A minimal dynamical model of North Atlantic Oscillation regimes

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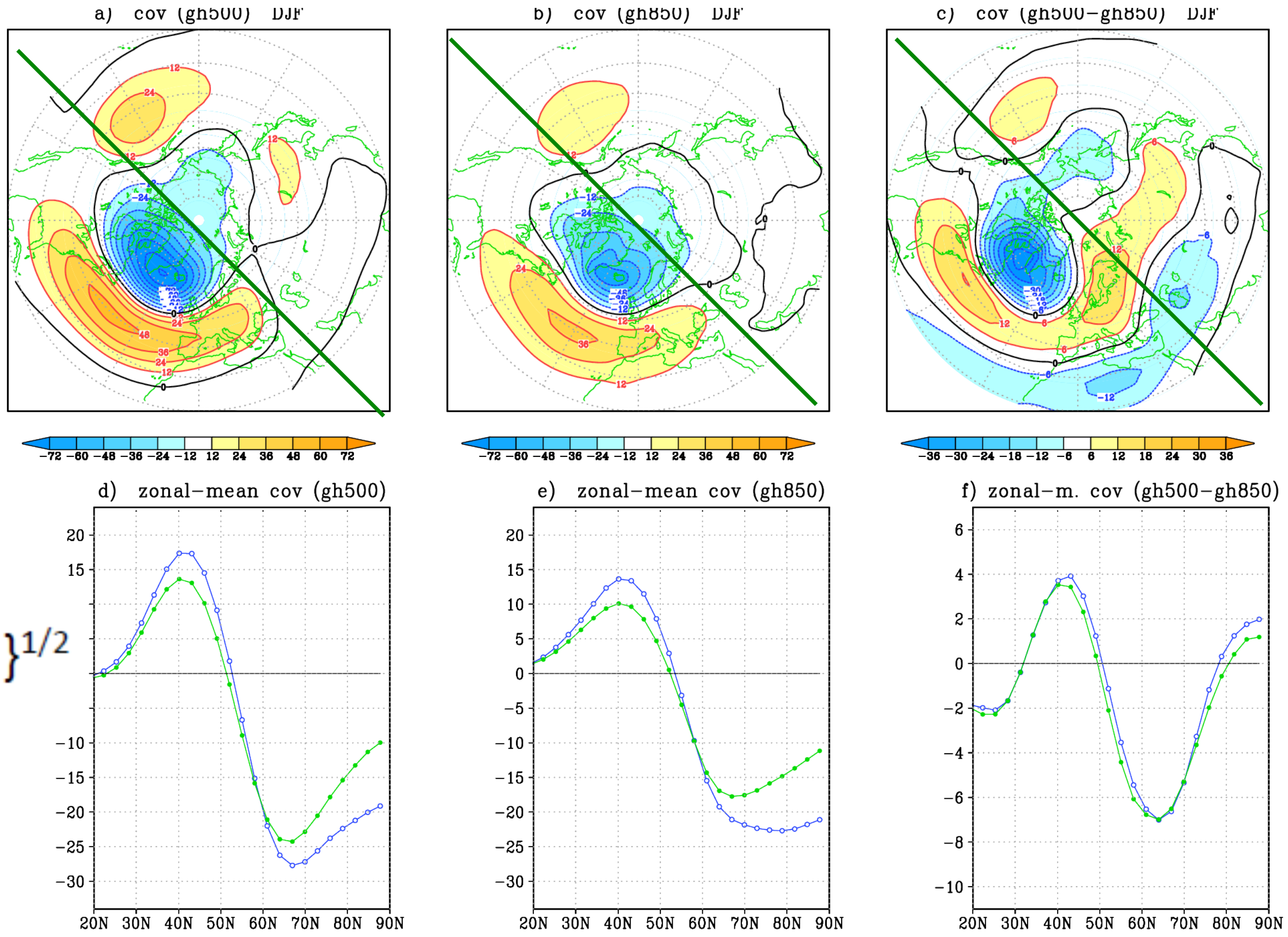
F. Molteni and F. Kucharski: A heuristic dynamical model of the North Atlantic Oscillation with a Lorenz-type chaotic attractor *(Climate Dynamics, under review)*

ECMWF

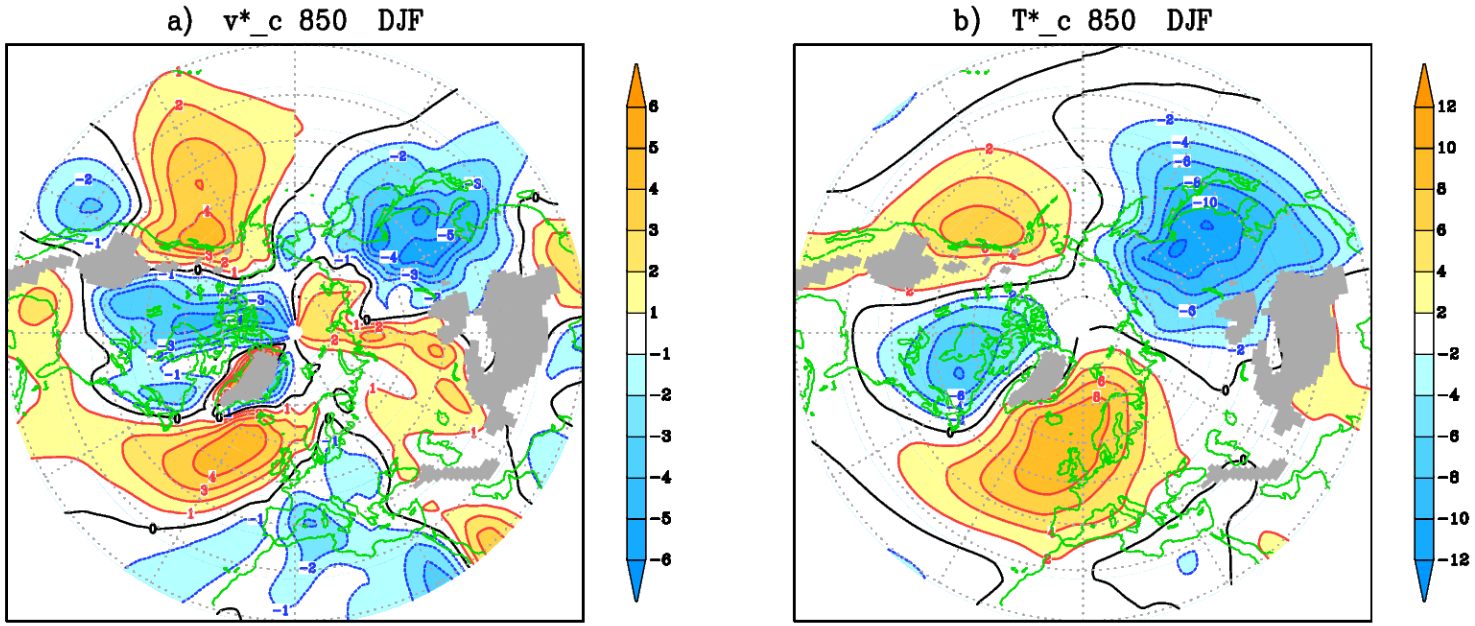
Covariances of geop. height with NAO index (from ERA-interim)

$$A_{nao} = [\, Z'_{500}\,(70W\text{-}20W,\ 30N\text{-}45N)] - [\, Z'_{500}\,(70W\text{-}20W,\ 55N\text{-}70N)]$$

$$C_{nao}(F) = \{ F' A_{nao} \} / \{ A^2_{nao} \}^{1/2}$$

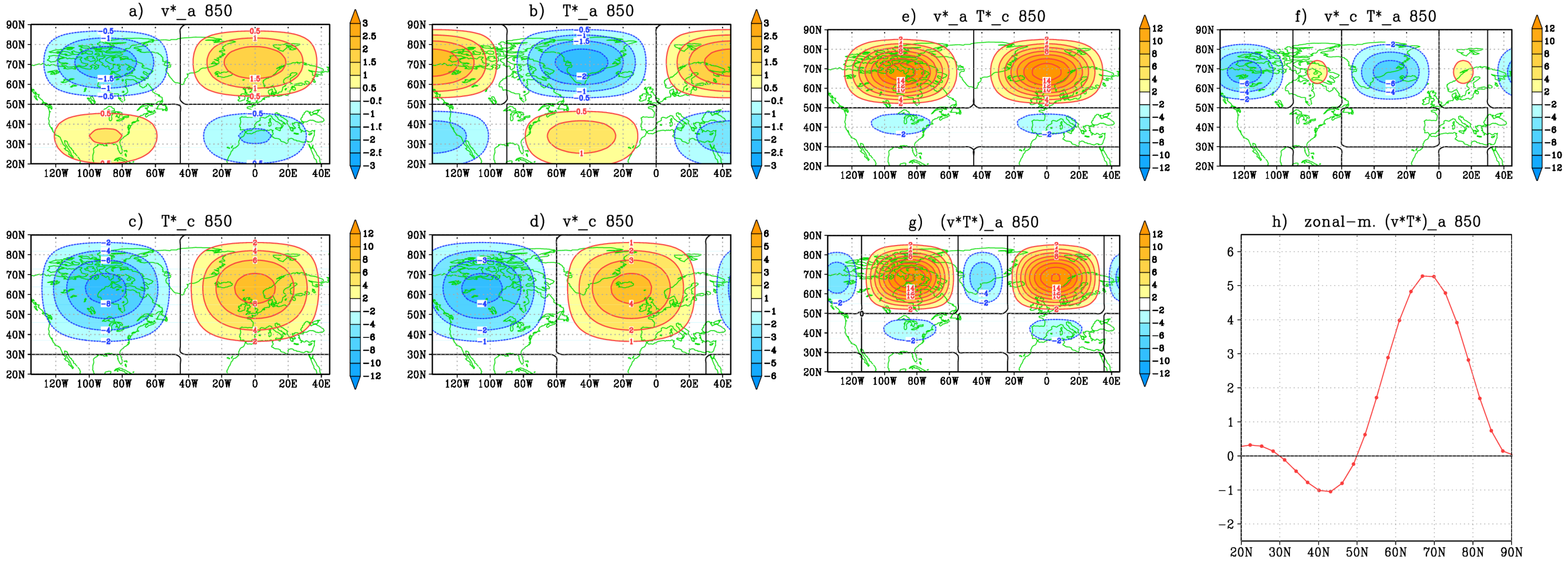


Statitionary waves at 850 hPa: meridional wind (v) and temperature (T)

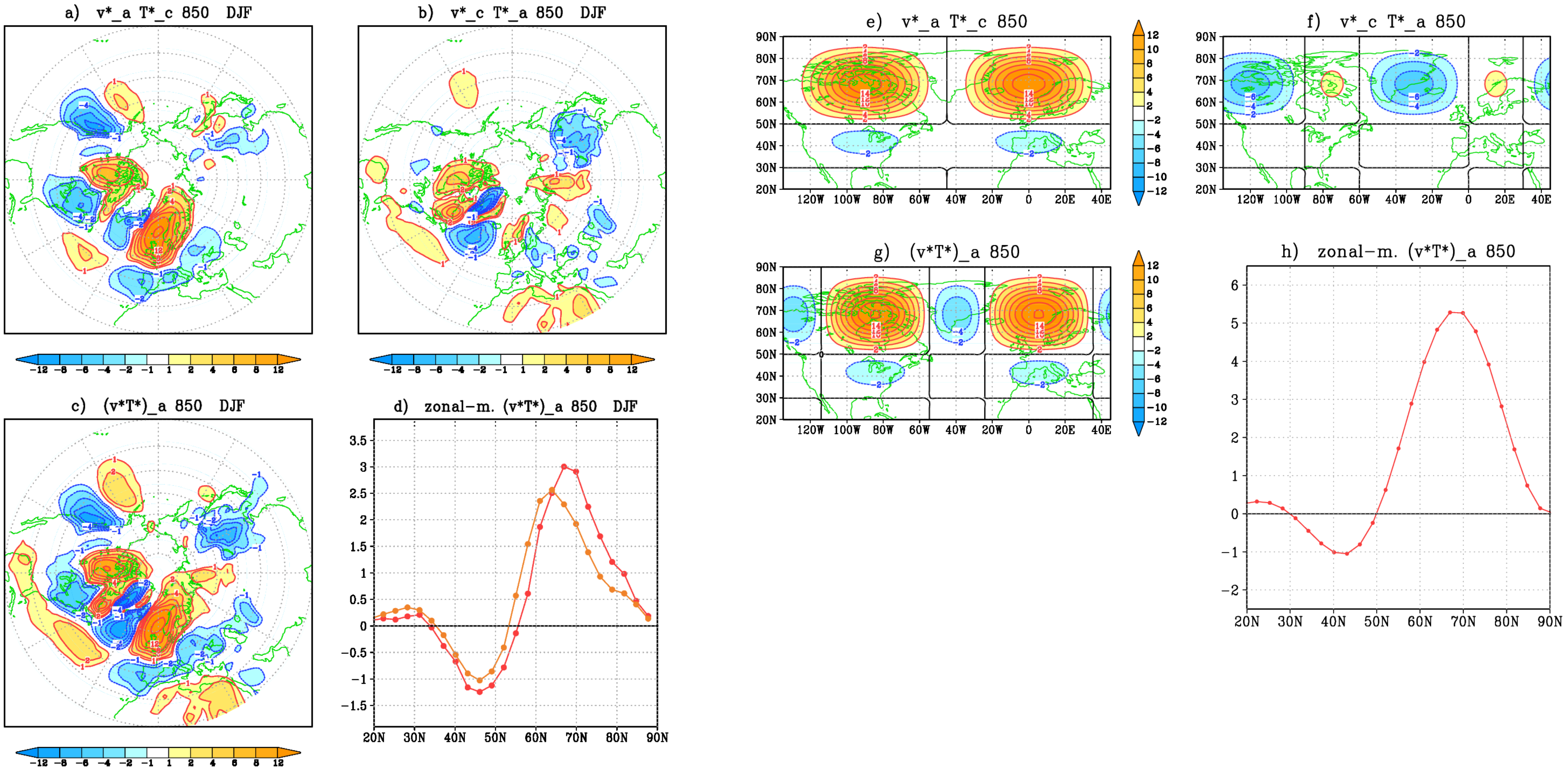


If the stationary waves had a geostrophic, equivalent barotropic structure, the v and T eddies would be orthogonal. In reality, v* and T* are positively correlated, so that heat is transported northward in mid-latitudes: [v*T*] > 0

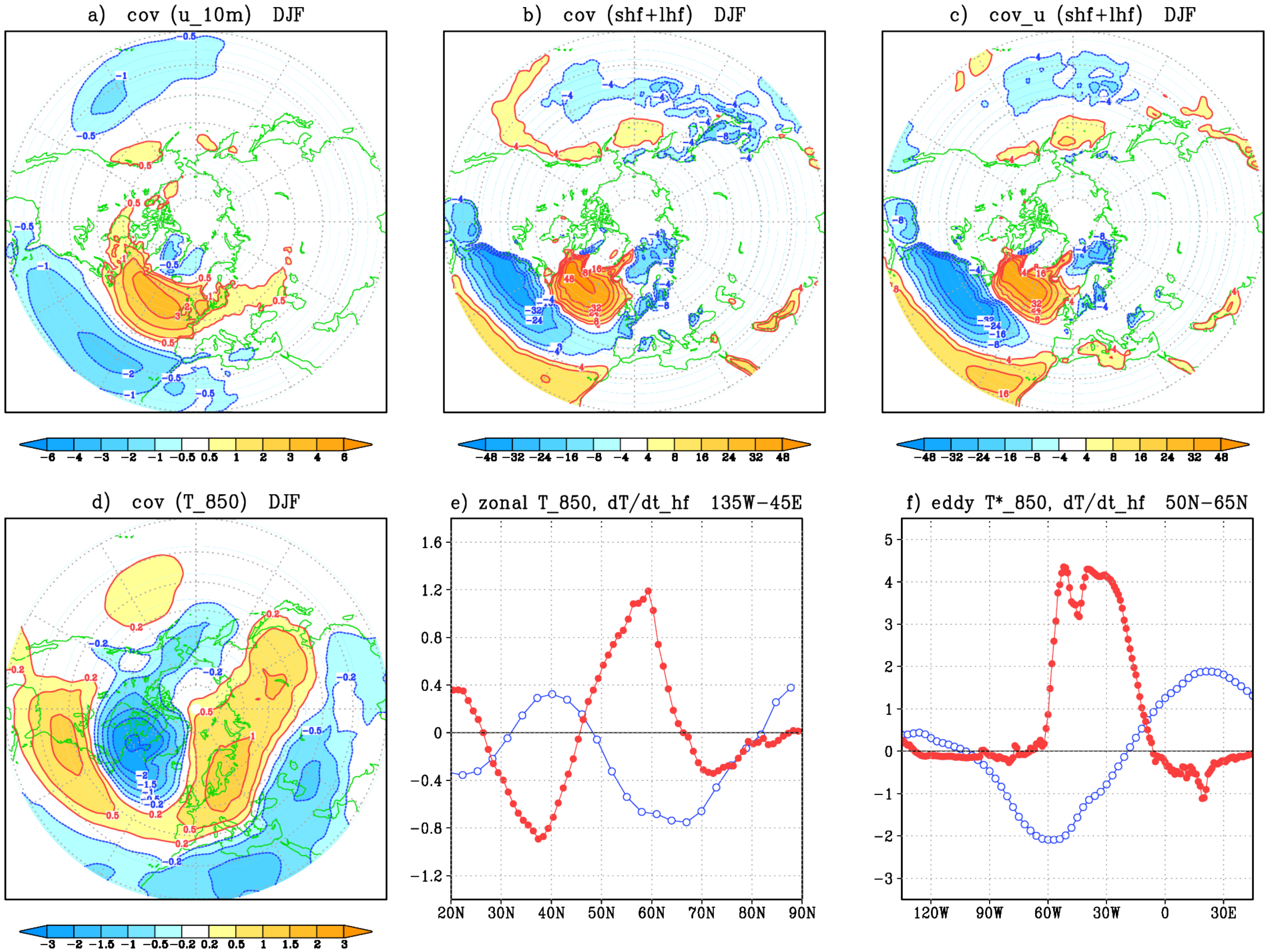
Interactions of an equiv. barotropic anomaly with baroclinic stationary waves



Meridional heat transport by the NAO anomaly



Covariances of surface heat fluxes with NAO anomaly



$$HF = c_H \, \rho \, |V_0| \, (H_s - H_a \, (z=0))$$

$$H = c_p T + L_c \, q$$

Over the North Atlantic, anomalies in $|V_0|$ give a large contribution to HF variability.

Assuming a uniform heating between the sfc and ~ 300hPa, a regression of the heating tendency as a linear function of the temperature tendency gives a damping time of ~ 10 days

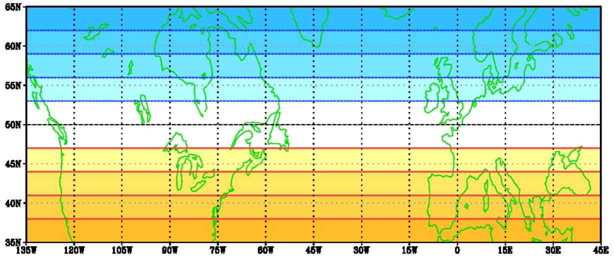
3-variable NAO model: definition of basic functions and DOF

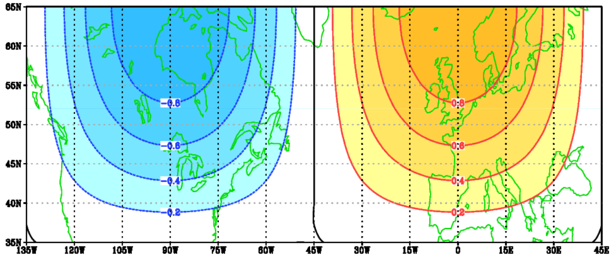
7a) $\Psi = - U\, y + \Psi^*_{NAO} + \Psi^*_{C}$

North America / Atlantic / Europe (NAE) channel:

135W-45E, 35N-65N

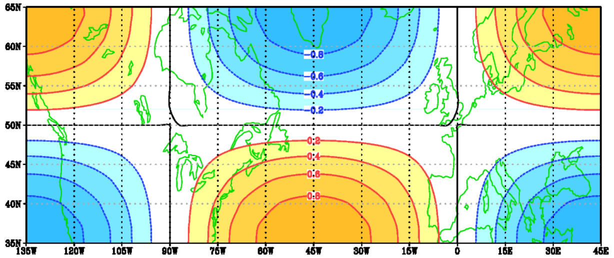
U



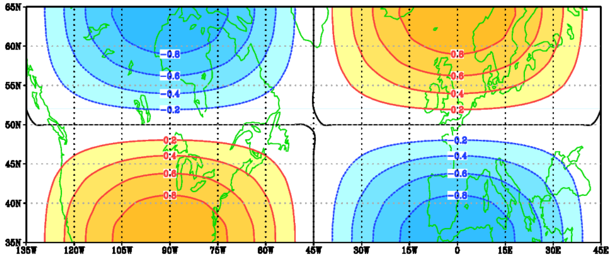


A

~ NAO index



B



7b) $\Psi' = - U\, y + \Psi^*_{NAO} = - U\, y + A\, F_1 + B\, F_2$

where:

8a) $F_1 = - (2/n) \cos(\pi\, x / L_x) \sin(\pi\, y / L_y)$

8b) $F_2 = (2/n) \sin(\pi\, x / L_x) \sin(\pi\, y / L_y)$

8c) $n^2 = (\pi / L_x)^2 + (\pi / L_y)^2$

3-variable NAO model: vorticity advection

9) $\partial \Delta\Psi^*_{NAO}/\partial t = - U \partial \Delta\Psi^*_{NAO} /\partial x - \beta \partial\Psi^*_{NAO} /\partial x$

where Δ is the horizontal laplacian and β is the gradient of planetary vorticity. Since, according to our definition, Ψ^*_{NAO} is an eigenfunction of the laplacian, Eq. (9) becomes:

10) $\partial\Psi^*_{NAO} /\partial t = - [U - \beta/n^2] \partial\Psi^*_{NAO} /\partial x$

If U' is defined as the deviation of the zonal mean wind from U_n, and t' as time normalised by a low-frequency time scale τ, Eq. 10 is reduced to the system:

11a) $dA/dt' = \alpha U' B$

11b) $dB/dt' = - \alpha U' A$

where $\alpha = (\pi \tau /L_x)$. With $L_x = 6400$ km, choosing $\tau = 23$ days gives $\alpha \approx 1$ s/m.

3-variable NAO model: meridional and vertical heat fluxes

12a) $d\,U'/dt' = \gamma A - H' - \kappa (U' - U^*)$

12b) $d\,A/dt' = U' B - H' - \kappa A$

12c) $d\,B/dt' = - U' A - \kappa (B - B^*)$

where:

12d) $H' = \sigma U'$

- Divergence of meridional heat transport
- Thermal dissipation due to surface heat fluxes
- Relaxation towards forced state driven by long-wave radiative damping

From NAO statistics: $\gamma \approx 2$, $\sigma \approx 2$, $\kappa \approx 0.5$

3-variable NAO model: equivalence to the Lorenz 1963 model

Setting:

$B' = B - B^*, \quad U^* = 0, \quad \gamma \approx \sigma$

12a) $d\,U'/dt' = \gamma A - H' - \kappa\,(U' - U^*)$

12b) $d\,A/dt' = U' B - H' - \kappa A$

12c) $d\,B/dt' = - U' A - \kappa\,(B - B^*)$

where:

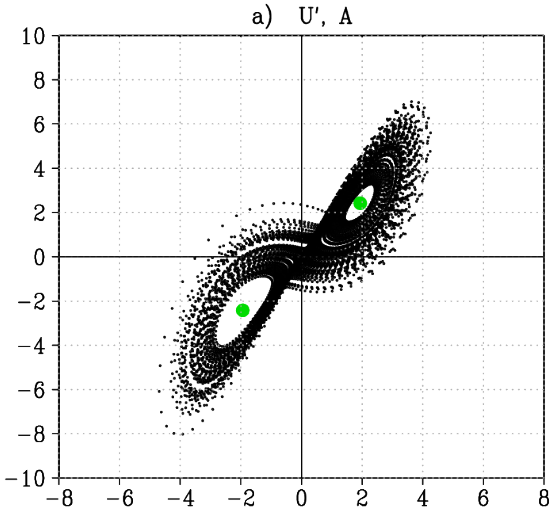
12d) $H' = \sigma U'$

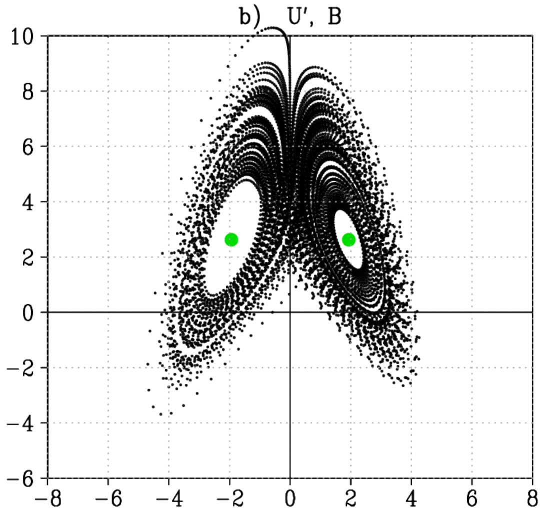
13a) $d\,U'/dt' = \sigma A - (\sigma + \kappa)\,U'$

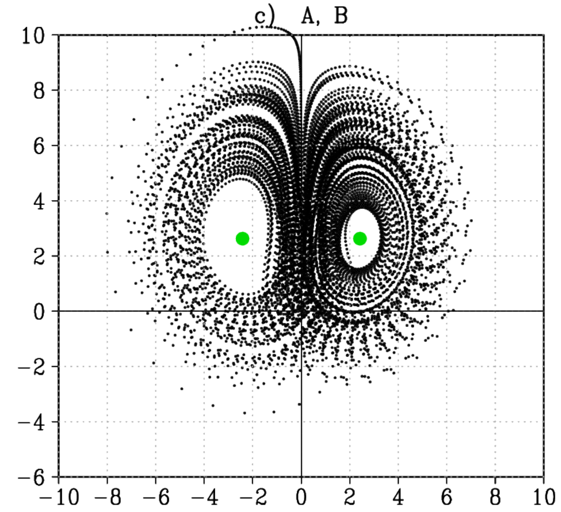
13b) $d\,A/dt' = U' B' + U' (B^* - \sigma) - \kappa A$

13c) $d\,B'/dt' = - U' A - \kappa B'$

Chaotic attractor of the 3-variable NAO model



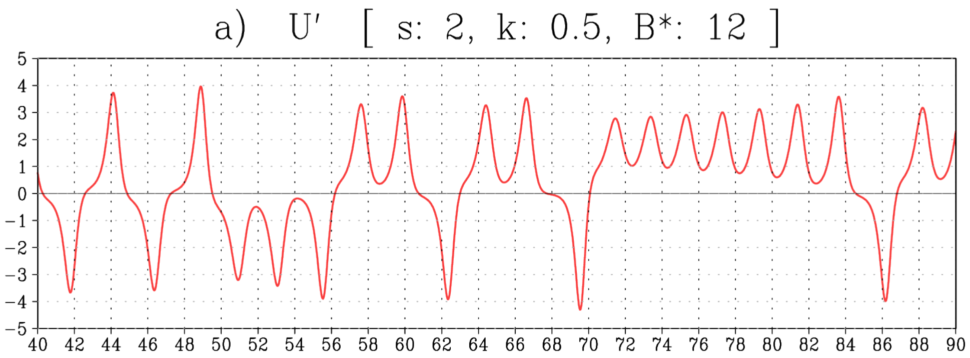


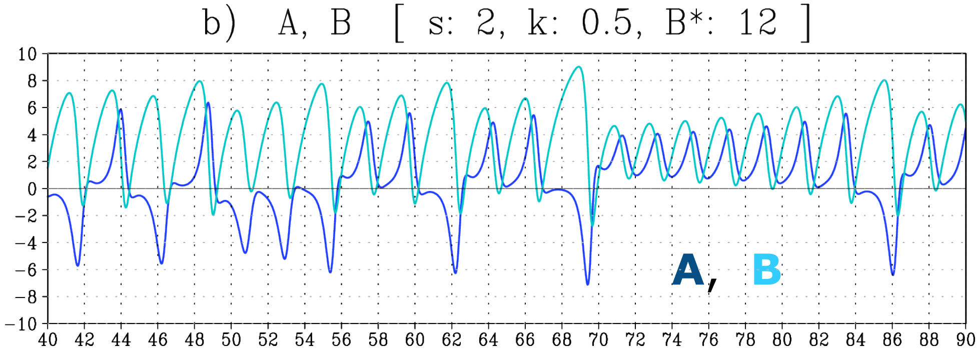


14a) $U'^2_s = [\, k\,\sigma / (\sigma + k) \,]\, (B^* - B_s)$

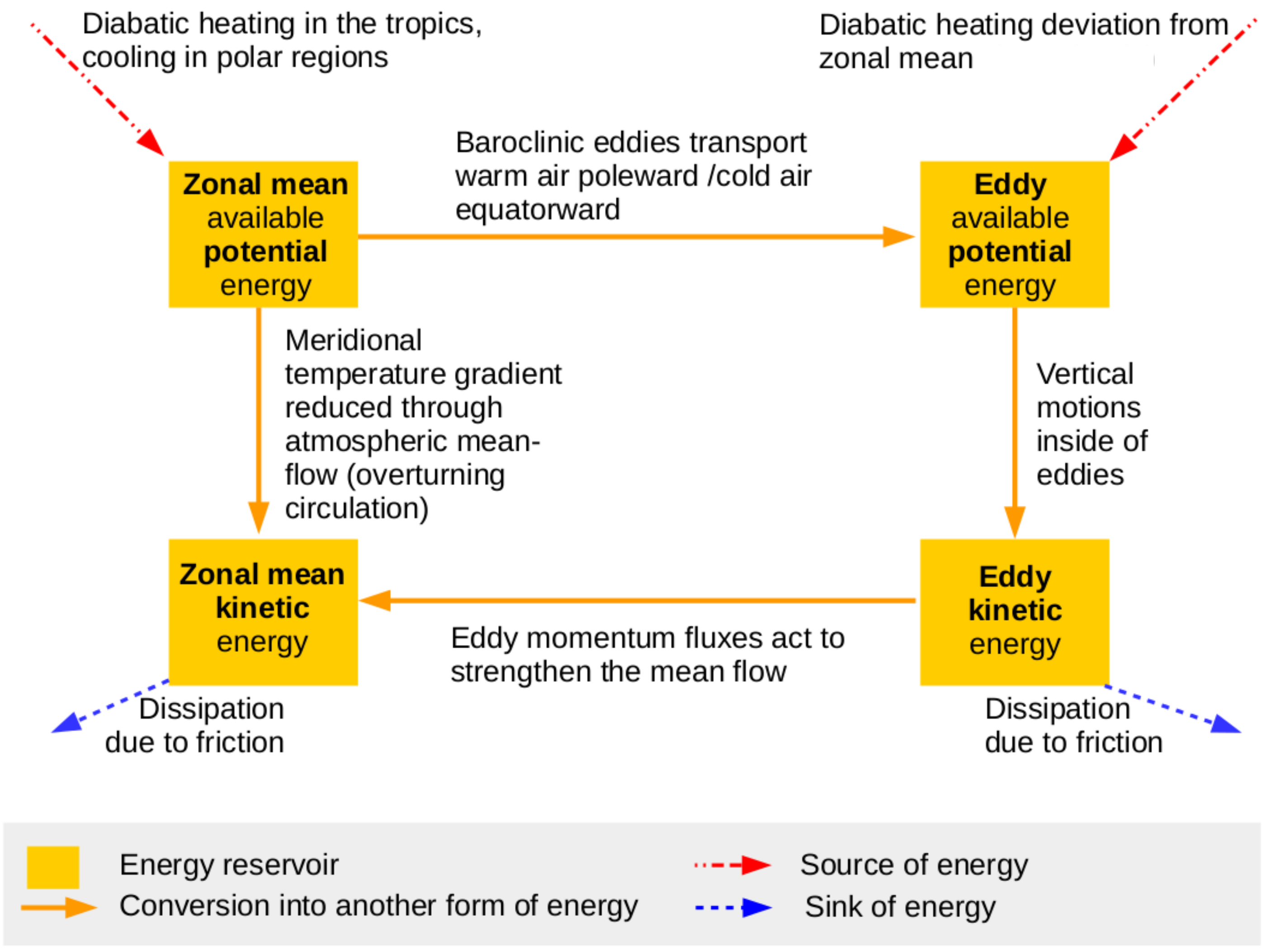
14b) $A^2_s = [\, k\, (\sigma + k) / \sigma \,]\, (B^* - B_s)$

14c) $B_s = \sigma + k + k^2/\sigma$





Atmospheric energy cycle (after Lorenz)



From Wikipedia

A non-linear oscillator model for high-frequency eddies

- Ambaum and Novak (*QJRMS* 2014)
- Novak et al. (*JAS* 2015)

$$\dot{s} = F - f,$$
$$\dot{f} = 2(s - s_0)\,f.$$

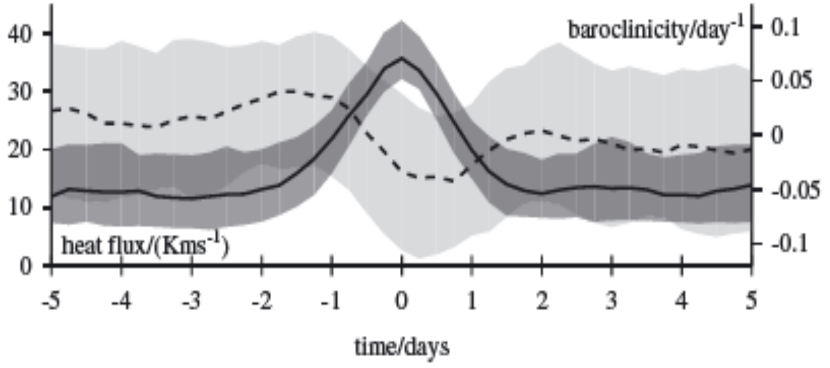


Figure 5. Composite of heat flux and baroclinicity for the winters of 1957–2001, centred around the maxima of the heat flux. The solid line is the median value of the heat flux and the dashed line is the median value of the baroclinicity. The shading corresponds to the interquartile range of each quantity. The anomalous (excess) baroclinicity has been plotted; the mean offset in the baroclinicity is 0.46 day^{-1}.

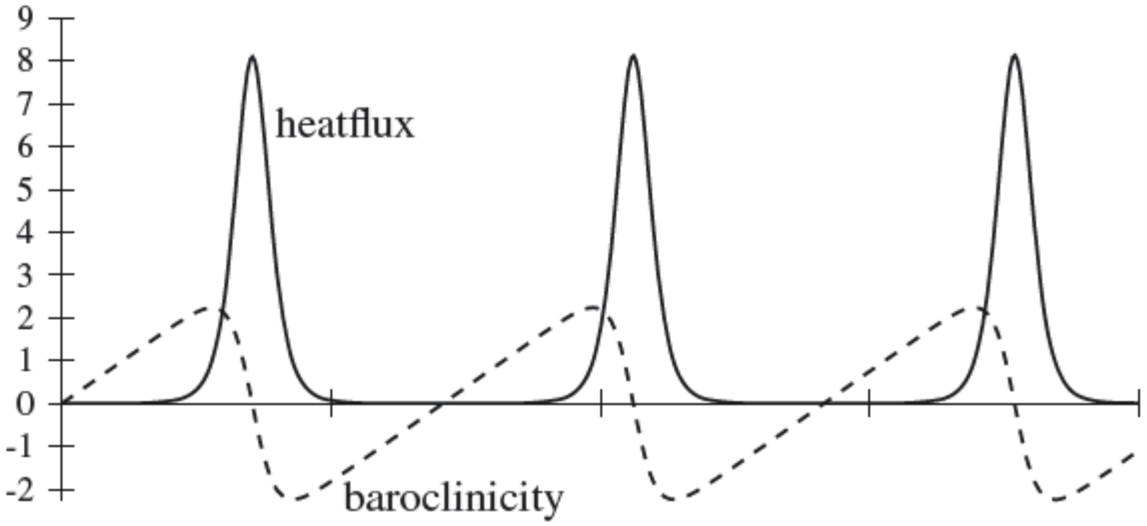
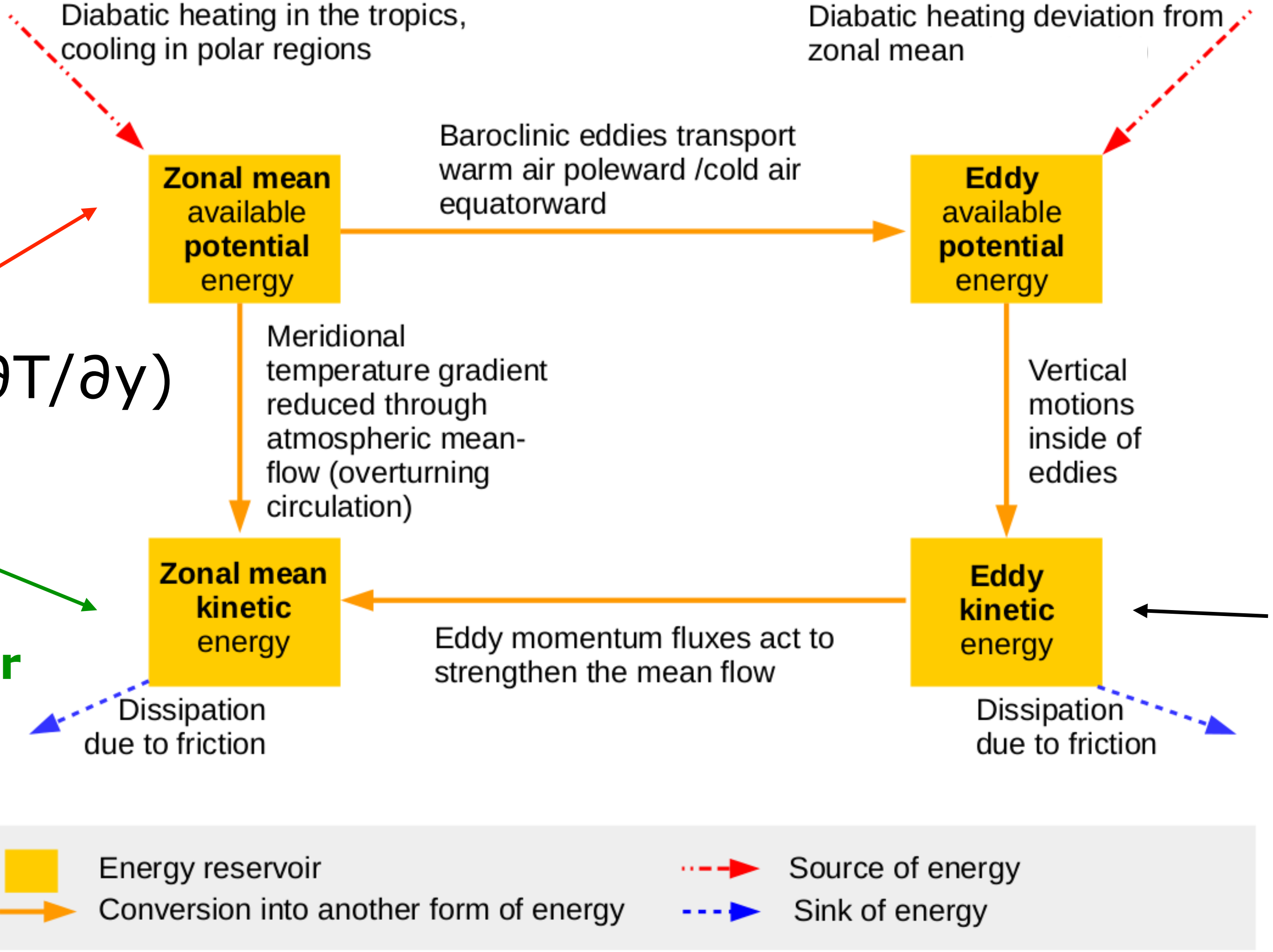


Figure 2. Time series of heat flux f (solid lines), rescaled with F, and excess baroclinicity $s - s_0$ (dashed lines), rescaled with $\sqrt{F}$. The time is rescaled with

Atmospheric energy cycle (after Lorenz)

$$U'(z) = U'_{th}\,(\partial T/\partial y) + U'_{btr}$$

$$U'(z=0) = U'_{btr}$$



E_{hf}

A 5-variable model with zonal flow/baroclinic eddies interactions

We can write the zonal wind at the equivalent barotropic level as the sum of a height-independent barotropic component and a thermal component:

15) $U' = U'_{btr} + U'_{th}$

High-frequency baroclinic eddies grow at the expense of zonal available potential energy and decay by surface drag and conversion of kinetic energy into the zonal-mean barotropic flow

16a) $d\,U'_{th}/dt' = \gamma A - H' - \kappa\,(U'_{th} - U^*) - c_a\,(E_{hf}^2 - E_0^2)$

16b) $d\,A/dt' = U'\,B - H' - \kappa\,A$

16c) $d\,B/dt' = - U'\,A - \kappa\,(B - B^*)$

16d) $d\,U'_{btr}/dt' = - \kappa_f\,U'_{btr} + c_k\,(E_{hf}^2 - E_0^2)$

16e) $d\,E_{hf}/dt' = (c_a\,U'_{th} + c_a\,U_0 - c_k\,U'_{btr} - \kappa_{ef})\,E_{hf}$

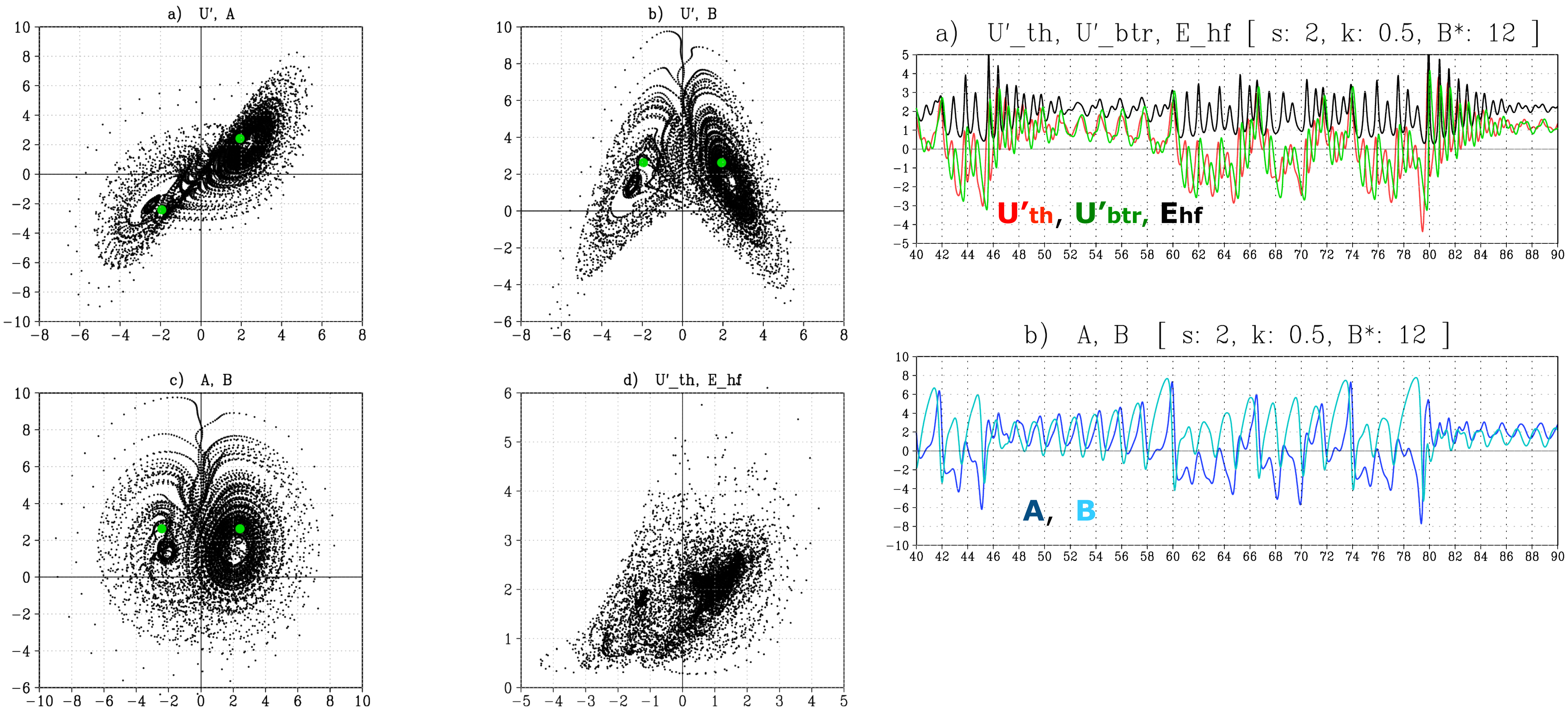
where U' is given by Eq. 15 and :

16f) $H' = \sigma\,U'_{btr}$

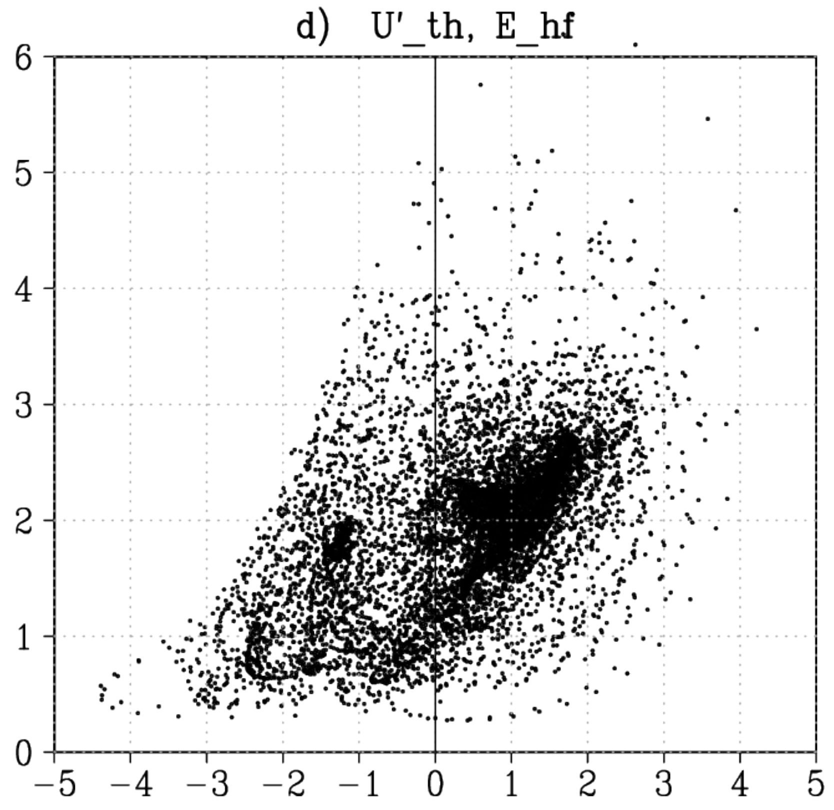
16g) $\kappa_{ef} = \kappa_f\,(1 + E_{hf}^2 / E_0^2)^{1/2}$

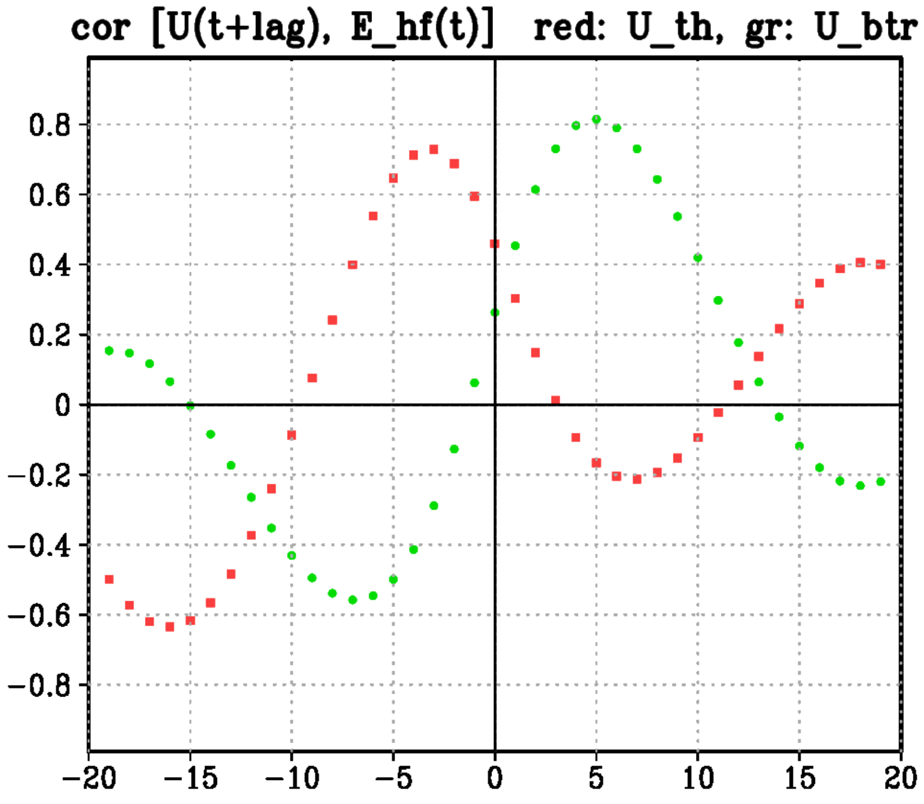
16h) $U_0 = 2^{1/2}\,\kappa_f / c_a$

Attractor of the 5-variable NAO model



Lead-lag relationship between zonal wind and h.f. eddy amplitude





Summary

- Regimes in the NAO can exist because of the balance of a positive and a negative feedback between the zonal and eddy component components of the NAO anomaly in the North America/Atlantic/European (NAE) sector, and the associated surface heat fluxes.
- The positive feedback is associated with the strengthening of the zonal-mean temperature gradient due to the interaction of the NAO anomaly with climatological stationary waves of wider meridional scale. In turn, vorticity advection by the increased zonal-mean wind forces a positive NAO anomaly.
- The negative feedback is due to thermal damping caused by the heating anomalies driven by surface fluxes. Over the North Atlantic, these fluxes are strongly controlled by the near-surface zonal wind speed.
- A simple 3-variable model including the effects of vorticity advection and the two feedbacks described above is formally equivalent to the Lorenz (1963) convection model, and has a chaotic attractor with two regimes.
- The model can be extended by incorporating non-linear oscillators that describe the energy conversions associated with the growth and decay of baroclinic eddies. The resulting 5-var model still shows a chaotic attractor with increased variability at sub-seasonal scale.