Dynamics, Symmetry and Asymmetry of Ancient Solutions to Mean Curvature Flow

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Lecture 1. ODE techniques in curvature flows.
Abstract: I will discuss the construction of self similar solutions (solitons) to MCF, under various symmetry assumptions (joint work with Ilmanen and Velazquez). These solutions can be used to construct examples of fattening and also turn out to be useful in obtaining estimates for solutions without symmetry.

Lecture 2. Uniqueness and Nonuniqueness of MCF from cones.
Abstract: Having constructed self similar Mean Curvature flows from cones, I will now look at solutions that are not necessarily self-similar, and show in particular that the set of varifold solutions to MCF with a minimal cone as initial condition is infinite dimensional.

Lecture 3. Velazquez example, and asymptotics for ancient ovals.
Abstract: Continuing from the previous lecture I will describe Velazquez example of a mean curvature flow that has type-2 blow-up, and in particular show how his construction implies that there exist solutions that become singular even though the mean curvature H itself remains bounded. This contrasts results of Sesum for Ricci flow, and Bing Wang for MCF in three dimensions. I will then also start the discussion of ancient convex solutions to MCF.

Lecture 4. Ancient convex solutions to MCF.
Abstract: After deriving the formal asymptotic description of ancient ovals, I will discuss recent work with Sesum and Daskalopoulos on the rigorous construction and classification of certain ancient ovals. The self similar solutions constructed in the first lecture will appear again.

Lecture 5. Dynamics and symmetry of ancient convex mean curvature flows.
Abstract: After explaining the concept of alpha non-collapsedness I will elaborate on Xu Jia Wang's work, and construct ancient convex mean curvature flows without any symmetry. Then I will explain the Brendle-Choi argument that proves ancient noncompact mean curvature flows are rotationally symmetric, and explain how this was adapted to the case of compact ancient flows in joint work with Daskalopoulos-Sesum.

References:

Tom Ilmanen: Lectures on Mean Curvature Flow and Related Equations, Lecture Notes for a Conference at ICTP, 1995, available at https://people.math.ethz.ch/~ilmanen/papers/notes.pdf The final section of the notes are an introduction to the first lecture.

J. Velazquez: "Curvature blow-up in perturbations of minimal cones evolving by mean curvature flow" Ann. Scuola Norm. Sup. Pisa 21 (1994) pp 595-628. This paper contains the examples in lecture 2. The fact that these singular solutions sometimes have bounded mean curvature does not seem to be well known.

For lectures 4 and 5 there are the recent preprints:
S. Angenent, P. Daskalopoulos, N. Sesum: Unique asymptotics of ancient convex mean curvature flow solutions, arXiv:1503.01178
S. Angenent, P. Daskalopoulos, N. Sesum: Uniqueness of two-convex closed ancient solutions to the mean curvature flow, arXiv:1804.07230
S. Brendle, K. Choi: Uniqueness of convex ancient solutions to mean curvature flow in R^3, arXiv:1711.00823