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Universal Scaling Laws for Correlation Spreading in Quantum Systems

Lorenzo Cevolani, Julien Despres, Giuseppe Carleo, Luca Tagliacozzo, Laurent Sanchez-Palencia

Summary

- **Background**
  - The relevant Hamiltonians and quenches
  - The Cardy-Calabrese picture
  - Pseudo-particles
- **Results**
  - Identifying different classes of LR
  - Universal picture
  - Short range systems
  - Long range systems

Background

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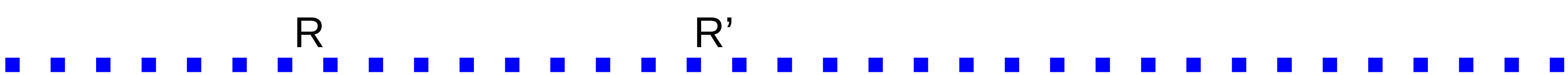
The relevant Hamiltonians

- 1D translational invariant system described with Hamiltonian

$$H = \sum_{R,R'} h_{R,R'}$$



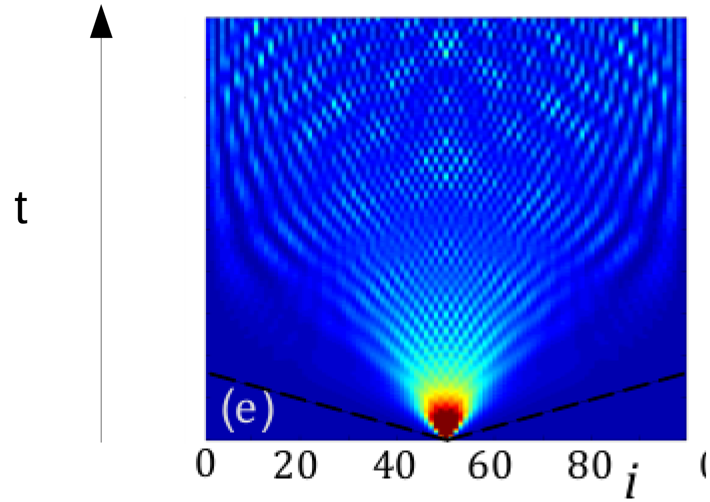
- Ex. short range Bose Hubbard

$$\hat{H} = -J \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \left(\hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}'} + \mathrm{H.c.} \right) + \frac{U}{2} \sum_i \hat{n}_{\mathbf{R}} \left(\hat{n}_{\mathbf{R}} - 1 \right)$$

The relevant quantum quench

- Start from the ground state $|\psi_0\rangle$ of H_0
- Put the system out of equilibrium by quenching the Hamiltonian to H_1 and let the system evolve

$$|\psi(t)\rangle = \exp\left[-iH_1 t\right]|\psi_0\rangle$$



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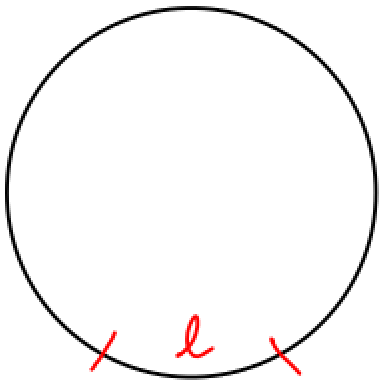
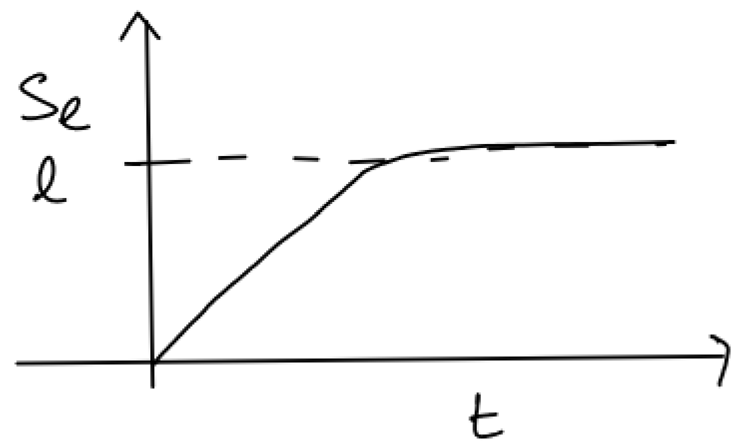
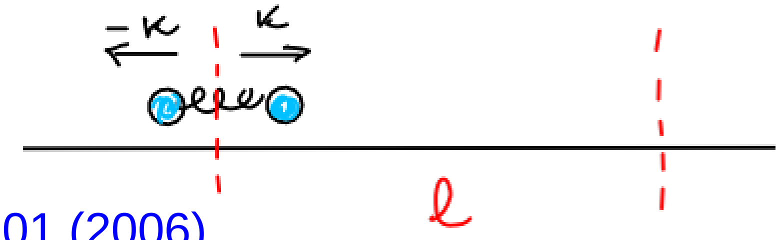
Calabrese Cardy

- After a quench, the excess energy produces a radiation of correlations





- Can be explained in terms of radiation of entangled pseudo-particles



Calabrese Cardy, Phys. Rev. Lett. 96, 136801 (2006).

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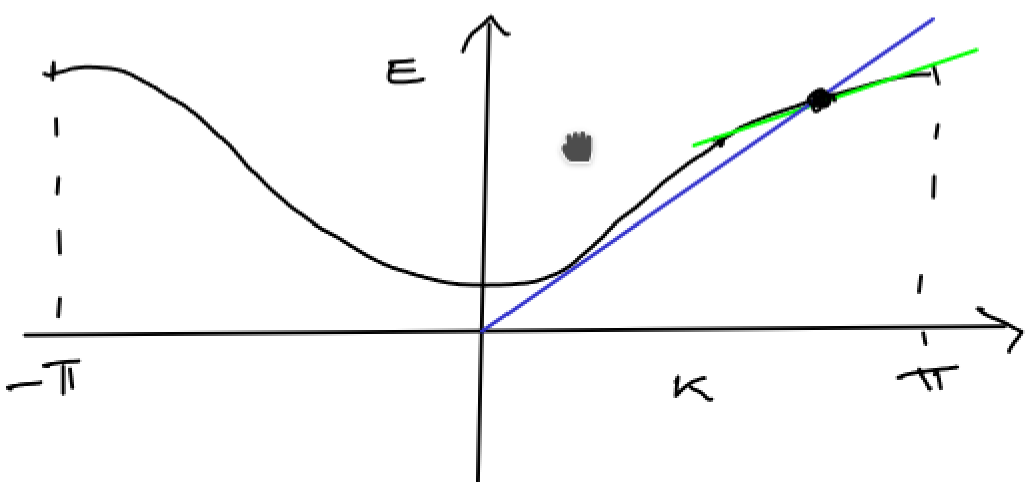
P(seudo)-Particles

- P-Particles are excitations

$$|A_k\rangle \propto \sum_R \exp[ikR] A_R |\psi_0\rangle$$

$$H|A_k\rangle = E(k)|A_k\rangle$$

- With well defined dispersion relation



$$V_\varphi = E(k)/k$$

$$V_g = \partial_k E(k)$$

Feynman, Statistical mechanics: a set of lectures ." (1998).

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Quantitative results in quenches using P-Particles

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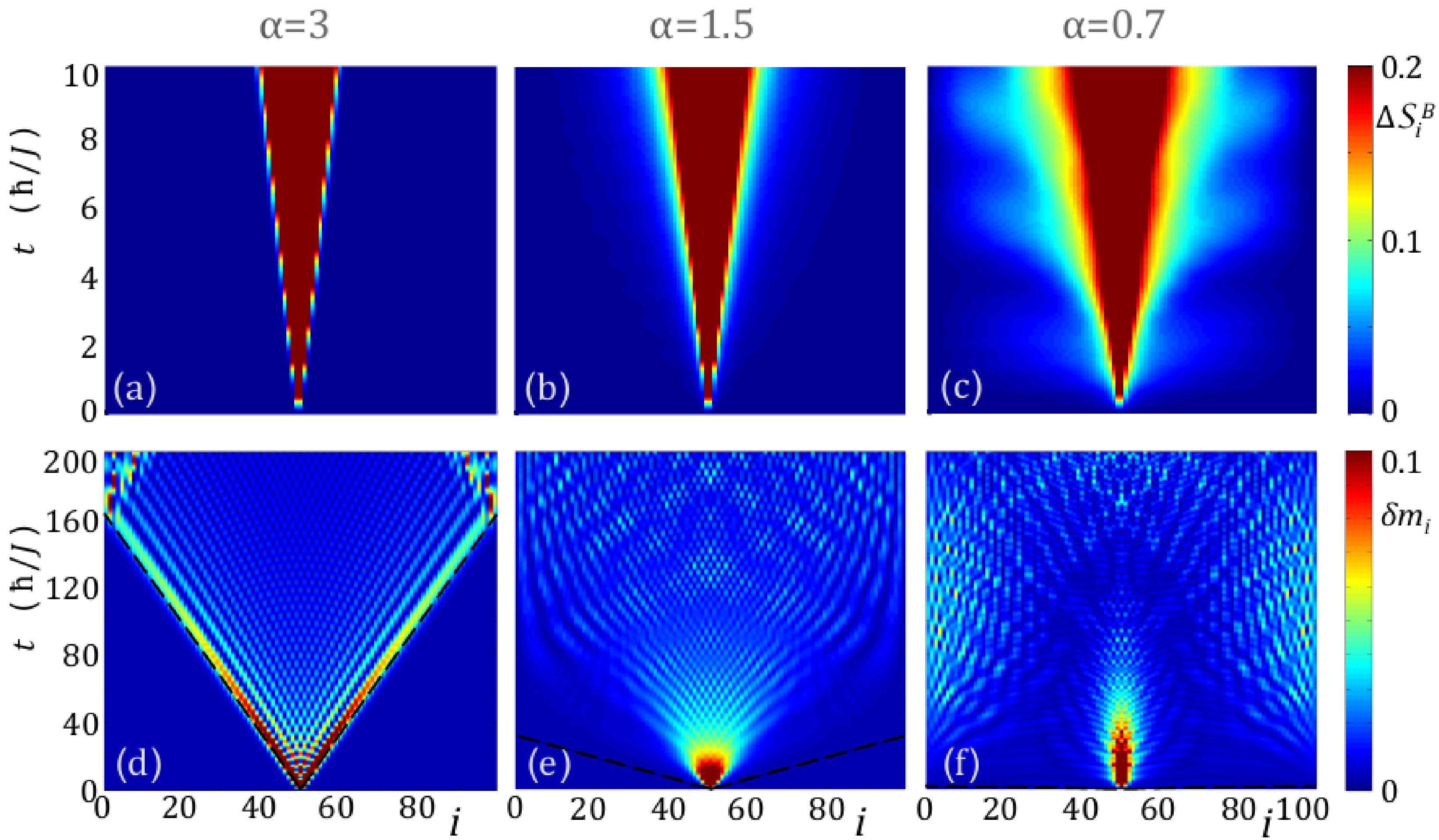
Correlations in Long Range Systems



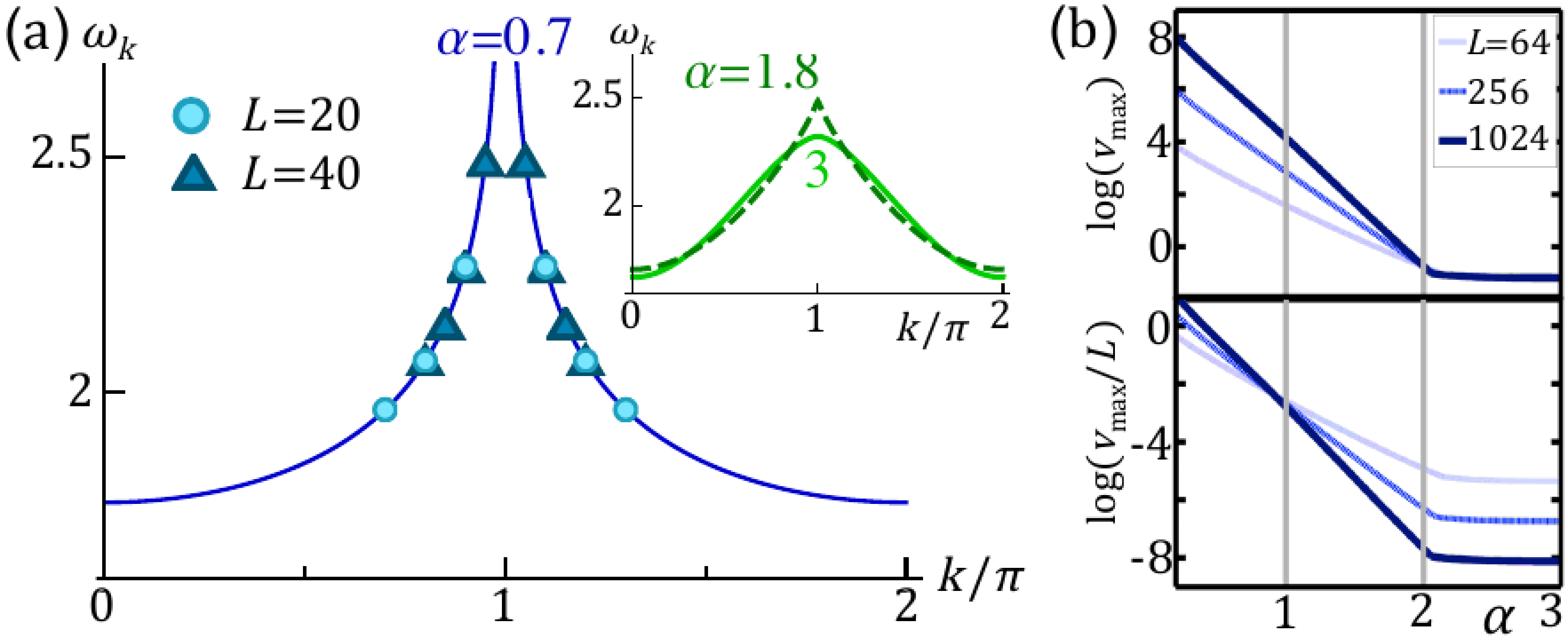
$$H = \sum_{i<j} \frac{\sin(\theta)}{|j-i|^{\alpha}} \sigma_x^i \sigma_x^j + \sum_i \cos(\theta) \sigma_z^i; 0 \leq \theta \leq \pi/2$$

Hauke, Tagliacozzo *Phys. Rev. Lett.* 111, 207202 (2013)

The role of P-Particles

- Regimes are well identified by looking at the P-Particle dispersion relation

$$t_b \propto L/v_{max} \propto L^{\alpha-1}$$



Hauke, Tagliacozzo, *Phys. Rev. Lett.* 111, 207202 (2013)

Further results

- Experiments with trapped ions
  Richerme et al. Nature (2014), Jurcevic et al. Nature (2014)
- XXZ models Cevolani et al. Phys Rev. A(R) (2015)...
- LR fermionic models Vodola et al. Phys. Rev. Lett. (2014), A. S. Buyskikh, et al Phys. Rev. (2016).....
- Higher D systems L. Cevolani, et al. NJP (2016).

....

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New results, universal scaling of correlations using P-Particles

Universal scaling law

- The general expression for correlation functions in terms of pseudo particles

$$\langle \hat{a}_R^\dagger(t)\hat{a}_0(t)\rangle$$

$$G(\mathbf{R},t)=g(\mathbf{R})-\int_{\mathcal{B}}\frac{d\mathbf{k}}{(2\pi)^D}\mathcal{F}(\mathbf{k})\,\frac{\mathrm{e}^{i(\mathbf{k}\cdot\mathbf{R}+2E_{\mathbf{k}}t)}+\mathrm{e}^{i(\mathbf{k}\cdot\mathbf{R}-2E_{\mathbf{k}}t)}}{2}$$

P-Particle momentum

P-Particle density

P-Particle energy

L. Cevolani, et al. NJP (2016).

Long time and space limit

- We can consider the stationary phase approx.

$$\nabla_k (kR \mp 2E_k t) = 0$$

$$2V_{\mathrm{g}}(k_{\mathrm{sp}}) = \pm R/t$$

$$V_{\mathrm{g}} = \nabla_k E_k$$

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Threshold effect, causal cone

- If the group velocity is bounded we can have no solution, to the stationary phase e.g.

$$R/t \;>\; 2V_{\mathrm{g}}^{\star}$$

  In this case the integral is exponentially small

- There are stationary-phase solutions for

$$R/t \;\leq\; 2V_{\mathrm{g}}^{\star}$$

Solution, short range systems

- If there is a solution it reads,

$$G(R,t) \propto \frac{\mathcal{F}(k_{\rm sp})}{\left(|\nabla_k^2 E_{k_{\rm sp}}|t\right)^{\frac{D}{2}}} \cos\left(k_{\rm sp}R - 2E_{k_{\rm sp}}t + \frac{\pi}{4}\right)$$

- Several features, several maxima-minima, at fixed

$$k^\star R - 2E^{\rm f}_{k^\star} t$$

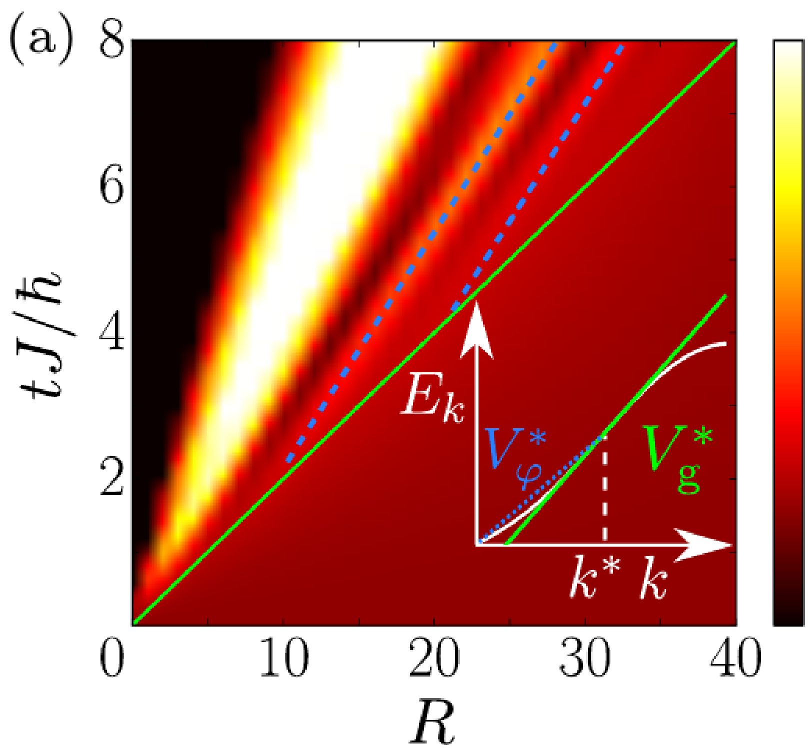
moving at characteristic speeds,

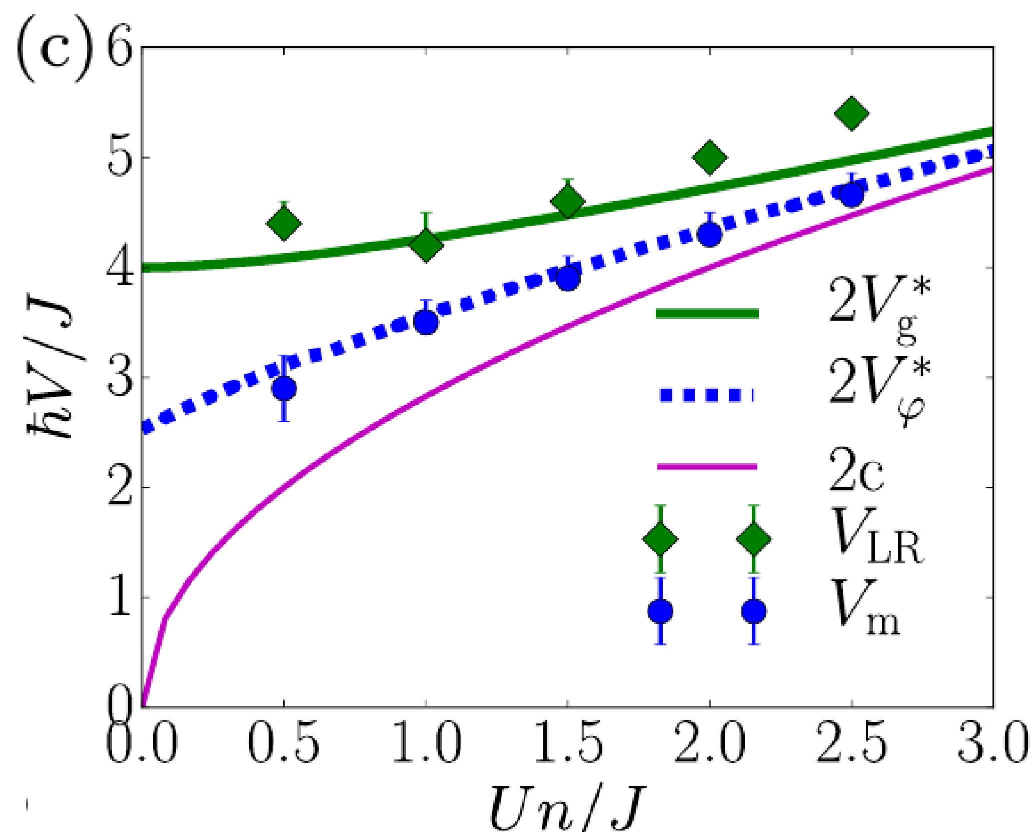
$$2V_\varphi^\star \equiv 2E_{k^\star}/k^\star$$

Ex. superfluid Bose Hubbard

$$\hat{H} = -J \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \left(\hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}'} + \mathrm{H.c.} \right) + \frac{U}{2} \sum_i \hat{n}_{\mathbf{R}} \left(\hat{n}_{\mathbf{R}} - 1 \right)$$

$U_{\mathrm{i}} n = J \longrightarrow U_{\mathrm{f}} n = 0.5 J.$ $\quad U_{\mathrm{i}} n = J \longrightarrow$



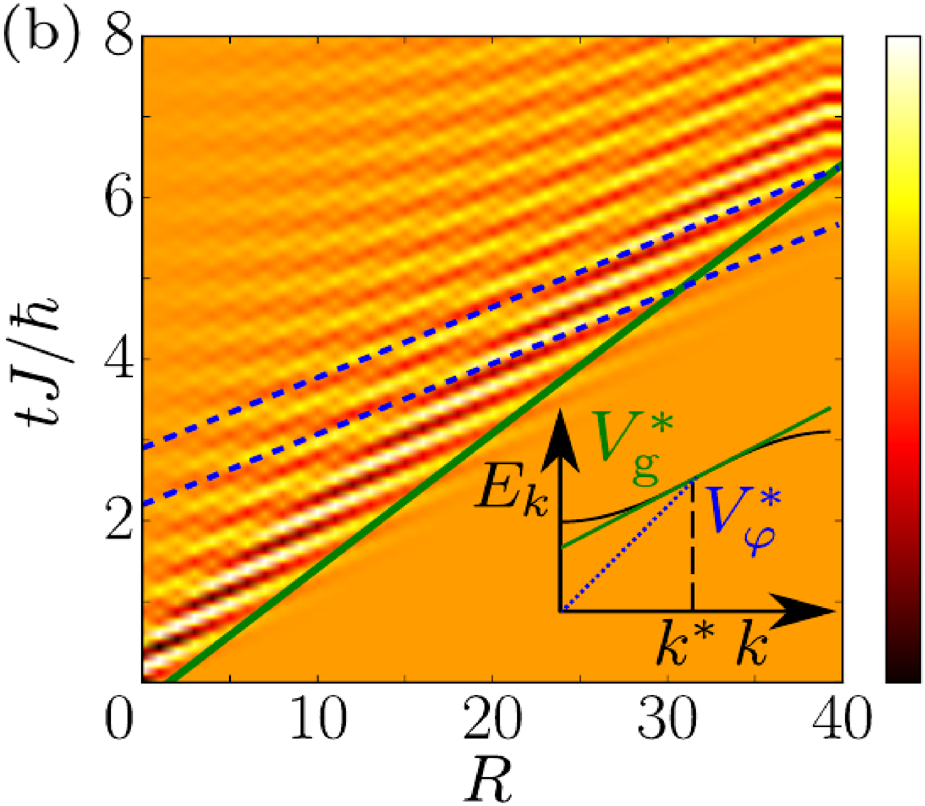


$$\langle \hat{a}^\dagger_R(t) \hat{a}_0(t) \rangle - \langle \hat{a}^\dagger_R(0) \hat{a}_0(0) \rangle$$

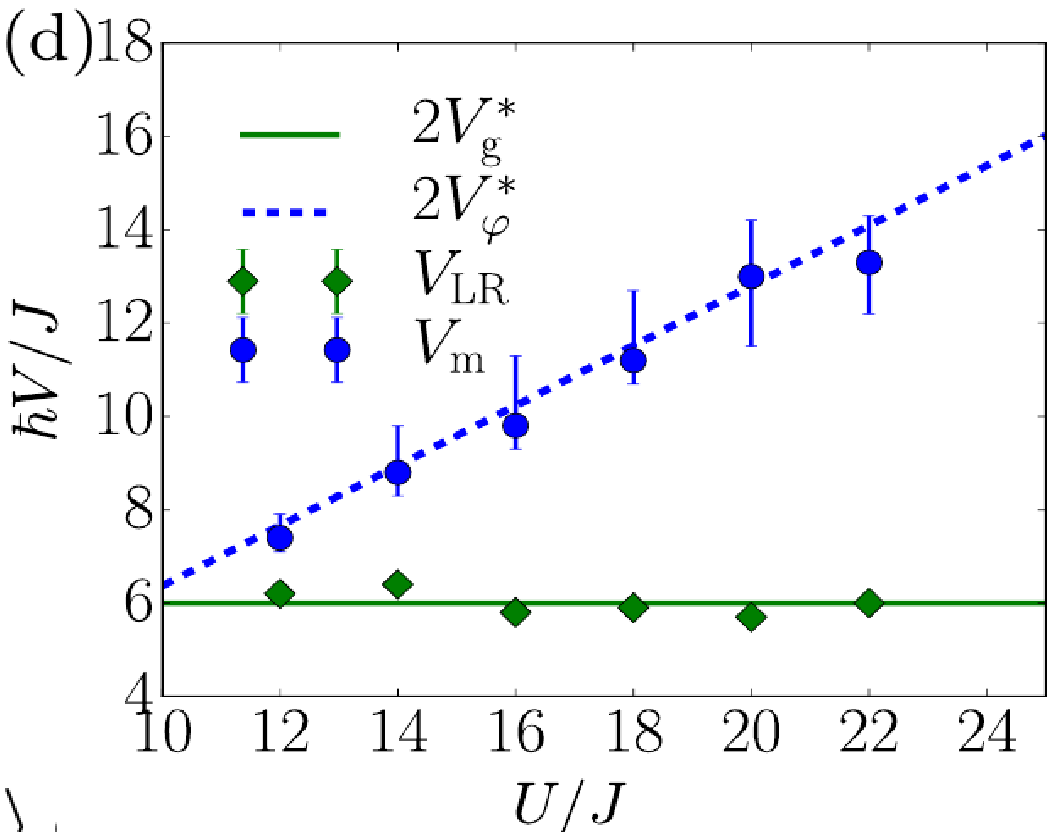
Group velocity larger than phase velocity

Ex. Bose Hubbard Mott phase

$U_{\mathrm{i}} = \infty \longrightarrow U_{\mathrm{f}} = 18J$ $\quad$ $U_{\mathrm{i}} = \infty \longrightarrow$



$\langle \hat{a}_R^\dagger(t)\hat{a}_0(t)\rangle - \langle \hat{a}_R^\dagger(0)\hat{a}_0(0)\rangle$



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LR systesm

- Ex. LR XXZ

$$\hat{H} = \sum_{\mathbf{R}\neq\mathbf{R}'} \frac{J/2}{|\mathbf{R}-\mathbf{R}'|^{\alpha}} \left[-\left(\hat{S}^x_{\mathbf{R}} \hat{S}^x_{\mathbf{R}'} + \hat{S}^y_{\mathbf{R}} \hat{S}^y_{\mathbf{R}'} \right) + \epsilon \hat{S}^z_{\mathbf{R}} \hat{S}^z_{\mathbf{R}'} \right]$$

- We focus on the regime where the
  - Energy is bounded
  - The group velocity diverges, e.g. at K=0

$$E_k \simeq \Delta + ck^z \qquad \mathcal{F}(k) \sim k^{\nu}$$

$$V_{\mathrm{g}}(k) = cz/k^{1-z}$$

LR, observable dependent algebraic cone

- There is always a stationary phase solution,

$$k_{\mathrm{sp}} = (2czt/R)^{1/(1-z)}$$

- The correlations in that limit read

$$G_{\mathrm{c}}(\mathbf{R}, t) \propto \frac{t^{\gamma}}{R^{\chi}} \cos\left[A_z \left(\frac{t}{R^z}\right)^{\frac{1}{1-z}} - 2\Delta t + \frac{\pi}{4}\right]$$

$$\gamma = \frac{\nu + D/2}{1-z} \qquad \chi = \frac{\nu + D(2-z)/2}{1-z}$$

- Not a sharp cone, but correlations of order one require

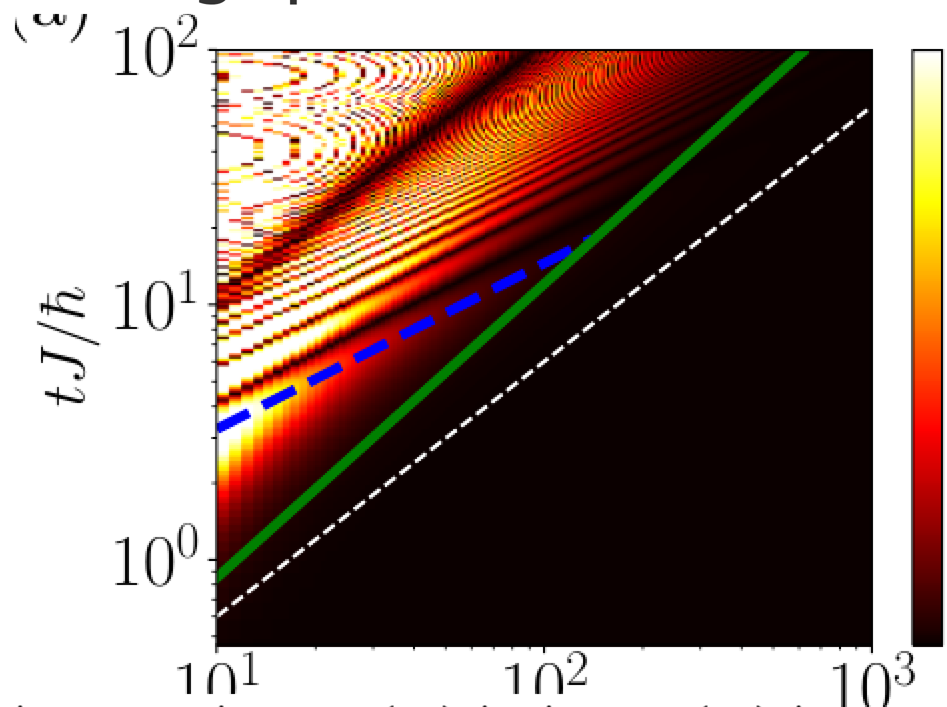
$$t^{\star} \propto R^{\beta_{\mathrm{LR}}}, \qquad \beta_{\mathrm{LR}} = \chi/\gamma$$

LR gapless (XXZ)

- The cone depends on the observable, while inner structure depends on gap
- In the gapless case

$$t_{\mathrm{m}} \propto R^{\beta_{\mathrm{m}}}$$

$$\beta_{\mathrm{m}} = z$$



$$\alpha = 2.3 \quad \langle S_R^z(t) S_0^z(t)\rangle - \langle S_R^z(t)\rangle\langle S_0^z(t)\rangle$$

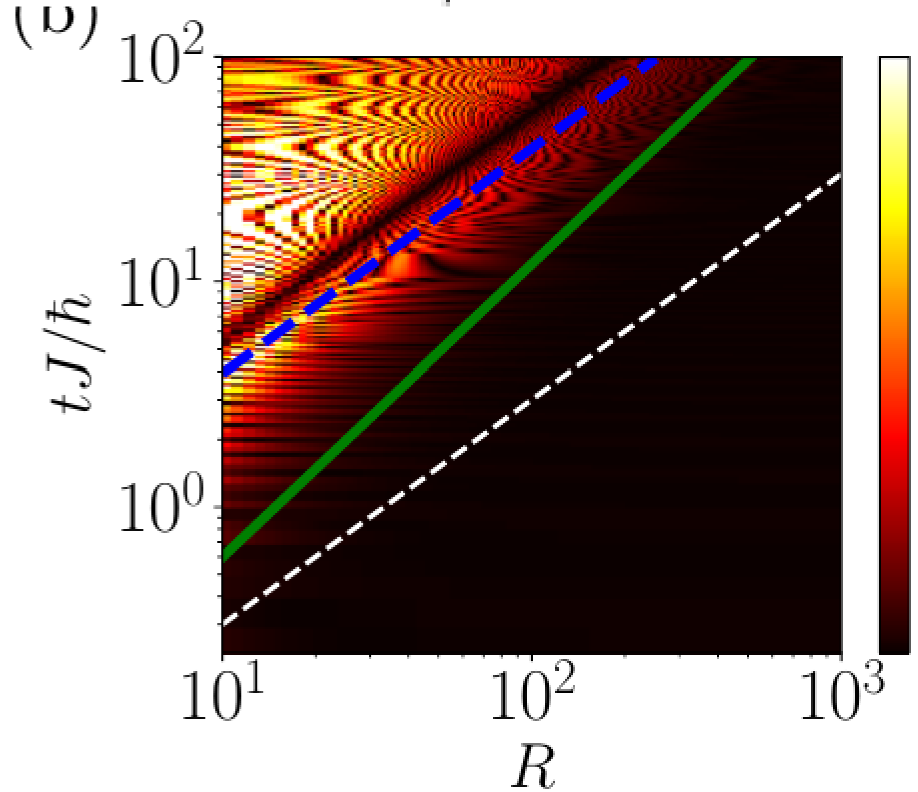
$$\hat{H} = \sum_{\mathbf{R}\neq\mathbf{R}'} \frac{J/2}{|\mathbf{R}-\mathbf{R}'|^{\alpha}} \left[-\left(\hat{S}^x_{\mathbf{R}}\hat{S}^x_{\mathbf{R}'} + \hat{S}^y_{\mathbf{R}}\hat{S}^y_{\mathbf{R}'}\right) + \epsilon \hat{S}^z_{\mathbf{R}}\hat{S}^z_{\mathbf{R}'}\right]$$

LR gapped

$J_{\mathrm{i}}/h = 0.25$ to $J_{\mathrm{f}}/h = 0.5$

- The outer cone is sub-ballistic

- The inner structure is ballistic.



$$\hat{H} = \sum_{\mathbf{R}\neq\mathbf{R}'} \frac{J/2}{|\mathbf{R}-\mathbf{R}'|^{\alpha}} \hat{S}^{\mathrm{x}}_{\mathbf{R}} \hat{S}^{\mathrm{x}}_{\mathbf{R}'} - h \sum_{\mathbf{R}} \hat{S}^{\mathrm{z}}_{\mathbf{R}}.$$

Conclusions

Conclusions

- P-Particles have been used in the past to provide a qualitative picture of the out-of-equilibrium dynamics
- Also some specific quantitative result
- We have provided a unified analysis that explain several features of the spread of correlations
- For SR systems, the results are universal and the structure only depends on the ratio between group and phase velocities
- For LR systems, the outer cone depends on the observable, but the inner structure only depends on the presence of a gap.

Thank you !!!

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