Boundary Conditions and Localization on AdS

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Workshop on Supersymmetric Localization and Holography:
Black Hole Entropy and Wilson Loops

ICTP, Trieste

References:

1) Localization on $\mathrm{AdS}_2 \times \mathrm{S}^1$

J.David, E.Gava, R.Gupta, K.Narain : JHEP 1703 (2017) 050

2) Boundary Conditions and Localization on AdS :Part 1

J.David, E.Gava, R.Gupta, K.Narain : arXiv:1802.00427

3) Boundary Conditions and Localization on AdS :Part 2

J.David, E.Gava, R.Gupta, K.Narain : to appear soon

Introductions and Motivations

Supersymmetric localization

- is a powerful technique to evaluate supersymmetric observables exactly.
- has provided non trivial checks of several conjectural dualities in susy QFT in various dim.
- has also provided highly non trivial checks for AdS/CFT.

(Dabholkar, Drukker,Gomes, Grassi, Marino, Putrov, Sen.)

- has been focussed mostly on susy QFT defined on curved but compact spaces without boundary.

Introductions and Motivations

Rigid susy QFT can also be defined on curved but non compact spaces.

Non compact spaces of the form $\mathrm{AdS}_n \times \mathrm{S}^m$ are relevant in evaluating black hole entropy as well as entanglement entropy in conformal field theories.

Localization on non compact spaces is a harder problem for two reasons:

1. needs to worry about boundary terms in susy variations,

2. needs to include boundary conditions.

On Boundary Conditions

On non compacts spaces the boundary conditions defines the problem.

These boundary conditions tell us which fluctuations to integrate over in the path integral to compute the observable.

Typically the natural boundary conditions one imposes on the quantum fluctuations are normalizable boundary conditions w.r.t

$$\int d^d x \sqrt{g}\, |\Phi|^2 < \infty \,.$$

Black Hole Entropy

Quantum entropy of an extremal black hole is given as

$$e^S = \int_{AdS_2,\text{finite}} [dg_{\mu\nu}][d\Phi]\, e^{-S_{\text{Eucld.}} - iq_i \oint_{\partial AdS_2} A^{(i)}}$$

(Sen)

One loop computations were performed with normalizable boundary conditions on all the fields.

(Banerjee,Gupta, Mandal,Sen)

Example:

$$S = \frac{A_H}{4} + C_1 \ln A_H + C_2 + \frac{C_3}{A_H} + \ldots + D_1\, A_H^n\, e^{-A_H} + ..$$

computation uses normalizable bdy. cond.

$\mathcal{N}$	8	4	2
C_1	-4	0	$2 - \frac{\chi}{24}$

$\boxed{\chi = 2(n_v - n_H + 1)}$

Entropy of black hole in AdS_4 and twisted partition function on $\mathrm{AdS}_2 \times \mathrm{S}^1$.

(Cabo-Bizet,Giraldo-Rivera, Pando Zayas)

Supersymmetric Localization

Basic idea : If there exist a fermionic symmetry Q such that

$Q^2 = \mathcal{L}_v$ is satisfied off shell,

and $QS = 0$

combinations of various symmetry like gauge transformation, R-symmetry etc.

Then $$< Q(...) >= \int e^{-S} Q(...) = 0$$

path integral measure is also invariant

$\Rightarrow$ Partition function does not change under $S \to S + t\,QV$ for any fermionic functional V such that $Q^2 V = 0$.

$$Z = \lim_{t\to\infty} \int e^{-S - t\,QV} = \int_{\mathcal{M}_{\rm loc}} e^{-S}$$

$$\mathcal{M}_{\rm loc} = \{QV = 0\}$$

Supersymmetry on $\mathrm{AdS}_2 \times \mathrm{S}^1$

We want to put $\mathcal{N}=2$ susy QFT with $U(1)_R$ on a background which admits one or more rigid supercharges.

(Closset,Dumitrescu, Festuccia,Komargodski)

The background metric is

$$ds^2 = d\tau^2 + L^2(dr^2 + \sinh^2 r\, d\theta^2)$$

We want to solve

$$(\nabla_\mu - iA_\mu)\,\epsilon = -\frac{1}{2}H\gamma_\mu\epsilon - iV_\mu\epsilon - \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu\gamma^\rho\epsilon\,,$$
$$(\nabla_\mu + iA_\mu)\,\tilde{\epsilon} = -\frac{1}{2}H\gamma_\mu\tilde{\epsilon} + iV_\mu\tilde{\epsilon} + \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu\gamma^\rho\tilde{\epsilon}$$

We require that the killing vector is

$$\widetilde{\epsilon}\gamma^\mu\epsilon\,\partial_\mu = K^\mu = \frac{\partial}{\partial\tau} + \frac{1}{L}\frac{\partial}{\partial\theta}$$

Killing Spinor

We need to turn on background value of some supergravity fields

$$A_\tau = V_\tau = \frac{1}{L} \qquad\qquad A_{r,\theta} = V_{r,\theta} = H = 0$$

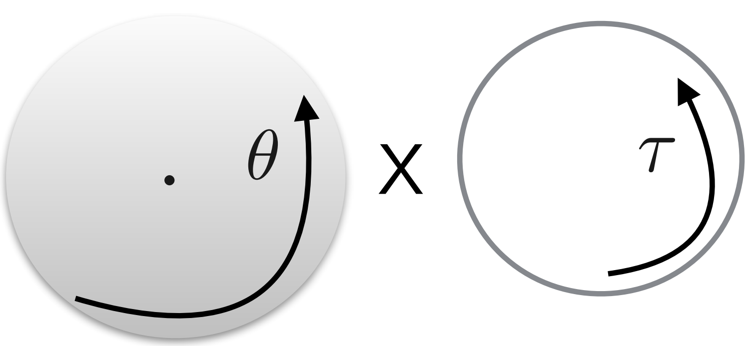
The killing spinors are given by

$$\epsilon = e^{\frac{i\theta}{2}} \begin{pmatrix} i\cosh(\frac{r}{2}) \\ \sinh(\frac{r}{2}) \end{pmatrix}, \qquad \tilde{\epsilon} = e^{-\frac{i\theta}{2}} \begin{pmatrix} \sinh(\frac{r}{2}) \\ i\cosh(\frac{r}{2}) \end{pmatrix}$$

The supersymmetry algebra on the background

$$Q^2 = \mathcal{L}_K + \delta_\Lambda^{\text{gauge transf}} + \delta_{\frac{1}{2L}}^{R-\text{symm}}.$$

Here $\qquad Q = \delta_\epsilon + \delta_{\tilde{\epsilon}}$



$\mathrm{AdS}_2 \times \mathrm{S}^1$

Chern-Simons+Matter

We consider Chern-Simons theory coupled to a fundamental matter.

(David,Gava,Gupta,Narain;16)

The theory is described by

$$S = S_{\mathrm{C.S.}} + S_{\mathrm{matter}}$$

where

$$S_{\mathrm{C.S.}} = \frac{k}{4\pi}\int d^3x\sqrt{g}\,\mathrm{Tr}\Big[i\varepsilon^{\mu\nu\rho}\Big(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\Big) - \tilde{\lambda}\lambda + \frac{i}{2}G\sigma\Big],$$

equivalent to boson theory

$$S_{\mathrm{matter}} = \int d^3x\sqrt{g}\Big[\mathcal{D}_\mu\bar{\phi}\mathcal{D}_\mu\phi + \Big(-\frac{1}{4}qG - \frac{\Delta}{4}R + \frac{1}{2}\Big(\Delta - \frac{1}{2}\Big)V^2 - q^2\sigma^2\Big)\bar{\phi}\phi - F\bar{F} + \ldots\Big]$$

Δ =R-charge of chiral multiplet

fermionic terms

SUSY of Action

The vector multiplet+chiral multiplet is supersymmetric upto boundary terms.

In particular, the action of the chiral multiplet is Q-exact.

$$S_{\text{matter}} = \text{Q-exact} + \text{boundary terms}$$

where

$$\text{boundary terms} = \int d^3x\sqrt{g}\,\nabla_\mu\Big[-\frac{1}{\cosh r}(\widetilde{\epsilon}\widetilde{\psi})(\epsilon\gamma^\mu\psi) - \frac{i}{2}V^\mu\bar{\phi}\phi - \frac{i}{\cosh r}\varepsilon^{\mu\rho\nu}(\epsilon\gamma_\nu\widetilde{\epsilon})\bar{\phi}\,D_\rho\phi + (\epsilon\gamma^\mu\widetilde{\epsilon})\frac{iq\sigma}{\cosh r}\bar{\phi}\phi\Big]\,.$$

The boundary terms go to zero with normalizable and smoothness conditions on all the fields.

Boundary Condition on Chiral Multiplet

The normalizable boundary conditions on $\mathrm{AdS}_2 \times \mathrm{S}^1$ are

$$\phi \sim e^{-\frac{r}{2}}, \quad \psi \sim e^{-\frac{r}{2}}$$

These conditions are not consistent with susy transformations

$$Q\phi = \epsilon\psi \qquad\qquad Q\psi = \Gamma^\mu \tilde{\epsilon}\, D_\mu \phi + \ldots$$

This is due to the fact that susy parameters grow exponentially for large r,

$$\epsilon \sim e^{\frac{r}{2}}, \quad \widetilde{\epsilon} \sim e^{\frac{r}{2}} \,.$$

This seems generic fact for any AdS_d .

Localization Manifold

We add following positive definite Q-exact terms

$$V_{\rm loc} = \int d^3x \sqrt{g} \frac{1}{(\tilde{\epsilon}\epsilon)^2} \mathrm{Tr}\left[\Psi^\mu (Q\Psi_\mu)^\dagger + \Psi(Q\Psi)^\dagger + \psi(Q\psi)^\dagger + \tilde{\psi}(Q\tilde{\psi})^\dagger\right]$$

Here $$\Psi = \frac{i}{2}(\tilde{\epsilon}\lambda + \epsilon\tilde{\lambda}) , \quad \Psi_\mu = \frac{1}{2}(\epsilon\gamma_\mu\tilde{\lambda} + \tilde{\epsilon}\gamma_\mu\lambda)$$

Solutions: $$a_\mu = 0 , \quad \sigma = \frac{i\alpha}{\cosh r} , \quad G = \frac{4i\alpha}{L\cosh^2 r} ,$$

real parameter

$$\phi = \bar{\phi} = F = \bar{F} = 0 .$$

Partition Function

Thus the partition function is given as

$$Z = \int_{\mathbb{R}} [d\alpha] \exp(-\pi i k L \mathrm{Tr}\alpha^2) Z^{\text{vec.}}_{1-\text{loop}}(\alpha) Z^{\text{chiral}}_{1-\text{loop}}(\alpha, \Delta)$$

The one loop determinant can be evaluated using Green's function of the kinetic operator.

$$\frac{\delta}{\delta\alpha} \ln Z_{1-\text{loop}}(\alpha) = \mathrm{Tr}[\underbrace{G_F}_{\text{fermionic Green's function}} \frac{\delta}{\delta\alpha} \mathcal{D}_F(\alpha)] - \frac{1}{2} \mathrm{Tr}[\underbrace{G_B}_{\text{bosonic Green's function}} \frac{\delta}{\delta\alpha} \mathcal{D}_B(\alpha)] \,.$$

$\mathcal{D}_F(\alpha)$: Fermionic kinetic operator

$\mathcal{D}_B(\alpha)$: Bosonic kinetic operator

Green's Function : Methodology 1

Green's function is a solution to the differential equation

$$\mathcal{D}(x)G(x,y) = \delta(x,y)$$

$\mathcal{D}(x)$: is a differential operator without zero modes.

We first find the solutions to the differential equation

$$\mathcal{D}(x)S(x) = 0$$

This does not have a global solution.

Let

$$\left.\begin{array}{l} S_1(x) : \text{is a valid solution near } x \to 0\,. \\ S_2(x) : \text{is a valid solution near } x \to \infty\,. \end{array}\right\}$$

Smoothness and bdy. conditions.

Then the Green's function is given as

$$G(x,y) = c\Big[\Theta(y-x)\,S_1(x)S_2(y) + \Theta(x-y)S_1(y)S_2(x)\Big]\,.$$

Green's Function : Methodology 2

The Green's function is

$$G(x,y) = c\Big[\Theta(y-x)\, S_1(x)S_2(y) + \Theta(x-y)S_1(y)S_2(x)\Big]\,.$$

Properties :

- The Green's function is continuous at $x = y$.
- The first derivative of the Green's function is discontinuous at $x = y$. The discontinuity fixes the constant c .

For fermionic case, the Green's function is discontinuous.

E.O.M

If bdy. conditions are consistent with susy, the Green's function for bosonic fields and fermionic fields are related.

Consequence of the supersymmetry: if X_0 satisfies

$$\mathcal{D}\, X_0 = 0,$$

for chiral $X_0 = \{\phi\}$

then its super partner also satisfies the same equation

$$\mathcal{D}\, Q X_0 = 0\ .$$

$$X_1 = \mathcal{M}\, Q X_0$$

for chiral, the fermions are $\{X_1, QX_0\}$

for

$$V_{\text{chiral}} = \begin{pmatrix} Q\bar{X}_0 & \bar{X}_1 \end{pmatrix} \begin{pmatrix} D_{00} & D_{01} \\ D_{10} & D_{11} \end{pmatrix} \begin{pmatrix} X_0 \\ QX_1 \end{pmatrix}$$

A solution for X_0 is also sol. for QX_0 provided it is consistent with the respective bdy. and smoothness conditions.

E.O.M 2

Explicitly the equation of motion for ϕ is

$$\mathcal{D}^\mu \mathcal{D}_\mu \phi - \Big(-\frac{1}{4} q G - \frac{\Delta}{4} R + \frac{1}{2}\left(\Delta - \frac{1}{2}\right) V^2 - q^2 \sigma^2 \Big)\phi = 0 \quad ,$$

and the remaining equation is

$$X_1 = \frac{iL}{(\Delta - 2) - 2Lq(\widetilde{\epsilon}\epsilon)\sigma + 2iQ^2} (\widetilde{\epsilon}\gamma^\mu \widetilde{\epsilon}) \mathcal{D}_\mu Q X_0$$

In terms of chiral multiplet fermion the twisted variables are

$QX_0 = \epsilon\psi$ and $X_1 = \widetilde{\epsilon}\psi$

Solutions to E.O.M

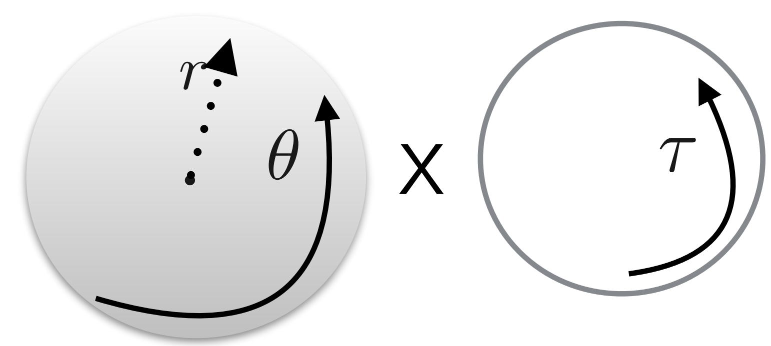
Explicitly obtaining the solutions for fields in chiral multiplet and analysing their asymptotic behaviour, we find that

$$\{f_{n,p}, (b_{n,p}, c_{n,p})\}|_{\text{normalizable}} = \begin{cases} \{f^+, (b^+_{n,p}, c^+_{n,p})\} & \text{for} \quad n > \frac{\Delta}{2L} \\ \{f^+, (b^-_{n,p}, c^-_{n,p})\} & \text{for} \quad \frac{\Delta-1}{2L} < n < \frac{\Delta}{2L} \\ \{f^-, (b^-_{n,p}, c^-_{n,p})\} & \text{for} \quad n < \frac{\Delta-1}{2L} \; . \end{cases}$$

Here the modes are coefficients of Fourier expansion:

$$X_0 \sim f_{n,p}\, e^{in\tau + ip\theta}, \qquad QX_0 \sim c_{n,p}\, e^{in\tau+ip\theta}$$

$$QX_1 \sim b_{n,p}\, e^{in\tau + i(p-1)\theta}$$



$\mathrm{AdS}_2 \times \mathrm{S}^1$

$$\boxed{e^{\frac{r}{2}} f_{n,p} \to 0, \qquad b_{n,p}(z) \to 0, \qquad c_{n,p}(z) \to 0} \qquad \text{for} \quad r \to \infty$$

Solutions to E.O.M:2

For susy bdy. conditions, we have

$$\{f_{n,p}, (b_{n,p}, c_{n,p})\}|_{\rm susy} = \begin{cases} \{f^+, (b^+_{n,p}, c^+_{n,p})\} & \text{for} \;\; n > \frac{\Delta-1}{2L} \\ \{f^-, (b^-_{n,p}, c^-_{n,p})\} & \text{for} \;\; n < \frac{\Delta-1}{2L} \end{cases} .$$

The susy boy. conditions are

$$e^{\frac{r}{2}} f_{n,p} \to 0, \quad e^{\frac{r}{2}} c_{n,p} \to 0, \quad e^{-\frac{r}{2}} b_{n,p} \to 0 \qquad \text{for} \quad r \to \infty$$

One Loop Determinant 1

We want to evaluate

$$\frac{\delta}{\delta\alpha}\ln Z_{1-\text{loop}}(\alpha) = \mathrm{Tr}[G_F\frac{\delta}{\delta\alpha}\mathcal{D}_F(\alpha)] - \frac{1}{2}\mathrm{Tr}[G_B\frac{\delta}{\delta\alpha}\mathcal{D}_B(\alpha)]\,.$$

For chiral multiplet

$$\frac{\delta}{\delta\alpha}\mathcal{D}_f(\alpha) = \frac{L^2 q}{2\sqrt{1-z}}\sigma_3, \qquad \frac{\delta}{\delta\alpha}\mathcal{D}_b(\alpha) = \frac{Lq(-i+2Lq\alpha)}{2\sqrt{1-z}}\,. \qquad z = \tanh^2 r$$

When normalizable bdy. conditions are consistent with susy, after integration by parts, we find

$$\boxed{\frac{\delta}{\delta\alpha}\ln \widetilde{Z}^{\text{chiral}}_{1-loop}(\alpha) = \int_0^1 dz\left[\frac{\partial}{\partial z}\mathrm{B.T.} + \mathrm{E.O.M}\right]}$$

$\mathrm{B.T.}$: bdy. terms, functions of solns. $S_1(z)$ and $S_2(z)$.

This is the case for $n > \dfrac{\Delta}{2L}$ and $n < \dfrac{\Delta-1}{2L}$.

One Loop Determinant 2

When normalizable bdy. conditions are not consistent with susy, after integration by parts, we find

$$\frac{\delta}{\delta\alpha}\ln\hat{Z}^{\mathrm{chiral}}_{1-loop}(\alpha)=\int_0^1 dz\left[\mathrm{Bulk\ Terms}+\frac{\partial}{\partial z}\mathrm{B.T.}+\mathrm{E.O.M}\right]$$

Bulk Terms : bulk terms, functions of solns. $S_1(z)$ and $S_2(z)$.

$$\mathrm{Bulk\ term}=-\int_0^1 dz\,\frac{Lq(-i+2Lq\alpha)S_{1-}(z)S_{2-}(z)}{4c_{1--}\sqrt{1-z}}+\int_0^1 dz\,\frac{Lq(-i+2Lq\alpha)S_{1-}(z)S_{2+}(z)}{4c_{1-+}\sqrt{1-z}}\,.$$

The complete partition function is

$$\ln Z^{\mathrm{chiral}}_{1-loop}(\alpha)=\ln\widetilde{Z}^{\mathrm{chiral}}_{1-loop}(\alpha)+\ln\hat{Z}^{\mathrm{chiral}}_{1-loop}(\alpha)$$

Chiral Multiplet Result

The one loop determinant is

Boundary terms from susy region

$$\ln Z^{\text{chiral}}_{1-\text{loop}}(\alpha,\Delta) = \sum_{p>0,\, n\geq\lceil \frac{\Delta}{2L}\rceil} \ln\Big(p+L(n+iq\alpha)-\frac{\Delta}{2}\Big) - \sum_{p\leq 0,\, n<\frac{\Delta-1}{2L}} \ln\Big(-p-L(n+iq\alpha)+\frac{\Delta}{2}\Big)$$

$$- \sum_{\substack{p\leq 0 \\ \lceil\frac{\Delta-1}{2L}\rceil\leq n<\frac{\Delta}{2L}}} \ln\Big(-p-L(n+iq\alpha)+\frac{\Delta}{2}\Big) - \sum_{\frac{\Delta-1}{2L}<n<\frac{\Delta}{2L}} \sum_{p\in\mathbb{Z}} \ln\frac{\Gamma(\frac{1}{2}+\frac{1}{4}\hat{x})\Gamma(\frac{1}{4}\hat{x}^*)}{\Gamma(\frac{1}{2}+\frac{1}{4}\hat{y})\Gamma(1+\frac{1}{4}\hat{y}^*)}$$

Boundary terms from non susy region

Bulk terms

Here $\hat{x} = 2|p| + \Delta - 2Ln + 2iLq\alpha\,, \quad \hat{y} = 2|p| - \Delta + 2Ln - 2iLq\alpha\,.$

Chiral Multiplet Result

The one loop determinant from another Q-exact deformations is

$$
\begin{aligned}
\ln Z(\alpha,\Delta) = & \sum_{p>0,n\geq\lceil\frac{\Delta}{2L}\rceil} \ln\left(p+L(n+iq\alpha)-\frac{\Delta}{2}\right) - \sum_{p\leq 0,n<\frac{\Delta-1}{2L}} \ln\left(-p-L(n+iq\alpha)+\frac{\Delta}{2}\right) \\
& - \sum_{p\leq 0,\lceil\frac{\Delta-1}{2L}\rceil\leq n<\frac{\Delta}{2L}} \ln\left(-p-L(n+iq\alpha)+\frac{\Delta}{2}\right) - \sum_{\frac{\Delta-1}{2L}<n<\frac{\Delta}{2L}} \sum_{p>0} \ln\frac{\Gamma(\widetilde{a}_2)\Gamma(b_2)}{\Gamma(\widetilde{b}_2)\Gamma(a_2)} \\
& - \sum_{\frac{\Delta-1}{2L}<n<\frac{\Delta}{2L}} \sum_{p\leq 0} \ln\frac{\Gamma(\widetilde{a}_2-p)\Gamma(b_2-p)}{\Gamma(\widetilde{b}_2-p)\Gamma(a_2-p)}
\end{aligned}
$$

Some Comments

- If the bdy. conditions consistent with susy, the variation is given in terms of bdy. terms. Boundary terms are independent of Q-exact action.

- Bulk terms depend on the Q-exact action.

- The result is consistent with explicit one loop calculation for free chiral multiplet using eigen function method.

Index Result

We also computed the one loop result using the index of D_{10}

$$\ln Z = -\frac{1}{2}\text{index}\, D_{10}\ln Q^2$$

$$\text{index}D_{10} = \text{dim. of kernel} - \text{dim. of cokernel}$$

In the case of chiral multiplet, we get

$$\ln Z^{\text{index}} = \sum_{p=1, n>\frac{\Delta-1}{2L}} \ln(p + L(n+iq\alpha) - \frac{\Delta}{2}) - \sum_{p=0, n>\frac{1-\Delta}{2L}} \ln(p + L(n-iq\alpha) + \frac{\Delta}{2})\,.$$

The one loop determinant of the vector multiplet using the index computation results

$$Z^{\text{vec}}_{\text{1-loop,index}} = \prod_{\rho>0}(\rho.\alpha)^2 \prod_{\rho}\sqrt{\prod_{n\neq 0}(n - i\rho\cdot\alpha)\prod_{p\neq 0}(\frac{p}{L} - i\rho\cdot\alpha)} = \prod_{\rho>0}\sinh(\pi\rho\cdot\alpha)\sinh(\pi L\rho\cdot\alpha)$$

ρ : roots of the Lie algebra

This is consistent with normalisable bdy. cond. if $L^2 > \frac{3}{4}$.

Hypermultiplet on $\mathrm{AdS}_2 \times \mathrm{S}^2$

We have computed partition function of a hypermultiplet on $\mathrm{AdS}_2 \times \mathrm{S}^2$ background with equal radius.

The metric background admits killing spinors with or without graviphoton background.

This computation is relevant for the black hole entropy.

The computations are performed using normalizable boundary conditions. Naively it is not consistent with susy.

However, we find that in this case there are no bulk terms.

Result on $\mathrm{AdS_2} \times \mathrm{S^2}$

When $T = \bar{T} = 0$, we find

$$\ln Z_{\text{hyper}} = -\sum_{\ell=0}^{\infty}\sum_{p>0}^{\infty} \ln(\ell + p - 2iq\alpha) - \sum_{\ell=0}^{\infty}\sum_{p<0} \ln(-\ell + p - 2iq\alpha)$$

$$= -\sum_{k\in\mathbb{Z}} |k| \ln(k - 2iq\alpha) \longrightarrow \text{same as } S^4$$

On the other hand for the black hole background i.e. $T = \bar{T} \neq 0$

$$\ln Z_{\text{hyper}} = \sum_{\ell=0}^{\infty}\sum_{p>0}^{\infty} \ln(\ell + p - 2iq\alpha) + \sum_{\ell=0}^{\infty}\sum_{p<0} \ln(-\ell + p - 2iq\alpha)$$

$$= \sum_{k\in\mathbb{Z}} |k| \ln(k - 2iq\alpha)$$

(Murthy, Reys)

Summary of Results : $\mathrm{AdS_2} \times \mathrm{S^1}$

- We have computed one loop partition function of Chern Simons theory coupled to a chiral multiplet on $\mathrm{AdS_2} \times \mathrm{S^1}$.
- One loop result of vector multiplet based on normalizable boundary conditions for gauge field is same as that on $\mathrm{S^3}$ with some conditions on L . Also consistent with explicit index calculations.
- One loop result for the chiral multiplet based on normalizable boundary conditions depends on the *Q*-exact actions.
- Disagreement if there exist an integer

$$n \in D : (\frac{\Delta - 1}{2L}, \frac{\Delta}{2L})$$

L : ratio of the radius of $\mathrm{AdS_2}$ to that of $\mathrm{S^1}$.

Δ : R-charge of the chiral multiplet

Summary of Results : $\mathrm{AdS}_2 \times \mathrm{S}^2$

- We have computed the one loop partition function of a hypermultiplet on $\mathrm{AdS}_2 \times \mathrm{S}^2$ using normalizable boundary conditions.

- In this case, we find that the normalizable boundary conditions are consistent with susy.

- The result of hypermultiplet is consistent with the index answer.

(Murthy, Reys)

Conclusion

- We developed Green's function method to compute one loop determinant and incorporate bdy. conditions on AdS-spaces.

- When bdy. conditions are consistent with susy, the variation of the one loop determinant is a total derivative.

A general proof is still missing.

- The Green's function is harder to compute for higher spins fields. The difficulty increases with space time dimension.

- One of the advantage of the Green's function method compared to index calculation is that one just needs asymptotic behaviour of the solutions.

Future Directions

- Next we want to apply the Green's function method to higher spin fields.
- It will be interesting to compute the one loop determinant in the case for black holes in AdS spaces.
- It will be interesting to generalize to higher dimensional AdS spaces.

Thank You.