Cosmology

Introduction

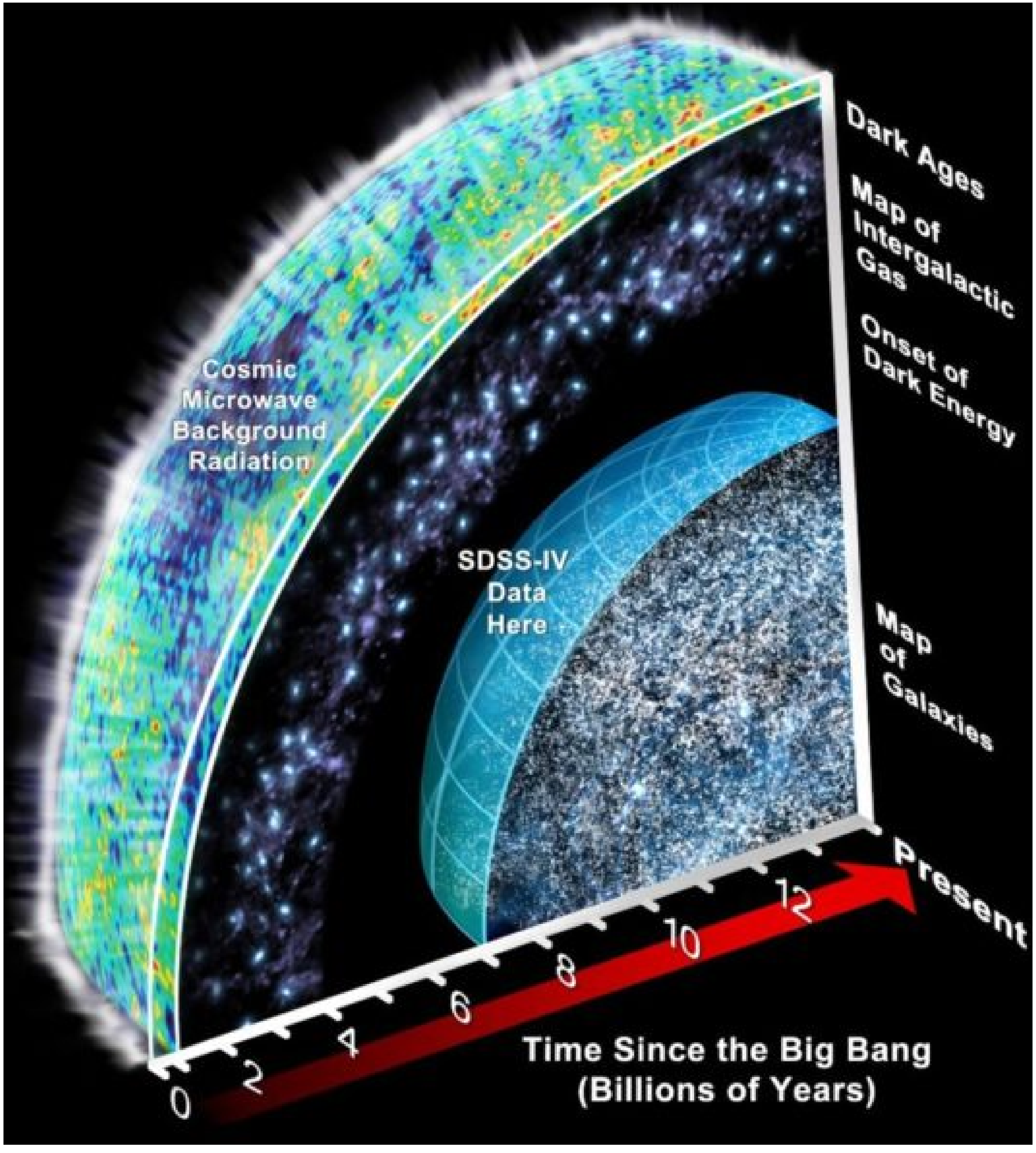
Geometry and expansion history

(Cosmic Background Radiation)

Growth

Secondary anisotropies

Large Scale Structure


Dark Ages
Map of Intergalactic Gas
Onset of Dark Energy
Cosmic Microwave Background Radiation
SDSS-IV Data Here
Map of Galaxies
Present
0
2
4
6
8
10
12
Time Since the Big Bang (Billions of Years)

Cosmology from Large Scale Structure Sky Surveys

- Supernovae Ia
- CMB
- Baryon Acoustic Oscillations
- Secondary anisotropies
- Cluster counts and clustering
- Redshift space distortions
- Weak gravitational lensing
- Your name here!

GEOMETRY

G
R
O
W
T
H

$cz \approx v \approx H_0 \, d$

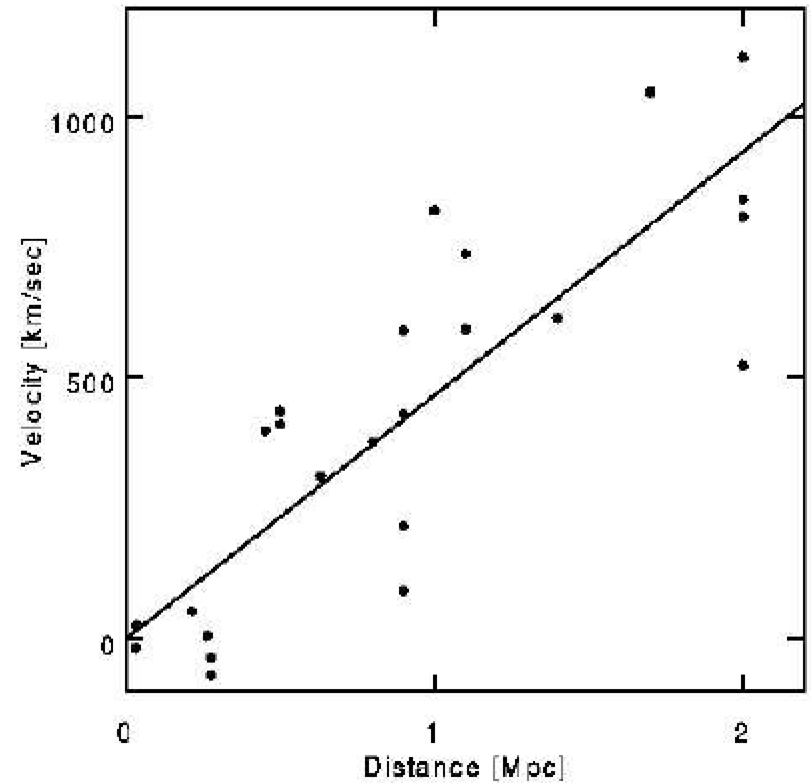
Speed = distance/time, so $1/H_0$ is a time:

(1 Mpc/500 km) sec

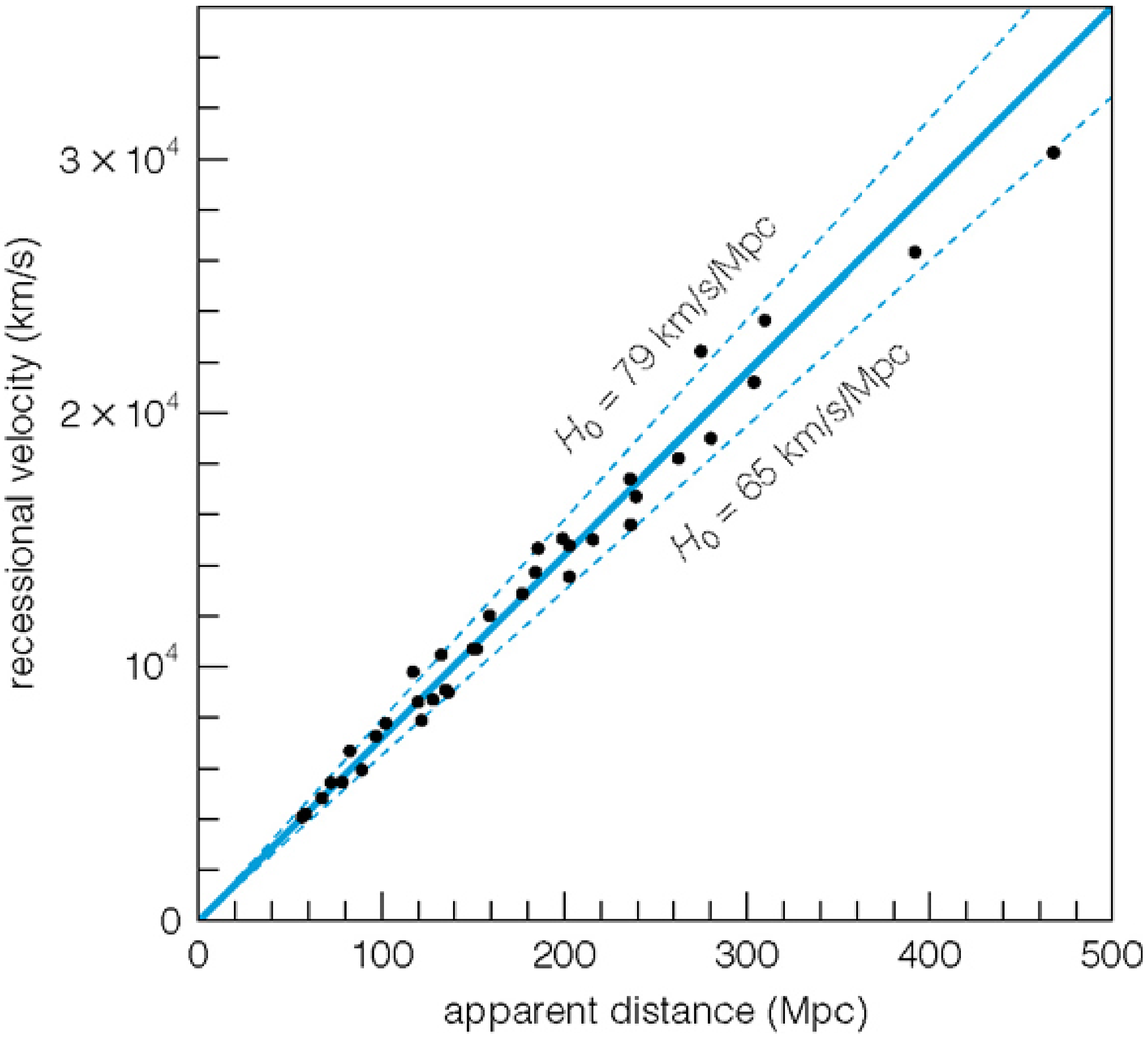
= $3.08 \times 10^{19}/500$ sec

= 2×10^9 years

This is the time since all separations = 0 (i.e. all objects were in same place)

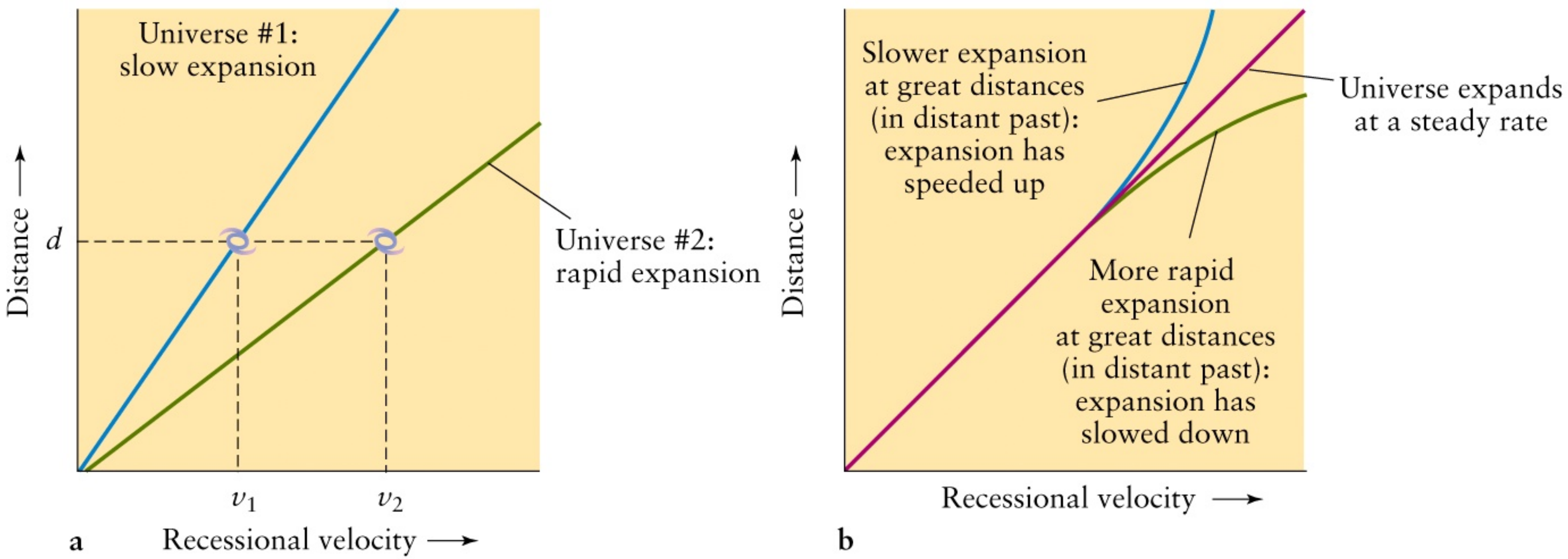


Slope of line gives $H_0 = 500$ (km/s)/Mpc.



Hubble's Law: velocity = H_0 x distance

Measuring the expansion



Expansion rate changes with time: Hubble's constant same at all positions in space, but may depend on time

Expect BIG BANG happened about ~14 Gyrs ago (assuming H_0~constant)

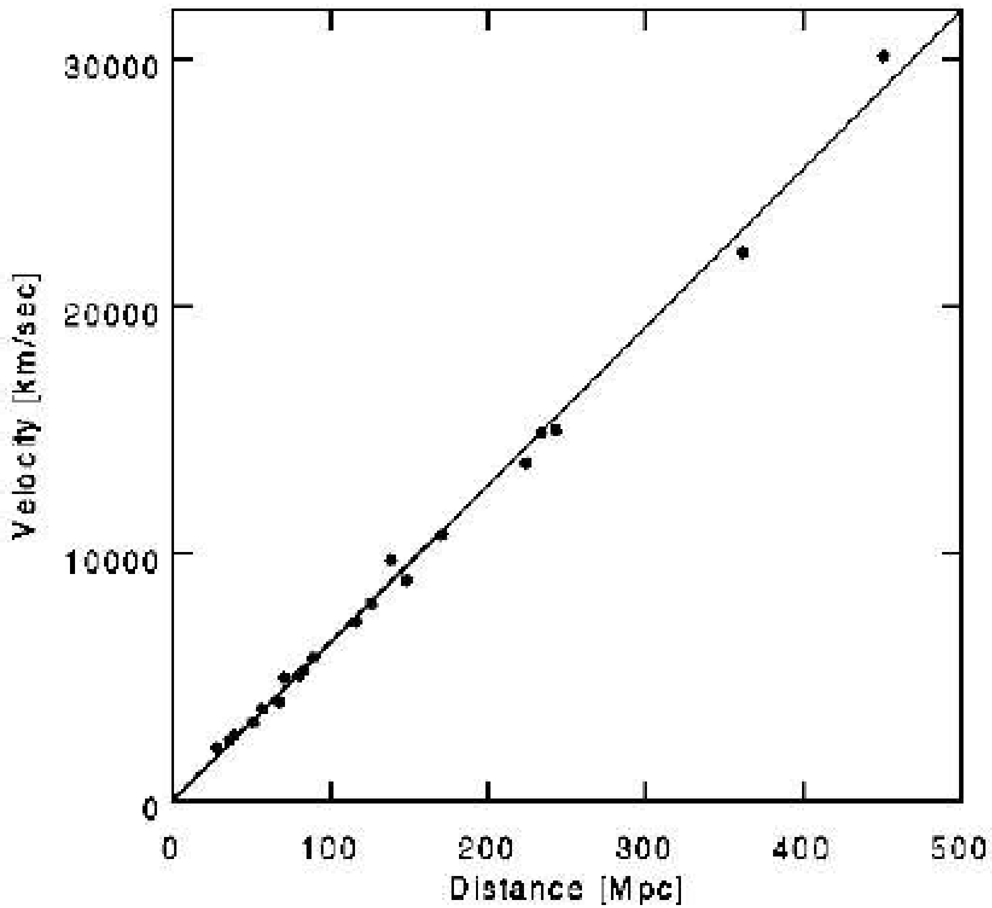
Expect observable scale of Universe:

$d_H = c/H_0$

$= (3\times10^5 \text{ km/s}) / (100h \text{ km/s/Mpc})$

$= 3000/h$ Mpc

(set h = 0.71)



Best current measurement

$H_0 = 71$ (km/s)/Mpc.

Age $\approx 1/H_0 = 14 \times 10^9$ years.

Three possible metrics for homogeneous and isotropic 3-space

$$ds^2 = dr^2 + S_\kappa(r)^2 d\Omega^2 \ ,$$

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

Changing from r to x = S_κ(r) makes this:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2/R^2} + x^2 d\Omega^2$$

Robertson-Walker metric

(If homogeneity and isotropy did not exist, it would be necessary to invent them!)

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$ Minkowski metric

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dx^2}{1 - \kappa x^2 / R_0^2} + x^2 d\Omega^2 \right]$$

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2 \right]$$

Much of Observational Cosmology dedicated to determining κ, $a(t)$, R_0

Distances in cosmology

$$ds^2 = a(t)^2[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

Along a spatial geodesic: $ds = a(t)dr$

'Proper' distance is d at fixed a: $d_p(t) = a(t)\int_0^r dr = a(t)r$

$$d_p(t) = a(t)r(x) = \begin{cases} a(t)R_0 \sin^{-1}(x/R_0) & (\kappa = +1) \\ a(t)x & (\kappa = 0) \\ a(t)R_0 \sinh^{-1}(x/R_0) & (\kappa = -1) \end{cases}$$

$$\dot{d}_p = \dot{a}r = \frac{\dot{a}}{a}d_p \qquad v_p(t_0) = H_0 d_p(t_0)$$

Note that $v_p > c$ for sufficiently large d_p

Redshift and expansion

Null-geodesic (light) has ds=0 so: $c^2 dt^2 = a(t)^2 dr^2$

Hence $c\dfrac{dt}{a(t)} = dr$ so $c\int_{t_e}^{t_0} \dfrac{dt}{a(t)} = \int_0^r dr = r$

But also $c\int_{t_e+\lambda_e/c}^{t_0+\lambda_0/c} \dfrac{dt}{a(t)} = \int_0^r dr = r$

Both equal same r, meaning interval between emission and observation always same

We had: $$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_{t_e+\lambda_e/c}^{t_0+\lambda_0/c} \frac{dt}{a(t)}$$

Subtract $\int_{t_e+\lambda_e/c}^{t_0} \frac{dt}{a(t)}$ from both to get:

$$\int_{t_e}^{t_e+\lambda_e/c} \frac{dt}{a(t)} = \int_{t_0}^{t_0+\lambda_0/c} \frac{dt}{a(t)}$$

Integral of dt/a(t) during emission = during observation.
But a ≈ constant during this short dt, so:

$\frac{1}{a(t_e)} \int_{t_e}^{t_e+\lambda_e/c} dt = \frac{1}{a(t_0)} \int_{t_0}^{t_0+\lambda_0/c} dt$ making $\frac{\lambda_e}{a(t_e)} = \frac{\lambda_0}{a(t_0)}$

But $z = (\lambda_0 - \lambda_e)/\lambda_e$ so $$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}$$

Luminosity distance

$$ds^2 = -c^2dt^2 + a(t)^2[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

How is flux = Luminosity/4π distance^2 modified?

flux = Luminosity/Area where: $A_p(t_0) = 4\pi\, S_\kappa(r)^2$

Luminosity = Energy/time, but $E_0 = E_e/(1+z)$

and $dt_0 = dt_e\,(1+z)$

So flux = Luminosity/4π $S_\kappa(r)^2\,(1+z)^2$.

Define luminosity distance: $d_L = S_\kappa(r)\,(1+z)$.

Even in flat space $d_L = r\,(1+z) = d_p(t_0)\,(1+z)$.

Angular diameter distance

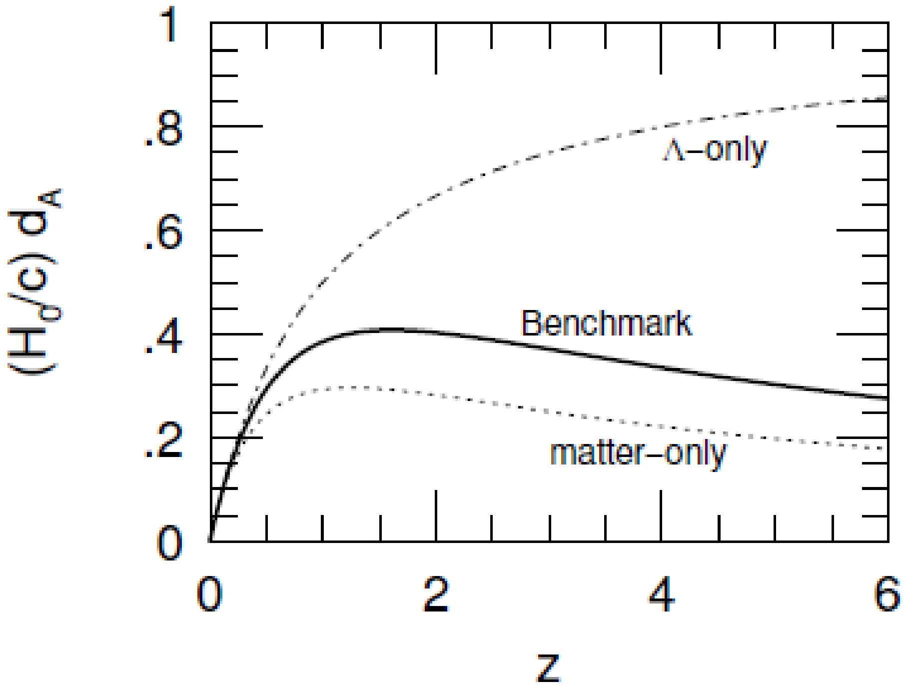
Light from (r,θ_1,ϕ_1) and (r,θ_2,ϕ_2) travels to origin:

$$ds = a(t_e)\, S_\kappa(r)\, \delta\theta$$

But ds = length ℓ, and $a(t_e) = 1/(1+z)$,

so $\ell = S_\kappa(r)\, \delta\theta/(1+z)$

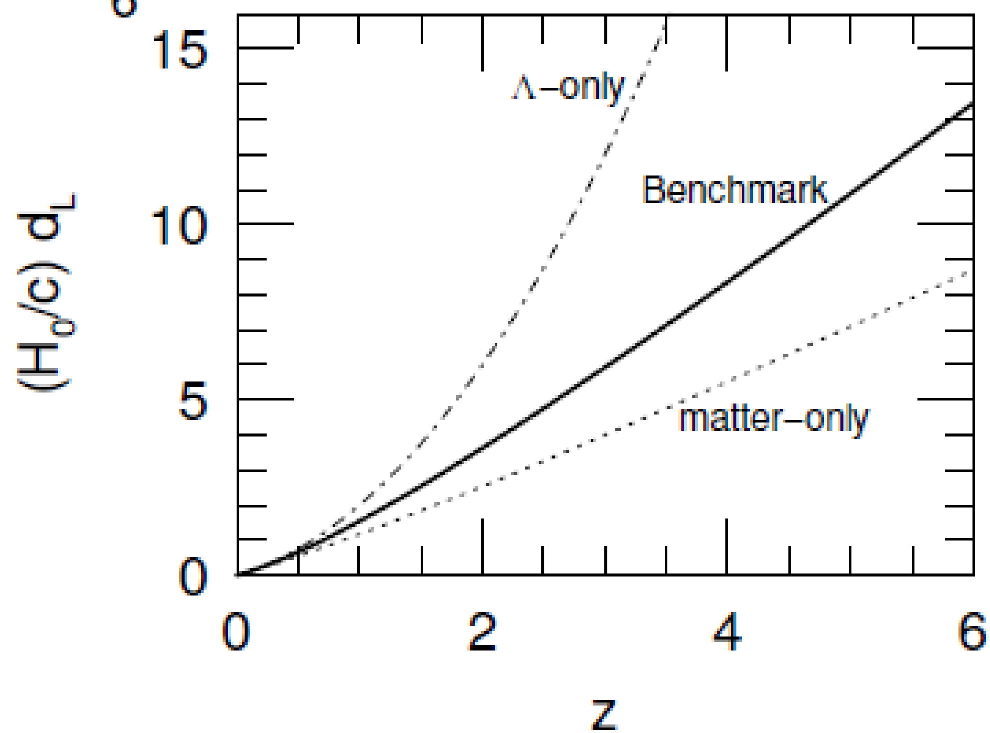
Hence $d_A = \ell/\delta\theta = S_\kappa(r)/(1+z) = d_L/(1+z)^2$



Angular diameter and luminosity distance

Age:

$H_0 t_0 = {}_0\int^1 da/a_0 \, (a_0 H_0 / aH)$



At small look-back times

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0}(t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0}(t-t_0)^2 + \dots$$

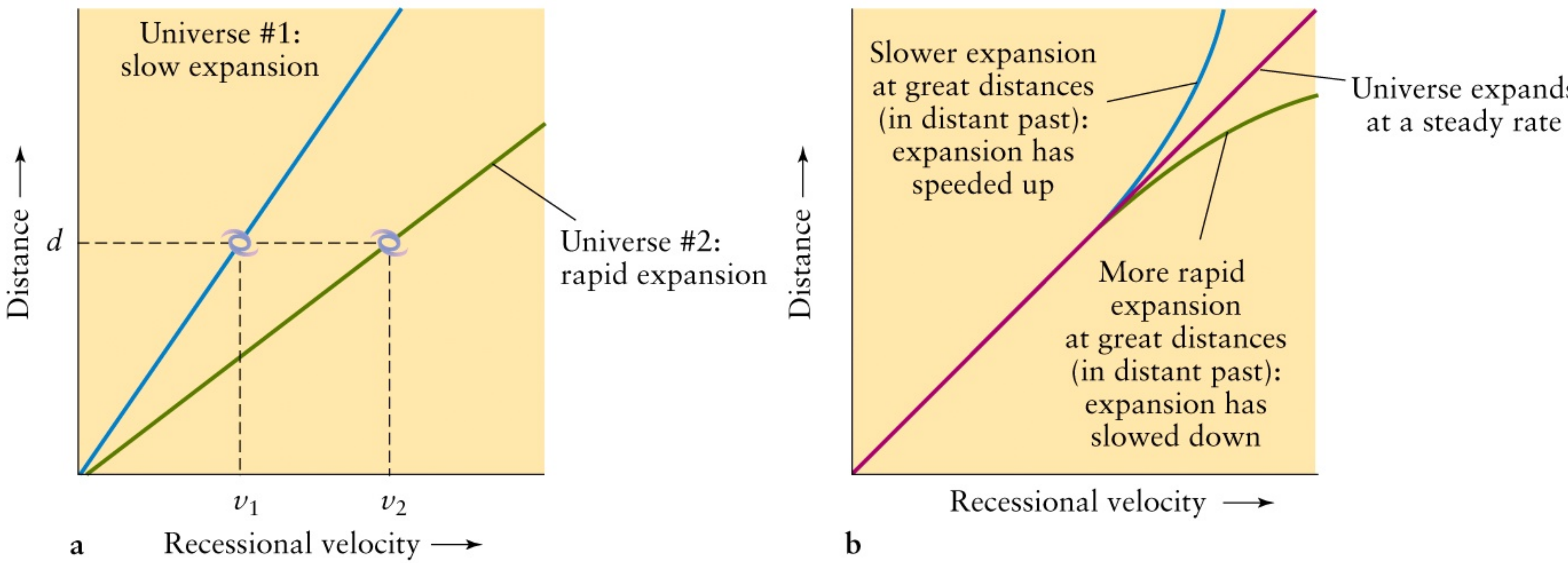
$$\frac{a(t)}{a(t_0)} \approx 1 + \left.\frac{\dot{a}}{a}\right|_{t=t_0}(t-t_0) + \frac{1}{2}\left.\frac{\ddot{a}}{a}\right|_{t=t_0}(t-t_0)^2$$

$$a(t) \approx 1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2$$

$$q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t=t_0} = -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0}$$

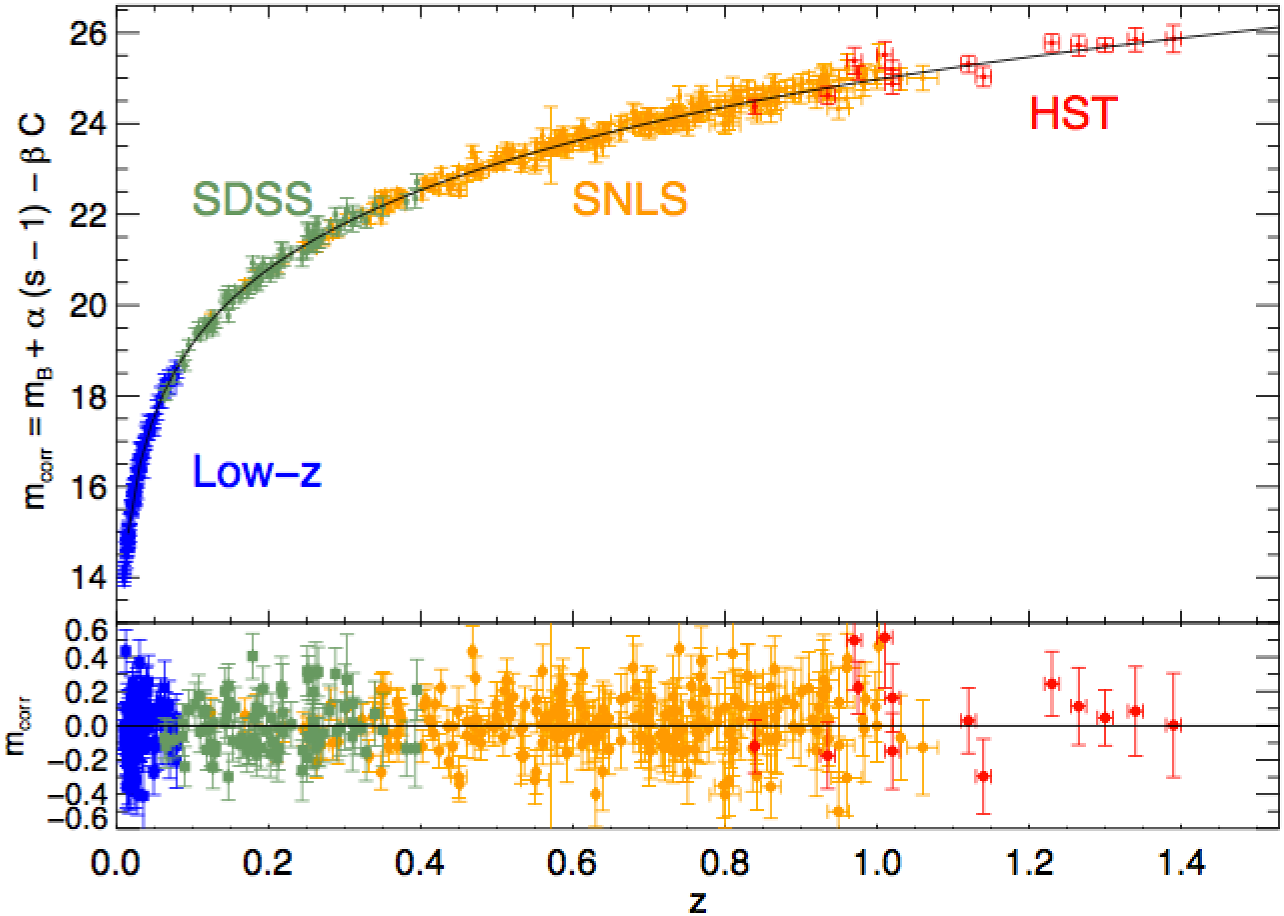
$$d_p(t_0) \approx \frac{c}{H_0}\left[z - (1+q_0/2)z^2\right] + \frac{cH_0}{2}\frac{z^2}{H_0^2} = \frac{c}{H_0}z\left[1 - \frac{1+q_0}{2}z\right]$$

Measuring the expansion



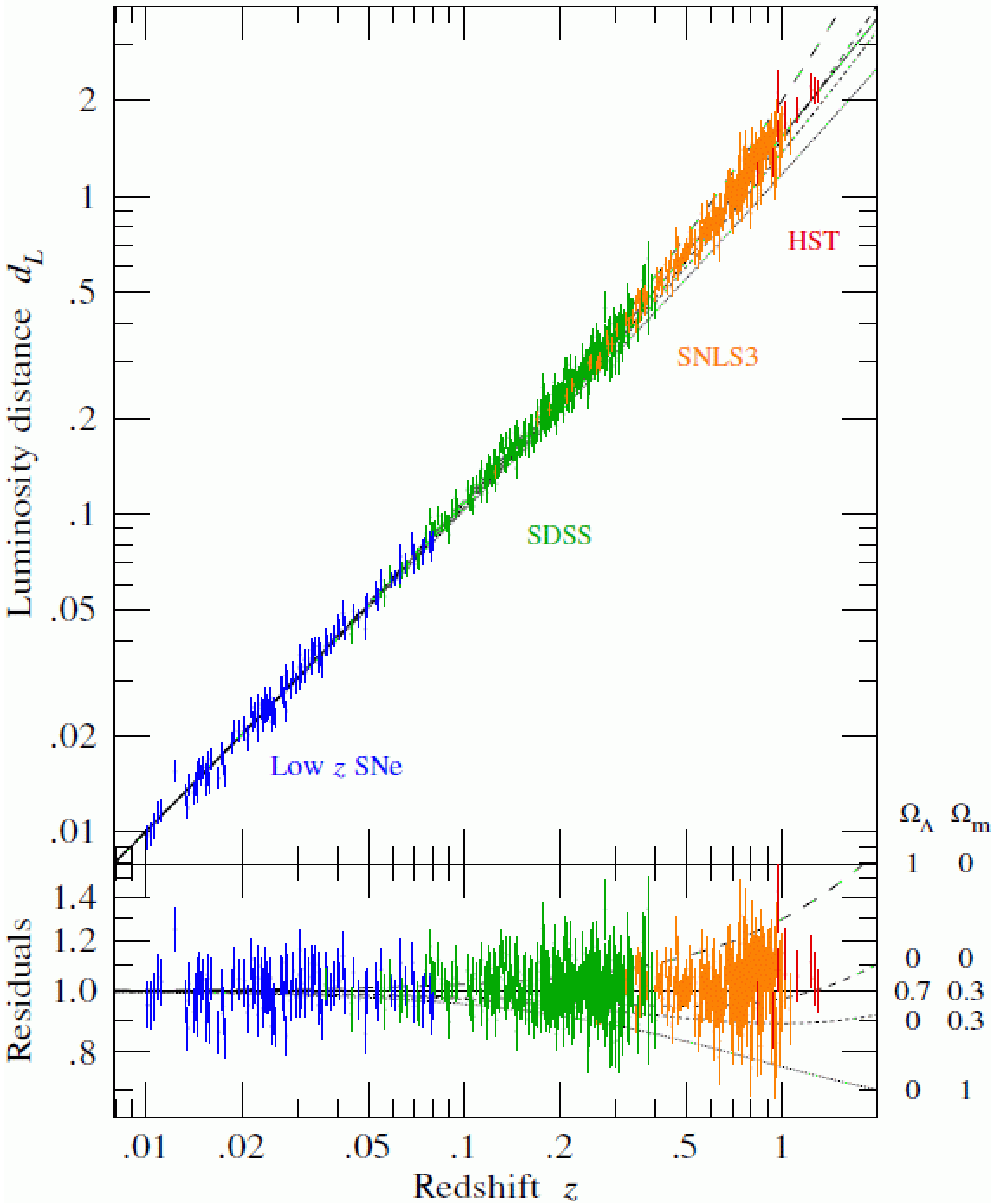
$$d_p(t_0) \approx \frac{c}{H_0}\left[z-(1+q_0/2)z^2\right]+\frac{cH_0}{2}\frac{z^2}{H_0^2}=\frac{c}{H_0}z\left[1-\frac{1+q_0}{2}z\right]$$

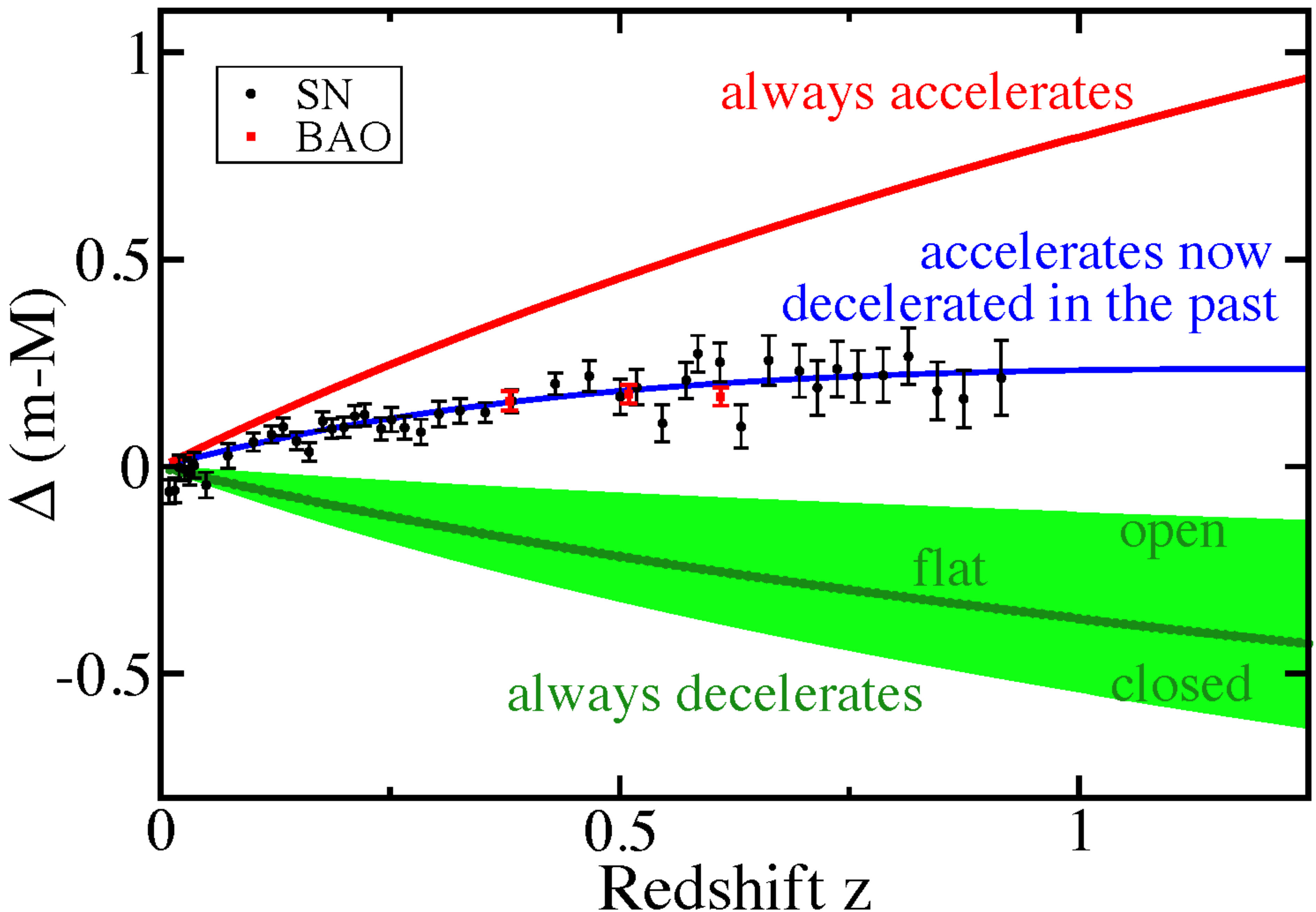
Standard Candles: SNIa



Supernova Cosmology:

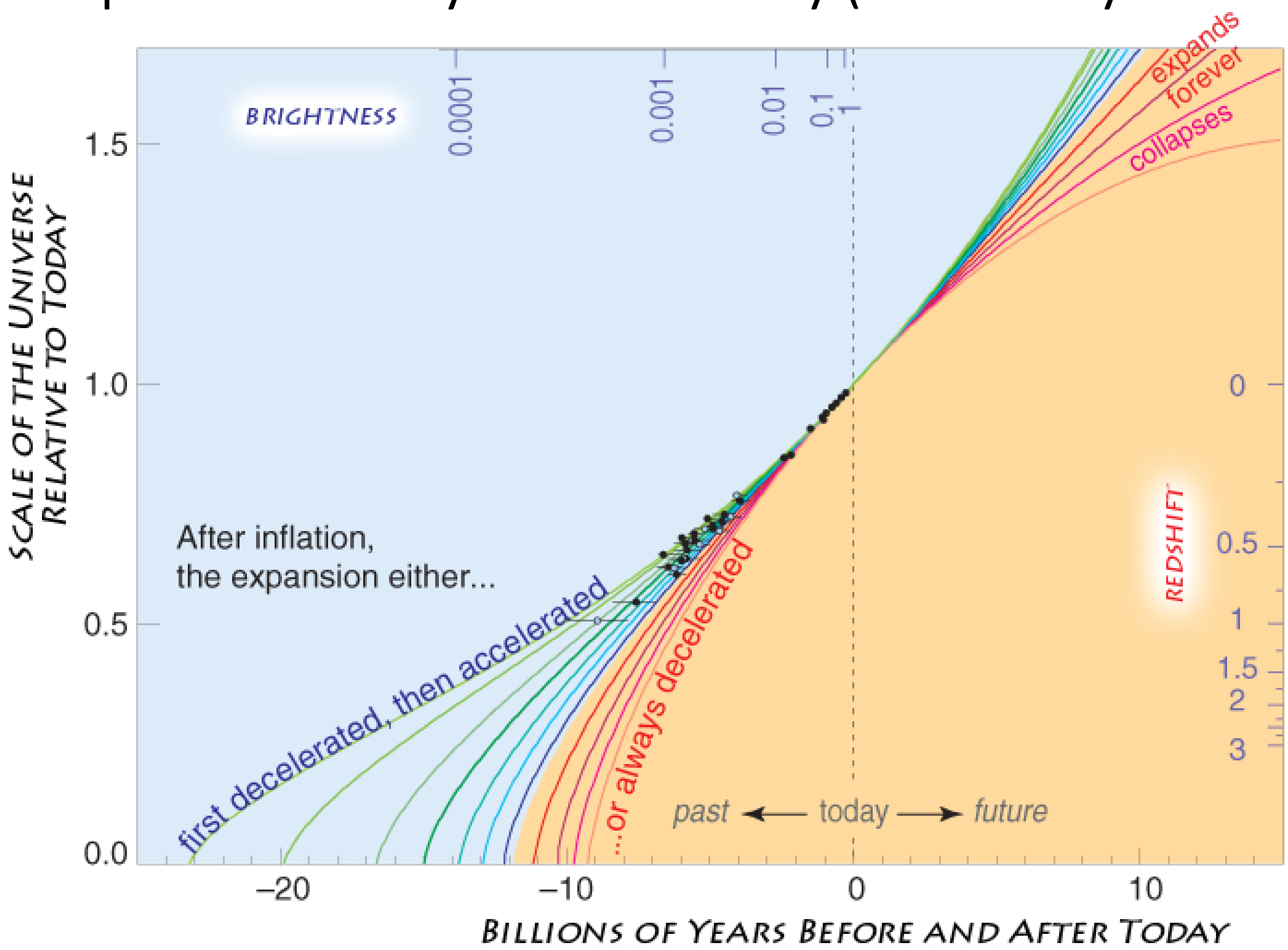
Evidence for a complex expansion history





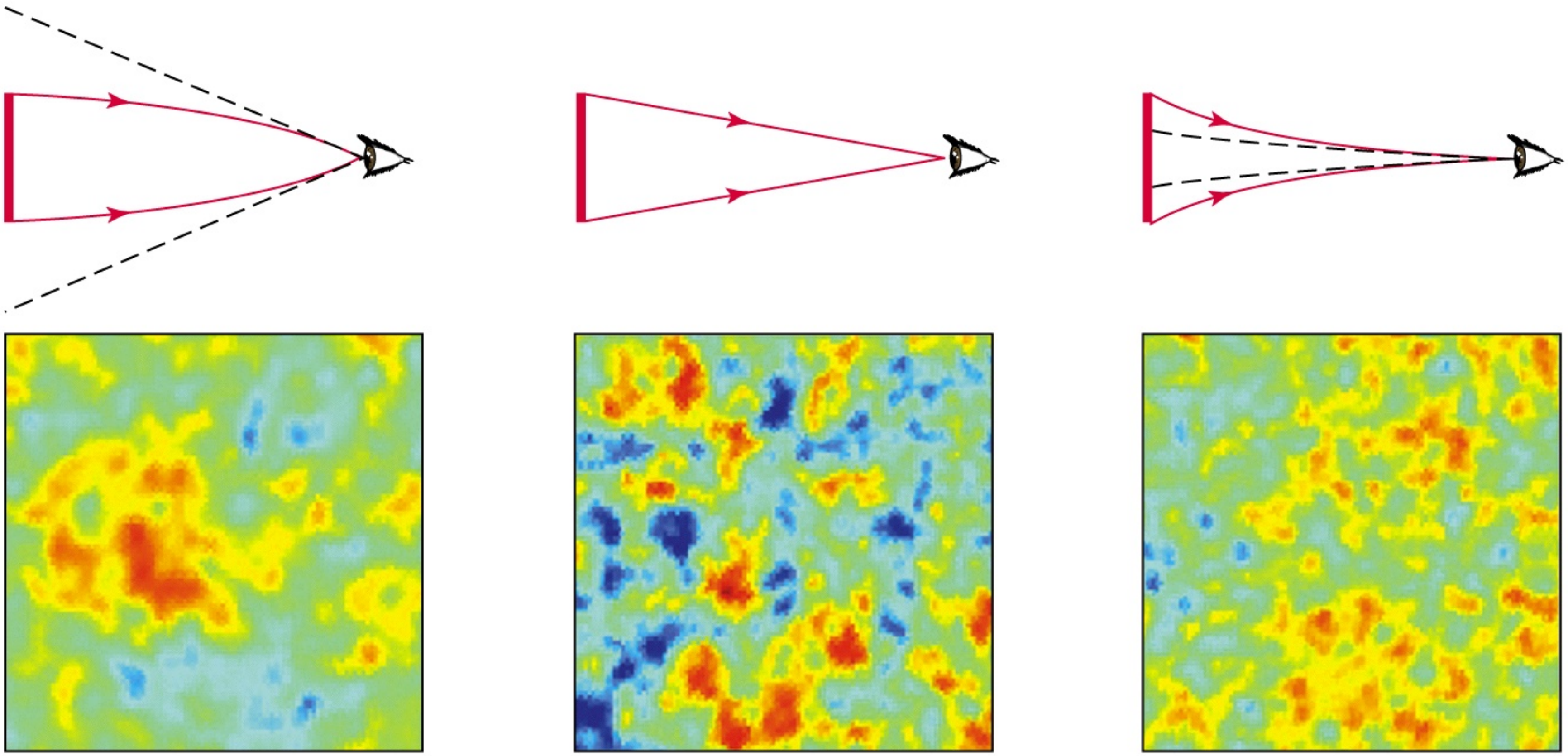
SN
BAO
always accelerates
accelerates now
decelerated in the past
open
flat
closed
always decelerates
Δ (m-M)
Redshift z
1
0.5
0
-0.5
0
0.5
1

Expansion history from Geomety (Luminosity distance)



Geometrical Test of curvature:

Standard Rod = Hubble volume at Last Scattering

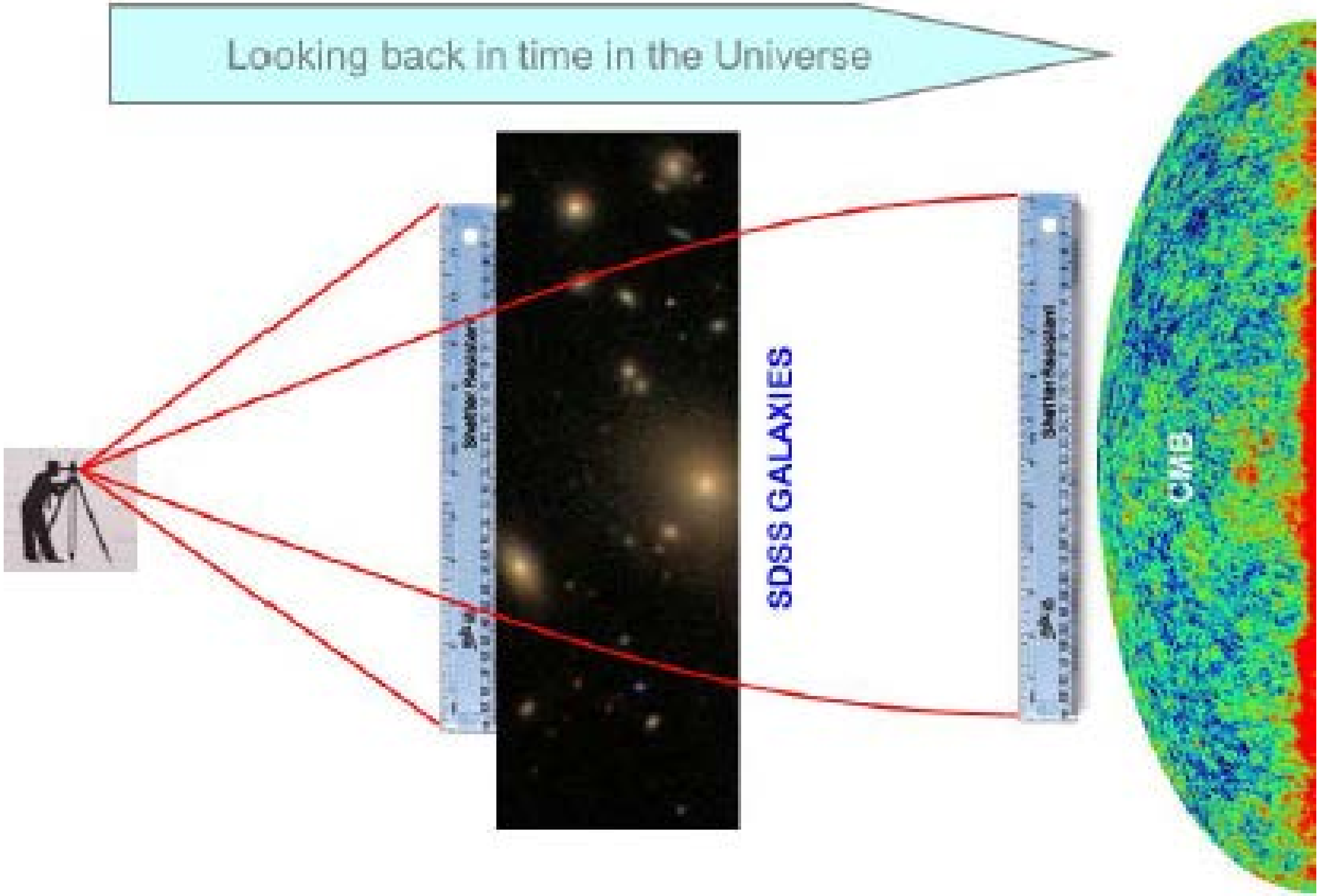


a If universe is closed, "hot spots" appear larger than actual size

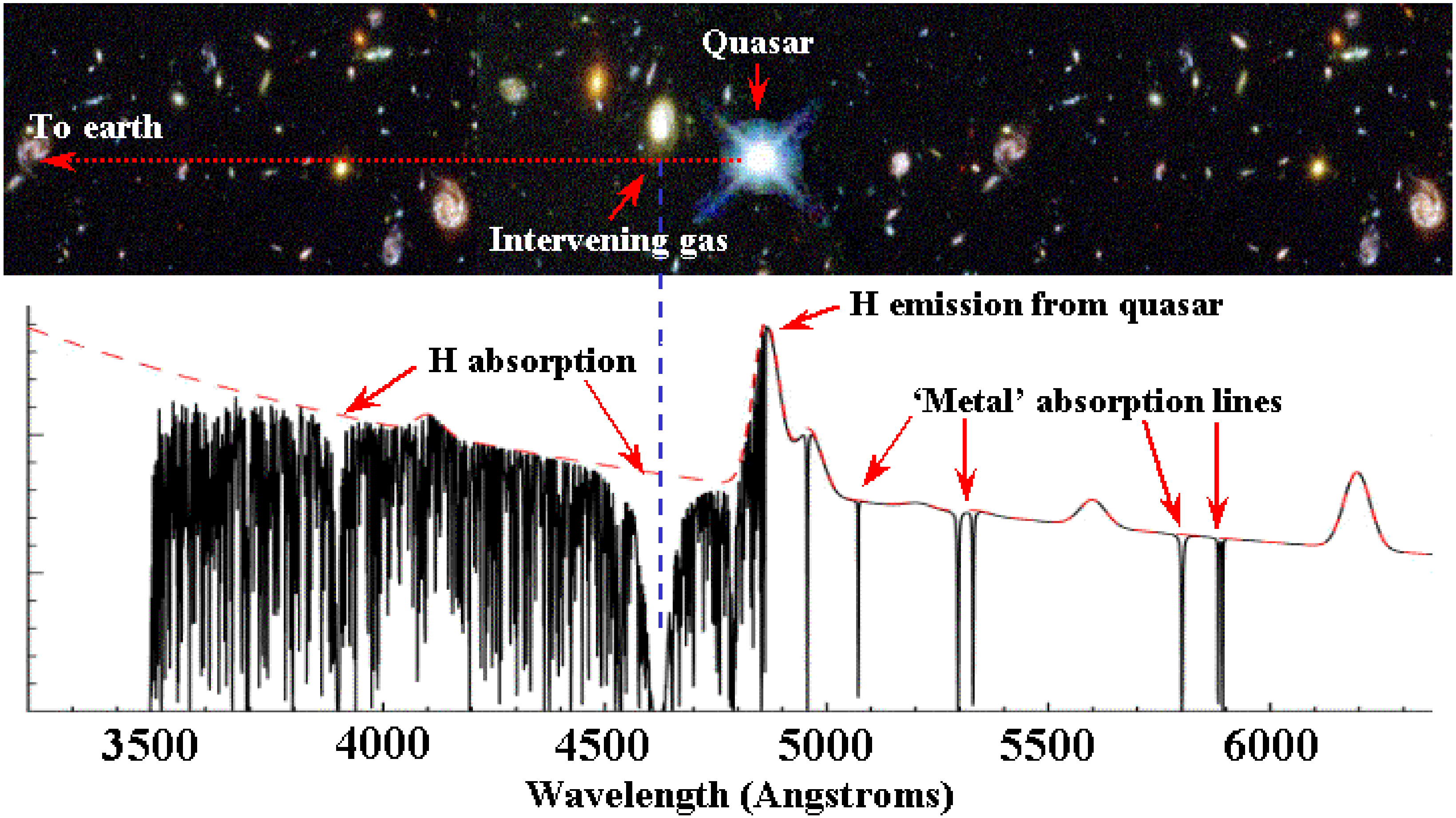
b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

CMB physics = geometry at late times: Baryon 'Acoustic' Oscillations in the Galaxy Distribution



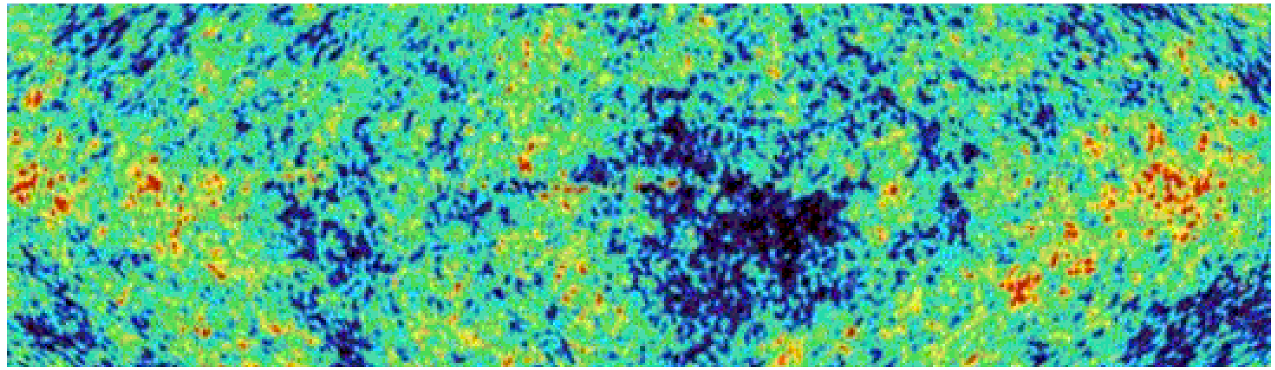
Can see baryons that are not in stars …



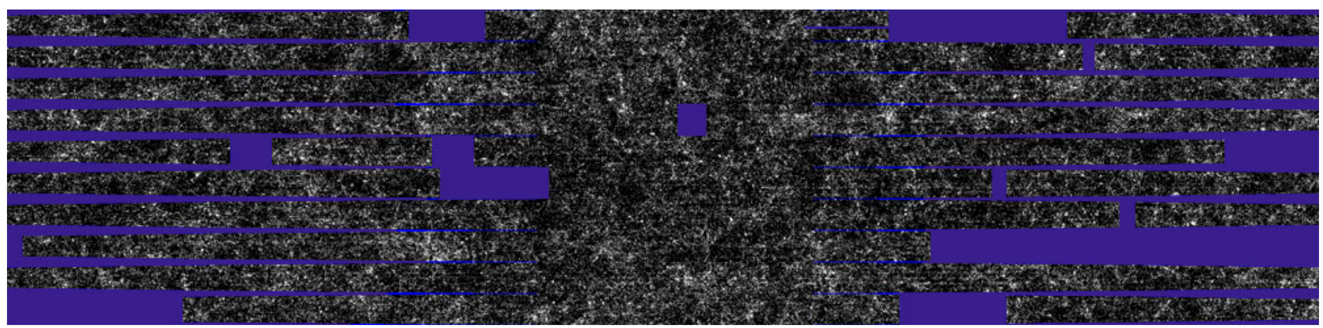
High redshift structures constrain neutrino mass

The ISW effect

Cross-correlate CMB and galaxy distributions

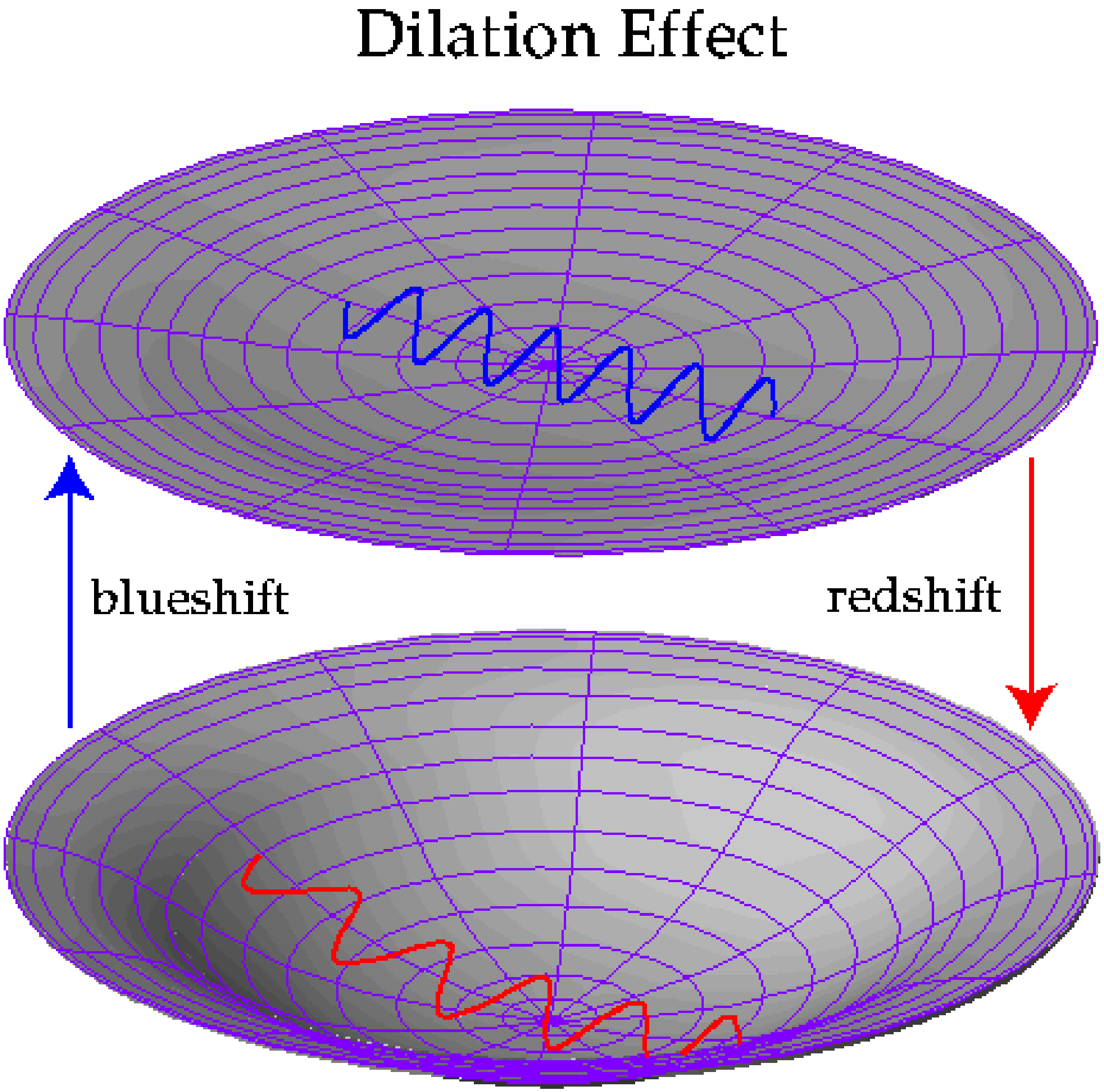


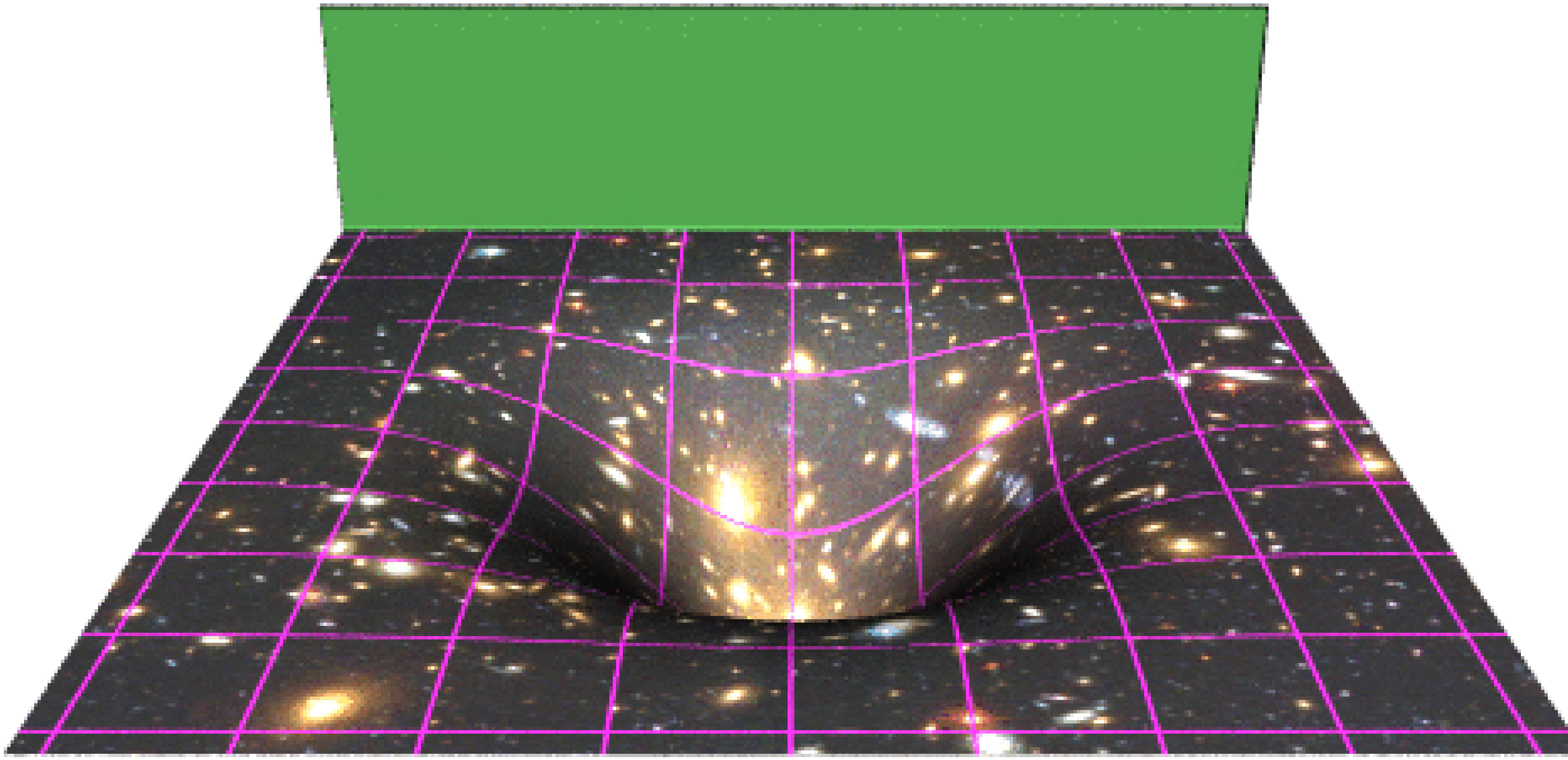
Interpretation requires understanding of galaxy population



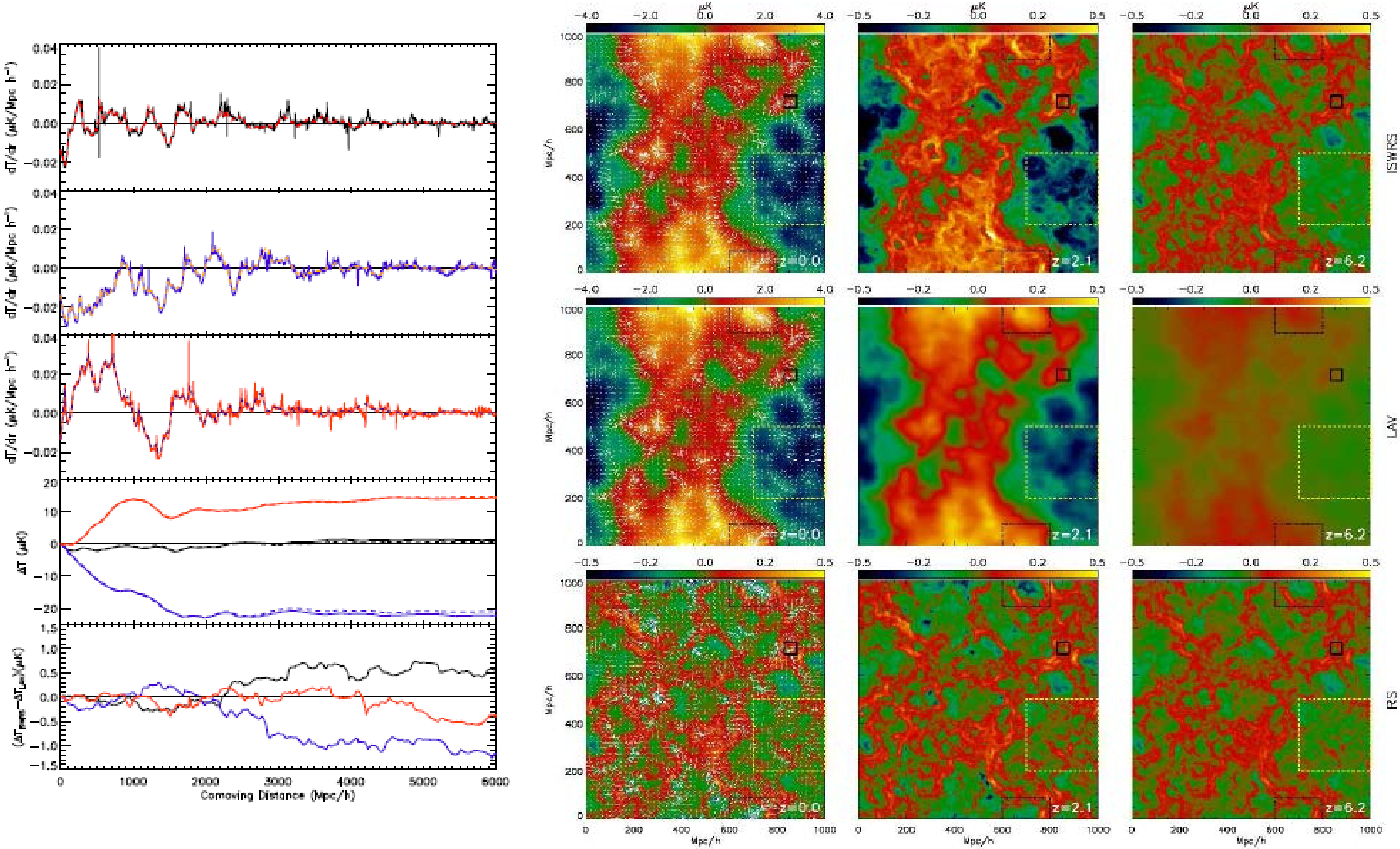
Cosmology from growth rate of gravitational instability (which must overcome expansion):

Signal depends on b(a) D(a) d/dt [D(a)/a]



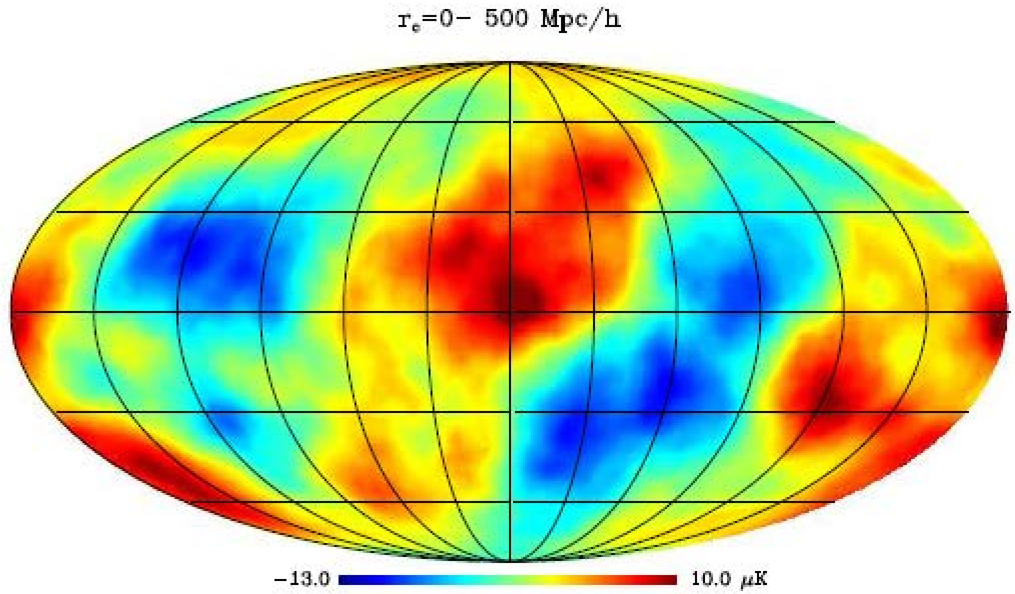


Effect mainly at later times, when Dark Energy begins to dominate



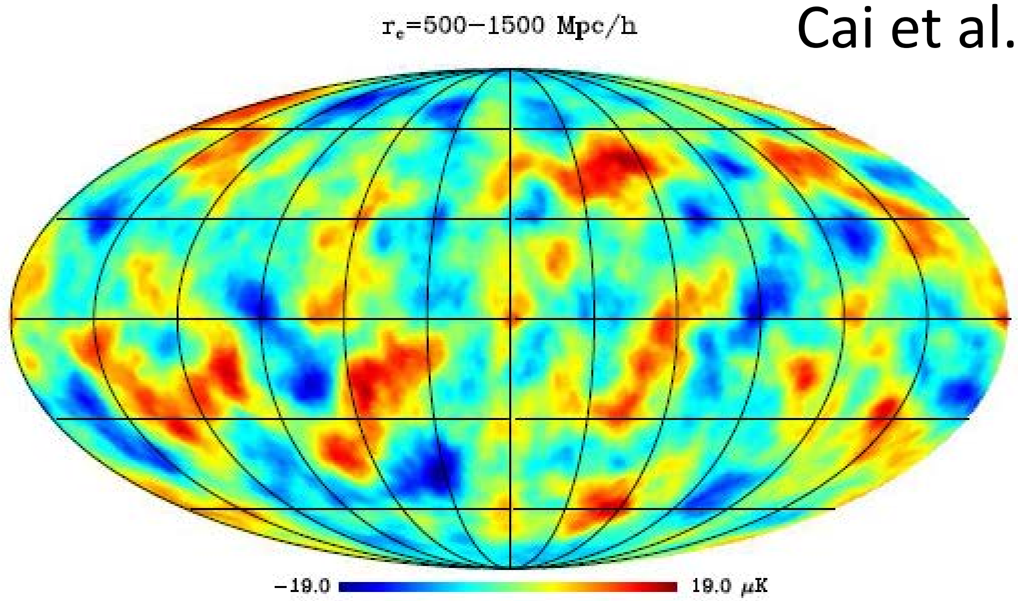
Cai et al. 2010

Linear
Effect
(smaller in
f(R) models)



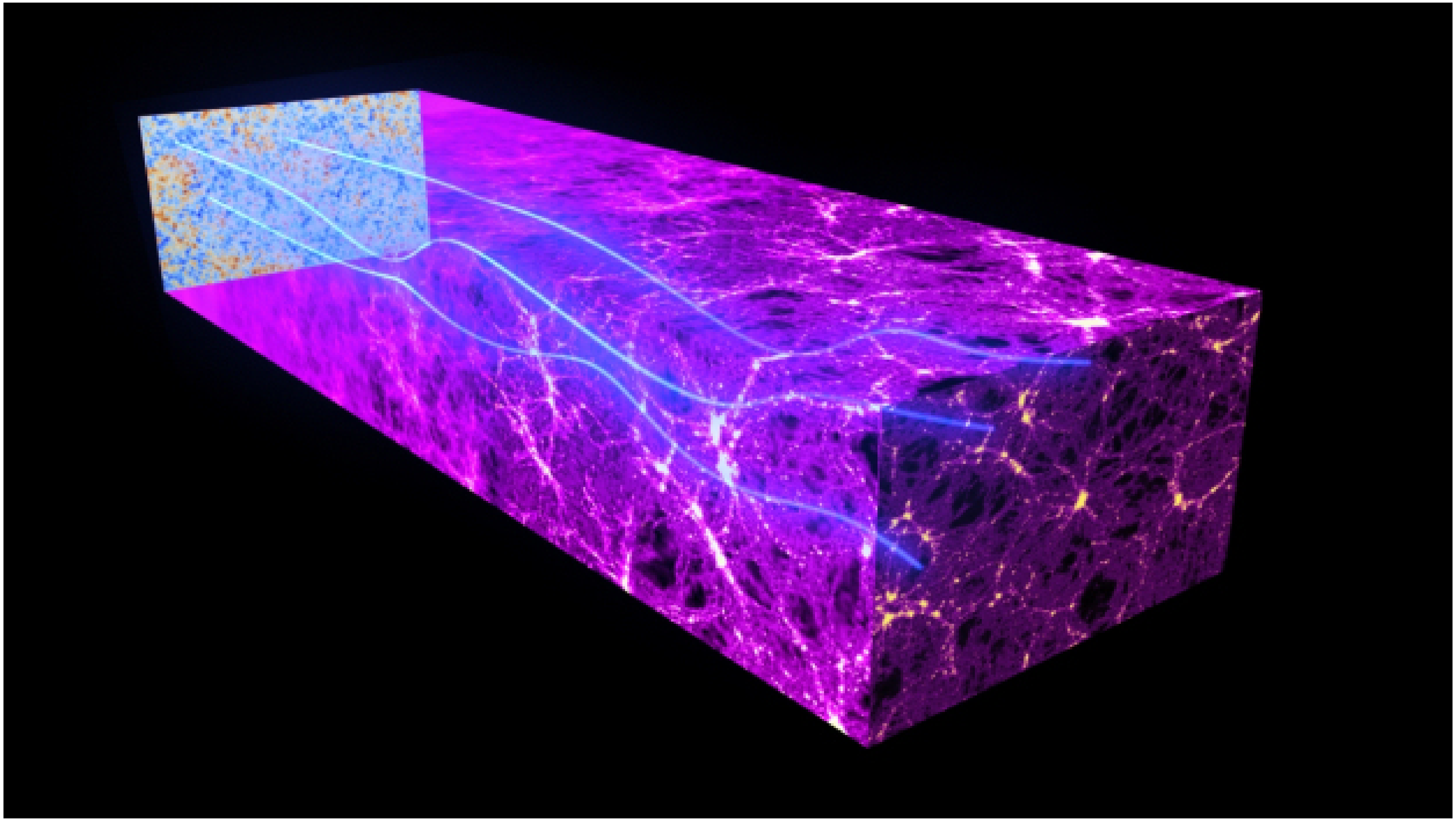
+

Cai et al. 2010, 2014

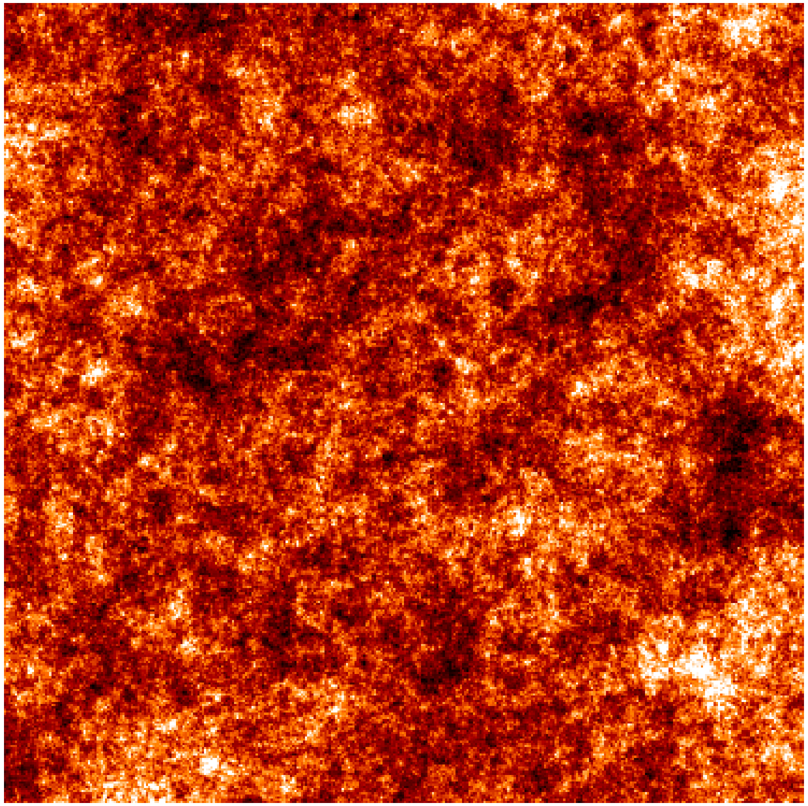


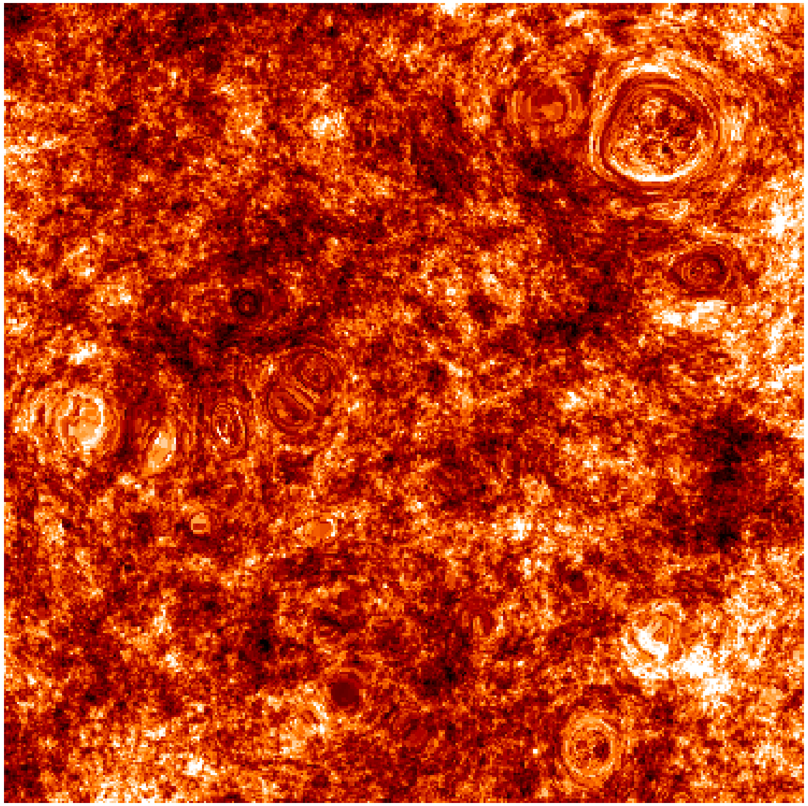
Nonlinear
effects
(bigger in
f(R) models)

Gravitational lensing



Lensing of the CMB

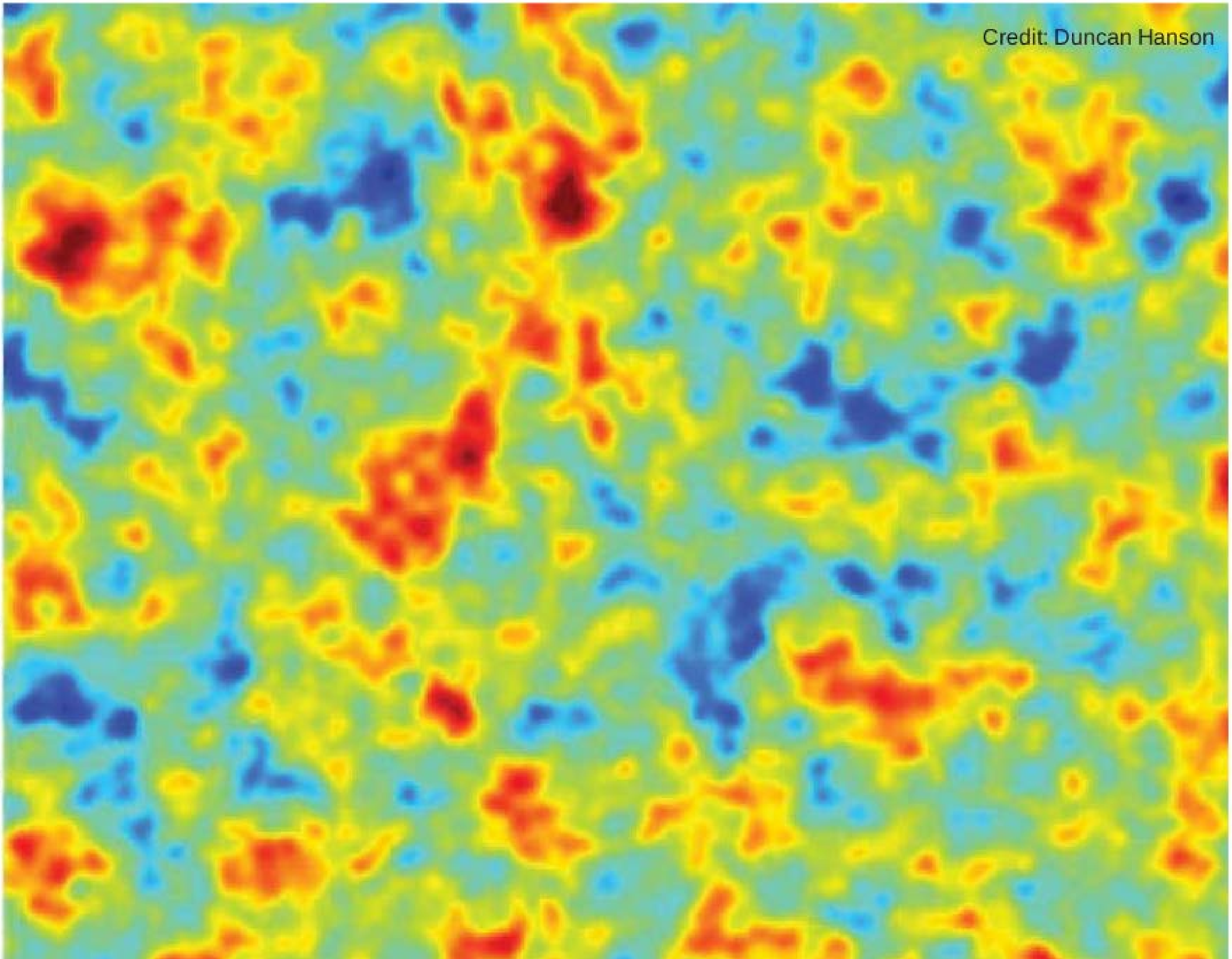




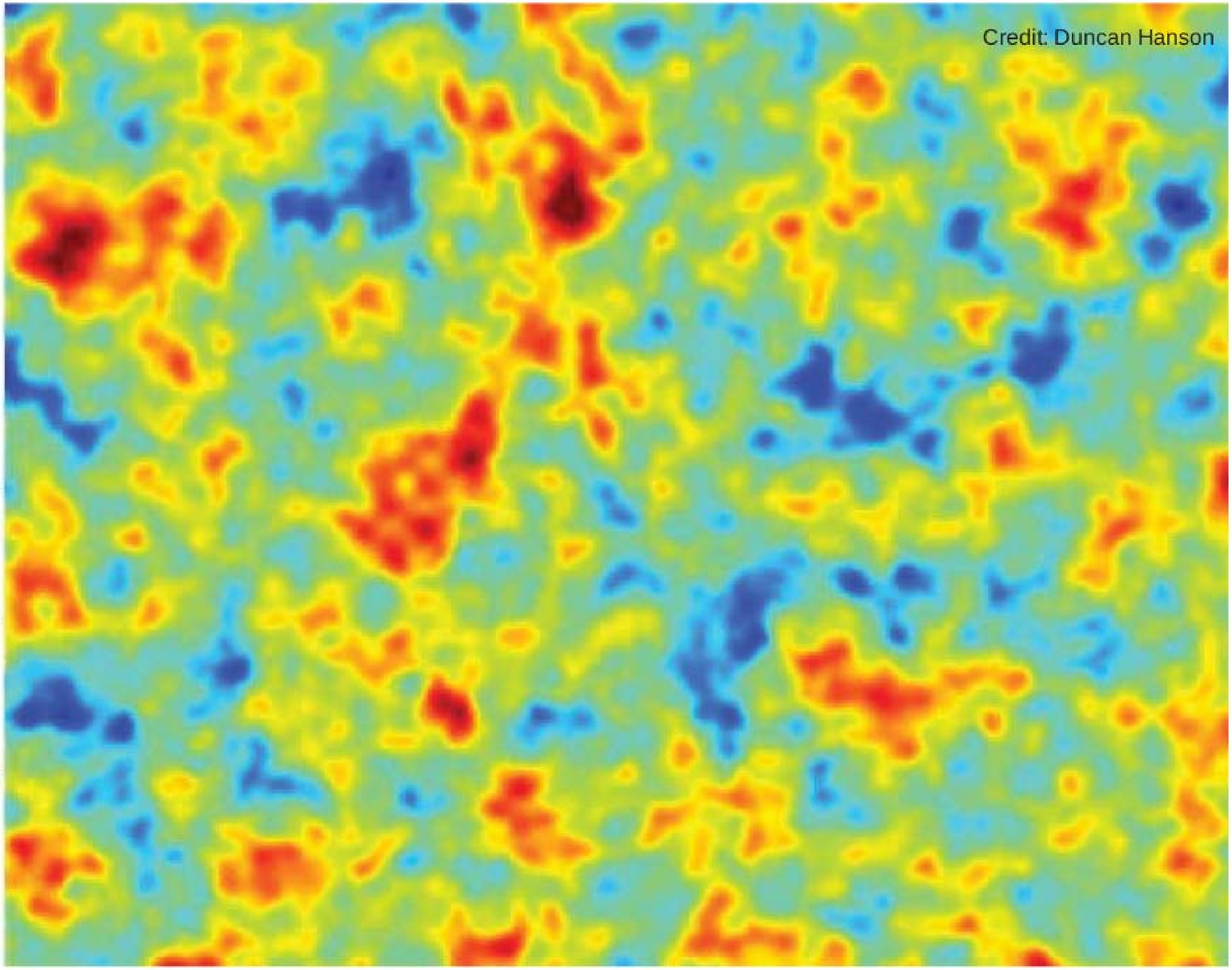
Primordial

Lensed

Experiments have just started measuring this effect



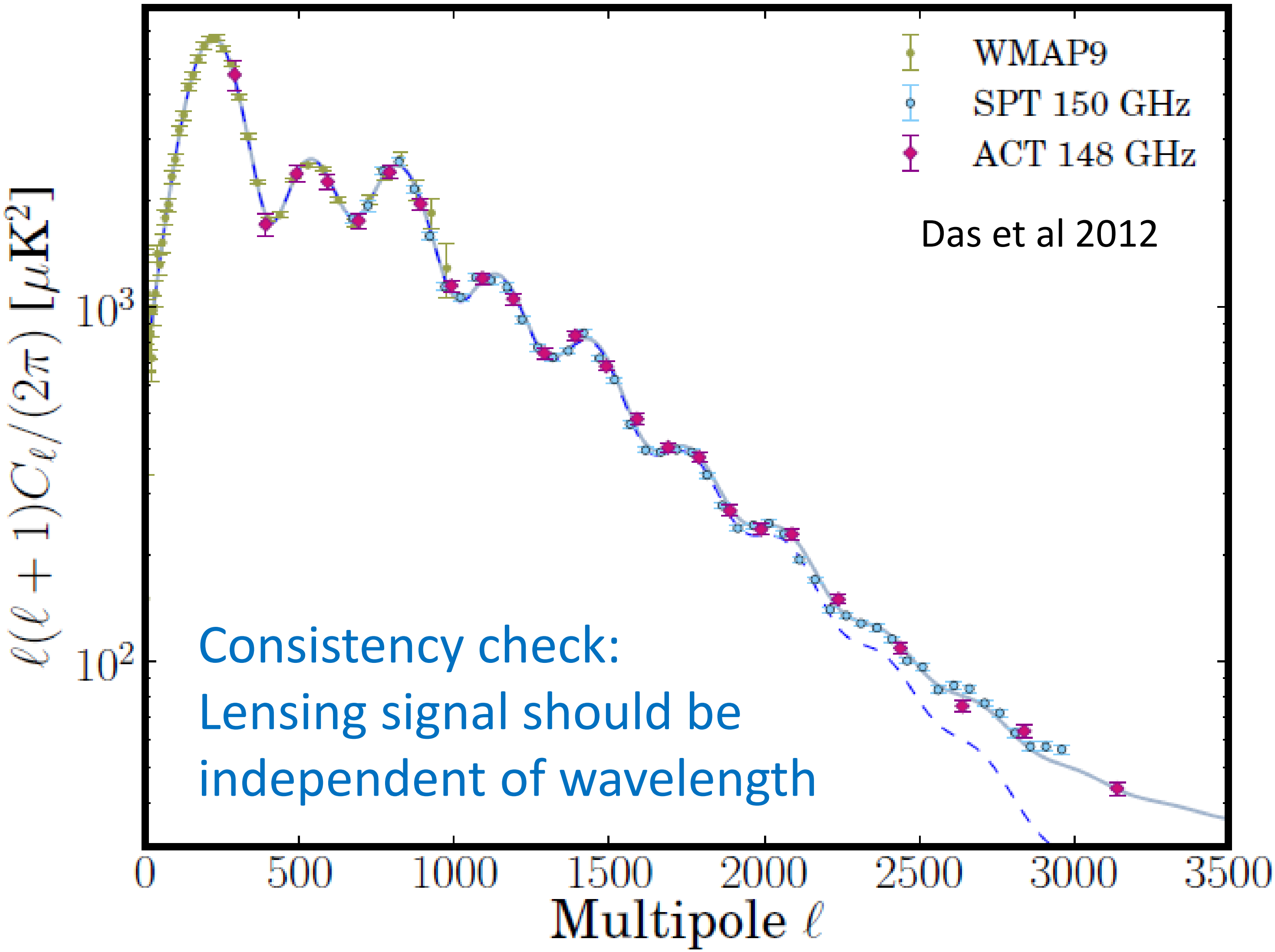
CMB Temperature (Unlensed)



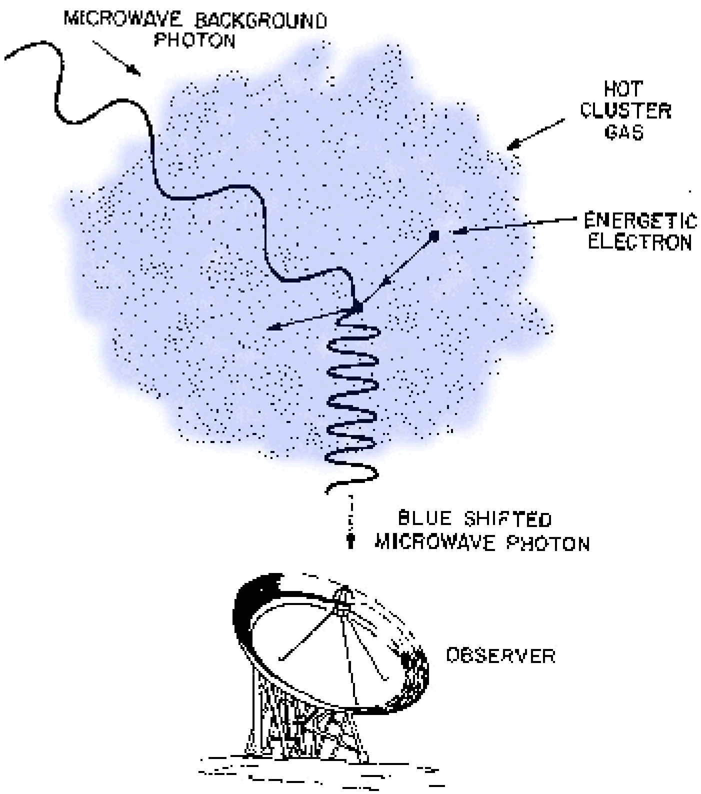
CMB Temperature (Lensed)

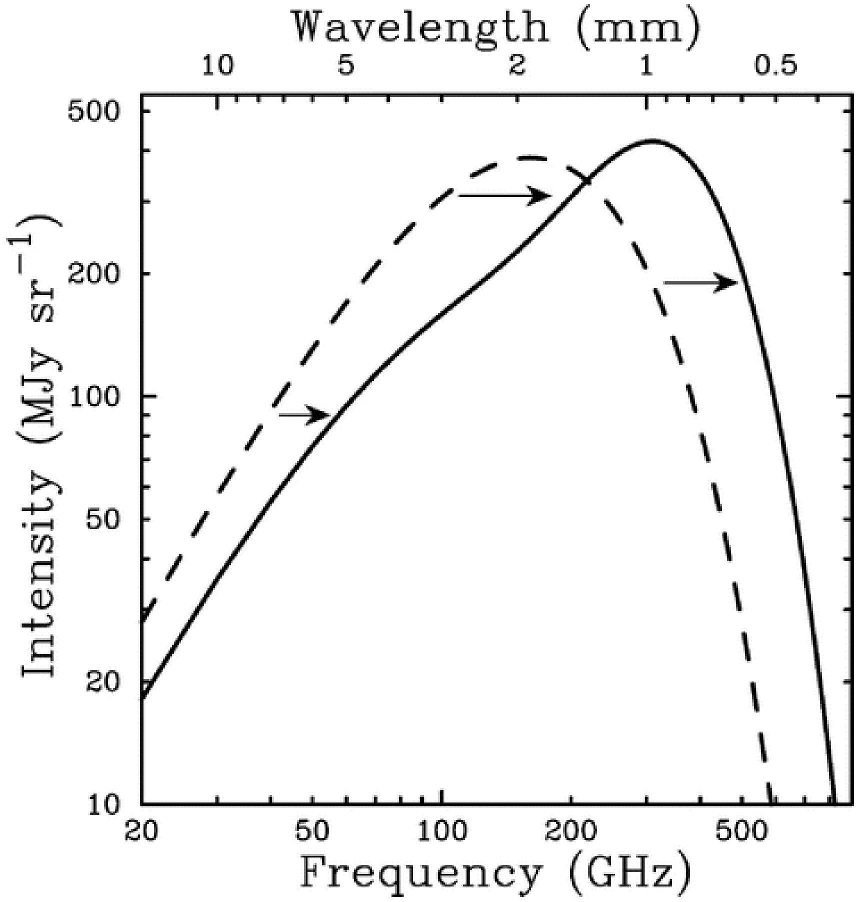
Order of magnitude

- GR lensing: 4Φ
- Potentials linear and small: $\Phi \sim 2 \times 10^{-5}$
- Deflection per lens: $\beta \sim 10^{-4}$
- Characteristic size from peak of Pk: L = 300 Mpc
- Comoving distance to CMB: D = 14000 Mpc
- Number of lenses: N ~ D/L ~ 50
- Total deflection: $\beta \sqrt{N} \sim$ 2 arcmins ~ $\ell = 3000$
- On these scales CMB smooth, so lensing dominates
- Attractive because single, distant source plane with smooth well-defined features

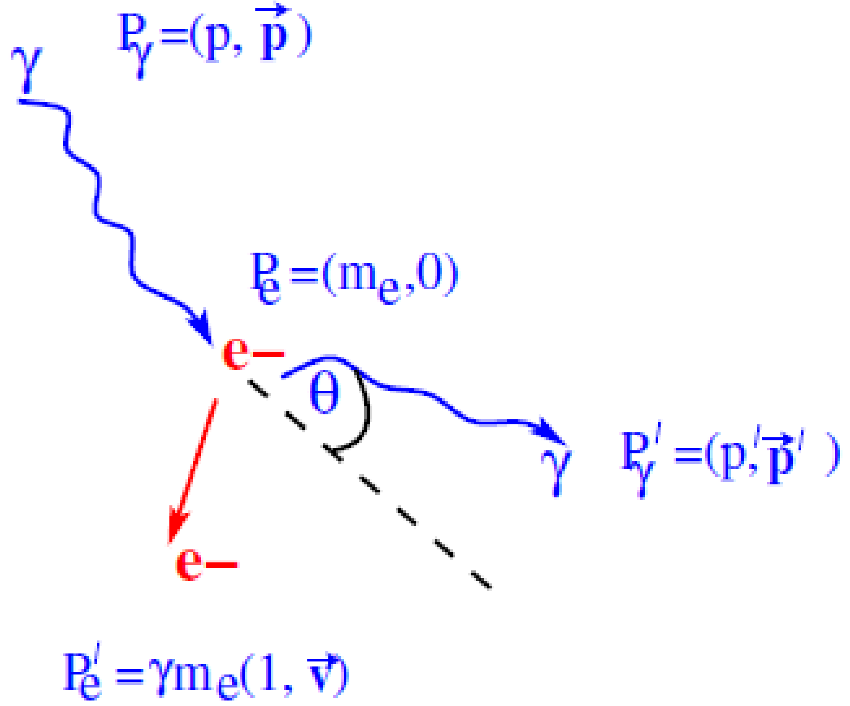

WMAP9
SPT 150 GHz
ACT 148 GHz
Das et al 2012
$\ell(\ell+1)C_\ell/(2\pi)$ [μK^2]
10^3
10^2
0
500
1000
1500
2000
2500
3000
3500
Multipole ℓ
Consistency check:
Lensing signal should be
independent of wavelength

The Sunyaev-Zeldovich effect(s)





Compton Scattering



$$\Delta p / p \approx -p / m_{\mathrm{e}} (1 - \cos\theta)$$

Recoil:

$$y_\gamma = \int \mathrm{d}t\, c\sigma_\mathrm{T} n_\mathrm{e} \frac{k_\mathrm{B} T_\gamma}{m_\mathrm{e} c^2}, \qquad T_\gamma = 2.725(1+z)$$

No. of scatterings   Energy transfer per scattering

Doppler effect:

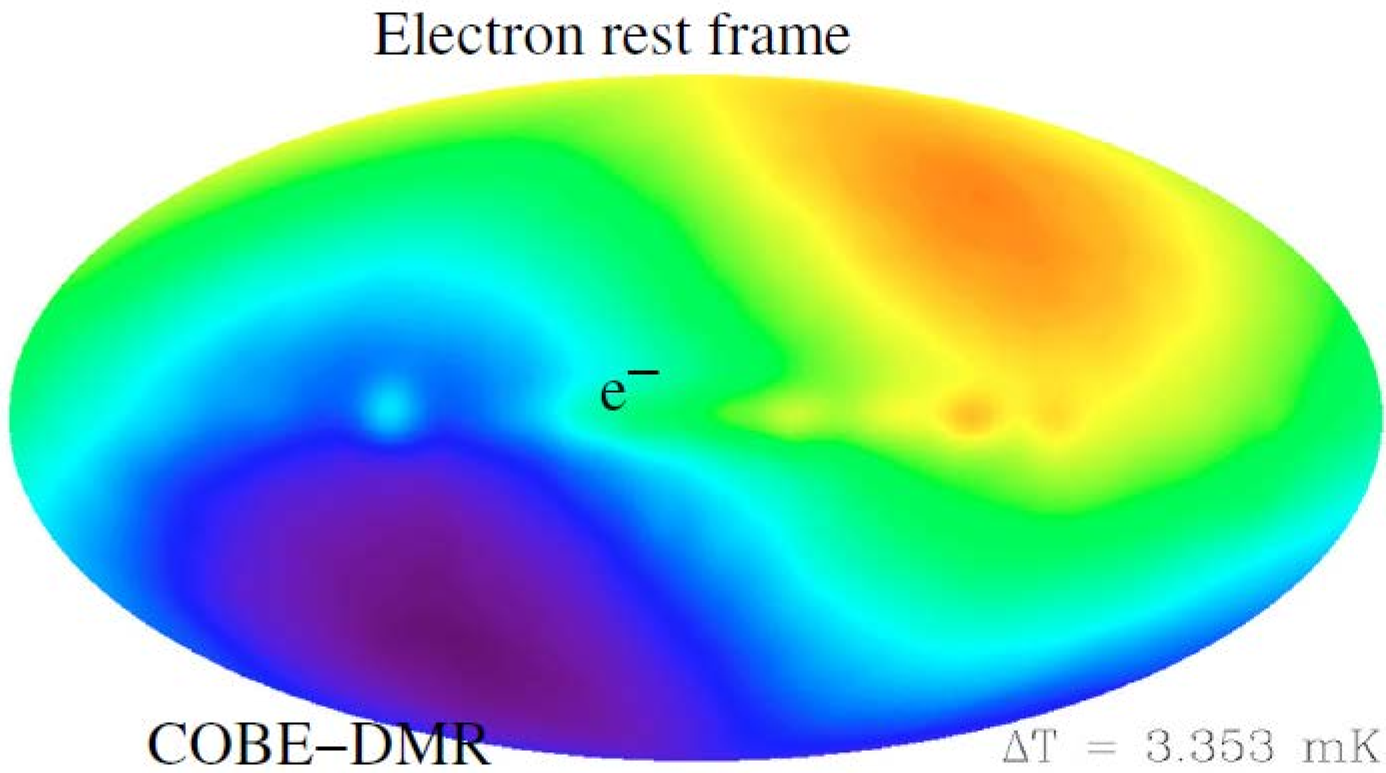
$$y_e = \int \mathrm{d}t\, c\sigma_\mathrm{T} n_\mathrm{e} \frac{k_\mathrm{B} T_\mathrm{e}}{m_\mathrm{e} c^2}$$

In early Universe $y_\gamma \approx y_\mathrm{e}$

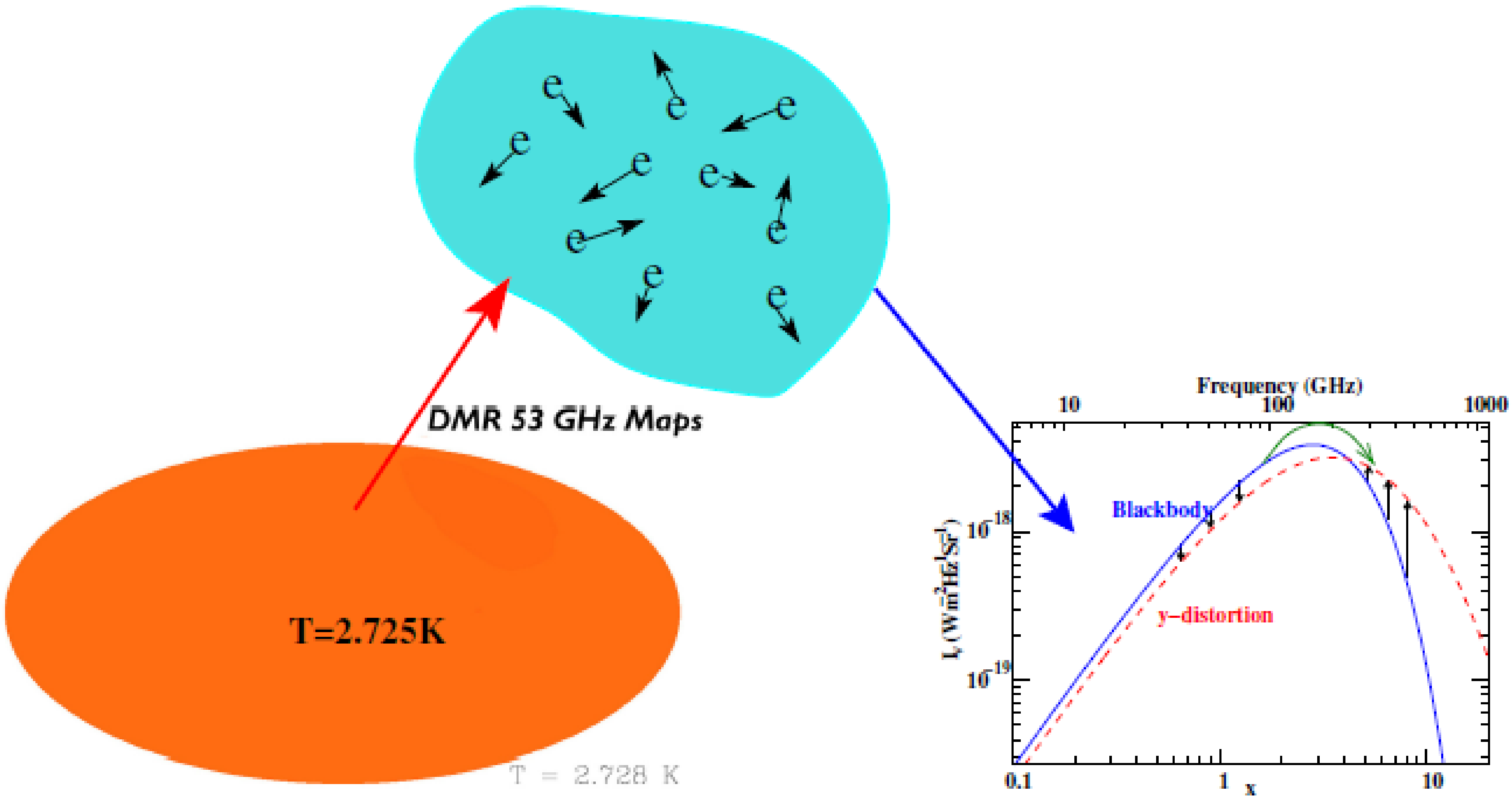
y: Amplitude of distortion

$$y = \int \mathrm{d}t\, c\sigma_\mathrm{T} n_\mathrm{e} \frac{k_\mathrm{B} \left(T_\mathrm{e} - T_\gamma\right)}{m_\mathrm{e} c^2}$$

CMB is dipole in e- restframe
SZ effect is ~ mixing of blackbodies

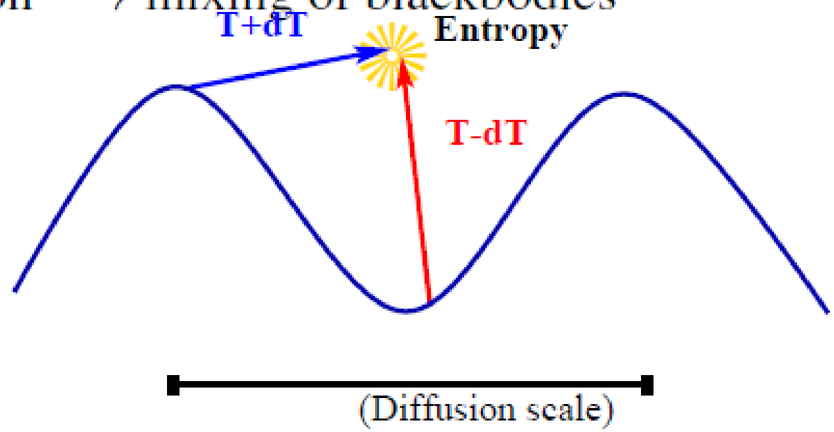


Resulting spectrum will not be blackbody



DMR 53 GHz Maps
T=2.725K
T = 2.728 K
Frequency (GHz)
10
100
1000
Blackbody
y−distortion
I_ν (W m^{-2}Hz^{-1}Sr^{-1})
10^{-18}
10^{-19}
0.1
1
x
10

Photon diffusion ⟶ mixing of blackbodies



Mixing of blackbodies gives y-type distortion

Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

$$< n_{\rm Planck} > = \frac{1}{e^{\frac{h\nu}{kT}}-1} + \left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle T \frac{\partial n_{\rm Pl}}{\partial T} + \frac{1}{2} \left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\rm Pl}}{\partial T}$$

(T + δT/T)

$$= n_{\rm Planck} \left(T + \left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle \right) + \frac{1}{2} \left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle n_{\rm SZ}$$

2/3 1/3

Black body Kompaneets operator/SZ

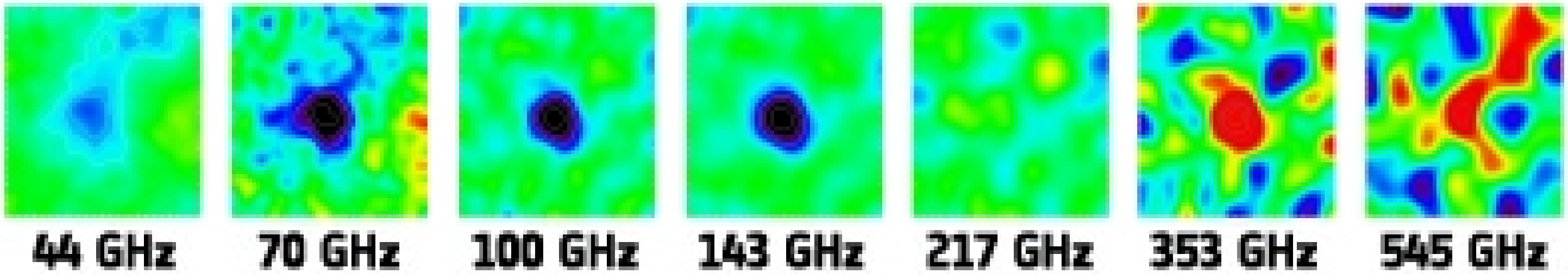
$$n_{SZ} = y \; T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\mathrm{Pl}}}{\partial T}$$

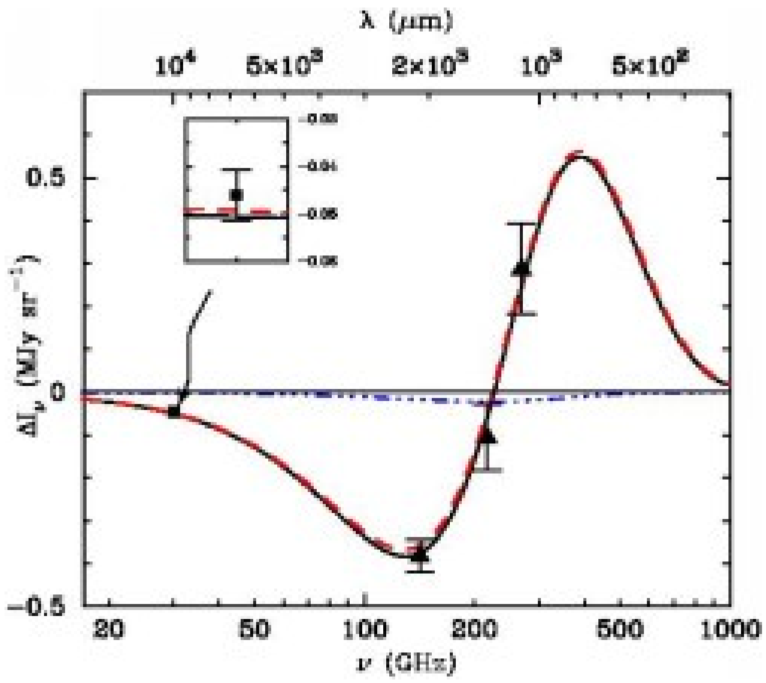
$$= \mathbf{y} \; \frac{\mathbf{x e^x}}{(\mathbf{e^x - 1})^2} \left(\mathbf{x} \frac{\mathbf{e^x + 1}}{\mathbf{e^x - 1}} - \mathbf{4} \right)$$

$$\Delta I_{sz} = I_{sz} - I_{planck} = \frac{2h\nu^3}{c^2} n_{sz}$$

$$y_\gamma \ll 1 \;,\; T_{\mathrm{e}} \sim 10^4$$

$$y = (\tau_{\mathrm{reionization}}) \frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2} \sim (0.1)(1.6 \times 10^{-6}) \sim 10^{-7}$$





- Unique spectral signature: decrease in the CMB intensity at frequencies below ~218 GHz, increase at higher frequencies.

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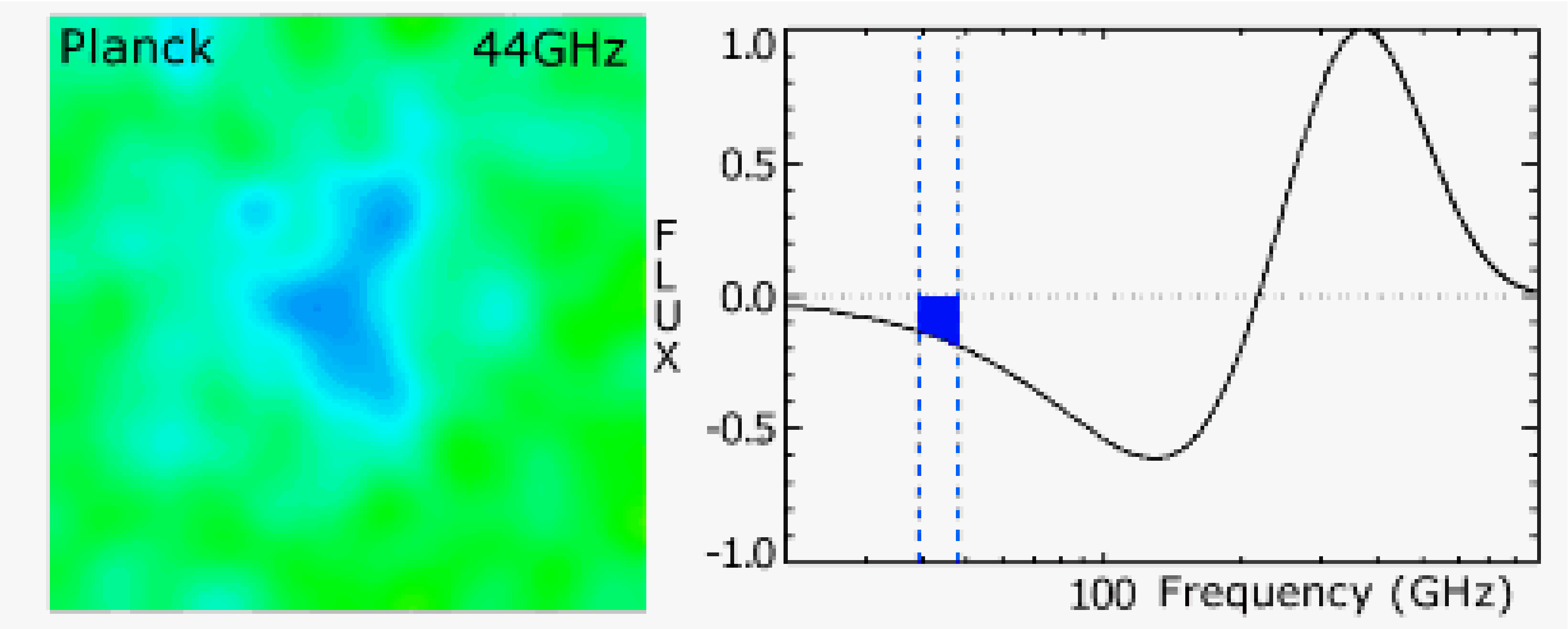
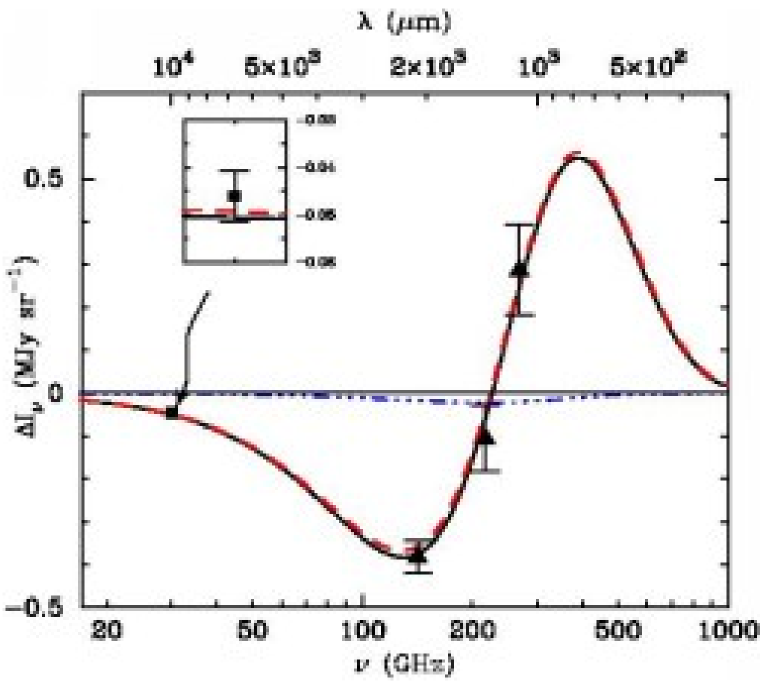
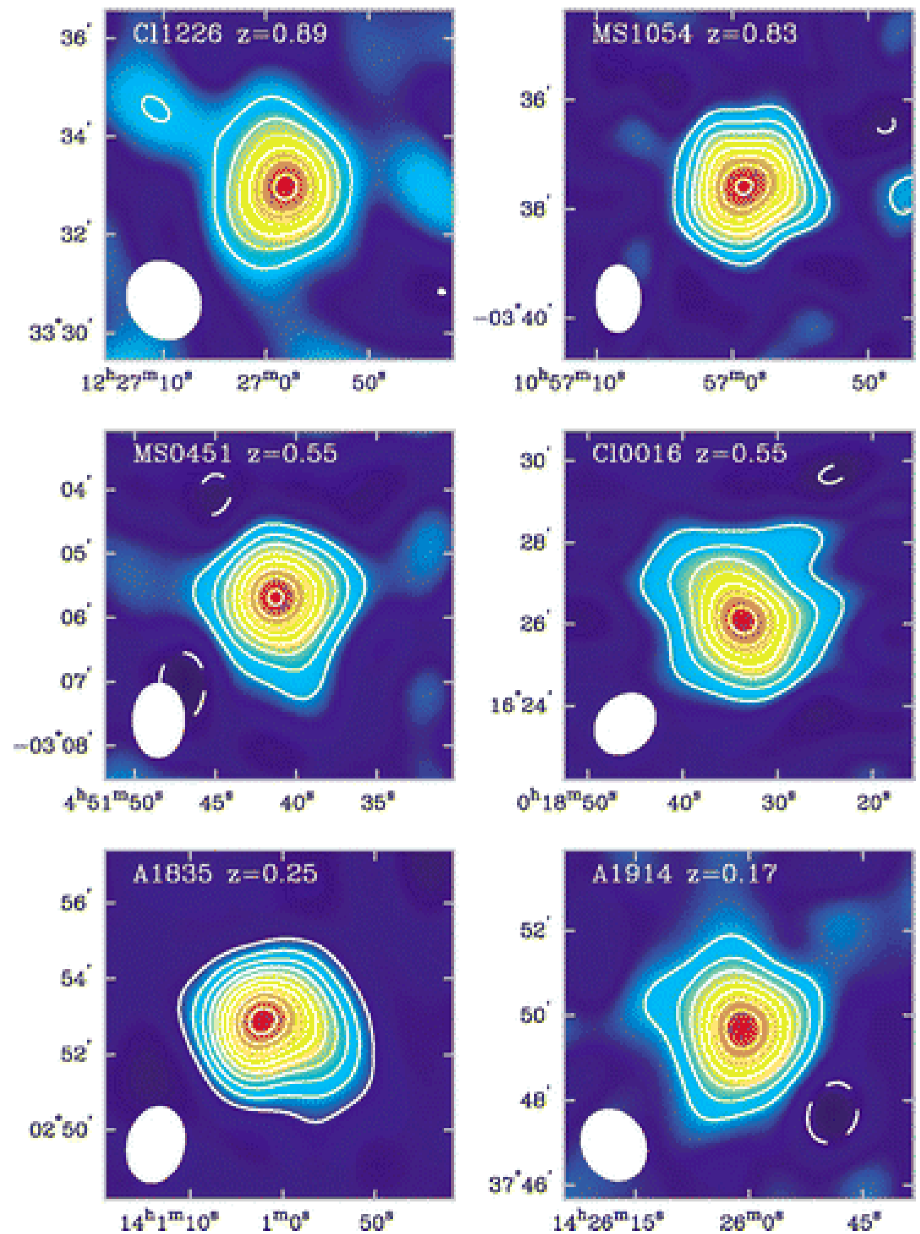


Approximately independent of redshift



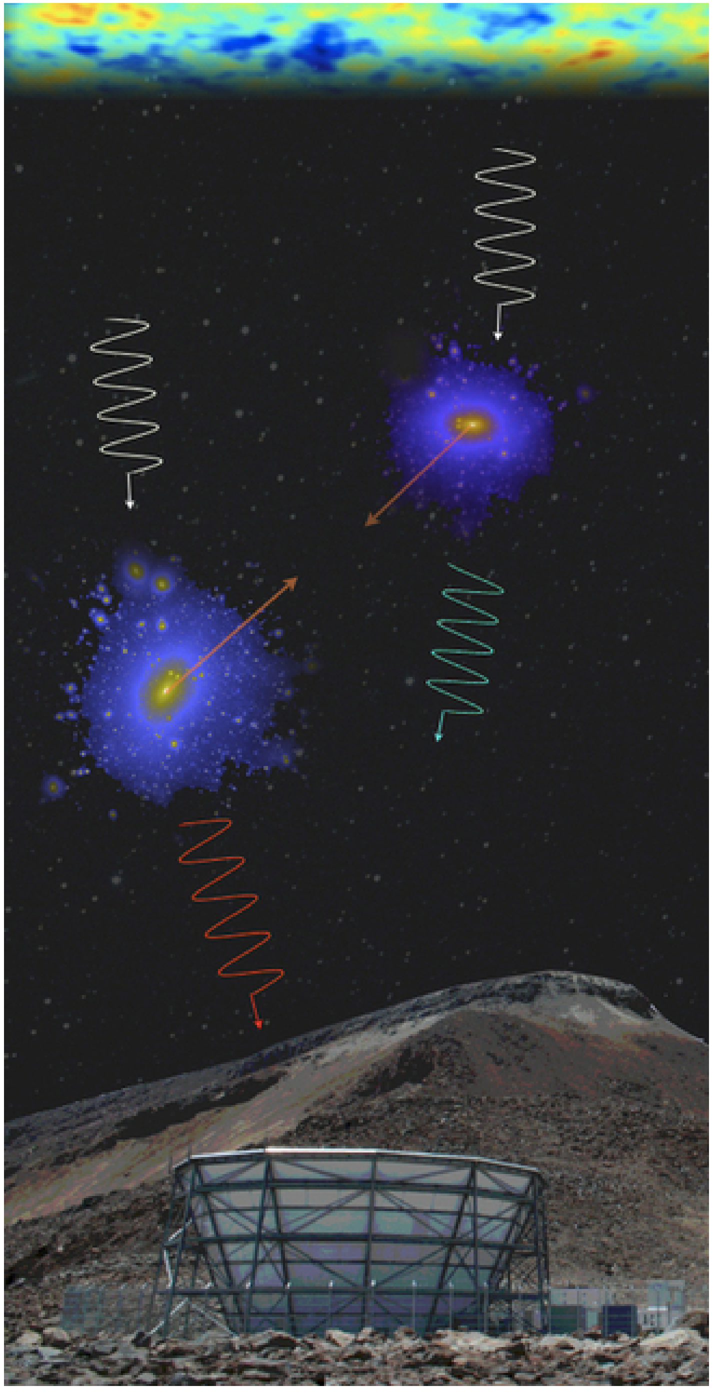


- Unique spectral signature: decrease in the CMB intensity at frequencies below ~218 GHz, increase at higher frequencies.
- Small (10^{-3} K) spectral distortion. At a given frequency, signal depends on the pressure of the cluster gas at each point in the cluster, so signal varies in strength over the face of a given cluster. Distortion is strongest in the center.
- Intensity summed over an entire cluster depends on the total mass of the cluster: lower mass clusters produce weaker signal. A single galaxy has insufficient mass to cause distortions in the cosmic background radiation.
- Independent of redshift.

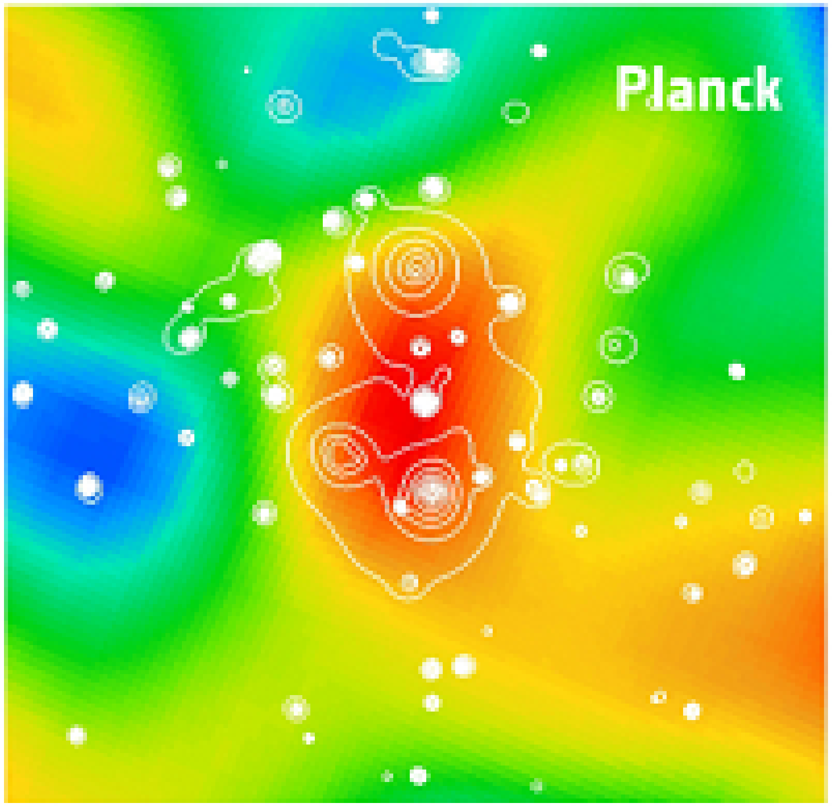
Kinetic SZ effect

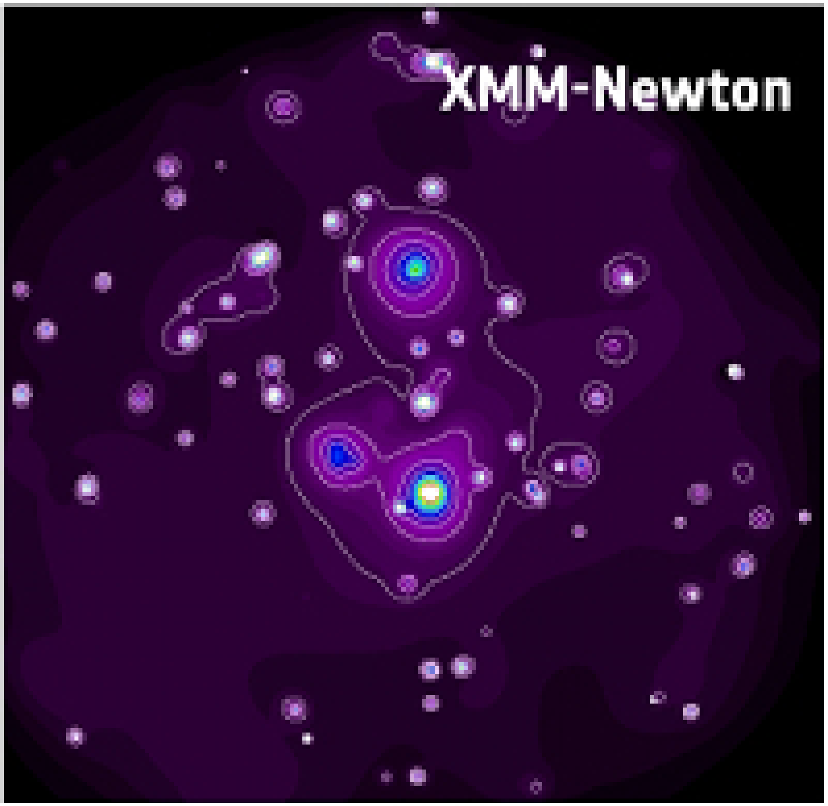
Both tSZ + kSZ give map of electrons = baryons

So they are nice probes of the gastrophysics of galaxy formation



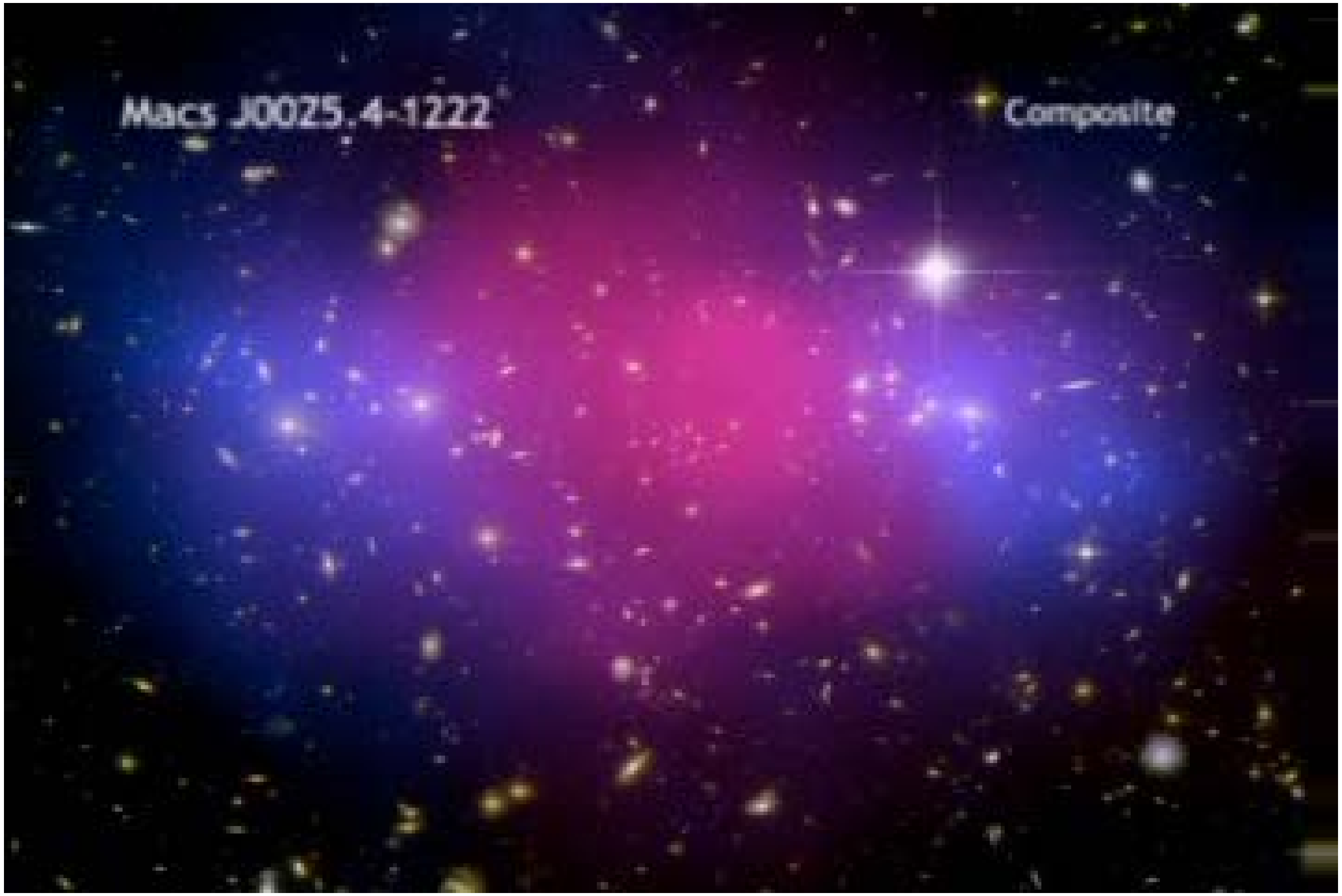
Can also look for the hot gas in X-rays





Crudely speaking: Lensing→Mass, SZ→pressure, Xray→Temperature

‘Bullet’-like clusters: Dark matter ~collisionless



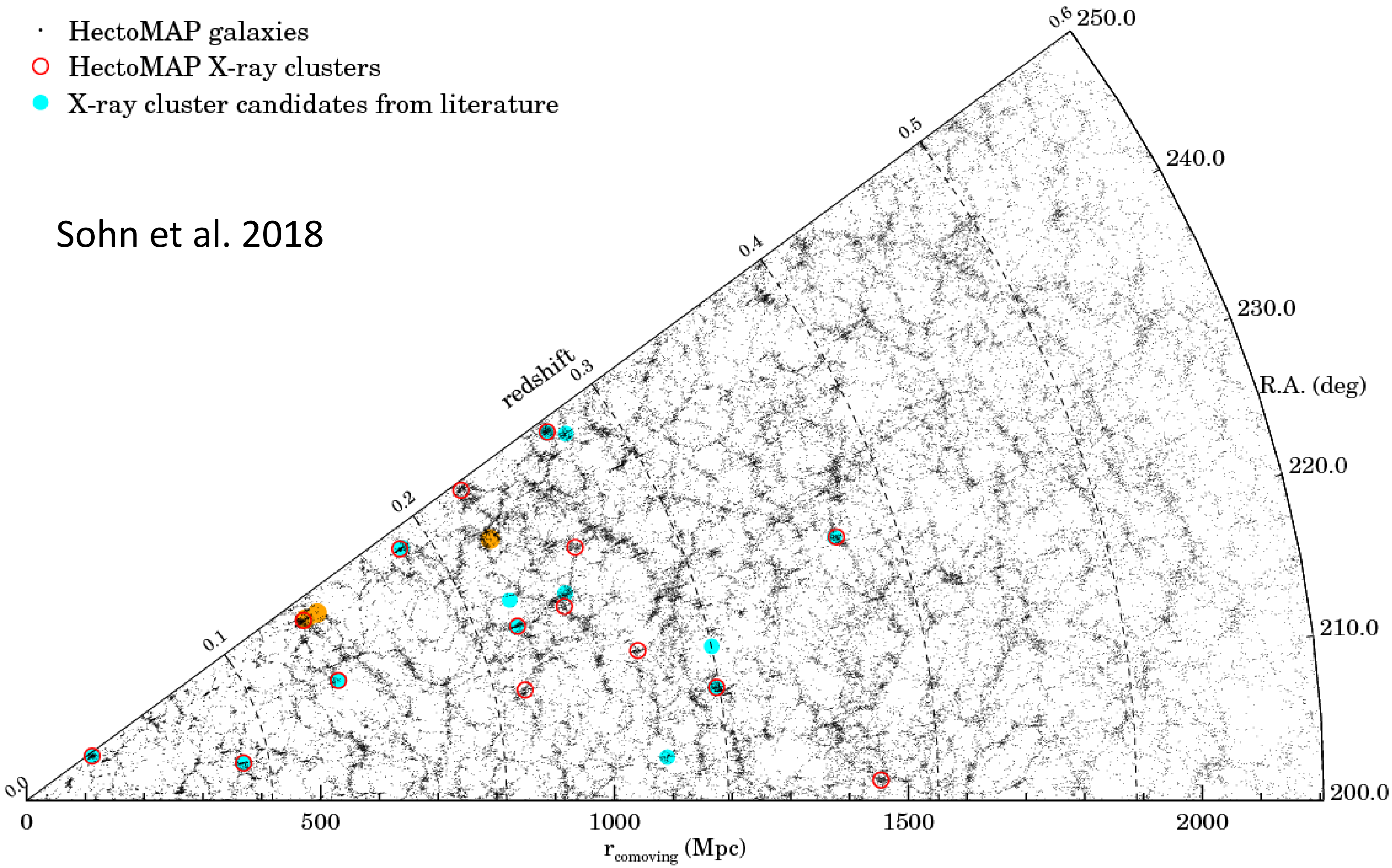
Lensing mass Xray photons

Why study clusters?

- Cluster counts contain information about volume and about how gravity won/lost compared to expansion
- Probe geometry and expansion history of Universe, and nature of gravity

Massive halo = Galaxy cluster

(Simpler than studying galaxies? Less gastrophysics?)



Sohn et al. 2018

$$d^2N/dzd\Omega = dV/dzd\Omega \times \int dm\ dn/dm$$

where

V is comoving volume

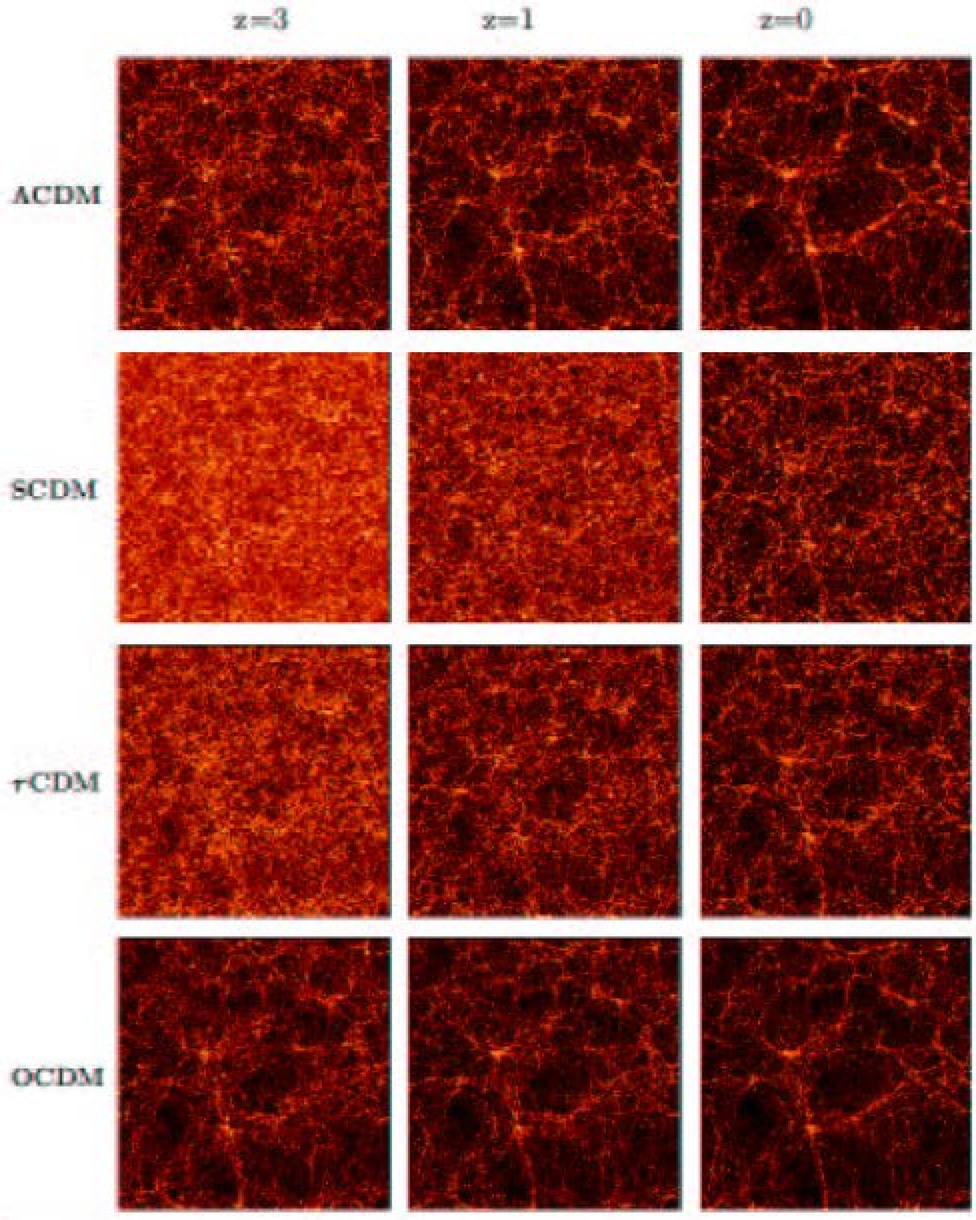
and

dn/dm is comoving number density.

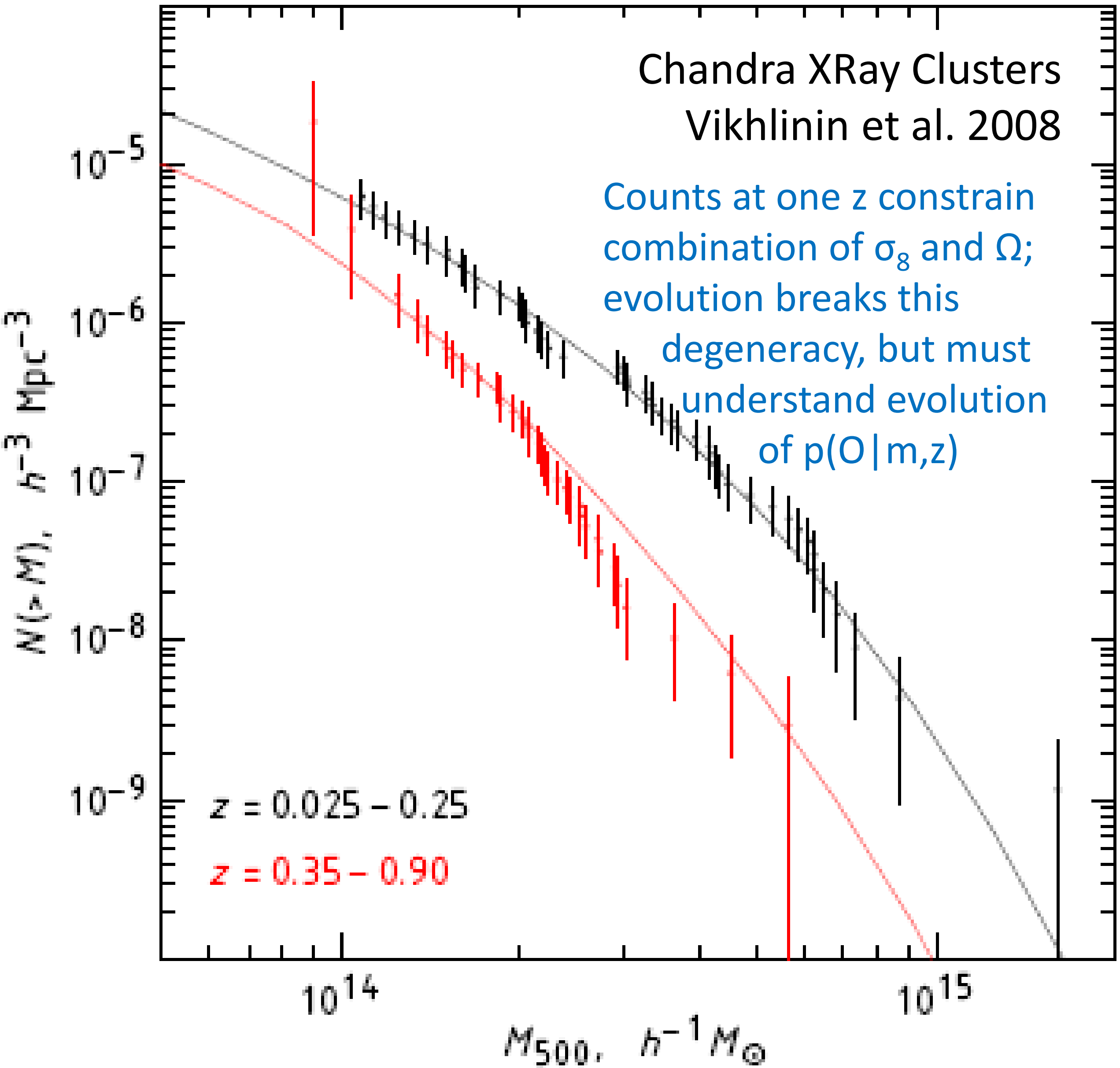
In practice, don't measure m, but an observable O (e.g. speeds of galaxies, Xray flux, SZ decrement) which is expected to correlate with m:

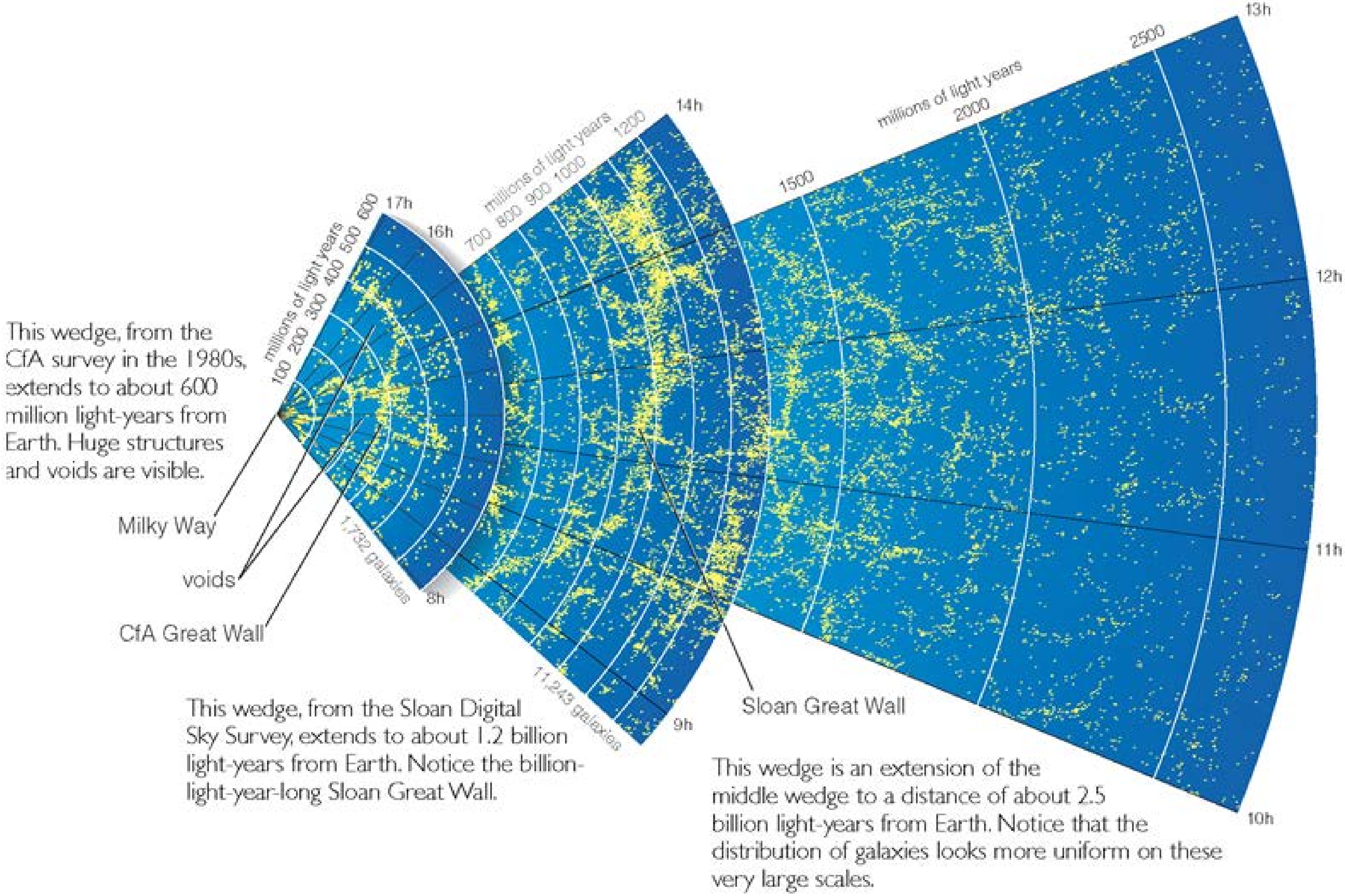
$$d^2N(O)/dzd\Omega = dV/dzd\Omega \times \int dm\ dn/dm\ p(O|m,z)$$

- Structure at a given time, and, more importantly, growth of structure, provides sharp constraints on models

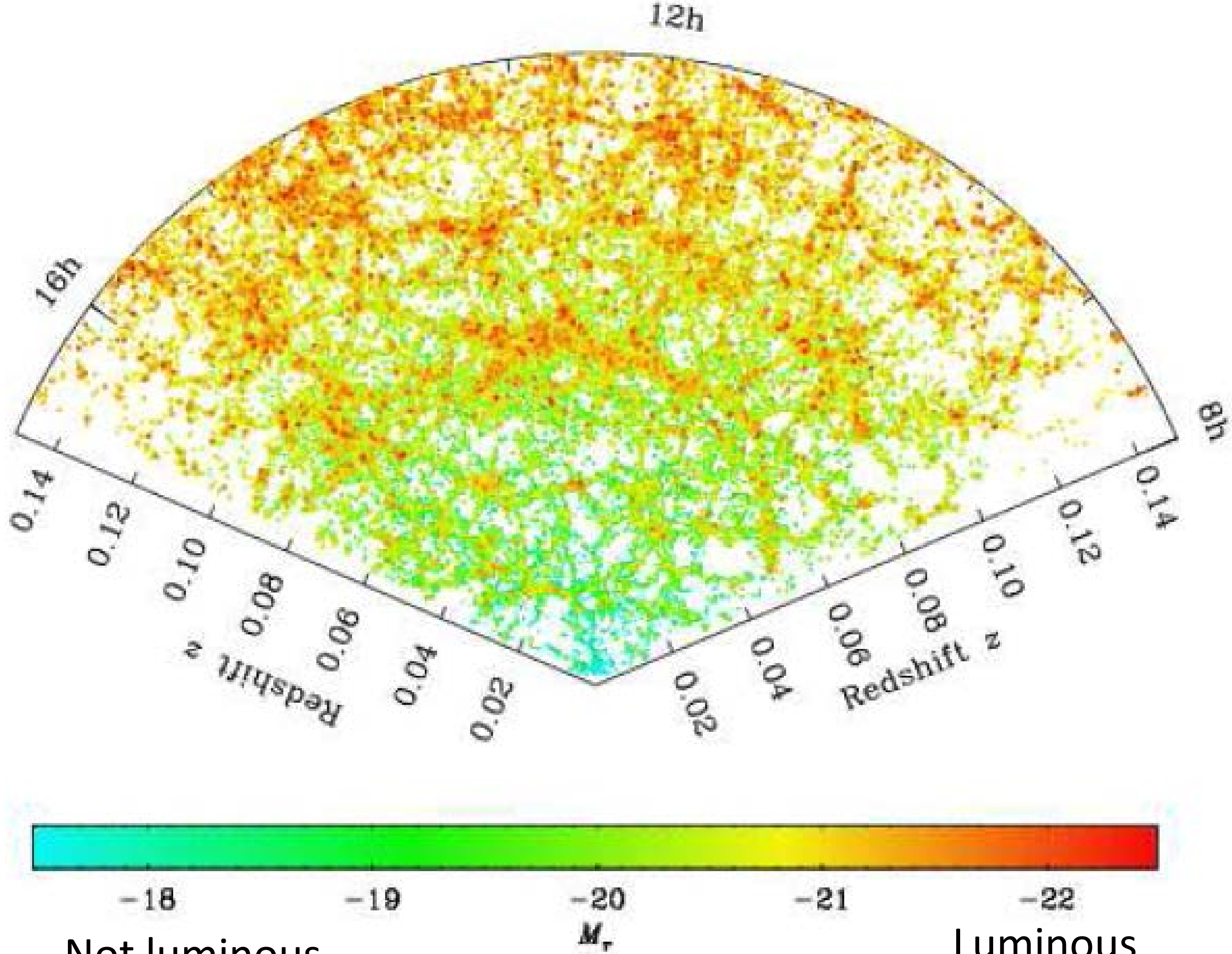


(Jenkins et al.: Virgo consortium)


Chandra XRay Clusters
Vikhlinin et al. 2008
Counts at one z constrain combination of σ_8 and Ω; evolution breaks this degeneracy, but must understand evolution of p(O|m,z)
z = 0.025 – 0.25
z = 0.35 – 0.90
$N(>M)$, h^{-3} Mpc^{-3}
M_{500}, $h^{-1} M_\odot$
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{14}
10^{15}



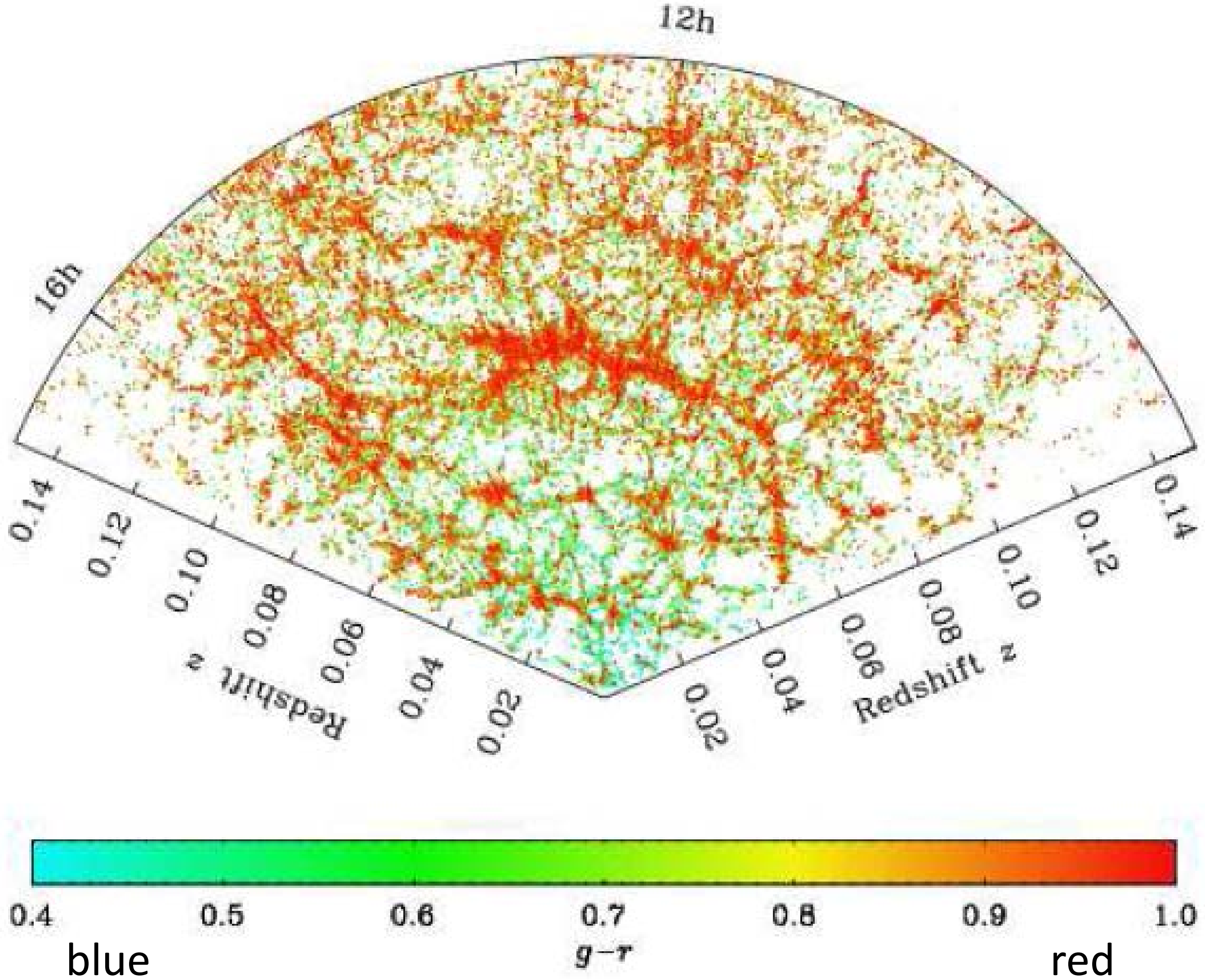
Structures in galaxy maps look very similar to the ones found in models in which dark matter is WIMPs



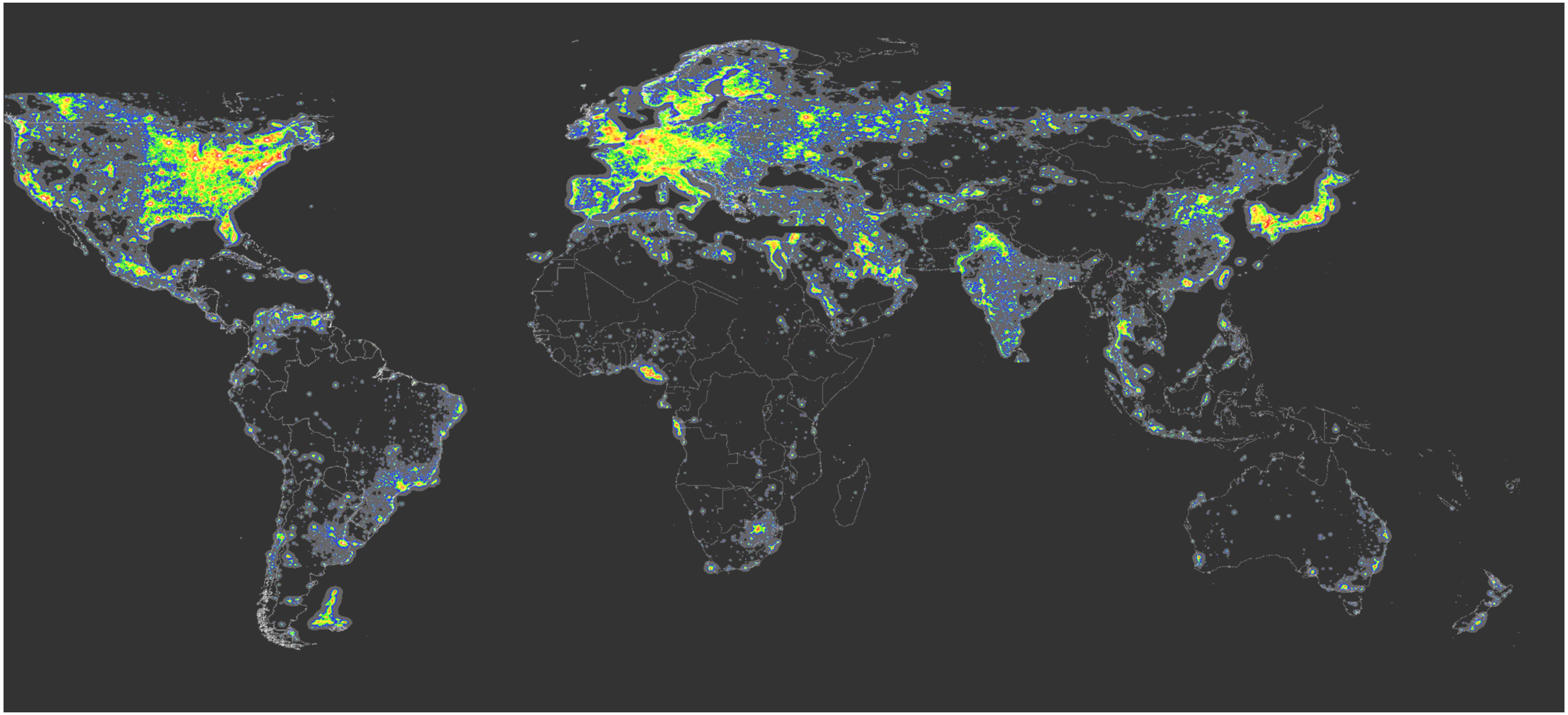
Not luminous

Luminous

Zehavi et al. 2010 (SDSS)


12h
16h
8h
0.14
0.12
0.10
0.08
0.06
0.04
0.02
Redshift z
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z
0.4
0.5
0.6
0.7
0.8
0.9
1.0
g−r
blue
red

Complication: Light is a biased tracer



Not all galaxies are fair tracers of dark matter;
To use galaxies as probes of underlying dark matter distribution, must understand 'bias'

How to describe different point processes which are all built from the same underlying density field?

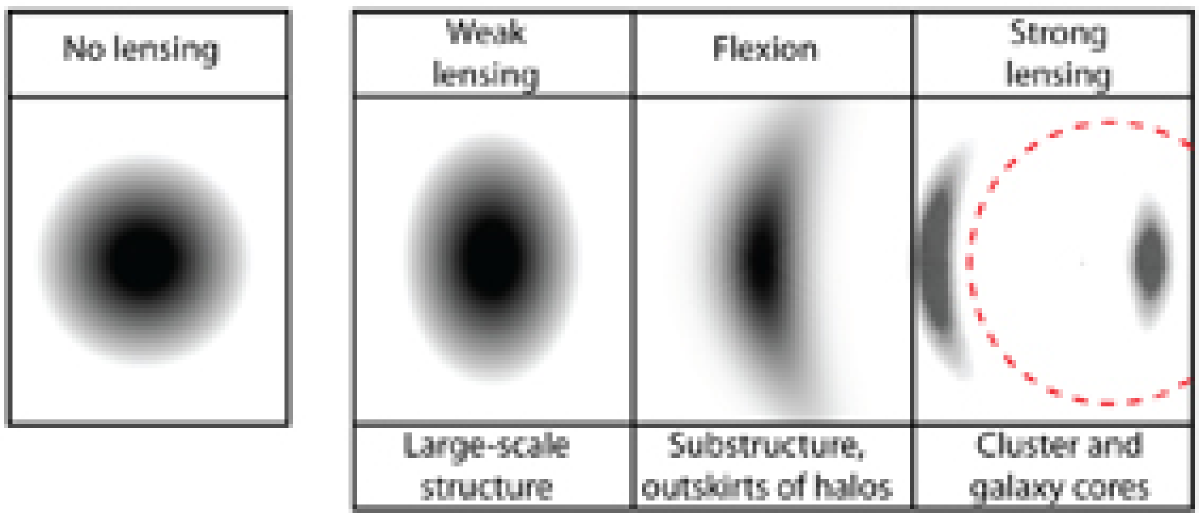
THE HALO MODEL

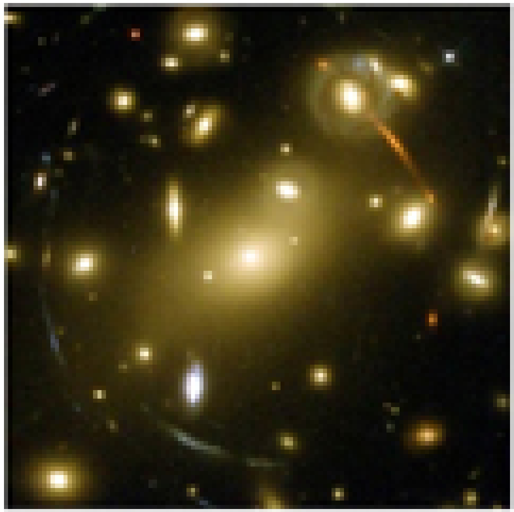
Review in Physics Reports (Cooray & Sheth 2002)

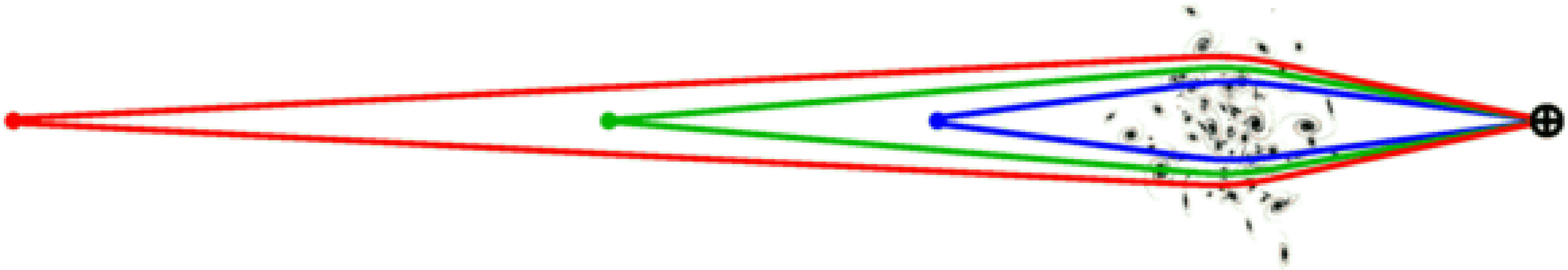
Cosmology from Gravitational Lensing

Volume as function of redshift

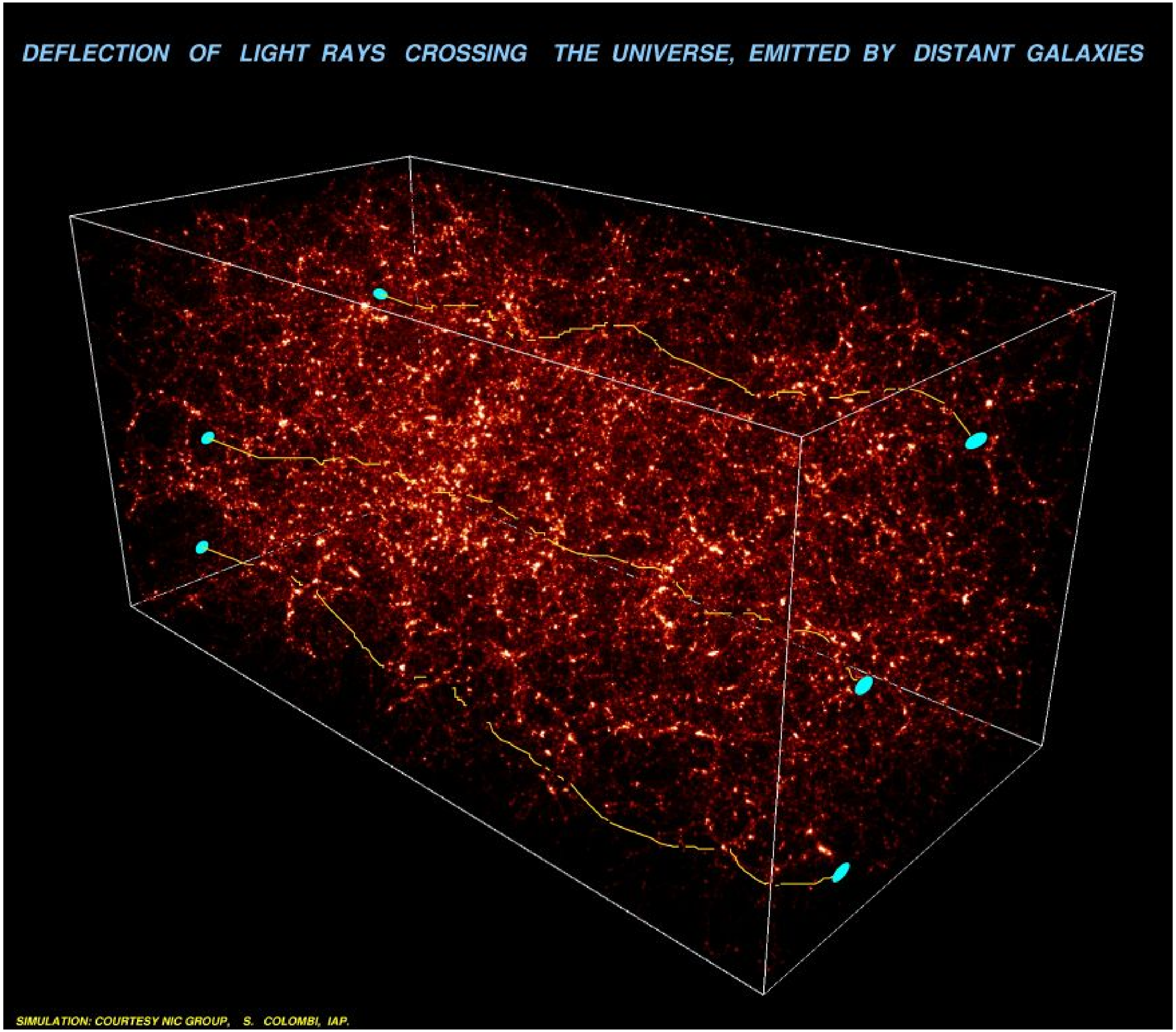
Growth of fluctuations with time







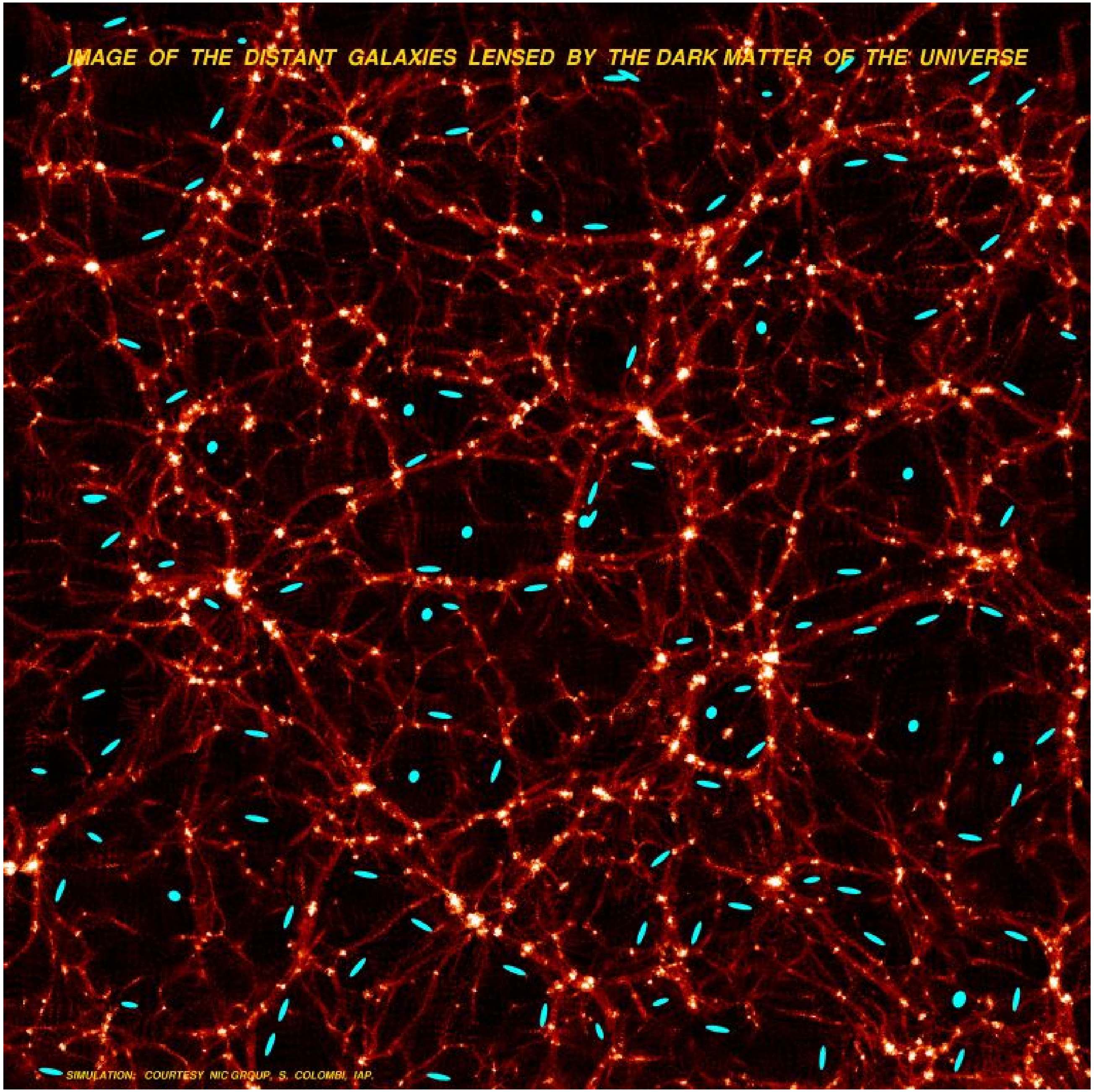
- Focal length strong function of cluster-centric distance; highly distorted images possible
- Strong lensing if source lies close to lens-observer axis; weaker effects if impact parameter large
- Strong lensing: Cosmology from distribution of image separations, magnification ratios, time delays; but these are rare events, so require large dataset
- Weak lensing: Cosmology from correlations (shapes or magnifications); small signal requires large dataset



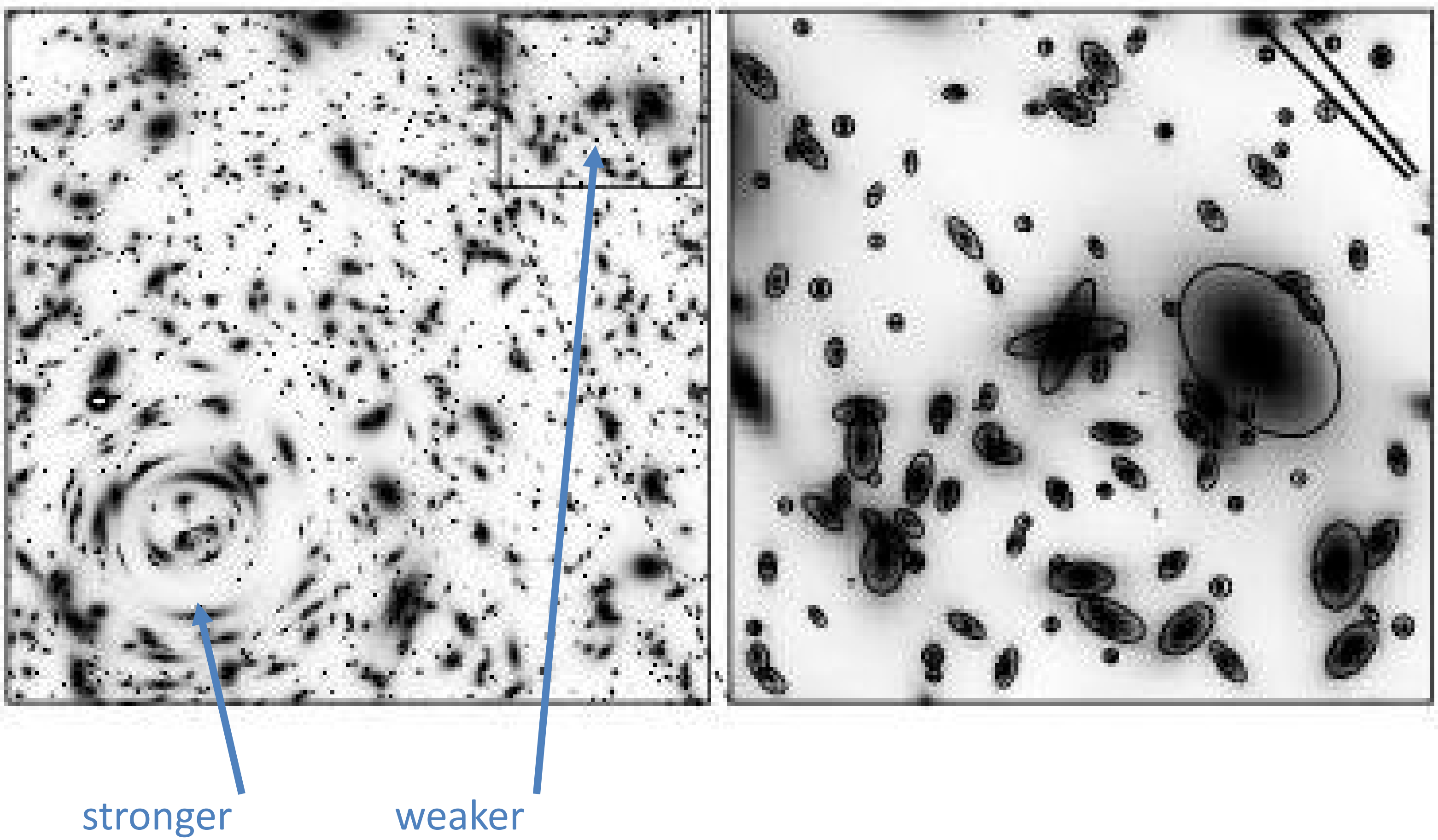
Lensing provides a measure of dark matter along line of sight

Weak lensing: Image distortions correlated with dark matter distribution

E.g., lensed image ellipticities aligned parallel to filaments, tangential to knots (clusters)

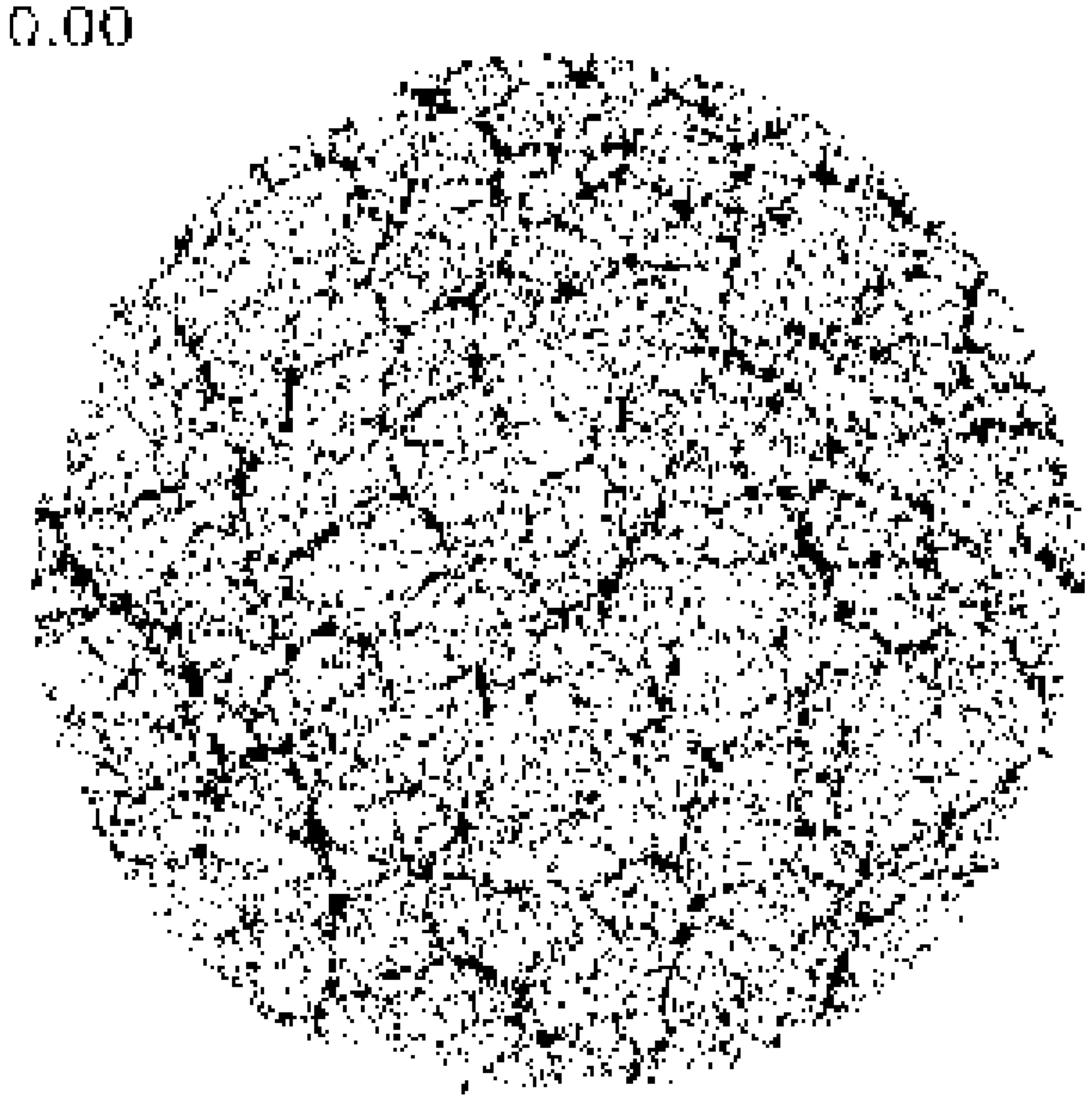


The shear power of lensing

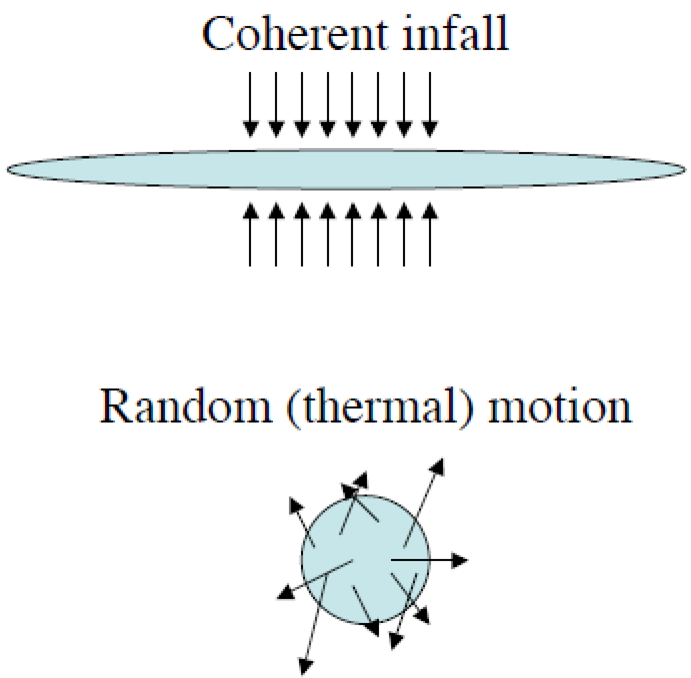


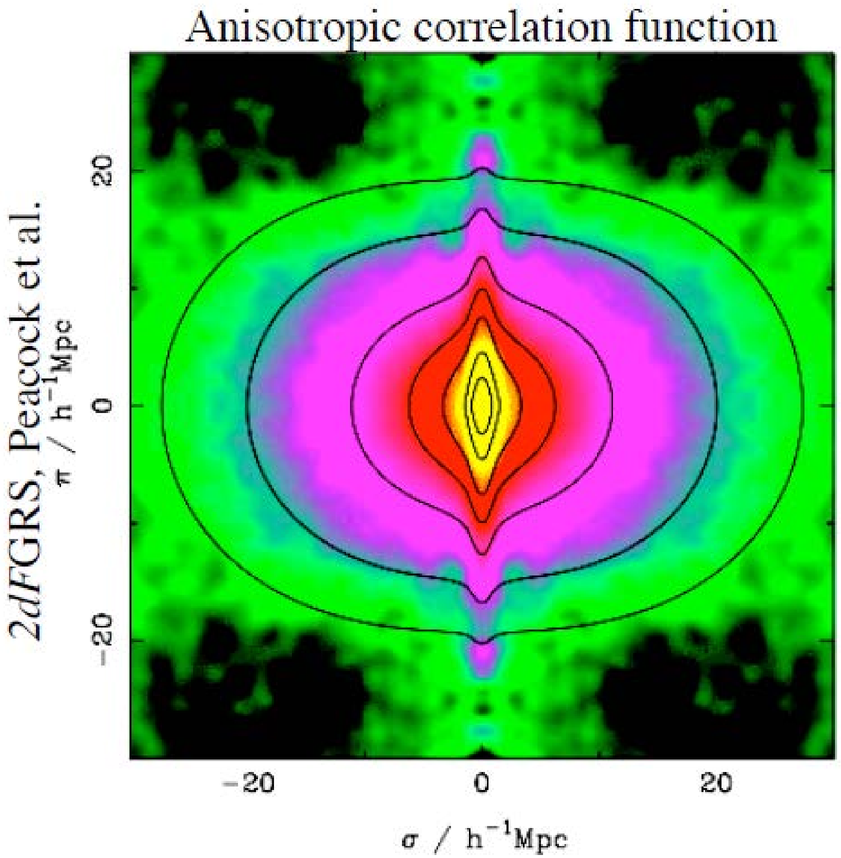
Cosmology from measurements of correlated shapes; better constraints if finer bins in source or lens positions possible

Redshift space distortions



Redshift space distortions





$$1 + \xi_s(s_\parallel, s_\perp) = \int_{-\infty}^{\infty} dr_\parallel \; [1 + \xi(r)] \; \mathcal{P}(\underbrace{r_\parallel - s_\parallel}_{\mathbf{v}_p}, \mathbf{r})$$

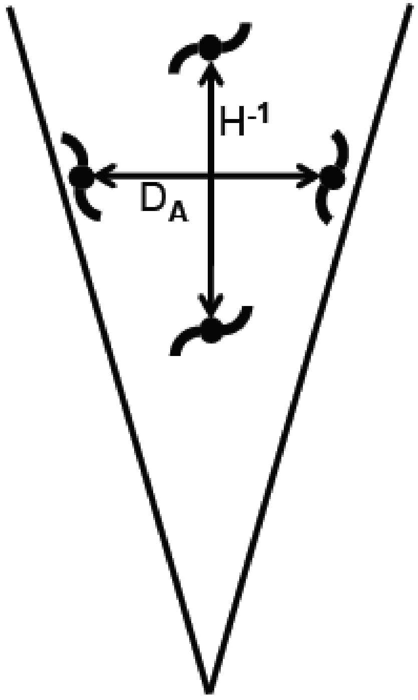
On large scales, use Gaussian statistics to compute (Fisher 1995)

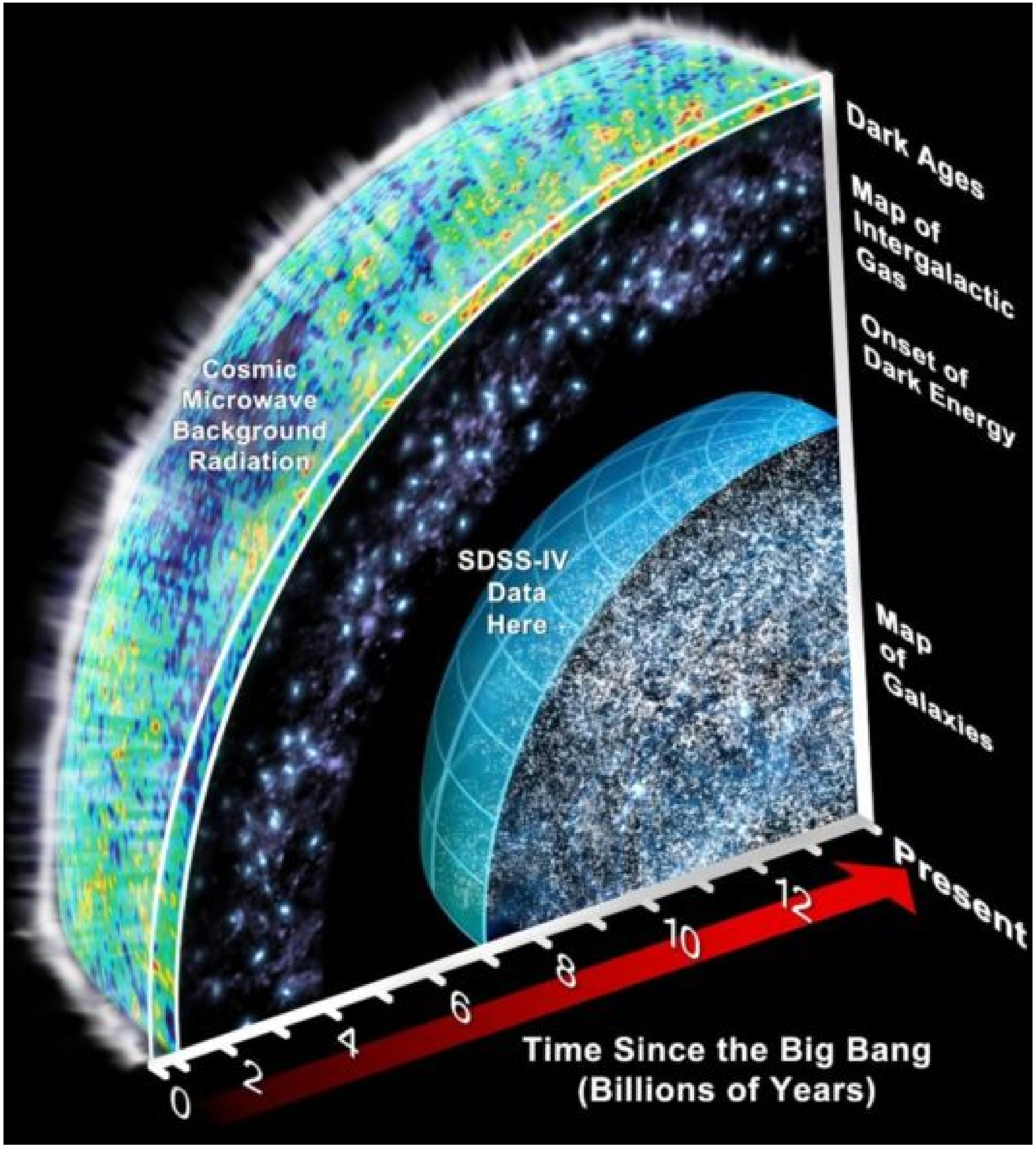
Alcock-Paczynski

- If the Universe is isotropic, clustering is same radial & tangential
- Stretching at a single redshift slice (for galaxies expanding with Universe) depends on

  $H^{-1}(z)$ (radial)

  $D_A(z)$ (angular)
- Analyze with wrong model -> see anisotropy
- AP effect measures $D_A(z)H(z)$
- RSD limits test to scales where can be modeled




Dark Ages
Map of Intergalactic Gas
Onset of Dark Energy
Map of Galaxies
Cosmic Microwave Background Radiation
SDSS-IV Data Here
0
2
4
6
8
10
12
Present
Time Since the Big Bang (Billions of Years)