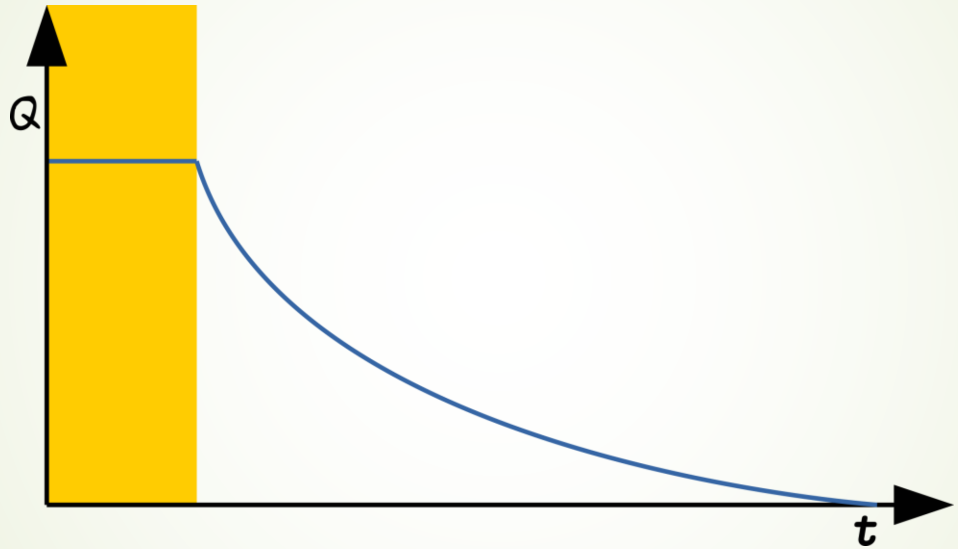
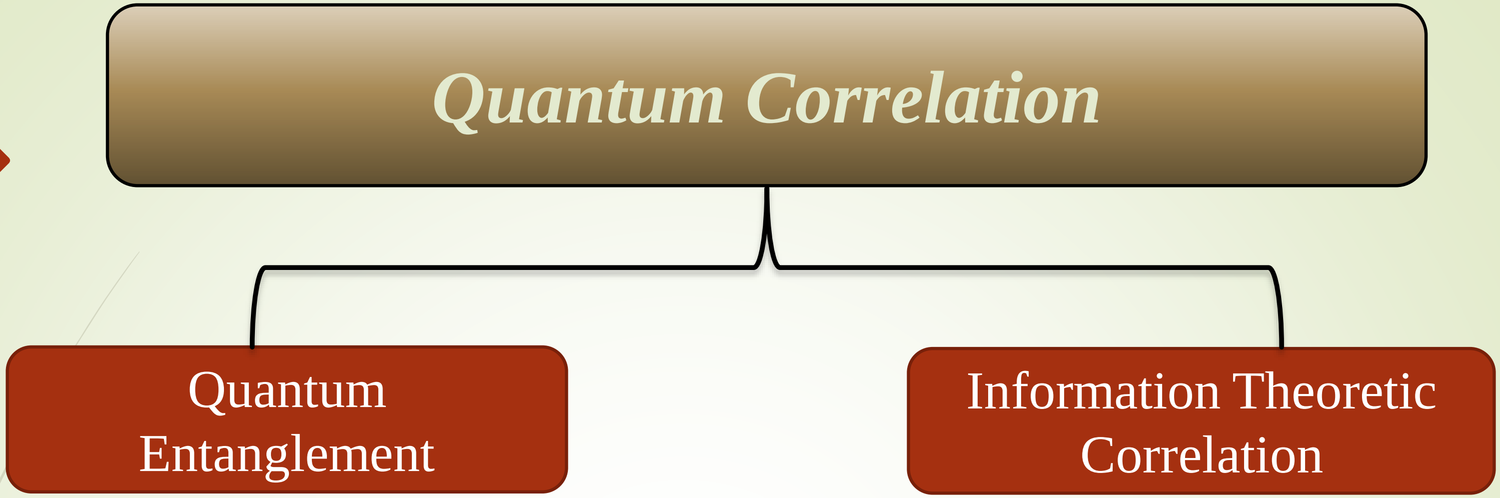
FROZEN QUANTUM CORRELATIONS



Titas Chanda

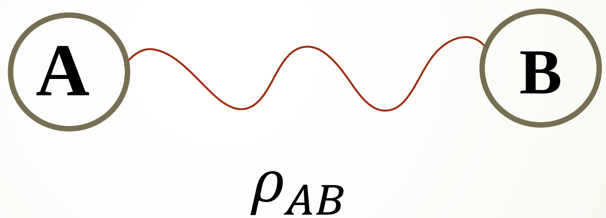
Harish-Chandra Research Institute, Allahabad, India


Quantum Correlation
Quantum Entanglement
Information Theoretic Correlation

Quantum Correlation

Quantum Entanglement

Information Theoretic Correlation



Quantum Correlation

Quantum Entanglement

Information Theoretic Correlation

ρ_{AB}

$$\rho_{AB} \neq \sum_i p_i \rho_A^i \otimes \rho_B^i$$

Quantum Correlation

Quantum Entanglement	Information Theoretic Correlation

A

B

ρ_{AB}

$$\rho_{AB} \neq \sum_i p_i \rho_A^i \otimes \rho_B^i$$

$$\rho_{AB} \neq \sum_i p_i \left| i_A \right\rangle \left\langle i_A \right| \otimes \rho_B^i$$

or

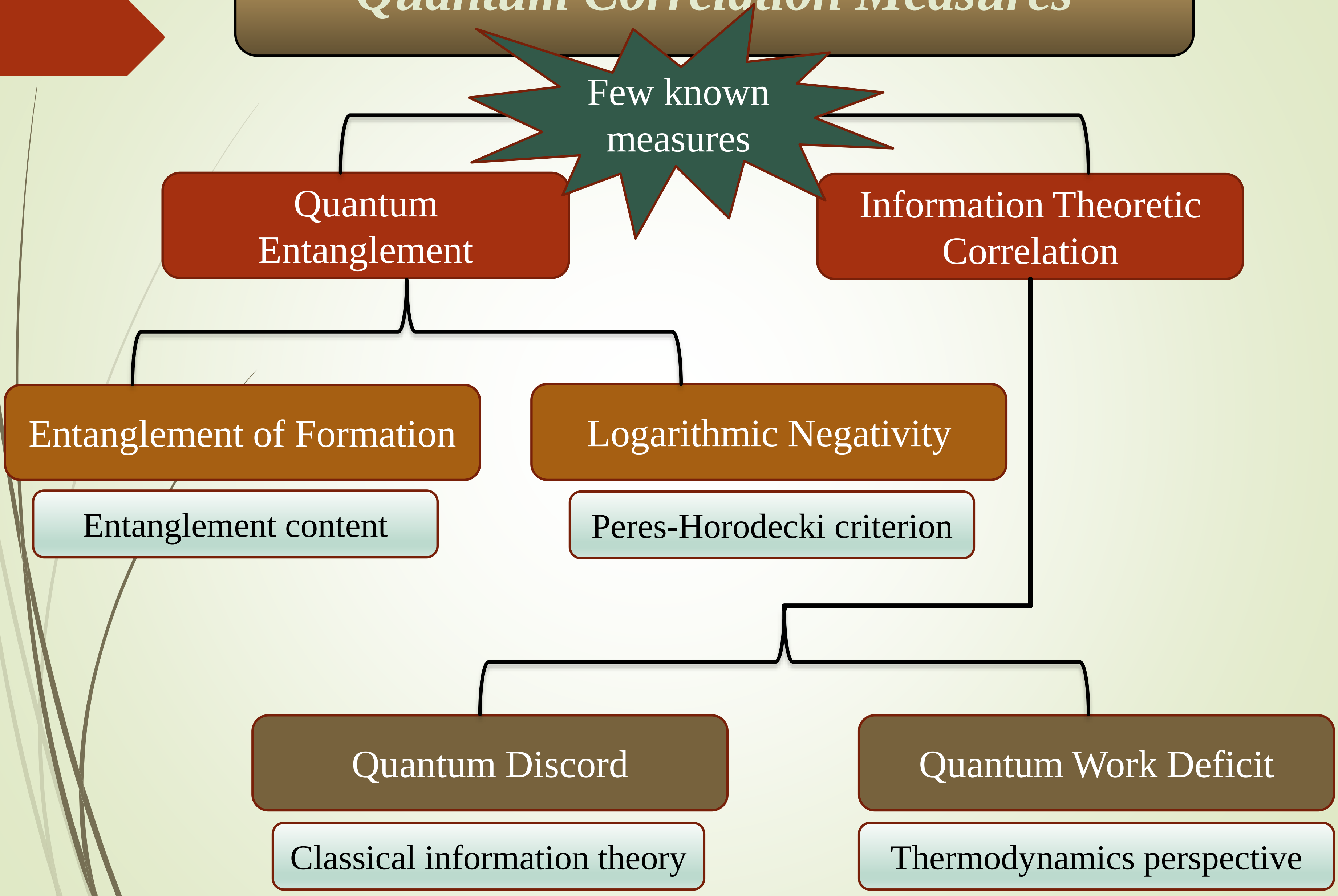
$$\rho_{AB} \neq \sum_i p_i \rho_A^i \otimes \left| i_B \right\rangle \left\langle i_B \right|$$

Quantum Correlation Measures

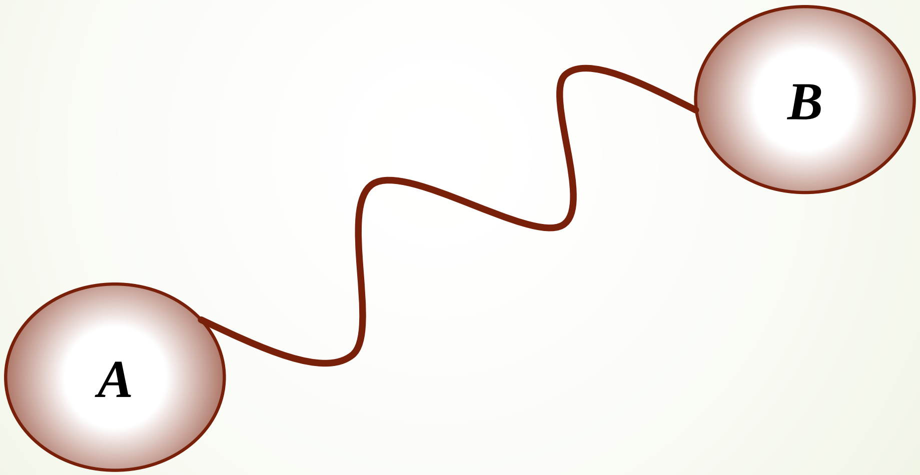
Few known measures

- Quantum Entanglement
  - Entanglement of Formation
    - Entanglement content
  - Logarithmic Negativity
    - Peres-Horodecki criterion
- Information Theoretic Correlation

Quantum Correlation Measures

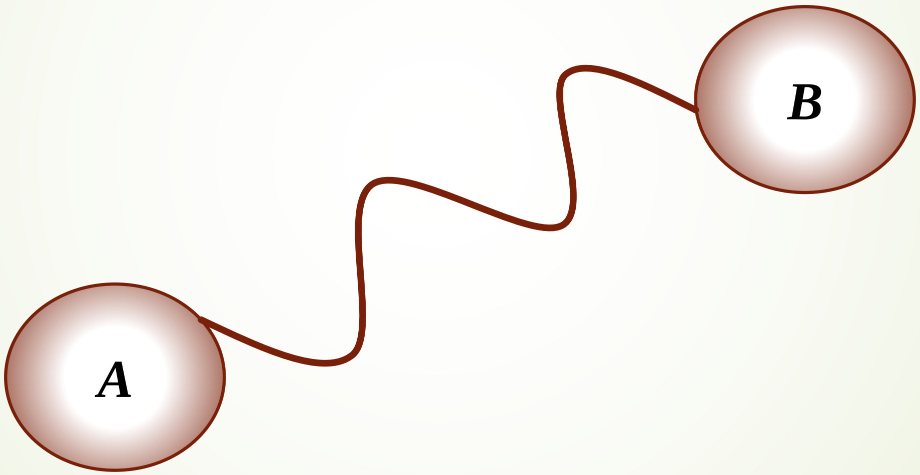


Open Quantum Systems



Open Quantum Systems

$$H_{total} = H_A + H_B + H^I_{AB}$$



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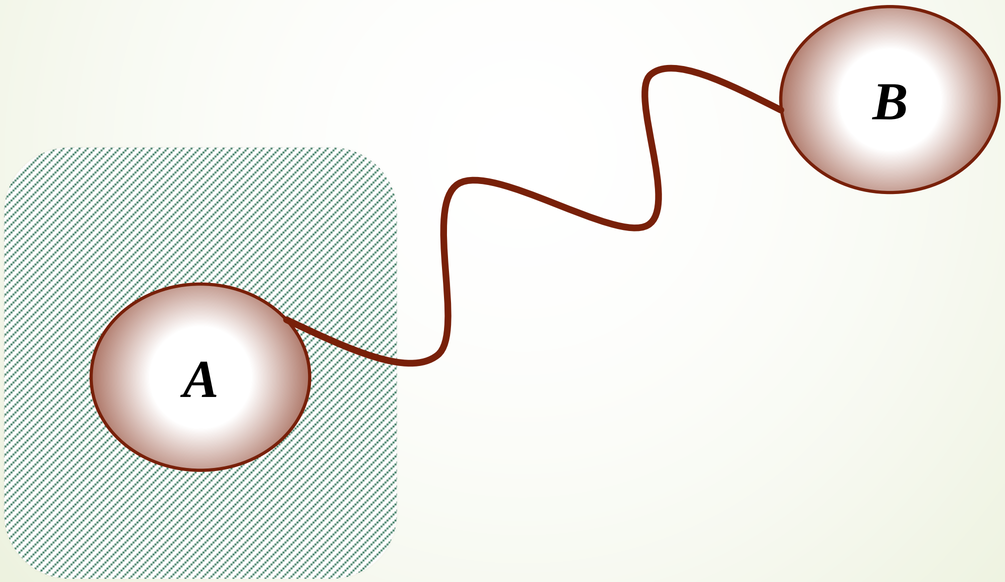

Open quantum system

Open Quantum Systems

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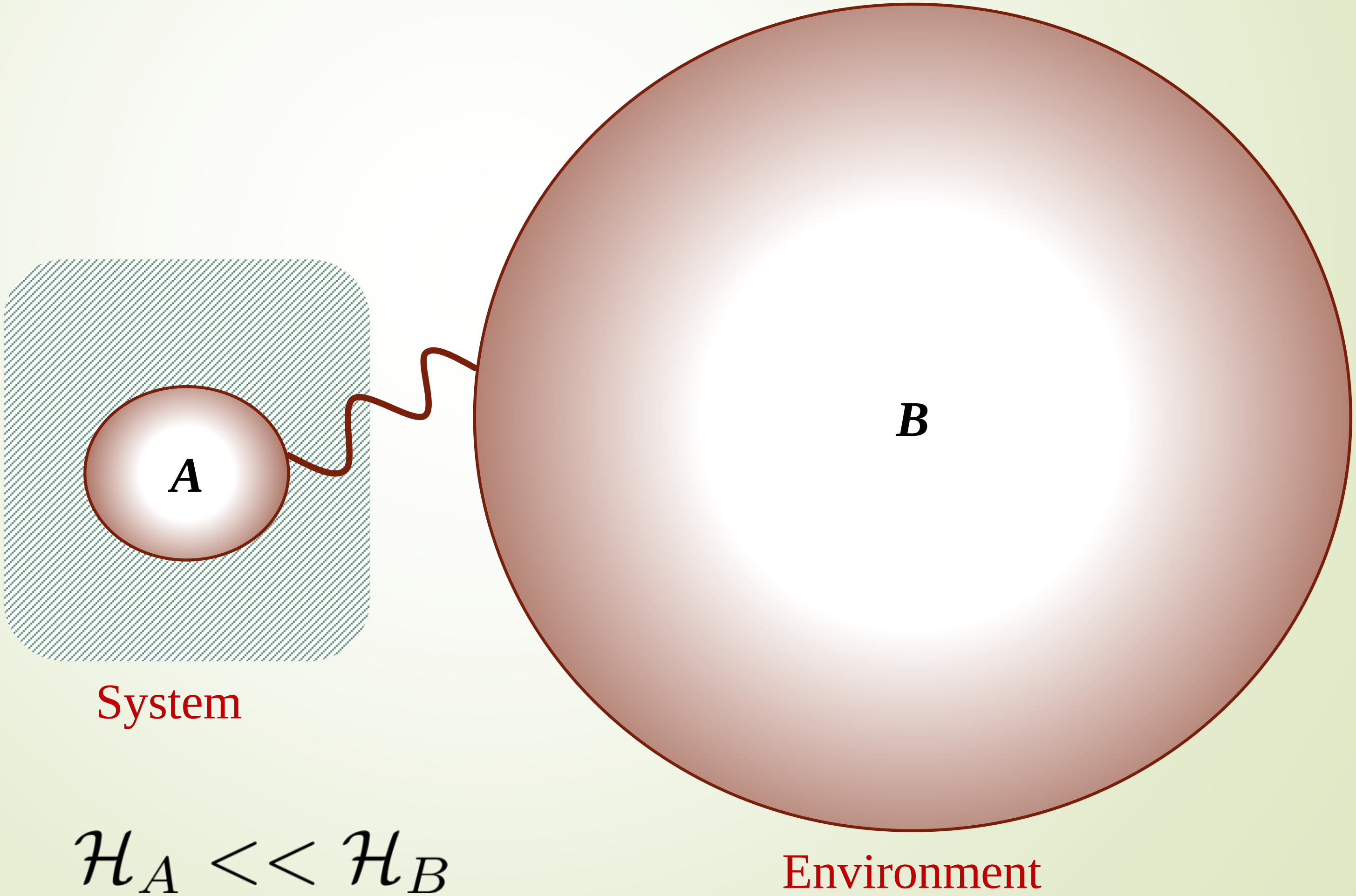

$$\mathcal{H}_A << \mathcal{H}_B$$

Open Quantum Systems

Dynamical evolution…

Kraus operator representation:

$$\rho_S(t) = \sum_i K_i(t)\rho_S(0)K_i(t)^\dagger$$

with

$$\sum_i K_i(t)^\dagger K_i(t) = \mathbb{I}$$

Open Quantum Systems

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Master equation:

$$\frac{d\rho_S(t)}{dt} = -\frac{i}{\hbar}[H_s, \rho_S(t)] + \mathcal{D}_t[\rho_S(t)]$$

Freezing of Discord and Its Allies

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- Initial two qubit state: ρ_{AB}

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- Bit-flip (BF), phase-flip (PF), bit-phase-flip channels

$$K_0(\gamma) = \sqrt{1-\gamma/2}\,\mathbb{I} \ \textbf{and} \ K_1 = \sqrt{\gamma/2}\,\sigma_\alpha$$

$$\alpha = 1 \text{ (bit-flip)}, \ \alpha = 2 \text{ (bit-phase-flip)}, \ \alpha = 3 \text{ (phase-flip)}$$

Freezing of Discord and Its Allies

Dynamics under noisy channels: there are several types...

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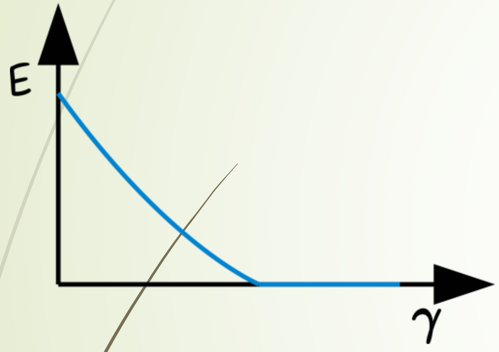
Entanglement usually decays and dies..

Yu & Eberly, Science (2009)



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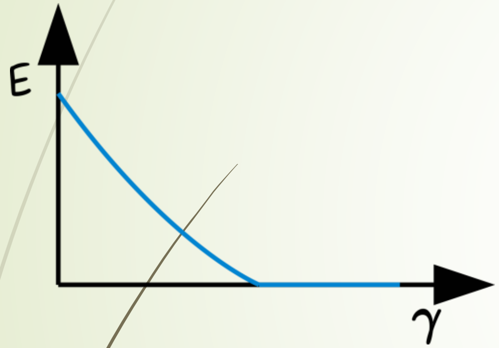
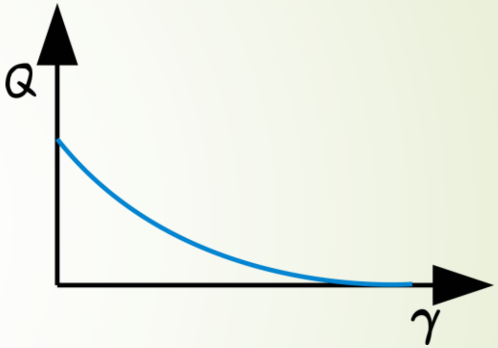


Quantum correlations are robust!
Werlang et. al., PRA (2009)



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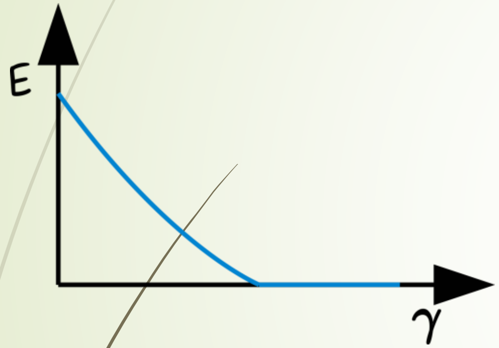

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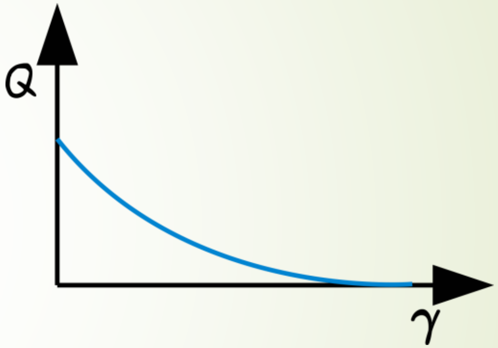

Play with the initial state
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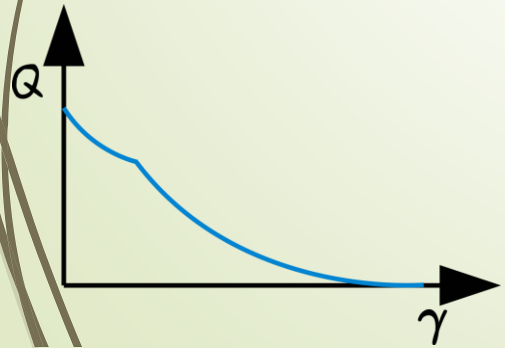

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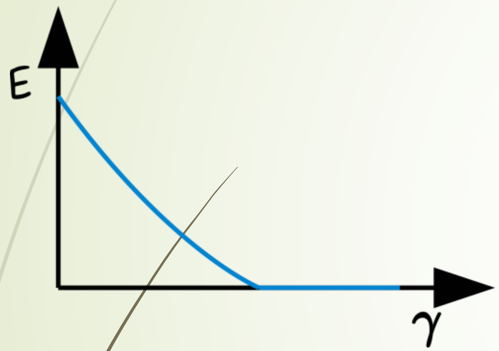

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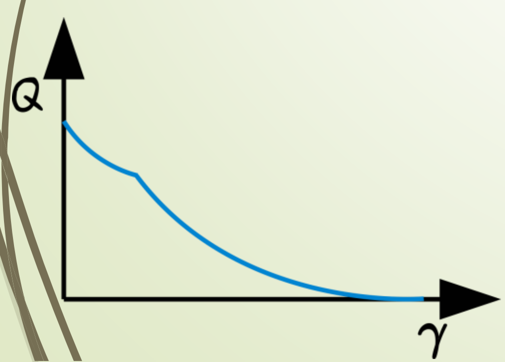

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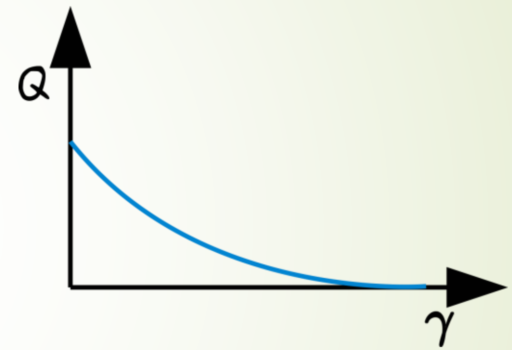
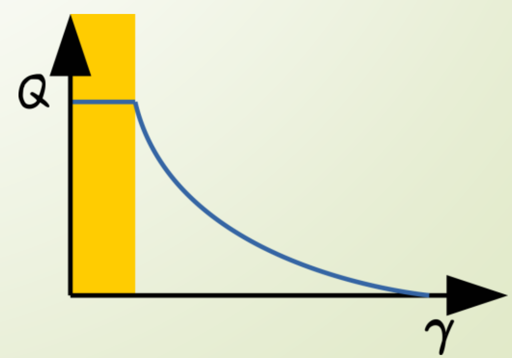


.., and there is **freezing!**

Mazzola et. al., PRL (2010),

Aaronson et. al., PRA (2013),

Cianciaruso et. al., Sci. Rep. (2015)



Freezing of Discord and Its Allies

Initial state → Bell diagonal (BD) state

$$\rho_{AB} = \frac{1}{4}\left[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^{\alpha} \otimes \sigma_B^{\alpha}\right]$$

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Two sets of conditions:

1. $c_{22}/c_{33} = -c_{11}, \textbf{ with } |c_{33}| < |c_{11}|$
2. $c_{33}/c_{22} = -c_{11}, \textbf{ with } |c_{22}| < |c_{11}|$

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Universal for (almost) all the discord-like measures…

Aaronson et. al., PRA (2013), Cianciaruso et. al., Sci. Rep. (2015)

Freezing of Discord and Its Allies

Most general two-qubit state (upto LU):

$$\rho_{AB} = \frac{1}{4}\Big[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^\alpha \otimes \sigma_B^\alpha + \sum_{\alpha=1}^{3} c_{\alpha 0}\sigma_A^\alpha \otimes \mathbb{I}_B + \sum_{\beta=1}^{3} c_{0\beta}\mathbb{I}_A \otimes \sigma_B^\beta\Big]$$

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Discord is hard to compute!!!
No analytical closed form!!!

Freezing of Discord and Its Allies

BD state + magnetization in *x* direction:

$$\rho_{AB} = \frac{1}{4}\Big[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^\alpha \otimes \sigma_B^\alpha + c_{10}\sigma_A^1 \otimes \mathbb{I}_B + c_{01}\mathbb{I}_A \otimes \sigma_B^1\Big]$$

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Closed form can be found for all (almost) parameter values

TC, T Das, D Sadhukhan, A K Pal, A Sen(De), U Sen, PRA (2015)

Freezing of Discord and Its Allies

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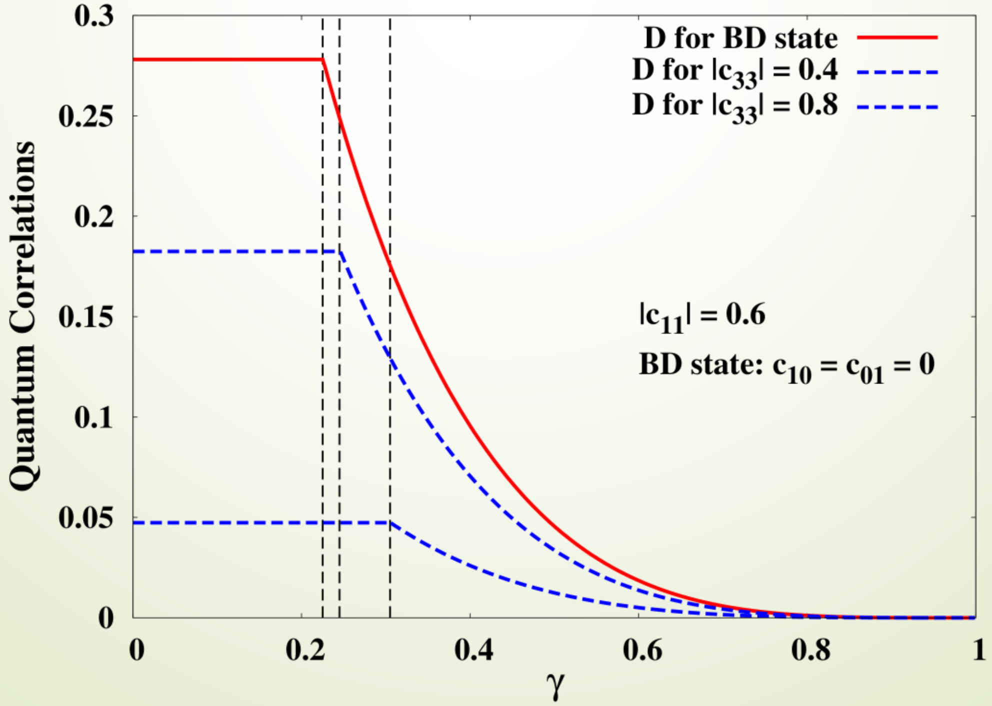

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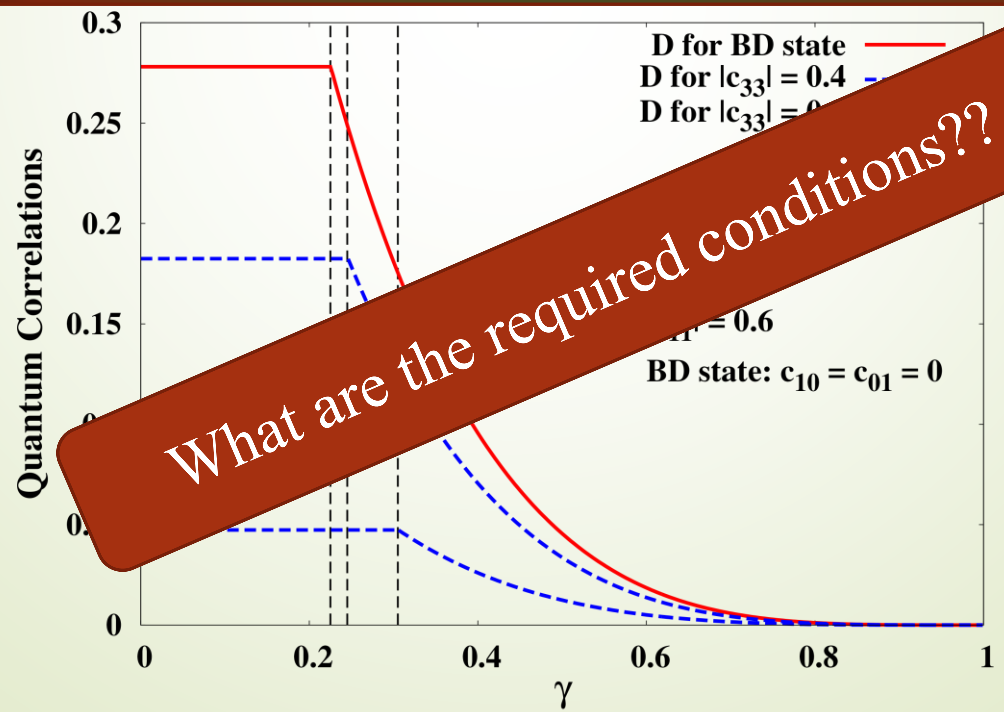

What are the required conditions??

Freezing of Discord and Its Allies

Two sets of conditions: Necessary and sufficient

Freezing of Discord and Its Allies

Two sets of conditions: Necessary and sufficient

(1) $(c_{22}/c_{33}) = -(c_{10}/c_{01}) = -c_{11}$

(2) $c_{33}^2 + c_{01}^2 \leq 1$

(3) $F(\sqrt{c_{33}^2 + c_{01}^2}) < F(c_{11}) + F(c_{01}) - F(c_{10})$

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$$\textbf{(3)}\ F(\sqrt{c_{22}^2 + c_{01}^2}) < F(c_{11}) + F(c_{01}) - F(c_{10})$$

$$F(y) = 2\left(H((1+y)/2) - 1\right)$$

$$H(\alpha) = -\alpha \log_2 \alpha - (1-\alpha)\log_2(1-\alpha)$$

Freezing of Discord and Its Allies

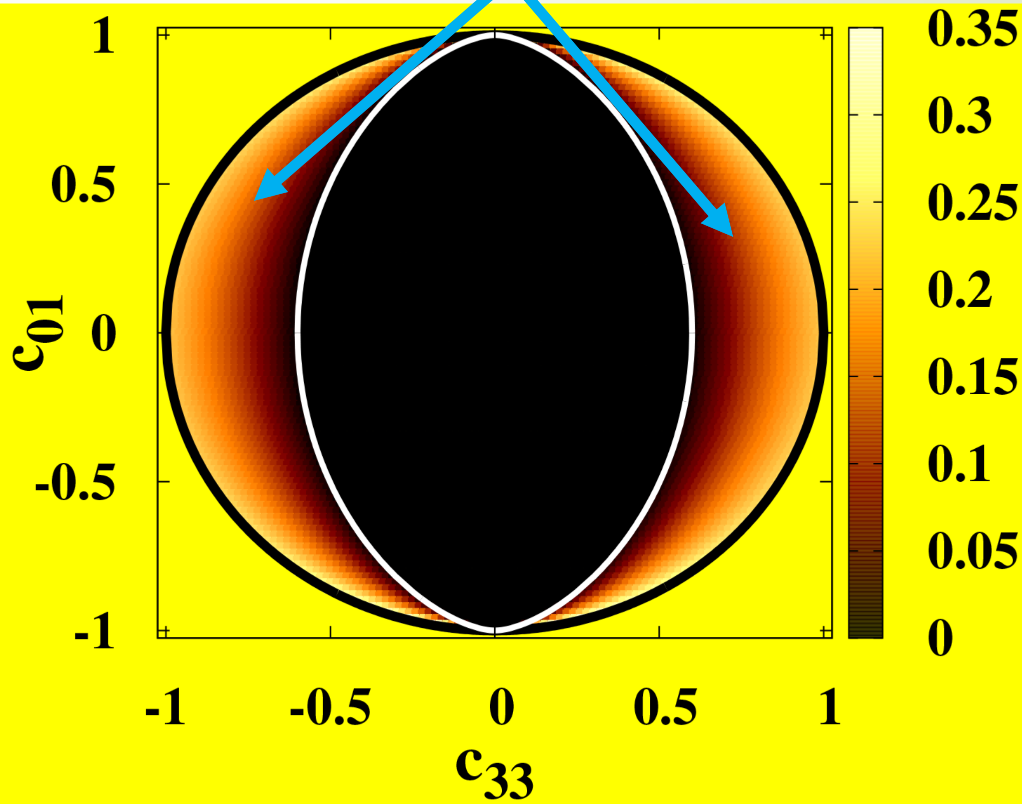
Two sets of conditions: Necessary and

Freezing

(1) $(c_{22}/c_{33}) =$

(2) $c_{33}^2 + c_{01}^2 \leq$

(3) $F(\sqrt{c_{33}^2 + c$



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Freezing of Discord and Its Allies

Universality no longer exists…

One-way quantum work deficit…

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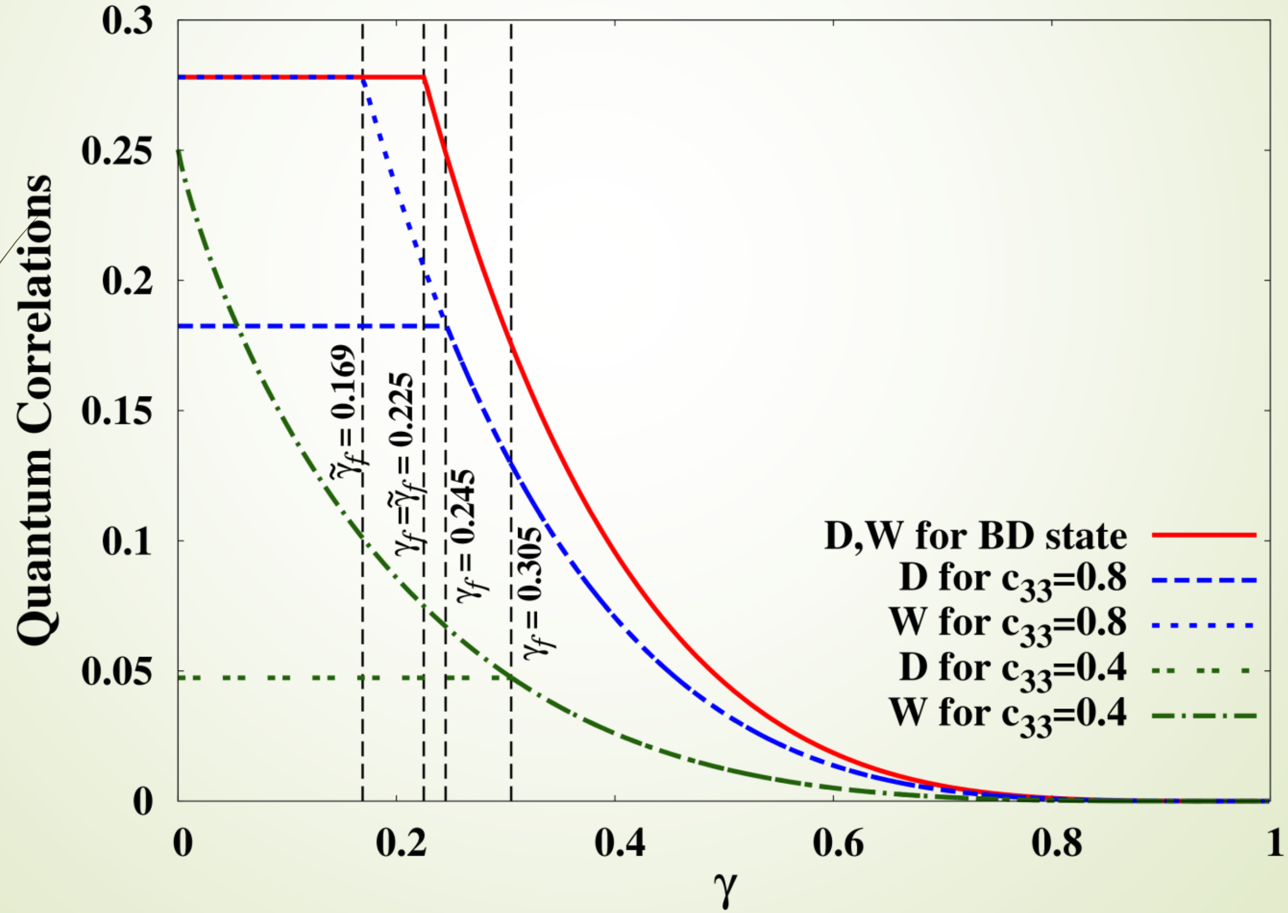

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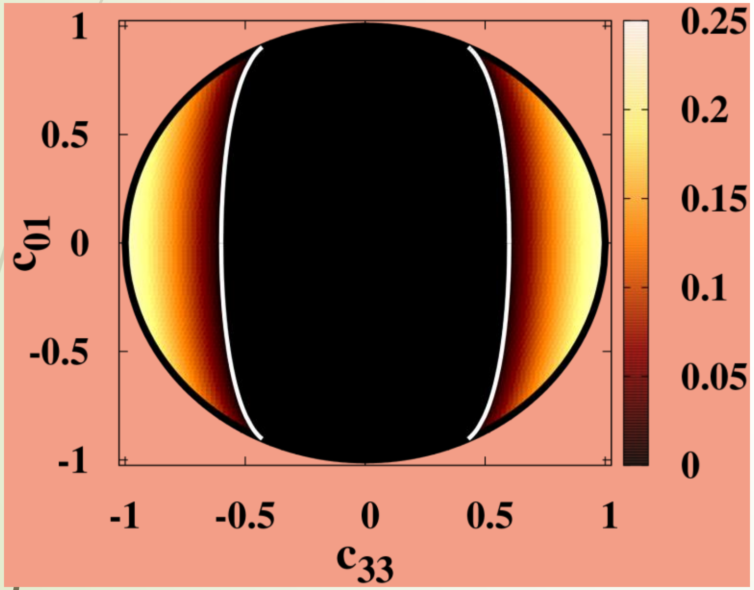
Work deficit



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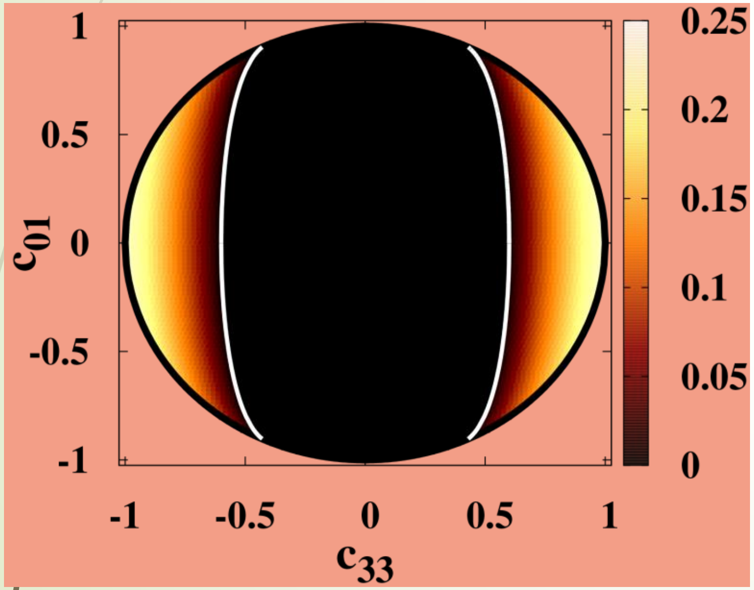
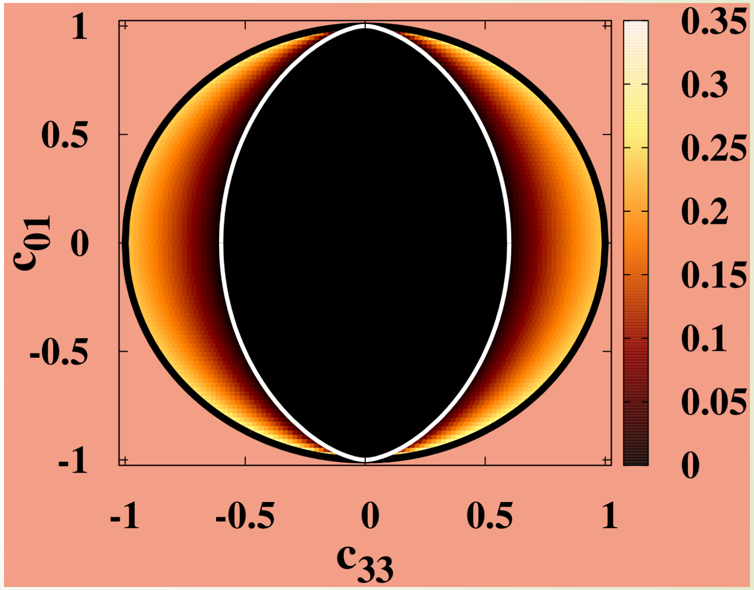


Discord



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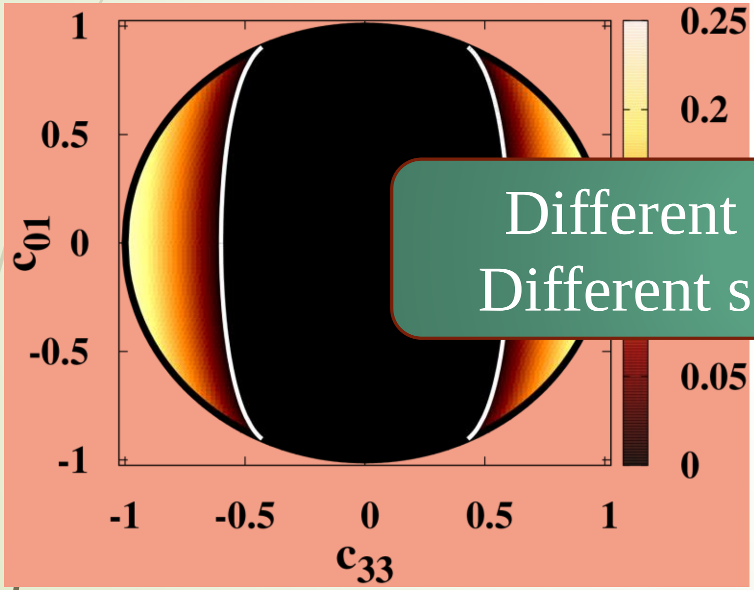

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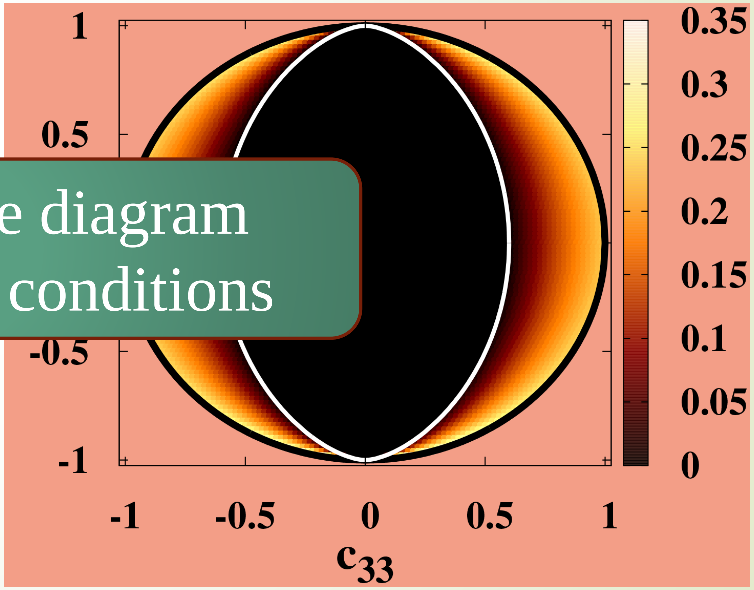

Different phase diagram
Different set of conditions

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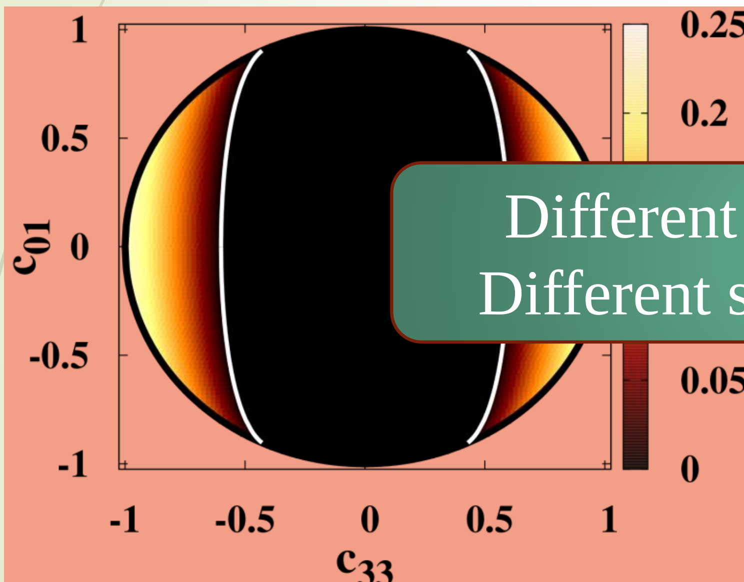

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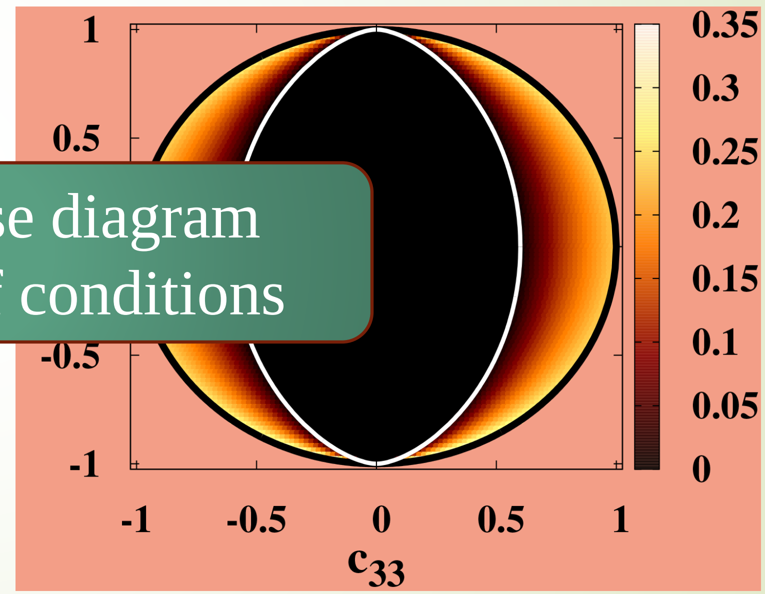


Different phase diagram
Different set of conditions

Freezing of entanglement??

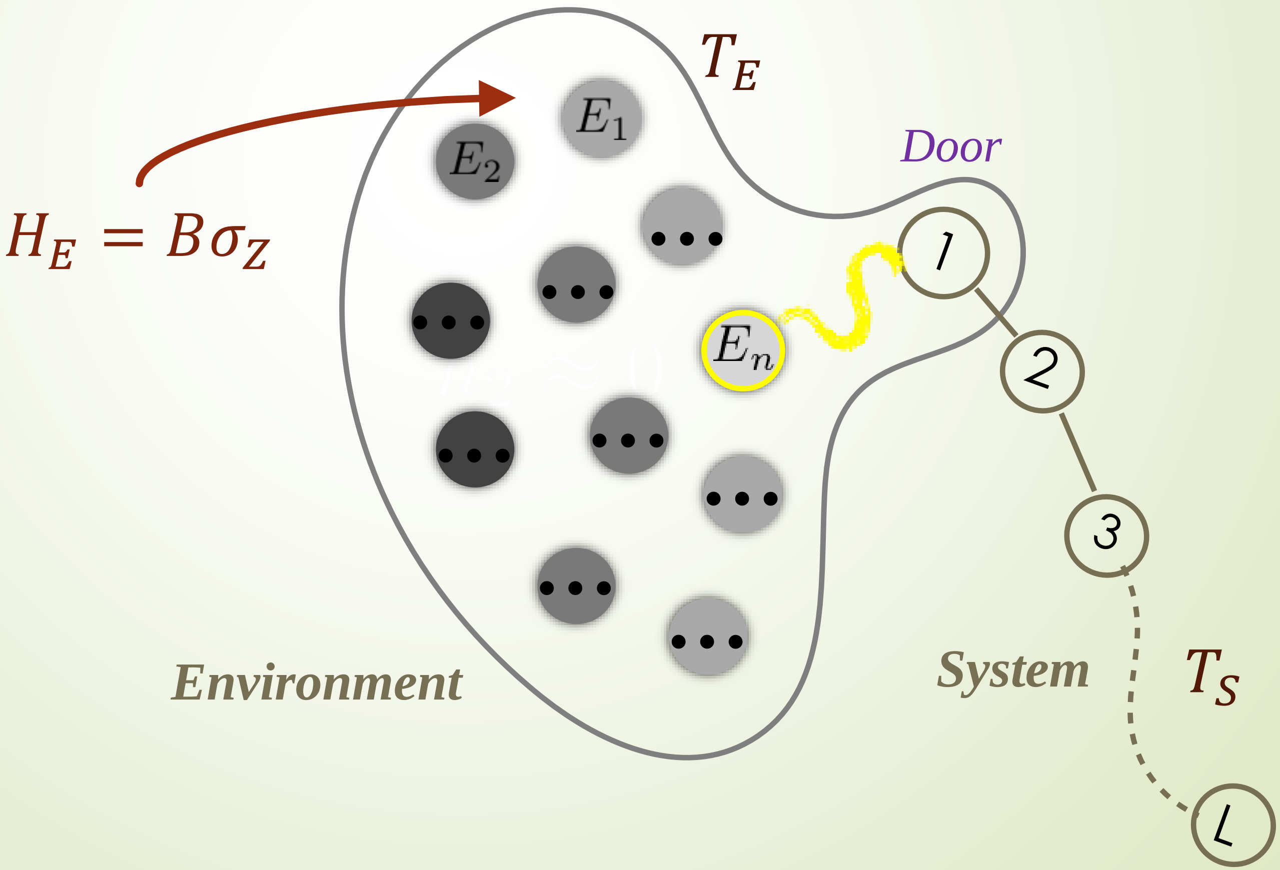
Freezing of Entanglement

Open evolution:

Freezing of Entanglement

Open evolution:

Repeated Quantum Interaction



Freezing of Entanglement

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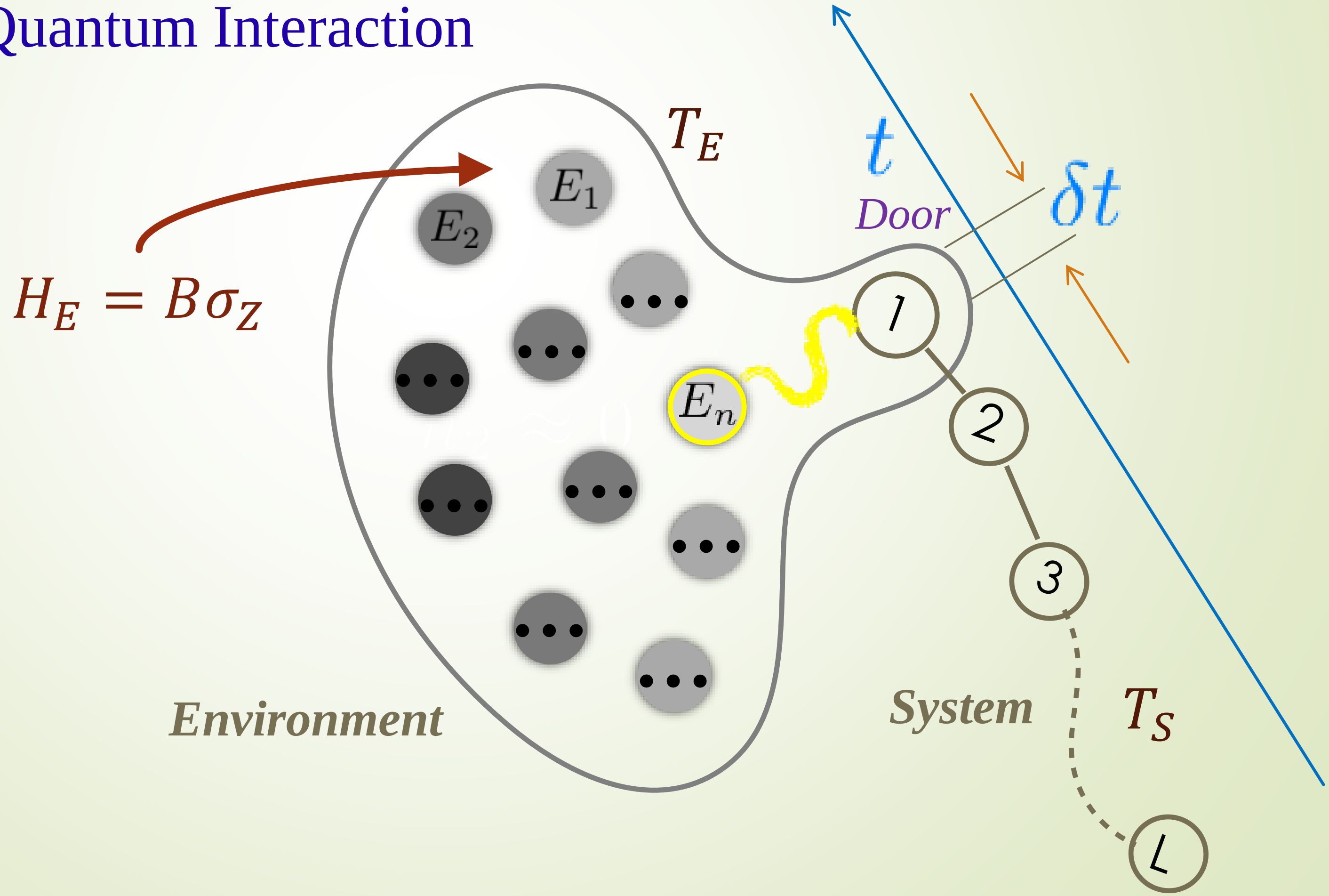
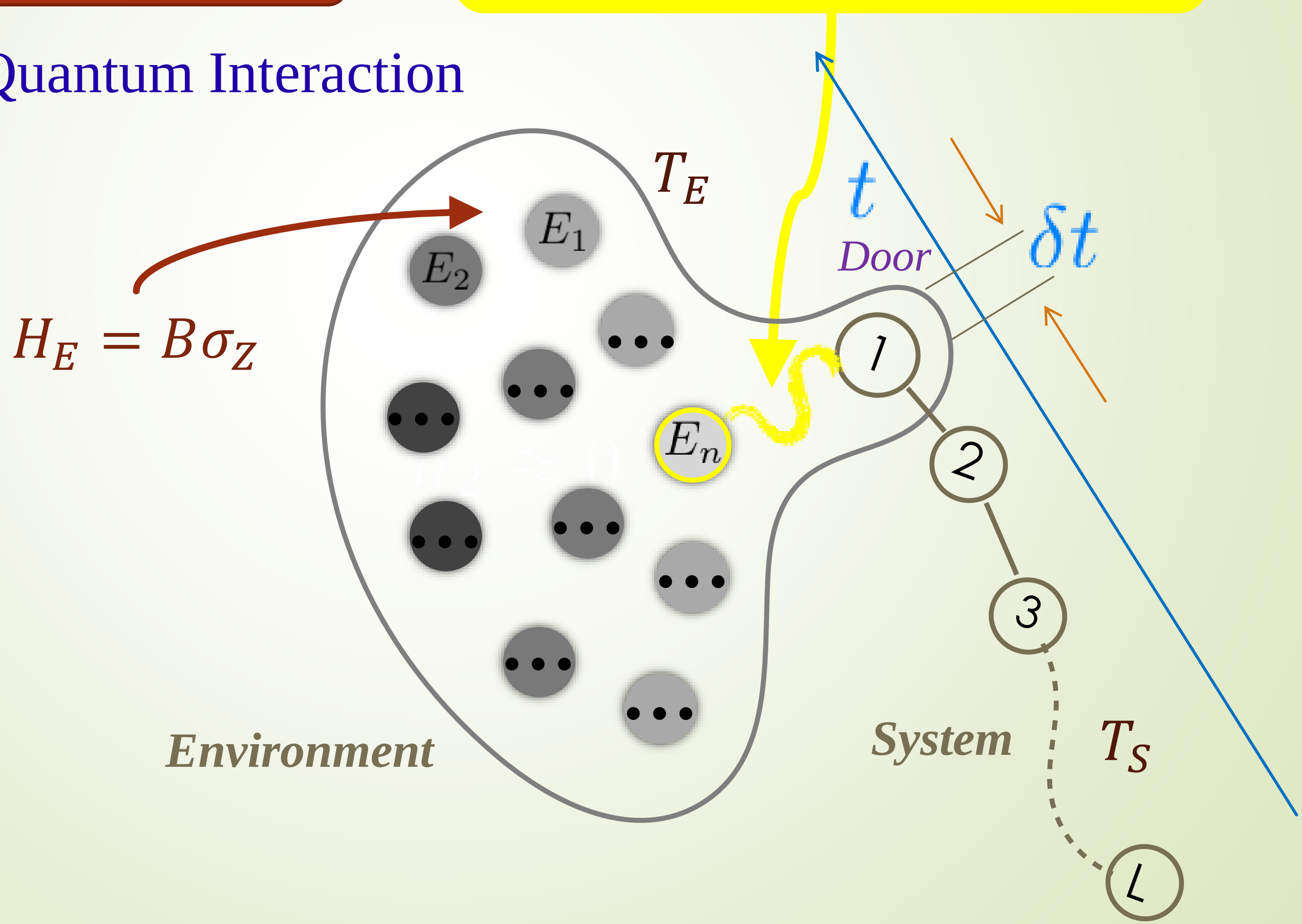


Freezing of Entanglement

Open evolution:

$$\hat{H}_{int} = k^{1/2}\delta t^{-1/2}(\hat{\sigma}_d^x\hat{\sigma}_E^x + \hat{\sigma}_d^y\hat{\sigma}_E^y)$$

Repeated Quantum Interaction



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Repeated Quantum Interaction

Quantum master equation:

$$\dot{\hat{\rho}}_S = -\frac{i}{\hbar}[\hat{H}_S, \hat{\rho}_S] + \mathcal{D}(\hat{\rho}_S)$$

$$\mathcal{D}(\hat{\rho}_S) = \frac{2k}{\hbar^2 Z_E}\sum_{l=1}^{N_d}\sum_{i=0}^{1} e^{(-1)^i\beta_E B}[2\hat{\eta}_{d_l}^{i+1}\hat{\rho}_S\hat{\eta}_{d_l}^{i} - \{\hat{\eta}_{d_l}^{i}\hat{\eta}_{d_l}^{i+1}, \hat{\rho}_S\}]$$

where…

$$Z_E = \mathbf{Tr}[\exp -\beta_E \hat{H}_E], \textbf{ and } \hat{\eta}_{d_l}^{\alpha} = \hat{\sigma}_{d_l}^x + (-1)^{\alpha}\hat{\sigma}_{d_l}^y$$

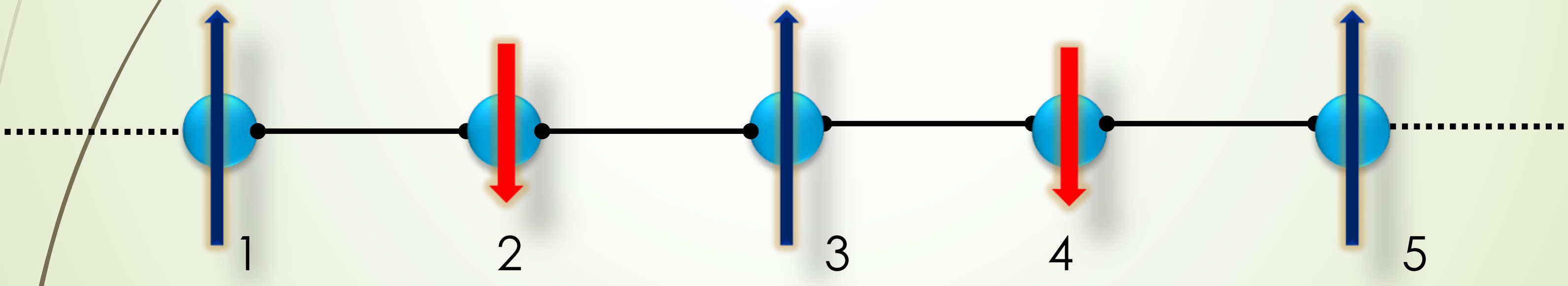
Freezing of Entanglement

System:

Freezing of Entanglement

System:

$$\hat{H} = \frac{1}{2}\sum_{i=1}^{N}\left[J\left\{\frac{1+\gamma}{2}\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right\} + \{h_1 + (-1)^i h_2\}\hat{\sigma}_i^z\right]$$



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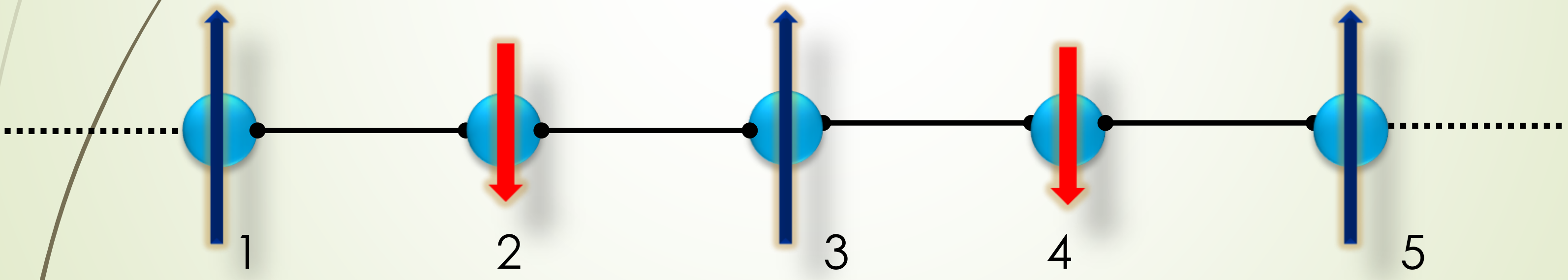


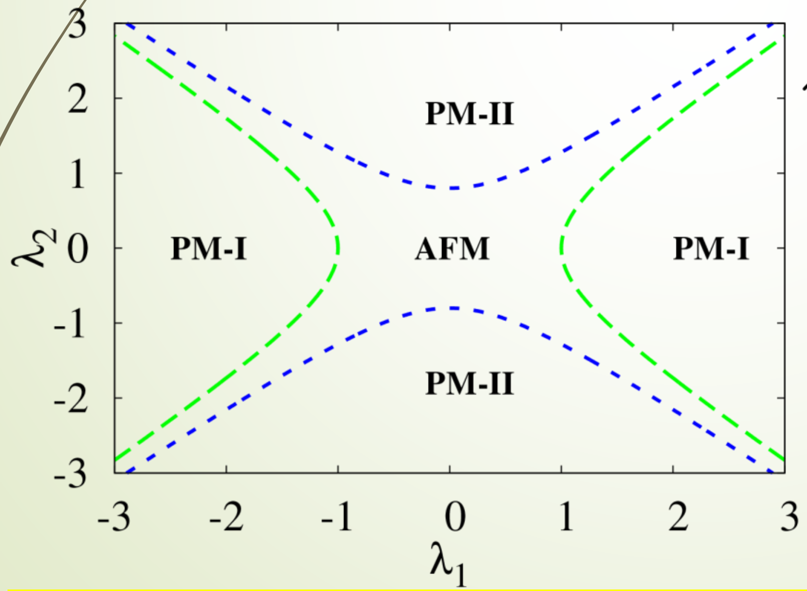
Can be solved "exactly" using successive Jordan-Wigner and Fourier transformation.

TC, T Das, D Sadhukhan, A K Pal, A Sen(De), U Sen, PRA (2016)

Freezing of Entanglement

System:

$$\hat{H} = \frac{1}{2}\sum_{i=1}^{N}\left[J\left\{\frac{1+\gamma}{2}\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right\} + \{h_1 + (-1)^i h_2\}\hat{\sigma}_i^z\right]$$

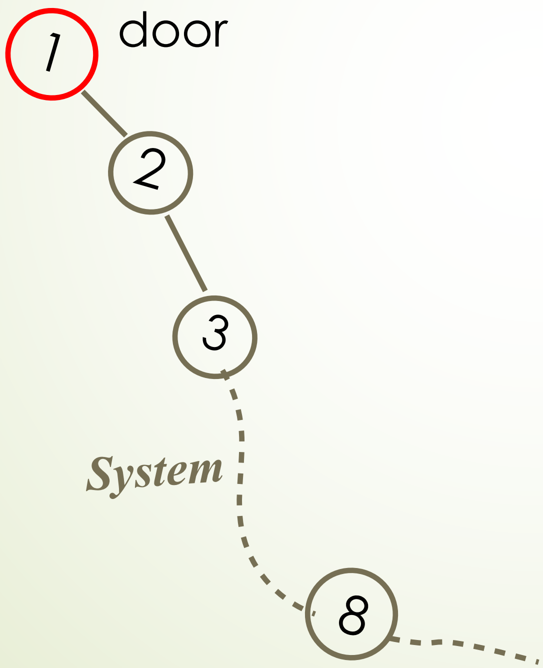


$\lambda_i = h_i/J$

TC, T Das, D Sadhukhan, A K Pal, A Sen(De), and U Sen, PRA (2016)

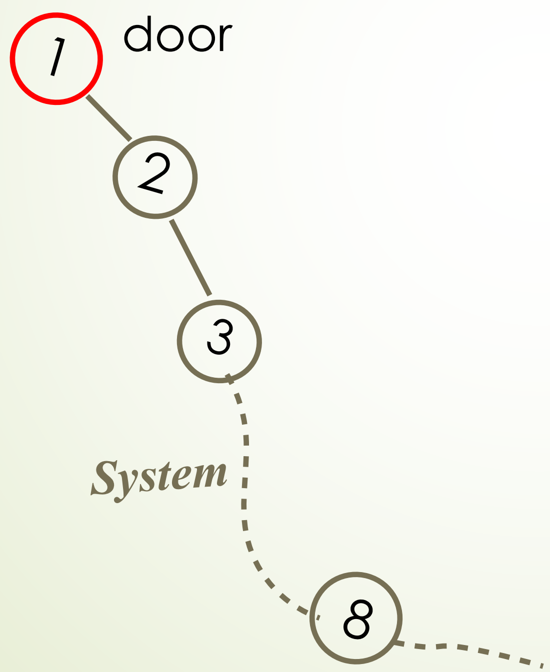
Freezing of Entanglement

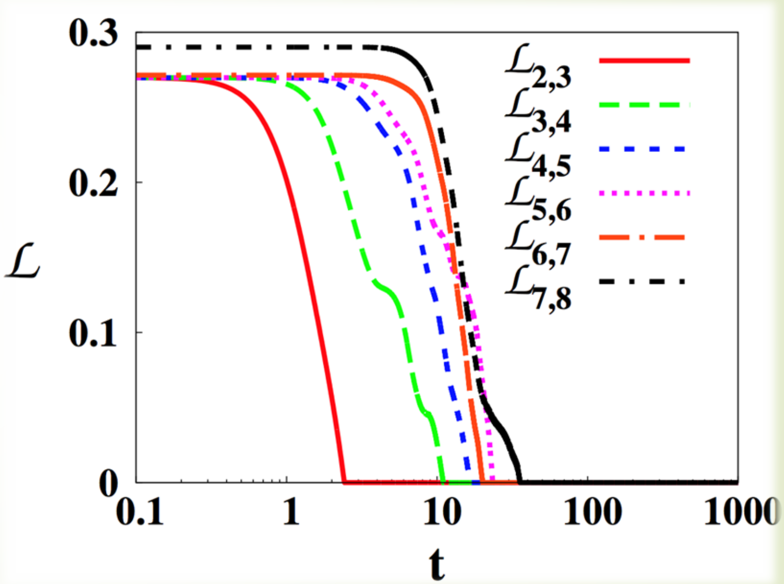
Freezing:



Freezing of Entanglement

Freezing:



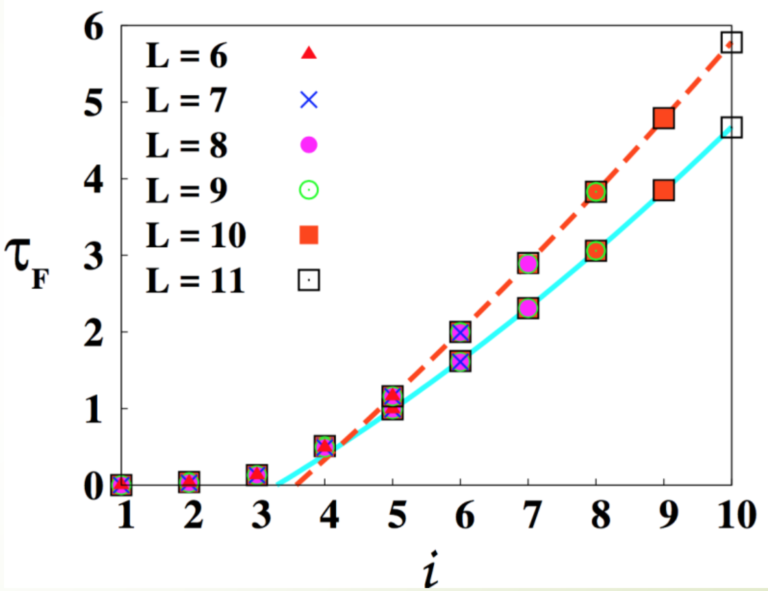


PM-II phase

Freezing of Entanglement

Freezing:

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair (i,i+1)



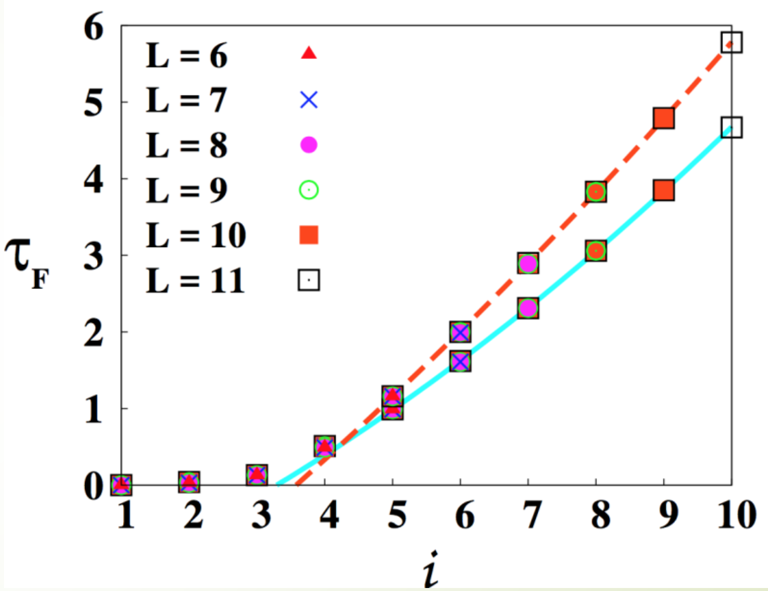
PM-II phase

Freezing of Entanglement

Freezing:

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair (*i*,*i*+1)

Monotonic: $\tau_F^{(i,i+1)} \geq \tau_F^{(j,j+1)}; i > j$



PM-II phase

Freezing of Entanglement

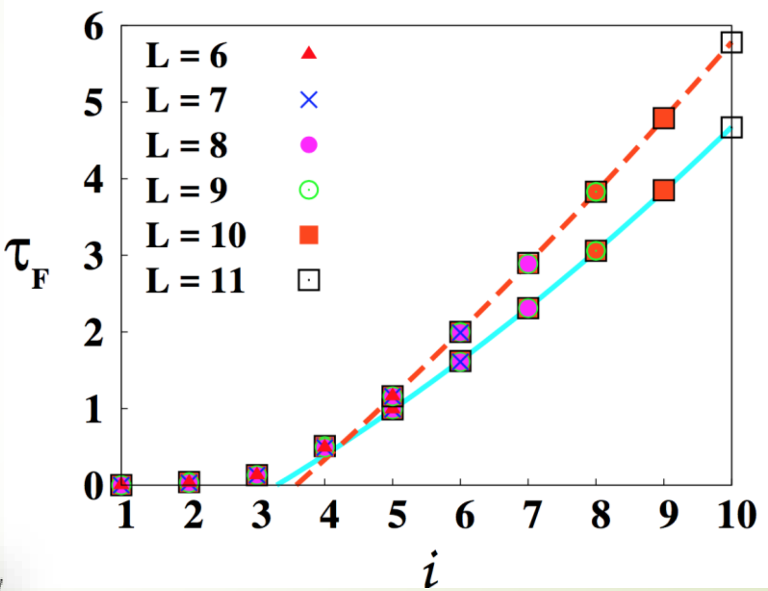
Freezing:

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair (i,i+1)

Monotonic: $\tau_F^{(i,i+1)} \geq \tau_F^{(j,j+1)}; i > j$

$$\tau_F^{i,i+1} = ai^2 + bi + c \quad \forall L$$

Scale invariance



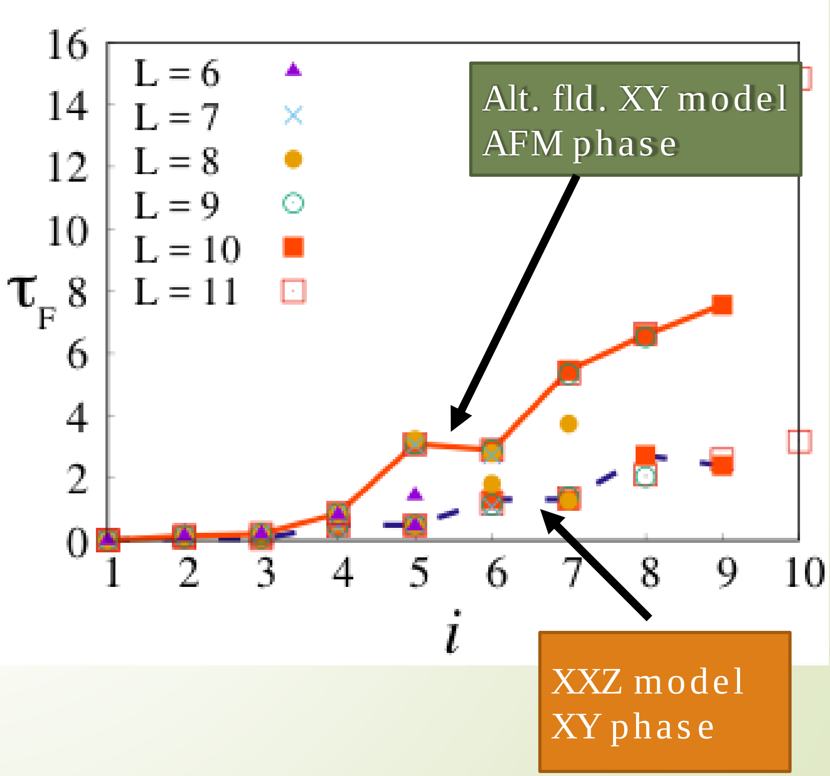
PM-II phase

Both in PM-I and PM-II phase

Freezing of Entanglement

Freezing:

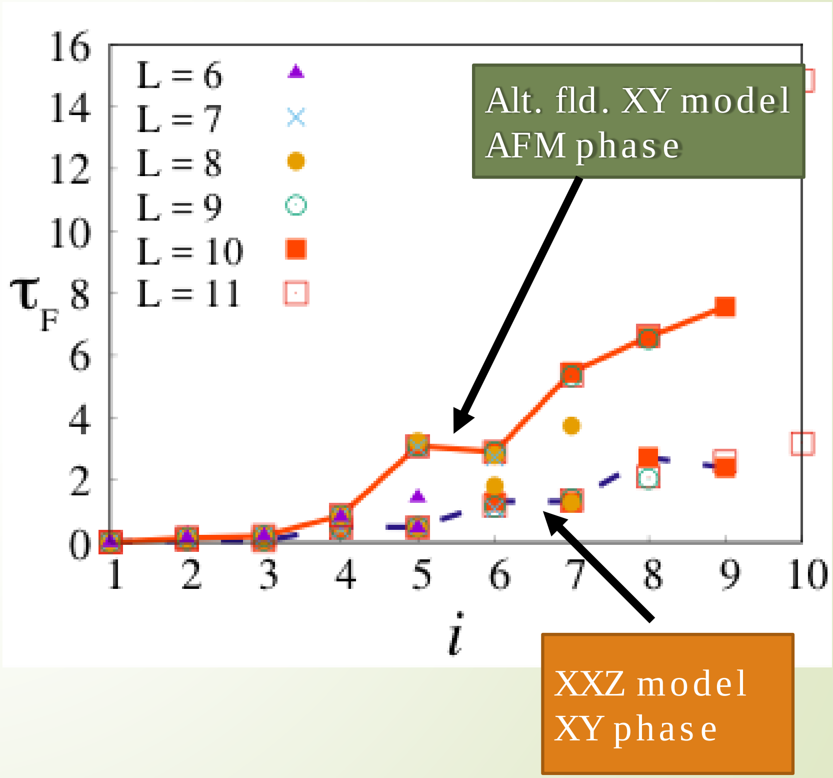
$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair (i,i+1)



Freezing of Entanglement

Freezing:

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair $(i,i+1)$

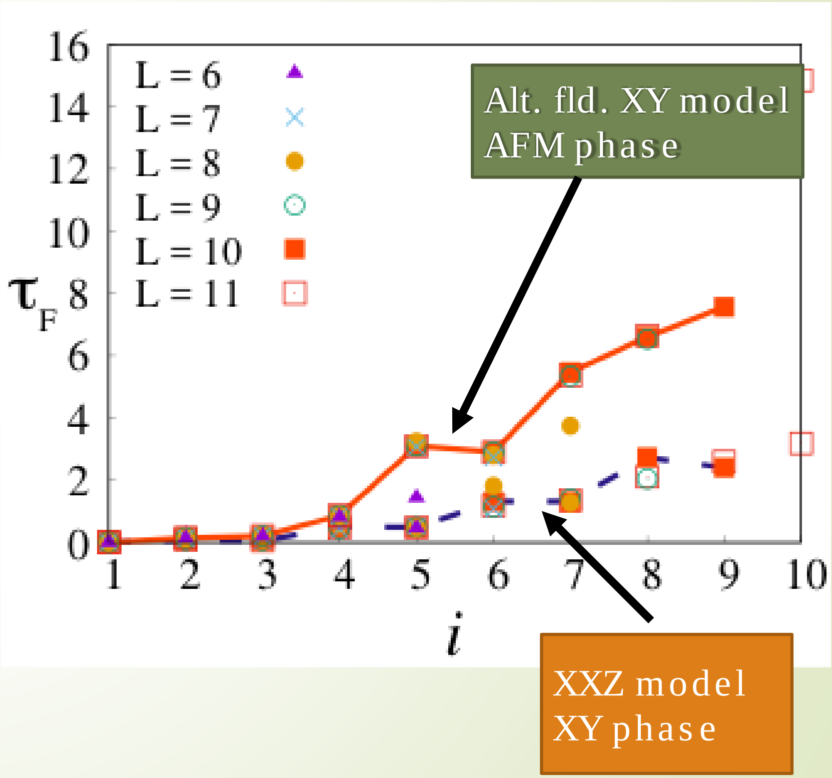


Non-Monotonic

Freezing of Entanglement

Freezing:

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair $(i,i+1)$



Non-Monotonic

No scale invariance

Freezing of Entanglement

Freezing:

$\tau^{(i,i+1)}$

To conclude:

1. Frozen discord and its allies
2. Scale invariant freezing of entanglement

In collaboration with:

Tamoghna Das, Debasis Sadhukhan, Amit Kumar Pal, Anindya Biswas, Aditi Sen(De), Ujjwal Sen

Ref:

1. **TC**, A K Pal, A Biswas, A Sen(De), U Sen, PRA (2015)
2. **TC**, T Das, D Sadhukhan, A K Pal, A Sen(De), U Sen, arXiv:1610.00730

A New Quantum Library

QIClib: **New general purpose quantum information and computing library written in C++11**

https://titaschanda.github.io/QIClib/

A New Quantum Library

QIClib: New general purpose quantum information and computing library written in C++11

https://titaschanda.github.io/QIClib/

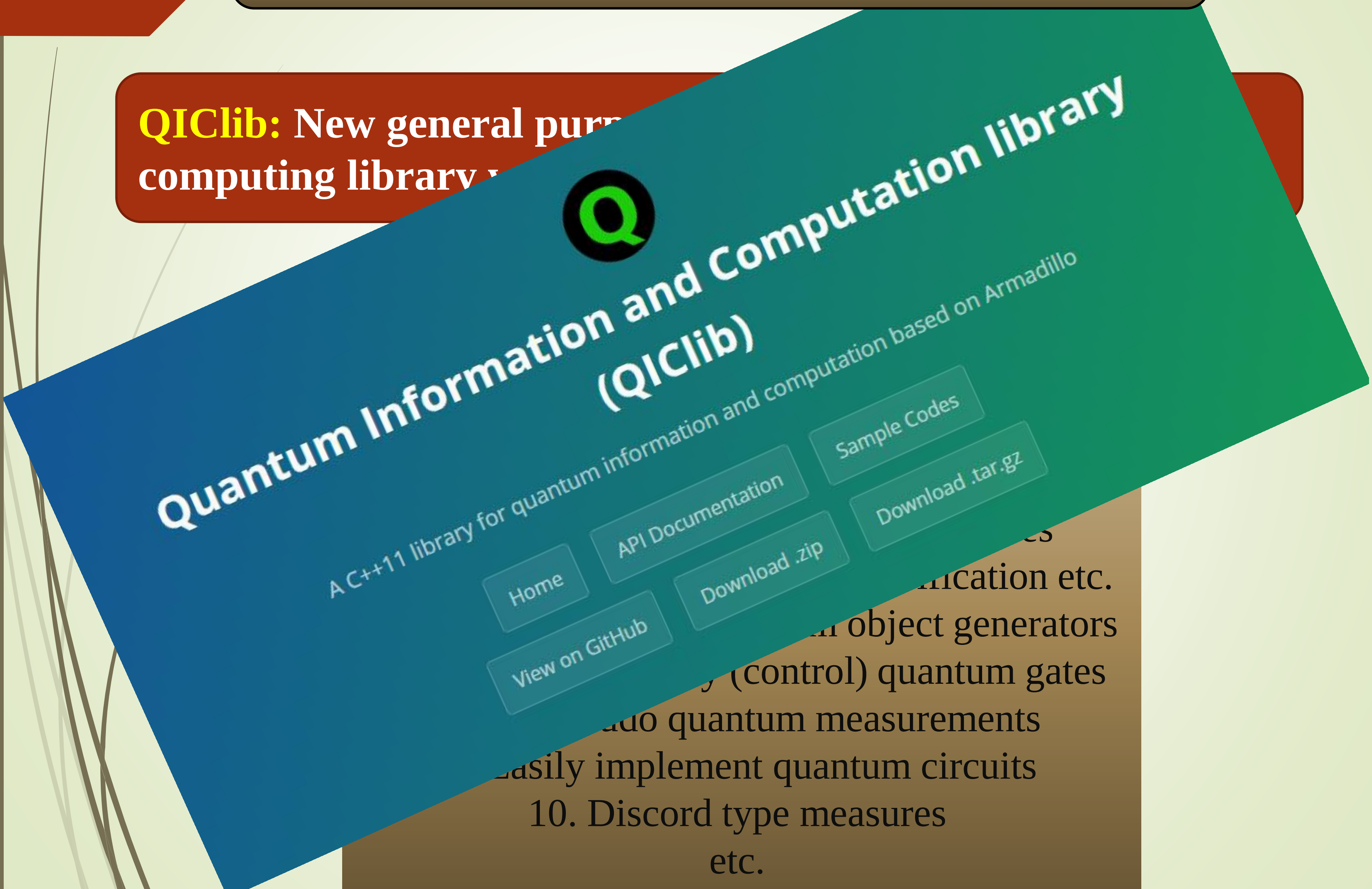
80+ features

1. Partial trace, partial transpose etc.
2. Matrix functions
3. Entropic functions
4. Different entanglement measures
5. Schmidt decomposition, purification etc.
6. State of the art random object generators
7. Apply arbitrary (control) quantum gates
8. Pseudo quantum measurements
9. Easily implement quantum circuits
10. Discord type measures

etc.

A New Quantum Library

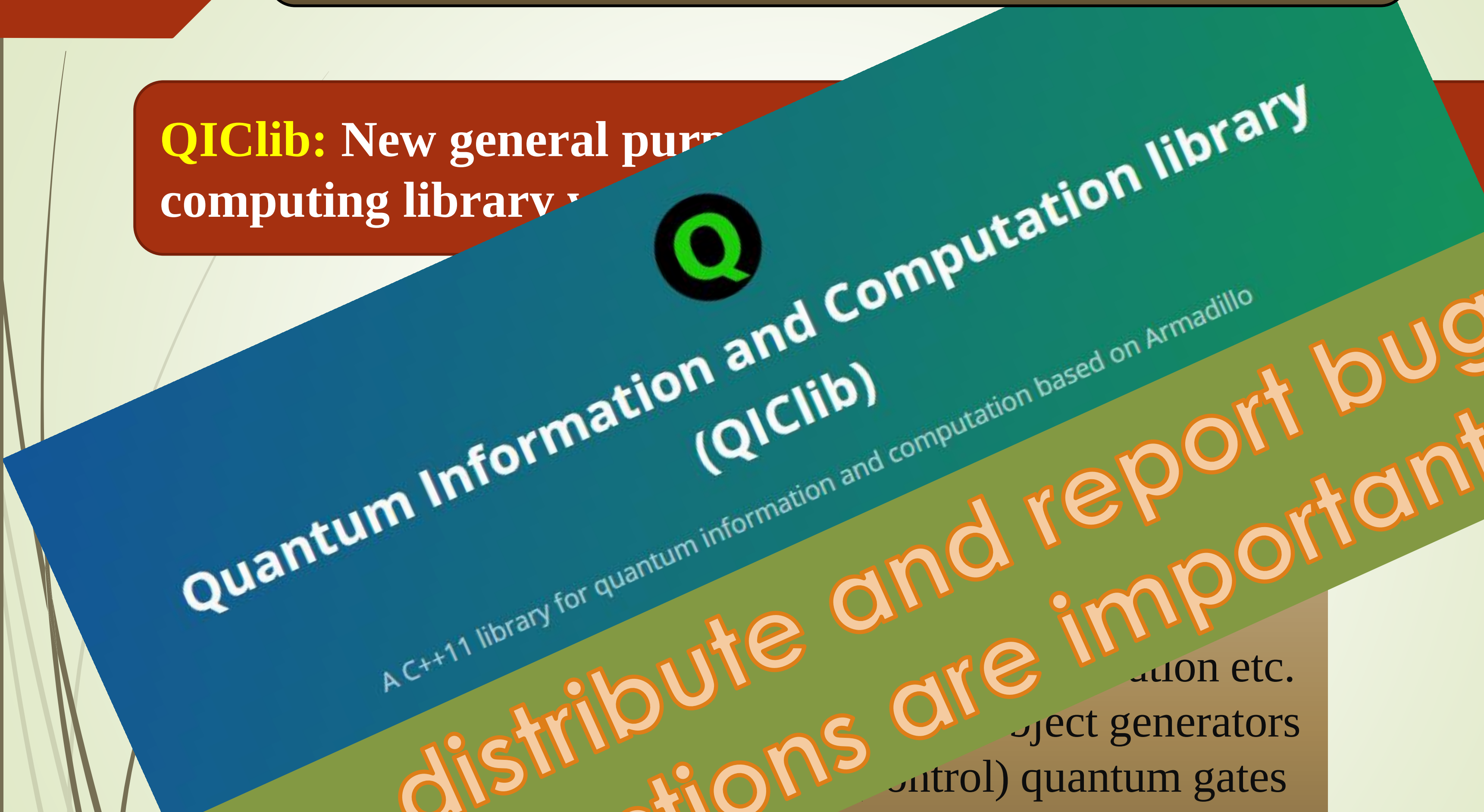
QIClib: **New general purp**
computing library w



es
ication etc.
object generators
(control) quantum gates
do quantum measurements
asily implement quantum circuits
10. Discord type measures
etc.

A New Quantum Library

QIClib: New general purp... computing library w...



Use, distribute and report bugs
Suggestions are important

...ation etc.
...bject generators
...ntrol) quantum gates
... quantum measurements
...y implement quantum circuits
10. Discord type measures
etc.

Thanking you…

Our group at HRI, India
Please visit us…