Sugra localization and AdS black holes (part II)

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Workshop on Susy Localization and Holography:
Black Hole Entropy and Wilson Loops
ICTP, Trieste, 9-13 July 2018

Based on..

- 1803.05920 with Ivano Lodato and Valentin Reys
- 1608.07294 with Francesco Benini and Alberto Zaffaroni

Important background literature

- Susy localization - *[Pestun'07]*
- (Quantum) entropy function - *[Sen'08]*
- Localization in supergravity - *[Dabholkar, Gomes, Murthy'10-11]*
- Topologically twisted index - *[Benini, Zaffaroni'15-16]*

Motivation

Quantum gravity?

- Supergravity can be seen as a toy model for quantum gravity at weak coupling
- Supersymmetric vacua are stable quantum states
- Supersymmetry allows for extrapolation of results from weak coupling (GR + matter) to strong coupling (novel quantum gravity effects)
- AdS/CFT gives a dual quantum picture, many exact results accessible via supersymmetric localization
- Existence of BPS (susy-preserving) black holes - an "integrable" sector of quantum gravity because of AdS_2 near-horizon

Black holes and susy holography

- "Black holes = statistical ensembles of gravitational degrees of freedom."
- What are the microscopic states that make up the black hole entropy?

$$S(q,p) = \log d(q,p)\ , \qquad d(q,p) \in \mathcal{Z}_+ \tag{1}$$

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- Make gradual progress, start with susy case with AdS_2 near-horizon geometry.
- Look at sugra solutions in various dimensions, assume $SU(1,1|1)$ near-horizon symmetry algebra ($U(1)_R$-symmetry, unlike $SU(2)_R$ of *[Strominger, Vafa'96]*).
- Holography suggests a dual field theory picture: a susy $\mathcal{N}=2$ quantum mechanics flowing to an IR conformal point.

The field theory perspective

- Dual field theory given by a Hamiltonian H_p depending on black hole magnetic charges p^i.
- Calculate grand-canonical susy partition function

$$Z(\Delta, p) = Tr_{\mathcal{H}}((-1)^F e^{i\Delta^i J_i} e^{-\beta H_p}) \ , \quad < J_i >= q_i \tag{2}$$

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- Find the microcanonical partition function via a Legendre transform,

$$Z(\Delta, p) = \sum_q Z(q, p) e^{i q_i \Delta^i} \ , \quad Z(q, p) = \int_\Delta Z(\Delta, p) e^{-i q_i \Delta^i} \qquad (3)$$

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- Assume no cancellation between bosonic and fermionic states in the large charge (large N) limit, find the BH entropy in the microcanonical ensemble by a saddle point approximation,

$$S_{BH}(q, p) \equiv \mathcal{I}(\dot{\Delta}) = \log Z(\dot{\Delta}, p) - i q_i \dot{\Delta}^i \ , \quad \frac{d\mathcal{I}}{d\Delta}|_{\dot{\Delta}} = 0 \tag{4}$$

Weak coupling: easy example

- Look at gapped $\mathcal{N}=2$ quantum mechanics with real masses σ^i: susy ground states $H=\sigma^i J_i$
- Free chiral multiplet ($y=e^{i(\Delta+i\beta\sigma)}$):

$$H_c=(a^\dagger a+b^\dagger b+1)|\sigma|-\frac{\sigma}{2}[\overline{\psi},\psi]\ ,\quad J_c=a^\dagger a-b^\dagger b+\frac{1}{2}[\overline{\psi},\psi] \tag{5}$$

$$Z_c(y)(\frac{(a^\dagger)^n}{n!}|0,\uparrow>)=\sum_{n=0}^{\infty}y^{n+1/2}=\frac{y^{1/2}}{1-y} \tag{6}$$

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- Free fermion multiplet:

$$H_F=\sigma J_F=\sigma\left(\lambda^\dagger\lambda-\frac{1}{2}\right) \tag{7}$$

$$Z_F(y)(|\uparrow>+|\downarrow>)=y^{-1/2}-y^{1/2}=\frac{1-y}{y^{1/2}} \tag{8}$$

Strong coupling: localization

- Twsited index on $S^1 \times \Sigma_g$ via Bethe potential / twisted superpotential of the 2d theory on Σ_g *[Hosseini, Zaffaroni'16]*:

$$Z(\Delta, p) = \int \mathrm{d}u \frac{Q(u,p)}{\prod_i \left(1 - e^{i\partial\mathcal{W}(\Delta,u)/\partial u}\right)} \tag{9}$$

- Large N evaluation, one leading solution $\bar{u}$ of $e^{i\partial\mathcal{W}/\partial u} = 1$,

$$\mathcal{W}(\Delta, \bar{u}) \sim F_{S^3}(\Delta) \tag{10}$$

$$\log Z(\Delta, p) = -\sum_i p^i \frac{\partial\overline{\mathcal{W}}}{\partial\Delta^i} \tag{11}$$

Localization matches with sugra at large N

- Twisted index of ABJM theory *[Benini, KH, Zaffaroni'15]* match to asymptotically $\mathrm{AdS}_4 \times \mathrm{S}^7$ black hole entropy *[Cacciatori, Klemm'09]* in 11d sugra (also with mass-deformation *[Bobev, Min, Pilch'18]*)

$$F_{S^3} \sim N^{3/2} \sqrt{\Delta^1 \Delta^2 \Delta^3 \Delta^4}$$

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- Twisted index of the $\mathrm{D2}_k$ theory *[Guarino, Jafferis, Varela'15; Hosseini, KH, Passias'17; Benini, Khachatryan, Milan'17]* match to asymptotically $\mathrm{AdS}_4 \times \mathrm{S}^6$ black hole entropy *[Guarino, Tarrio'17]* in massive IIA 10d sugra

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- Twisted index of $\mathcal{N} = 4$ SYM theory *[Hosseini, Nedelin, Zaffaroni'16]* match to asymptotically $AdS_5 \times S^5$ black string entropy *[Benini, Bobev'13]* in type IIB 10d sugra

$$F_{S^3} \sim N^2 \frac{\Delta^1\Delta^2\Delta^3}{\Delta^0}$$

More matches with sugra

- Universal twist $\bar{\Delta}^i \sim \bar{p}^i$ RG flows match to many 10d and 11d sugra black holes *[Azzurli, Bobev, Crichigno, Min, Zaffaroni'17]*

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- Evidence of field theory matches to rotating black hole entropy via anomaly coefficients: $\mathcal{N} = 4$ SYM theory *[Hosseini, KH, Zaffaroni'17]* to rotating $\mathrm{AdS}_5 \times \mathrm{S}^5$ black holes *[Gutowski, Reall'04]*; 6d $(2, 0)$ theory *[Hosseini, KH, Zaffaroni'18]* to rotating $\mathrm{AdS}_7 \times \mathrm{S}^4$ black holes *[Cvetic, Gibbons, Lu, Pope'05; Chow'07]*

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- Subleading corrections to large N results: $\log N$ corrections from localization computed numerically *[Liu, Pando Zayas, Rathee, Zhao'17]*

Entropy at finite N?

- No cancellation between bosons and fermons assumption!(?)
- Finite N field theory (microscopic) entropy

$$d^{SUSY}_{micro} \equiv Z(q,p) = \int \left(\prod_i \mathrm{d}\Delta^i \right) \delta(\sum_i \Delta^i - 1) Z(\Delta, p) e^{-iq_i \Delta^i} \tag{12}$$

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- Infer the exact (macroscopic) black hole entropy via the holographic dictionary

$$e^{S(q,p)} = d_{macro} = d_{micro} \stackrel{?}{=} d^{SUSY}_{micro} \in \mathcal{Z}_+ \tag{13}$$

- Any putative quantum gravity calculation must lead to d_{macro}
- However, holographically $d^{SUSY}_{macro} = d^{SUSY}_{micro}$, no assumptions!

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- Field theory: at weak coupling states in the grand-canonical ensemble; at strong coupling use localization / anomalies.
- Quantum gravity: at weak coupling sugra calculation (?); at strong coupling string theory (definition via the field theory dual?)
- In QFT well-defined microcanonical and grand-canonical ensemble; in quantum gravity so far only microcanonical ensemble?

Quantum entropy - see V. Reys' talk!

- Formal definition:

$$d_{macro} \equiv \left\langle \exp\left(4\pi q_i \int_0^{2\pi} W_\tau^i \mathrm{d}\tau \right) \right\rangle_{EAdS_2=\mathcal{H}_2}^{finite} \tag{14}$$

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- Explicit approach for d_{macro}^{SUSY}: pick a sugra theory and a black hole near-horizon geometry (a BPS *solution*). Assume/prove the gravitational background to be fixed (*freeze* gravity multiplet) and perform susy localization on a curved background of the remaining (vector-, hyper-, tensor-) multiplets in the bulk.

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- Conceptual issues (see *[de Wit, Murthy, Reys'18]*): 1-loop determinant of the gravity multiplet, non-compact space (Euclidean AdS_2), exact integration measure. Let's neglect them at a first aproximation!

4d $\mathcal{N} = 2$ off-shell gauged supergravity

- Conformal sugra formalism developed in *[de Wit, van Holten, van Proeyen'80], ...*
- use Euclidean version for full consistency *[de Wit, Reys'17]*
- Weyl multiplet: metric $g_{\mu\nu}$, auxiliary tensor $T^{\pm}_{ab}$ and scalar D, gauge fields b_μ (dilatation), A_μ $(SO(1,1)_R)$, $\mathcal{V}^{i}_{\mu j}$ $(SU(2)_R)$, gravitini and dilatini

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- (compensating) hypermultiplet: four real scalars A_i^α, hyperini
- after gauge fixing equivalent to Poincare sugra with n_V vector multiplets and FI gauging ξ_I

On-shell solution and superalgebra

- half-BPS near-horizon geometry in off-shell formalism *[de Wit, van Zalk'12], [KH, Katmadas, Lodato'16]*
- gravity: $\mathrm{d}s^2 = v_1 \mathrm{d}s^2_{AdS_2} + v_2 \mathrm{d}s^2_{S^2}$, $b_\mu = A_\mu = 0$, $D = -(v_1^{-1} + 2v_2^{-2})/6$, $T^{\pm}_{12} = w^{\pm} = \pm 2 v_1^{-1/2}$

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- vectors: $\dot{F}^I_{12} = e^I, \dot{F}^I_{34} = p^I$, $\dot{Y}^{ij,I}(\dot{X}^I_{\pm})$, scalars $X^I_{\pm}(\xi_I, q_I, p^I)$ subject to attractor mechanism
- hypers: gauge fix to break $SU(2)_R$ to $U(1)_R$, $\mathcal{V}^i_{\mu j} = -2i\xi_I \dot{W}^I_\mu \sigma^i_{3j}$, twisting condition $\xi_I p^I = 1/2$

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- $SU(1,1|1)$ superalgebra, bosonic subgroups $SU(1,1) \times U(1)_R$
- pick localizing supercharge $\mathcal{Q}$, s.t. $\mathcal{Q}^2 = \mathcal{L}_\tau + \delta_{U(1)_R} + \delta_{gauge}$ equivariant differential

Localization locus

- Weyl multiplet frozen
- vector multiplet: arbitrary functions $C_k^I(\theta,\varphi), D_k^I(\theta,\varphi)$, $Y^{ij,I}(C_k^I, D_k^I)$

$$\delta X_\pm^I = \sum_{k=1}^{\infty} (D_k^I \pm C_k^I) r^{-k} \tag{15}$$

$$\delta W_\tau^I = \sqrt{v_1} \sum_{k=2}^{\infty} (D_{k-1}^I + C_k^I)(r^{1-k} - 1) \tag{16}$$

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- hypermultiplet extra constraint $\xi_I \delta X_\pm^I = 0$
- use integration variables $\phi_+^I \equiv 2\dot{X}_+^I \sum_{k=1}^\infty (C_k^I + D_k^I)$ and $\phi_\perp^I(C_k, D_k)(\theta,\varphi)$:

$$\partial_{\theta,\varphi}\phi_+^I = 0 = \partial_{\theta,\varphi}\delta W_\tau^I \ , \qquad \xi_I \phi_+^I = 1 \tag{17}$$

Classical action

- two-derivative + Wilson line action:

$$S^{2d} + S^{W} = 2\pi r_0 \left(p^I (\dot{\mathcal{F}}_I^+ + \dot{\mathcal{F}}_I^- \right) + q_I e^I) - 2\pi \left(p^I \mathcal{F}_I^+ (\phi_+) + q_I \phi_+^I \right) \tag{18}$$

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- reinstate Newton's constant,

$$d_{macro}^{SUSY} = \int_{\phi_+} \exp\left[-\frac{\pi}{2\mathcal{G}_N} \left(p^I \mathcal{F}_I^+(\phi_+) + q_I \phi_+^I \right) \right] Z_{\text{ind}}^{\text{reg}}(\phi_+) \tag{19}$$

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- 1-loop, measure, gravity localization - hidden inside $Z_{\text{ind}}^{\text{reg}}(\phi_+)$
- higher derivative F-terms: additional $256 F_A$ to the classical action, string theory origin?

Saddle point = attractor mechanism

- Saddle point evaluation

$$\frac{\partial}{\partial \phi_+^I}\left(p^I \mathcal{F}_I^+(\phi_+) + q_I \phi_+^I\right)\Big|_{\dot{\phi}_+^I} = 0\,, \quad S_{BH} = -\frac{\pi}{2\mathcal{G}_N}\left(p^I \mathcal{F}_I^+(\dot{\phi}_+) + q_I \dot{\phi}_+^I\right)$$

- Precise match with attractor mechanism (Lorentzian) of *[Cacciatori, Klemm'09], [Dall'Agata, Gnecchi'10]* after Wick rotation.

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- Precise match with attractor mechanism (Lorentzian) of *[Cacciatori, Klemm'09], [Dall'Agata, Gnecchi'10]* after Wick rotation.
- Saddle point agreement between d_{macro}^{SUSY} and $d_{micro}^{SUSY}=Z(q,p)$ in all known examples:

$$\mathcal{F}(11d/S^7)\sim\sqrt{\phi^0\phi^1\phi^2\phi^3}$$

$$\mathcal{F}(mIIA/S^6)\sim(\phi^1\phi^2\phi^3)^{2/3}$$

$$\mathcal{F}(IIB/S^5\times S^1)\sim\frac{\phi^1\phi^2\phi^3}{\phi^0}$$

Grand-canonical ensemble?

- Field theory microcanonical partition function

$$d_{micro}^{SUSY} = \int \left(\prod_i \mathrm{d}\Delta^i \right) \delta(\sum_i \Delta^i - 1) Z(\Delta, p) e^{-iq_i \Delta^i} \tag{20}$$

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- Supergravity localization result

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- Supergravity localization result

$$d^{SUSY}_{macro} = \int \left(\prod_i \mathrm{d}\phi^I_+ \right) \delta(\sum_i \phi^I_+ - 1) e^{-\frac{\pi}{2\mathcal{G}_N} p^I \mathcal{F}^+_I} Z^{\mathrm{reg}}_{\mathrm{ind}} \; e^{-\frac{\pi}{2\mathcal{G}_N} q_I \phi^I_+} \tag{21}$$

- Define grand-canonical ensemble in sugra,

$$Z^{sugra}(\phi_+, p) = \exp\Big[-\frac{\pi}{2\mathcal{G}_N} \big(p^I \mathcal{F}^+_I(\phi_+) \big) \Big] Z^{\mathrm{reg}}_{\mathrm{ind}}(\phi_+) \tag{22}$$

Speculations

What are the microscopic states that make up the black hole entropy?

- What does sugra localization count? Is it just a calculational trick or allows for a deeper interpretation?
- Precise holographic match between states in the grand-canonical ensemble?

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- What does sugra localization count? Is it just a calculational trick or allows for a deeper interpretation?
- Precise holographic match between states in the grand-canonical ensemble?
- Fuzzball proposal for AdS black holes, explicit classical geometries counted by $Z^{sugra}(\phi_+, p)$?

Speculations

What are the microscopic states that make up the black hole entropy?

- What does sugra localization count? Is it just a calculational trick or allows for a deeper interpretation?
- Precise holographic match between states in the grand-canonical ensemble?
- Fuzzball proposal for AdS black holes, explicit classical geometries counted by $Z^{sugra}(\phi_+, p)$?
- Look for the answer in Euclidean theory, *[Freedman, Pufu'13], [Bobev, Elvang, Freedman, Pufu'13]* and *[Cassani, Martelli'14]*?

Future work

- Continue the sugra localization program: 1-loop contribution, localization measure...possible hints on finite N evaluation of the matrix model in field theory?
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- Go beyond susy and extremality - near AdS_2 geometries, SYK, ...