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Analog and digital-analog quantum simulation of the Quantum Rabi Model

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Trieste, September 2017


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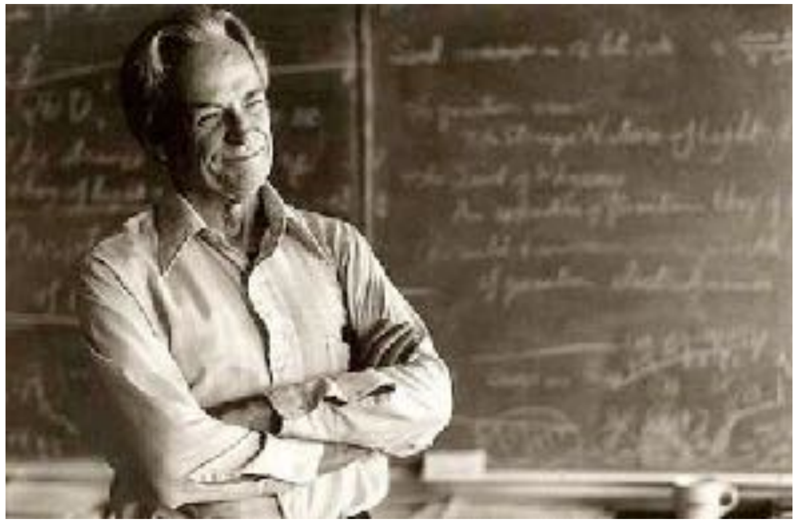
1) Introduction to quantum simulations

What is a quantum simulation?

Definition

Quantum simulation is the intentional reproduction of the quantum aspects of a physical or unphysical model onto a typically more controllable quantum system.

Richard Feynman



Let nature calculate for us

<=>

Greek theatre

Mimesis or imitation is always partial,
this is the origin of creativity in science and arts

Quantum simulation <=> Quantum theatre

Why are quantum simulations relevant?

a) Because we can discover analogies between unconnected fields, producing a flood of knowledge in both directions, e.g. black hole physics and Bose-Einstein condensates.

b) Because we can study phenomena that are difficult to access or even absent in nature, e.g. Dirac equation: *Zitterbewegung* & Klein Paradox, unphysical operations.

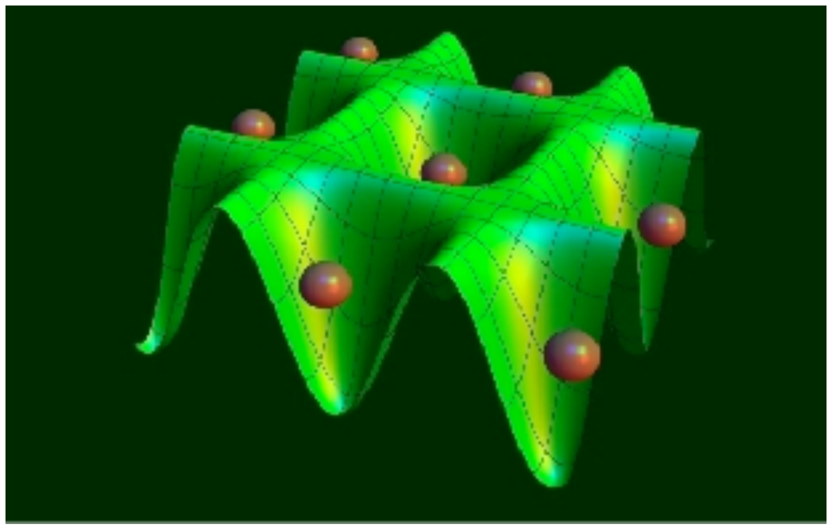
c) Because we can predict novel physics without manipulating the original systems, some experiments may reach quantum supremacy: CM, QChem, QFT, ML, AI & AL.

d) Because we can contribute to the development of novel quantum technologies via scalable quantum simulators and their merge with quantum computing.

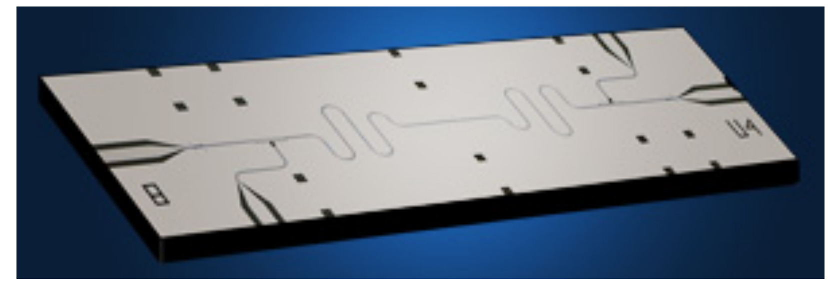
e) Because we are unhappy with reality, we enjoy arts and fiction in all its forms: literature, music, theatre, painting, quantum simulations.

Quantum Platforms for Quantum Simulations

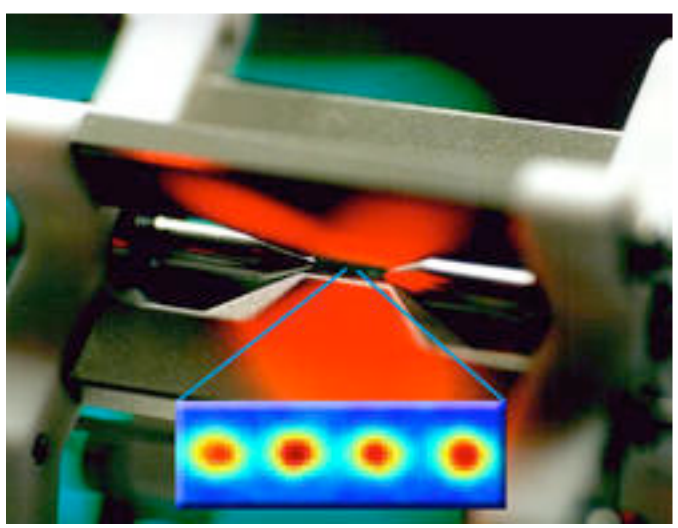
Optical lattices



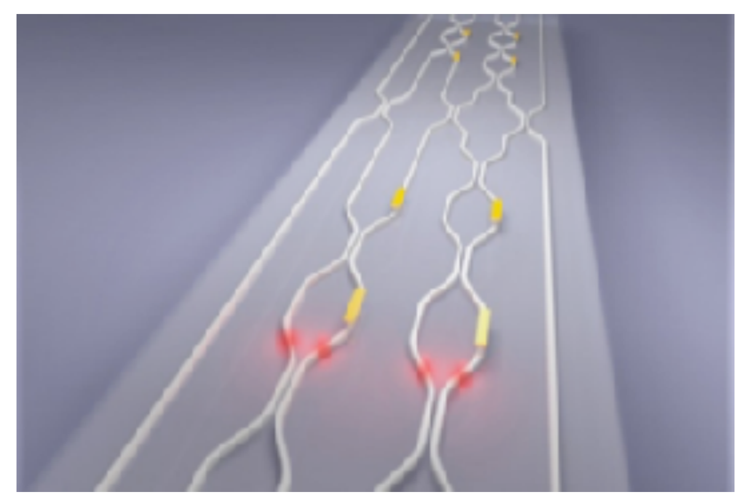
Superconducting circuits



Trapped ions



Quantum photonics



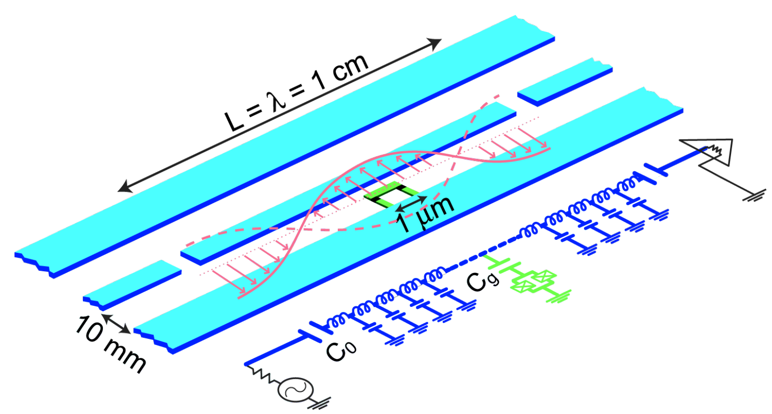
… among several others!

2) The Jaynes-Cummings model in circuit QED and trapped ions

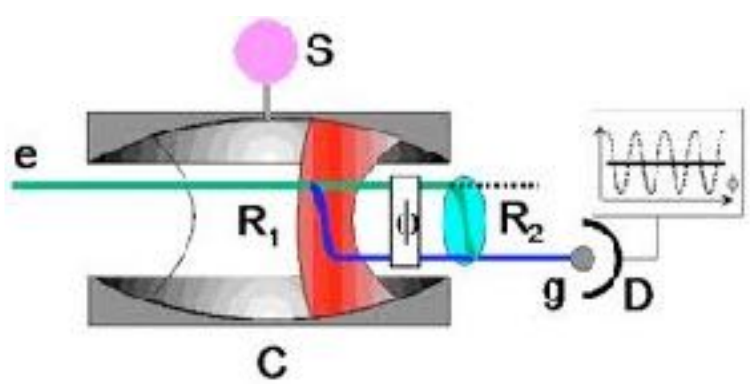
Quantum simulation of the Jaynes-Cummings model in circuit QED

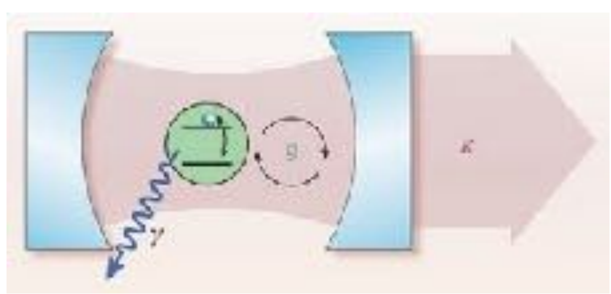
We could also see the JC model in circuit QED as a quantum simulation:
the two-level atom is replaced by a superconducting qubit, called artificial atom.

$$H_{JC} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g\left(\sigma^+ a + \sigma^- a^\dagger\right)$$



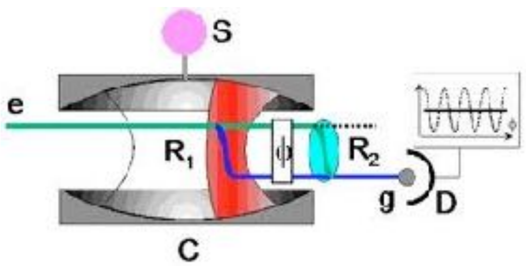
Quantum simulations are never a plain analogy, cQED has advantages in qubit control
as in microwave CQED, but also longitudinal and transversal driving as in optical CQED.



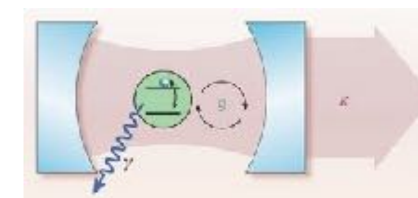


Quantum simulation of the Jaynes-Cummings model in ion traps

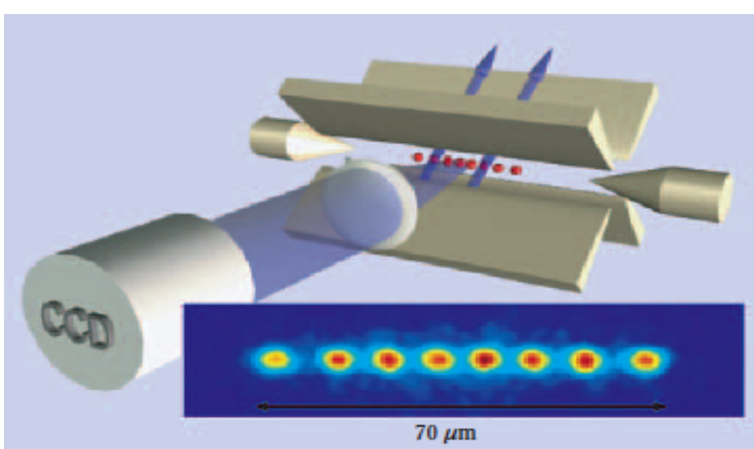
The simplest and most fundamental model describing the coupling between light and matter is the Jaynes-Cummings (JC) model in cavity QED.



$$H_{JC} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^{\dagger}a + \hbar g\left(\sigma^{+}a + \sigma^{-}a^{\dagger}\right)$$



We could consider the implementation of the JC model in trapped ions as (one of) the first nontrivial quantum simulation(s).



$$H_0 = \hbar\nu(a^{\dagger}a + \frac{1}{2})$$

$$H_r = \hbar\eta\tilde{\Omega}_r\left(\sigma^{+}a e^{i\phi_r} + \sigma^{-}a^{\dagger}e^{-i\phi_r}\right)$$

Red sideband excitation of the ion = JC interaction

$$H_b = \hbar\eta\tilde{\Omega}_b\left(\sigma^{+}a^{\dagger}e^{i\phi_b} + \sigma^{-}a e^{-i\phi_b}\right)$$

Blue sideband excitation of the ion = anti-JC interaction

The quantized electromagnetic field is replaced by quantized ion motion

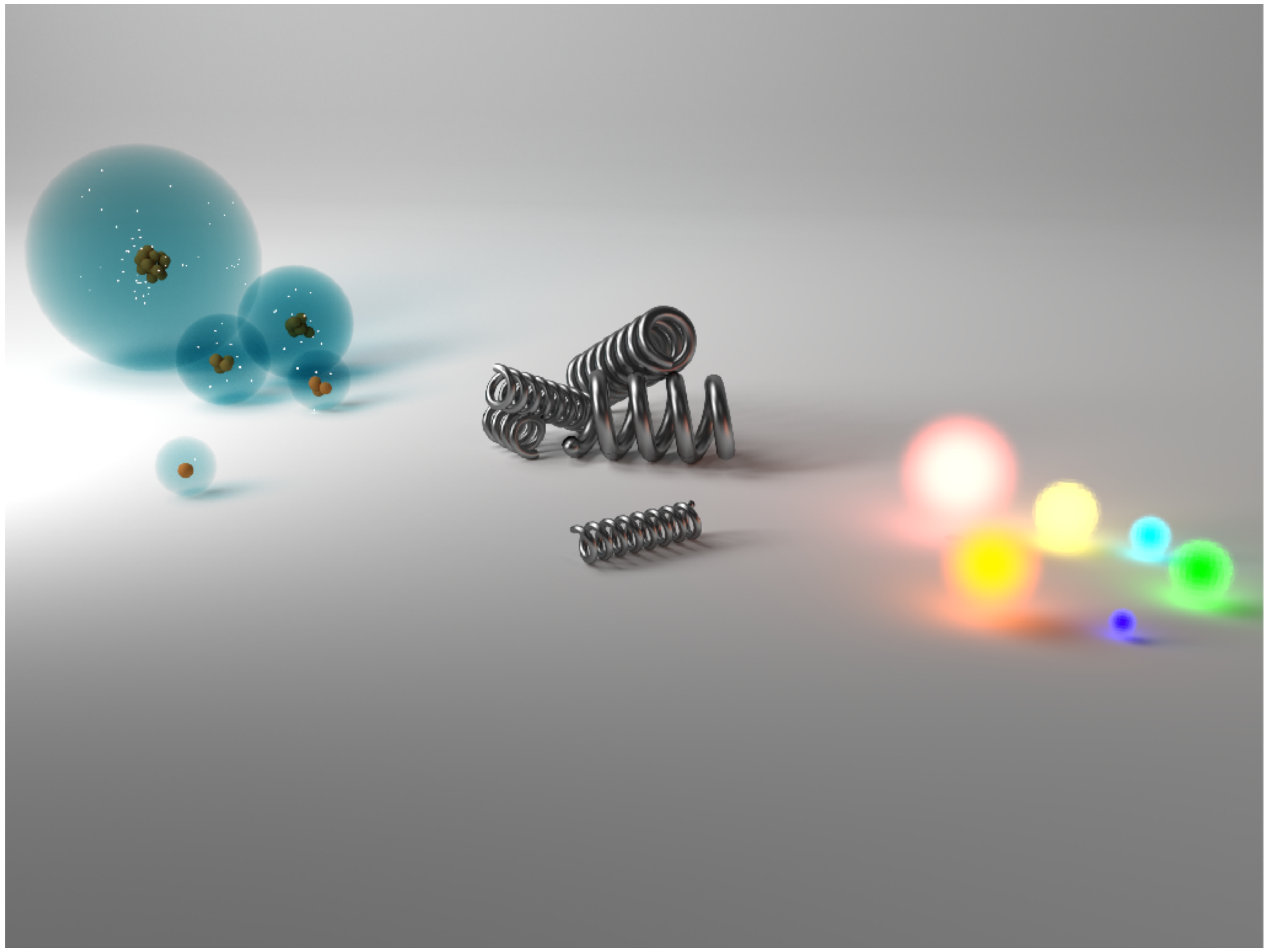
3) Analog quantum simulation of the quantum Rabi model in circuit QED

The quantum Rabi model: USC and DSC regimes

The quantum Rabi model (QRM) describes the dipolar light-matter coupling.
The JC model is the QRM after RWA, it is the SC regime of cavity/circuit QED.

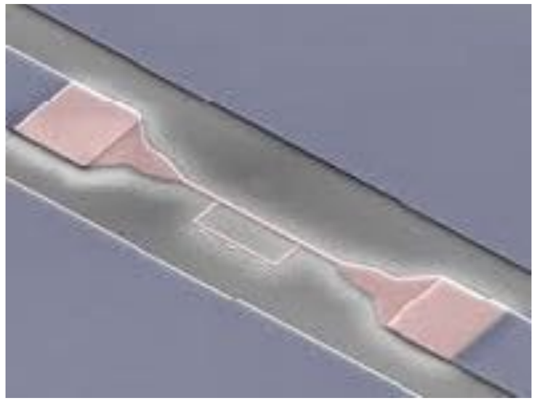
$$H_{Rabi} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g\left(\sigma^+ + \sigma^-\right)\left(a + a^\dagger\right)$$

The QRM is not used for describing usual experiments because the RWA is valid in the microwave and optical regimes in quantum optics, where the JC model is enough.



Ultrastrong coupling regime of the QRM

We have recently seen the advent of the ultrastrong coupling (USC) regime of light-matter interactions in cQED, where $0.1 < g/w < 1$, and RWA is not valid.



T. Niemczyk et al., Nature Phys. **6**, 772 (2010)

P. Forn-Díaz et al., PRL **105**, 237001 (2010)

- Current experimental efforts reach perturbative and nonperturbative USC regimes where $g/w \sim 0.1$-1.0
- The analytical solutions of the QRM were presented: D. Braak, PRL **107**, 100401 (2011).

There are interesting and novel physical phenomena in the USC regime of the QRM:

a) Physics beyond RWA: Bloch-Siegert shifts, entangled ground states, among others.

$$\sigma^\dagger a + \sigma a^\dagger + \sigma^\dagger a^\dagger + \sigma a$$

b) Faster and stronger quantum operations

b.1) Ultrafast quantum gates (CPHASE) that may work at the subnanosecond scale

b.2) New regimes of light-matter coupling: Deep strong coupling (DSC) regime of QRM.

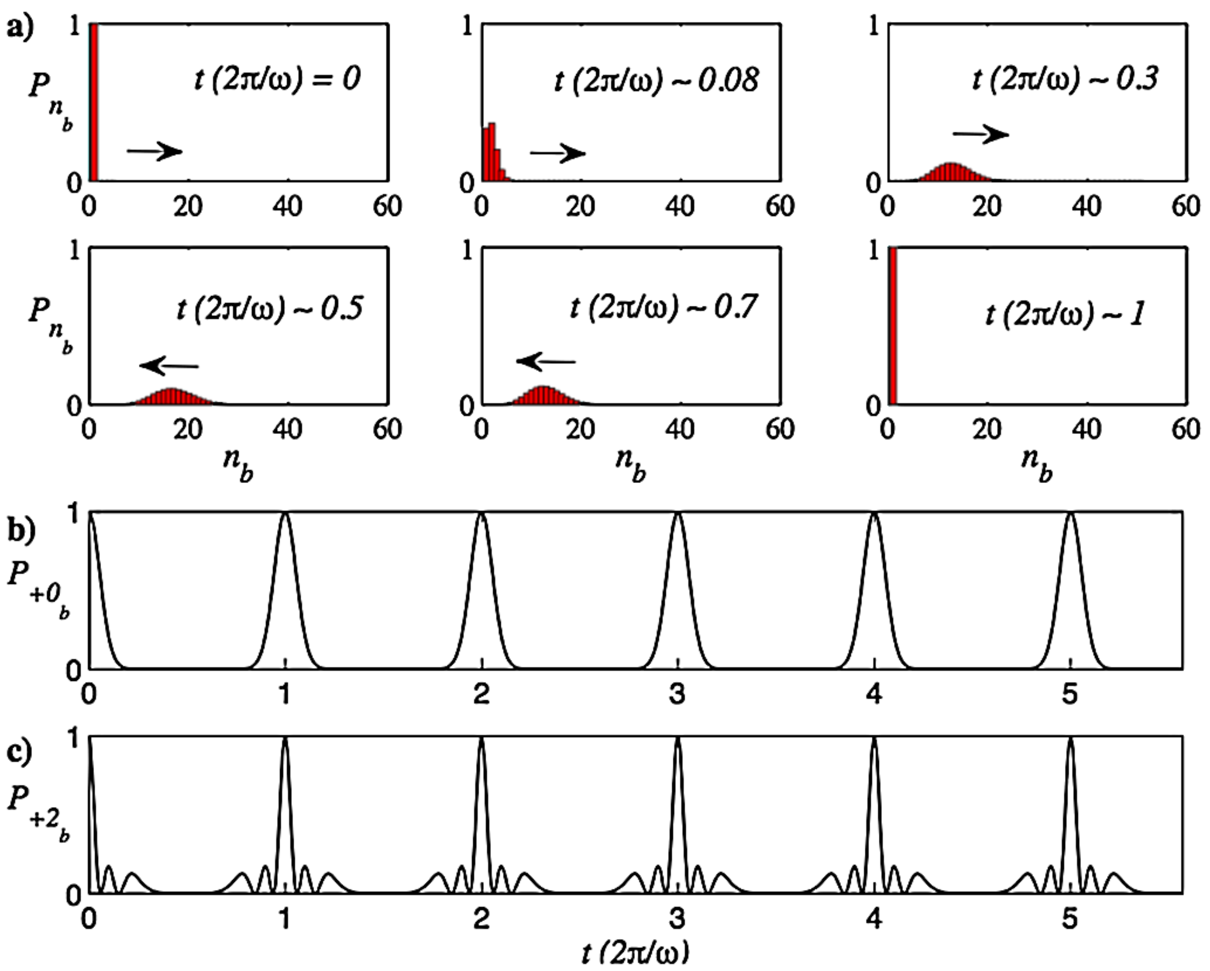
Deep strong coupling regime of the QRM

The DSC regime of the JC model happens when g/w > 1.0, and we can ask whether such a regime could be experimentally reached or ever exist in nature.

$$\Pi = -\sigma_z(-1)^{n_a} = -(|e\rangle\langle e| - |g\rangle\langle g|)(-1)^{a^\dagger a}$$

$$|g0_a\rangle \leftrightarrow |e1_a\rangle \leftrightarrow |g2_a\rangle \leftrightarrow |e3_a\rangle \leftrightarrow \ldots (p = +1)$$
$$|e0_a\rangle \leftrightarrow |g1_a\rangle \leftrightarrow |e2_a\rangle \leftrightarrow |g3_a\rangle \leftrightarrow \ldots (p = -1)$$

Forget about Rabi oscillations or perturbation theory: parity chains and photon number wavepackets define the physics of the DSC regime.



J. Casanova, G. Romero, et al., PRL **105**, 263603 (2010)

Is it possible to cheat technology or nature?

We may reach USC/DSC regimes in the lab but be unable to observe predictions, mainly due to the difficulty in ultrafast on/off coupling switching.

What can we do then? Here, we propose two options:

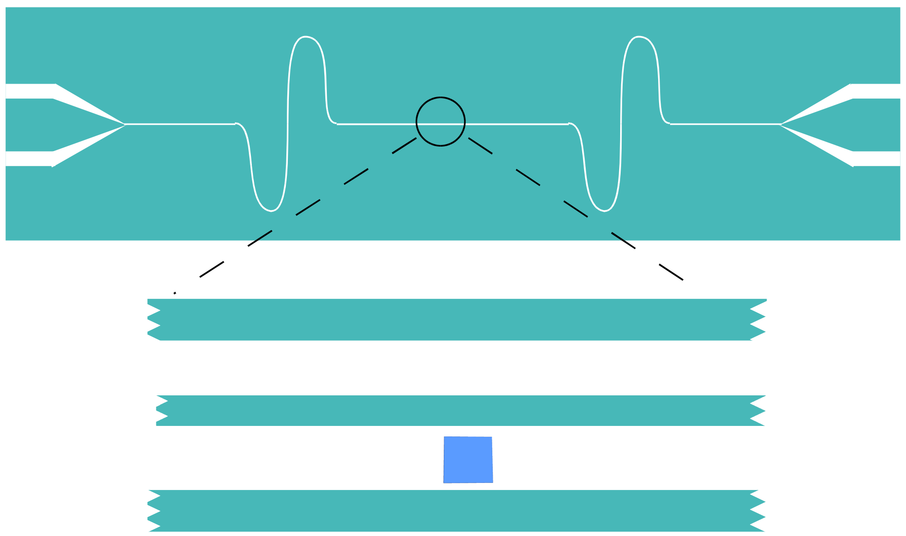
a) We go brute force and try to design ultrafast switching techniques that allow us to design a quantum measurement of relevant observables.

b) We could also reveal these regimes via quantum simulations.

b.1) Recently appeared several experiments realizing the quantum Rabi model and light-matter coupling in USC/DSC regimes

b.2) Is it possible a quantum simulation of the QRM with access to all regimes?

Simulating USC/DSC regimes of the QRM



$$\mathcal{H}_{\rm JC} = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g(\sigma^\dagger a + \sigma a^\dagger)$$

Two-tone microwave driving

$$\mathcal{H}_D = \hbar\Omega_1(e^{i\omega_1 t}\sigma + \text{H.c.}) + \hbar\Omega_2(e^{i\omega_2 t}\sigma + \text{H.c.})$$

Leads to the effective Hamiltonian: QRM in all regimes

$$\mathcal{H} = \hbar(\omega - \omega_1)a^\dagger a + \frac{\hbar\Omega_2}{2}\sigma_z + \frac{\hbar g}{2}\sigma_x(a + a^\dagger)$$

A two-tone driving in cavity QED or circuit QED can turn any JC model into a USC or DSC regime of the QRM model.

D. Ballester, G. Romero, et al., PRX **2**, 021007 (2012)

Quantum simulation of relativistic quantum mechanics

1+1 Dirac equation $$i\hbar\frac{d\psi}{dt} = (c\sigma_x p + mc^2\sigma_z)\psi$$

$$\omega_{\text{eff}} = \omega - \omega_1 = 0 \longrightarrow \mathcal{H}_{\text{D}} = \frac{\hbar\Omega_2}{2}\sigma_z + \frac{\hbar g}{\sqrt{2}}\sigma_x p$$

$$\mathcal{H}_D = \hbar\sum_j \Omega_j(e^{i(\omega_j t+\phi)}\sigma + \text{H.c.}) \qquad \phi = \pi/2$$

Zitterbewegung, via measuring $\langle X\rangle(t)$

R. Gerritsma et al., Nature **463**, 68 (2010)

1+1 Dirac particle + Potential

Add a classical driving to the cavity

$$\mathcal{H} = \mathcal{H}_{JC} + \hbar\sum_{j=1,2}(\Omega_j e^{-i(\omega_j t+\phi_j)}\sigma^\dagger + \text{H.c.}) + \hbar\xi(e^{-i\omega_1 t}a^\dagger + \text{H.c.})$$

$$\mathcal{H}_{\text{eff}} = \frac{\hbar\Omega_2}{2}\sigma_z - \frac{\hbar g}{\sqrt{2}}\sigma_y\hat{p} + \hbar\sqrt{2}\xi\hat{x}$$

Klein paradox

R. Gerritsma et al., PRL **106**, 060503 (2011)

Measuring $\langle X\rangle$ to observe these effects

Quadrature moments have been measured at ETH and WMI:

E. Menzel et al., PRL **105**, 100401(2010); C. Eichler et al., PRL **106**, 220503 (2011)

Experimental AQS of QRM in Karlsruhe group

Simulation scheme

Ballester PRX **2** (2012)

$$\hat{H}/\hbar = \frac{\omega_q}{2}\hat{\sigma}_z + \omega_r \hat{b}^\dagger \hat{b} + g\left(\hat{\sigma}_- \hat{b}^\dagger + \hat{\sigma}_+ \hat{b}\right) + \hat{\sigma}_x \left(\eta_1 \cos\omega_1 t + \eta_2 \cos\omega_2 t\right)$$

transversal microwave drives

- rotating frame with respect to ω_1

$$\hat{H}_1/\hbar = (\omega_q - \omega_1)\frac{\hat{\sigma}_z}{2} + (\omega_r - \omega_1)\hat{b}^\dagger \hat{b} + g\left(\hat{\sigma}_- \hat{b}^\dagger + \hat{\sigma}_+ \hat{b}\right) + \frac{\eta_1}{2}\hat{\sigma}_x + \frac{\eta_2}{2}\left(\hat{\sigma}_+ e^{i(\omega_1 - \omega_2)t} + \hat{\sigma}_- e^{-i(\omega_1 - \omega_2)t}\right)$$

- interaction picture in $\eta_1/2\, \sigma_x$, basis change via Hadamard transformation, constraint: $\omega_1 - \omega_2 = \eta_1$

  → effective Hamiltonian with $\omega_{eff} \equiv \omega_r - \omega_1 \approx MHz$

$$\hat{H}_{eff}/\hbar = \frac{\eta_2}{2}\frac{\hat{\sigma}_z}{2} + \omega_{eff}\hat{b}^\dagger \hat{b} + \frac{g}{2}\sigma_x\left(\hat{b}^\dagger + \hat{b}\right)$$

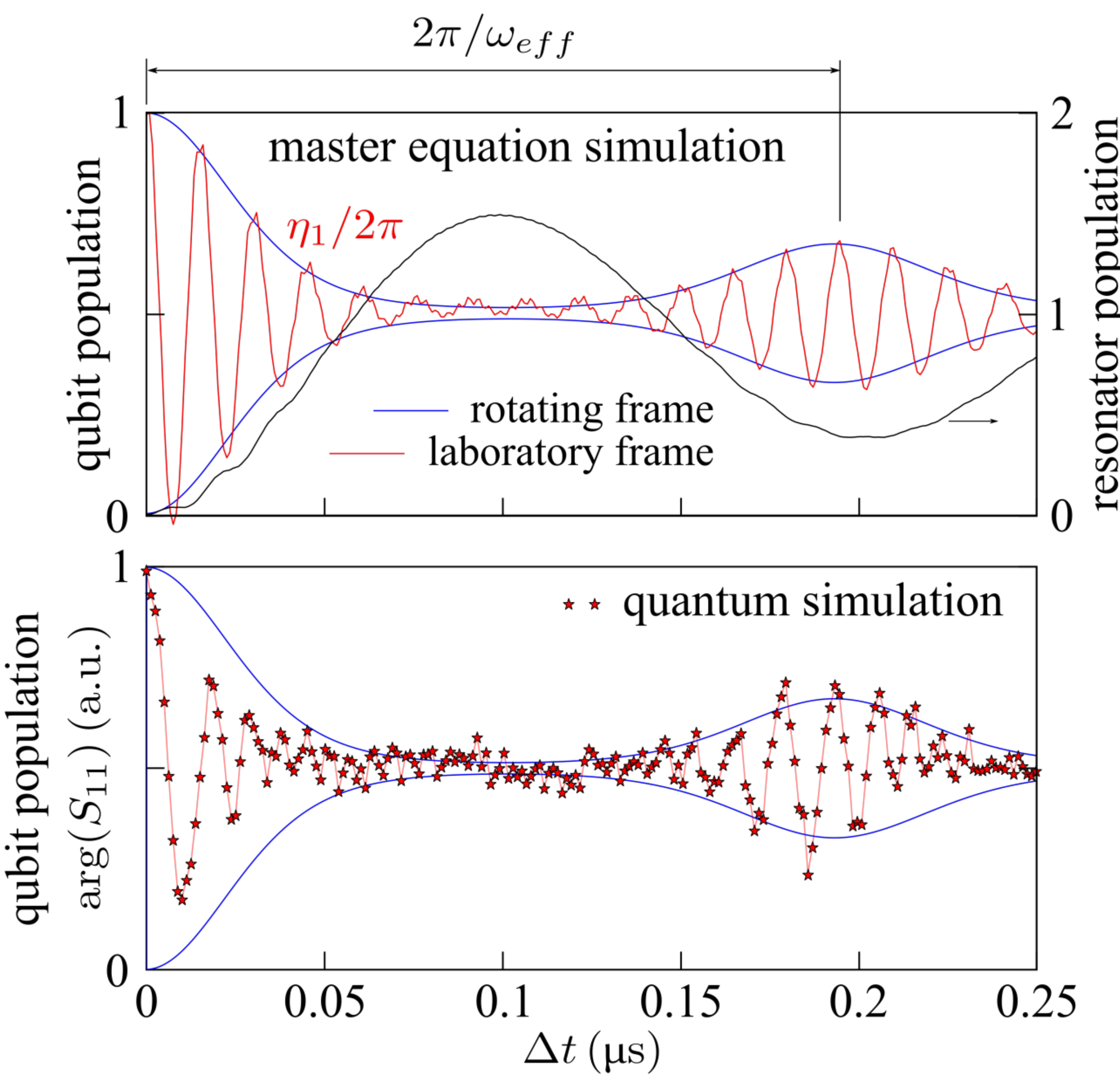
~ MHz ~ MHz 5 MHz

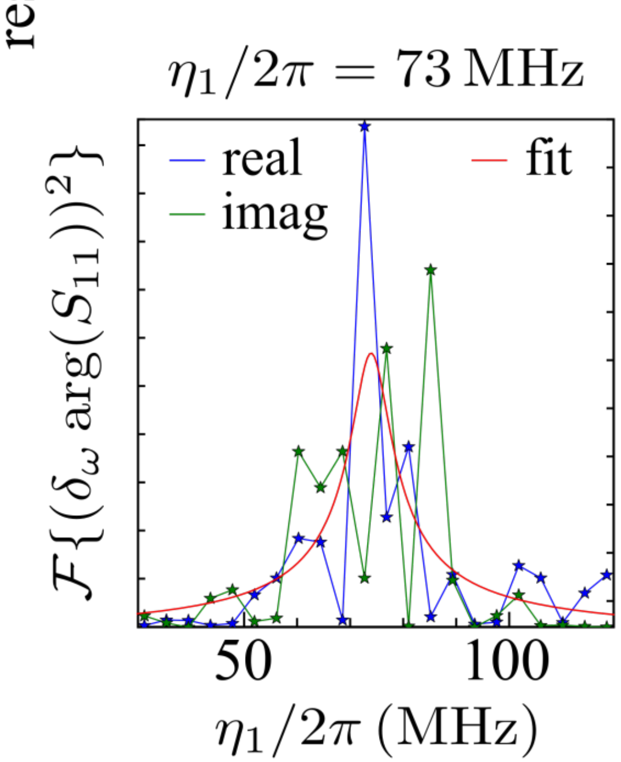
Experimental AQS of QRM in Karlsruhe group

Quantum state collapse and revival

$$\hat{H}_{eff}/\hbar = \omega_{eff}\hat{b}^{\dagger}\hat{b} + \frac{g}{2}\sigma_x\left(\hat{b}^{\dagger} + \hat{b}\right)$$

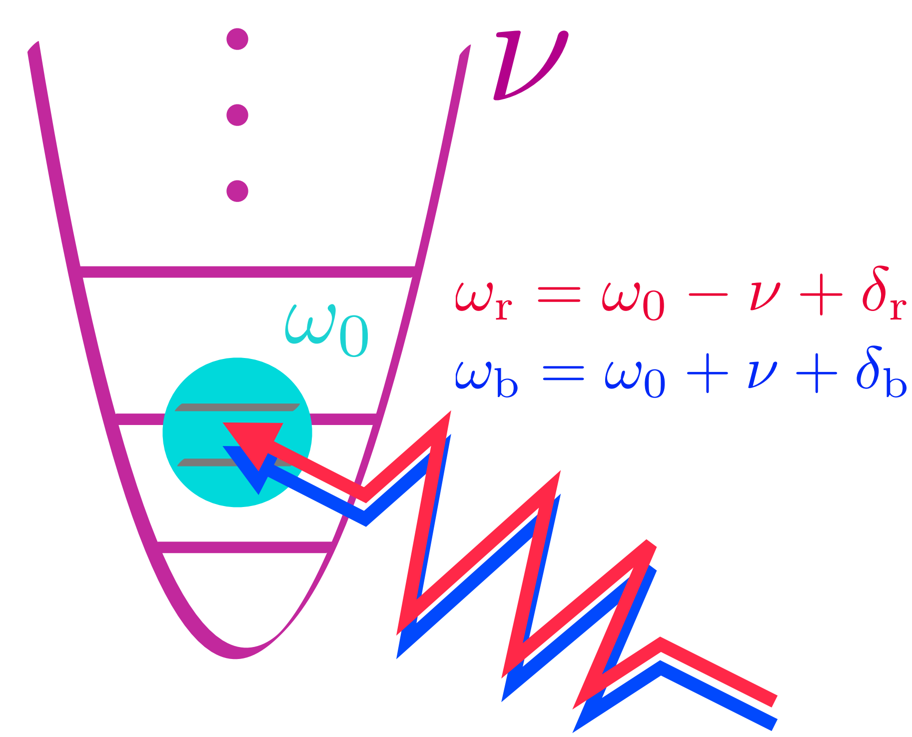
5 MHz 2.5 MHz





$T↓1 = 5\ \mu s$
$Q↓L = 13000$

Analog quantum simulation of QRM in trapped ions



$$H = \hbar \frac{i\eta\Omega}{2}(a\sigma^+ e^{-i\delta_{\mathrm{r}}t} + \mathrm{H.c.}) + \hbar \frac{i\eta\Omega}{2}(a^\dagger\sigma^+ e^{-i\delta_{\mathrm{b}}t} + \mathrm{H.c.})$$

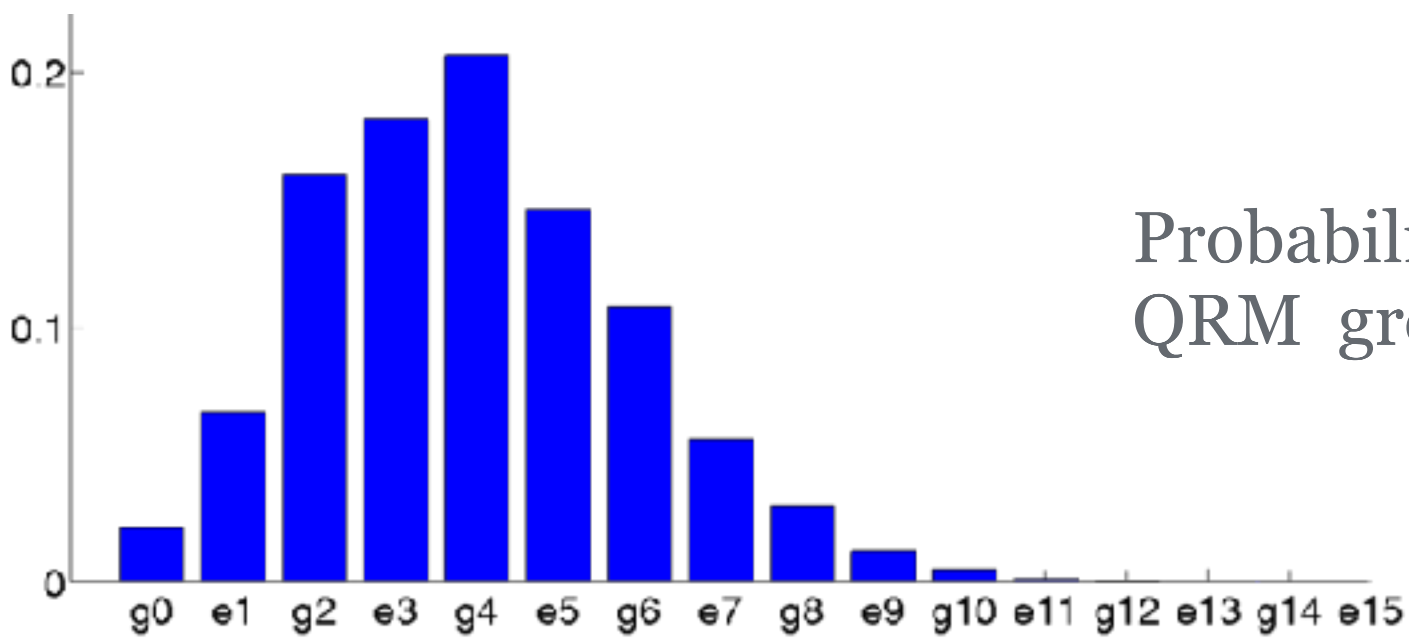
Interaction picture

$$\boxed{H = \hbar \frac{\delta_{\mathrm{r}} - \delta_{\mathrm{b}}}{2} a^\dagger a - \hbar \frac{\delta_{\mathrm{r}} + \delta_{\mathrm{b}}}{4}\sigma_z + \hbar \frac{i\eta\Omega}{2}(a + a^\dagger)(\sigma^+ - \sigma^-)}$$

High tunability $\quad \omega_0^R = -\frac{1}{2}(\delta_r + \delta_b), \ \omega^R = \frac{1}{2}(\delta_r - \delta_b), \ g = \frac{\eta\Omega}{2}$

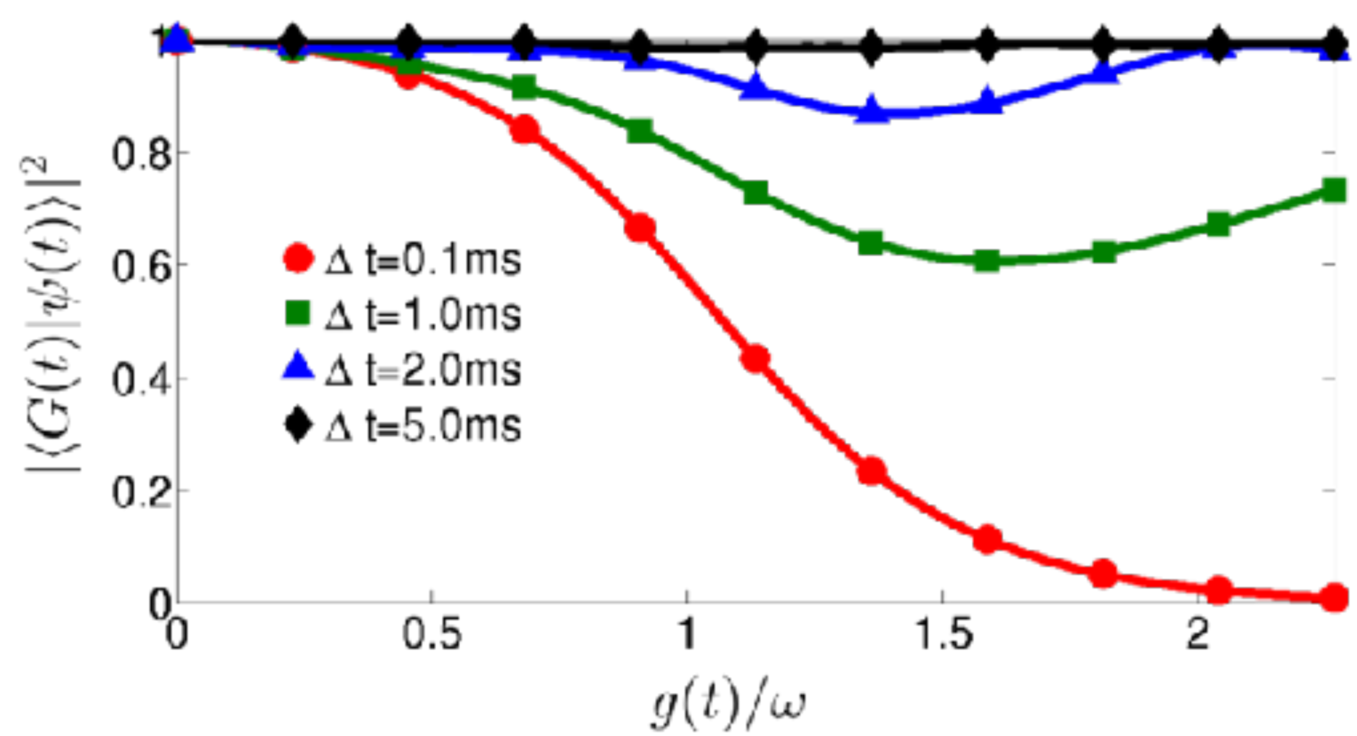
Interaction picture transformation commutes with the observables of interest $\sigma_z, \ a^\dagger a$

J. S. Pedernales et al., Sci. Rep. 5, 15472 (2015)

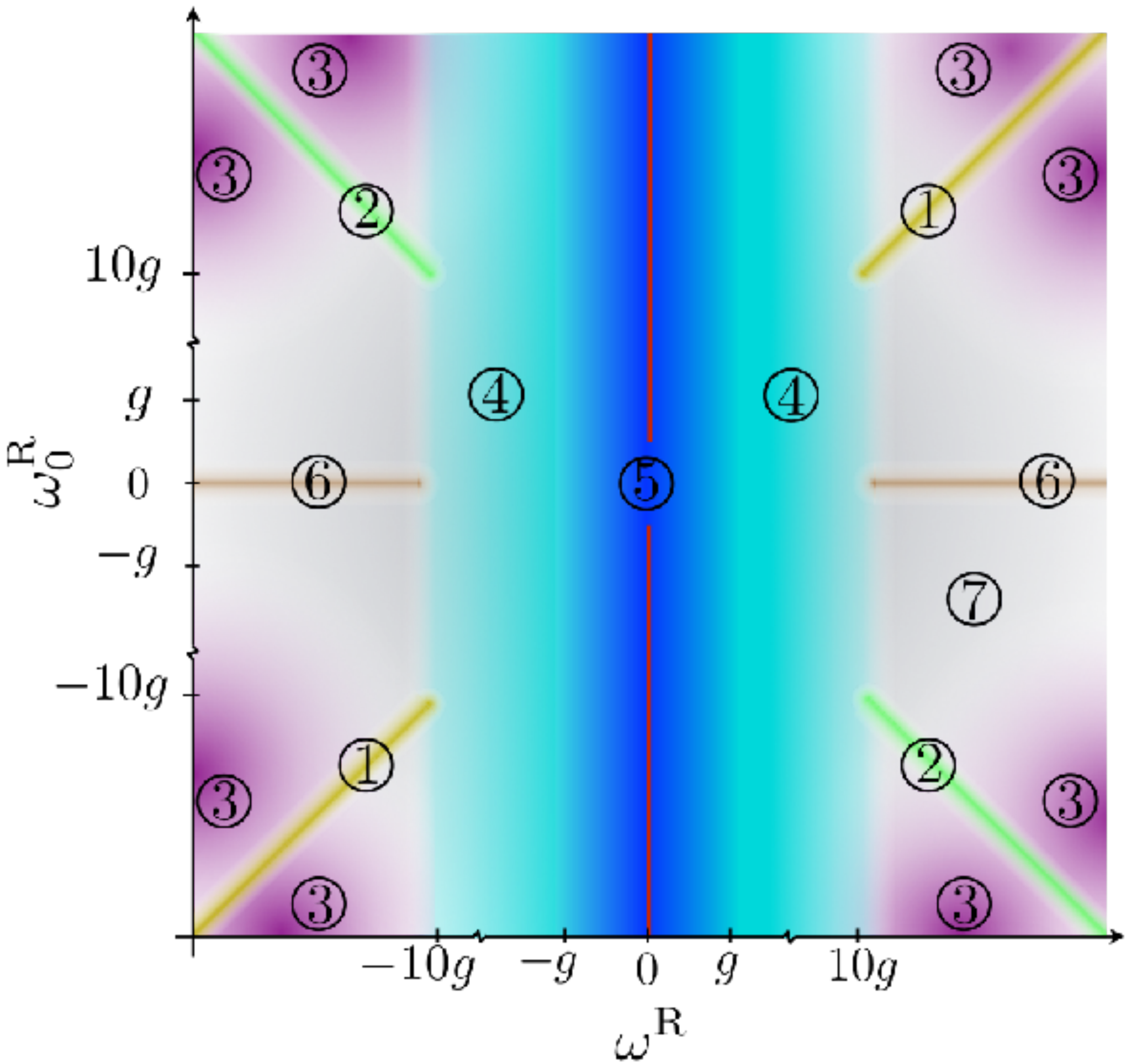


Probability distribution of the QRM ground state for $g/\omega = 2$

Adiabatic generation of entangled ground state of QRM



Coupling regimes of the QRM



1. JC $g \ll |\omega^{\mathrm{R}}|, |\omega_0^{\mathrm{R}}|$
$|\omega^{\mathrm{R}} - \omega_0^{\mathrm{R}}| \ll |\omega^{\mathrm{R}} + \omega_0^{\mathrm{R}}|$

2. AJC $g \ll |\omega^{\mathrm{R}}|, |\omega_0^{\mathrm{R}}|$
$|\omega^{\mathrm{R}} - \omega_0^{\mathrm{R}}| \gg |\omega^{\mathrm{R}} + \omega_0^{\mathrm{R}}|$

3. Dispersive regime
$g < |\omega^{\mathrm{R}}|, |\omega_0^{\mathrm{R}}|, |\omega^{\mathrm{R}} - \omega_0^{\mathrm{R}}|, |\omega^{\mathrm{R}} + \omega_0^{\mathrm{R}}|$

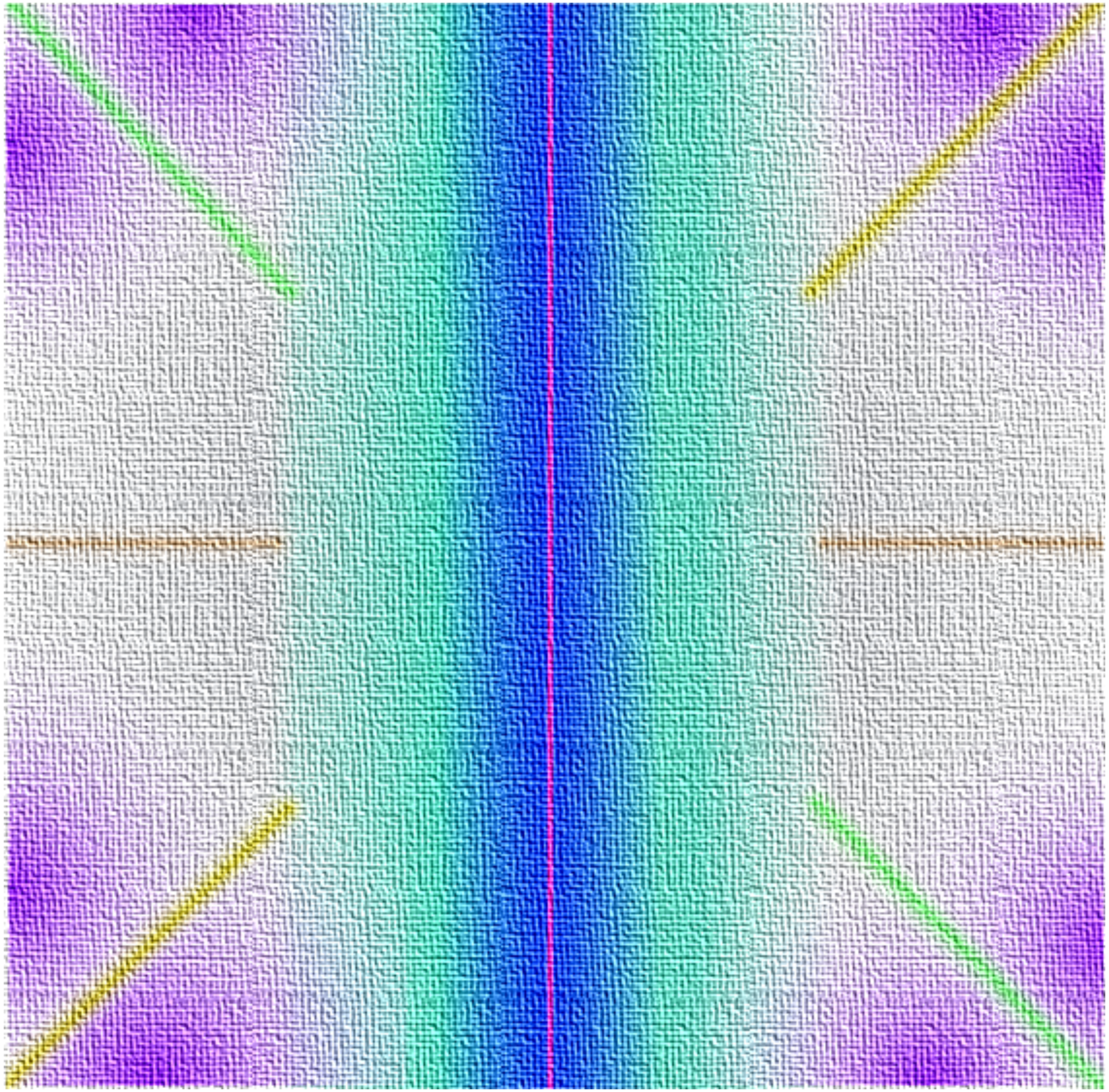
4. USC $g < |\omega^{\mathrm{R}}| < 10g$

5. DSC $|\omega^{\mathrm{R}}| < g$

6. Decoupling regime
$|\omega_0^{\mathrm{R}}| \ll g \ll |\omega^{\mathrm{R}}|$

7. Open to study
$|\omega_0^{\mathrm{R}}| \sim g \ll |\omega^{\mathrm{R}}|$

Dirac equation $\omega^{\mathrm{R}} = 0$



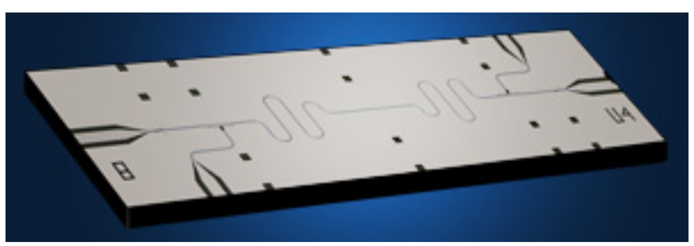
Cover of the special issue on the quantum Rabi model
in Journal of Physics A, 2016-17

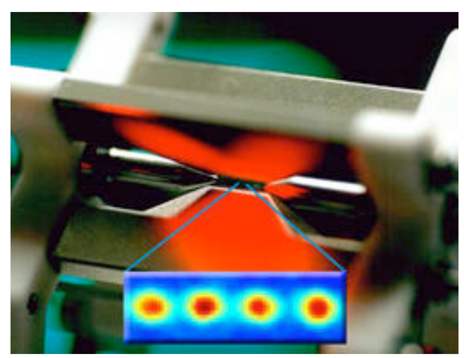
4) Digital-analog quantum simulation of the quantum Rabi model

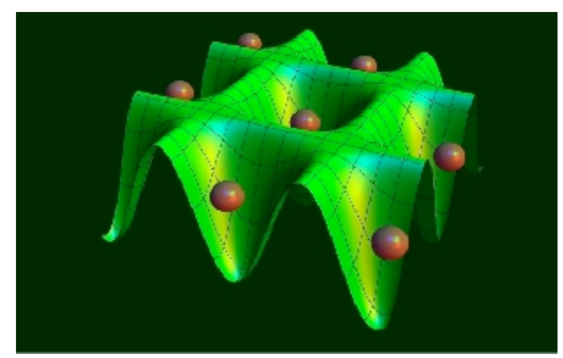
Analog or Digital Quantum Simulations?

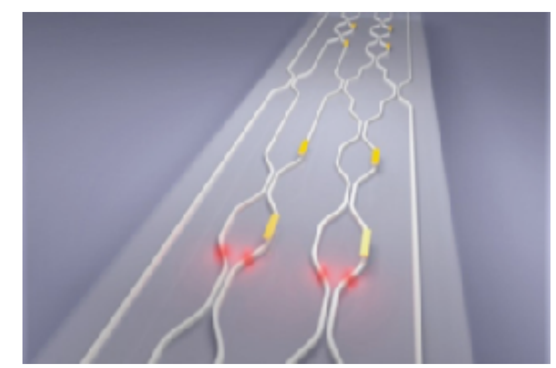
a) Analog quantum simulators (AQS) map qubits onto qubits, bosonic modes onto bosonic modes, involving always-on interactions and accumulating tiny errors that are not easy to correct.

b) Digital quantum simulators (DQS) discretize the time evolution with single/multiqubit gates. They are considered as universal quantum simulators allowing for error correction protocols.









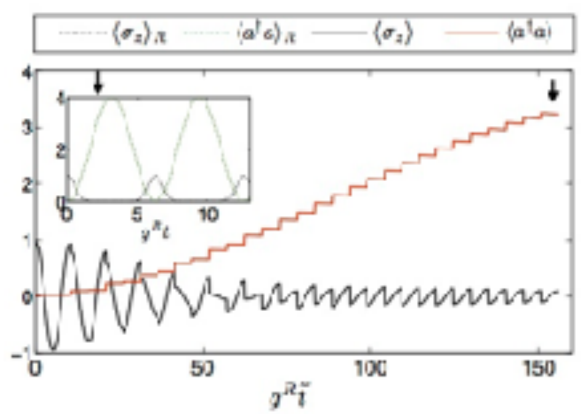
c) We propose to integrate DQS & AQS into **digital-analog quantum simulators (DAQS)** to develop a modular approach of analog blocks combined with digital techniques.

Complexity Simulating Complexity

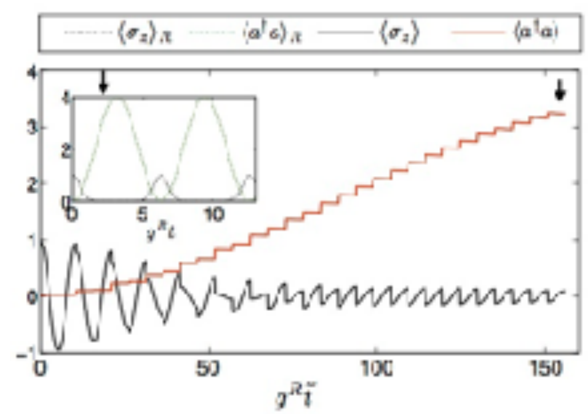
A fist experiment in DAQS for superconducting circuits

Bilbao theory + Delft experiment

Digital quantum Rabi and Dicke models
Mezzacapo et al., Sci. Rep. 2014



Experiment at TU Delft
Langford et al., Nat. Comm. 2017



In DAQS, **analog blocks** are combined sequentially with **digital steps**.

Analog blocks are made of collective quantum gates, that is, in-built complex operations.
Digital steps are local quantum operations that may act also in a global manner.

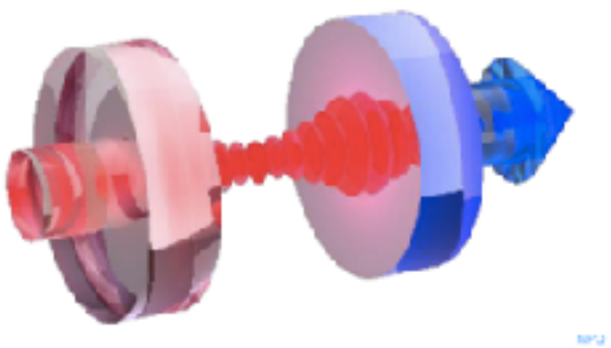
Analog blocks provide the complexity of the simulated model, **digital steps provide flexibility**.
Similar spirit can be followed by introducing digital-adiabatic quantum computers (DAQC).

How DAQS works in superconducting circuits?

Digital-analog quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

Quantum Rabi model: most fundamental light-matter interaction



$$H_R = \omega_r^R a^\dagger a + \frac{\omega_q^R}{2}\sigma^z + g^R\sigma^x(a^\dagger + a)$$

Small coupling as compared to mode & qubit frequencies: Jaynes-Cummings model

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g(a^\dagger\sigma^- + a\sigma^+)$$

Digital-analog quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

Interaction available in cQED: Jaynes-Cummings model

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g(a^\dagger \sigma^- + a\sigma^+)$$

$$\Downarrow$$

Digital decomposition: JC + local rotations

$$H_R = H_1 + H_2,$$

$$H_1 = \frac{\omega_r^R}{2} a^\dagger a + \frac{\omega_q^1}{2}\sigma^z + g(a^\dagger \sigma^- + a\sigma^+),$$

$$H_2 = \frac{\omega_r^R}{2} a^\dagger a - \frac{\omega_q^2}{2}\sigma^z + g(a^\dagger \sigma^+ + a\sigma^-),$$

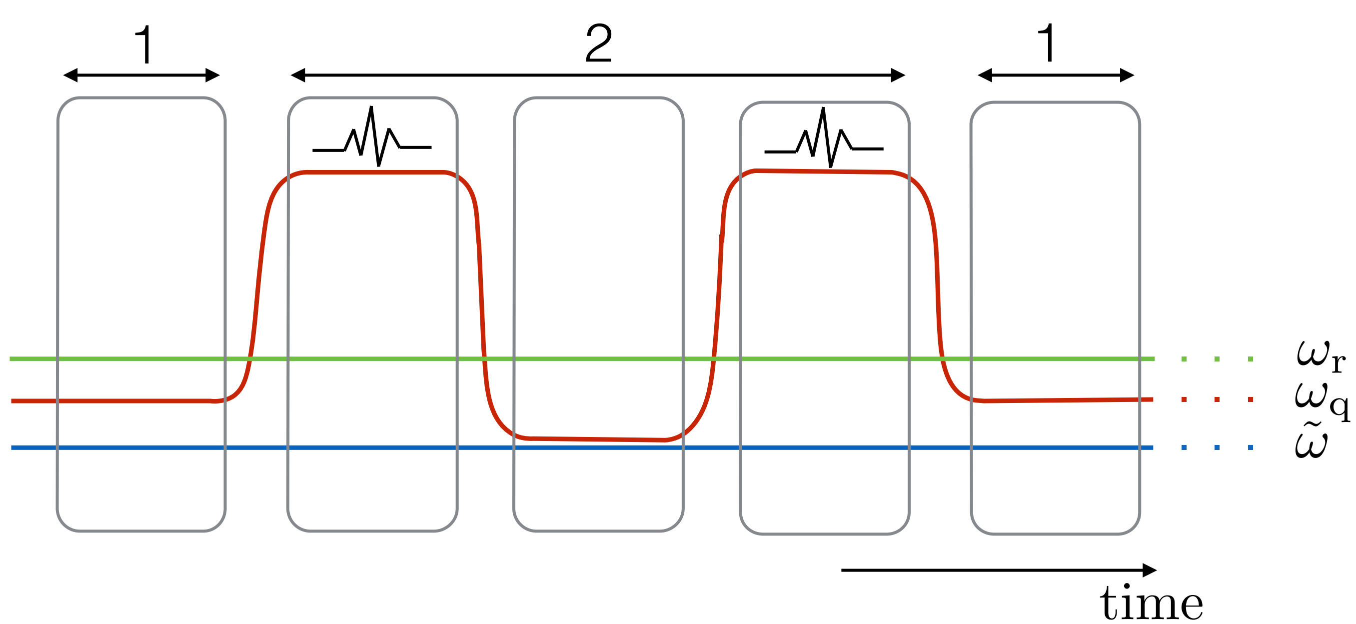
Digital-analog quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

JC in interaction picture $\quad \tilde{H} = \tilde{\Delta}_r a^\dagger a + \tilde{\Delta}_q \sigma^z + g(a^\dagger \sigma^- + a\sigma^+),$

and we get AJC $\quad e^{-i\pi\sigma^x/2} \tilde{H} e^{i\pi\sigma^x/2} = \tilde{\Delta}_r a^\dagger a - \tilde{\Delta}_q \sigma^z + g(a^\dagger \sigma^+ + a\sigma^-).$

$\longrightarrow$ Trotterization

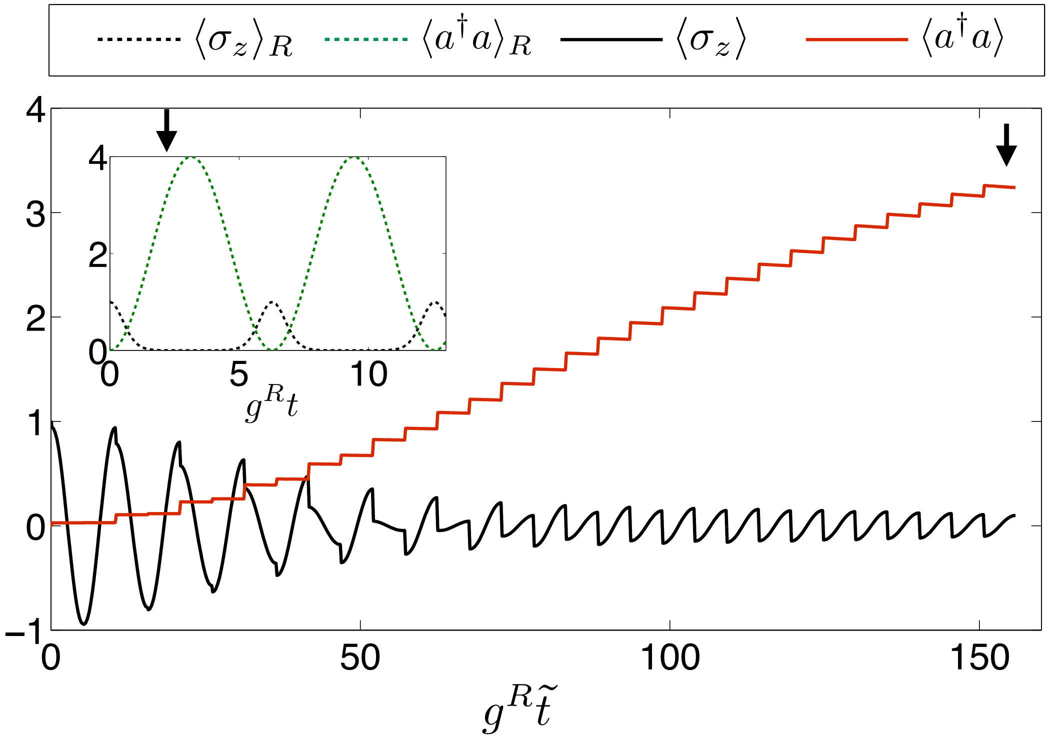


Digital-analog quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

$g^R = \omega_q^R/2 = \omega_r^R/2$	$\tilde{\omega} = 7.4$ GHz, $\omega_q^1 - \omega_q^2 = 200$ MHz
$g^R = \omega_q^R = \omega_r^R$	$\tilde{\omega} = 7.45$ GHz, $\omega_q^1 - \omega_q^2 = 100$ MHz
$g^R = 2\omega_q^R = \omega_r^R$	$\tilde{\omega} = 7.475$ GHz, $\omega_q^1 - \omega_q^2 = 100$ MHz

Some parameters…



$$\omega_q^R = 0, \text{ and } g^R = \omega_r^R.$$

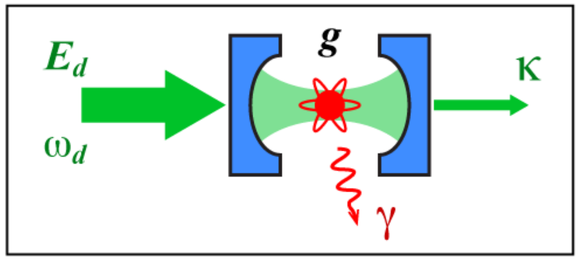
USC & DSC regimes are simulated. Move now towards to the Dicke model!

Experimental DQS of the quantum Rabi model: Delft

N K Langford *et al.*, in preparation (2016)

Trotter decomposition

Rabi, Phys Rev (1936)
Mezzacapo *et al.*, Sci Rep (2014)



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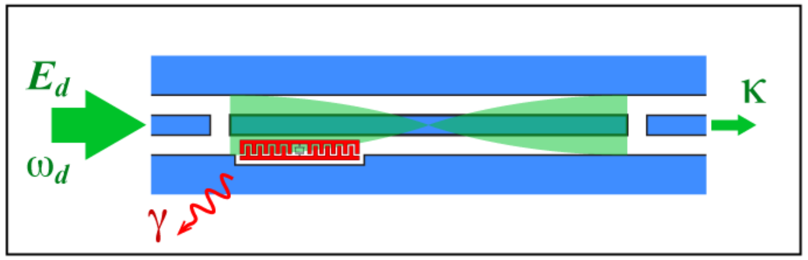
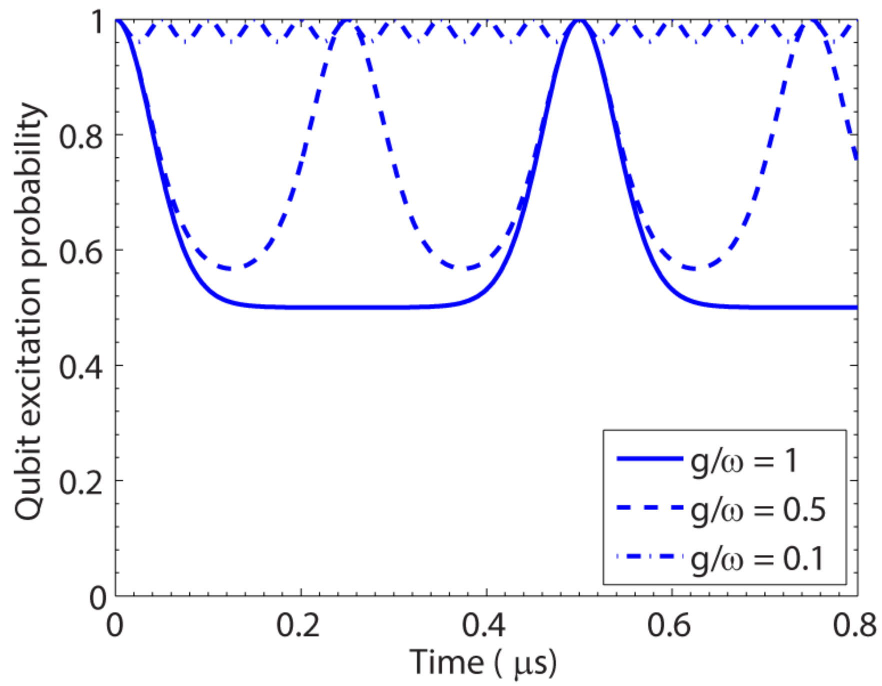


$$H = \hbar\omega_r \, a^\dagger a + \tfrac{1}{2}\hbar\omega_q \, \sigma_z + \hbar g \left(\sigma_+ + \sigma_-\right)\left(a + a^\dagger\right) = H_{\mathrm{JC}} + H_{\mathrm{AJC}}$$

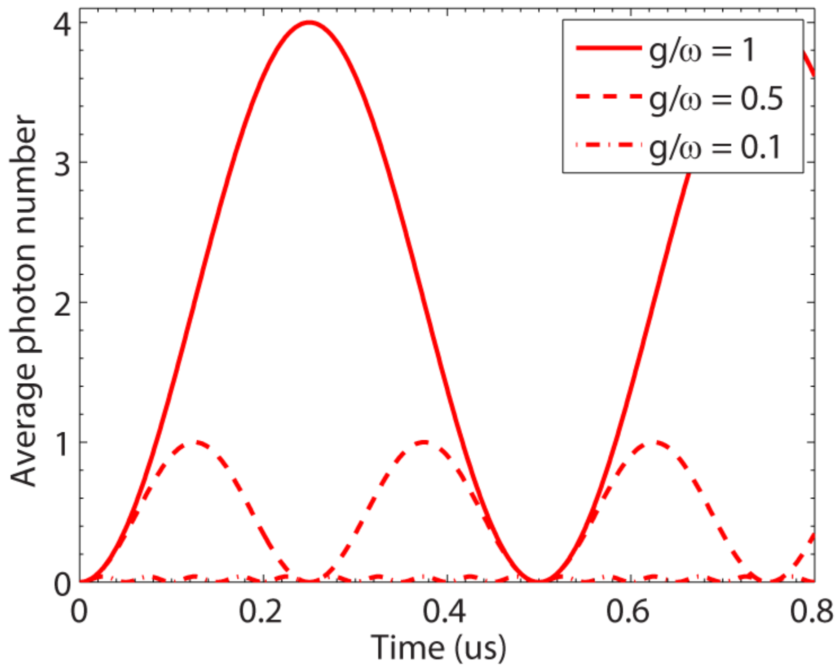
$$H_{\mathrm{JC}} = \hbar\Delta_r \, a^\dagger a + \tfrac{1}{2}\hbar\Delta_q \, \sigma_z + \hbar g \left(a\sigma_+ + a^\dagger\sigma_-\right)$$

$$H_{\mathrm{AJC}} = X \, H_{JC} \, X$$

Expected dynamics for g = 2 MHz



qubit



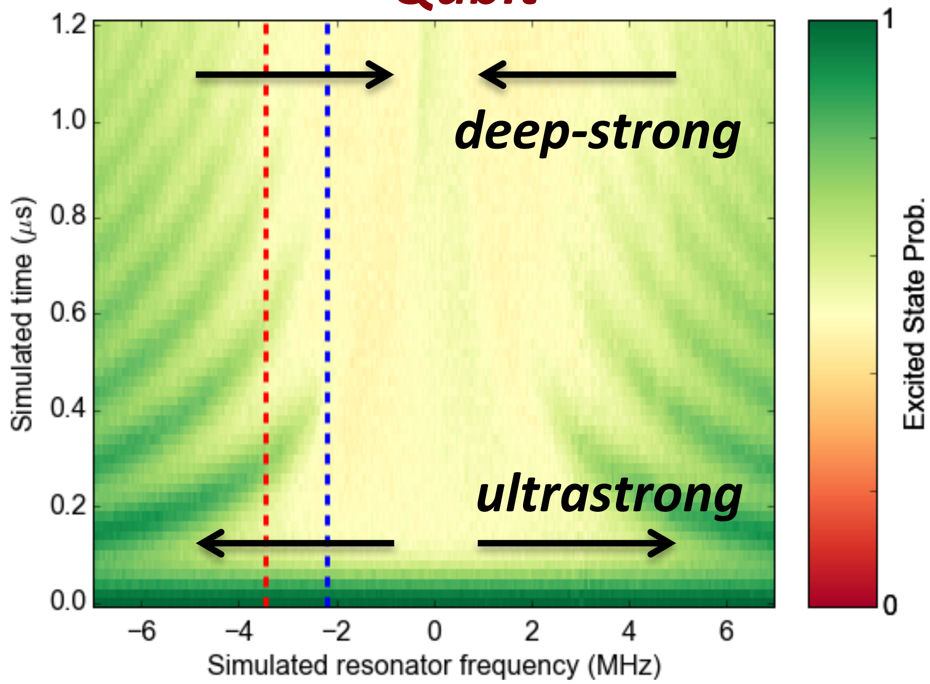
resonator

DQS of the QRM with transmons: Delft

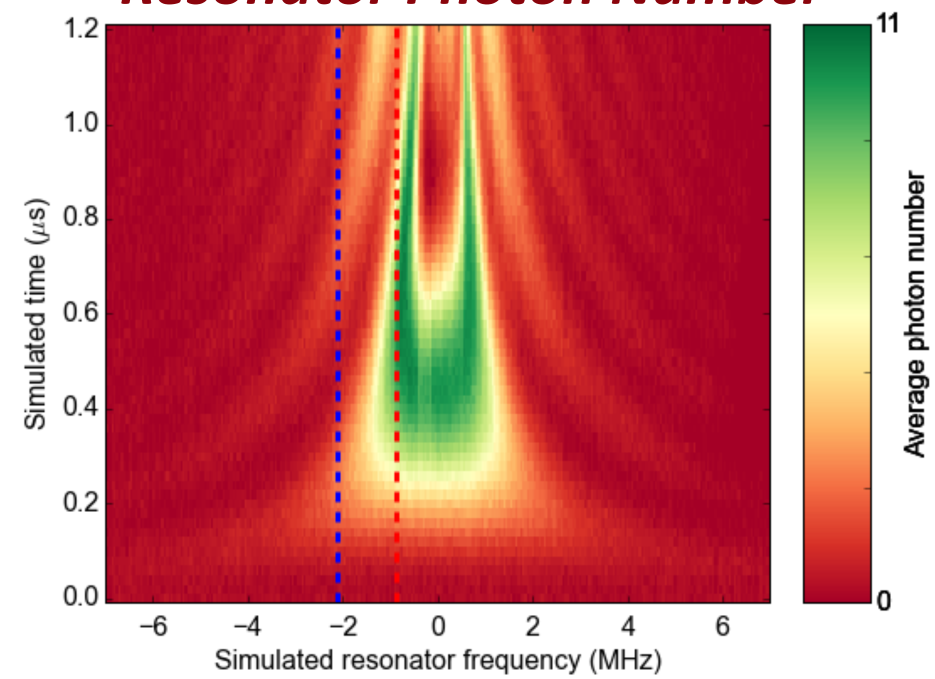
N K Langford *et al.*, in preparation (2016)

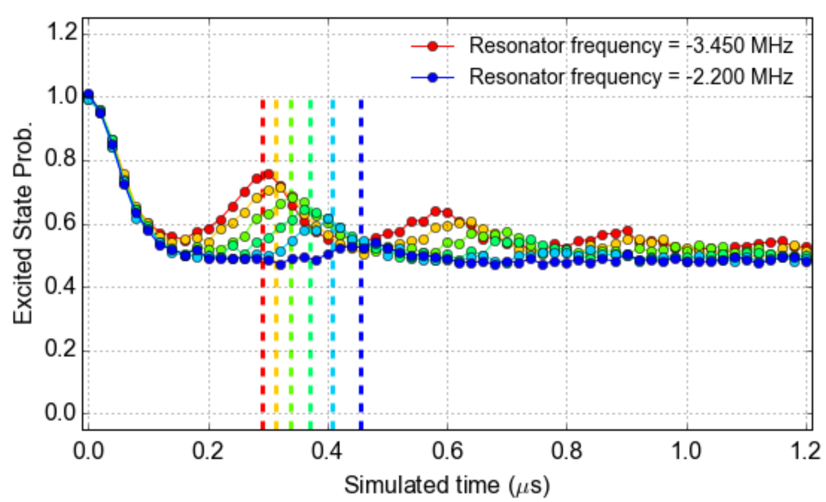
- For g ~ 1.95 MHz, g/ω = 1 gives expected qubit revival at 0.51 microseconds
- Qubit revivals beyond 0.4 us, photon number oscillations beyond 1.1 us (g/ω > 2)

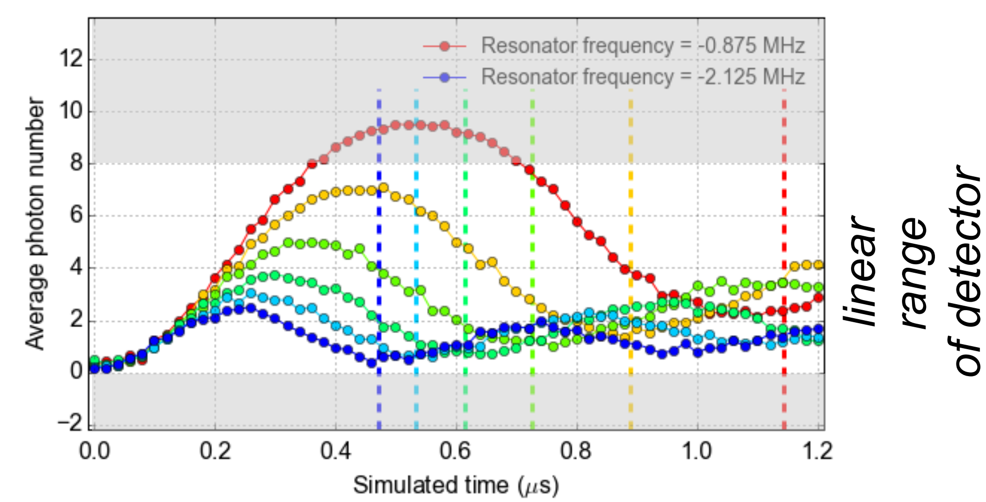
Qubit



Resonator Photon Number



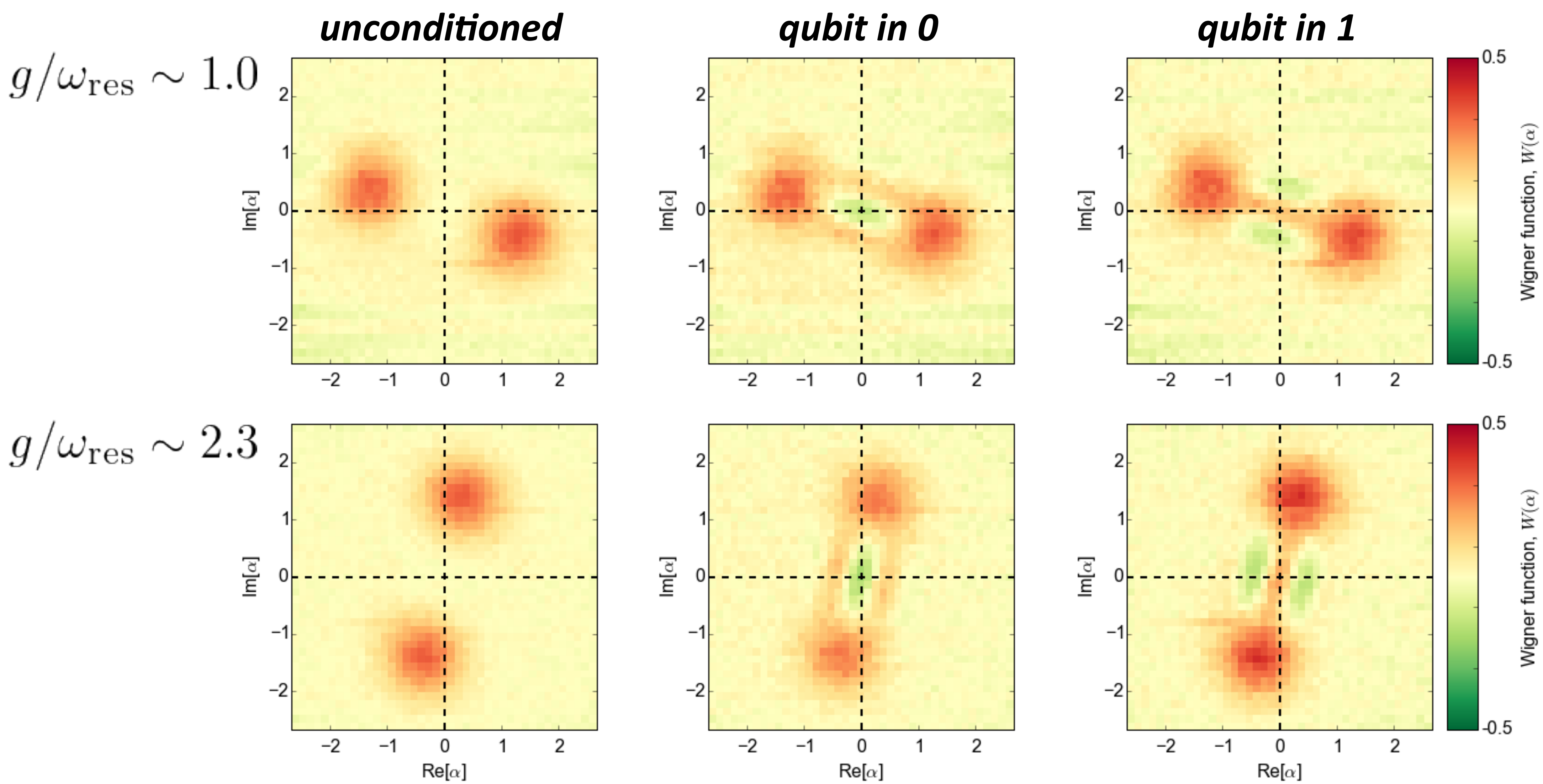




"Schrödinger cats" in DSC regime of QRM: Delft

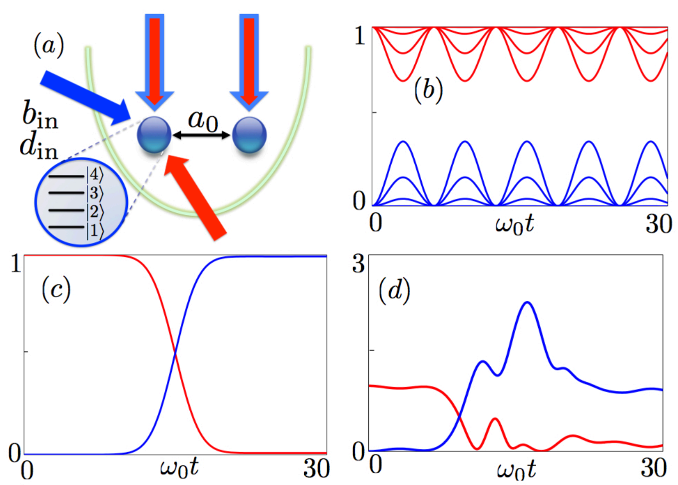
N K Langford *et al.*, in preparation (2016)

- DSC regime leads to "Schroedinger cat"-like entanglement between qubit & resonator
- Witnessed by observing negativity in both conditional cavity Wigner functions (state conditioned on measuring the qubit in 0 or 1) – smoking gun for deep-strong coupling



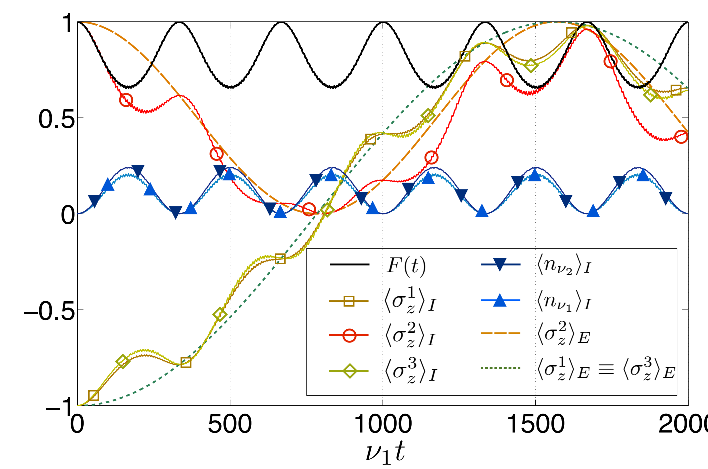
Further works involving DAQS concepts

Quantum Field Theory models
Casanova et al., PRL 2011



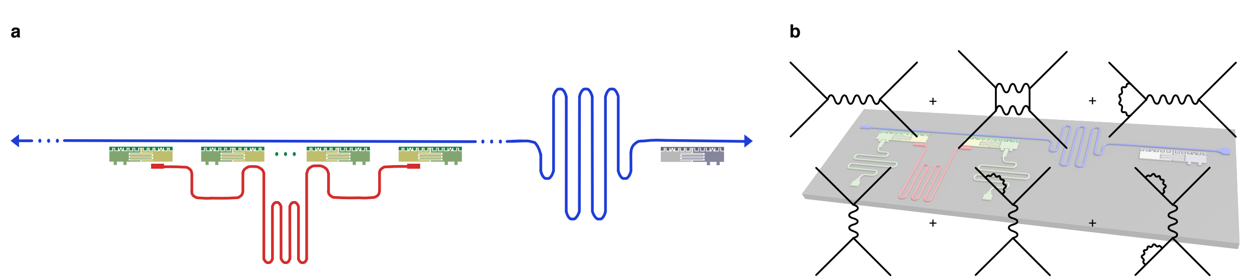
Trapped ions

Holstein Models
Mezzacapo et al., PRL 2012



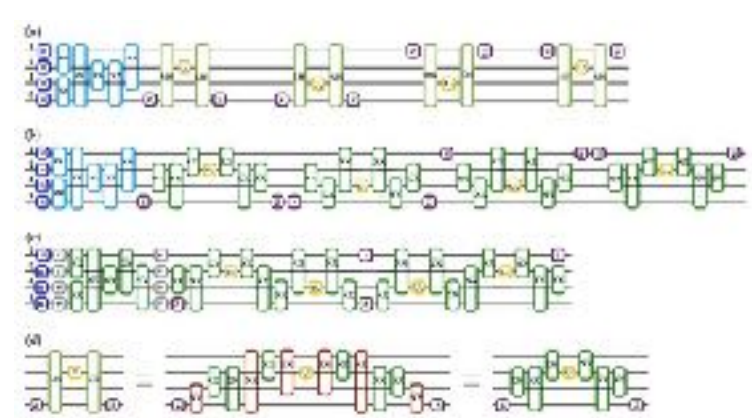
Trapped ions

Quantum field theory models
L. García-Álvarez et al., PRL 2015



Superconducting circuits

Quantum chemistry models
L. García-Álvarez et al., SciRep 2016



Superconducting circuits