta(3))^{1/3} \simeq 1$
  Metal-Insulator transition in 2D

Two-subattice models (Chiral classes)

(Gade, Wegner, NPB (1991))



The Hamiltonian is defined on a bipartite lattice

$$H = -\sum_{\langle ij\rangle\,\sigma} t_{ij}\, e^{i\phi_{ij}}\, c^{\dagger}_{i\sigma} c_{j\sigma} - \cancel{\sum_{i,\sigma} \mu\, c^{\dagger}_{i\sigma} c_{i\sigma}}$$

- $t_{ij} = t_{ji}$ random hopping,
- $\phi_{ij} = -\phi_{ji}$, if $\neq 0$, breaks time reversal symmetry $(\mathcal{T})$,
- $\mu \neq 0$ breaks sublattice symmetry $(\mathcal{S})$

The effective action

$$\begin{aligned} S[Q] \;&=\; \frac{\pi}{16}\,\sigma \int dR\; Tr\left(\nabla Q\,\nabla Q^{\dagger}\right) - 4\nu\, Tr(\hat{\omega} Q) \\ &\quad -\frac{\pi}{8}\,\Pi \int dR\,\left[Tr\left(Q^{\dagger}(R)\vec{\nabla} Q(R)\right)\right]^2 \end{aligned}$$

(for $\mu \neq 0 \Rightarrow \Pi = 0$)

Results with and without sublattice symmetry in 2D

	Coset space	Symm. class	$\beta(g)$
$\mu \neq 0,\ \phi_{ij} = 0$	$\mathrm{Sp}(4n)/\mathrm{Sp}(2n)\times\mathrm{Sp}(2n)$	AI	g^2
$\mu \neq 0,\ \phi_{ij} \neq 0$	$\mathrm{U}(4n)/\mathrm{U}(2n)\times\mathrm{U}(2n)$	A	$O(g^3)$
$\mu = 0,\ \phi_{ij} = 0$	$\mathrm{U}(8n)/\mathrm{Sp}(4n)$	BDI	0
$\mu = 0,\ \phi_{ij} \neq 0$	$\mathrm{U}(4n)\times\mathrm{U}(4n)/\mathrm{U}(4n)$	AIII	0

- without sublattice symmetry ($\mu \neq 0$):

$$\sigma = \sigma_0 - \frac{1}{2\pi^2}\log(\tau_\varphi/\tau)$$

(insulator, like for the on-site disorder)

- with sublattice symmetry ($\mu = 0$):

$$\sigma = \sigma_0$$

(conductor, Gade-Wegner criticality) at any order in g

$\beta(g) = 0$ also for CII (*Fabrizio, Dell'Anna, Castellani, PRL (2002)*)

Superconductors (Bogoliubov-de Gennes classes)

(Altland, Zirnbauer, PRB(1997))

For BdG Hamiltonians, since $U(1)$ is not preserved, charge diffusion is massive. The scaling parameter is the spin (or heat) conductivity:

- Classes C and CI: positive corrections $\beta(g) \sim g^2$ $\Rightarrow$ weak localization
- Classes D, DIII: negative corrections $\beta(g) \sim -g^2$ $\Rightarrow$ weak anti-localization (spin-metal - spin-insulator transition)

(Senthil, Fisher, Balents, Nayak, PRL (1998); Fabrizio, Dell'Anna, Castellani, PRL (2002))

Class C can be obtain also from random hopping Hamiltonian with magnetic impurities *(Dell'Anna, AdP (2017))*

Topological terms

For almost all classes (except for AI and BDI) in 2D the non-linear σ-model can be supplemented by a topological term:

- θ-term for A, C, D (like the Pruisken term for the Integer Quantum Hall, with $\theta = \sigma_{ij}/8$) or AII, CII

$$S_\theta = \theta \int dR\, Tr \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q$$

- WZW-term for AIII, CI, DIII (chiral anomaly).

$$S_{WZ} = \frac{k}{24\pi} \int dR^2 \int_0^1 d\bar{R}\; Tr \epsilon_{\mu\nu\lambda} (Q^{-1}\partial_\mu Q)(Q^{-1}\partial_\nu Q)(Q^{-1}\partial_\lambda Q)$$

We can get WZW term taking the imaginary part of the action, left over in the σ-model derivation.
(Dell'Anna, Fabrizio, Castellani, JSTAT (2007))

Anderson criticality in 2D (summary)

- Metal-Insulator transitions breaking spin-rotation invariance: classes AII, D, DIII
- Gade-Wegner criticality, line of fixed-points: $\beta(g) = 0$ for chiral classes: AIII, BDI, CII
- Criticality from topological terms
  - θ-term: $\mathbb{Z}_2$ topology ($\theta = \pi$) for classes AII and CII. Two hypotheses: attractive fixed point to *(i)* finite or (ii) ∞-(ideal) conductivity *(Ostrovsky, Gorny, Mirlin, PRL (2007))*
  - θ-term: $\mathbb{Z}$ topology for classes A, C, D. IQHE-like classes $\Rightarrow$ fixed point between localized to localized
  - WZW terms: Classes AIII, CI, DIII.

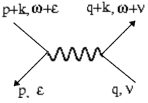
Only one symmetry class AI is always in the localized phase.

Interacting systems

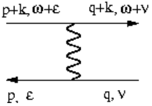
(Altshuler, Aronov, SSC 1983; Finkel'stein, ZETF 1983; Castellani, Di Castro, PRB 1984)

3 scattering amplitudes *(Finkel'stein, JETP 1984)*

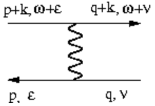
Γ_s in p-h singlet channel:



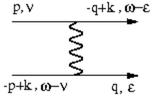
$-\frac{1}{2}$



Γ_t in p-h triplet channel:



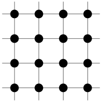
Γ_c in p-p Cooper channel:



6 scattering amplitudes with chiral symmetry *(Dell'Anna, NPB 2006)*

- Γ_s^0 Γ_t^0 Γ_c^0 previous scattering terms
- Γ_s^3 Γ_t^3 Γ_c^3 with $k \to k + q_\pi$, where $q_\pi = (\pi, \pi)$

Lattice model with disorder and interactions



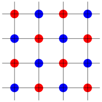
The interacting Hamiltonian is

$$H = -\sum_{\langle ij\rangle\,\sigma} t_{ij}\, c^\dagger_{i\sigma} c_{j\sigma} - \sum_{i,\sigma} \mu_i c^\dagger_{i\sigma} c_{i\sigma} + \frac{1}{2} \sum_{|k|\ll k_F} \sum_{p_1 p_2 \omega n m}$$

$$\Big\{\Gamma^0_s\, c^\dagger_n(p_1)\,\sigma_0\, c_{n+\omega}(p_1+k)\, c^\dagger_m(p_2)\,\sigma_0\, c_{m-\omega}(p_2-k)$$
$$-\Gamma^0_t\, c^\dagger_n(p_1)\,\vec{\sigma}\, c_{n+\omega}(p_1+k)\, c^\dagger_m(p_2)\,\vec{\sigma}\, c_{m-\omega}(p_2-k)$$
$$+\Gamma^0_c \sum_{\sigma\neq\sigma'} c^{\dagger\sigma}_n(p_1)\, c^{\dagger\sigma'}_{\omega-n}(k-p_1)\, c^{\sigma'}_m(p_2)\, c^{\sigma}_{\omega-m}(k-p_2)\Big\}$$

(Finkel'stein, JETP 1984)

Lattice model with disorder and interactions



The interacting Hamiltonian is

$$H = -\sum_{\langle ij\rangle\,\sigma} t_{ij}\, c^\dagger_{i\sigma} c_{j\sigma} \qquad + \frac{1}{2} \sum_{|k|\ll k_F} \sum_{p_1 p_2 \omega n m}$$

$$\Big\{ \Gamma^0_s\, c^\dagger_n(p_1)\,\sigma_0\, c_{n+\omega}(p_1+k)\, c^\dagger_m(p_2)\,\sigma_0\, c_{m-\omega}(p_2-k)$$
$$-\Gamma^0_t\, c^\dagger_n(p_1)\,\vec\sigma\, c_{n+\omega}(p_1+k)\, c^\dagger_m(p_2)\,\vec\sigma\, c_{m-\omega}(p_2-k)$$
$$+\Gamma^0_c \sum_{\sigma\neq\sigma'} c^{\dagger\sigma}_n(p_1)\, c^{\dagger\sigma'}_{\omega-n}(k-p_1)\, c^{\sigma'}_m(p_2)\, c^{\sigma}_{\omega-m}(k-p_2)$$

$$+\Gamma^3_s\, c^\dagger_n(p_1)\,\sigma_0\, c_{n+\omega}(p_1+k+q_\pi)\, c^\dagger_m(p_2)\,\sigma_0\, c_{m-\omega}(p_2-k-q_\pi)$$
$$-\Gamma^3_t\, c^\dagger_n(p_1)\,\vec\sigma\, c_{n+\omega}(p_1+k+q_\pi)\, c^\dagger_m(p_2)\,\vec\sigma\, c_{m-\omega}(p_2-k-q_\pi)$$
$$+\Gamma^3_c \sum_{\sigma\neq\sigma'} c^{\dagger\sigma}_n(p_1)\, c^{\dagger\sigma'}_{\omega-n}(k-p_1+q_\pi)\, c^{\sigma'}_m(p_2)\, c^{\sigma}_{\omega-m}(k-p_2+q_\pi) \Big\}$$

(Dell'Anna, NPB 2006)

Interacting effective action

(Finkel'stein, JETP (1984); Dell'Anna, NPB (2006))

The corresponding effective action can be renormalized and reads

$$S[Q] = S_{NLSM}$$

$$- \sum_{\alpha=0,3} \Gamma_s^{\alpha} \sum_{\ell=0,3} \int' Tr\left(Q^{aa}_{n,\,n+\omega} \tau_\ell \, \sigma_0 \, \gamma_\alpha\right) Tr\left(Q^{aa}_{m+\omega,\,m} \tau_\ell \, \sigma_0 \, \gamma_\alpha\right)$$
$$+ \sum_{\alpha=0,3} \Gamma_t^{\alpha} \sum_{\ell=0,3} \int' Tr\left(Q^{aa}_{n,\,n+\omega} \tau_\ell \, \vec{\sigma} \, \gamma_\alpha\right) Tr\left(Q^{aa}_{m+\omega,\,m} \tau_\ell \, \vec{\sigma} \, \gamma_\alpha\right)$$
$$+ \sum_{\alpha=0,3} \Gamma_c^{\alpha} \sum_{\ell=1,2} \int' Tr\left(Q^{aa}_{n+\omega,\,-n} \tau_\ell \, \sigma_0 \, \gamma_\alpha\right) Tr\left(Q^{aa}_{m+\omega,\,-m} \tau_\ell \, \sigma_0 \, \gamma_\alpha\right)$$

τ_i, σ_i, γ_i Pauli matrices in particle-hole, spin and sublattice spaces and $\int' = \frac{\pi^2 \nu^2}{32} \int dR \sum_{nm\omega a}$

Results with interactions

Very rich and not universal behaviors of the β-functions, not uniquely determined by symmetry classes *(Dell'Anna, AdP (2017))*

- Class A
  - yes $\mathcal{S}$, no $\mathcal{T}$, SU(2) $\rightarrow$ U(1)
    Antiferromagnetic fluctuations induce by disorder
  - no $\mathcal{S}$, no $\mathcal{T}$, yes SU(2)
    RG $\rightarrow$ clean system with long-range interaction
  - no $\mathcal{S}$, no $\mathcal{T}$, no SU(2)
    Interaction is RG irrelevant, RG $\rightarrow$ free case
- Class AIII
  - yes $\mathcal{S}$, no $\mathcal{T}$, yes SU(2)
    Antiferromagnetic fluctuations induce by disorder
  - no $\mathcal{S}$, no $\mathcal{T}$, SU(2) $\rightarrow$ U(1)
    Localization (unlike free case), interactions $\rightarrow$ scale invariants
- Class C
  - yes $\mathcal{S}$, no SU(2) (broken by magnetic impurities)
    Localization or Anti-localization, depending on the interaction

Results with interactions

Class AI and Class BDI

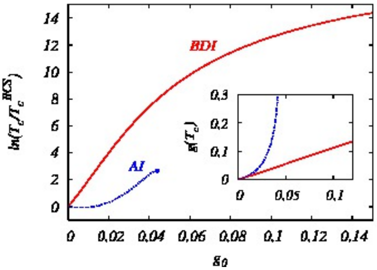
- Far from instabilities
  for $\Gamma_c^0 > 0$, (and $\Gamma_3^s > 0$, $\Gamma_3^t < 0$ for BDI)

	No Interaction	Yes Interaction
AI	Anderson Insulator	delocalization (Finkel'stein)
BDI	Metal (Gade-Wegner)	Anderson-Mott Insulator

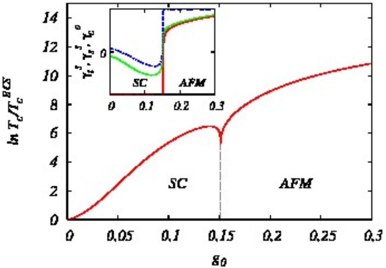
- Close to instabilities
  - $\Gamma_c^0 < 0$ can diverge under RG $\Rightarrow$ Superconductivity (SC)
  - $\Gamma_s^3 < 0$ can diverge under RG $\Rightarrow$ Charge density wave (CDW)
  - $\Gamma_t^3 > 0$ can diverge under RG $\Rightarrow$ Antiferromagnetism (AFM)

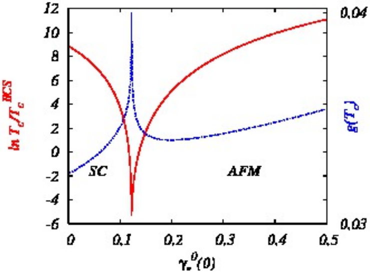
Since the dephasing time (time scale for the coherence to be destroyed by inelastic processes) is $\tau_\varphi \sim T^{-1}$
$\Rightarrow$ temperature T is the IR cutoff
(Burmistrov, Gornyi, Mirlin, PRL 2012 (AI); Dell'Anna, PRB 2013 (BDI))

Solving RG equations



$$T_c \gg T_c^{BCS}$$





Enhancement of T_c for class BDI

Two counterintuitive results in the presence of disorder (g_0) and interactions (γ_0) (with $\gamma_0 \ll g_0 \ll 1$) in the presence of short-range repulsive interaction

- Enhancement of superconductivity by disorder

$$T_c \sim (T_c^{BCS})^{-\frac{\gamma_{c0}}{g_0}} \gg T_c^{BCS} \quad d = 2$$

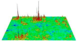
$$T_c \sim (T_c^{BCS})^{1-g_0} \gg T_c^{BCS} \quad d = 3$$

- Antiferromagnetic fluctuations driven by random hopping

$$T_c \sim (T_c^{N})^{-\frac{2\gamma_{t0}}{3g_0}} \gg T_c^{N} \quad d = 2$$

$$T_c \sim (T_c^{N})^{1-\frac{3g_0}{2}} \gg T_c^{N} \quad d = 3$$

(Dell'Anna, PRB (2013))

Multifractal wavefunctions  $\Rightarrow$ inomogeneity of the pairing Δ $\Rightarrow$ enhancement of T_c *(Feigelman, Ioffe, Kravtsov, Cuevas, AoP (2010))*

Thank you for your attention