The Abdus Salam International Centre for Theoretical Physics

INFINITE PHASE SPACE and THE TWO-HEADED ARROW OF TIME

Abdus Salam Lectures 2018: Lecture 3
Budinich Lecture Hall
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Trieste, Italy
January 31, 2018

— Alan Guth —

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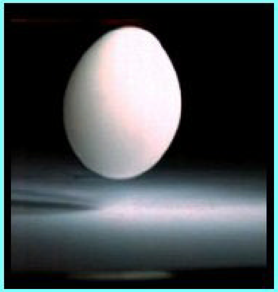

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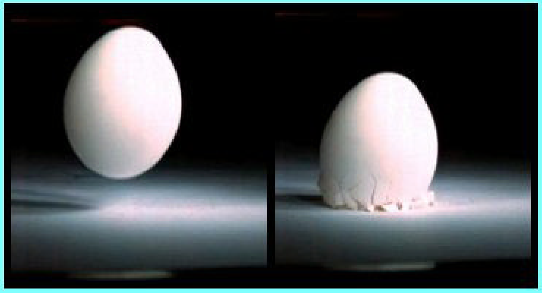

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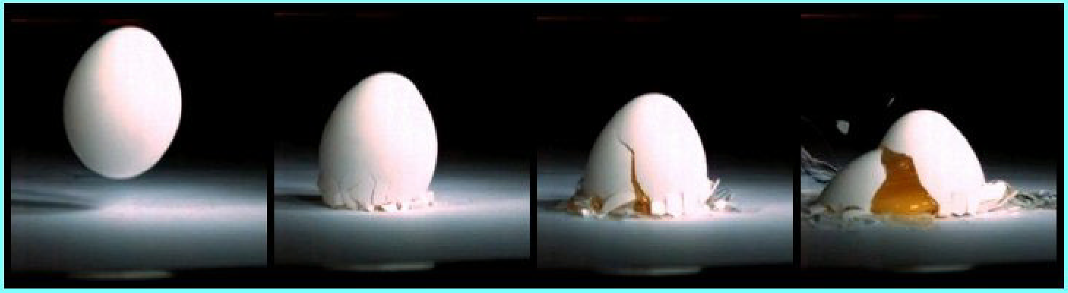

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Mystery of the Arrow of Time

Real Events:

Mystery of the Arrow of Time

Real Events:

Arrow of time

Mystery of the Arrow of Time

Real Events:

Arrow of time

Laws of Physics:

Mystery of the Arrow of Time

Real Events:

Arrow of time

Laws of Physics:

Time symmetric

But What About CP Violation?

Since 1964 and the famous work of Fitch and Cronin, we have known that CP symmetry is violated. Since CPT is a valid symmetry in any Lorentz-invariant local quantum field theory, we assume that T must also be violated.

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But we still claim that the laws of physics make NO DISTINCTION between the past and the future.

Our point is that CPT, which is exact, is a time-reversal operator.

For every state, there exists a correponding time-reversed state which will evolve along exactly the same trajectory as the original state, but backwards in time.

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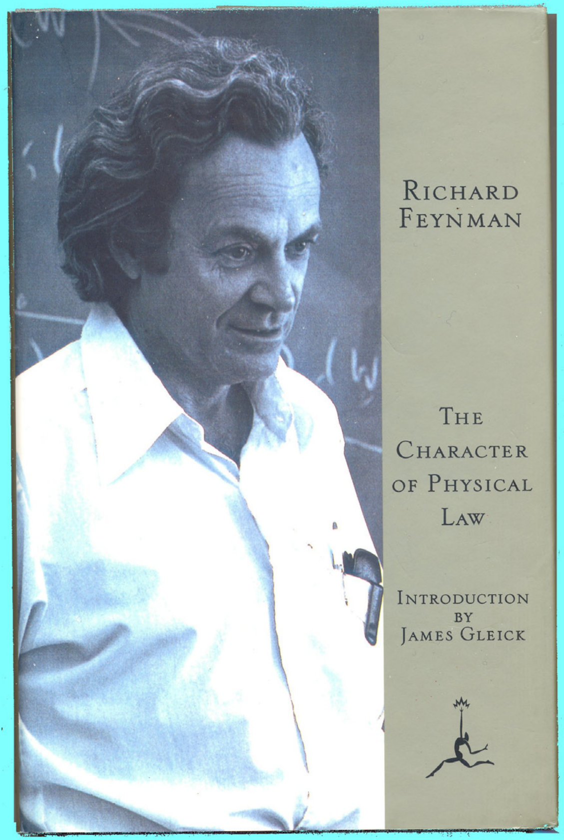
When T was believed exact, we thought that the time-reversed state could be achieved by reversing the momenta and spins of all particles. Now we know that we must also replace all particles by their antiparticles (C transformation), and reflect the state in a mirror (P transformation). But the existence of such a time-reversed state has not been questioned.

Feynman and the Distinction of Past and Future

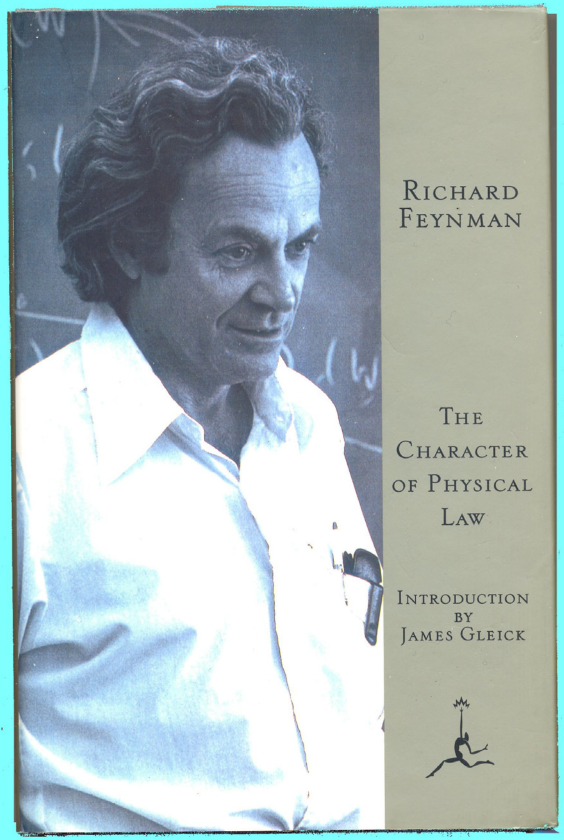
The most obvious interpretation of this evident distinction between past and future, and this irreversibility of all phenomena, would be that some laws, some of the motion laws of the atoms, are going one way. . . . There should be somewhere in the works some kind of a principle that uxles only make wuxles, and never vice versa, and so the world is turning from uxley character to wuxley character all the time — and this one-way business of the interactions of things should be the thing that makes the whole phenomena of the world seem to go one way.



— *The Character of Physical Law, 1965*

Feynman and the Distinction of Past and Future

But



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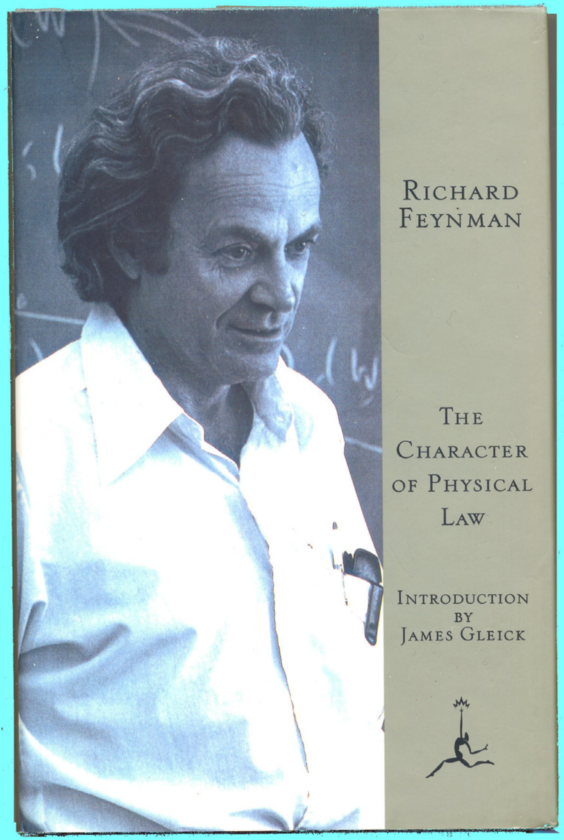
Feynman and the Distinction of Past and Future

But we have not found this yet. That is, in all the laws of physics that we have found so far there does not seem to be any distinction between the past and the future.



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Entropy and the Arrow of Time

- We don't know what causes the arrow of time, but we can describe it: ordered systems tend to evolve into disordered systems, so the world is turning from an ordered state to a disordered state.
- *Entropy* is a measure of disorder. We always see entropy increase, and never decrease (2nd law of thermodynamics).

Classic Example: Gas in a Box

- ☆ If a gas is placed in the corner of an evacuated box, it will spread to fill the box. Entropy increases. Once the box is filled the gas is in equilibrium, in a state of maximum entropy — it will stay that way forever, with random fluctuations about the equilibrium state.

- ☆ The opposite motion — i.e., the return of all the molecules to the corner of the box — is certainly possible, but it is highly unlikely that the positions and velocities of the gas molecules will be in just the right configuration to do that. Statistically, we understand how highly ordered states are likely to become disordered.

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- ☆ **BUT:** If the gas in a box is a metaphor for the universe, how did it get into the highly ordered (low entropy) initial state??

Could the Low Entropy Initial State Be a Rare Fluctuation from Equilibrium?

Ludwig Boltzmann (1895):

Yes! "Assuming the universe great enough, the probability that such a small part of it as our world should be in its present state is no longer small." *Nature vol. 51, p. 413, 1895.*

Could the Low Entropy Initial State Be a Rare Fluctuation from Equilibrium?

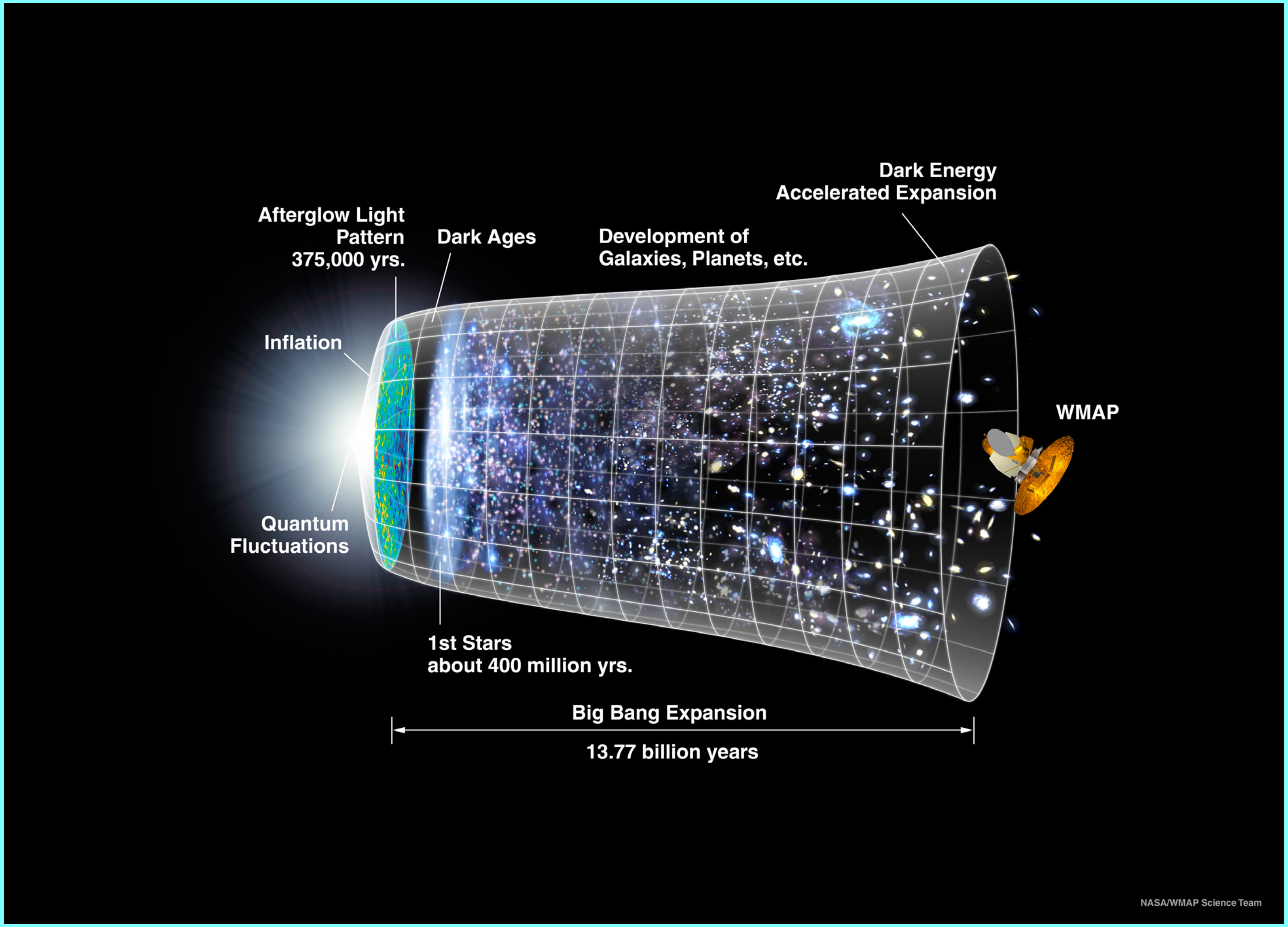
Richard Feynman (1965):

No! If the order in the universe were a fluctuation in equilibrium, then it would not extend so far. Boltzmann was right that we don't know how big the universe is, so no matter how improbable it may be for a galaxy like ours to form as a fluctuation, it may still be likely that it happens. But, Feynman argued, we still know something about *relative probabilities*. It will be vastly more likely to form an isolated galaxy, surrounded by equilibrium gas, than to form two galaxies. And the probability of seeing 10^{11} galaxies in the visible region is too small to even think about.

Cosmological "Solution"

For lack of any other explanation, it is usually assumed that the low entropy initial state was fixed by whatever unknown physics determined the initial conditions for the universe.

Does Inflation Explain the Arrow of Time?



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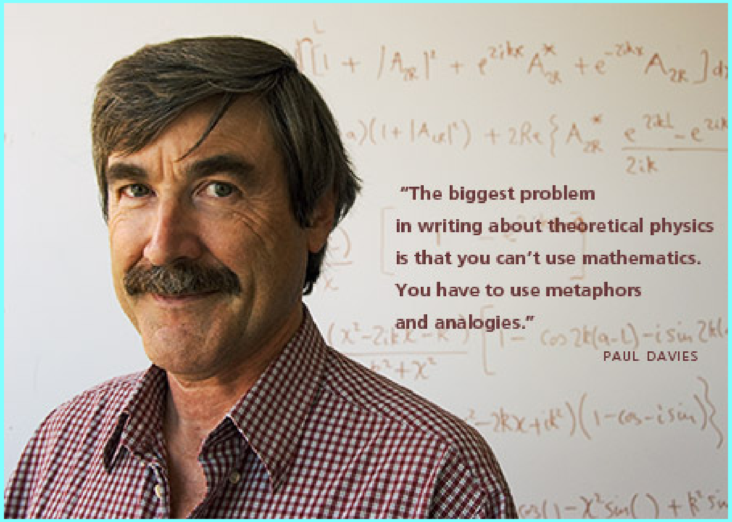

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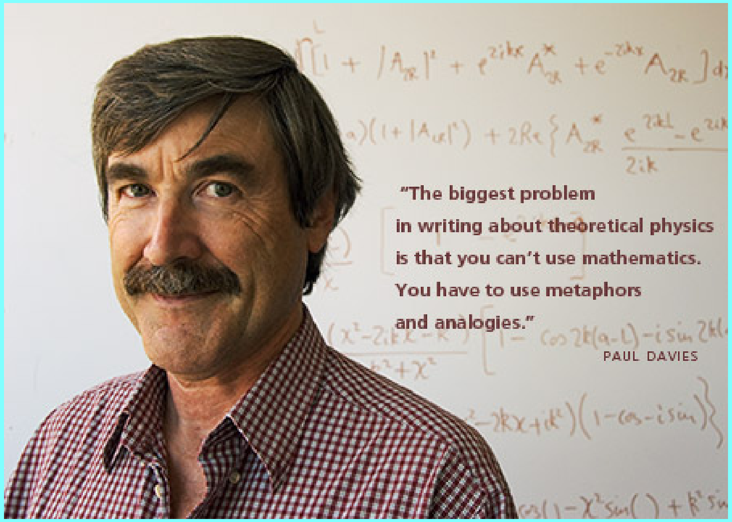

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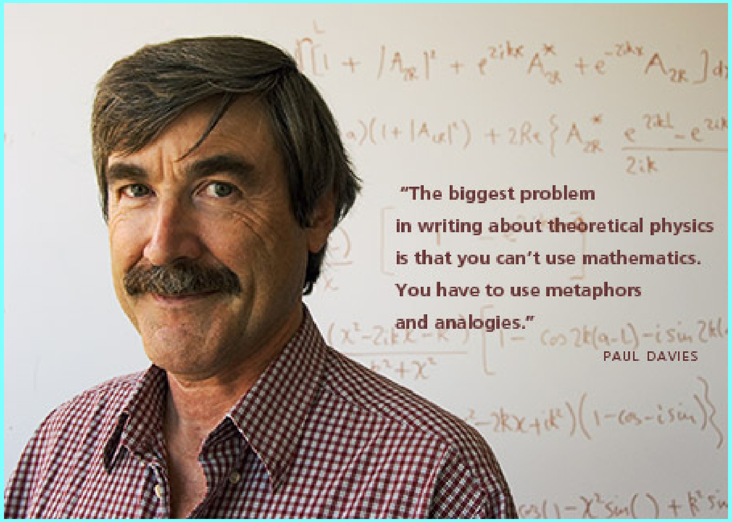

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Inflation and time asymmetry in the Universe

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398 NATURE VOL. 301 3 FEBRUARY 1983

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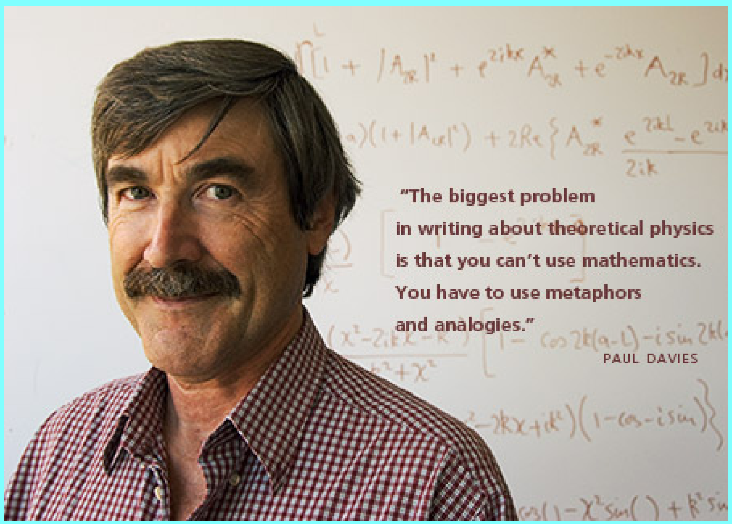

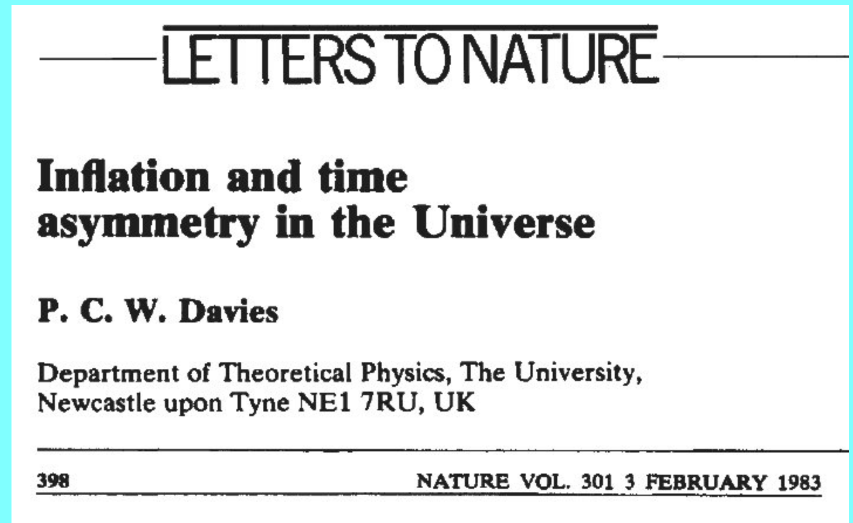

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"The recently proposed inflationary Universe scenario explains several of the mysteries of modern cosmology. I argue here that it also provides a natural explanation for the origin of time asymmetry ('time's arrow') in the Universe."

Don Page:

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THE UNITED STATES OF AMERICA
ONE DOLLAR
How Predictable Is Quantum Gravity?
Don Page bets Stephen Hawking one pound Sterling that strong quantum cosmic censorship holds, namely that a pure initial state composed entirely of regular field configurations on complete, asymptotically flat hypersurfaces will have a unique S-matrix evolution under the laws of physics to a pure final state composed entirely of regular field configurations on complete, asymptotically flat hypersurfaces.
Stephen Hawking bets Don Page $1.00 that in quantum gravity the evolution of such a pure initial state can be given in general only by a $-matrix to a mixed final state and not always by an S-matrix to a pure final state.
Don N. Page
I concede in light of the weakness of the $
Stephen Hawking, 23rd April 2007

Don Page:

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Inflation does not explain time asymmetry

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Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA, and Center for Theoretical Physics, University of Texas, Austin, Texas 78712, USA

39 NATURE VOL. 304 7 JULY 1983

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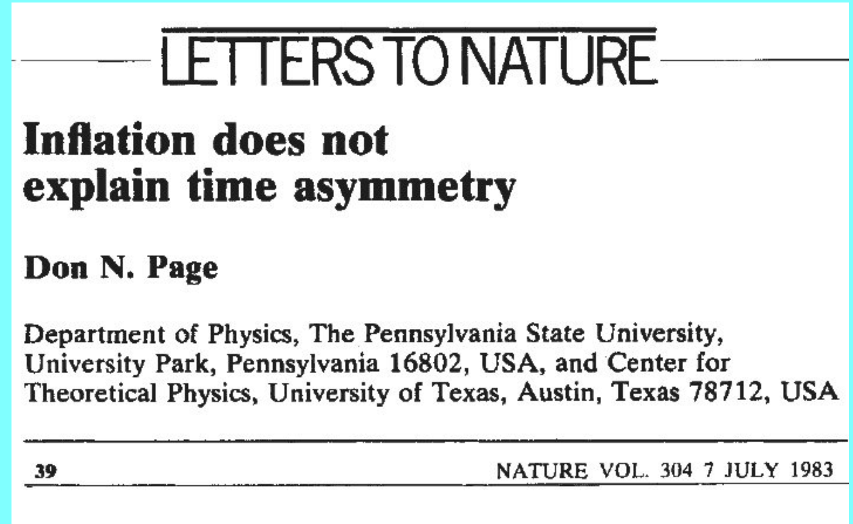

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"Davies has argued that the inflationary cosmological scenario provides a natural explanation for the time asymmetry of the Universe. Here I dispute this argument by noting that the inflationary scenario implicitly invokes time asymmetry with the assumption of the absence of initial spatial correlations. No scenario based on CPT-invariant dynamical laws can explain the time asymmetry apart from postulating or explaining these special initial conditions, as Penrose has emphasized."

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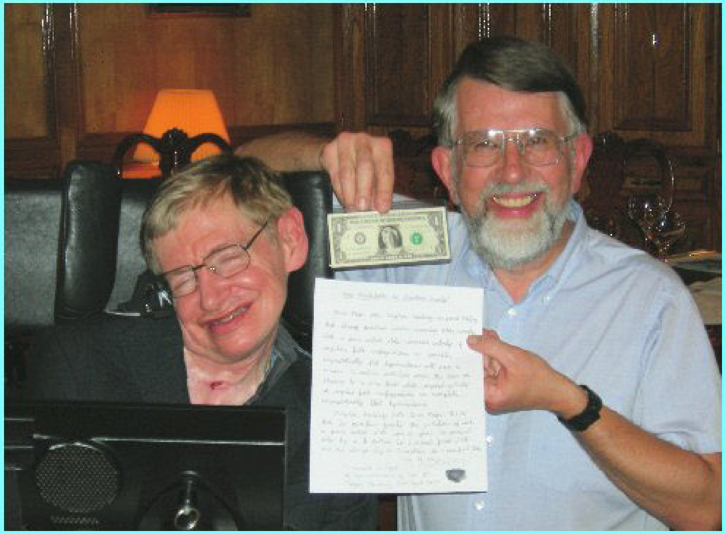

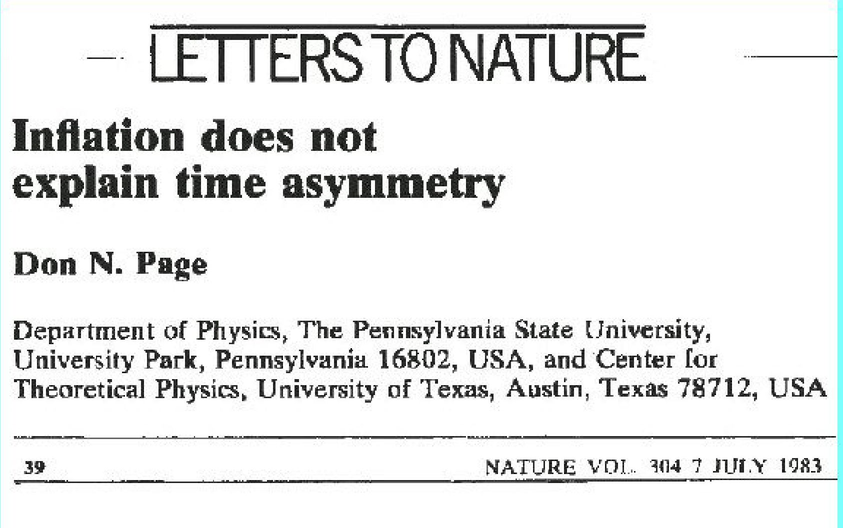
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We disagree:

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☆ **KEY IDEA:** If the maximum possible entropy is **INFINITE**, then any state of finite entropy is a state of low entropy! The entropy can increase from any given starting point. The metaphor of a gas in a box becomes a gas without a box.

Toy Model: A Gas Without a Box

- Purpose of this toy model: to show that it is possible to create an arrow of time from time-symmetric laws of physics **AND** time-symmetric initial conditions.

- The model: Consider a gas of N (large number) non-interacting particles, moving in empty space. Choose the initial conditions by making up a probability distribution for positions and velocities, and use a random number generator with these probabilities to fix the initial positions and velocities for the N particles.

Toy Model: Initial Conditions

- Insist that the probabilities be normalizable — probabilities must add up to one. This rules out ill-defined options, such as a uniform probability to be anywhere.

★ **YES,**

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- Contradictions like this show why a uniform probability distribution is simply **not conceivable.** (I learned about this particular paradox from Aron Wall.)

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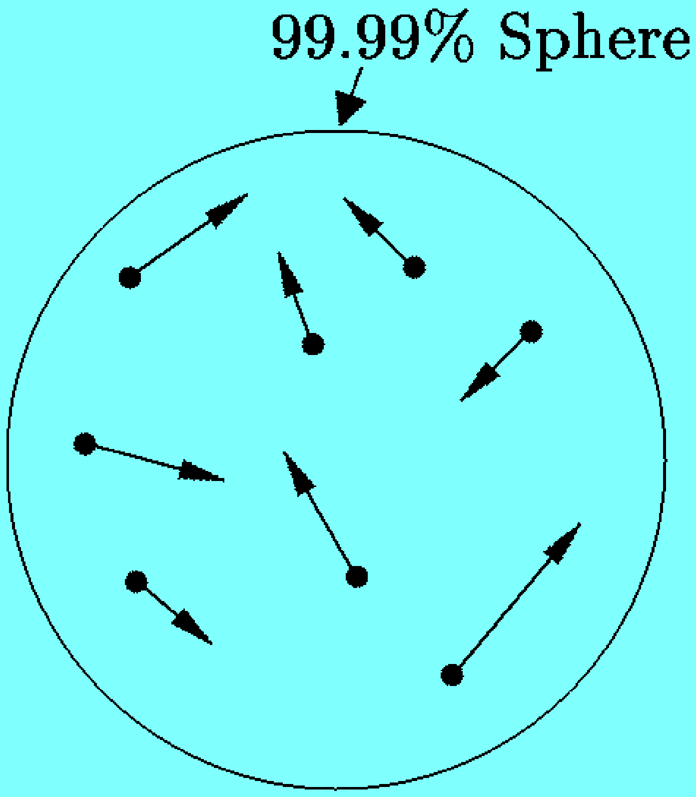
- Normalizability implies that the distribution of particles is localized — it must be possible to draw a sphere that is big enough so that the probability that all N particles are inside the sphere is 99.99% (or any number you choose).

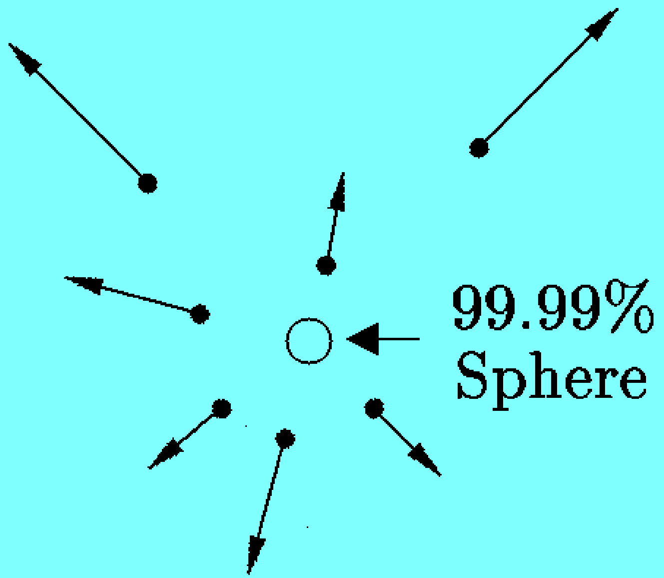
Toy Model: Behavior Near the Starting Point

- Let the system evolve. Particles move at constant velocities.
- Initially some particles are moving in, others are moving out, entropy might be going up or down. For a while we do not expect an arrow of time.

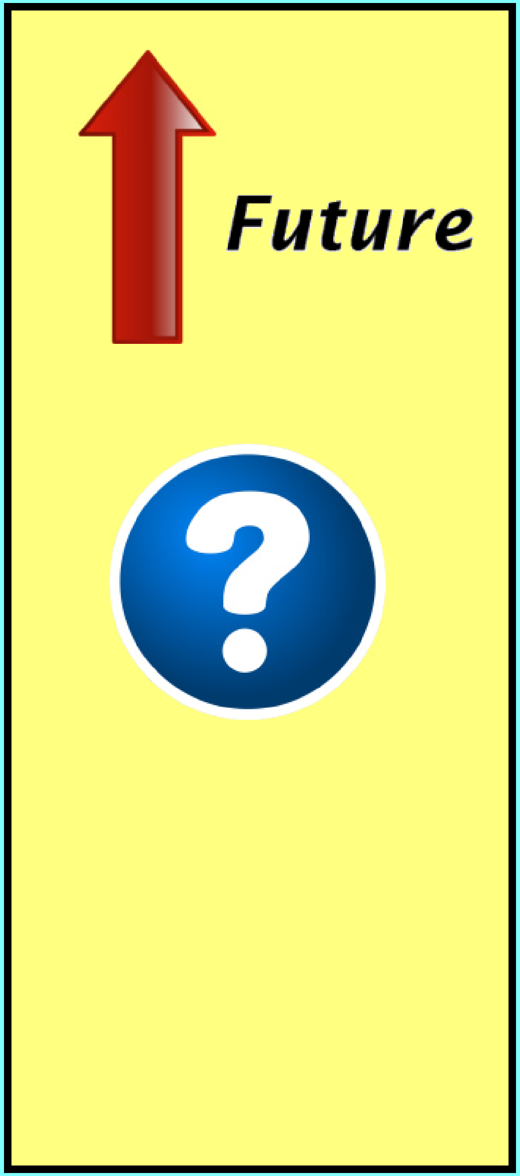


Toy Model: Behavior at Late Times

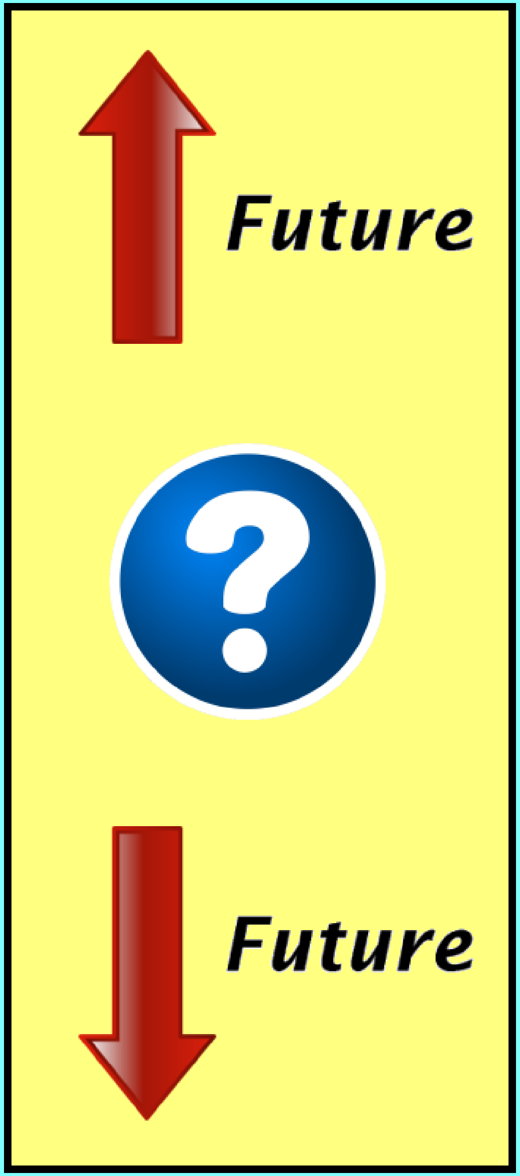
- Particles have fixed velocities, so after a long time they will have moved a large distance, far outside the 99.99% sphere.
- The picture will look like the diagram, with a visually clear arrow of time. Coarse-grained entropy will grow indefinitely as the gas spreads out through the infinite space.



☆ If we evolve the system forward in time, entropy will start to grow, approaching its maximum value of infinity, and an arrow of time will develop.



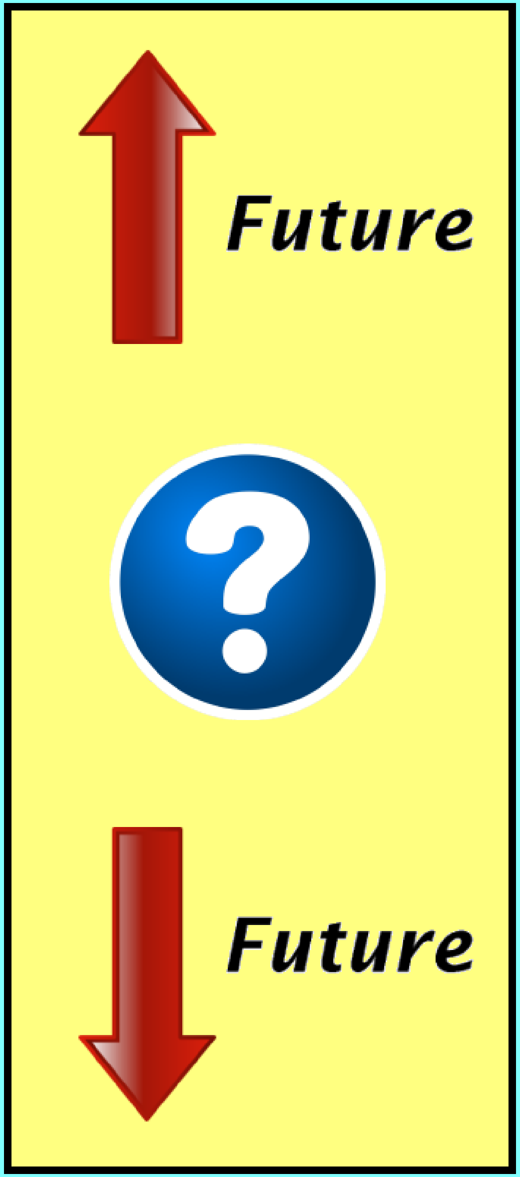
- ☆ If we evolved the system backwards in time, it would behave the same way, but at large negative times the arrow of time would point the other way!
- ☆ Bottom line: for a finite amount of time near the starting point, there is no arrow of time. But for infinite periods of time in the future and in the past, the arrow of time is well-defined.



★ The ever-expanding gas is a metaphor for (eternal) inflation. In many inflationary models, once inflation starts it never completely stops. It stops in places but is always continuing in other places.

SUMMARY SO FAR

- ☆ I have tried to show that time-symmetric laws of evolution, with time-symmetric initial conditions, can nonetheless produce an arrow of time.
- ☆ A key requirement is a system with an infinite maximum possible entropy.
- ☆ The arrow of time is two-headed, pointing to the future in the future and to the past in the past.



Can fine-tuning be explained in the context of Hamiltonian evolution?

For finite phase space, NO:

Due to Liouville's theorem, if a system is known to lie within a specific volume of phase space, that volume does not change as the system evolves. So time evolution merely pushes the probability distribution around in phase space. What is improbable at one time, because it corresponds to a small volume in phase space, is equally improbable at all times.

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BUT: if the available phase space is infinite, the answer is YES!!

Example of a Hamiltonian leading to fine-tuning:

$$\boxed{H = -pq \ .}$$

Then

$$\dot{p} = -\frac{\partial H}{\partial q} = p \qquad \dot{q} = \frac{\partial H}{\partial p} = -q \ .$$

So, as time passes, q is fine-tuned to become arbitrarily close to 0. (Similarly q can be fine-tuned to any value, and any function of p and q can be fine-tuned.)

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suggested by Larry Guth. This gives

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- How effective is the fine-tuning?

  It is as effective as you want! Suppose you want to fine-tune q so that $|q| < \epsilon$, where ϵ is some small number that you specify. Suppose you want this fine-tuning to hold with probability $1 - \delta$, where δ is some small number that you specify. Then, if the initial state is chosen, there is always some time T such that for any $t > T$, $P(|q| < \epsilon) > 1 - \delta$.

If we assume that the universe has a finite available phase space, do we get into trouble?

- Suppose, for example, that reality can be described by some quantum system with a maximum possible entropy. Then the system will reach thermal equilibrium and undergo Poincaré recurrences *forever*, and all microstates will occur and re-occur with equal probability.

- Life (including observers like us) will continue to occur in the thermal equilibrium phase, but with overwhelming probability the worlds that they will observe will look nothing like ours. Boltzmann brains.

- ☆ Our view of the world is historical. We understand the universe in terms of how it evolved from its big-bang origin. But in thermal equilibrium, probabilities are determined **ONLY** by state counting. For example, a state that looks just like our world except that $T_{\rm CMB} = 10$ K would have more microstates, and would be much more likely than 2.7 K. [Ref: Dyson, Kleban, & Susskind (2002).] So, if the entropy has an upper limit, we would expect that 10 K would be much more likely than 2.7 K.

- ☆ However, if the semiclassical global picture of eternal inflation is valid, then new pocket universes are constantly being created and new regions of phase space are constantly being explored. Poincaré recurrences do not happen, and our historical understanding of the world is justified.

SUMMARY

1) I claim to have shown that time-symmetric laws of evolution, with time-symmetric initial conditions, can nonetheless produce an arrow of time, if the available phase space is infinite.

2) I claim to have shown that Hamiltonian evolution, if the available phase space is infinite, can lead to the fine-tuning of dynamical variables.

3) I claim to have shown that if the universe is described by underlying physics with only a finite available phase space, then we would expect a thermal equilibrium world that would be very different from what we observe.