Curve Shortening Flow for Embedded and Unknotted Curves in $\mathbb{R}^3$.

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Let $\gamma : S^1 \times [0, \omega) \to \mathbb{R}^3$ be an smooth family of embedded space curves. We say that γ evolves by the curve shortening flow if

$$\frac{\partial}{\partial t}\gamma(\cdot, t) = k(\cdot, t)N(\cdot, t), \tag{CSF}$$

where $k(\cdot, t)$ and $N(\cdot, t)$ are the curvature and the normal vector of $\gamma(\cdot, t)$, respectively.

A fundamental problem within the curve shortening flow is to study the singularities that the curves may develop during the evolution. For instance, the formation of singularities for embedded planar curves have been fully understood. In contrast, to study the behavior of curves in $\mathbb{R}^3$ (codimension 2) which evolves by (CSF) is more difficult than the planar case (for example, in $\mathbb{R}^3$, the curves may not remain embedded). Consequently, fewer results are known in that context and usually it is necessary to preserve certain geometric quantities during the flow.

Thus, in this talk I will consider an special case of embedded curves in $\mathbb{R}^3$: curves with total curvature less than 4π. If γ has these characteristics, there exists a unique minimal surface X with boundary γ. Following the work of Huisken, I will define intrinsic and extrinsic distance functions to analyze the formation of singularities, particularly, type II singularities.