The Abdus Salam International Centre for Theoretical Physics

Workshop on Quantum Science and Quantum Technologies

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The Wigner function as a measure of quantum correlations

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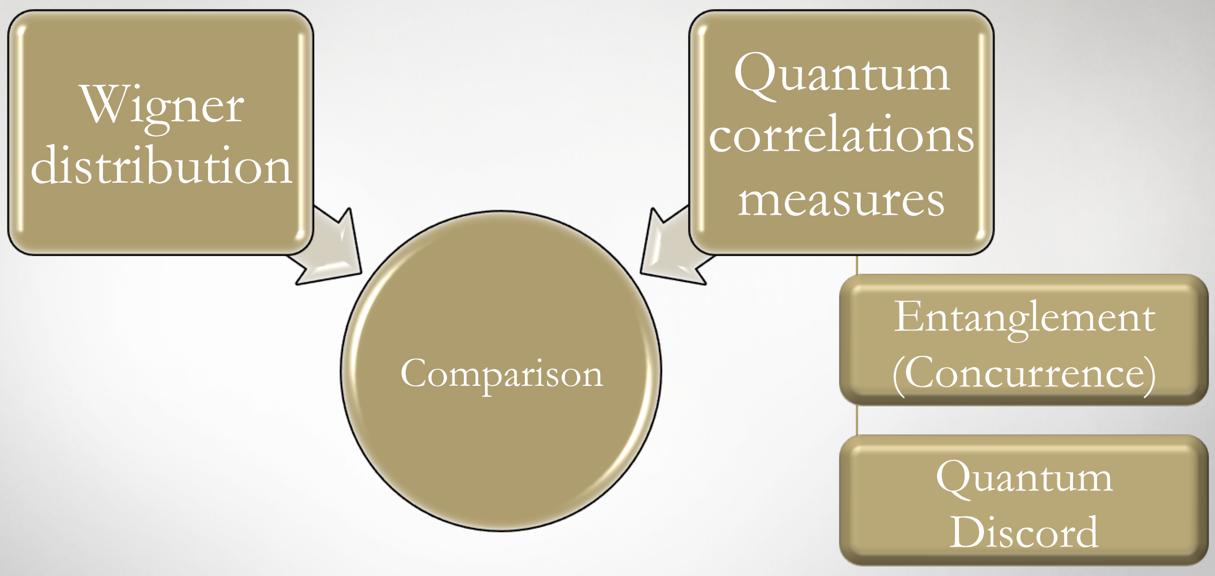


F. Siyouri, M. Ziane, Y. Hassouni, M.E.B.

Quantum correlations: Entanglement

Entanglement measures : Concurrence

Bipartite entanglement in the state ρ_{XY}

$$C(\rho_{XY}) = \left(\max\left[0, \left(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)\right]\right)$$

with λ_1, λ_2, λ_3 and λ_4 are the eigenvalues of $\rho_{XY}\tilde{\rho}_{XY}$ arranged in decreasing order.

'Spin flipped' density matrix is defined as $\tilde{\rho}_{XY} = (\sigma_y \otimes \sigma_y)\rho^*_{XY}(\sigma_y \otimes \sigma_y)$

Entanglement measures : Entanglement of formation

$$E(\rho_{XY}) = H(x) = -x \log x - (1-x)\log(1-x)$$

$$x = \frac{1}{2}(1 + \sqrt{1 - C^2})$$

Quantum correlations: Quantum discord

Total correlations in a bipartite system ρ_{XY}

$$I(X:Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{X,Y})$$

Quantum correlations in ρ_{XY}

$$J(X:Y)_{\{\prod_j^Y\}} = S(\rho_X) - S(\rho_{X|\{\prod_j^Y\}})$$

Quantum discord in a bipartite system ρ_{XY}

$$D(X:Y)_{\{\prod_j^Y\}} = I(X:Y) - \max_{\{\prod_j^Y\}}\left[J(X:Y)_{\{\prod_j^Y\}}\right]$$

$$= S(\rho_Y) - S(\rho_{X,Y}) + \min_{\{\prod_j^Y\}} S(\rho_{X|\{\prod_j^Y\}})$$

Wigner Function

Phase space representation of the state ρ:

$$W(q,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} dy\, \mathrm{e}^{2ipy}\langle q-y|\rho|q+y\rangle$$

Properties:

♫- $W(q,p)$ is real.

♫- $\int\int W(q,p)dqdp = 1$

♫- $\langle A\rangle = Tr\left[\rho A\right] = \int\int W(q,p)\tilde{A}\left(q,p\right)dqdp$

with $\tilde{A}(q,p) = \int_{-\infty}^{\infty} dy\, \mathrm{e}^{2ipy}\langle q-y|A|q+y\rangle$ Weyl transform of operator A.

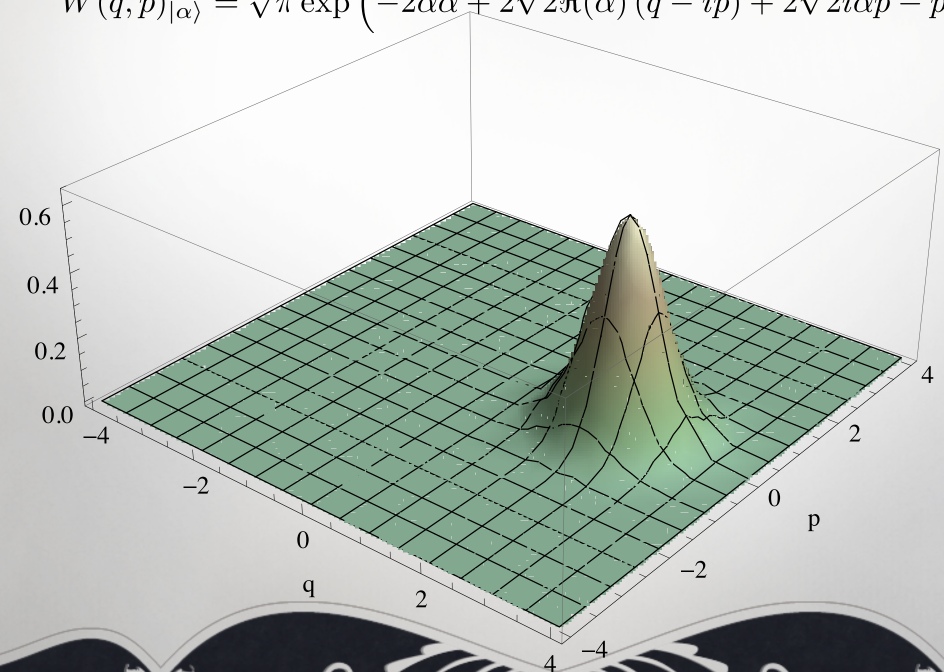
♫- The Wigner function can be negative ➔ A quasiprobability distribution.

Wigner Function : (*Examples*)

Wigner Function : (Examples)

♫- Coherent states : $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}} |n\rangle$

$$W(q,p)_{|\alpha\rangle} = \sqrt{\pi} \exp\left(-2\alpha\bar{\alpha} + 2\sqrt{2}\Re(\alpha)(q - ip) + 2\sqrt{2}i\alpha p - p^2 - q^2\right)$$



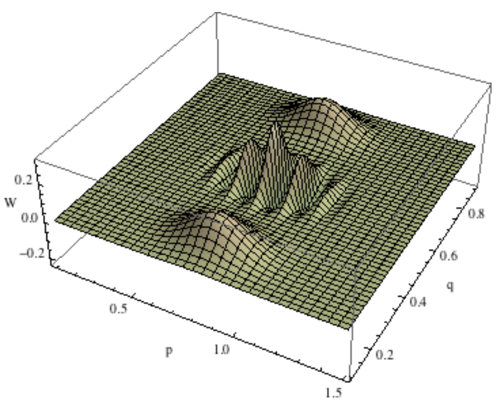
Wigner Function : (*Examples*)

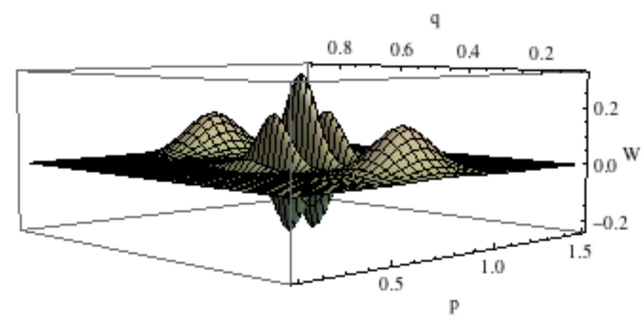
♫- Superposition of Coherent states (cat states) : $|\pm_\alpha\rangle = N_\pm(\alpha)\Big[|\alpha\rangle \pm |-\alpha\rangle\Big]$

$$N_\pm(\alpha) = \frac{1}{\sqrt{2\left(1 \pm \mathrm{e}^{-2|\alpha|^2}\right)}}$$

$$W(q,p) = n\,\pi^{\frac{-3}{2}}\left(W_{\alpha\,\alpha} + W_{\alpha\,-\alpha} + W_{-\alpha\,\alpha} + W_{-\alpha\,-\alpha}\right)$$

$$W_{ij} = \sqrt{\pi}\exp\Big[-(p^2) + (I\sqrt{2}p(\alpha_j - \alpha_i^*)) - (\frac{1}{2}(|\alpha_i|^2 + |\alpha_j|^2)) - q^2 + (\sqrt{2}q(\alpha_j + \alpha_i^*)) - (\alpha_i^*\alpha_j)\Big]$$





Bipartite coherent states - Werner states

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}} |n\rangle$$

♫- Superposition of two bipartite coherent states : $|\psi^{\pm}\rangle_{XY} = n_{\pm}\Big[|\alpha,\beta\rangle \pm |-\alpha,-\beta\rangle\Big]_{XY}$

$$n_{\pm}(\alpha) = \frac{1}{\sqrt{2\left(1 \pm e^{-2|\alpha|^2} e^{-2|\beta|^2}\right)}}$$

Remember : coherent states $|+\alpha\rangle$ and $|-\alpha\rangle$ are not mutually orthogonal.

But : cat states $|+\rangle$ and $|-\rangle$ are.

We rewrite $|\psi^{\pm}\rangle_{XY}$ using $|\pm\rangle_{\alpha} = N_{\pm}(\alpha)\Big[|\alpha\rangle \pm |-\alpha\rangle\Big]$ and $|\pm\rangle_{\beta} = N_{\pm}(\beta)\Big[|\beta\rangle \pm |-\beta\rangle\Big]$

We find

$$|\psi^{+}\rangle_{XY} = \frac{n^{+}}{2}\left[\frac{|+_{\alpha},+_{\beta}\rangle}{N_{+}^{\alpha}N_{+}^{\beta}} + \frac{|-_{\alpha},-_{\beta}\rangle}{N_{-}^{\alpha}N_{-}^{\beta}}\right]_{XY} \qquad |\psi^{-}\rangle_{XY} = \frac{n^{-}}{2}[\frac{|+_{\alpha},-_{\beta}\rangle}{N_{+}^{\alpha}N_{-}^{\beta}} + \frac{|-_{\alpha},+_{\beta}\rangle}{N_{-}^{\alpha}N_{+}^{\beta}})]_{XY}$$

Bipartite coherent states - Werner states

Bipartite coherent states - Werner states

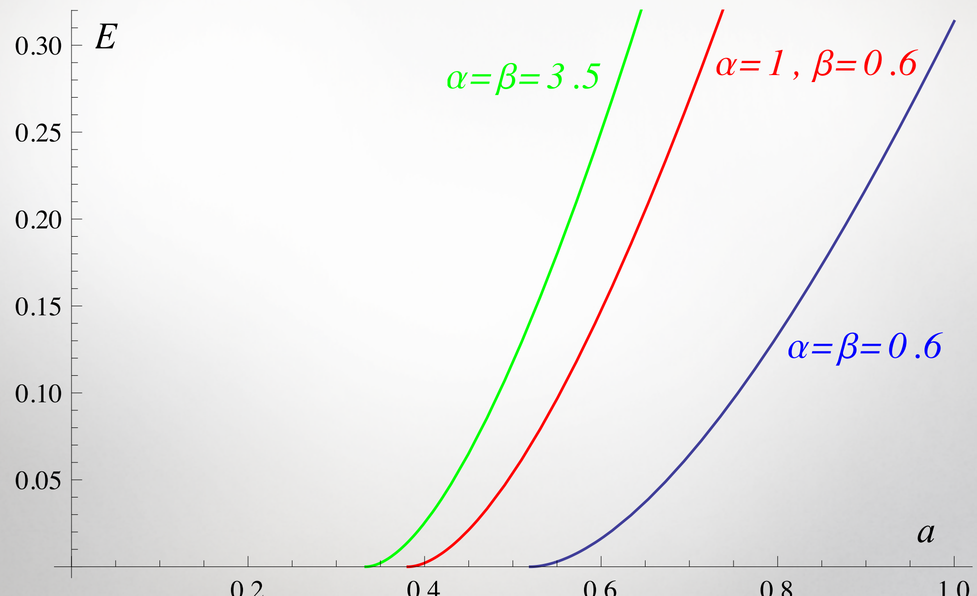
♬- Werner superposed coherent states :

$$\rho(\psi^{\pm}, a) = (1-a)\frac{I}{4} + a\left|\psi^{\pm}\right\rangle\left\langle\psi^{\pm}\right|$$

Behavior of quantum correlations

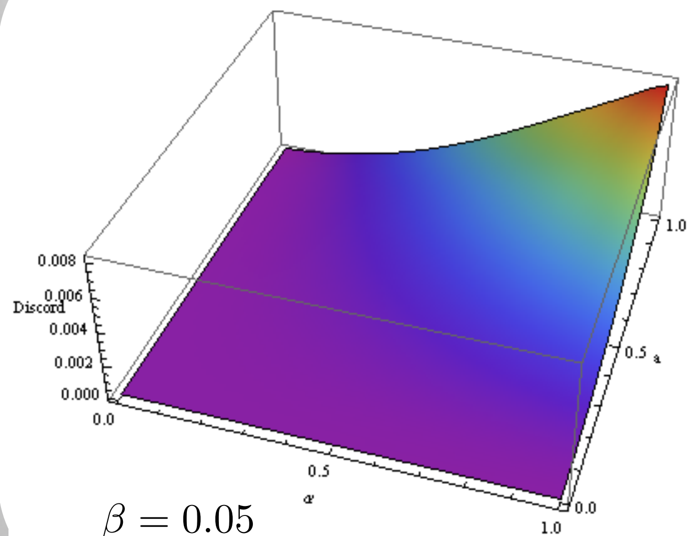
Behavior of quantum correlations

♫- Entanglement (Entanglement of formation):

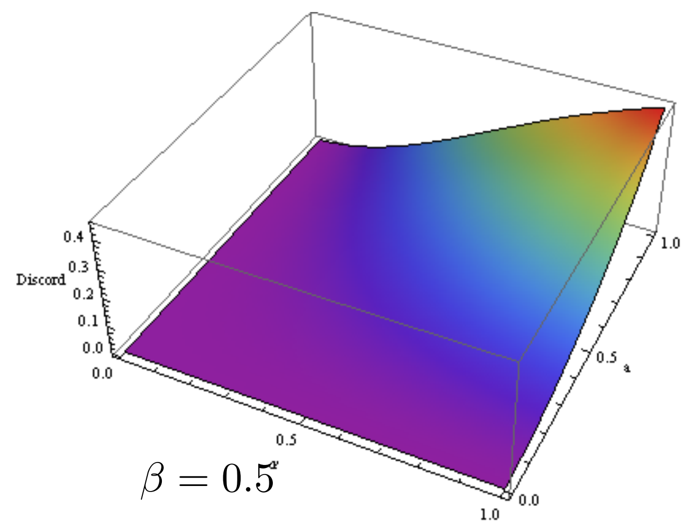


Behavior of quantum correlations

♫- Quantum discord : $D\left(\rho\left(\psi^{+}\right)\right)$



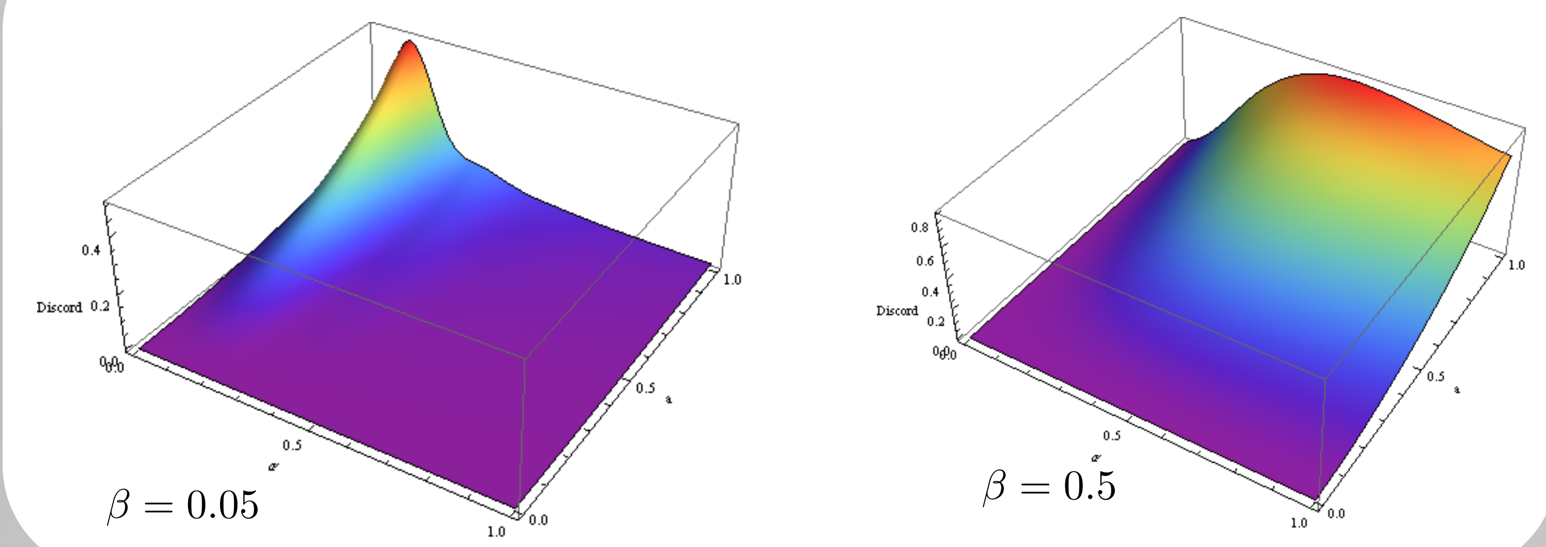
$\beta = 0.05$



$\beta = 0.5$

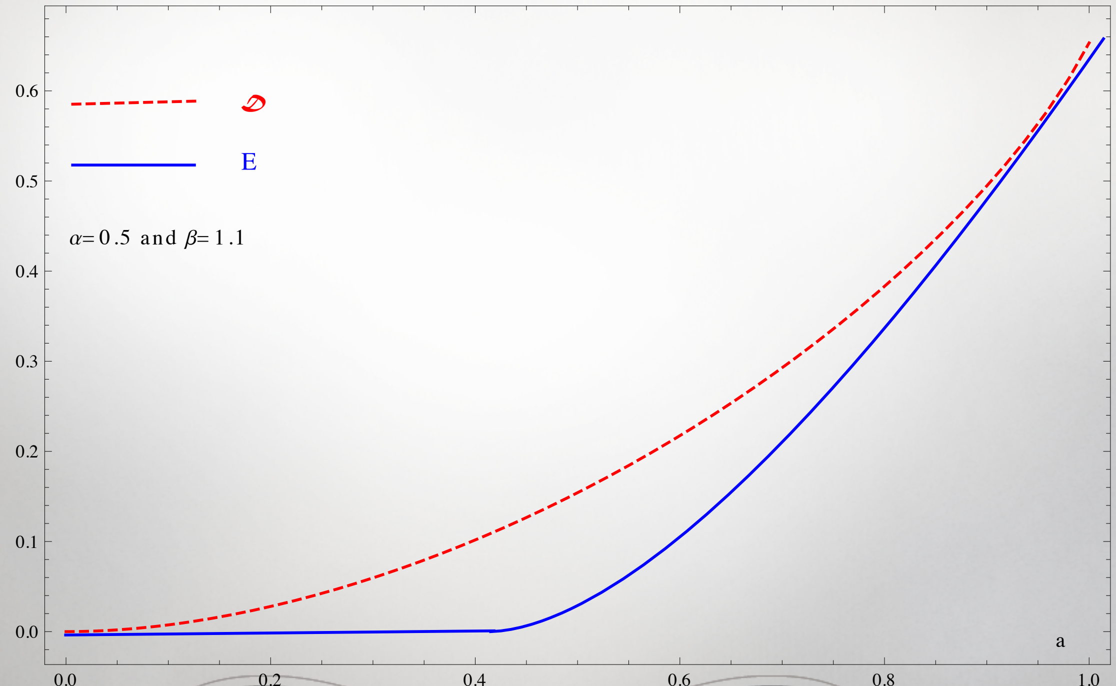
Behavior of quantum correlations

♫- Quantum discord : $D\left(\rho\left(\psi^{-}\right)\right)$



Behavior of quantum correlations

♫- Quantum discord / Entanglement of formation :



Behavior of the Wigner function

♫- Wigner function of a bipartite state : $W(q_1, p_1, q_2, p_2) = W(q_1, p_1)\, W(q_2, p_2)$

♫- Wigner function of the bipartite superposed coherent states :

$$W(q_1, p_1, q_2, p_2) = n^2(\pi^{\frac{-3}{2}})^2 \Big(W_{\alpha\ \alpha}\, W_{\beta\ \beta} + W_{\alpha\ -\alpha}\, W_{\beta\ -\beta} + W_{-\alpha\ \alpha}\, W_{-\beta\ \beta} + W_{-\alpha\ -\alpha}\, W_{-\beta\ -\beta} \Big)$$

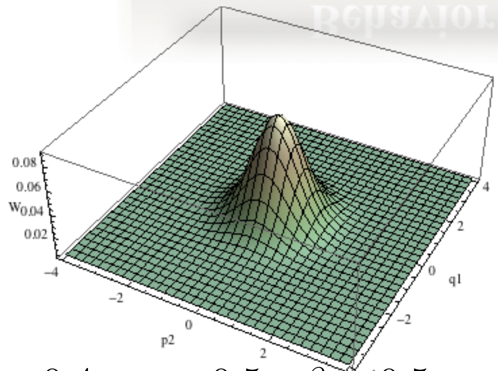
♫- Wigner function of the Werner superposed coherent states:

$$W_{2mcs}(q_1, p_1, q_2, p_2) = \frac{(1-a)}{4\pi^2} + aW(q_1, p_1, q_2, p_2)$$

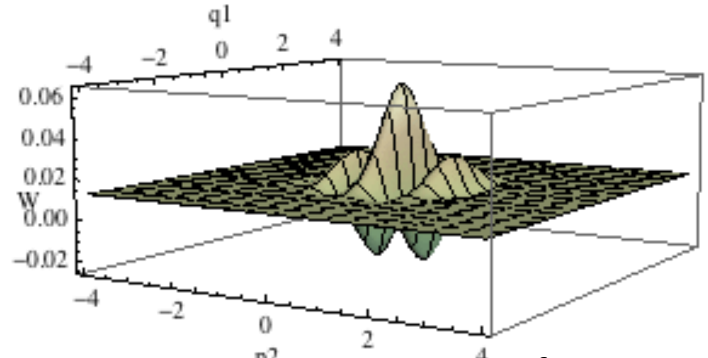
♫- Negativity of the Wigner function:

$$\delta = \int\int |(W_{2mcs}(q_1, p_2))| - W_{2mcs}(q_1, p_2)\ dp_1\ dq_2$$

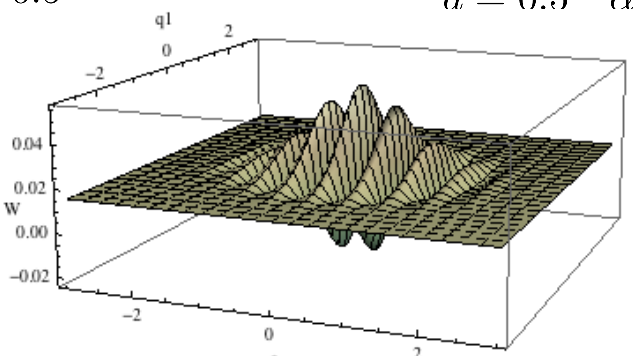
Behavior of the Wigner function



$a = 0.4 \quad \alpha = 0.5 \quad \beta = 0.5$



$a = 0.5 \quad \alpha = 3.5 \quad \beta = 2$



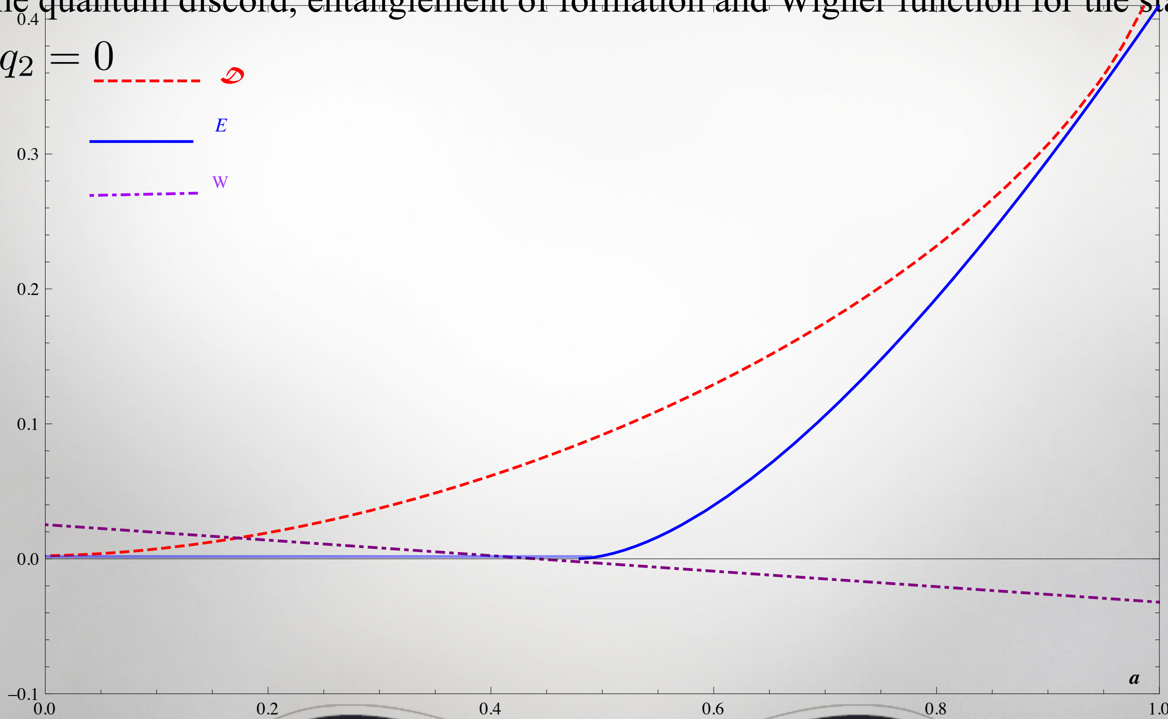
$a = 0.4 \quad \alpha = 3.5 \quad \beta = 4.5$

Quantum correlations vs Wigner function

Quantum correlations vs Wigner function

♫- Plot the quantum discord, entanglement of formation and Wigner function for the state $\rho(\psi^+, a)$

$p_1 = q_2 = 0$



Quantum correlations vs Wigner function

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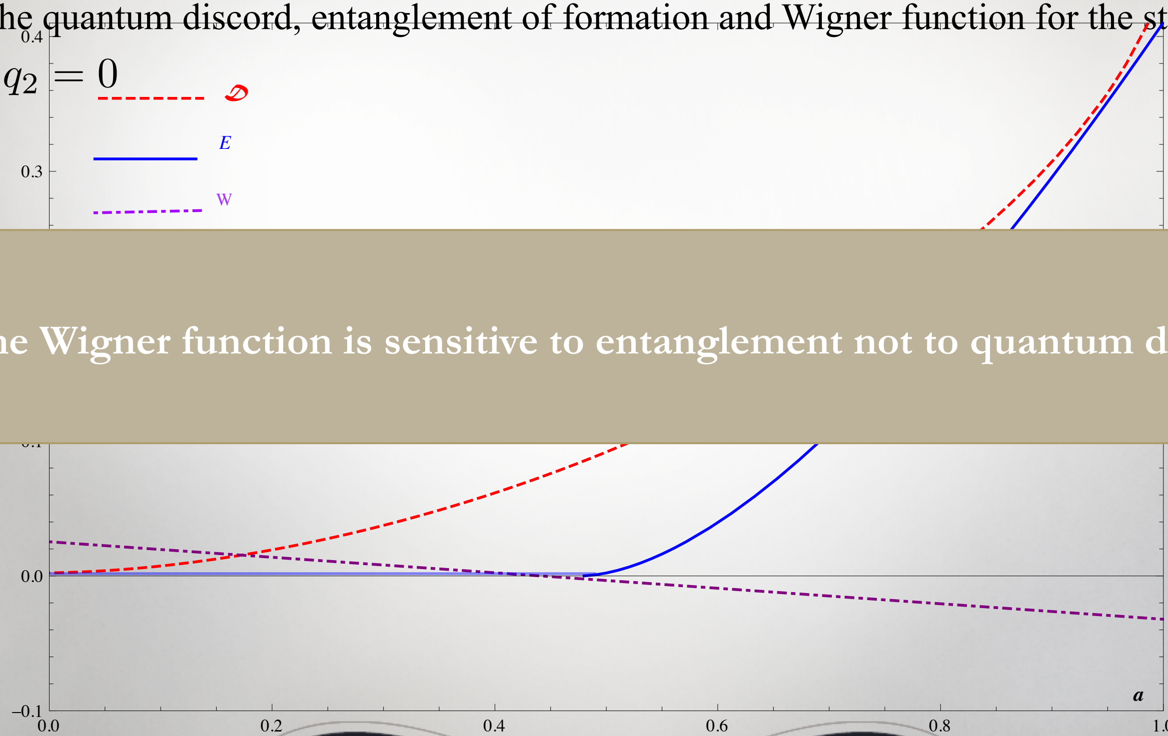
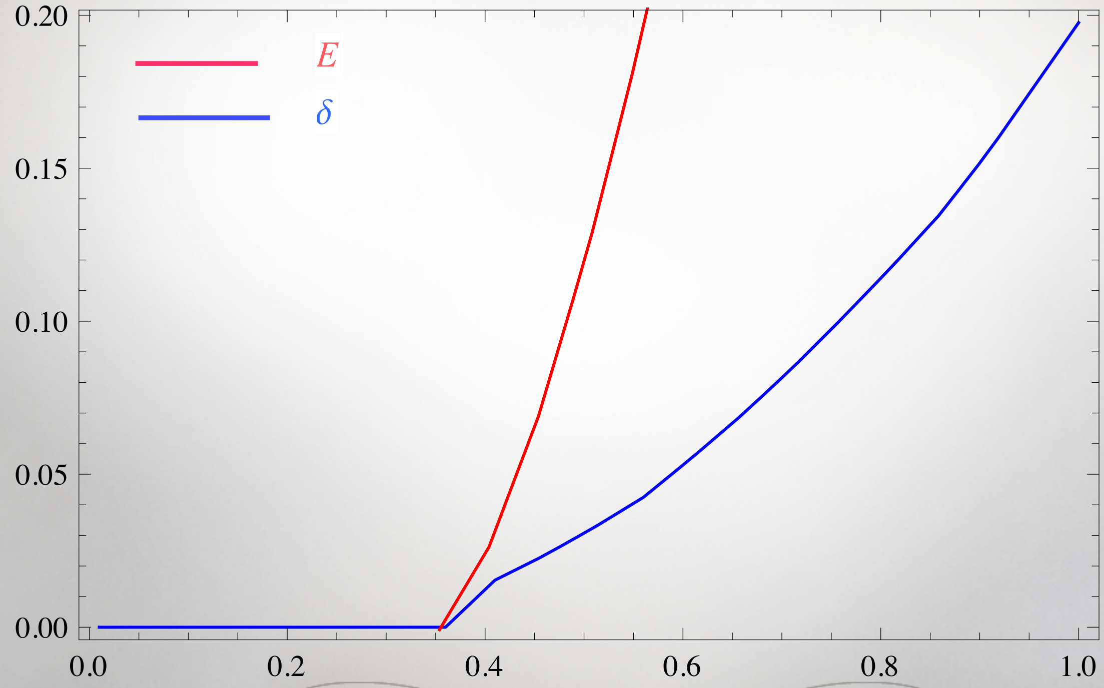


The Wigner function is sensitive to entanglement not to quantum discord

Quantum correlations vs Wigner function

Quantum correlations vs Wigner function

♬- Can the Wigner function provide a measure for Entanglement?

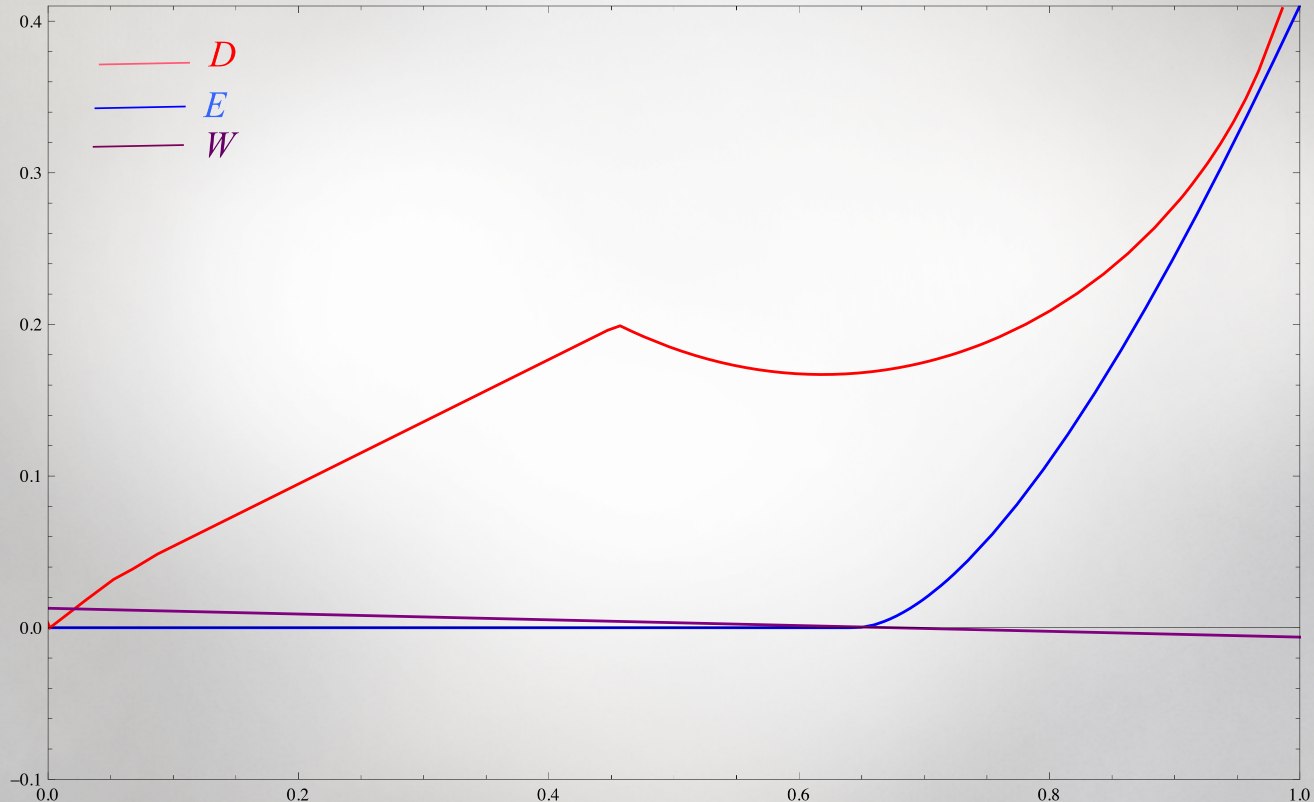


Wigner function does detect and measure quantum correlations!

But not all quantum correlations!!

Only Entanglement!!!

What about multipartite correlations?


D
E
W
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.2
0.4
0.6
0.8
1.0

What's next?

Study the possibility of using Wigner function to detect/ measure multipartite quantum correlations.

Apply all this in a truly continuous variable framework.

F. Siyouri, Y. Hassouni & M.E.B. quant. Inf. Proc. 15 (2016) 4237

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Thank you

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