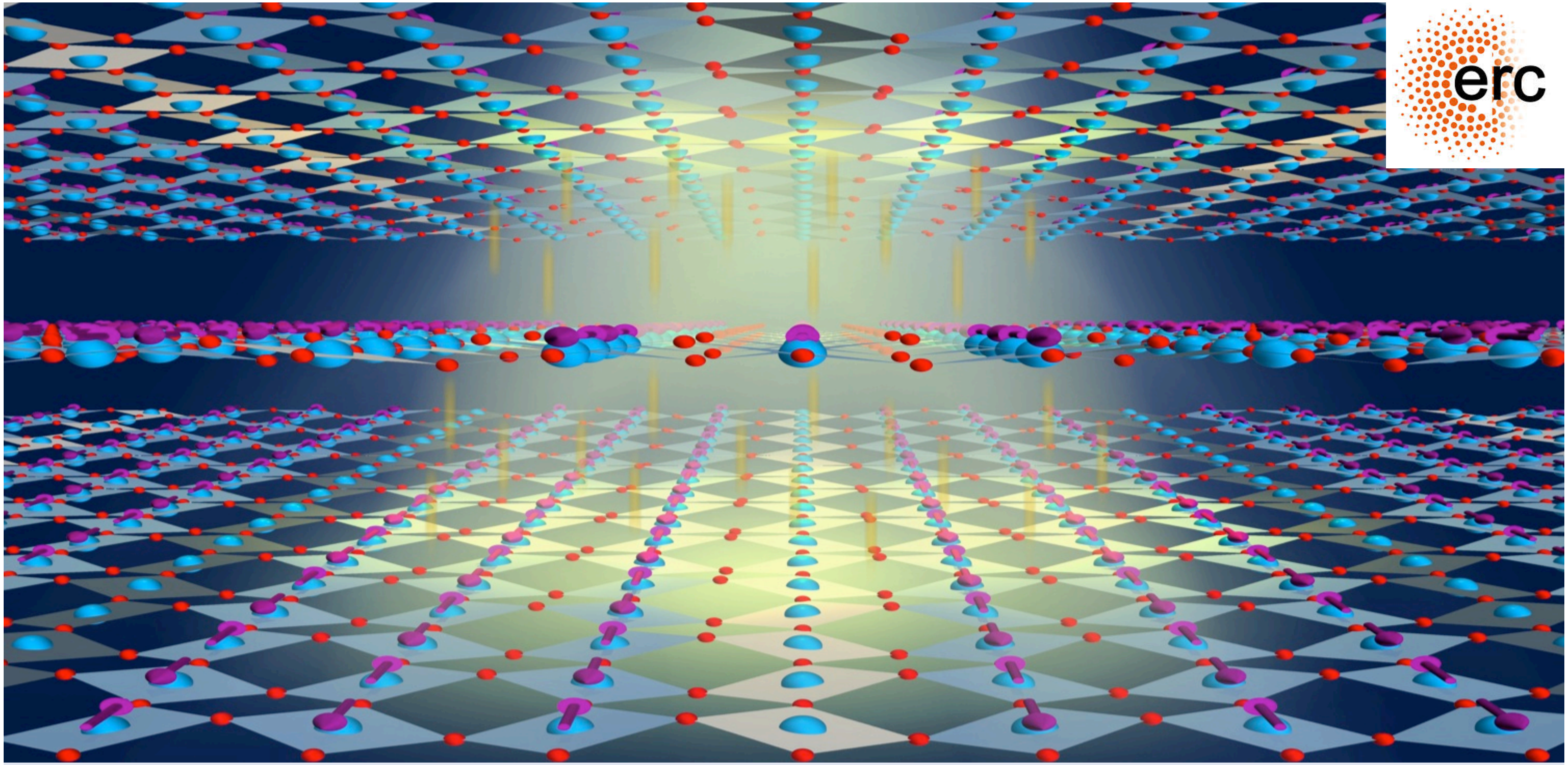
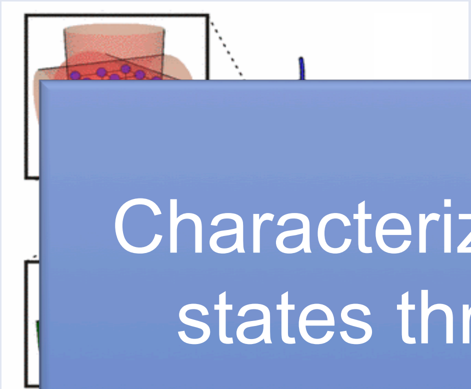


Quantum Fluctuation Relations in the Presence of Conserved Quantities

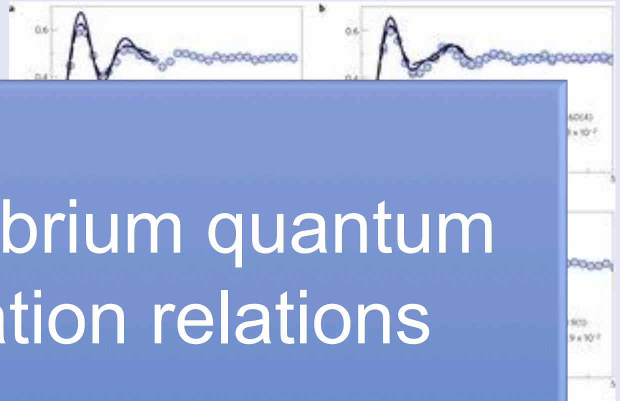
Dieter Jaksch | Clarendon Laboratory, University of Oxford | 11th Sept. 2017

Non-equilibrium dynamics

Driven optical lattices



Relaxation in optical lattices

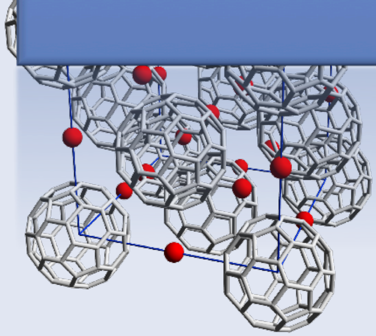


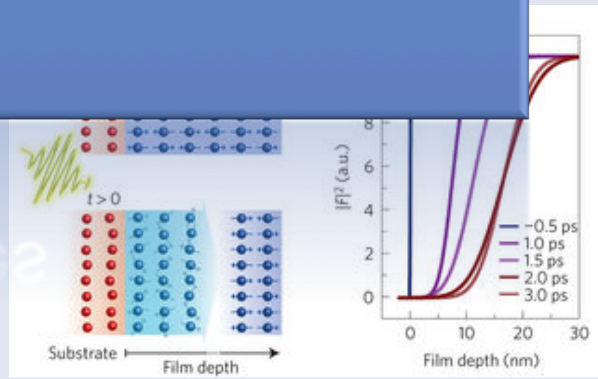
A. Zenesi … , 325 (2012)

Characterize non-equilibrium quantum states through fluctuation relations

Detect dynamics induced symmetry broken states

Driven … $_3$ film





M. Mitrano *et al.*, Nature **530,** 461 (2016)

M. Foerst *et al.*, Nature Materials **14,** 883 (2015)

Classical fluctuation relations

- Thermodynamics

$$w \geq \Delta F, \quad \Delta S \geq 0$$

- Fluctuation relations

  → Jarzynski equality

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

  → Crooks relation

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

- These relations constrain $\mathrm{PDF}(w)$

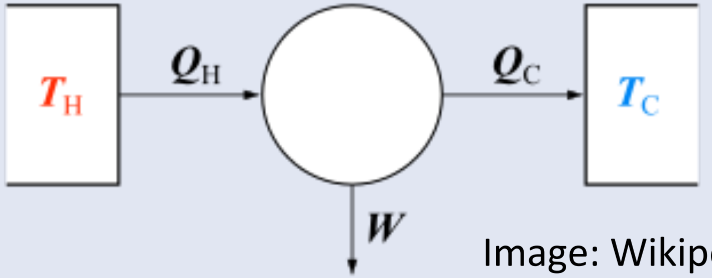
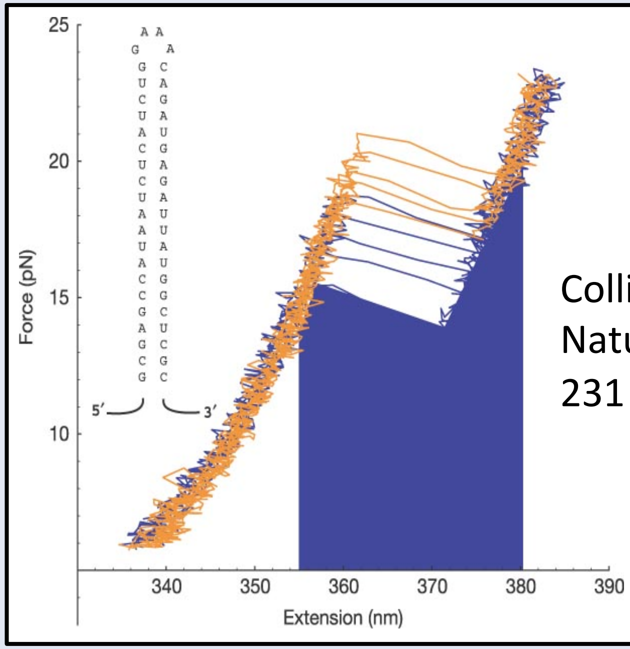


Image: Wikipedia



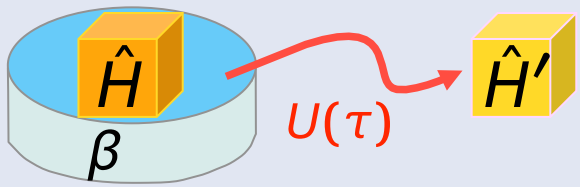
Collin et al., Nature **437** 231 (2005)

C. Jarzynski, PRL **78**, 2690 (1997); G.E. Crooks, J. Stat. Phys., **90**, 1481 (1998).

Quantum fluctuation relations

→ Quantum Jarzynski equality

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

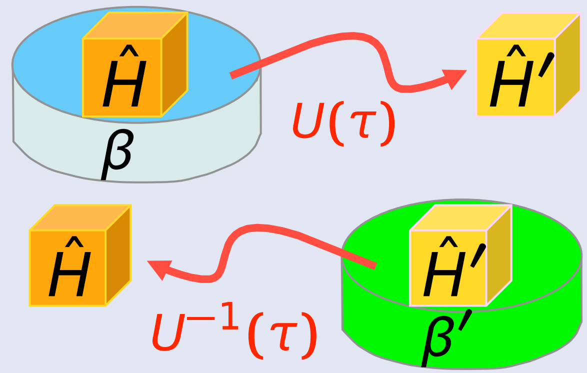


→ Tasaki-Crooks relation

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

FW:

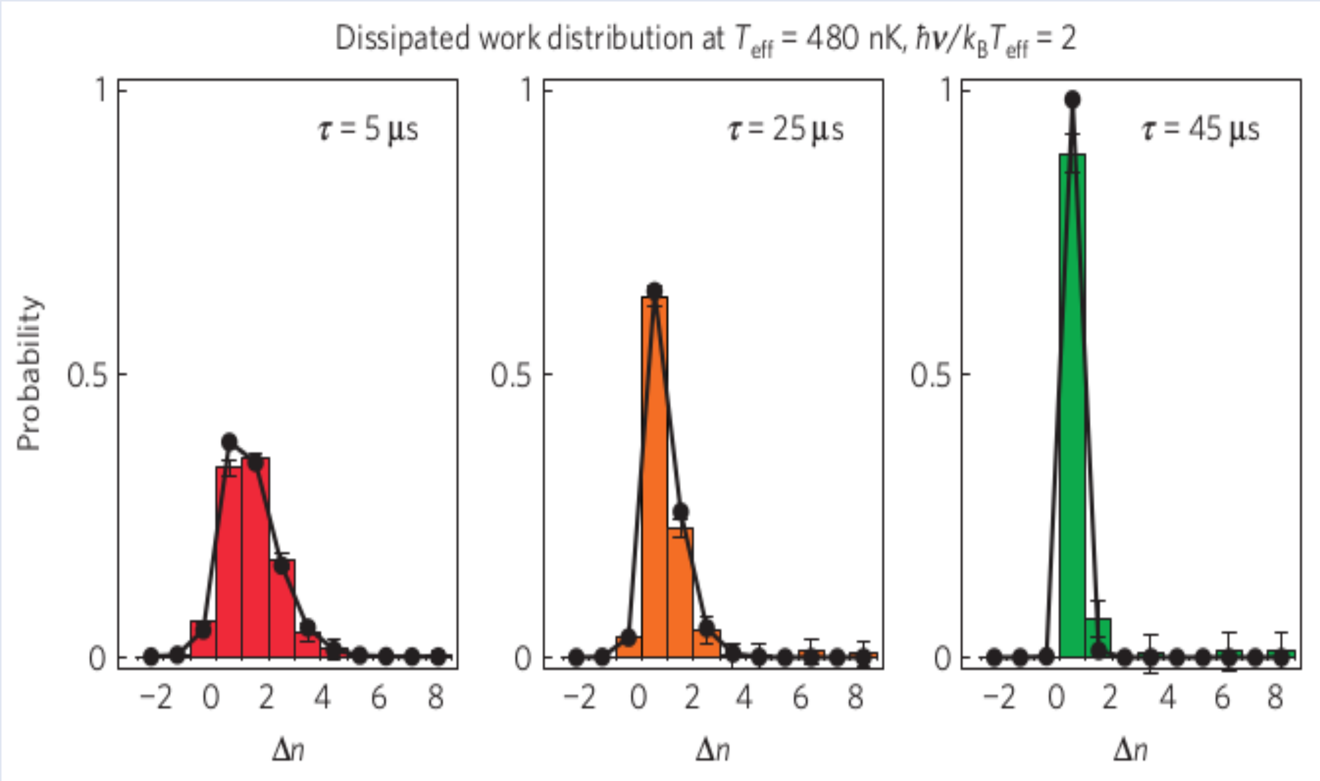
BW:



H. Tasaki, arXiv:cond-mat/0009244

Quantum fluctuation relations

→ Testing the Quantum Jarzynski equality



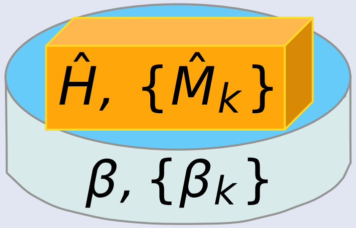
Fast quench

Slow quench

An et al., Nat. Phys. **11** 193 (2015) [one trapped ion]

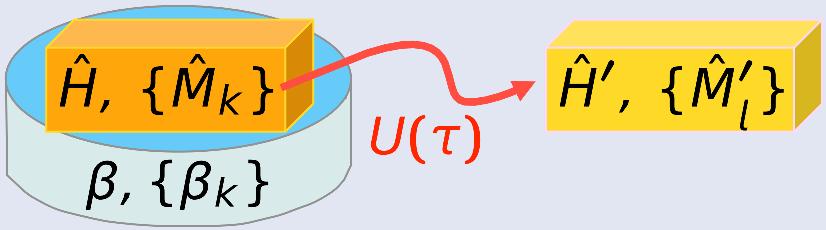
Conserved quantities M_k

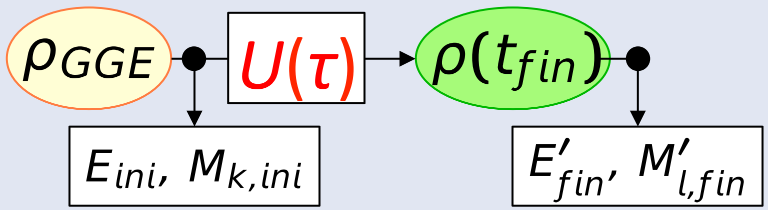
- Generalized Gibbs ensemble ρ_{GGE}



$$\rho_{GGE} = \frac{1}{Z_{GGE}} e^{-\beta H - \sum_k \beta_k M_k}$$

- Generalized work W





$$A_{ini} = \beta E_{ini} + \sum_k \beta_k M_{k,ini}$$

$$A_{fin} = \beta' E'_{fin} + \sum_l \beta'_l M'_{l,fin}$$

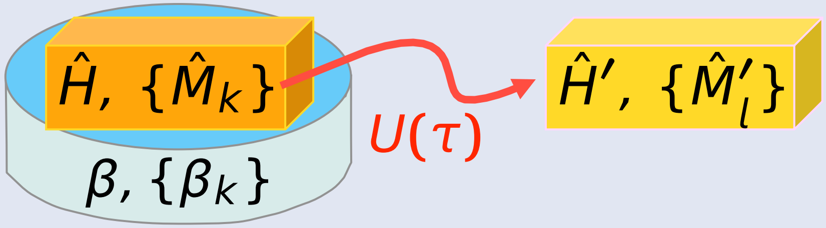
$$W = A_{fin} - A_{ini}$$

→ replace w by W

Generalized fluctuation relations

→ Generalized Jarzynski equality

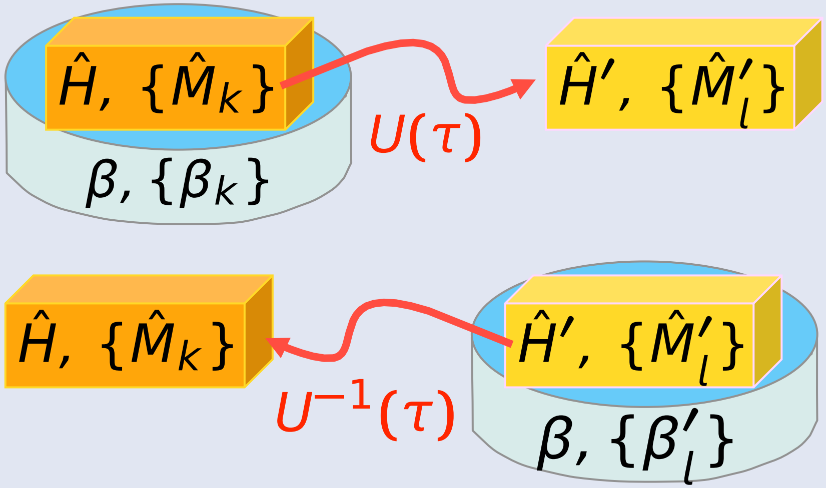
$$\langle e^{-W} \rangle = e^{-\Delta F_{GGE}}$$



→ Generalized Tasaki-Crooks relation

$$P_f(W) = e^{W - \Delta F_{GGE}} P_b(-W)$$

with $F_{GGE} = -\ln(Z_{GGE})$



J. Mur Petit, A. Relano, R. Molina, DJ, submitted.

Example I: The Dicke model

- Bosonic field a coupled to an ensemble of N spin ½ systems σ_i

$$H = \omega_a a^\dagger a + \omega_s J_z + H_{\mathrm{int}}$$

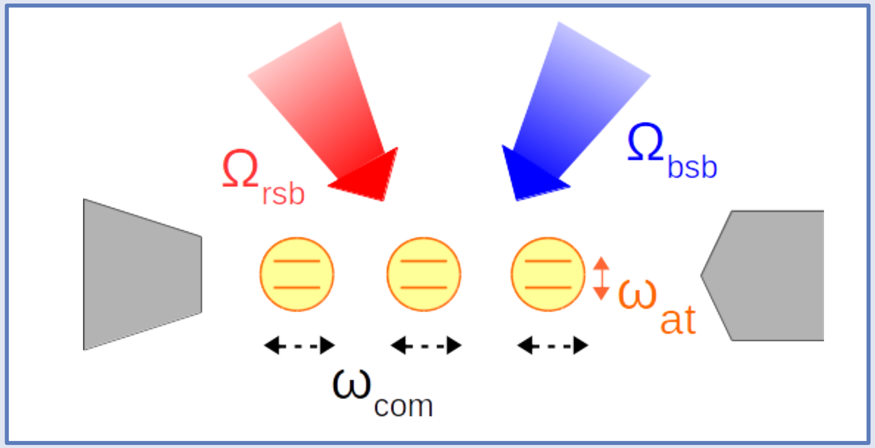
with $J = 1/2 \sum_{i=1}^{N} \sigma_i$, and

$$H_{\mathrm{int}} = g(1-\alpha)(J_+ a + J_- a^\dagger) + g\alpha(J_+ a^\dagger + J_- a)$$

- For $\alpha = 0$ the quantity

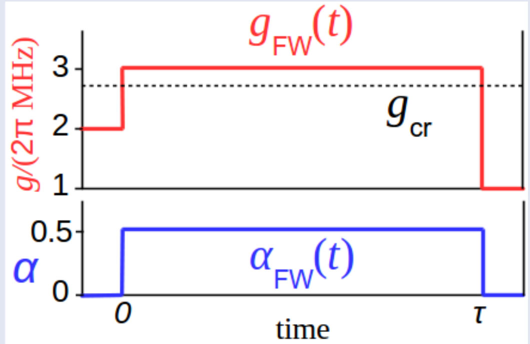
$$M = J_z + a^\dagger a$$

is conserved.

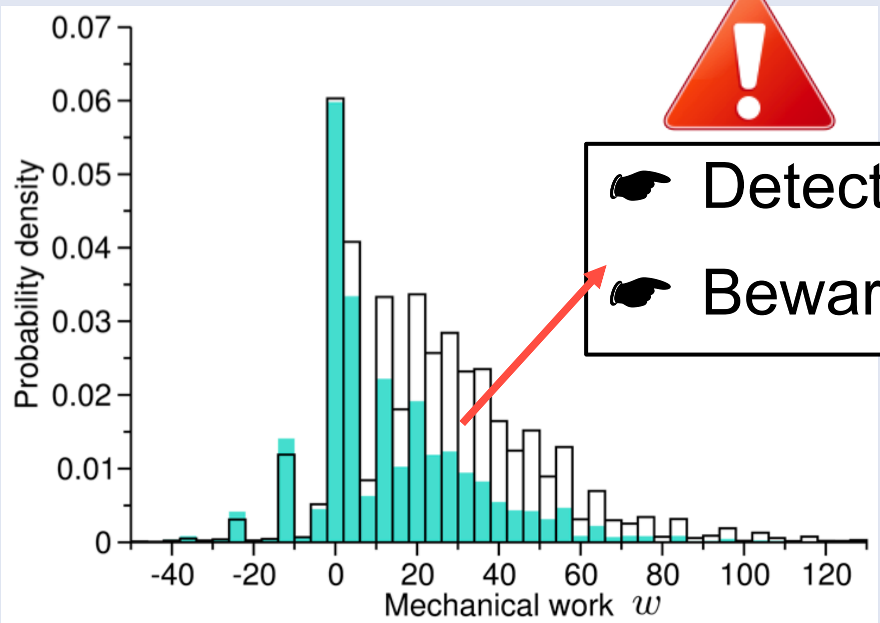


Tasaki-Crooks relation results

$$(*)\ P_{FW}(w) = e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$$



Work PDFs:



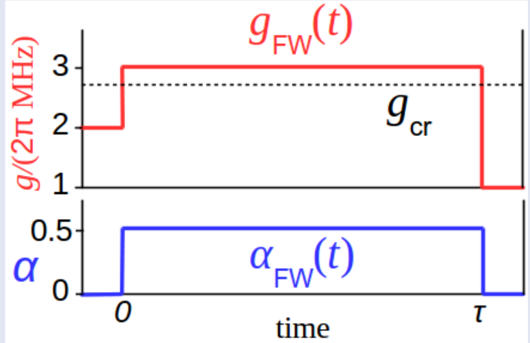
- Detect if ρ is missing charges
- Beware wrong estimates of $\beta, \Delta F$

$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$$

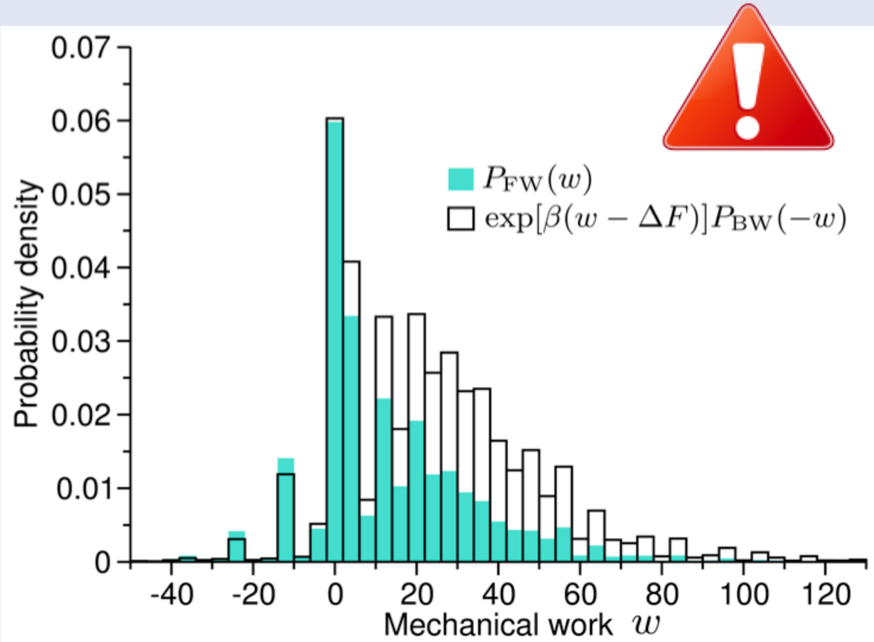
Tasaki-Crooks relation results

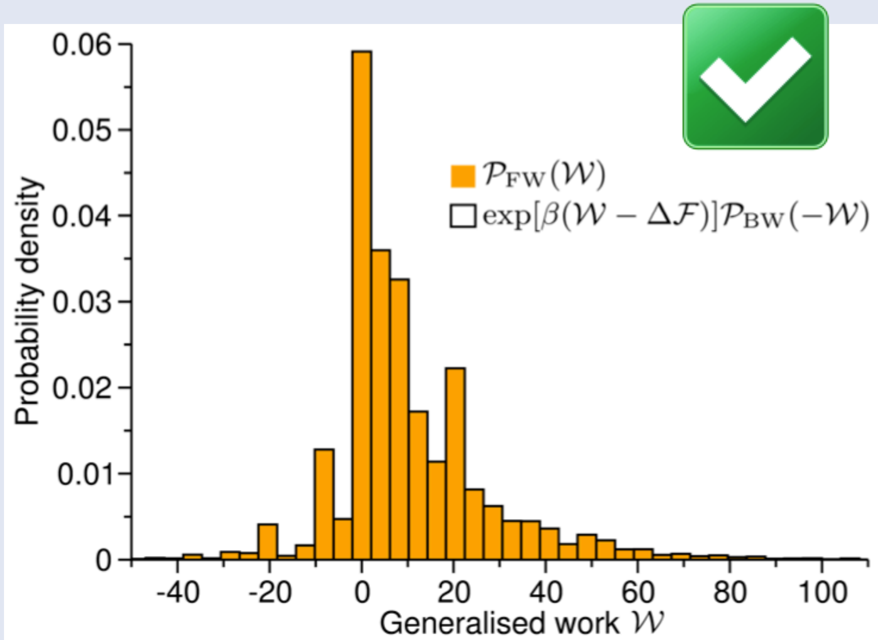
$$(*)\ P_{FW}(w) = e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$$

$$P_{FW}(W) = e^{W-\Delta F_{GGE}} P_{BW}(-W)$$



Work PDFs:





$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$$

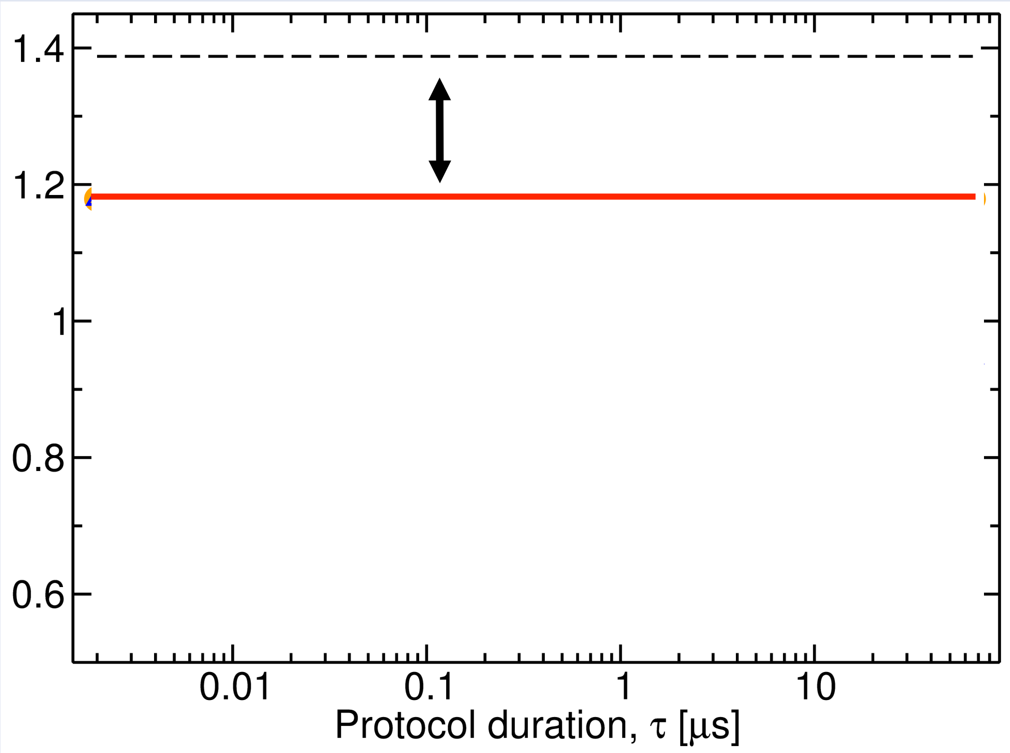
Jarzynski equation results

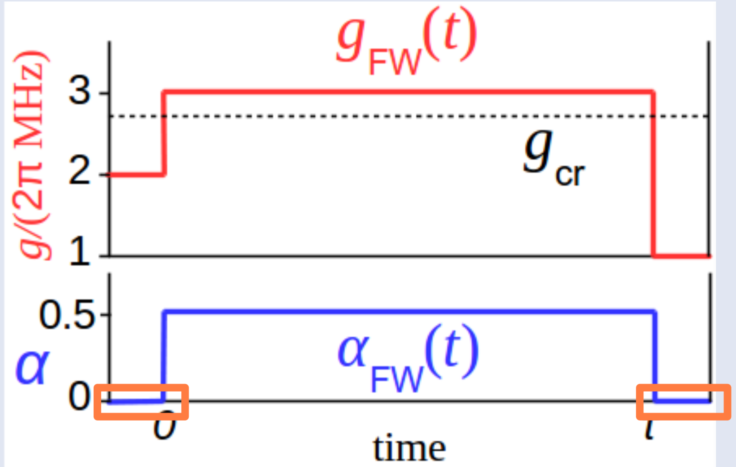
Varying protocol duration τ:

$$\langle \exp(-W) \rangle = \exp(-\Delta F_{GGE})$$

$$\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{Gibbs})$$

$$\beta = 0.1,\ \beta_M = 0.3$$





$\exp(-\beta \Delta F_{Gibbs})$

$\exp(-\Delta F_{GGE})$

Jarzynski equation results

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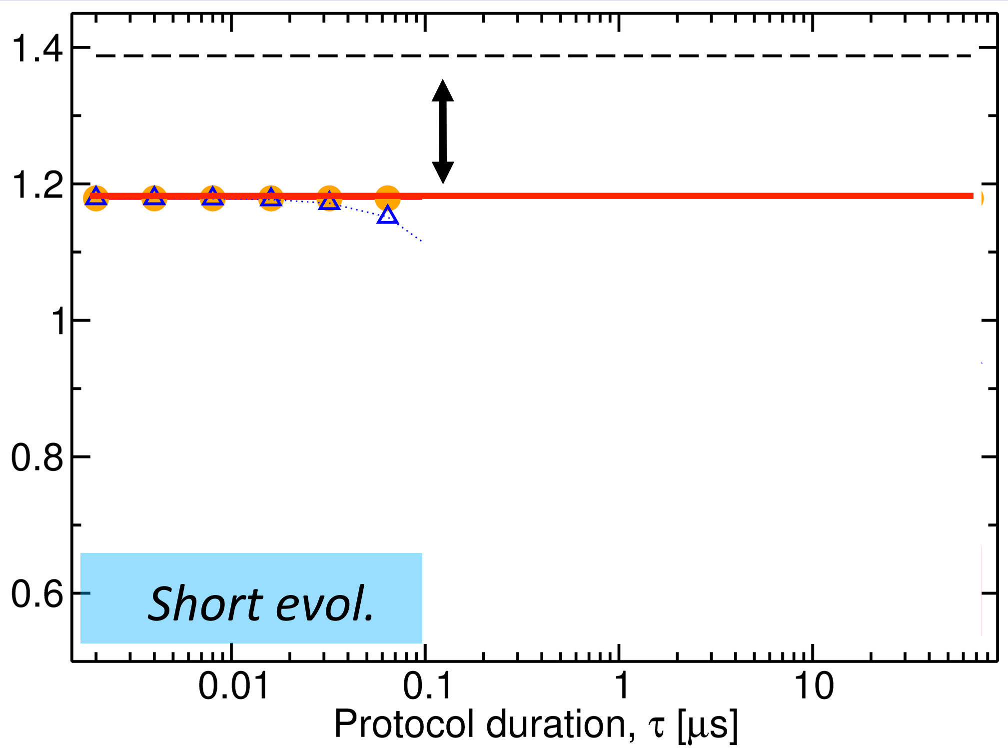

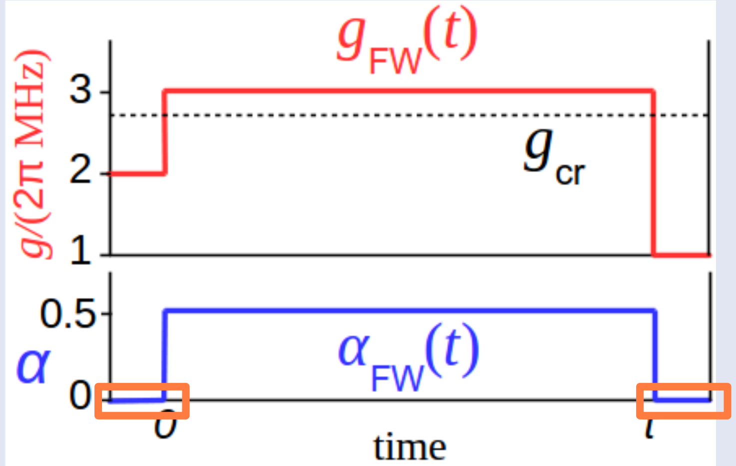

☛ Beware wrong estimate of $\beta\Delta F$

- ⋯⋯ $\exp(-\beta \Delta F_{Gibbs})$
- —— $\exp(-\Delta F_{GGE})$
- ● $\langle \exp(-W) \rangle$
- △ $\langle \exp(-\beta w) \rangle$

Jarzynski equation results

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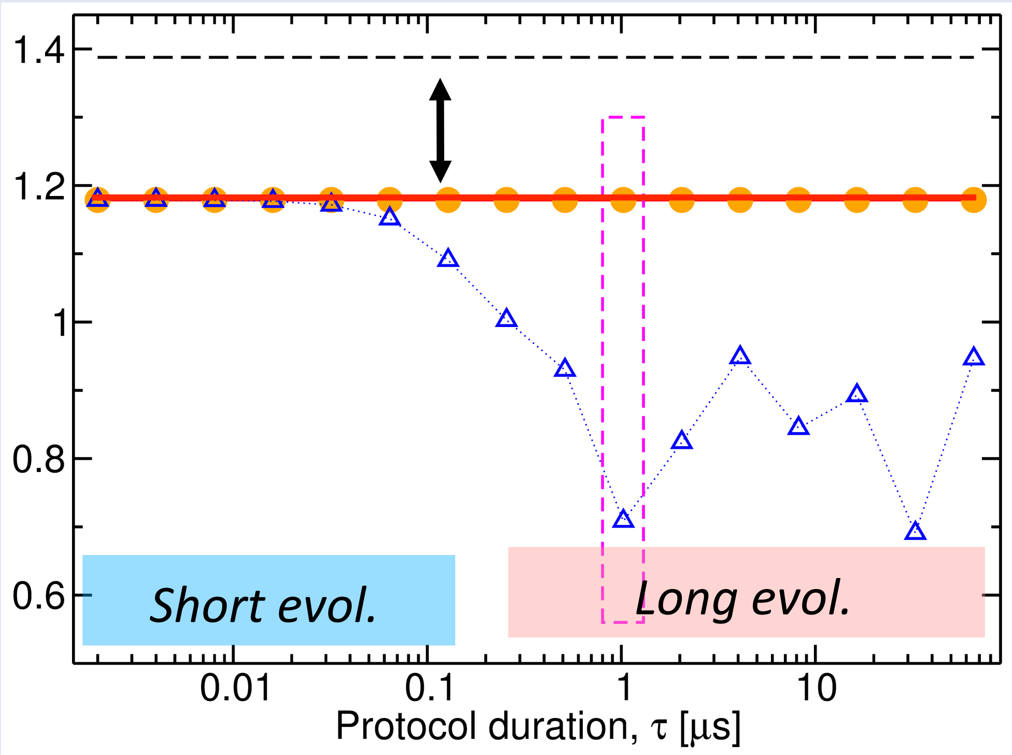
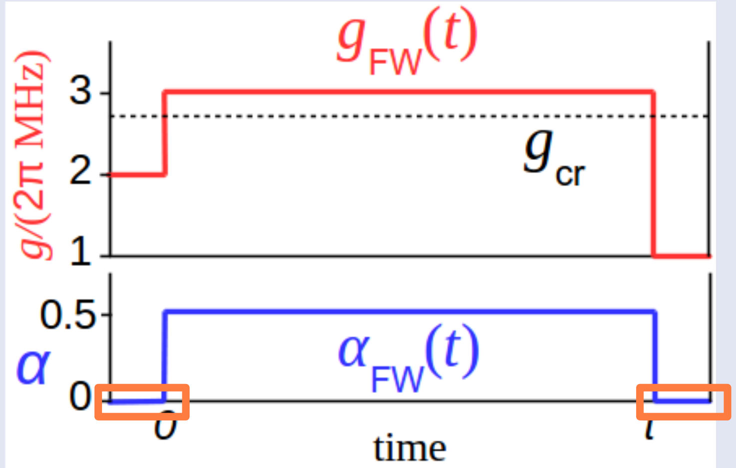




☛ Track missing charges relevant to dynamics

- ········ $\exp(-\beta \Delta F_{Gibbs})$
- —— $\exp(-\Delta F_{GGE})$
- ● $\langle \exp(-W) \rangle$
- △ $\langle \exp(-\beta w) \rangle$

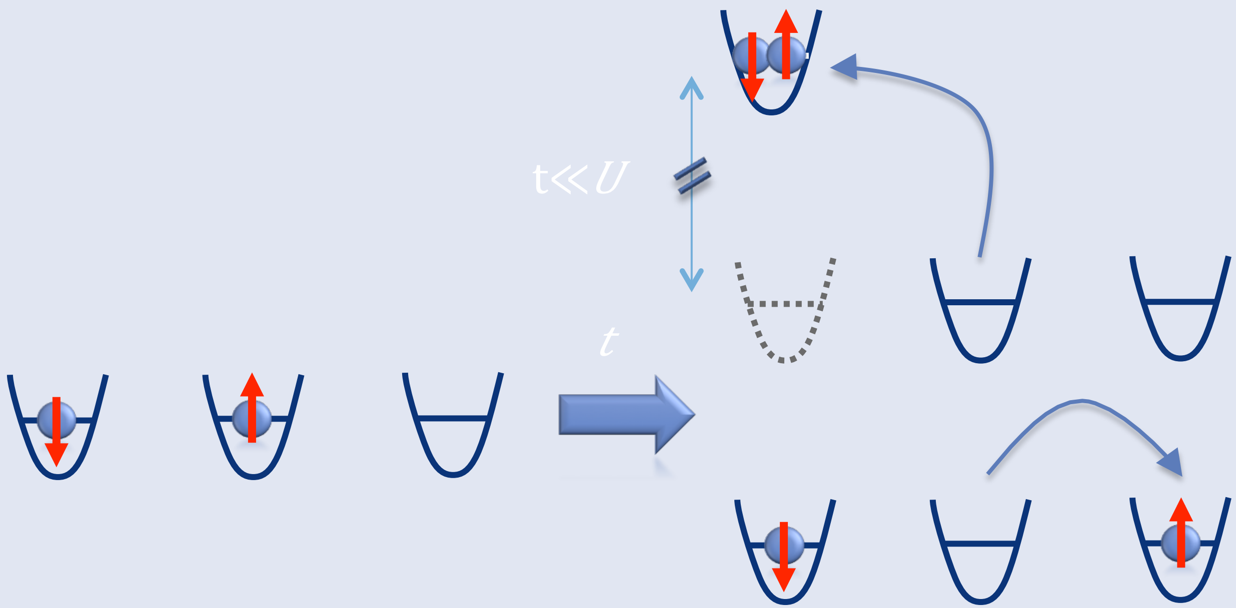
Example II: Driven Hubbard model

- Hubbard Hamiltonian

$$H_{\text{Hub}} = -t\sum_{\langle i,j\rangle,\sigma}(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + U\sum_{i} n_{i\uparrow} n_{i\downarrow} = H_{\text{hop}} + H_{U}$$

with $c_{i\sigma}$ destroying a fermion with spin σ in site i.

- We consider the strongly correlated limit $U \gg t$

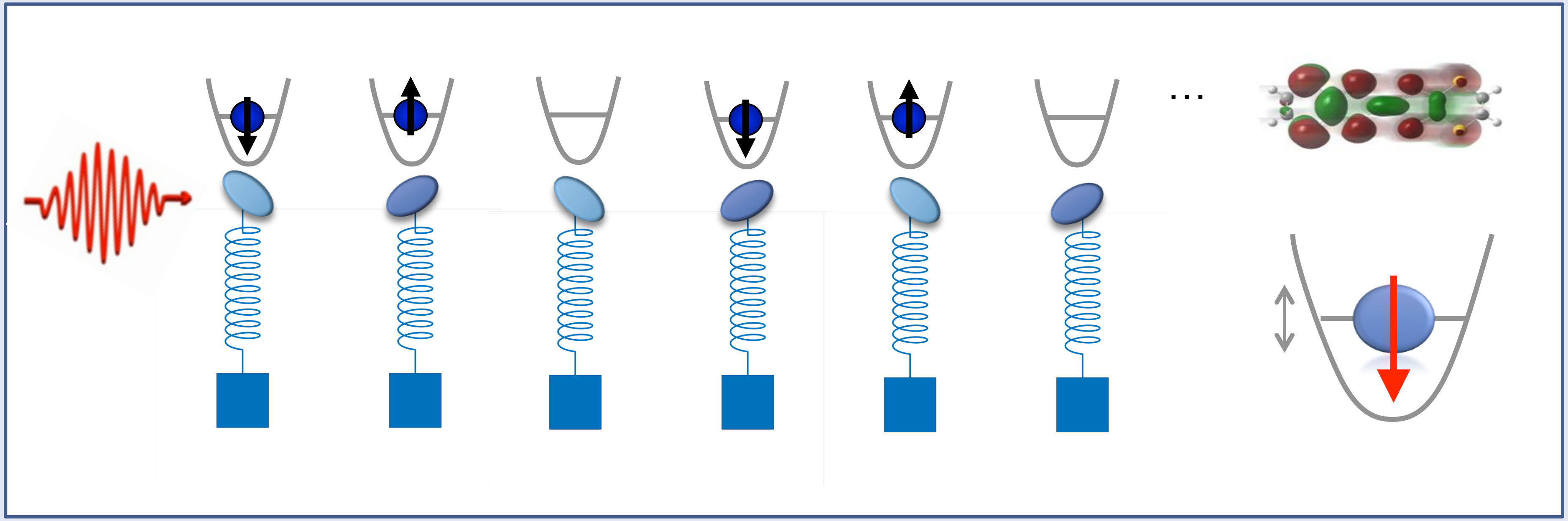


Driving local vibrations

- Include driving of a-b lattice with frequency Ω

$$H_d(\tau) = V/2 \sum_{j\in a} \sin(\Omega\tau - \phi)\, n_j + V/2 \sum_{j\in b} \sin(\Omega\tau + \phi)\, n_j$$

we assume the driving to be out of phase $\phi = \pi/2$



Driving local vibrations

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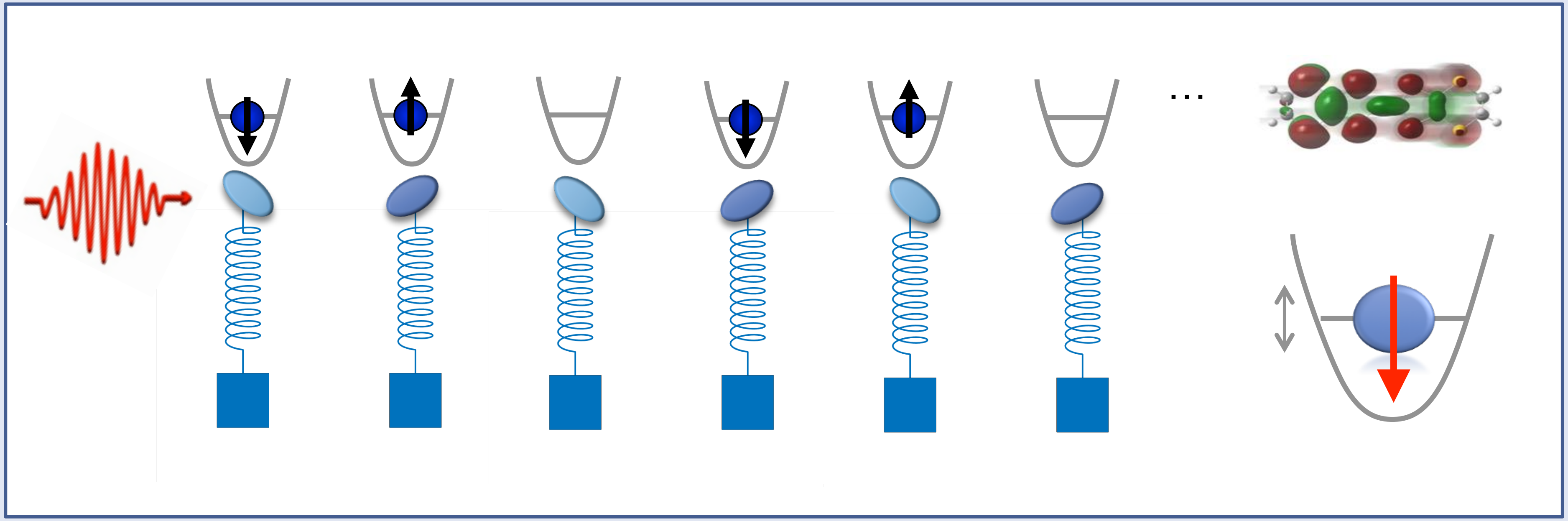

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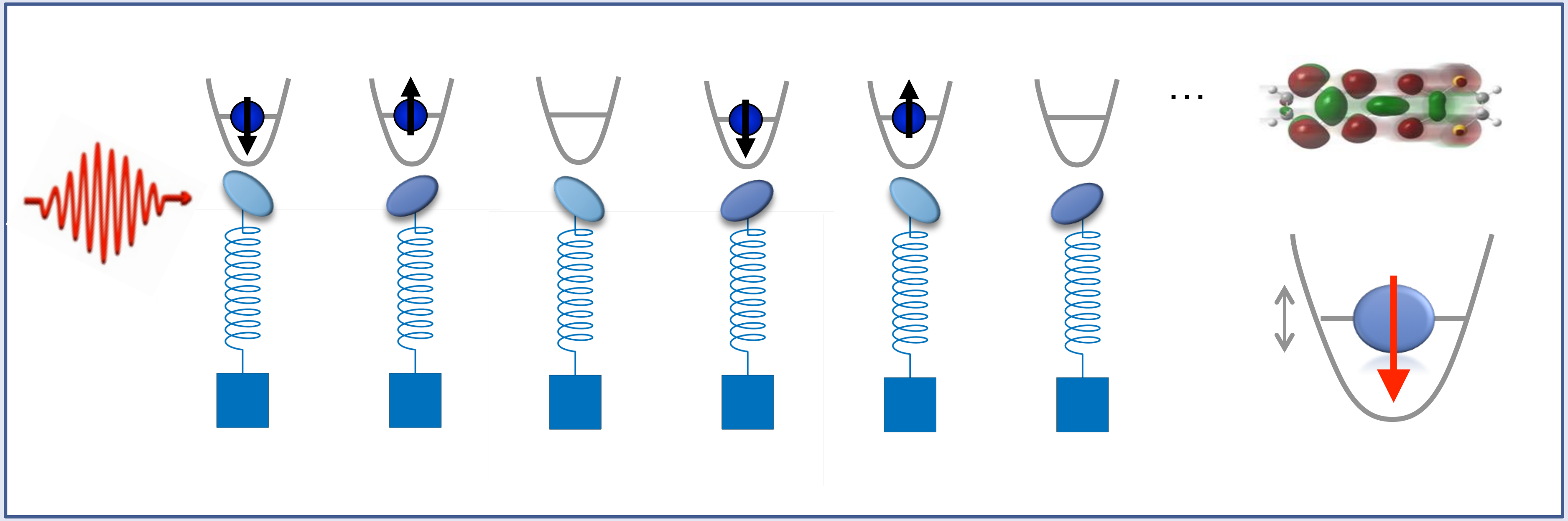
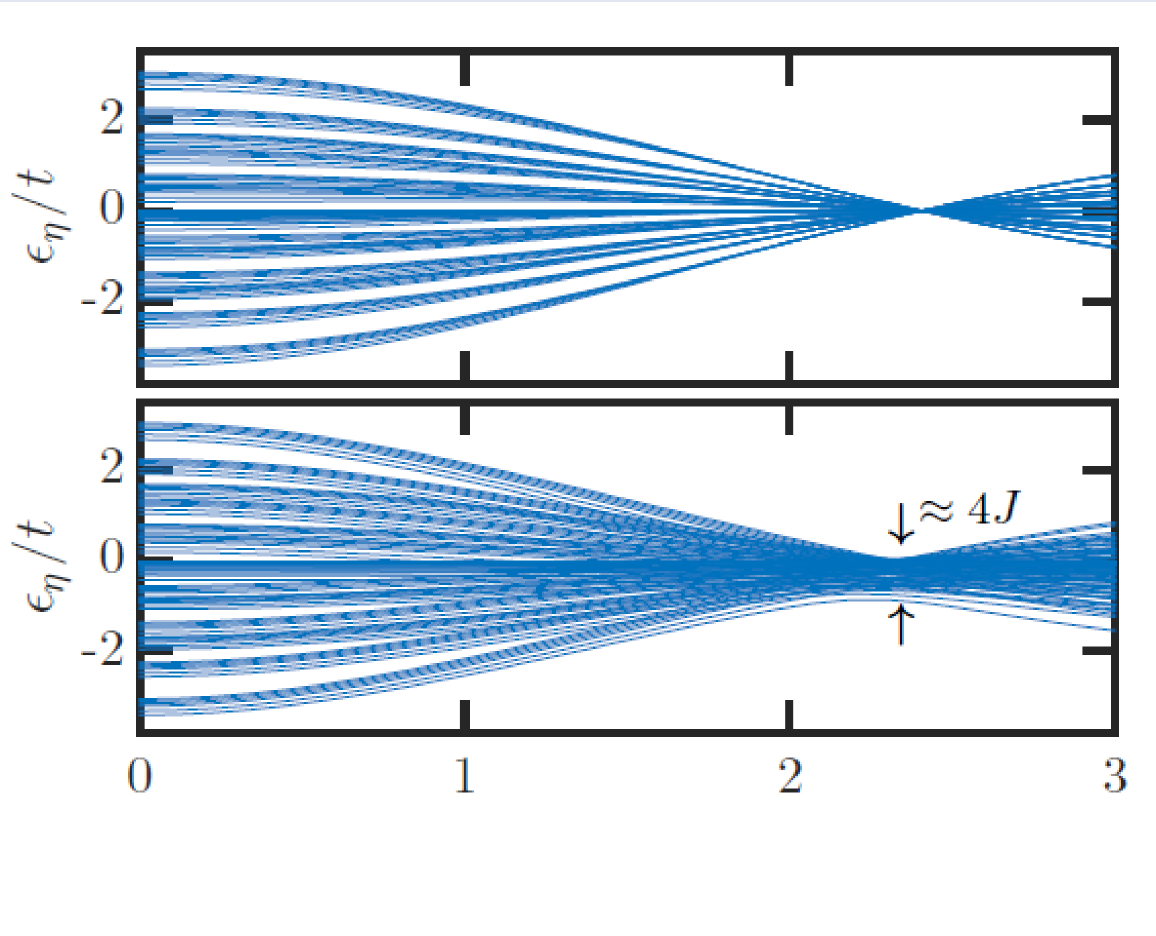


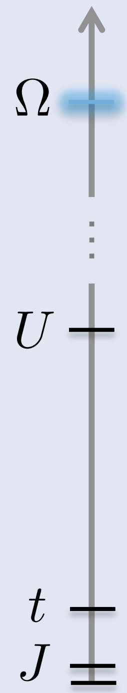
Quasi-energies in a small driven system



$\Omega \gg U \gg t$

$t \approx \mathcal{J}_0(\nu)t$

$t < \Omega < U$

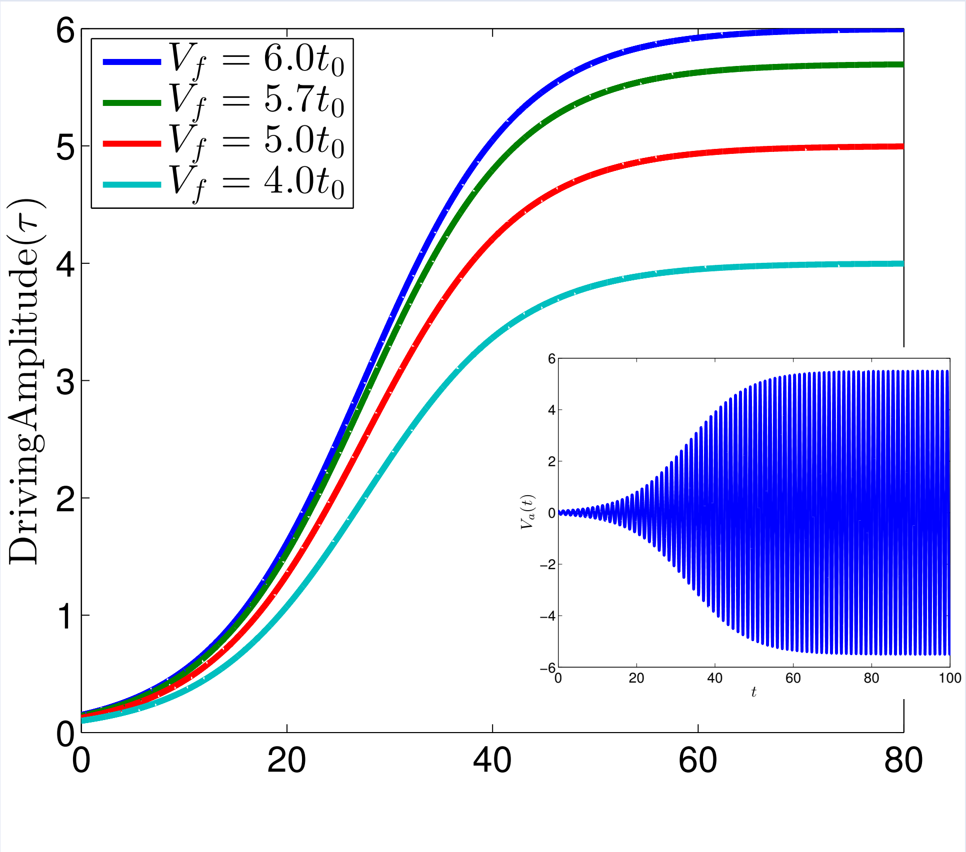


When driving in the gap $t < \Omega < U$ a finite bandwidth $J = 4t^2/U$ remains for any driving

J. Coulthard, SR Clark, S. Al-Assam, A. Cavalleri, DJ, Phys. Rev. B **96**, 085104 (2017).

Dynamically ramped up driving

We use td-iDMRG to study the dynamics for a slowly increasing drive and starting from the ground state



We look at correlation functions and their structure factors

Density-density correlations

$N_{ij} = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$

Spin-spin correlations

$S_{ij} = \langle S_i^z S_j^z \rangle$

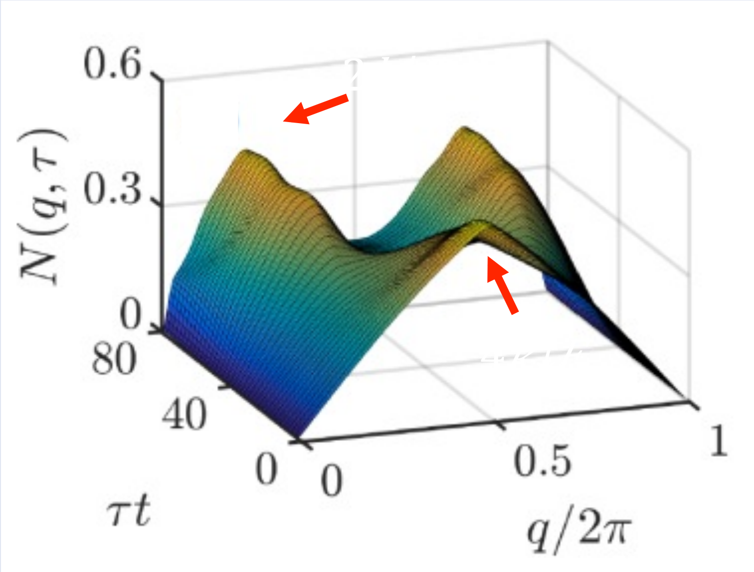
where $S_i^z = (n_{i\uparrow} - n_{i\downarrow})/2$

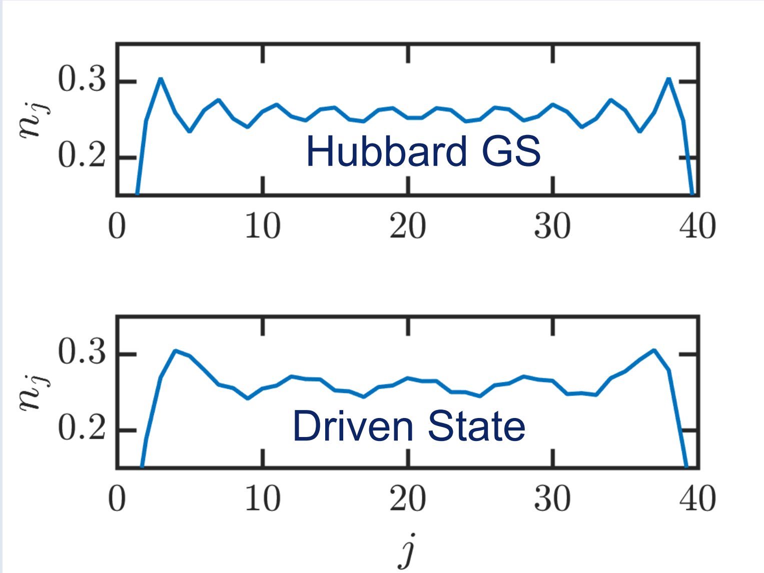
Pair correlations

$P_{ij} = \langle b_{i,i+1}^\dagger b_{j,j+1} \rangle$

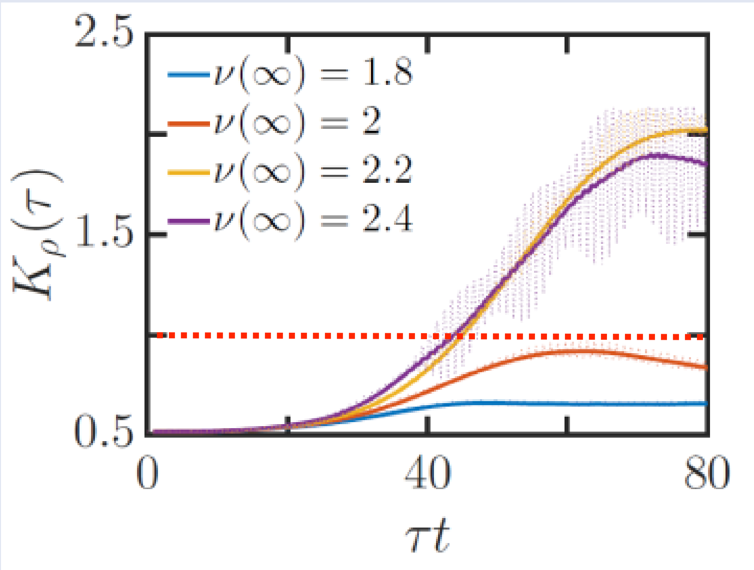
where $b_{ij} = (c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow})/\sqrt{2}$

Density structure factor





$U=20t$
$\Omega=6t$
$L=\infty$
$n=1/2$
$k_F=\pi/4$

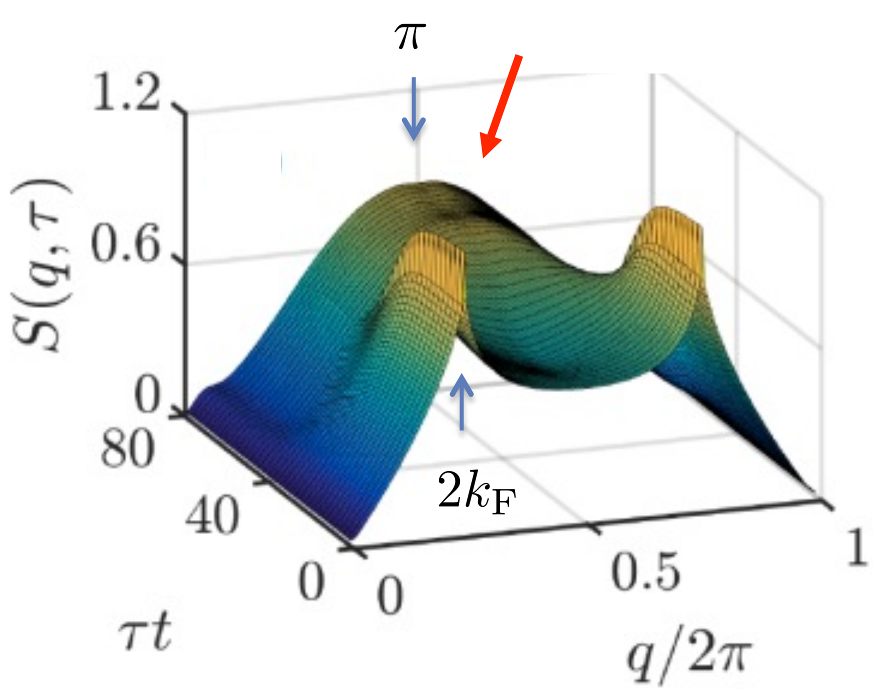


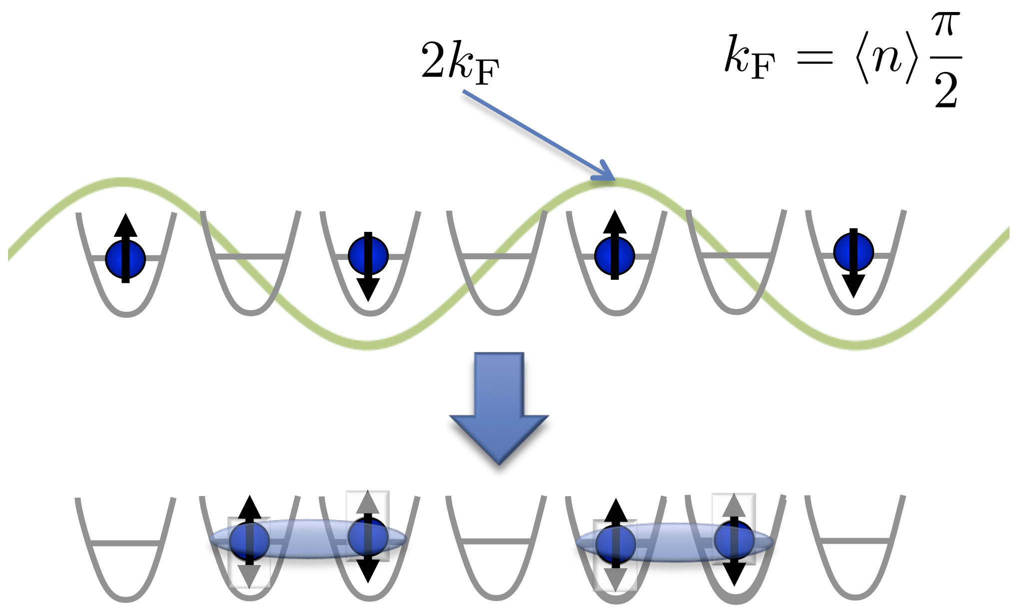
For small quasi-momentum q

$N(q)\approx K_\rho\, q/\pi$

Repulsive Luttinger liquid (M) for $K_\rho<1$
Attractive Luttinger liquid (SC) for $K_\rho>1$

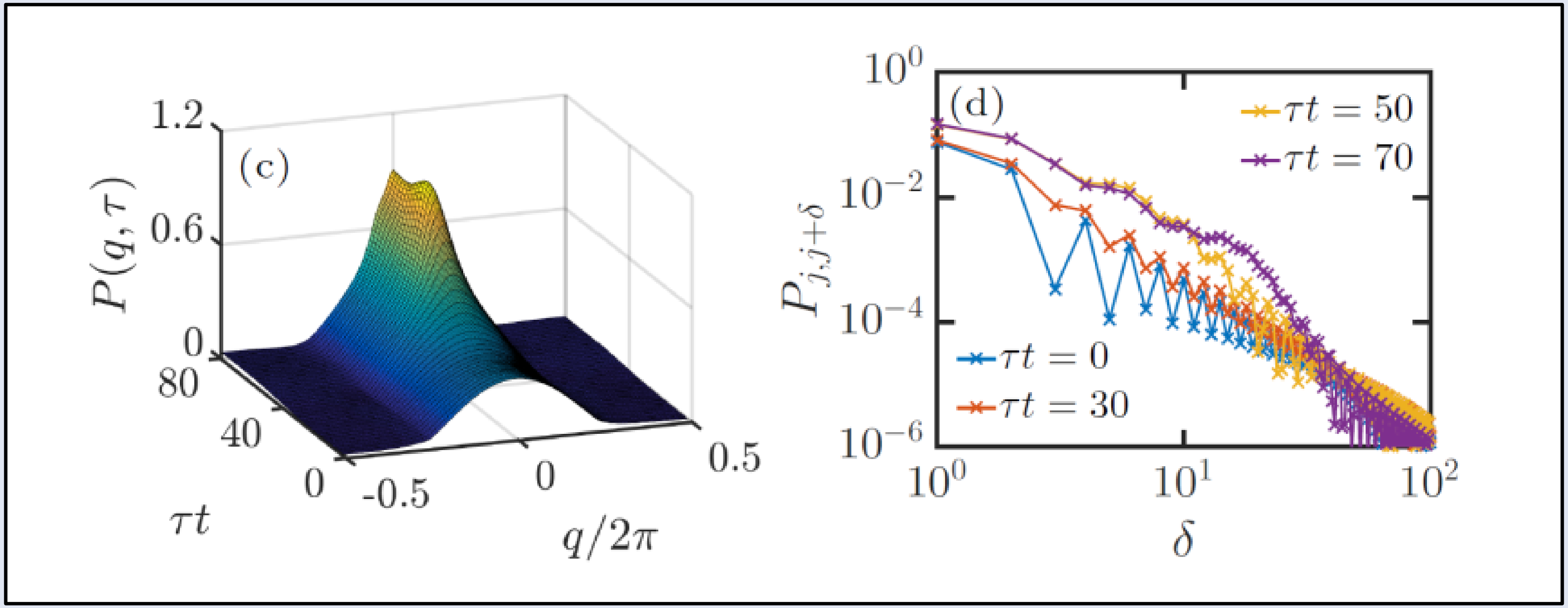
Spin structure factor





The $q=2k_F$ spin wave in the Hubbard ground state gives way to AFM bound pairs.

Pair correlations



Long-range pair correlations are enhanced by the driving

Correlations spread with speed $t/4 \approx J$

Low energy *tJ*-model

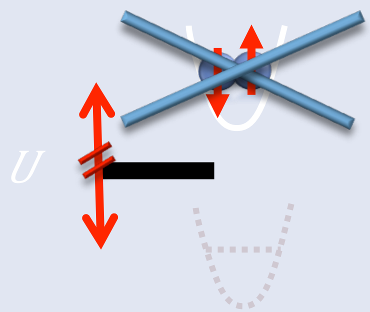
- We project double occupancies out of the Hubbard model in second order to obtain an effective low energy model valid in the limit $t \ll U$

$$H_{tJ} = \mathcal{P}\left[H_{hop} - J\sum_{\langle i,j\rangle} b_{ij}^{\dagger} b_{ij} - \alpha J \sum_{\langle i,j,k\rangle} (b_{ij}^{\dagger} b_{jk} + h.c.)\right]\mathcal{P} = \mathcal{P}\left[H_{hop} + H_{pair}\right]\mathcal{P}$$

Single pa… NN singlet pairs … Singlet pair hopping $\alpha = 1/2$

Project out double occupancies $\mathcal{P} = \prod_j (1 - n_{j\uparrow} n_{j\downarrow})$

The number of NN pairs $M = \sum_{\langle i,j\rangle} b_{ij}^{\dagger} b_{ij}$ is conserved for $t = 0$



J. Coulthard, SR Clark, S. Al-Assam, A. Cavalleri, DJ, Phys. Rev. B **96**, 085104 (2017).

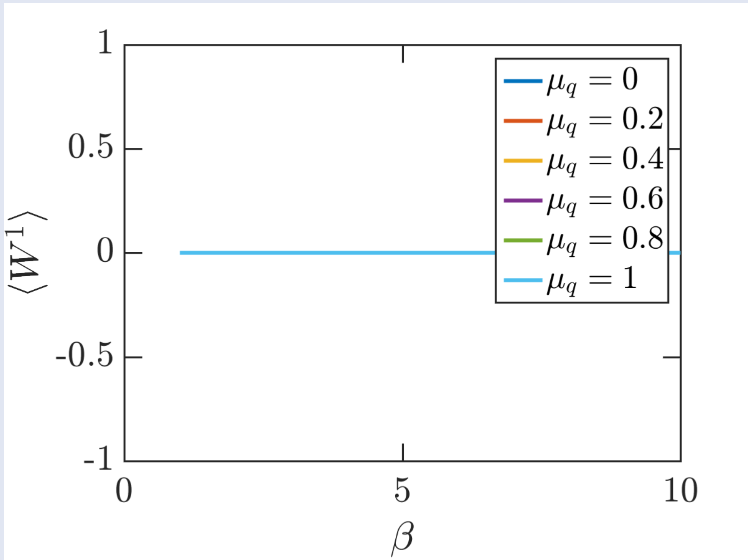
Results (preliminary)

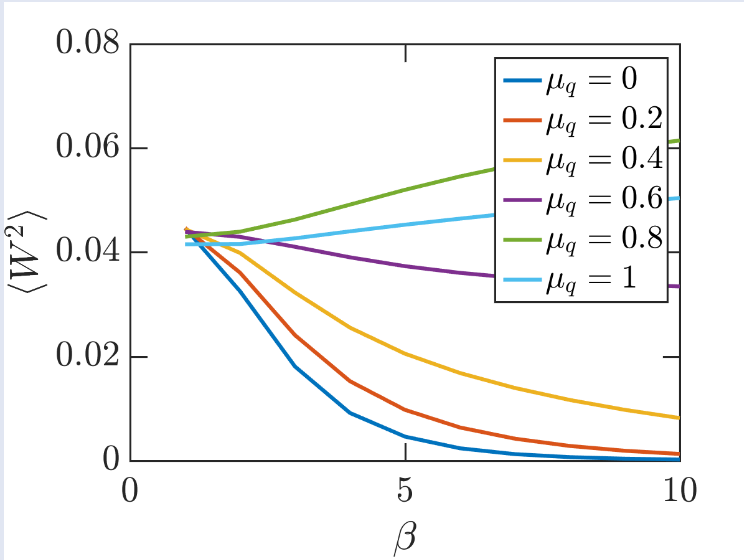
Initial state at time τ=0: $$\rho(\tau=0)=\frac{1}{Z}\exp\left[-\beta\left(H_{0J}-\mu M\right)\right]$$

Process: $t=0\rightarrow 0.1$

Standard work: $$\langle w\rangle=\langle H_{tJ}-H_{0J}\rangle=-t\sum_{j,\sigma}\langle c_{j,\sigma}^{\dagger}c_{j+1,\sigma}+H.c.\rangle\equiv 0$$

$$\langle w^2\rangle=\langle H_{tJ}^2-2H_{tJ}H_{0J}+H_{0J}^2\rangle$$





Calculations: Jonathan Coulthard [L=16, J=1, μ=0.8, χ=100]

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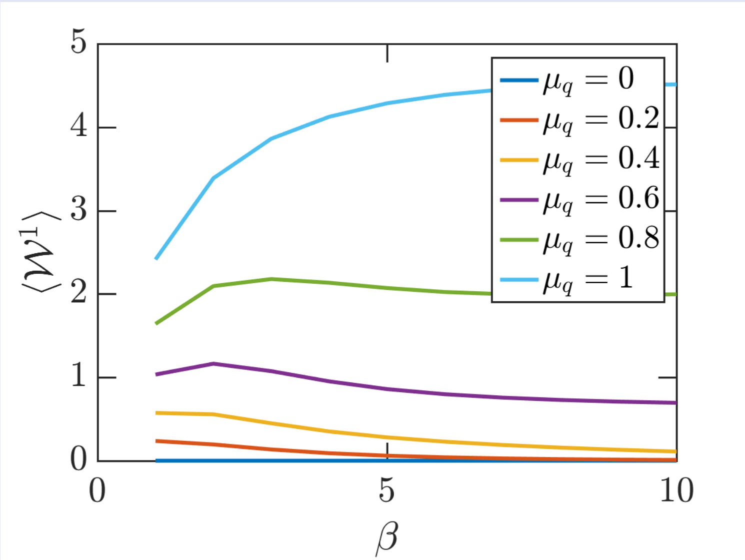

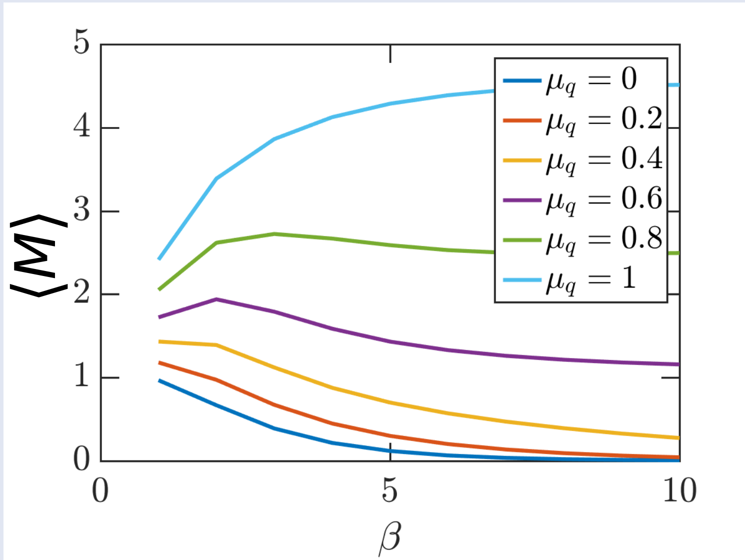

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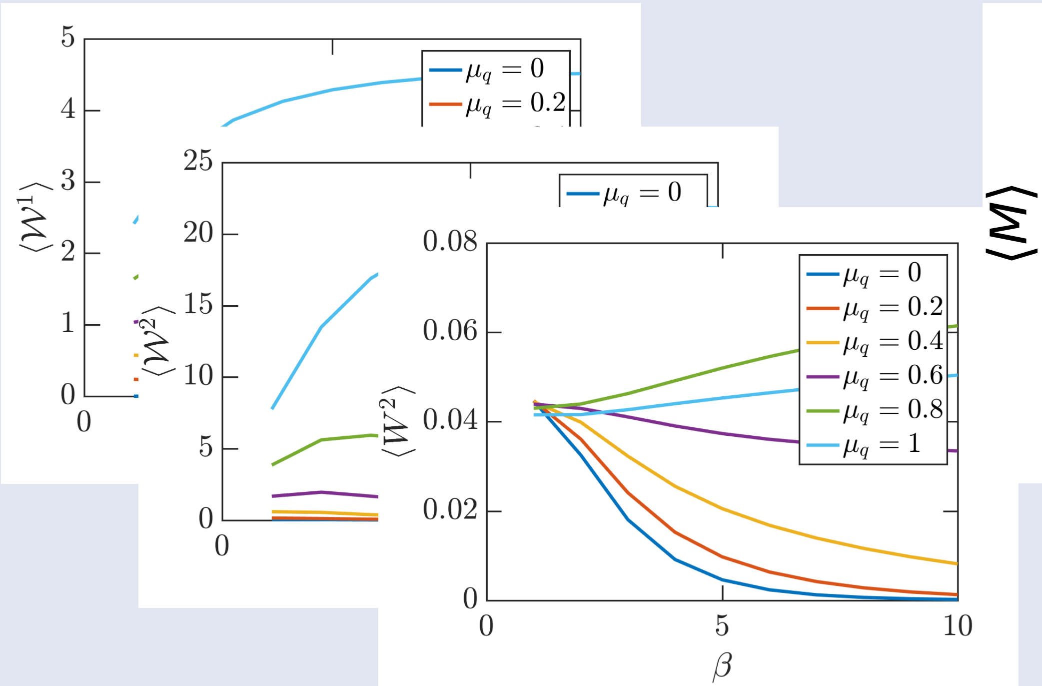
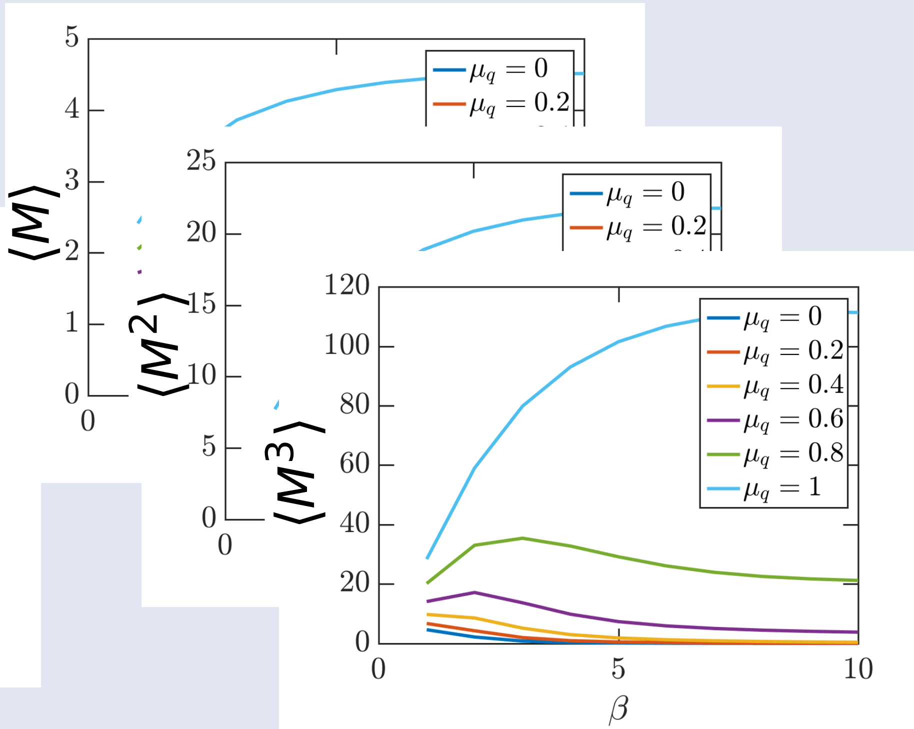
Process: $t=0\to0.1$

Generalized work: $\langle W\rangle=\beta\langle H_{tJ}-(H_{0J}-\mu M)\rangle\equiv\beta\mu\langle M\rangle$

$$\langle W^2\rangle=\beta^2\langle H_{tJ}^2-2H_{tJ}(H_{0J}-\mu M)+(H_{0J}-\mu M)^2\rangle$$





Summary

- Use extended quantum fluctuation relations to
  - Track formation and destruction of charges in quantum quenches
  - Detect pre-thermalized GGE states and parameters
  - Identify non-thermalized (conserved) degrees of freedom
  - Indirectly observe dynamically induced symmetry breaking

- References

  - T.H. Johnson *et al.*, Phys Rev. A **93**, 053619 (2016)
  - J. Coulthard *et al.*, Phys. Rev. B **96**, 085104 (2017)
  - J. Mur Petit, A. Relano, R. Molina, DJ, submitted.

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