Topological properties and many-body phases of synthetic Hofstadter strips

ICFO Institut de Ciències Fotòniques

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EU STREP EQuaM

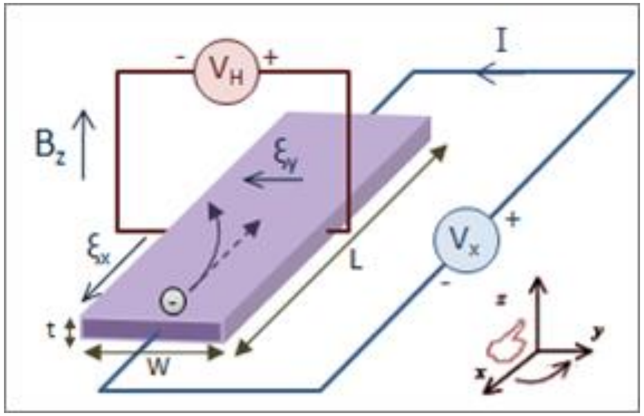
Workshop on Quantum Science and Quantum Technologies – ICTP Trieste 13/09/2017

Plan

- Integer Quantum Hall systems and Edge states
- Cold atom realizations: synthetic gauge field
  - Synthetic lattice (Extradimension)
    Topology in narrow strips
- Dimerized interacting ladder
  - Meissner/Vortex phase (in analogy to type II superconductors)
  - Effect of the dimerization
    Reverse of chiral current (single particle)
    Commensurate-Incommensurate transition (strong interactions)
- Prospects

Quantum Hall effect

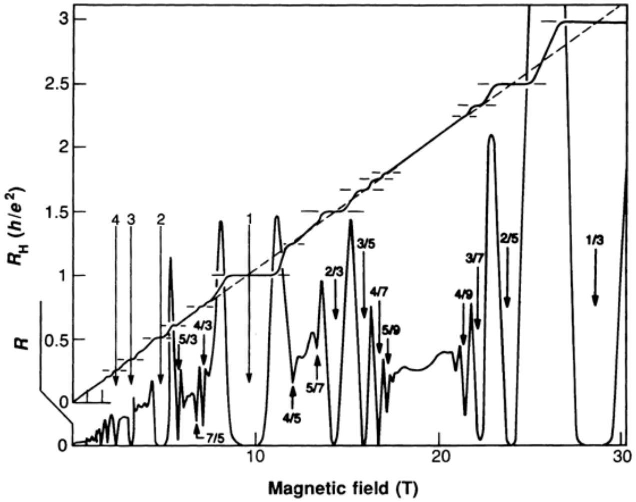
1879: Classical Hall effect (consequence of Lorentz force)



E. Hall

1980: Quantum Hall effect: Electric conductivity quantized

$$\sigma = \frac{I_{\text{channel}}}{V_{\text{Hall}}} = \nu \frac{e^2}{h}$$



K. Von Klitzing

Integer Quantum Hall effect in a lattice

IQH explained in terms of single particle physics (Landau level filling)

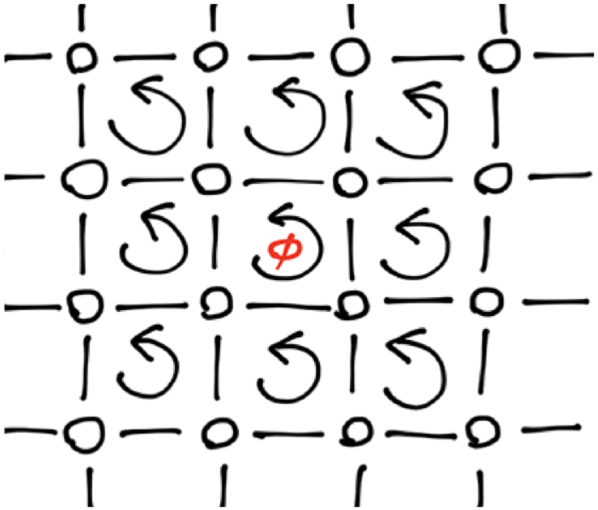
$$H = -\sum_{n,m} (J a^\dagger_{n+1,m} + J' e^{i\Phi n} a^\dagger_{n,m+1}) a_{n,m} + h.c.$$

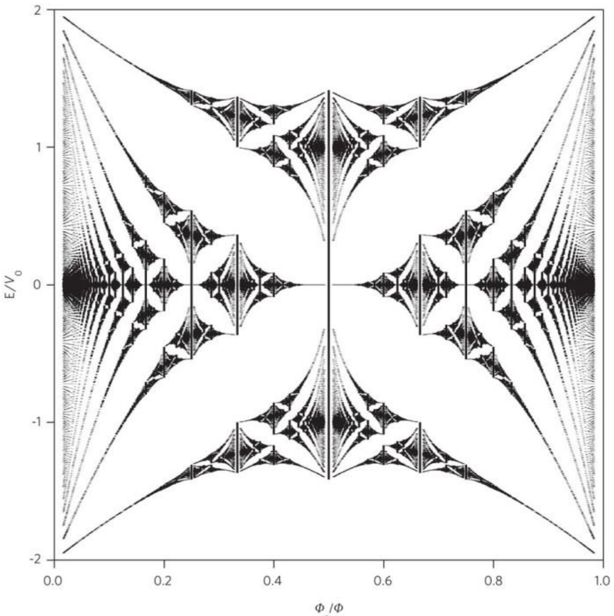
Quantization determined by topology of filled bands (1-Chern number)

Bulk-boundary correspondence (Topological insulator prototype)

Edge states determined by spectrum of periodic system

On square lattice simple formulation: Hofstadter model





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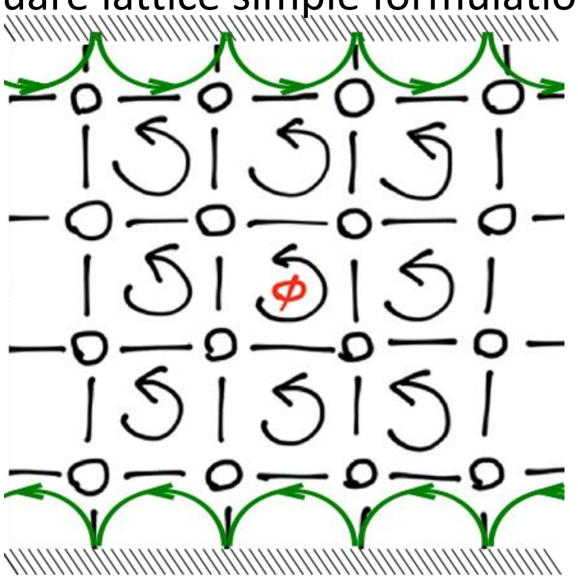
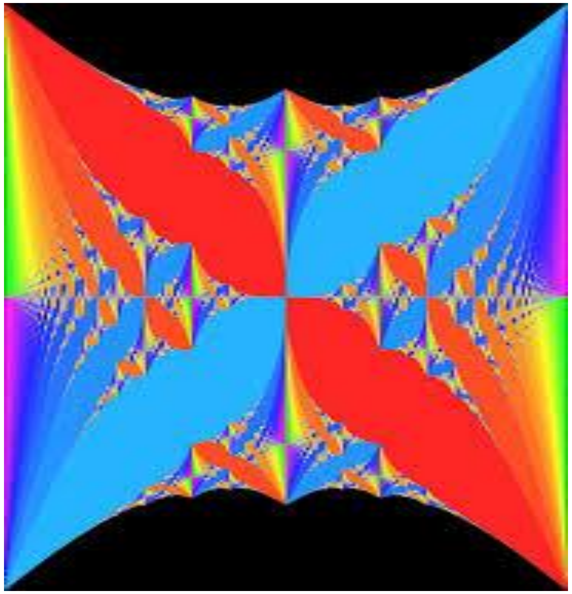
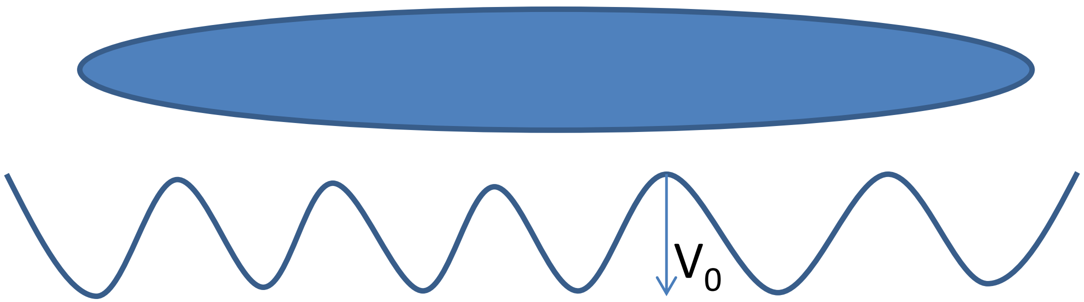
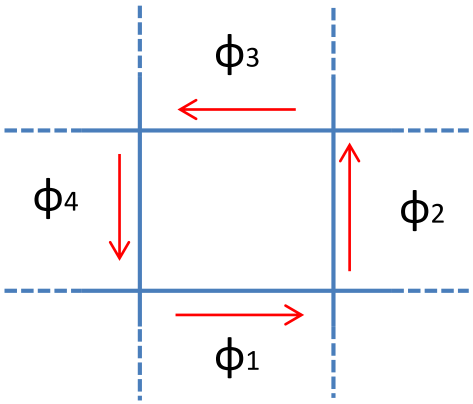




Cold atoms in optical lattices as charged particles



Hopping with phases ➡ Synthetic magnetic field for neutral atoms



Synthetic Aharonov-Bohm effect

$\phi = \Sigma_i \phi_i$ = magnetic flux

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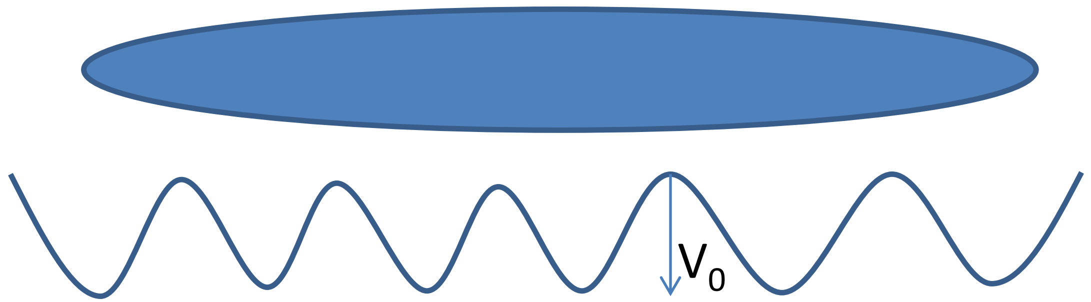

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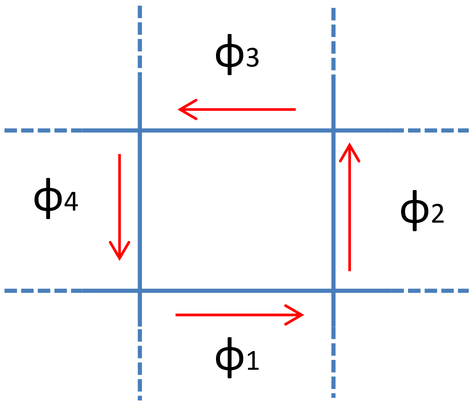

Synthetic Aharonov-Bohm effect

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Several ways: here *"Extradimension"* + Raman laser = synthetic lattices

Simulating an extra dimension

[Boada,AC,Latorre,Lewenstein, PRL 108, 133001 (2012)]

In optical lattices 1D-3D Hubbard model by tuning optical potential

And > 3D? In a lattice Dimensionality ≡ Connectivity

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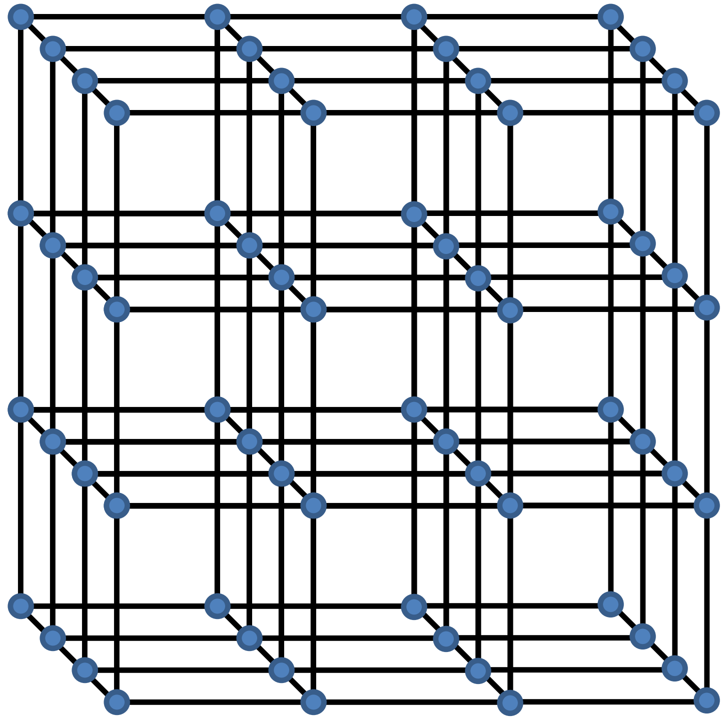
Hopping in D+1 hypercubic lattice as



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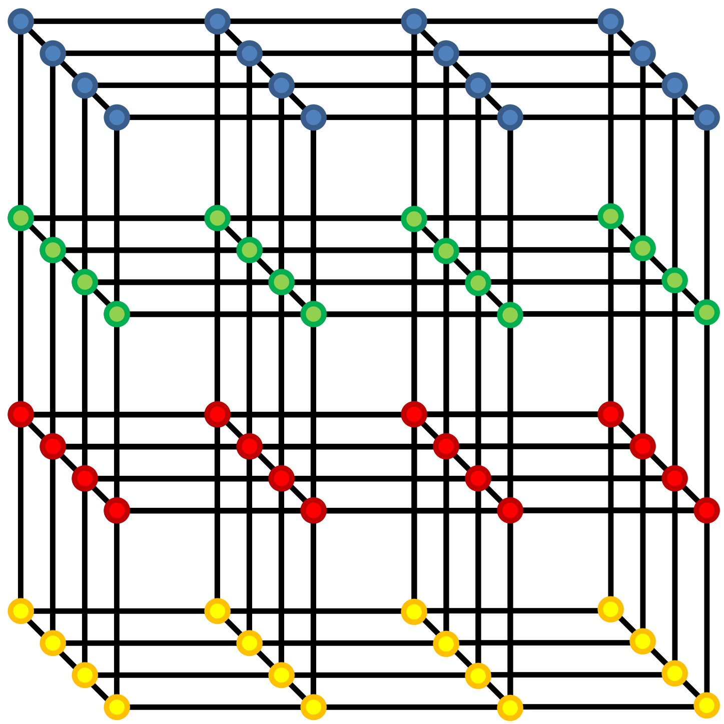

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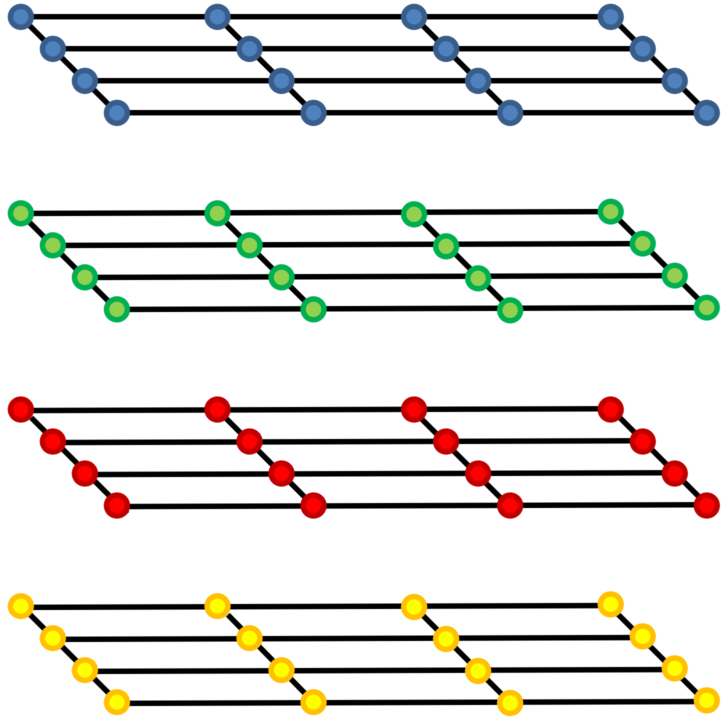

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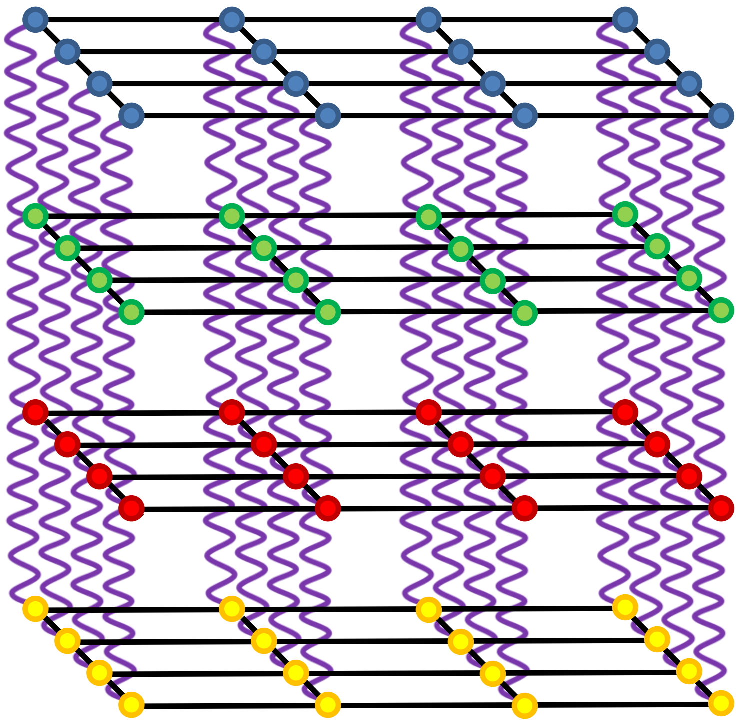
Coupled atomic states hopping in *D*-lattices



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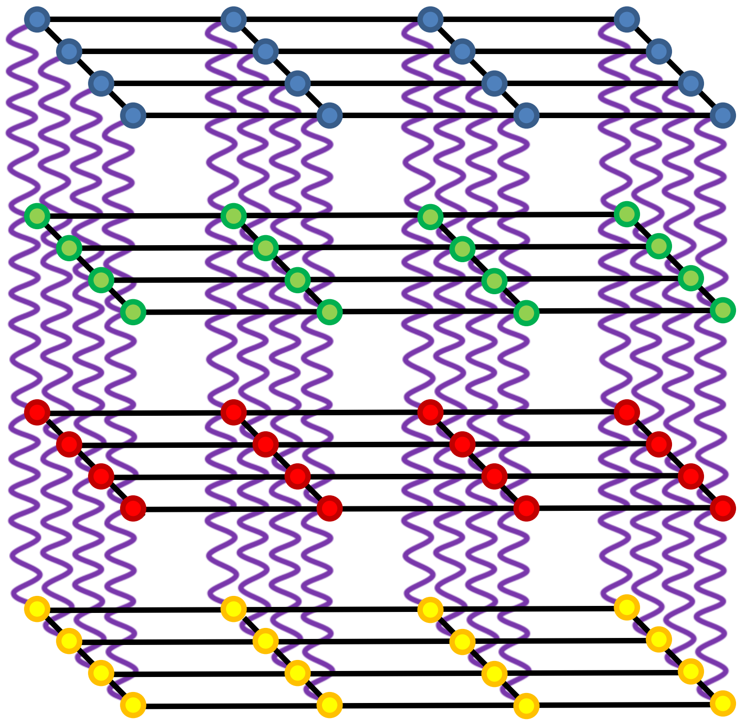

Not only spin states
Momentum states
Trap modes…

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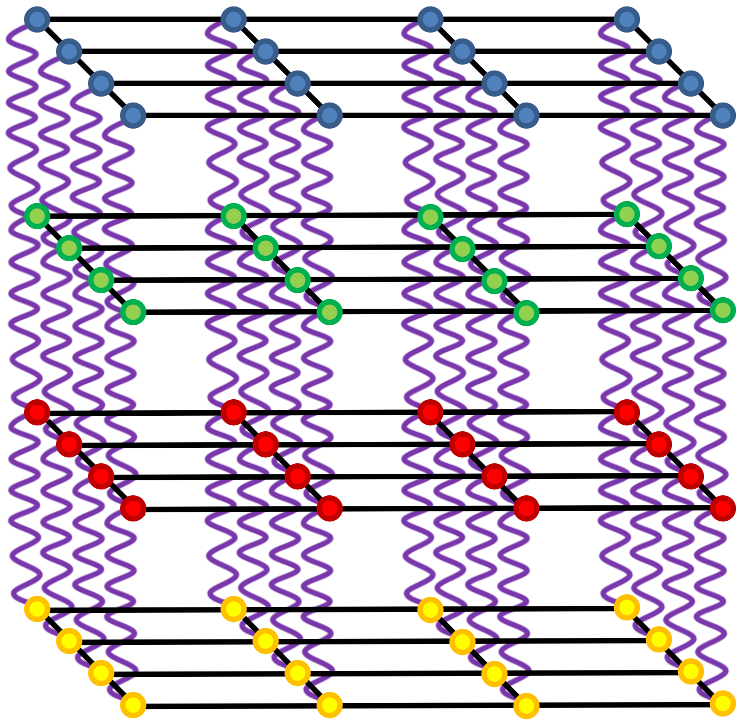

Not only spin states
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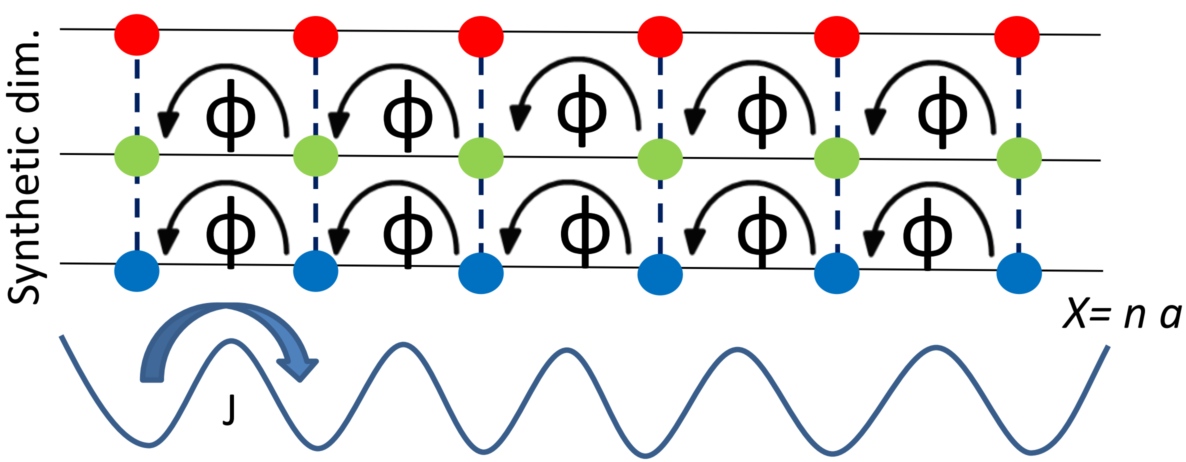
Not only atoms
Cold molecules,
Photonic crystal,
Ring resonators...

Synthetic gauge fields in synthetic dimension

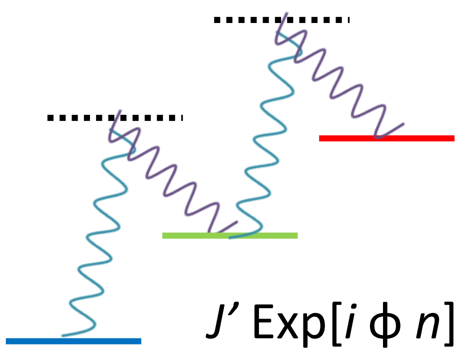
[AC *et al* PRL 112 , 043001 (2014)]



1d-lattice loaded e.g. with ^{87}Rb (F=1, m=-1,0,1)

+

Raman dressing



J' Exp[*i* ϕ *n*]

Constant magnetic flux ϕ!

Sharp Boundaries ➡ Edge currents (hard to get in real 2d lattice)
signal of Topological nature of quantum Hall
(bulk-boundary correspondence)

Synthetic gauge fields in synthetic dimension

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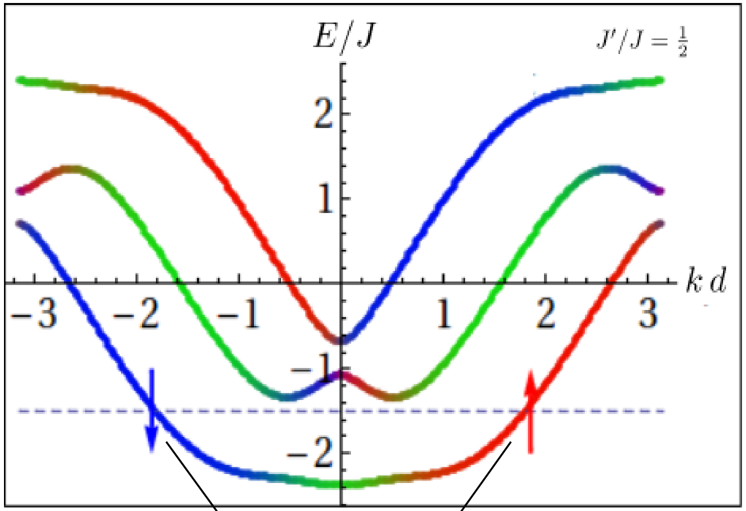
Spectrum



"Genuine" Edge states for small *J'/J*:

-live in the gap,

-have linear dispersion

-have well defined spin

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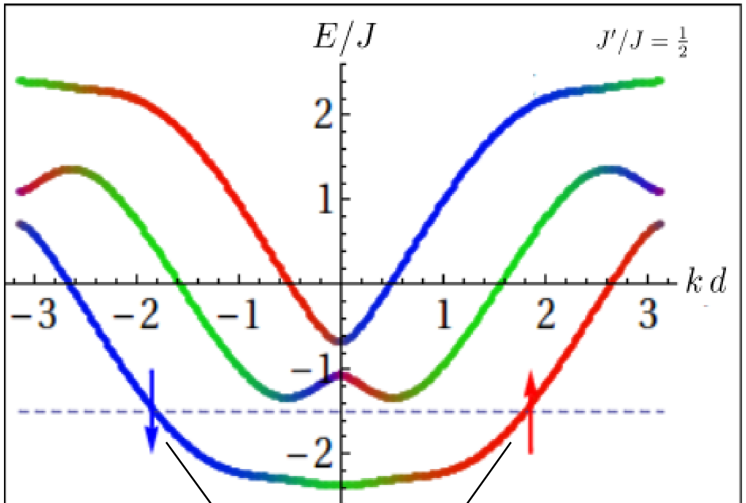

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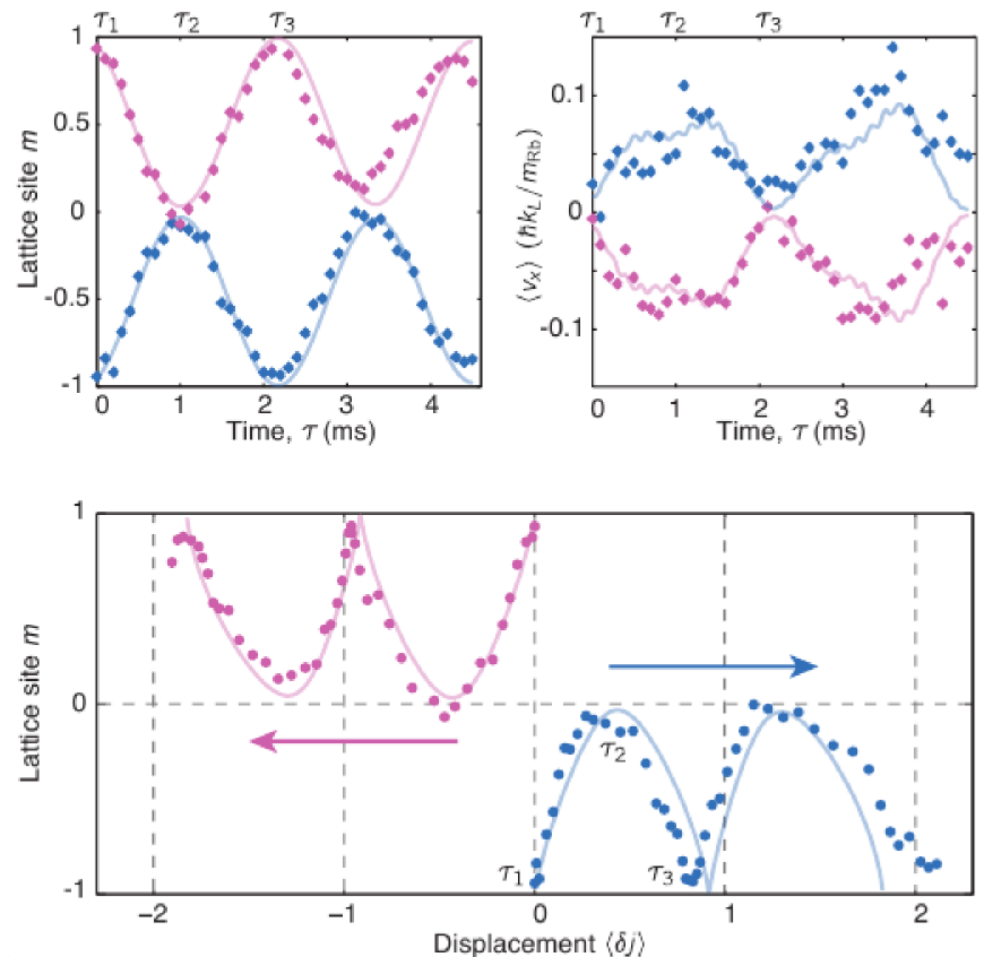

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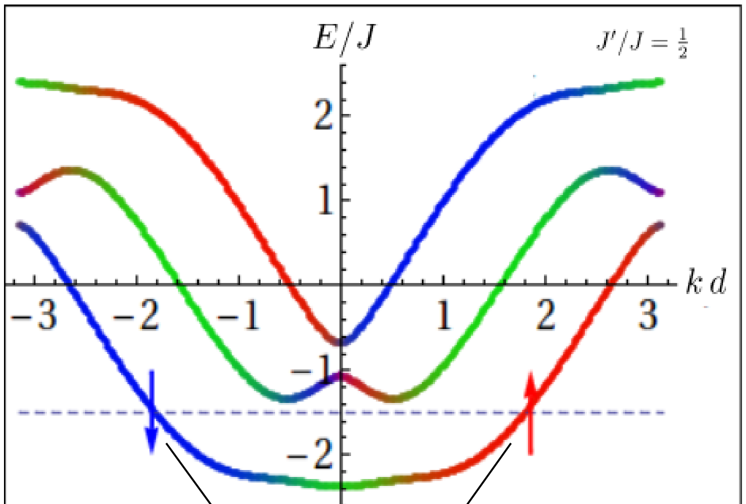

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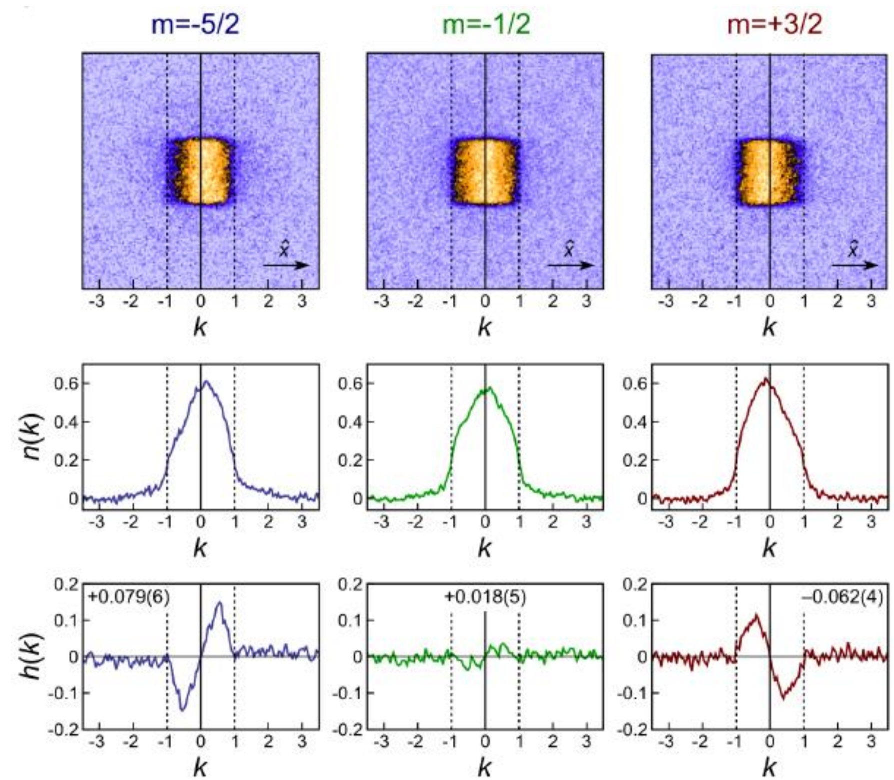

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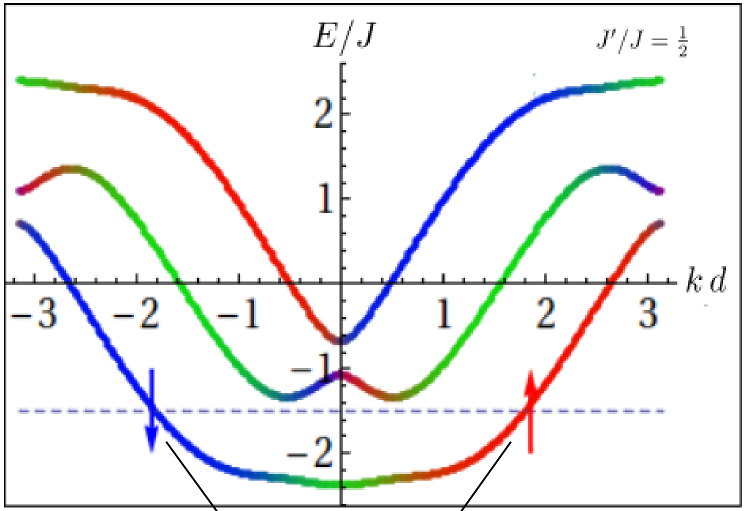

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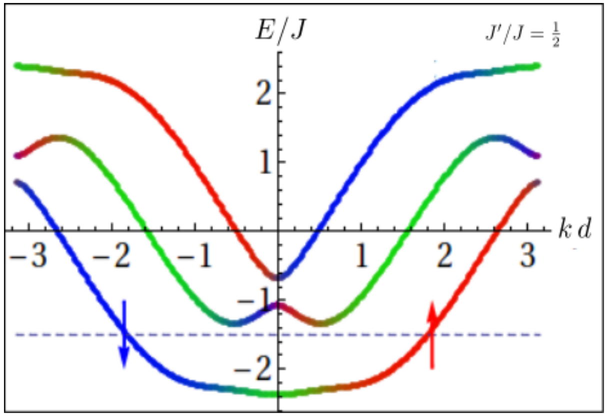
Also with clock states (ladder)

LENS: Livi et al. PRL 117, 220401 (2016)

JILA: Kolkowitz et al. Nature 542 66 (2017)

Topology in narrow strips

Narrow Hofstadter strips have edge states



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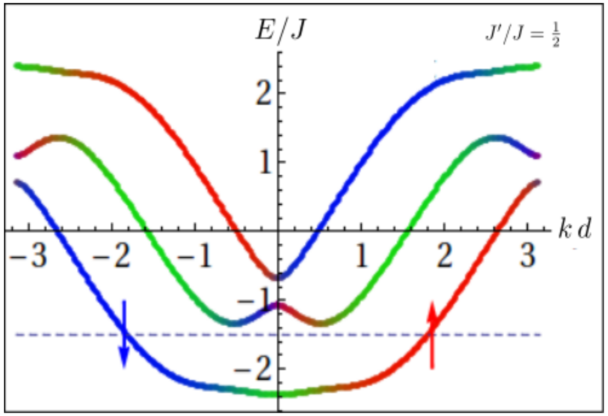

What about the “bulk”?

How big should it be to display topological properties?

Is there some reminiscence of open/closed boundary correspondence?

Is Chern number defined? / Can we measure it?

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC, SciPost 3, 012, (2017)

Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

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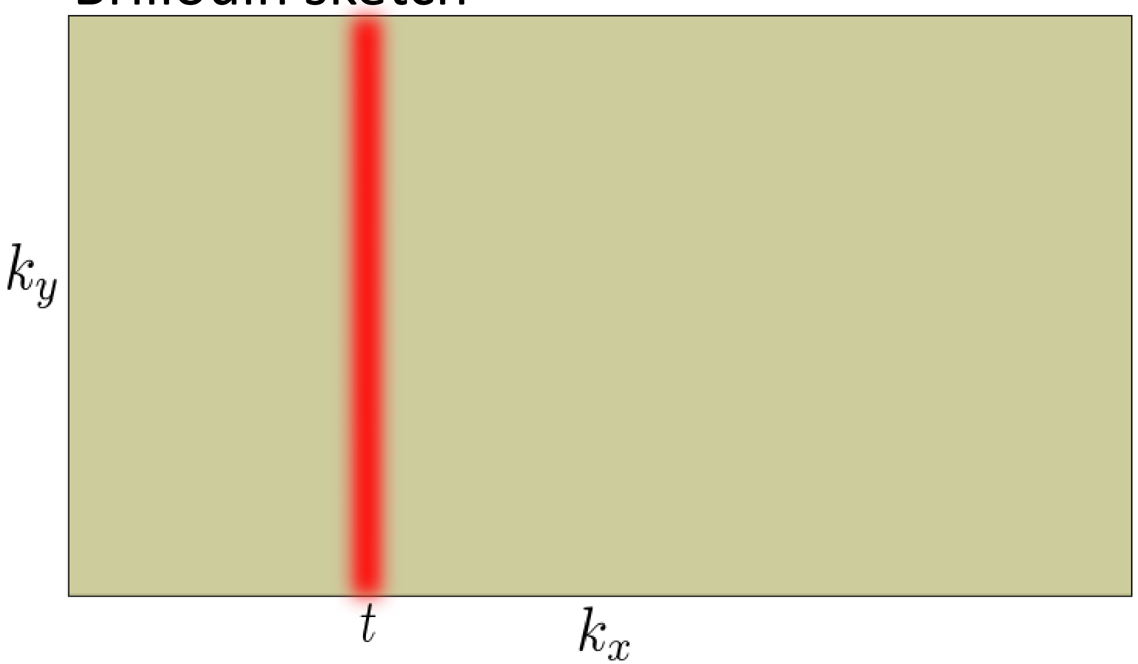
- Large (periodic) system, a lowest band state well localized in *y* and spread in *x*
- Apply a force along *x*

Brillouin sketch



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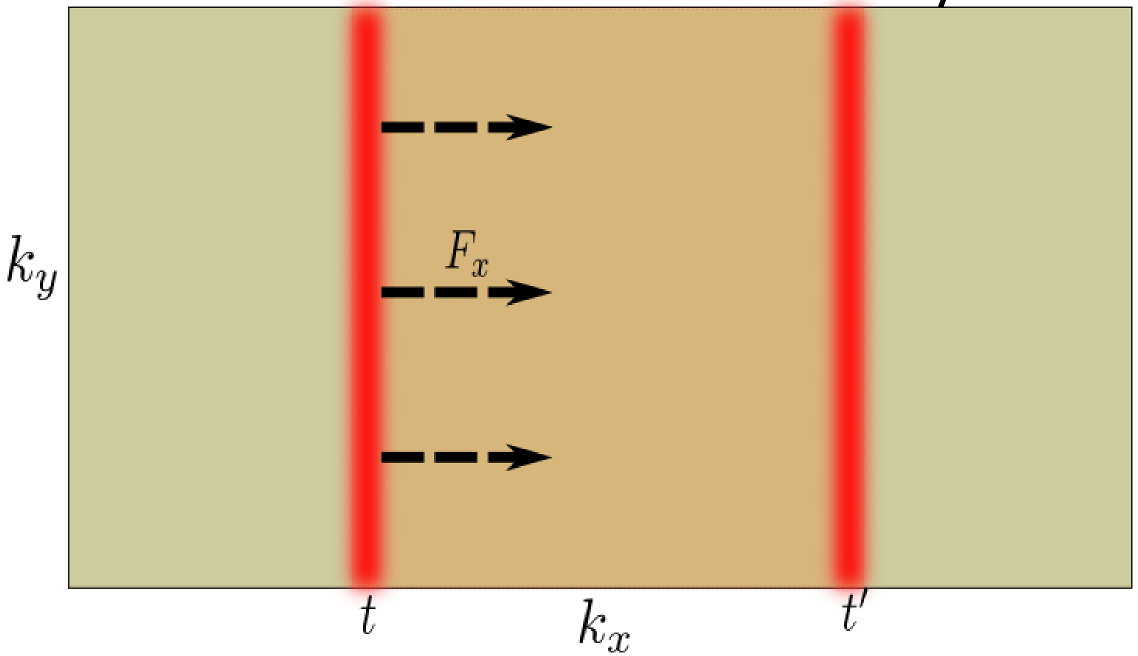
Brillouin sketch of semiclassical dynamics



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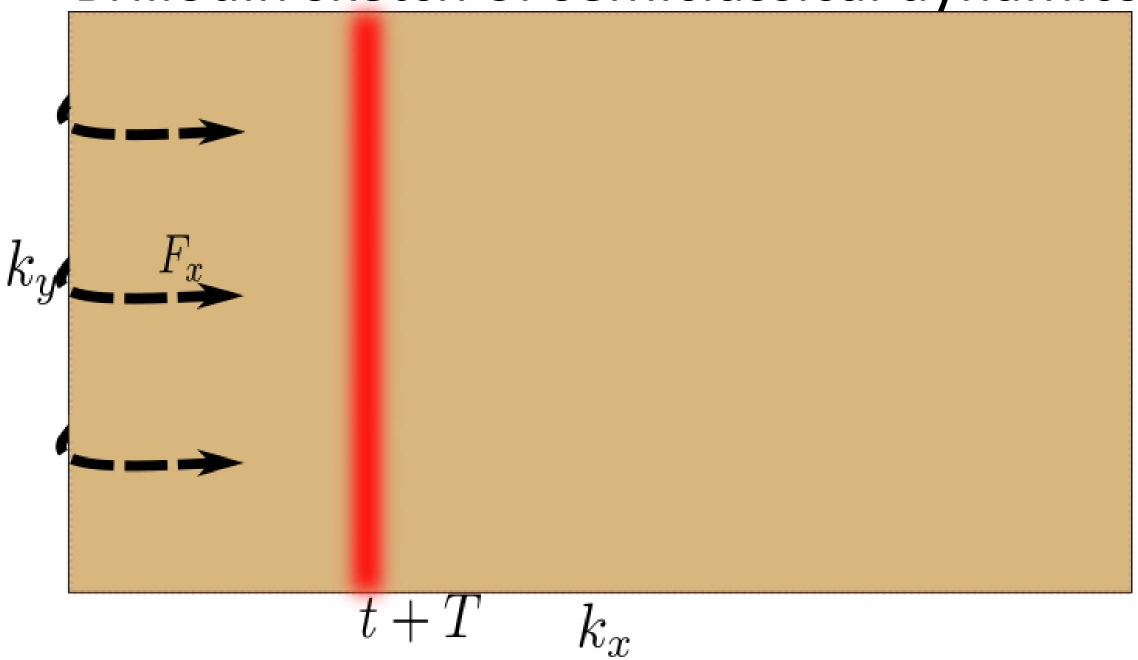
- After a Bloch oscillation observe the displacement

Brillouin sketch of semiclassical dynamics



Displacement in *y* due to anomalous velocity!

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In formulae: semiclassical approach

$$\mathbf{k}(t) = \mathbf{k}_0 + \frac{t}{\hbar d} F_x \, \mathbf{e}_x \longrightarrow \mathbf{v}(\mathbf{k}) = \frac{1}{\hbar}\partial_{\mathbf{k}} E(\mathbf{k}) + \frac{F_x}{\hbar d}\mathcal{F}(\mathbf{k})\mathbf{e}_y$$

$$|\psi(\mathbf{k})|^2 \sim \tfrac{1}{A_{BZ}}\delta(k_x - k_x(t))$$

Wave packet:

$$\langle \mathbf{r}(T) - \mathbf{r}(0)\rangle = \int_0^T \langle \mathbf{v}(t)\rangle = \tfrac{\hbar d}{|F_x|}\int_{BZ} \mathbf{v}(\mathbf{k}) = \mathrm{sgn}(F_x)\mathcal{C}\, d\, \mathbf{e}_y$$

State easy to prepare if the coupling $J_y \ll J_x$

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Applicable also to strips until we don't reach the boundary…

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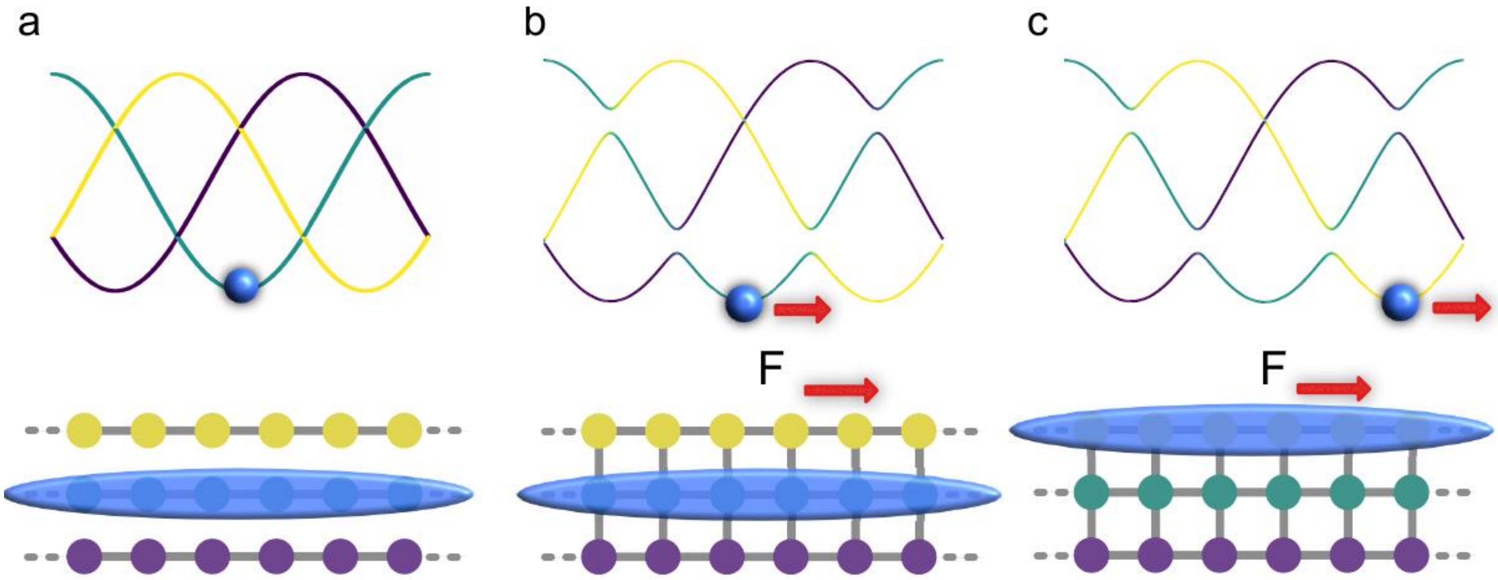
Scheme:



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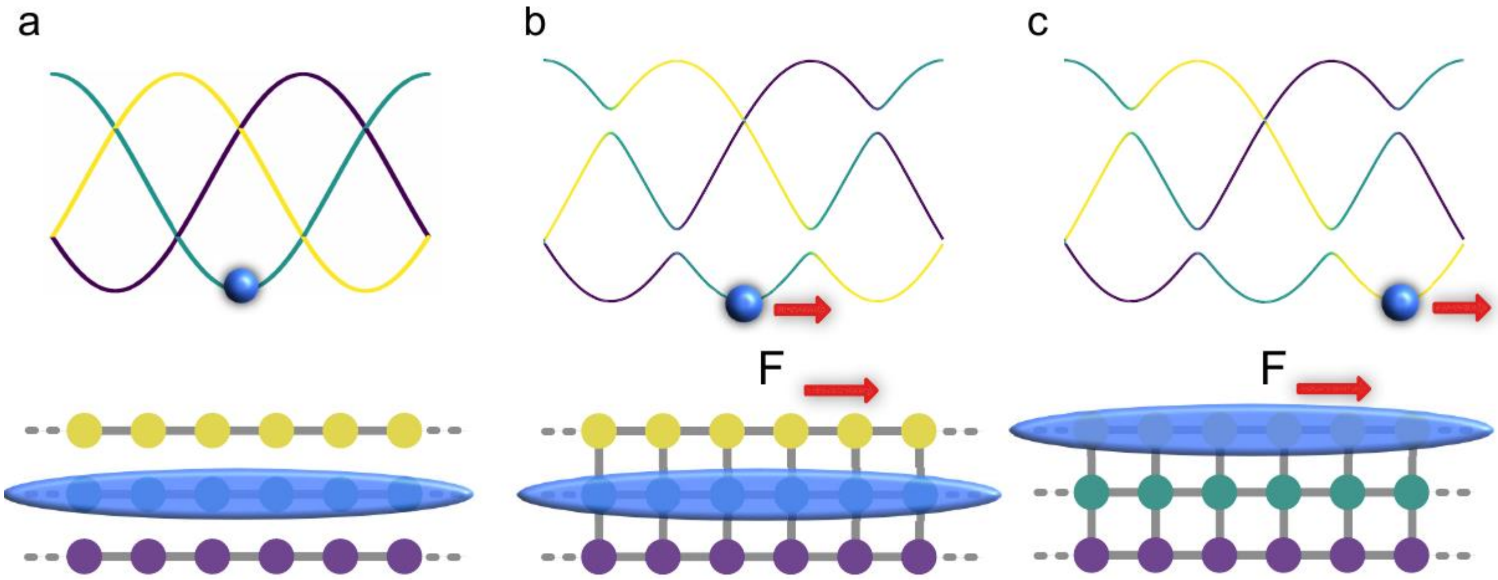
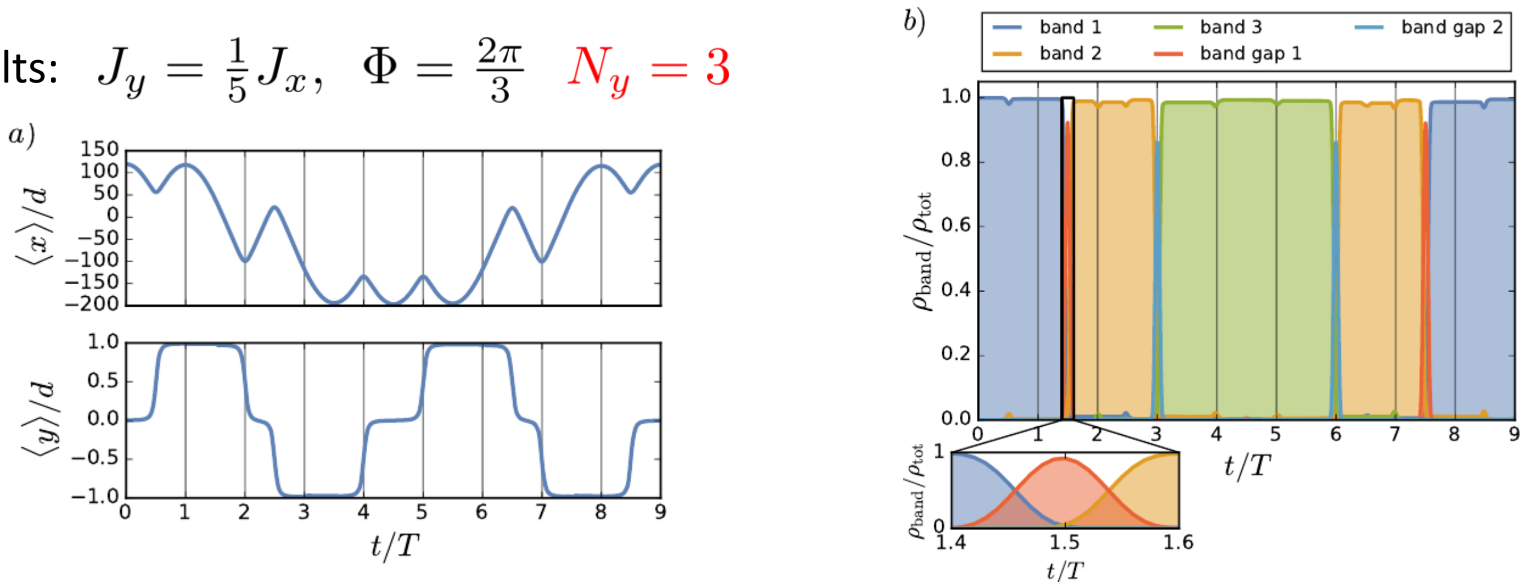


Results: $J_y = \frac{1}{5} J_x, \quad \Phi = \frac{2\pi}{3} \quad N_y = 3$



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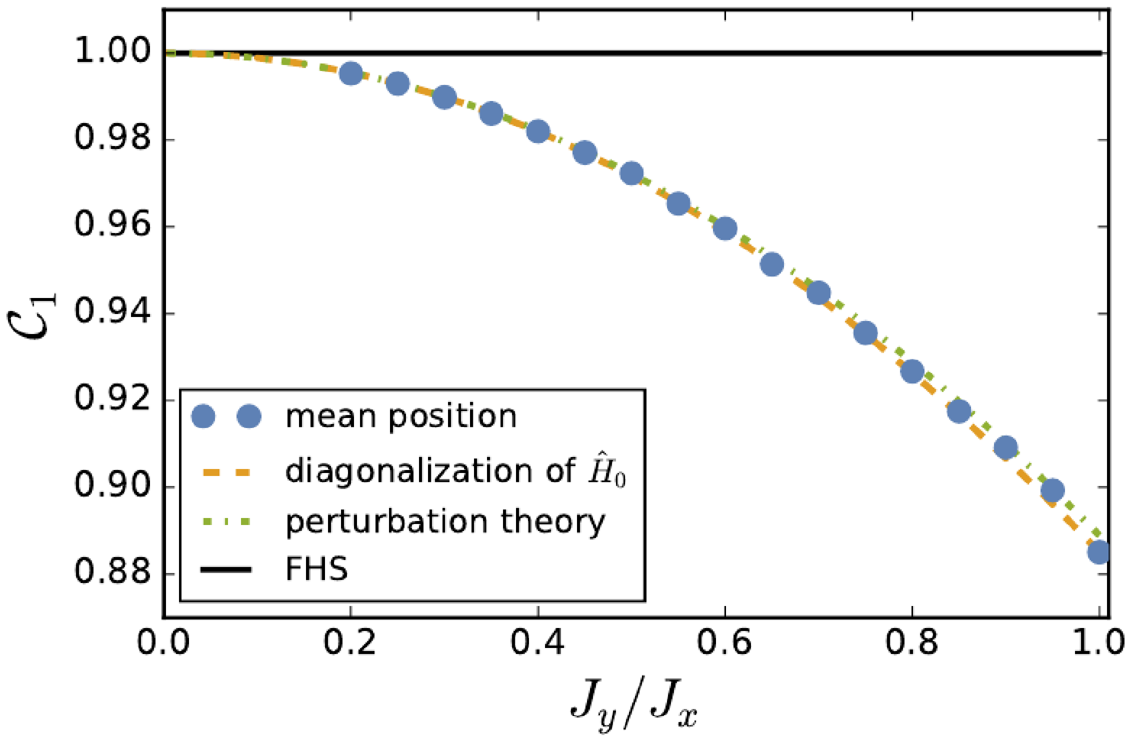
Why does it work? Perturbative argument also for edge states:

- Gap linear in J_y/J_x
- Hybridization spin states (spreading in y) quadratic in J_y/J_x



Quadratic degradation of the measurement

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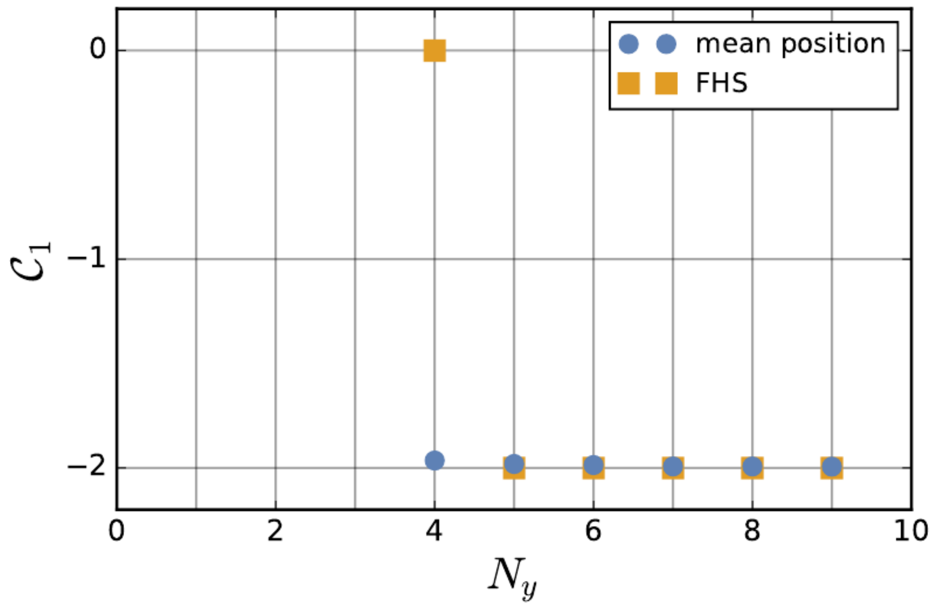
Higher $\mathcal{C}$ possible for $N_y \geq \mathcal{C} + 2$

Ex: $\Phi = \frac{4\pi}{5} \rightarrow \mathcal{C}_1 = -2$



"Better" than Fukui-Hatsugai-Suzuki algorithm J. Phys. Soc. Jpn. (2005)

Robust to disorder (< gap) and typical harmonic confinement

Interactions? Gap small (although may hold thought adiabatic argument, see later)

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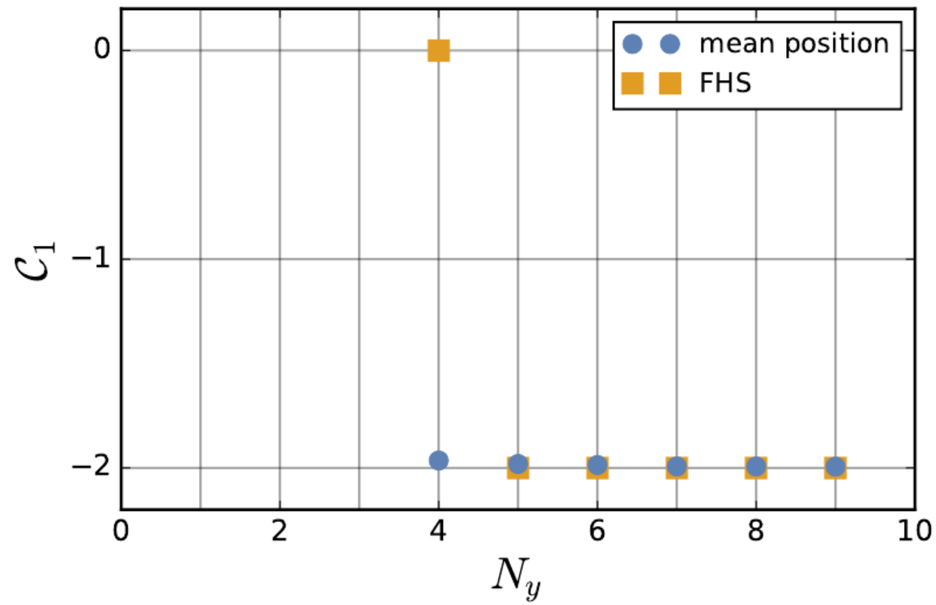

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Narrow Hofstadter strips have also sym. prot. 1D topology [Barbarino *et al., arXiv:1708:02929]*

Synthetic lattices in interaction

- Interesting route to interaction -> Fractional QH effect?!
  No heating expected

- Peculiarity: Interactions are naturally long range
  in the synthetic dimension

- Quasi 1D approach to 2D interesting both
  theoretically & practically

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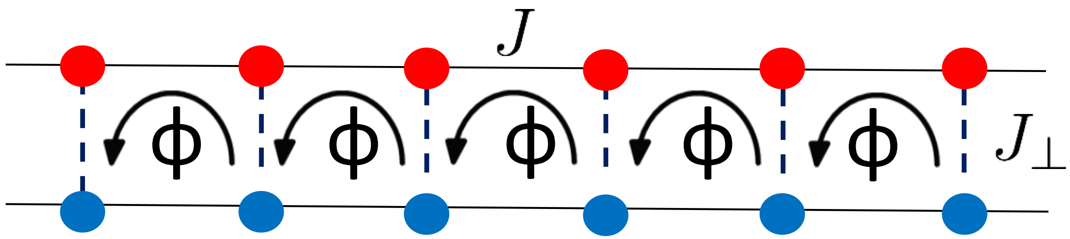
Many studies: Meissner-vortex and commensurable incommensurable transitions, Fractional pumping, Laughlin like states, pseudo Majorana… [a lot here in Trieste!]

Here: effect of dimerization on synthetic Hofstadter ladder

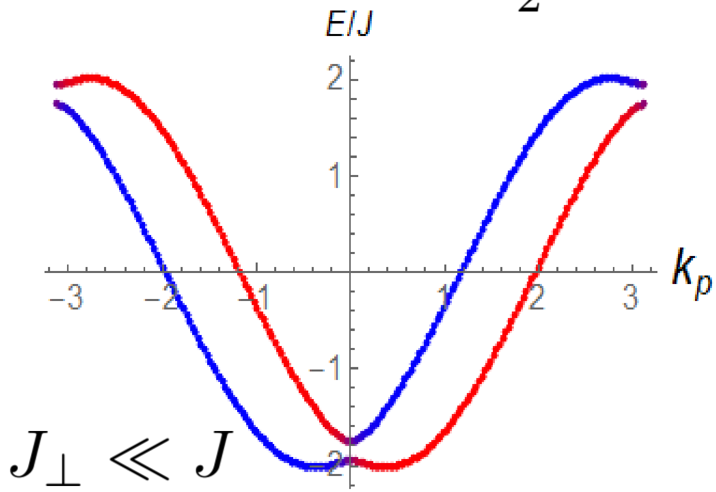
Meissner/Vortex phase in flux ladder



No interactions: real = synthetic ladder

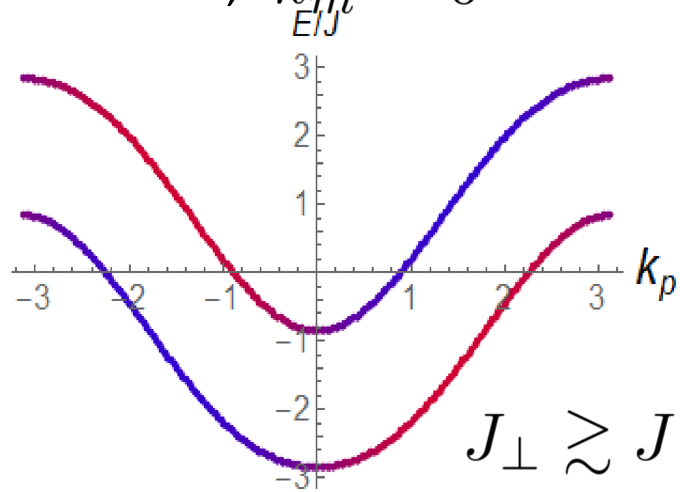
Weak interleg (Raman) coupling:

2 minima, $k_m \sim \pm\frac{\phi}{2}$



Strong interleg (Raman) coupling:

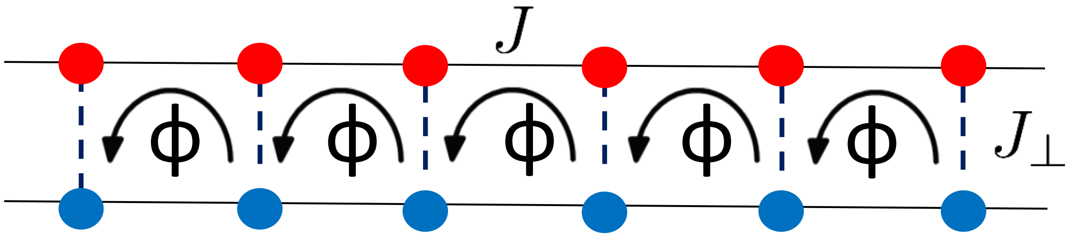
1 minima, $k_m = 0$



[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions

Real ladder experiment [Atala et, Nature Phys. 2014]

Meissner/Vortex phase in flux ladder



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Observables

$$J_c(j,m) = i\,\langle \hat{a}^\dagger_{j+1,m}\hat{a}_{j,m}\rangle + H.c.$$

$$J_\perp(j) = i\,\langle \hat{a}^\dagger_{j,1/2}\hat{a}_{j,-1/2}\rangle + H.c.$$

$J_\perp \ll J$

$J_\perp \gtrsim J$

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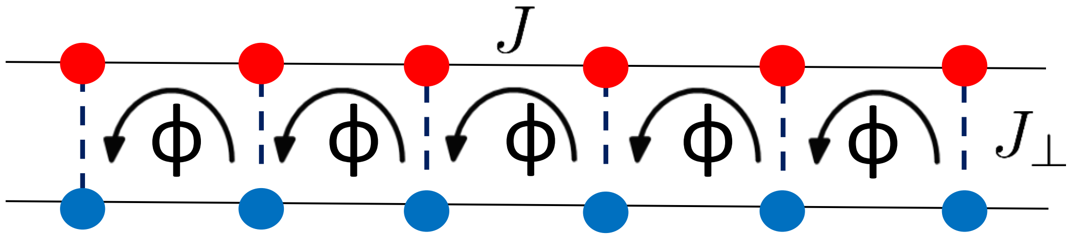
Real ladder experiment [Atala et, Nature Phys. 2014]

Meissner/Vortex phase in flux ladder



No interactions: real = synthetic ladder

Weak interleg (Raman) coupling:

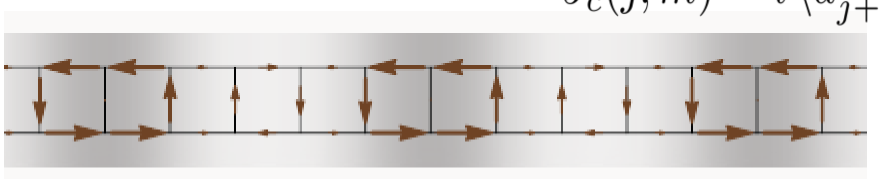
2 minima, $k_m \sim \pm\frac{\phi}{2}$

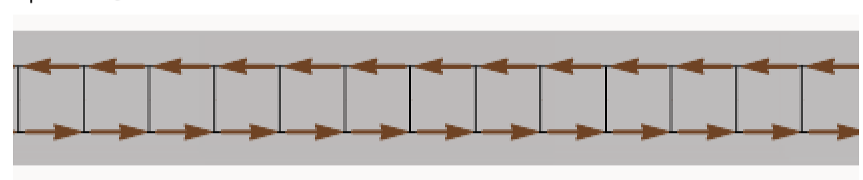
Strong interleg (Raman) coupling:

1 minima, $k_m = 0$

Observables

$$J_c(j,m) = i\,\langle \hat{a}^\dagger_{j+1,m}\hat{a}_{j,m}\rangle + H.c.$$





$$J_\perp(j) = i\,\langle \hat{a}^\dagger_{j,1/2}\hat{a}_{j,-1/2}\rangle + H.c.$$

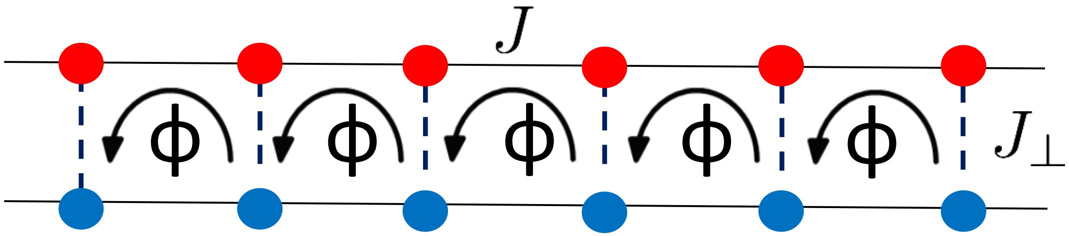
$J_\perp \ll J$

$J_\perp \gtrsim J$

[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions

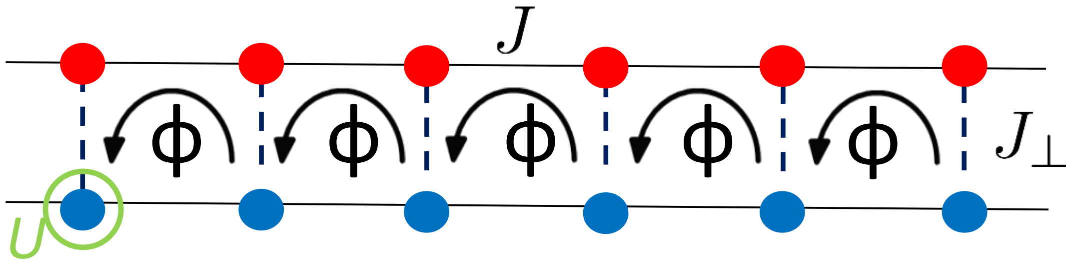
Real ladder experiment [Atala et, Nature Phys. 2014]

Meissner/Vortex phase in flux ladder



Effect of interactions: suppress vortex phase

Meissner/Vortex phase in flux ladder



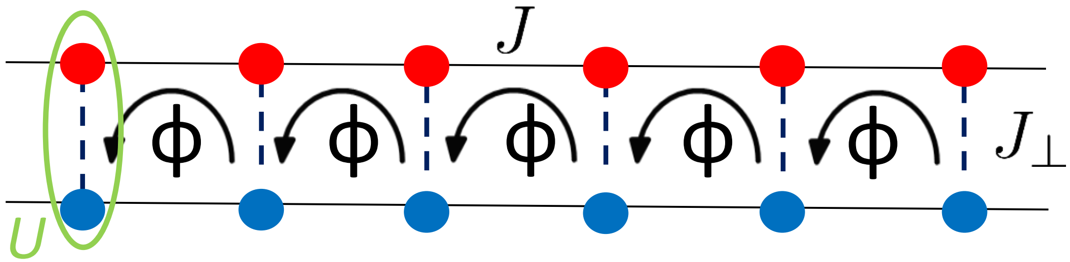
Effect of interactions: suppress vortex phase

Real ladder: vortex phase survives in the hard-core limit for ϕ large

more phases at $U \neq \infty$

see [Petrescu, Le Hur,PRL 2013]
[Piraud et al, PRB 2015]

Meissner/Vortex phase in flux ladder



Effect of interactions: suppress vortex phase

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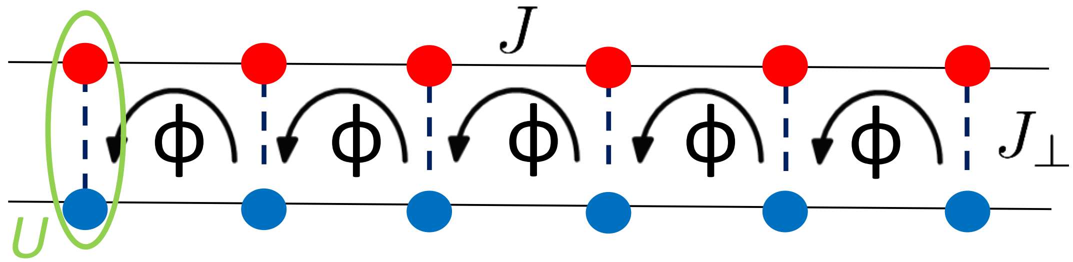
Synthetic ladder: vortex phase disappears in the hard-core limit

more phases at $U \neq \infty$ ....

Meissner/Vortex phase in flux ladder



Effect of interactions: suppress vortex phase

Real ladder: vortex phase survives in the hard-core limit for ϕ large

more phases at $U \neq \infty$

see [Petrescu, Le Hur,PRL 2013]
[Piraud et al, PRB 2015]

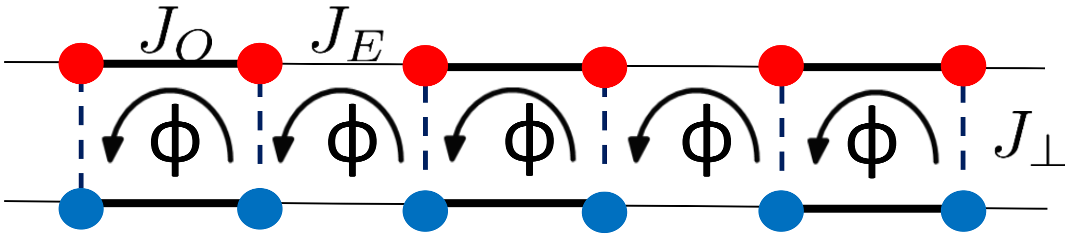
Synthetic ladder: vortex phase disappears in the hard-core limit

more phases at $U \neq \infty$ ....

Idea: nucleate vortices by dimerizing the lattice ("easy" exp. handle)

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

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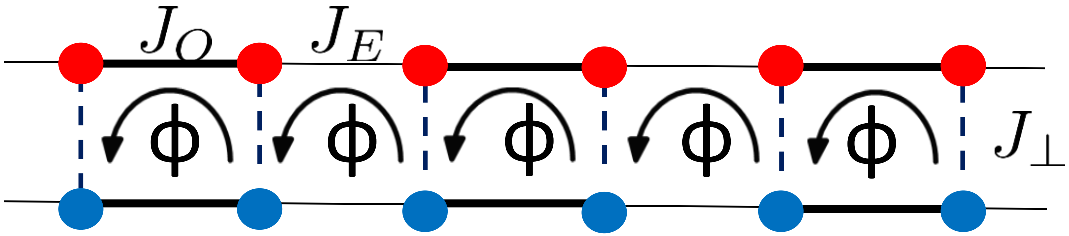
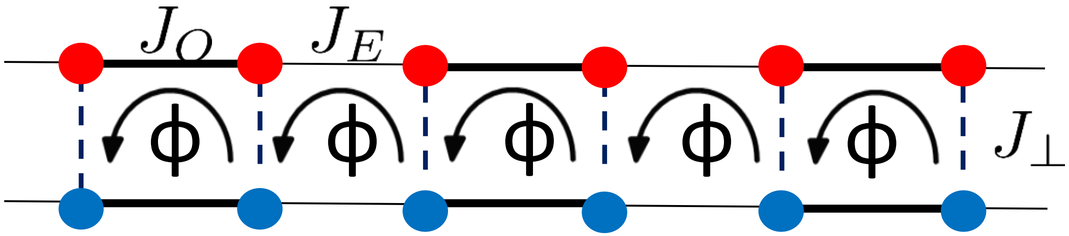


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

No interactions: 4 bands

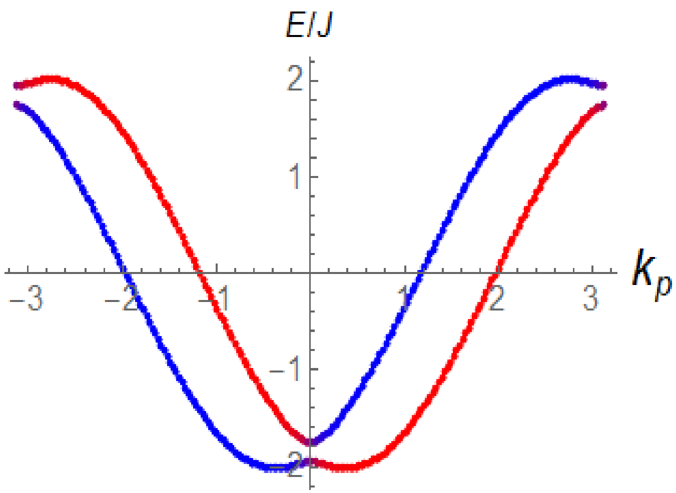
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

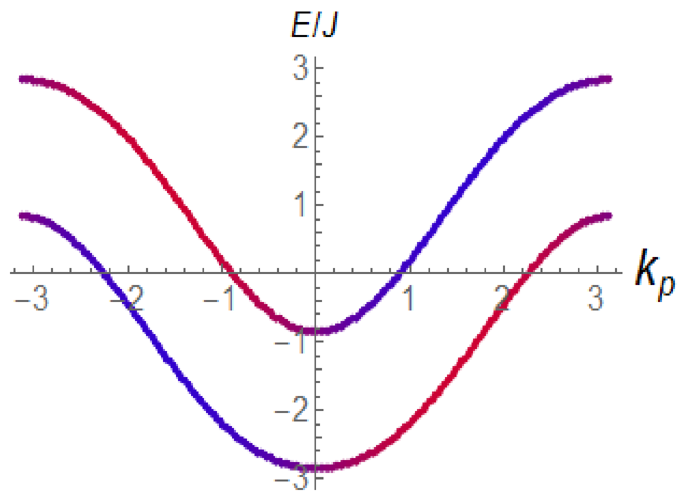
No interactions: 4 bands



$J_\perp \ll J$

$\Delta = 0$

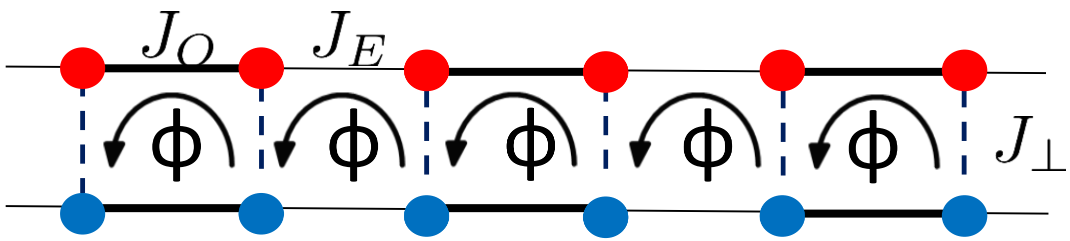
Just band folding



$J_\perp \gtrsim J$

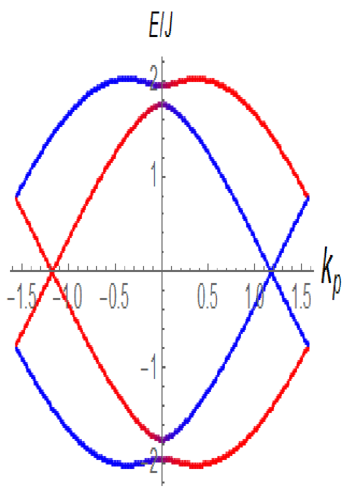
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

No interactions: 4 bands



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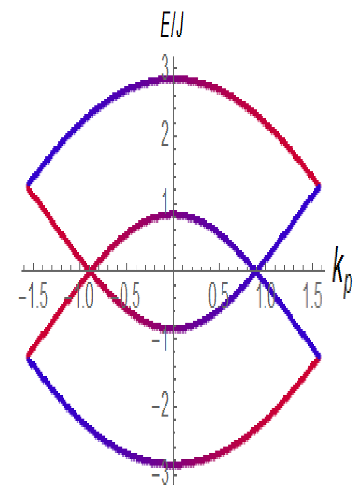
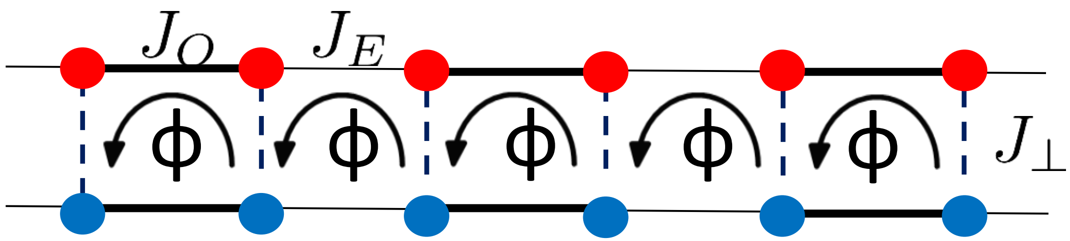


$J_\perp \gtrsim J$

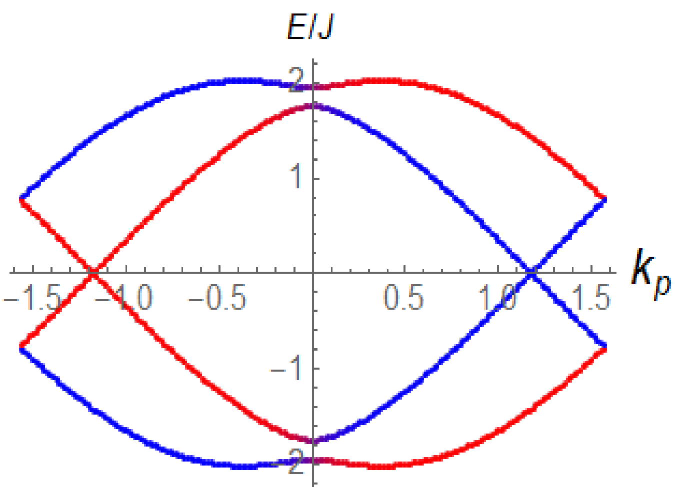
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$$J = \frac{J_O + J_E}{2}$$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

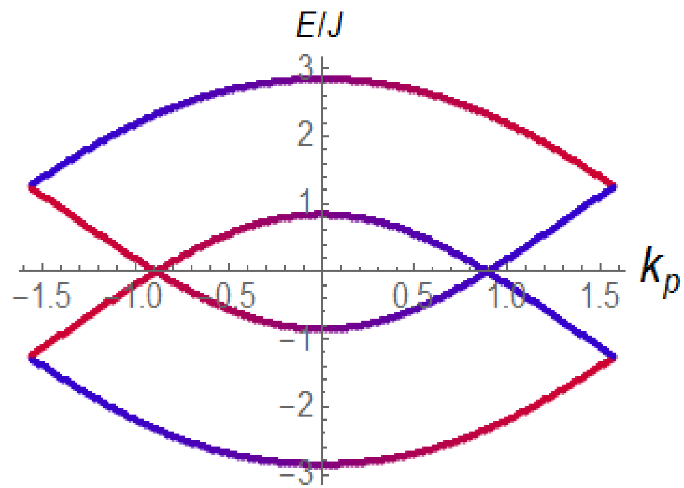
No interactions: 4 bands



$J_\perp \ll J$

$\Delta = 0$

Just band folding



$J_\perp \gtrsim J$

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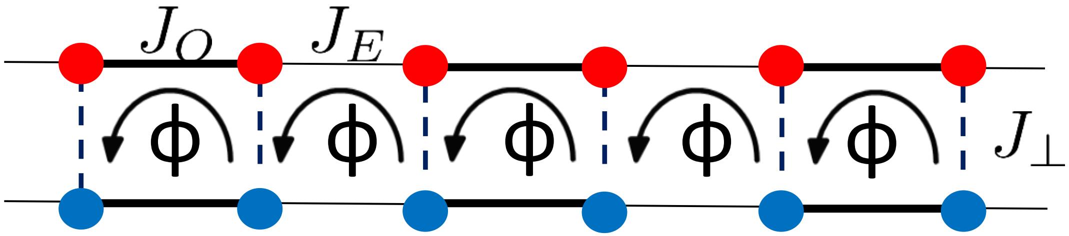
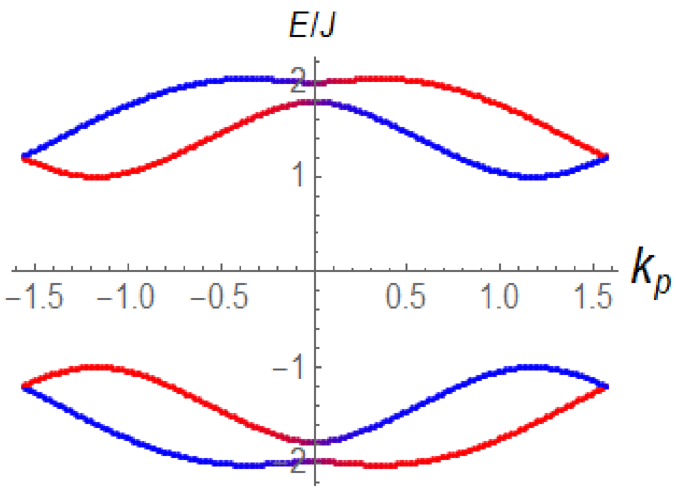


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

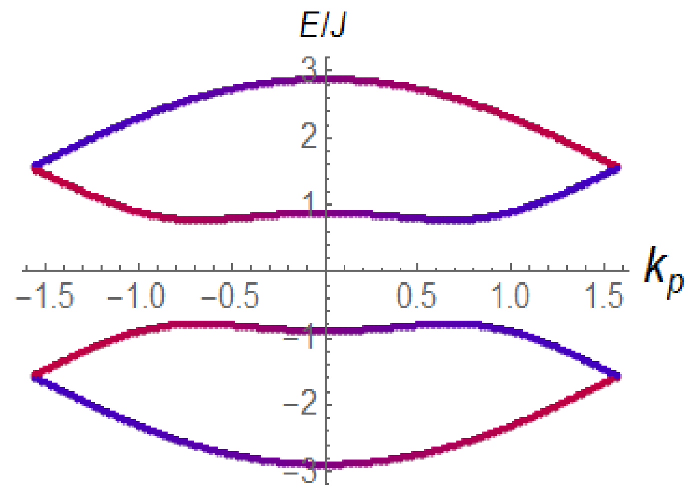
No interactions: 4 bands



$J_\perp \ll J$

$\Delta = 0.5$

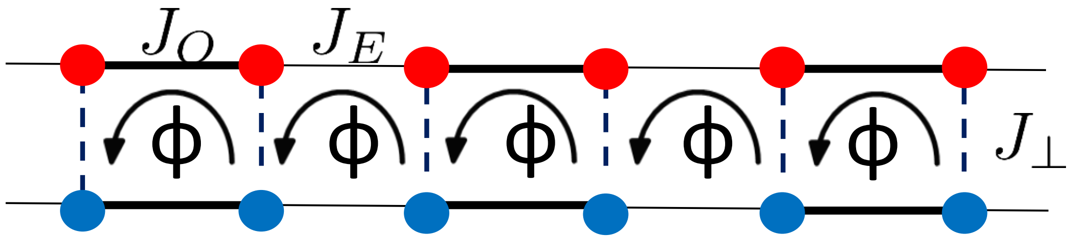
Bands deform & mix



$J_\perp \gtrsim J$

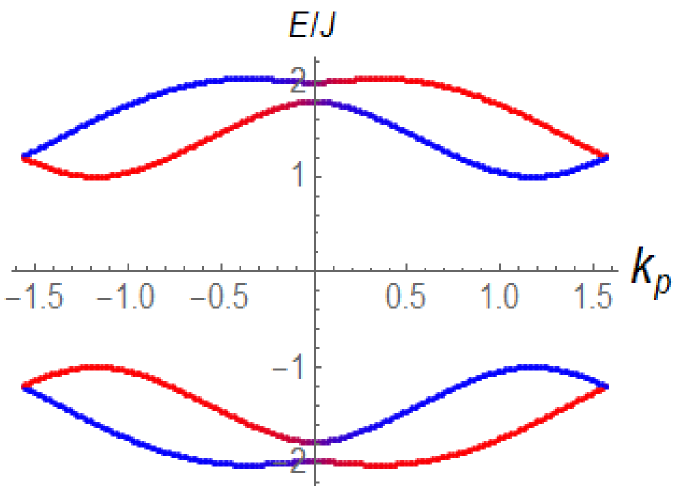
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

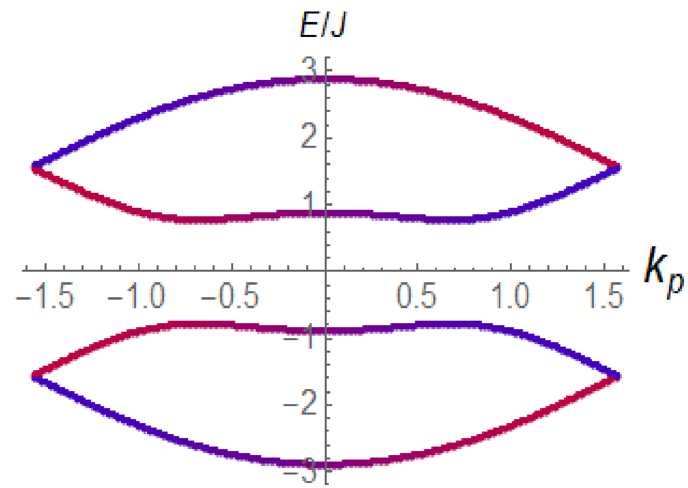
No interactions: 4 bands



$J_\perp \ll J$

$\Delta = 0.5$

Bands deform & mix

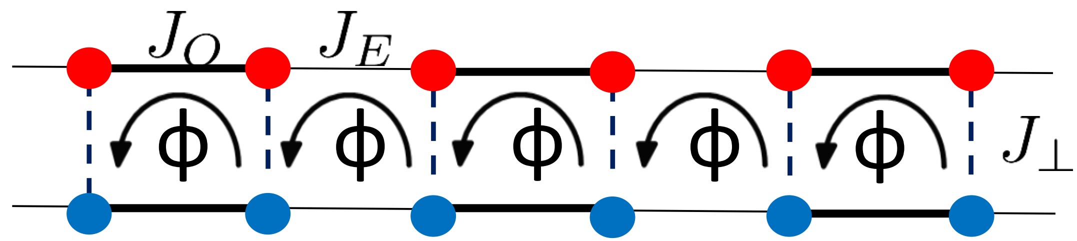


$J_\perp \gtrsim J$

Minima separate: dimerization enhances vortex phase!

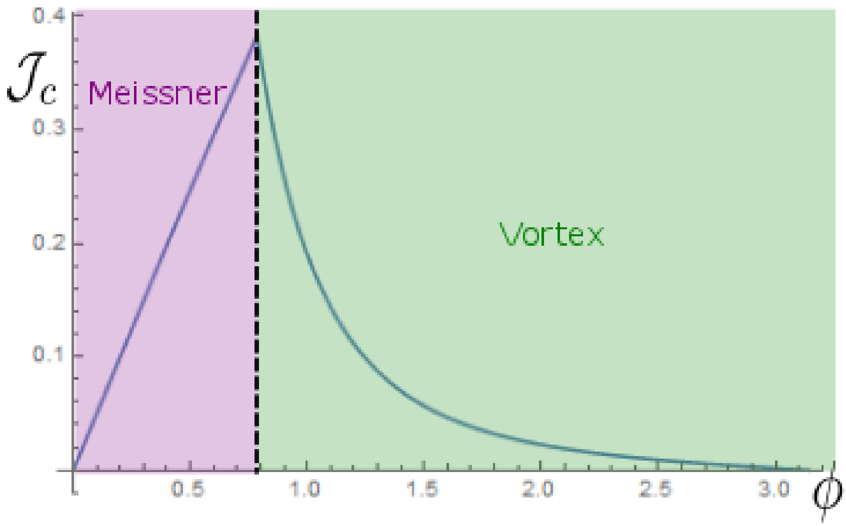
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

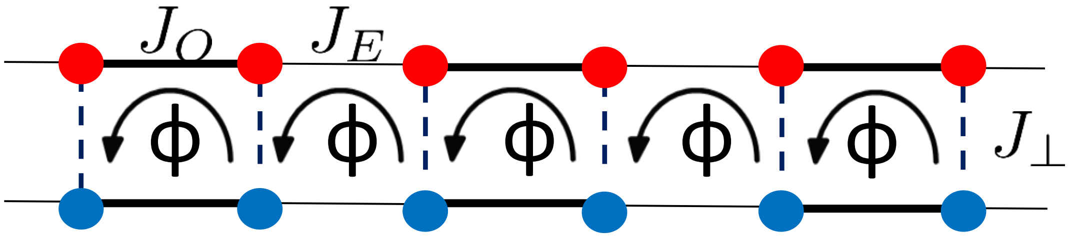
No interactions: Reverse of chiral current



$\Delta = 0$

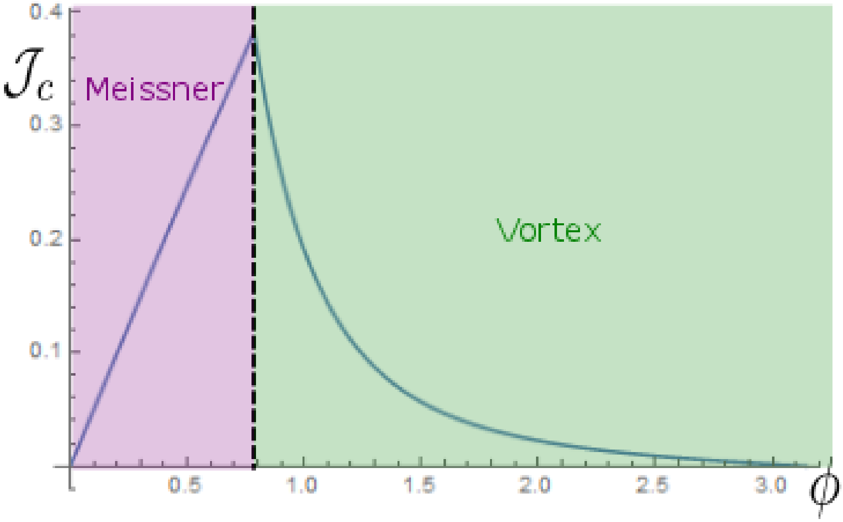
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$

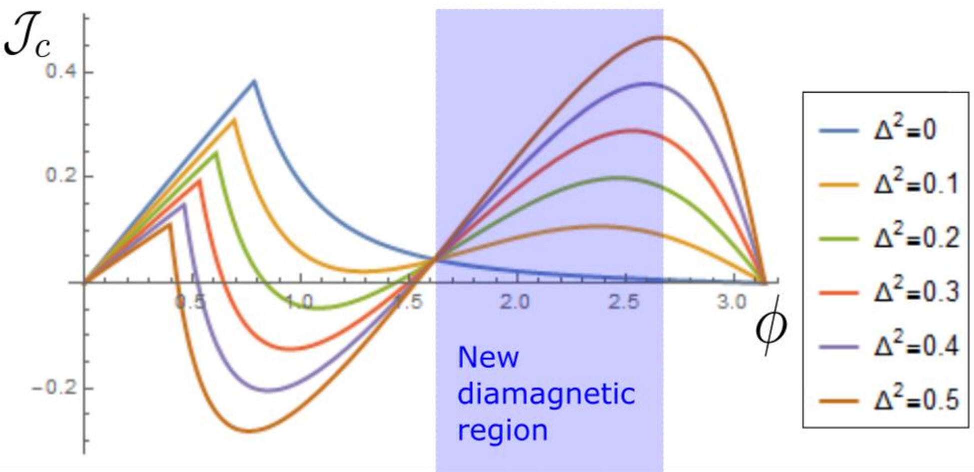


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

No interactions: Reverse of chiral current



$\Delta = 0$

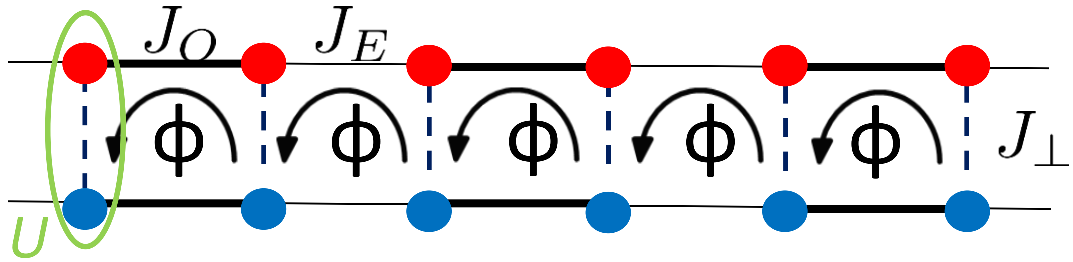


$\Delta \neq 0$

Current behavior confirms vortex enhancement!

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$

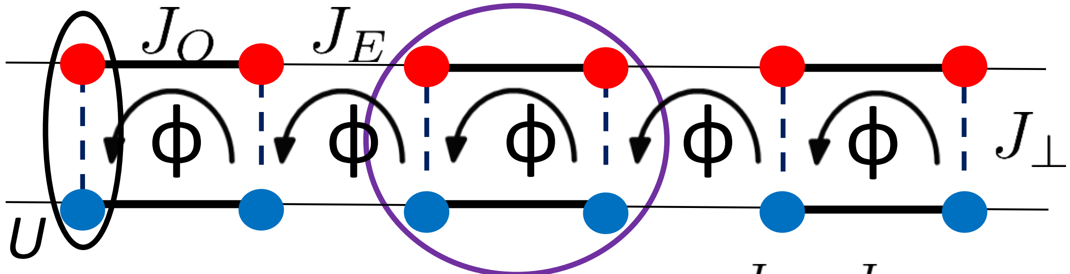


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Interactions: $U \rightarrow \infty$ 3 states per rung

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



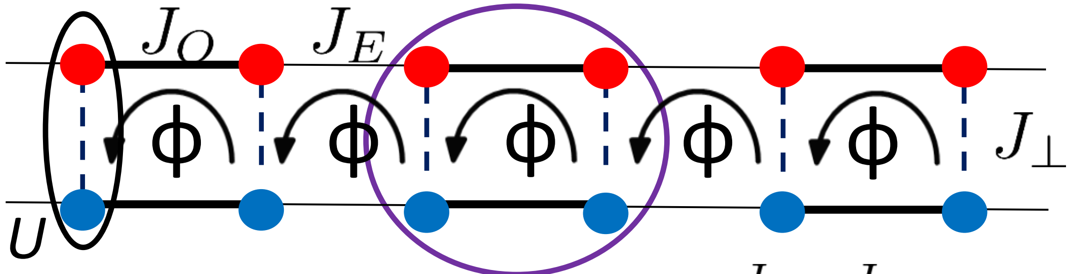
Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Interactions: $U \to \infty$ 3 states per rung

$J_E \ll J_O$ 9 states per plaquette

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

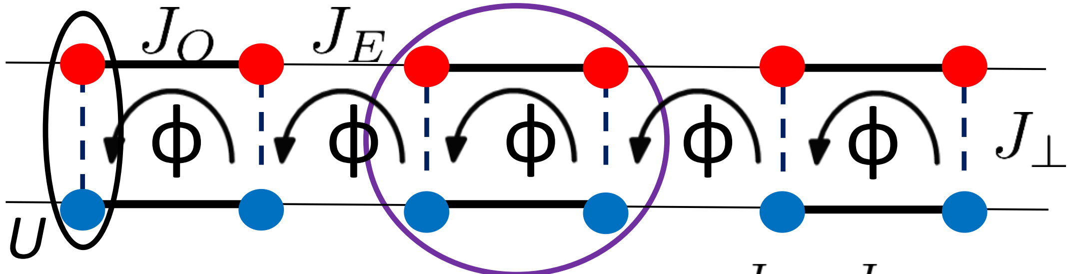
Interactions: $U \to \infty$ 3 states per rung

$J_E \ll J_O$ 9 states per plaquette

	1 *n=0,*	4 *n=1,*	4 *n=2*
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm 2J_\perp$
		$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	± 0

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Interactions: $U \to \infty$ 3 states per rung

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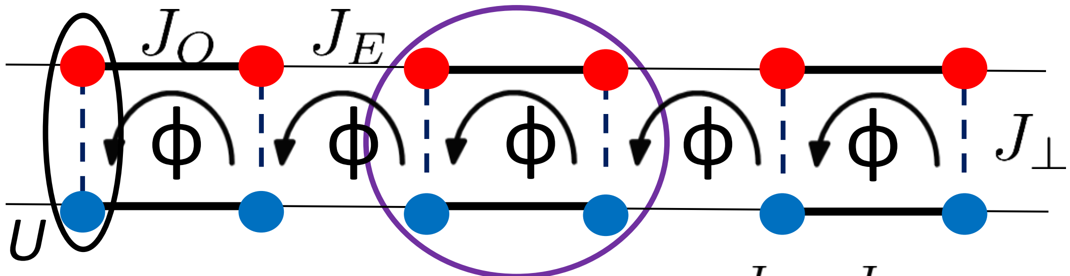
	1 *n=0,*	4 *n=1,*	4 *n=2*
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$ $\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm 2J_\perp$ ± 0

$J_\perp \gtrsim J_O$ Plaquette in *n=2* → Band insulator

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Interactions: $U \to \infty$ 3 states per rung

$J_E \ll J_O$ 9 states per plaquette

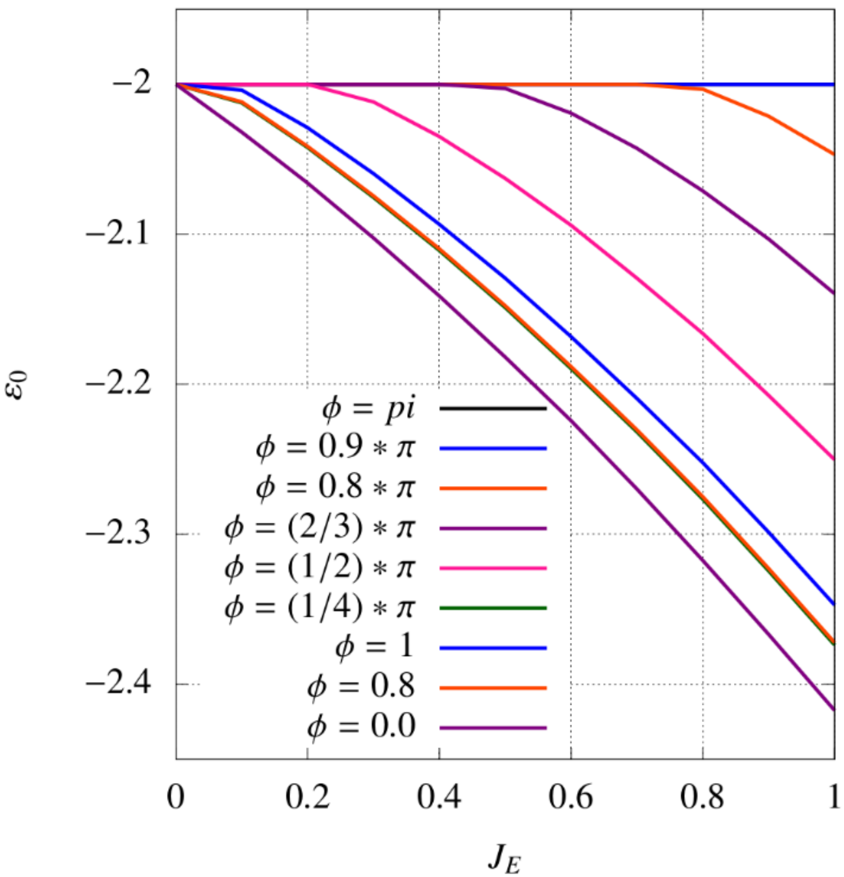
	1 *n=0,*	4 *n=1,*	4 *n=2*
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm 2J_\perp$
		$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	± 0

$J_\perp \gtrsim J_O$ Plaquette in *n=2* → Band insulator

$J_\perp < J_O$ Plaquette in *n=1* → Imprinted vortex

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

DMRG calculations confirm perturbative expectations Ex. $J_\perp = J_O = 1$



Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

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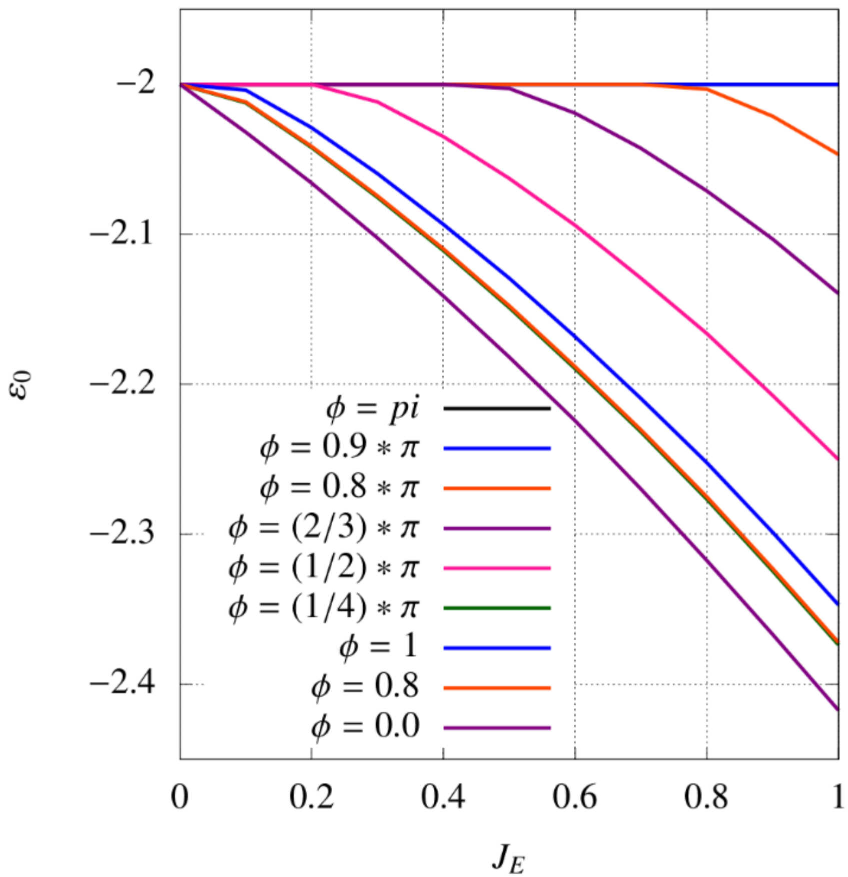
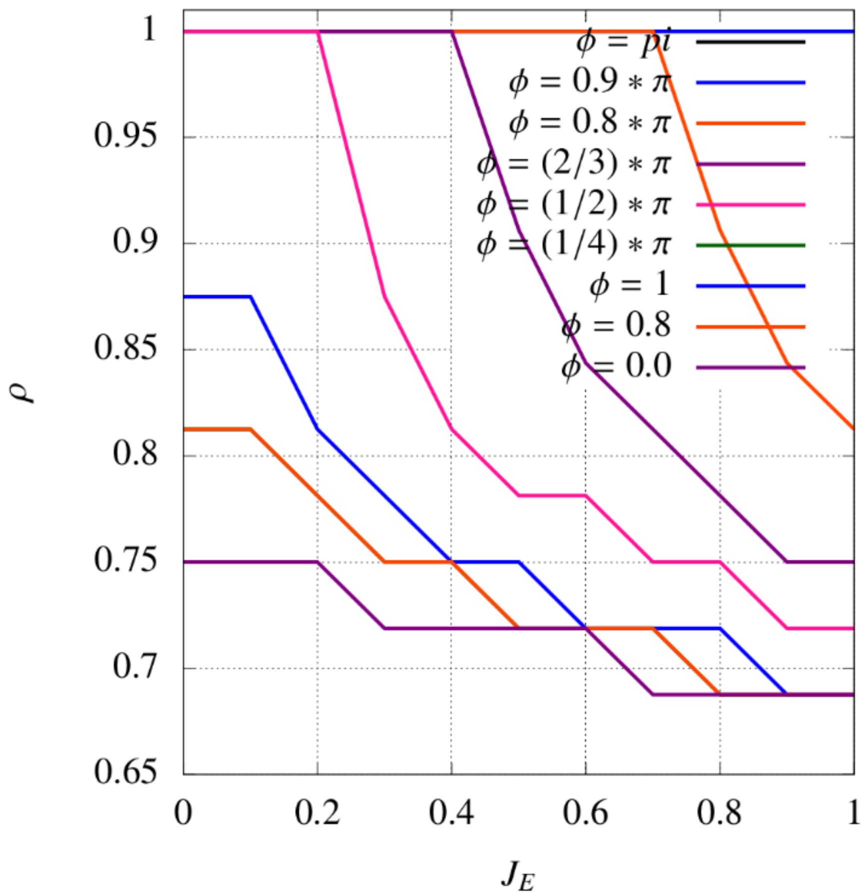
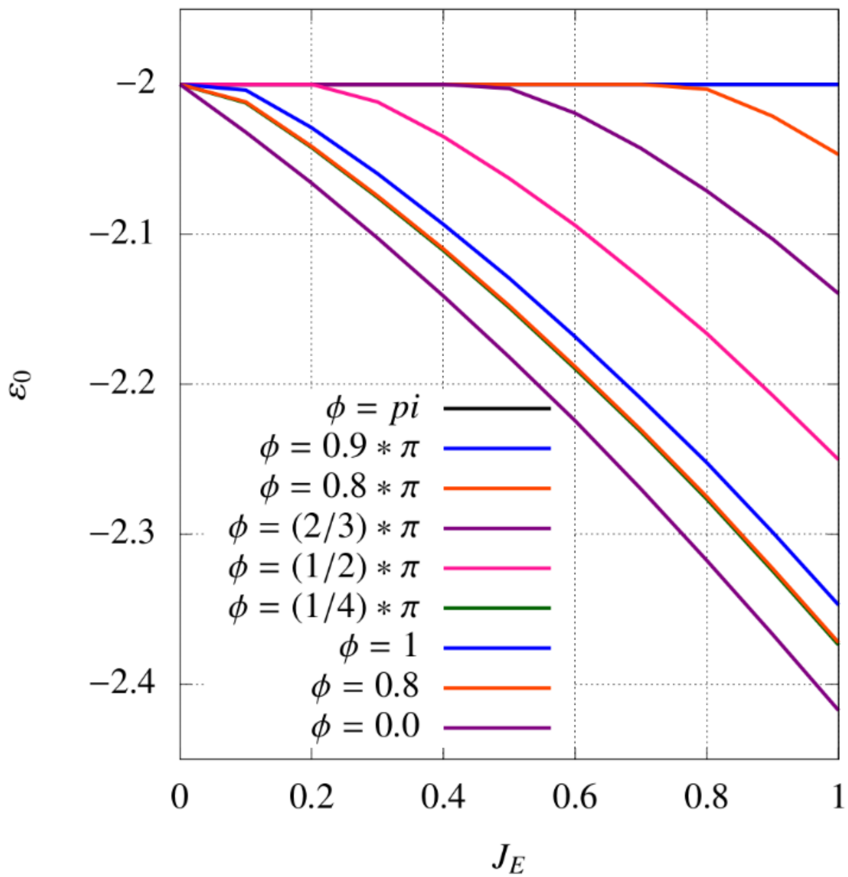
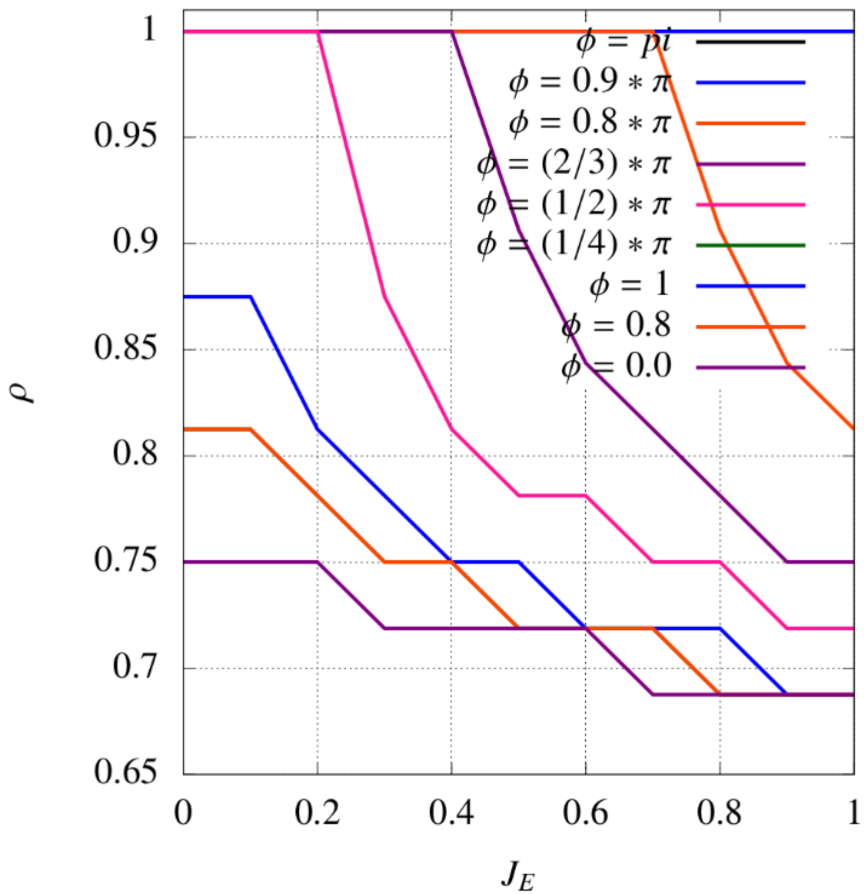




Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

DMRG calculations confirm perturbative expectations Ex. $J_\perp = J_O = 1$

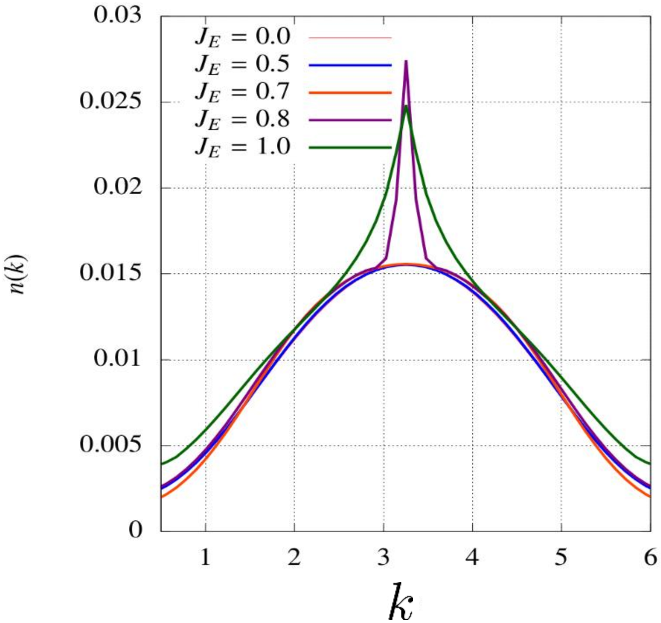


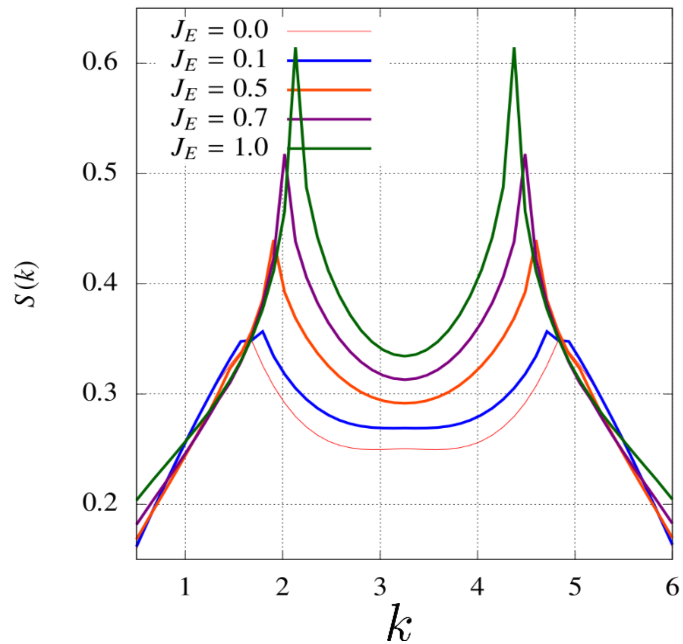


Phase diagram through calculation of currents and structure factors

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J_\perp = J_O = 1$ Ex. $\phi = \pi/3$



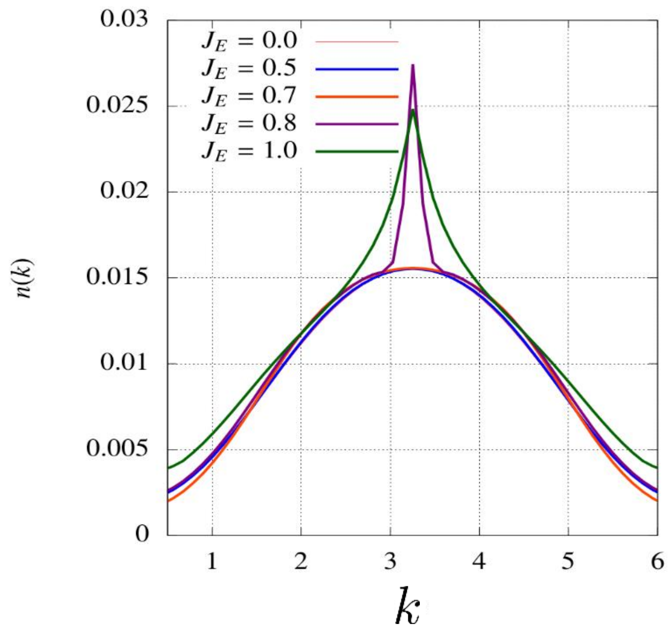


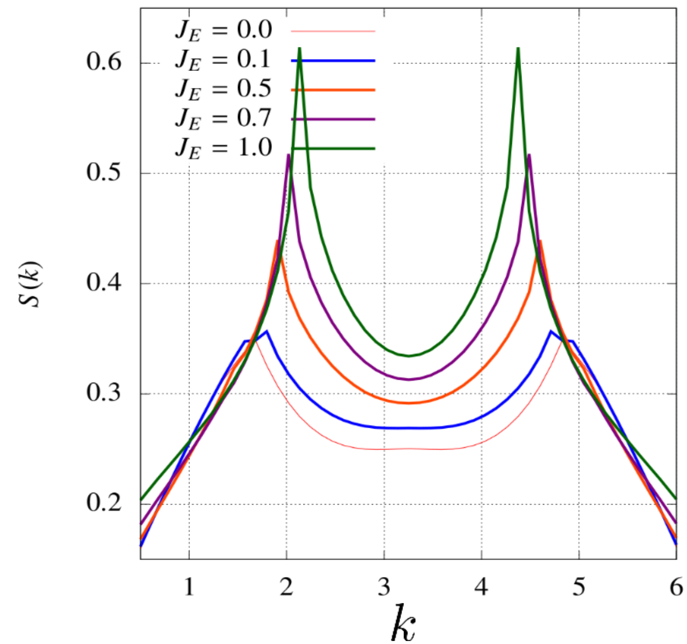
0 → J_E → J_O

Melted vortex → Meissner charge density wave → Meissner

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J_\perp = J_O = 1$ Ex. $\phi = \pi/3$





$0 \longrightarrow J_E \longrightarrow J_O$

Melted vortex ⟶ Meissner charge density wave ⟶ Meissner

Similar to attractive interleg interaction, [Orignac et al, PRB 96, 014518 (2017)]

Evidence of commensurate-incommensurate transition

Further steps

- No hard-core boson limit: bosons different fermions

- Study the accessible experimental parameters

- Search for "visible" Laughlin-like states in such regimes

cf. [Calvanese et al, PRX 7, 021033 (2017)],[Petrescu et al, PRB 96, 014524 (2017)]

….. *Hopefully many more*

Summary

- Synthetic edge state in synthetic Hofstadter strips
- “Bulk topology” in synthetic Hofstadter strips
- Effect of dimerization in synthetic Hofstadter ladder w/o interactions

Outlook

- Better understanding decoupling argument with interactions
- Achieve different interaction patterns in synthetic lattices than *SU(N)*
- Comparison between dimerized ladders and 4-leg strips
- …

“Extradimensional” collaborators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O. Boada


M. Lewenstein

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O. Boada


M. Lewenstein


T. Grass

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P. Massignan


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J. Rodriguez-Laguna


C. Muschik


R.W. Chhajlany


S. Mugel


J. Asboth


C. Lobo

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