The thresholding scheme for mean curvature flow as minimizing movement scheme

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Abstract: The thresholding scheme of Bence-Merriman-Osher is a appealingly simple and computationally relevant time-discretization of motion by mean curvature, which easily extends to the multi-phase case. In the two-phase case, this discretization is easily seen to preserve the comparison principle, and it has been shown to converge to the viscosity solution of mean-curvature flow.
In [1], we have pointed out that this scheme preserves the gradient flow structure of mean curvature flow in the sense that it can be interpreted as a minimizing movement scheme in the language of de Giorgi.
Based on this interpretation, in [2] we give a convergence proof in the multi-phase case. This convergence proof is a conditional one, in the spirit of the convergence proof of Luckhaus & Sturzenhecker for the Almgren-Taylor-Wang scheme. In particular, the limiting notion of solution is based on Caccioppoli sets, and the additional condition amounts to rule out ghost interfaces.
In [3], we give another (conditional) convergence proof, which is even closer to the gradient flow structure: We show that de Giorgi's tools of ``metric slope'' and ``variational interpolation'' are just taylor-made to show that one obtains the family of localized dissipation inequalities that Brakke used to characterize mean-curvature flow.

References:
[1] S. Esedoglu, F. Otto.
Threshold dynamics for networks with arbitrary surface tensions.
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[2] T. Laux, F. Otto.
Convergence of the thresholding scheme for multi-phase mean-curvature flow.
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[3] T. Laux, F. Otto.
Brakke's inequality for the thresholding scheme.
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