Majorana fermions and the topological Kondo effect

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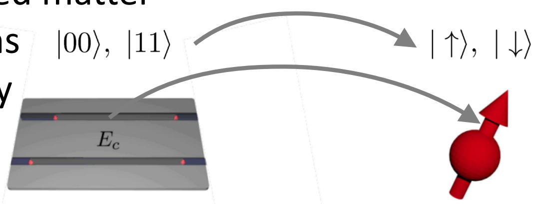
Topological Kondo: idea in 1 slide

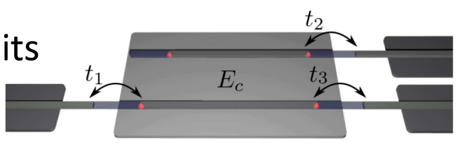
[BB, N. R. Cooper, PRL **109**, 156803 (2012)]

- Fact 1: conduction electrons + quantum spin with degenerate levels
  ➡ **Kondo effect**
  - key paradigm in strong correlations
  - arising from the q. dynamics of a spin qubit

- Fact 2: Majorana fermions in condensed matter
  ➡ **topological qubits** $\sim$nonlocal spins
  - level degeneracy $\sim$ top. degeneracy



- Idea: coupling conduction electrons to topological qubits?
  ➡ **topological Kondo effect**
  - Majorana induced strong correlations
  - demonstrates q. dynamics of top. qubits via transport



Outline

• Intro to Majorana fermions
 - what are they?
 - how do they emerge?
 - key features & potential uses
 - some of the experimental signatures

• Topological Kondo effect
 - from Majoranas to Kondo – the topological Kondo idea
 - transport signatures, incl. NFL features
 - topological Kondo beyond the minimal setup
 - (exact) scaling functions for nonequilibrium transport

Further reading

Reviews on Majorana fermions:

J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
M. Leijnse, K. Flensberg, Semicond. Sci. Technol. **27**, 124003 (2012)
C. W. J. Beenakker, Annu. Rev. Con. Mat. Phys. **4**, 113 (2013)
R. M. Lutchyn *et al*. Nat. Rev. Mater. **3**, 52 (2018)

Background on the Kondo effect:

A. C. Hewson, *The Kondo Problem to Heavy Fermions* (CUP 1997)
L. P. Kouwenhoven and L. I. Glazman, Physics World **14**, 33 (2001)
M. Pustilnik and L. I. Glazman, J. Phys. Condens. Matter **16**, R513 (2004)

Background on field theory/CFT approaches:

I. Affleck, Acta Phys. Polon. B26, 1869 (1995)
I. Affleck *et al*. Phys. Rev. B **45**, 7918 (1992)
M. Oshikawa, C. Chamon, and I. Affleck, J. Stat. Mech. P02008 (2006)

Majorana fermions

Consider an arbitrary fermion problem with operators

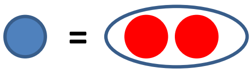
$$c_j : \quad \{c_j, c_k^\dagger\} = \delta_{jk}, \{c_j, c_k\} = 0$$

We can always take the Hermitian & anti-Hermitian parts:

$$\gamma_{j1} = c_j + c_j^\dagger, \quad \gamma_{j2} = -i(c_j - c_j^\dagger)$$

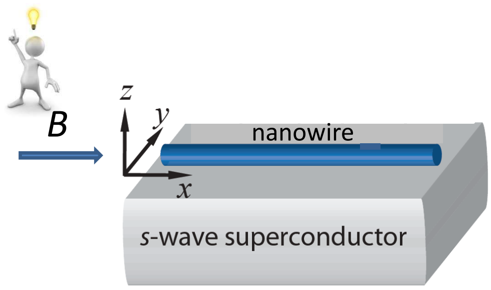
$$\Rightarrow \boxed{\gamma_\alpha = \gamma_\alpha^\dagger, \quad \{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha\beta}}$$

Always works as a maths trick...



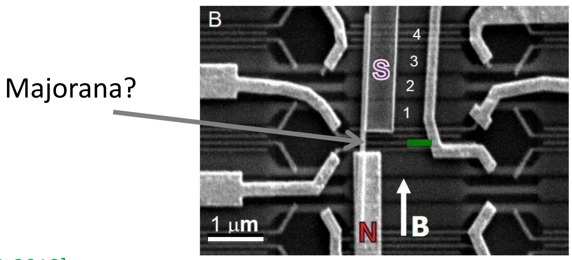
But can also emerge as a form of fractionalisation :

[A. Kitaev, Phys.-Usp. 2001]



[Fu&Kane, PRL 2008; J. Sau *et al. 2010,* R. Lutchyn *et al.* PRL 2010, Y. Oreg *et al.* PRL 2010]

Delft experiment: InSb nanowire



[V. Mourik *et al.*, Science, 2012]

Superconductors & E-H symmetry

BCS mean field description

$$H = \sum_{\alpha\beta} h_{\alpha\beta} c_\alpha^\dagger c_\beta + \tfrac{1}{2}\Delta_{\alpha\beta} c_\alpha^\dagger c_\beta^\dagger + \tfrac{1}{2}\Delta_{\alpha\beta}^* c_\beta c_\alpha \qquad \Delta_{\alpha\beta} = -\Delta_{\beta\alpha}$$

$$H = \tfrac{1}{2}(\mathbf{c}^\dagger \quad \mathbf{c}) \underbrace{\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}}_{H_{\mathrm{BdG}}} \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix} + \text{const.}$$

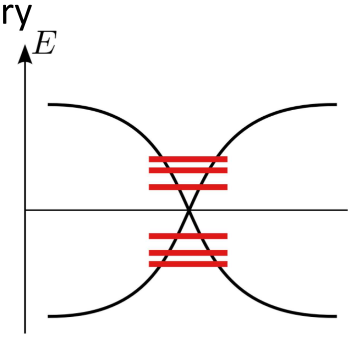
$$\mathcal{C} H_{\mathrm{BdG}} \mathcal{C}^{-1} = -H_{\mathrm{BdG}} \qquad \mathcal{C} = K\Sigma_1$$: e-h symmetry

⇒ Spectral symmetry:

$$H_{\mathrm{BdG}}\psi = E\psi$$

$$\underbrace{\mathcal{C} H_{\mathrm{BdG}} \mathcal{C}^{-1}}_{-H_{\mathrm{BdG}}} \mathcal{C}\psi = E\mathcal{C}\psi$$

$$H_{\mathrm{BdG}}(\mathcal{C}\psi) = -E(\mathcal{C}\psi)$$



E-H symmetry & negative energy "modes"

$$H = \tfrac{1}{2}(\mathbf{c}^\dagger \;\; \mathbf{c}) H_{\mathrm{BdG}} \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix}$$

$$\mathcal{C} H_{\mathrm{BdG}} \mathcal{C}^{-1} = -H_{\mathrm{BdG}} \;\Rightarrow\; H_{\mathrm{BdG}}\psi = E\psi,\; H_{\mathrm{BdG}}(\mathcal{C}\psi) = -E(\mathcal{C}\psi),\; \mathcal{C} = K\Sigma_1$$

$$H_{\mathrm{BdG}} = U \begin{pmatrix} E_1 & & & \\ & \ddots & & \\ & & -E_1 & \\ & & & \ddots \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} | & & | & \\ \psi_{E_1} & \cdots & \mathcal{C}\psi_{E_1} & \cdots \\ | & & | & \\ | & & | & \end{pmatrix}$$

$$H = \tfrac{1}{2}(\mathbf{c}^\dagger \;\; \mathbf{c}) U \,\mathrm{diag}(E_j \ldots, -E_j, \ldots)\, U^\dagger \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix}$$

$$(\boldsymbol{\alpha}^\dagger_{E>0} \;\; \boldsymbol{\alpha}^\dagger_{E<0}) = (\mathbf{c}^\dagger \;\; \mathbf{c}) U$$

Leads to an apparently unusual form (note the 1/2, negative energies):

$$H = \frac{1}{2}(\boldsymbol{\alpha}^\dagger_{E>0} \;\; \boldsymbol{\alpha}^\dagger_{E<0}) \begin{pmatrix} E_1 & & & \\ & \ddots & & \\ & & -E_1 & \\ & & & \ddots \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_{E>0} \\ \boldsymbol{\alpha}_{E<0} \end{pmatrix} = \frac{1}{2}\sum_j E_j \alpha^\dagger_j \alpha_j$$

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$$(\boldsymbol{\alpha}^\dagger_{E>0} \;\; \boldsymbol{\alpha}^\dagger_{E<0}) = (\mathbf{c}^\dagger \;\; \mathbf{c}) U$$

$$\Rightarrow \quad \alpha^\dagger_{E_1} = (\mathbf{c}^\dagger \;\; \mathbf{c}) \cdot \psi_{E_1}, \quad \alpha^\dagger_{-E_1} = (\mathbf{c}^\dagger \;\; \mathbf{c}) \cdot \underbrace{\Sigma_1 \psi^*_{E_1}}_{\mathcal{C}\psi_{E_1}} = (\mathbf{c} \;\; \mathbf{c}^\dagger) \cdot \psi^*_{E_1}$$

Redundancy relation:

$$\boxed{\alpha^\dagger_{-E_1} = \alpha_{E_1}}$$

Hamiltonian diagonalises to the usual form:

$$\begin{aligned} H = \frac{1}{2}\sum_j E_j \alpha^\dagger_{E_j}\alpha_{E_j} &= \frac{1}{2}\sum_{E_j>0} E_j \left(\alpha^\dagger_{E_j}\alpha_{E_j} - \alpha^\dagger_{-E_j}\alpha_{-E_j}\right) \\ &= \frac{1}{2}\sum_{E_j>0} E_j \left(\alpha^\dagger_{E_j}\alpha_{E_j} - \alpha_{E_j}\alpha^\dagger_{E_j}\right) = \sum_{E_j>0} E_j \alpha^\dagger_{E_j}\alpha_{E_j} + \mathrm{const} \end{aligned}$$

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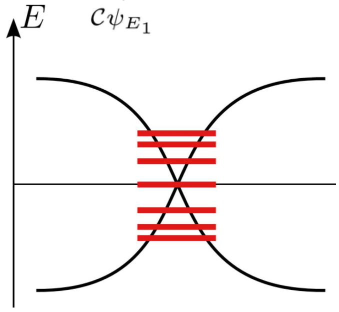
For zero modes this suggests:

$$\boxed{\alpha^\dagger_{E=0} = \alpha_{E=0}}$$



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(Locally) nondegenerate zero mode:

$$\mathcal{C}\psi_0 = e^{i\chi}\psi_0 \quad \Rightarrow \quad e^{-i\chi/2}\mathcal{C}\psi_0 = \mathcal{C}e^{i\chi/2}\psi_0 = e^{i\chi/2}\psi_0$$

Can choose: $\boxed{\mathcal{C}\psi_0 = \psi_0}$

$$\gamma = (\mathbf{c}^\dagger \;\; \mathbf{c}) \cdot \psi_0 = (\mathbf{c}^\dagger \;\; \mathbf{c}) \cdot \underbrace{\Sigma_1 \psi^*_0}_{\mathcal{C}\psi_0} = (\mathbf{c} \;\; \mathbf{c}^\dagger) \cdot \psi^*_0 = \gamma^\dagger$$

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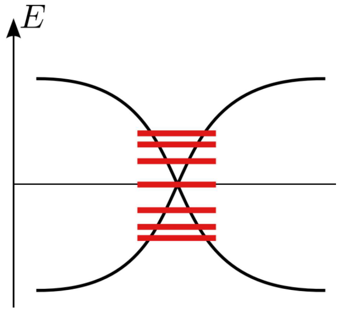

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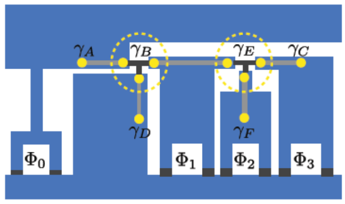
$$\boxed{\gamma = \gamma^\dagger}$$

With more spatially separated zero modes:

$$\psi_0^{(j)} \to \gamma_j, \;\; \langle \psi_0^{(j)}, \psi_0^{(k)} \rangle = \delta_{jk} \;\; \Rightarrow \;\; \boxed{\{\gamma_j, \gamma_k\} = 2\delta_{jk}}$$

(Locally) nondegenerate zero mode in superconductor: guaranteed to be Majorana mode



[Beenakker group. PRB 2013]

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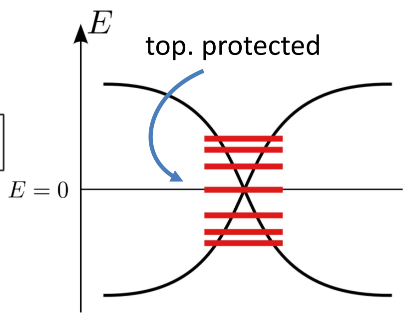

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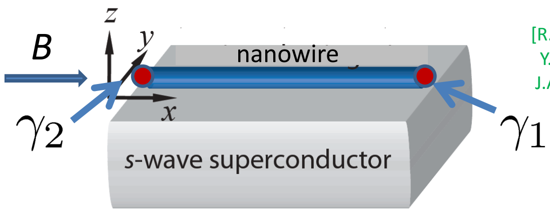
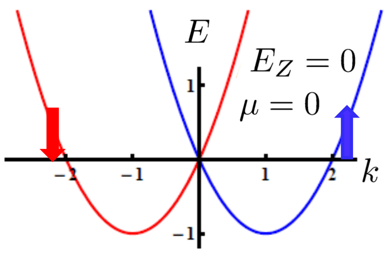
Nanowire realisation



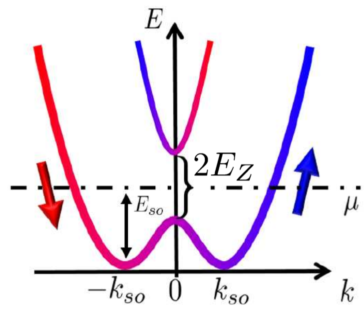
[R. Lutchyn *et al.* PRL 2010,
Y. Oreg *et al.* PRL 2010,
J.Alicea *et al.* Nat. Phys. 2011]

$$H_{\mathrm{BdG}} = \left(\frac{p^2}{2m} - \mu\right)\Sigma_3 + \alpha p \sigma_3 \Sigma_3 + E_Z \sigma_1 + \Delta \Sigma_1$$

$$H_0 = \frac{p^2}{2m} - \mu + \alpha p \sigma_3 + E_Z \sigma_1$$



[Adapted from: Oreg *et al.* Phys. Rev. Lett. 2010]



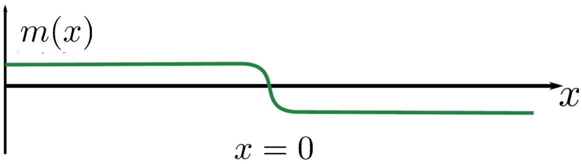
[Adapted from: Lutchyn *et al.* Nat. Rev. Mater. 2018]

Jackiw-Rebbi-type picture

Linear (Dirac/Majorana) gap closing described by

$$H_{\text{eff}} = -iv\partial_x\tau_1 + m\tau_3 \qquad E = \pm\sqrt{(vk)^2 + m^2} \qquad \tau_1(H_{\text{eff}})^*\tau_1 = -H_{\text{eff}}$$

Consider an interface across which the gap parameter changes sign:



$$H_{\text{eff}} = -iv\partial_x\tau_1 + m(x)\tau_3$$

Jackiw-Rebbi: interface binds a zero mode.

$$H_{\text{eff}}\Psi = 0$$

$$i\partial_x\tau_1\Psi = \mu(x)\tau_3\Psi \qquad \mu(x) = m/v$$

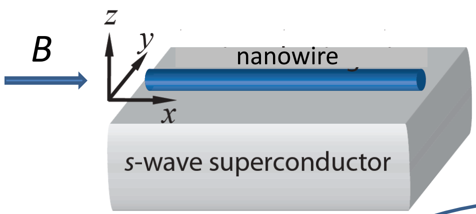
$$\Rightarrow \quad \partial_x\Psi = -i\tau_1\tau_3\mu(x)\Psi = -\tau_2\mu(x)\Psi$$

$$\Psi = e^{-\int_0^x \mu(x')dx'\tau_2}\Psi(0) \Rightarrow \Psi = e^{\mp\int_0^x \mu(x')dx'}\psi_\pm \qquad (\tau_2\psi_\pm = \pm\psi_\pm)$$

The convergent one for the profile above: $\Psi = e^{\int_0^x \mu(x')dx'}\psi_-$

NB: exponentially localised to interface

Nanowire realisation



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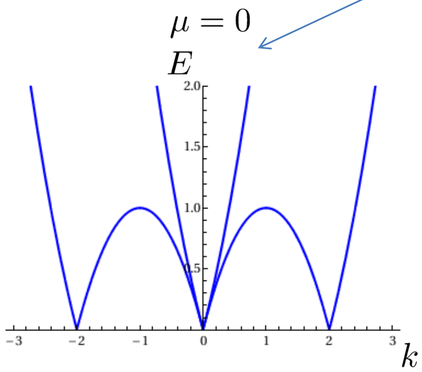
$$H_{\text{BdG}} = \left(\frac{p^2}{2m} - \mu\right)\Sigma_3 + \alpha p\sigma_3\Sigma_3 + E_Z\sigma_1 + \Delta\Sigma_1$$



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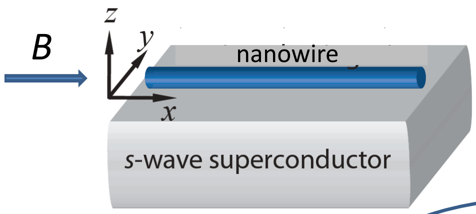

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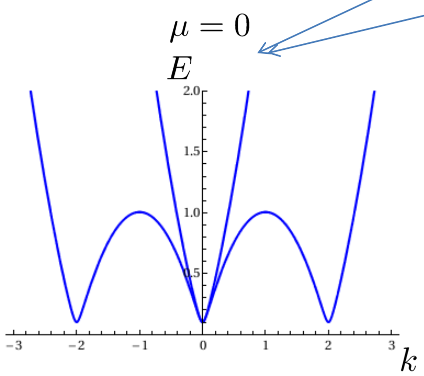
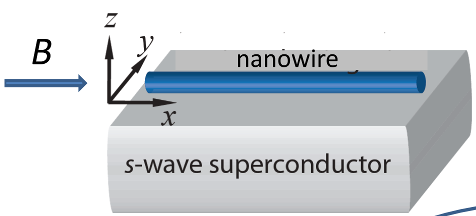


Gapped but top. trivial:

- TR inv ➡ levels degenerate

Nanowire realisation



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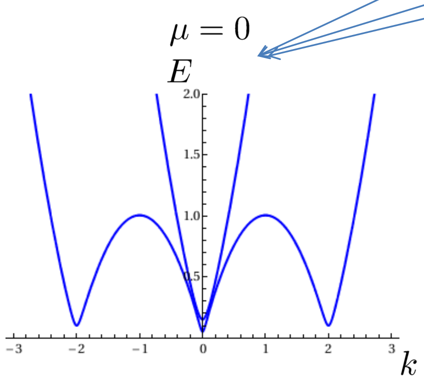

Zeeman breaks TR invariance.
Top. regime? Look for linear gap closing.

$$H(p=0) = -\mu\Sigma_3 + E_Z\sigma_1 + \Delta\Sigma_1$$

➡ $$E = \pm\left(\sqrt{\Delta^2+\mu^2} \pm' |E_Z|\right)$$

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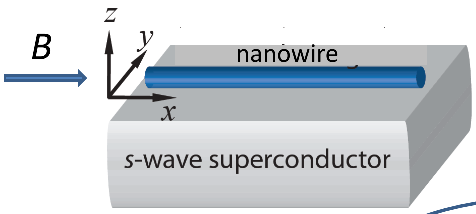

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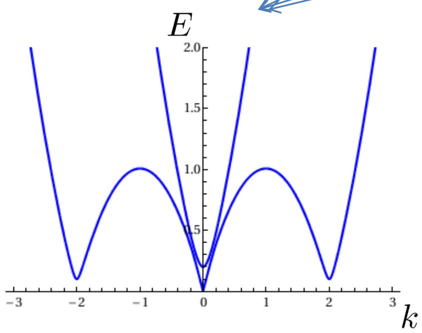

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Gap closing @ $|E_Z| = \sqrt{\Delta^2+\mu^2}$

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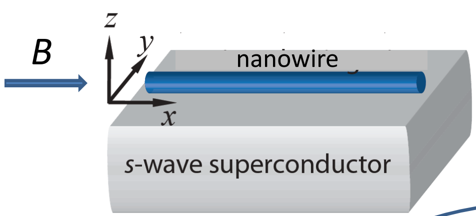

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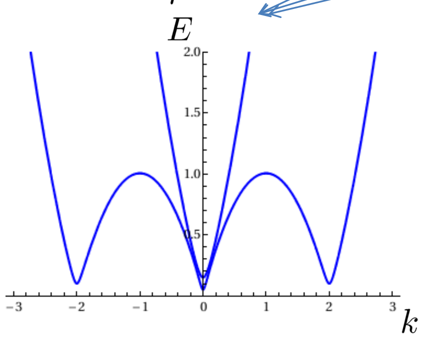

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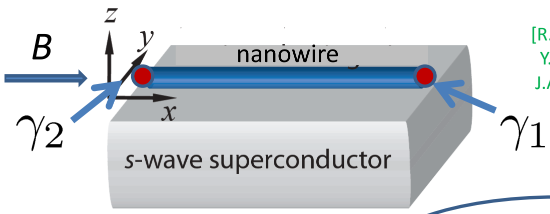
Top. regime? Look for linear gap closing.

$$H(p=0) = -\mu\Sigma_3 + E_Z\sigma_1 + \Delta\Sigma_1$$

$$\Rightarrow E = \pm\left(\sqrt{\Delta^2+\mu^2} \pm' |E_Z|\right)$$

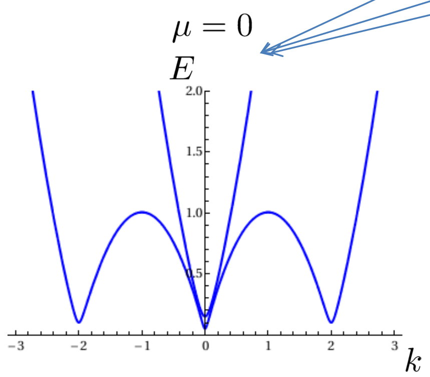
$$H_{\mathrm{eff}} = vp\tau_1 + \left(\sqrt{\Delta^2+\mu^2} - |E_Z|\right)\tau_3$$

Nanowire realisation



[R. Lutchyn *et al.* PRL 2010,
Y. Oreg *et al.* PRL 2010,
J.Alicea *et al.* Nat. Phys. 2011]

$$H_{\rm BdG} = \left(\frac{p^2}{2m} - \mu\right)\Sigma_3 + \alpha p \sigma_3 \Sigma_3 + E_Z \sigma_1 + \Delta \Sigma_1$$



Zeeman breaks TR invariance.
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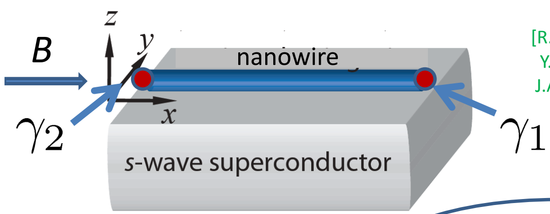
$|E_Z| > \sqrt{\Delta^2 + \mu^2}$: topological phase

Nanowire realisation



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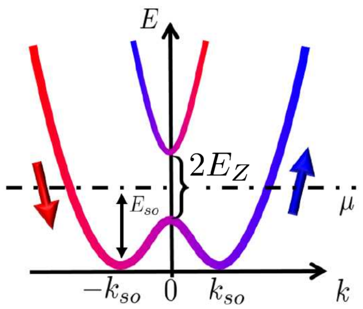

[Adapted from: Lutchyn *et al.* Nat. Rev. Mater. 2018]

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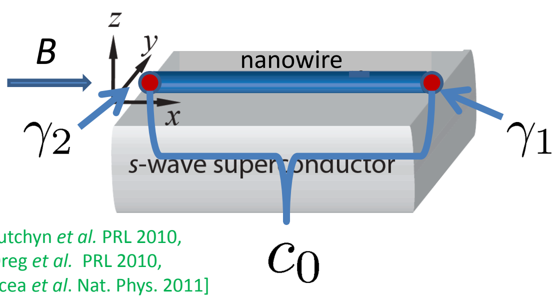
Majorana fermions: key features



[R. Lutchyn *et al.* PRL 2010,
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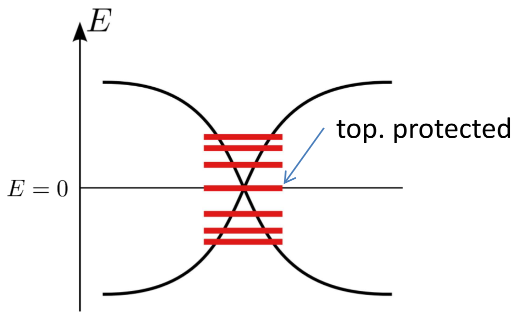


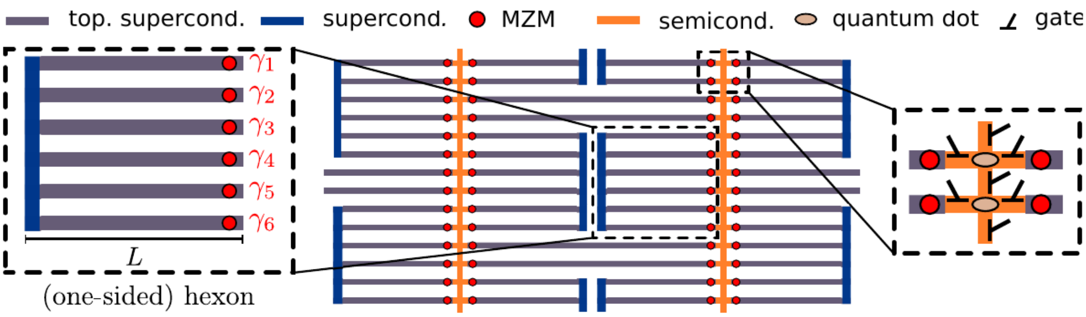
$$\gamma_j = \gamma_j^\dagger, \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$$c_0 = \tfrac{1}{2}(\gamma_1 + i\gamma_2)$$ (recall: Majoranas as Hermitean and anti-Hermitean parts of fermions)

More generally:
1 fermion per 2 Majorana; system of ordinary fermions

⇒ Majoranas must come in pairs!

Majorana fermions in nanodevices: envisioned applications



[T. Karzig *et al*. PRB 2017]

$$\gamma_j = \gamma_j^\dagger, \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$$c_0 = \tfrac{1}{2}(\gamma_1 + i\gamma_2)$$

- $c_0^\dagger$ costs no energy ➡ **topological** GS degeneracy
- $|0\rangle, |1\rangle$ topological qubit
- more Majoranas ➡ qubit operations

[Bravyi&Kitaev,,Ann.Phys 2002, D.A.Ivanov PRL 2001]

Envisioned applications: some underlying principles

Preliminary considerations:

- Groundstate degeneracy for N Majoranas:

  N Majoranas; 1 fermion per pair ➡ *N/2* zero energy fermions

  ➡ $2^{N/2}$- fold GS degeneracy

  However, overall parity is conserved (in a closed system)

  ➡ $2^{N/2-1}$ - fold degenerate space to operate on

- Fermion parity in terms of Majoranas: parity of the pair i, j:

$$c_{ij} = \tfrac{1}{2}(\gamma_i + i\gamma_j) \quad ➡ \quad \pi_{ij} = 1 - 2c_{ij}^\dagger c_{ij} = i\gamma_j\gamma_i$$

  - Overall fermion parity:

$$\Pi_{\text{tot}} = \pi_{12}\pi_{34}\ldots\pi_{N-1,N}$$

NB: even (odd) products of Majoranas preserve (flip) overall parity

Envisioned applications: some underlying principles

Majorana advantages include:

1) **Topologically protected information storage**:

- low energy (subgap), fermion parity conserving operators

$$\mathcal{O} = \sum_{ij} a_{ij}\gamma_j\gamma_j + \sum_{ijkl} b_{ijkl}\gamma_i\gamma_j\gamma_k\gamma_l + \ldots \text{(even powers)}$$

- low energy, **local**, fermion parity conserving operators

$$\mathcal{O}_{\text{loc}} = \sum_l a_l\gamma_l^2 + b_l\gamma_l^4 \propto \mathbb{I}$$

➡ resilience against local, parity conserving, perturbations

2) **Topologically protected gates** (though not universal set), e.g., via **non-Abelian statistics**: exchanging Majorana *i* and *j* implements

$$|GS\rangle \to \frac{1}{\sqrt{2}}(1 \pm \gamma_i\gamma_j)|GS\rangle$$

Non-Abelian statistics

Exchanging Majorana *i* and *j* implements

$$|GS\rangle \to \frac{1}{\sqrt{2}}(1 \pm \gamma_i\gamma_j)|GS\rangle$$

How does his come about and what is non-Abelian about it?

- Exchanging γ_i and γ_j:

$$U_{ij}^\dagger\gamma_i U_{ij} = \eta_j\gamma_j, \quad U_{ij}^\dagger\gamma_j U_{ij} = \eta_i\gamma_i \qquad (1)$$

Most general unitary involving only γ_i and γ_j:

$$U_{ij} = a + b\ \gamma_i\gamma_j, \quad |a|^2 + |b|^2 = 1, \quad a^*b = b^*a \qquad (2)$$

(1) & (2) ➡ $U_{ij}^\pm = \frac{e^{i\alpha_{ij}}}{\sqrt{2}}(1 \pm \gamma_i\gamma_j)\,, \qquad \eta_j = -\eta_i = \pm 1$

- Non-Abelian because successive exchanges do not commute:

$$U_{12}^\pm U_{23}^\pm \neq U_{23}^\pm U_{12}^\pm$$

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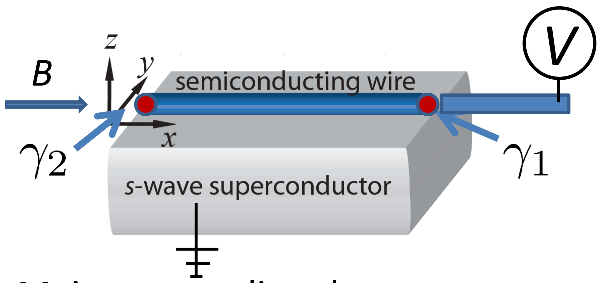
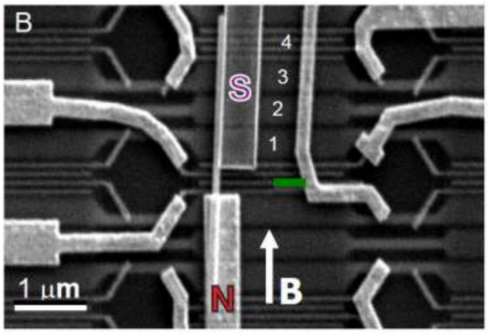
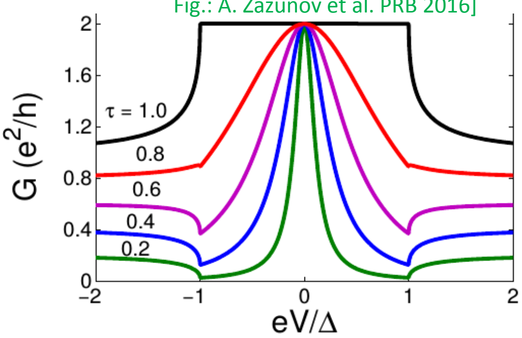
Majorana fermions in nanodevices: first signatures: zero energy nature



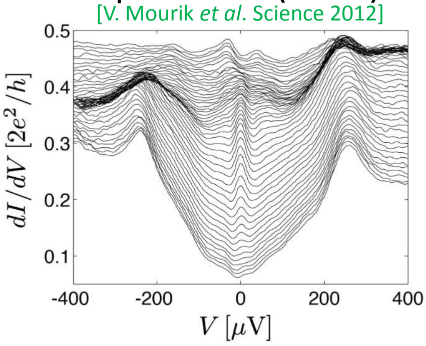


Majorana mediated resonant transport (resonant Andreev reflection)

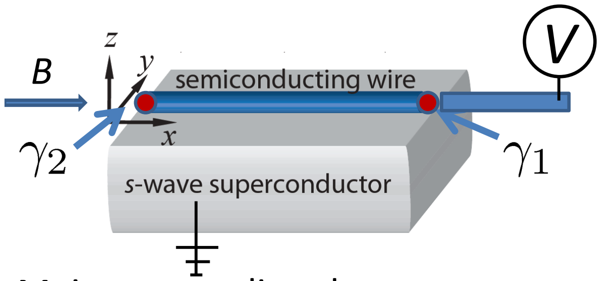
Theory [K. T. Law & P. A. Lee, PRL 2009; Wimmer *et al.* NJP 2011; Fig.: A. Zazunov et al. PRB 2016]

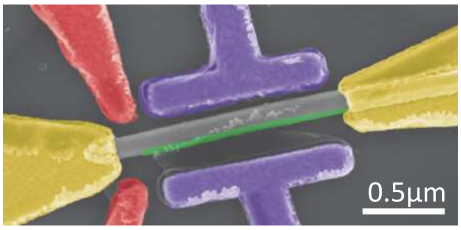


Experiment (2012) [V. Mourik *et al.* Science 2012]



Majorana fermions in nanodevices: zero energy nature – recent demonstration





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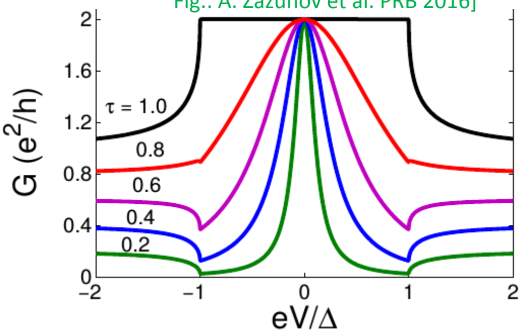
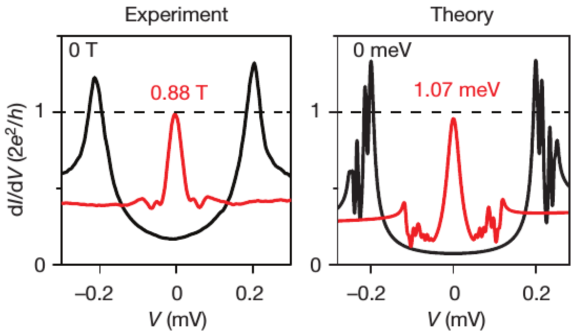


Experiment (2017)

[H. Zhang et al., Nature 2018]

$2e^2/h$ peak finally seen:



Majorana fermions in nanodevices: some of the confirmed features

Zero energy nature via $2e^2/h$ conductance peak in hard gap

[H. Zhang et al., Nature 2018 (Kouwenhoven group); F. Nichele et al. PRL 2018 (Marcus group)]

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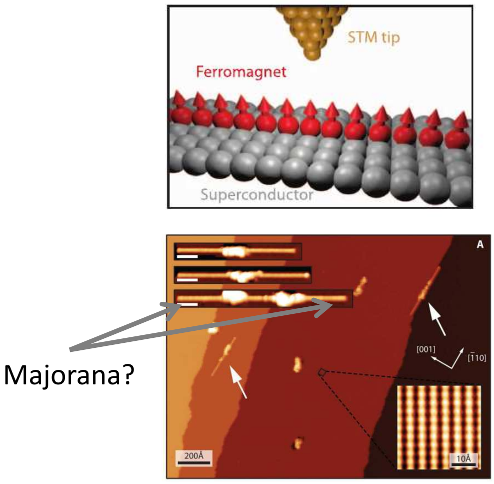
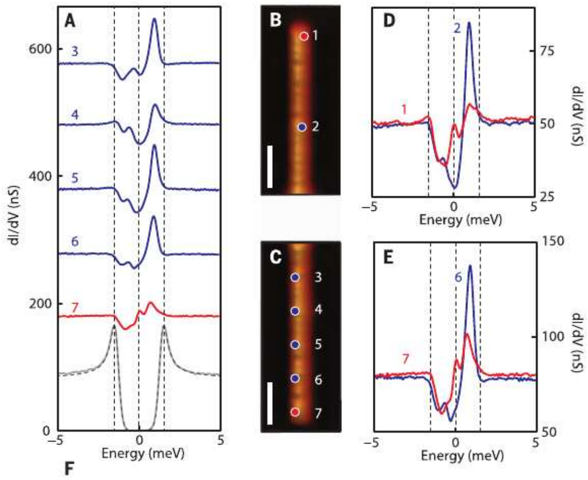
Localised end-mode nature of state





[S. Nadj-Perge *et al*., Science, 2014]

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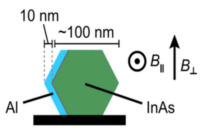
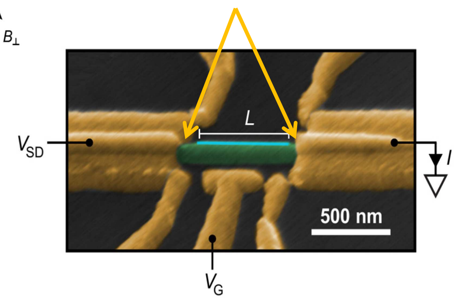
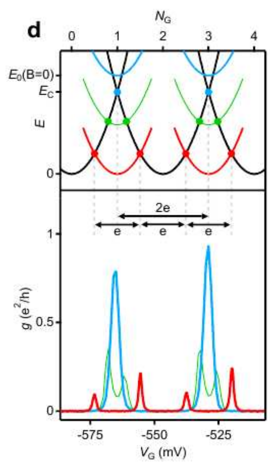
Exponential protection against level splitting
[S. M. Albrecht *et al*. Nature 2016]







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Exponential protection against level splitting

[S. M. Albrecht *et al*. Nature 2016]

… all via various forms of conductance measurements

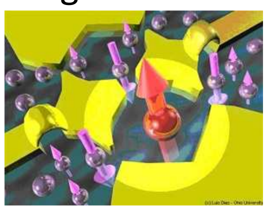
But yet untested: **nonlocal topological qubit**

… can one see this via conductance?

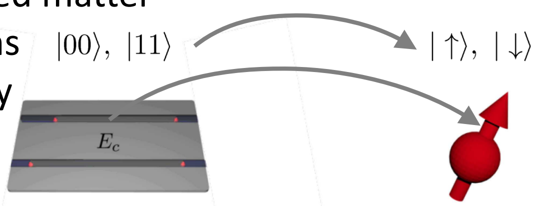
Topological Kondo: idea in 1 slide

[BB, N. R. Cooper, PRL **109**, 156803 (2012)]

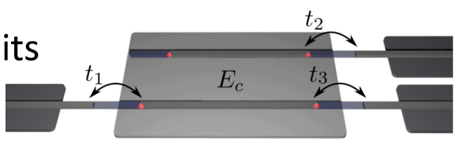
- Fact 1: conduction electrons + quantum spin with degenerate levels
  - ⇨ **Kondo effect**
    - key paradigm in strong correlations
    - arising from the q. dynamics of a spin qubit



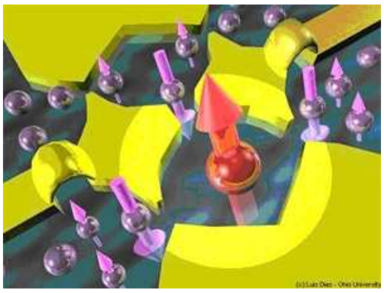
- Fact 2: Majorana fermions in condensed matter
  - ⇨ **topological qubits** $\sim$nonlocal spins
    - level degeneracy $\sim$top. degeneracy



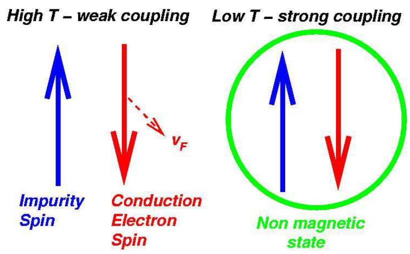
- Idea: coupling conduction electrons to topological qubits?
  - ⇨ **topological Kondo effect**
    - Majorana induced strong correlations
    - demonstrates q. dynamics of top. qubits via transport



Kondo reminder



[fig:Luis Dias/Ohio University]



[fig:Wikipedia]

$$H_K = J\mathbf{S}_{\rm im}\cdot\mathbf{S}_{\rm cond}(0)$$

Scattering of conduction electrons on a spin with degenerate levels
- spin often “effective” spin (e.g., quantum dot/island)
- strong correlation paradigm: “asymptotic freedom”
- non-Fermi liquid behaviour possible (e.g., multichannel Kondo)

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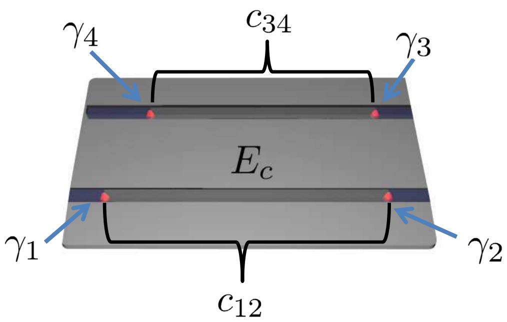
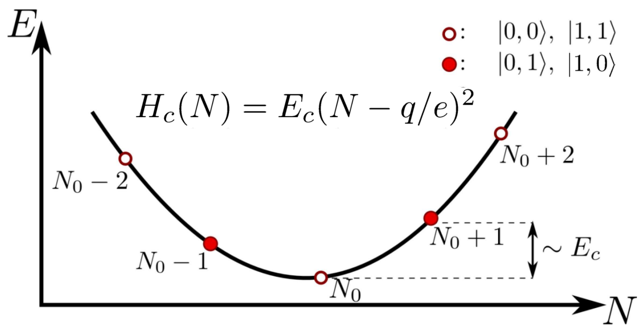
From Majoranas to Kondo: effective spin

top. degeneracy ➡ "spin" degeneracy

top. qubit ➡ nonlocal spin





4 Majoranas:

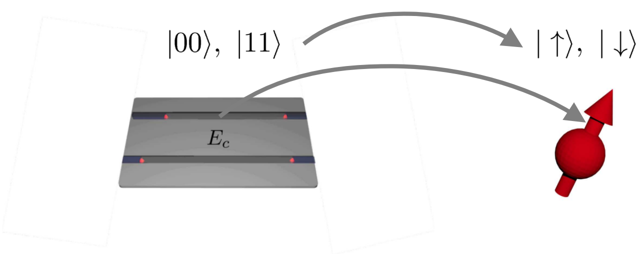
➡ GS twofold degenerate $|n_{12}, n_{34}\rangle = |0,0\rangle,\ |1,1\rangle \equiv |\uparrow\rangle,\ |\downarrow\rangle$

Considerations also apply to setups with TIs; Fe adatom chains, etc.

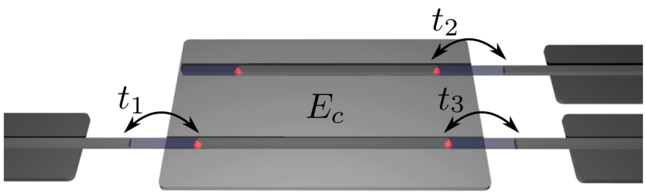
[Mourik *et al.* Science (2012); I. Knez *et al.* PRL (2012); S. Nadj-Perge *et al.*, Science, 2014; S. M. Albrecht *et al*. Nature, 2016]

From Majoranas to Kondo

We have our effective spin



Kondo: couple this to conduction electrons

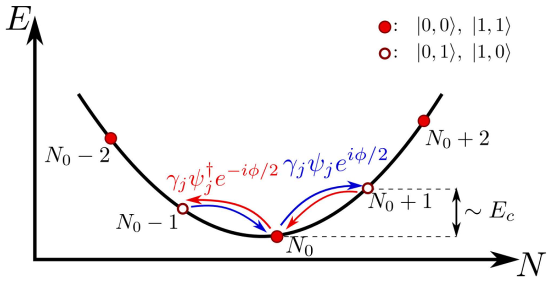


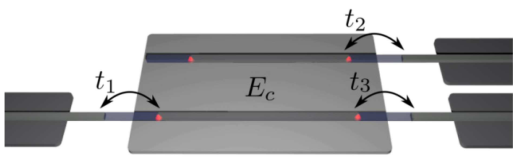
Topological Kondo effect

spinless

$$H = H_{\text{lead}} + H_c(N) + \sum_j t_j\, \gamma_j e^{i\phi/2}\, \psi_j + \text{H.c.}$$

($e^{\pm i\phi/2}|N\rangle = |N \pm 1\rangle$) [L. Fu PRL 2010]



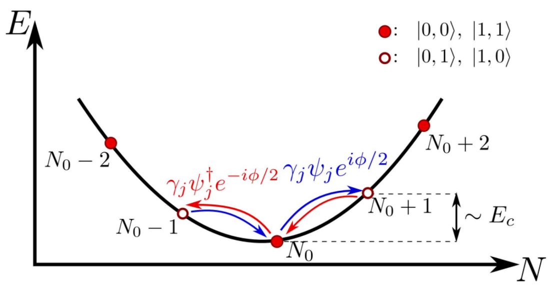


Topological Kondo effect: lead-island term

spinless

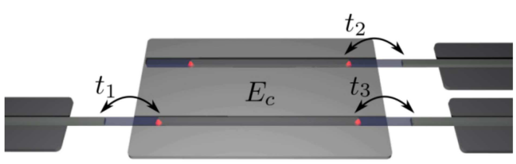
$$H = H_{\text{lead}} + H_c(N) + \sum_j t_j\, \gamma_j e^{i\phi/2}\, \psi_j + \text{H.c.}$$

($e^{\pm i\phi/2}|N\rangle = |N \pm 1\rangle$) [L. Fu PRL 2010]



Tunneling at j-th lead: $H_T^{(j)} = \tau_j \psi_S^\dagger(x_j)\psi_j + \text{H.c.}$

$\psi_S^\dagger(x_j)$ in terms of excitations?



Recall: BdG vs electron operators

$$H = \frac{1}{2}(\mathbf{c}^\dagger \;\; \mathbf{c}) H_{\rm BdG} \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix}$$

$$H_{\rm BdG} = U \begin{pmatrix} E_1 & & & \\ & \ddots & & \\ & & -E_1 & \\ & & & \ddots \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} \psi_{E_1} \Big| & \cdots & \Big| \mathcal{C}\psi_{E_1} \Big| & \cdots \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{\alpha}_{E\geq 0} \\ \boldsymbol{\alpha}_{E<0} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_{E>0} \\ \boldsymbol{\gamma} \\ \boldsymbol{\alpha}^\dagger_{E>0} \end{pmatrix} = U^\dagger \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \mathbf{c} \\ \mathbf{c}^\dagger \end{pmatrix} = U \begin{pmatrix} \boldsymbol{\alpha}_{E>0} \\ \boldsymbol{\gamma} \\ \boldsymbol{\alpha}^\dagger_{E>0} \end{pmatrix}$$

$$\boldsymbol{\psi}_{E_l>0} = \begin{pmatrix} \boldsymbol{u}_{E_l} \\ \boldsymbol{v}_{E_l} \end{pmatrix} \qquad \boldsymbol{\psi}_{E_l=0} = \begin{pmatrix} \boldsymbol{\xi}_l \\ \boldsymbol{\xi}^*_l \end{pmatrix}$$

$$c_j = \sum_l U_{jl} \begin{pmatrix} \boldsymbol{\alpha}_{E>0} \\ \boldsymbol{\gamma} \\ \boldsymbol{\alpha}^\dagger_{E>0} \end{pmatrix}_l = \sum_{E_l=0} (\xi_l)_j \gamma_l + \sum_{E_l>0} \left[(u_{E_l})_j \alpha_{E_l} + (v^*_{E_l})_j \alpha^\dagger_{E_l} \right]$$

$$\longrightarrow \sum_l \xi_l(x_j)\gamma_l + \sum_{E_l>0} \left[u_{E_l}(x_j)\alpha_{E_l} + v^*_{E_l}(x_j)\alpha^\dagger_{E_l} \right] \equiv \sum_l \xi_l(x_j)\gamma_l + c_>(x_j)$$

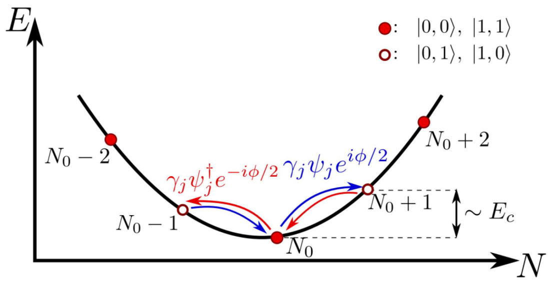
Topological Kondo effect: lead-island term

spinless

$$H = H_{\rm lead} + H_c(N)$$

$$+ \sum_j t_j \, \gamma_j e^{i\phi/2} \, \psi_j + \text{H.c.}$$

$(e^{\pm i\phi/2}|N\rangle = |N \pm 1\rangle)$ [L. Fu PRL 2010]



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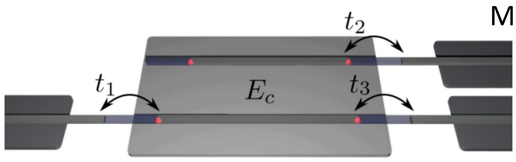
$\psi_S^\dagger(x_j)$ in terms of excitations?

$$\psi_S^\dagger(x_j) = e^{i\phi/2}[\textstyle\sum_l \xi_l^*(x_j)\gamma_l + \psi^\dagger_{S,>}(x_j)]$$

Majorana wavefn. Part with operators above the gap



Majoranas exponentially localized:

$$\xi_l(x_j) \approx \xi_j(x_j)\delta_{lj}$$

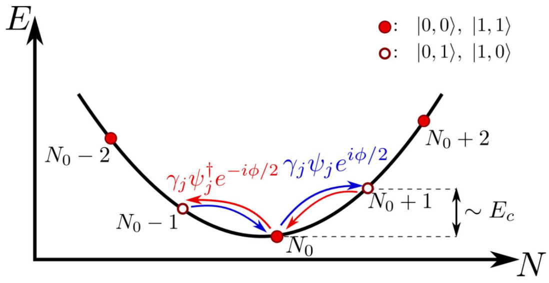
working much below the gap ➡ $\psi_S^\dagger(x_j) \approx e^{i\phi/2}\xi_j^*(x_j)\gamma_j$

Topological Kondo effect

spinless

$$H = H_{\rm lead} + H_c(N) + \sum_j t_j \, \gamma_j e^{i\phi/2} \, \psi_j + \text{H.c.}$$

($e^{\pm i\phi/2}|N\rangle = |N \pm 1\rangle$) [L. Fu PRL 2010]



- **Kondo regime** $T, V, t_j \ll E_c$: virtual transitions $N_0 \leftrightarrow N_0 \pm 1$

➡ **effective Hamiltonian** in the space spanned by $|m\rangle = |\sigma\rangle|\Psi\rangle$

top. qubit state lead state

NB above is shorthand for state with N_0 ; more generally:

$$|j\rangle = |n_{12}, n_{34}; N; \Psi\rangle \qquad n_{12} + n_{34} = N \bmod 2$$

Topological Kondo effect

spinless

$$H = H_{\rm lead} + H_c(N) + \sum_j t_j \, \gamma_j e^{i\phi/2} \, \psi_j + \text{H.c.}$$

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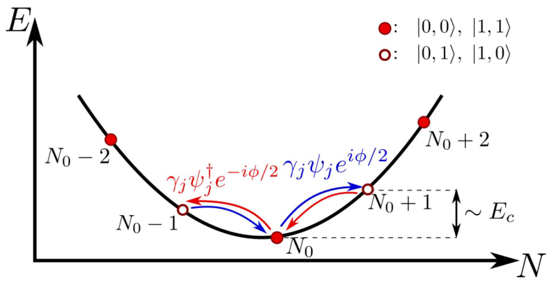

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top. qubit state lead state

from 2nd order p.t. (Schrieffer-Wolff):

$$H = H_0 + V, \quad H_0|j\rangle = E_j|j\rangle, \quad \langle j|V|j\rangle = 0 \quad \Rightarrow \quad H_{\rm eff} = H_0 + \delta H$$

$$\langle m|\delta H|n\rangle = -\frac{1}{2}\left[\sum_{j\neq m} \frac{\langle m|V|j\rangle\langle j|V|n\rangle}{E_j - E_m} + \sum_{j\neq n} \frac{\langle m|V|j\rangle\langle j|V|n\rangle}{E_j - E_n}\right]$$

simplifies if $E_j - E_{m,n} \gg |E_m - E_n|$

here: $\sim E_c \gg \sim \delta\varepsilon_{\rm leads} \sim T, V$

Topological Kondo effect

spinless

$$H = H_{\text{lead}} + H_c(N)$$
$$+ \sum_j t_j\, \gamma_j e^{i\phi/2}\, \psi_j + \text{H.c.}$$

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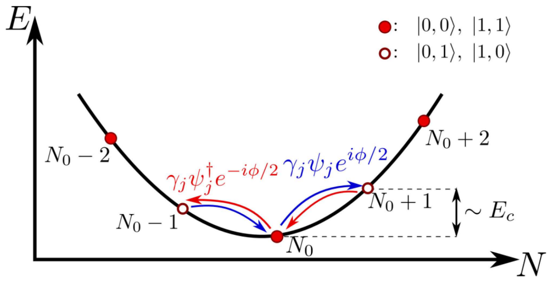

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If $E_j - E_{m,n} \gg |E_m - E_n|$

$$\Rightarrow \quad H_{\text{eff}} = H_0 - \sum_{E_j \neq E_m} \frac{V|j\rangle\langle j|V}{E_j - E_m}$$

Topological Kondo effect

spinless

$$H = H_{\text{lead}} + H_c(N)$$
$$+ \sum_j t_j\, \gamma_j e^{i\phi/2}\, \psi_j + \text{H.c.}$$

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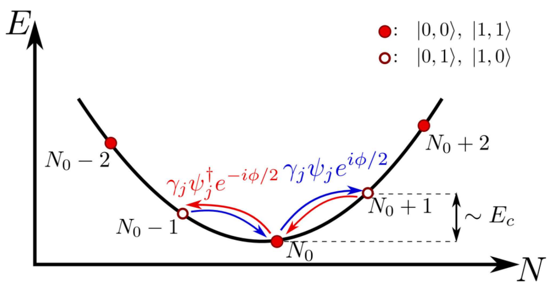
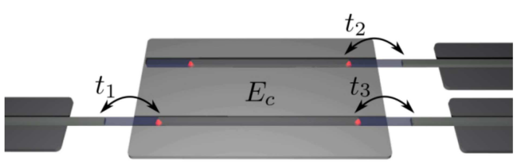


- **Kondo regime** $T, V, t_j \ll E_c$: virtual transitions $N_0 \leftrightarrow N_0 \pm 1$

$$H_{\text{eff}} = H_{\text{lead}} - \tfrac{1}{U_+} \sum_{ij} t_i t_j \psi_i^\dagger \gamma_i \gamma_j \psi_j - \tfrac{1}{U_-} \sum_{ij} t_i t_j \gamma_j \psi_j \psi_i^\dagger \gamma_i$$

$(U_\pm \approx H_c(N_0 \pm 1) - H_c(N_0) > 0)$



$$H_{\text{eff}} = H_{\text{lead}} + \sum_{i\neq j} \lambda_{ij}^+ \gamma_j \gamma_i \psi_i^\dagger \psi_j - \sum_i \lambda_{ii}^- \psi_i^\dagger \psi_i$$

$$\left(\lambda_{ij}^\pm = \left(\tfrac{1}{U_+} \pm \tfrac{1}{U_-}\right) t_i t_j \sim \tfrac{t_i t_j}{E_c}\right)$$

Topological Kondo effect

$$H_{\rm eff} = H_{\rm lead} + \sum_{i\neq j} \lambda^+_{ij}\gamma_j\gamma_i\psi_i^\dagger\psi_j - \sum_i \lambda^-_{ii}\psi_i^\dagger\psi_i, \qquad \left(\lambda^\pm_{ij}\sim\frac{t_it_j}{E_c}\right).$$

3 Majoranas ➡ σ_α : $\quad \sigma_1 = -i\gamma_2\gamma_3, \quad \sigma_2 = -i\gamma_3\gamma_1, \quad \sigma_3 = -i\gamma_1\gamma_2$

↳ 3 leads

$$\left.\begin{array}{r}\lambda^+_{12} = \lambda^+_{21}\\ \gamma_1\gamma_2 = -\gamma_2\gamma_1\end{array}\right\} \lambda^+_{12}\gamma_1\gamma_2(\psi_2^\dagger\psi_1 - \psi_1^\dagger\psi_2) = \lambda^+_{12}\sigma_3\, i(\psi_2^\dagger\psi_1 - \psi_1^\dagger\psi_2)$$

$$i(\psi_2^\dagger\psi_1 - \psi_1^\dagger\psi_2) = \boldsymbol{\psi}^\dagger J_3\boldsymbol{\psi} = \hat{J}_3 \qquad J_3 = i\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \qquad \boldsymbol{\psi} = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}$$

spin-1 generator

↳ $\hat{J}_3$: spin-1 density

$$\lambda^+_{12}\gamma_1\gamma_2(\psi_2^\dagger\psi_1 - \psi_1^\dagger\psi_2) = \lambda^+_{12}\sigma_3\hat{J}_3$$

Topological Kondo effect

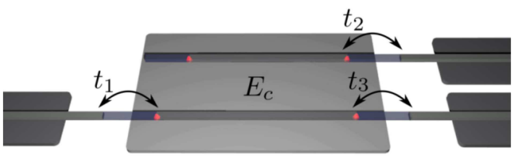
$$H_{\rm eff} = H_{\rm lead} + \frac{1}{2}\sum_\alpha \lambda_\alpha\sigma_\alpha\hat{J}_\alpha - \sum_i \lambda^-_{ii}\psi_i^\dagger\psi_i, \qquad \left(\lambda_\alpha = \sum_{ab}|\varepsilon_{\alpha ab}|\lambda^+_{ab}\right).$$

$\hat{J}_\alpha$ ← $\boldsymbol{\psi}^\dagger J_\alpha\boldsymbol{\psi}$

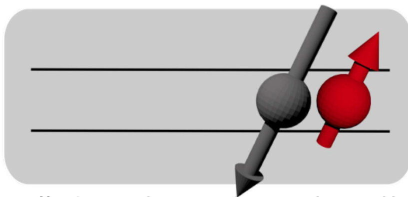
Antiferromagnetic Kondo

- $S = \frac{1}{2}$ impurity coupled to
- $S = 1$ conduction electrons.

[Fabrizio&Gogolin PRB 1994, Sengupta&Kim, PRB 1996]



- Overscreened Kondo



- "Spin" distributed nonlocally
  ➡ curious transport features
  (e.g., G_{ij} ⬄ Kondo anisotropy)
- 3 leads is minimal for Kondo:
  ➡ "smoking gun" for non-locality

Outline

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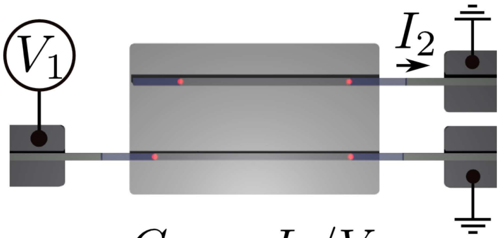
  - transport signatures, incl. NFL features
  - topological Kondo beyond the minimal setup
  - (exact) scaling functions for nonequilibrium transport

Transport signatures



$$G_{21} = I_2/V_1$$

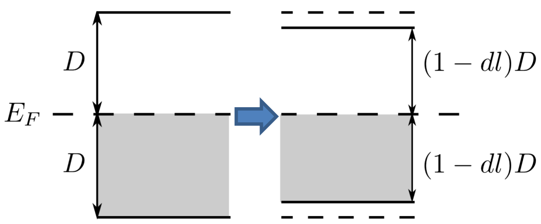
Focus on T-dep. of linear conductance

$$H_{\rm eff} = H_{\rm lead} + \sum_{i\neq j} \lambda^+_{ij}\gamma_j\gamma_i\psi^\dagger_i\psi_j - \sum_i \lambda^-_{ii}\psi^\dagger_i\psi_i$$

$$G_{21}(T) \sim (\lambda^+_{12})^2 + \dots$$

Log-singularities $\sim \left[\ln\left(\frac{D}{T}\right)\right]^n$

Sum up by RG



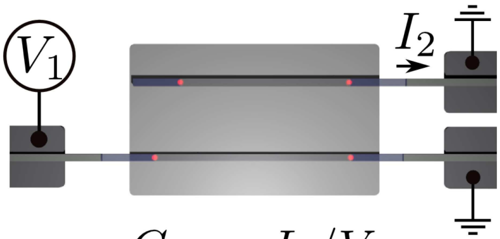
Standard Kondo flow:

$$\frac{d\lambda_1}{dl} = \rho\lambda_2\lambda_3, \quad \text{cycl. perm.,}$$

$$\left(\lambda_\alpha = \sum_{ab} |\varepsilon_{\alpha ab}|\lambda^+_{ab}\right)$$

$$\frac{d\lambda^-_{kk}}{dl} = 0$$

Transport signatures



$G_{21} = I_2/V_1$

Focus on T-dep. of linear conductance

$$H_{\rm eff} = H_{\rm lead} + \sum_{i\neq j} \lambda^+_{ij}\gamma_j\gamma_i\psi^\dagger_i\psi_j - \sum_i \lambda^-_{ii}\psi^\dagger_i\psi_i$$

$$G_{21}(T) \sim (\lambda^+_{12})^2 + \ldots$$

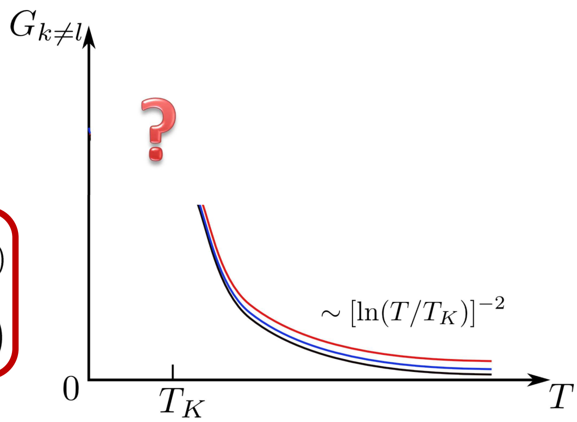
Log-singularities $\sim \left[\ln\left(\frac{D}{T}\right)\right]^n$

Conductance from RG:

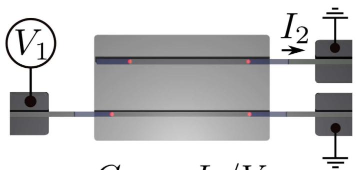
$$\lambda^+(D) \sim \frac{1}{\ln(D/T_{\rm K})} \qquad \lambda^+_\alpha/\lambda^+_\beta \to 1$$

$$G_{21}(T) \sim \frac{1}{\ln^2(T/T_{\rm K})} \quad (T \gg T_{\rm K})$$

$$G_{21}/G_{31} \to 1 \quad \left(T_{\rm K} \sim E_c e^{-1/\rho\bar\lambda}\right)$$



Majorana-Klein hybridization



$G_{21} = I_2/V_1$

[B. Béri, PRL **110**, 216803 (2013)]

Bosonization approach

$$H^{(j)}_{\rm lead} = H_0(\rho_j, \theta_j) \qquad [\rho(x), \theta(y)] = i\delta(x-y)$$

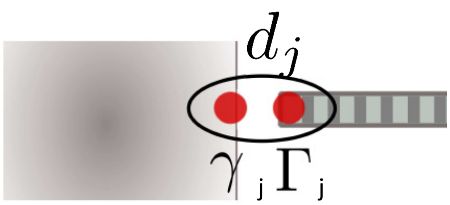
$\psi_j(0) \sim \Gamma_j e^{i\theta_j(0)}$ Klein factors = **auxiliary Majoranas**

Coupling:

$$\gamma_j\psi_j \sim \gamma_j\Gamma_j e^{i\theta_j(0)}$$

Parity of hybrid fermion $d_j = \frac{1}{2}(\gamma_j + i\Gamma_j)$

$$\pi_j = 1 - 2d^\dagger_j d_j = -i\gamma_j\Gamma_j$$



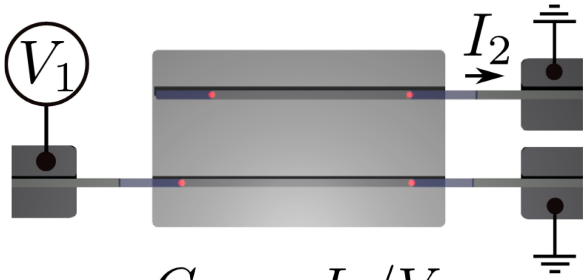
➡ problem effectively in terms of ρ_j, θ_j only, related to QBM.

Topological Kondo solved for

- arbitrary # of leads
- Luttinger liquid leads

[see also Altland&Egger PRL 2013, A. Zazunov *et al.* arXiv:1307.0210]

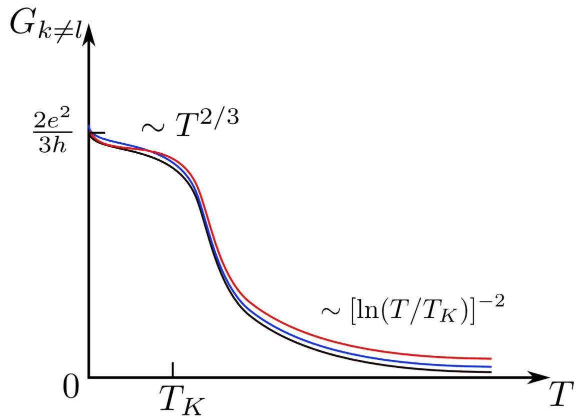
Transport signatures



$$G_{21} = I_2/V_1$$

Conductance for $T \ll T_{\rm K}$

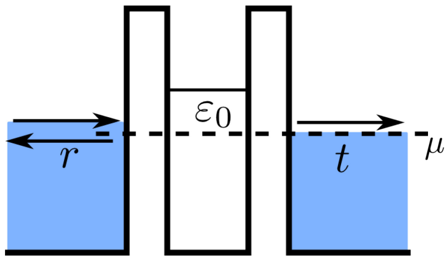
$$G_{k\neq l}(T) = \frac{2e^2}{3h} + c_{kl}T^{2/3}$$

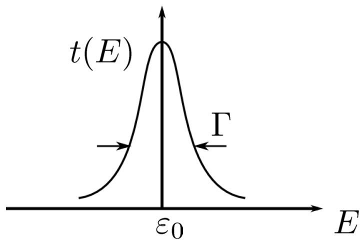


- Noninteger power law:
  NFL physics - w/o fine tuning!

Fermi liquid transport

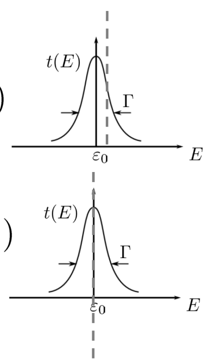
Resonant tunneling





$$t(E) = \frac{\Gamma^2}{(E-\varepsilon_0)^2 + \Gamma^2}$$

$$G(T) \sim \int dE\ t(E)\ \frac{\partial f_\beta(E-\mu)}{\partial E} \sim \begin{cases} G_0 + aT & (\mu \neq \varepsilon_0) \\ G_0 + aT^2 & (\mu = \varepsilon_0) \end{cases}$$



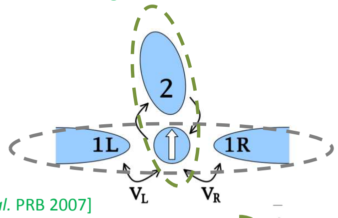
Fermi liquid physics ➡ integer exponents

$$G(T) \sim \int dE\ [t(\mu) + cE^\alpha]\ \frac{\partial f_\beta(E-\mu)}{\partial E} = G_0 + cT^\alpha \int d(\beta E)\ (\beta E)^\alpha \frac{\partial f_\beta(E-\mu)}{\partial \beta E}$$

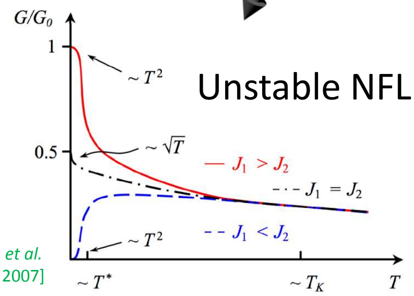
NFL & Overscreened Kondo

Multichannel Kondo

[Nozieres&Blandin J. Phys 1980, Affleck&Ludwig, Nucl Phys. B 1991, Oreg&Goldhaber-Gordon 2003 ...]



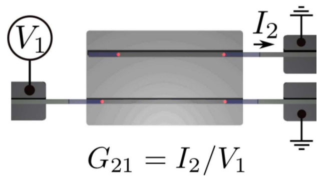
[fig:A Toth *et al.* PRB 2007]

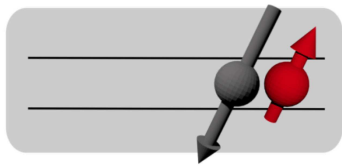


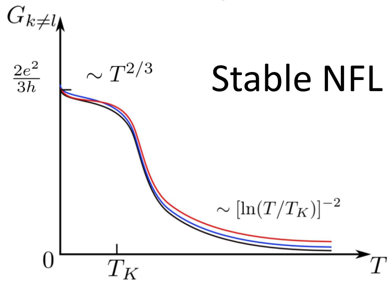
Unstable NFL

[fig:AlToth *et al.* PRB 2007]

Topological Kondo

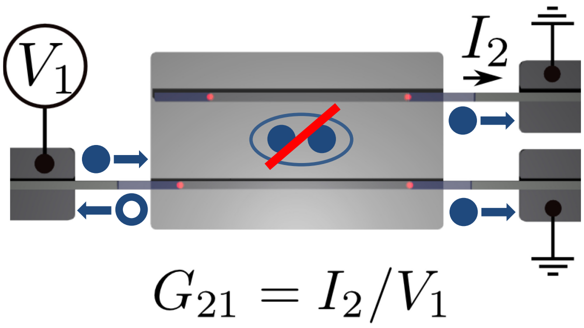






Stable NFL

Transport signatures

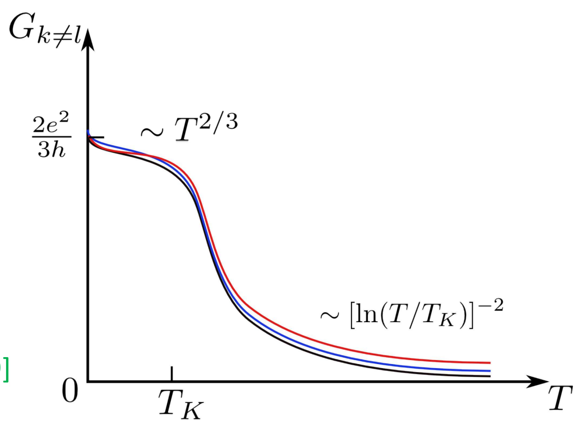


Conductance for $T \ll T_K$

$$G_{k\neq l}(T) = \frac{2e^2}{3h} + c_{kl}T^{2/3}$$

- Noninteger power law:
  NFL physics - w/o fine tuning!
- $T \to 0$
  - isotropic, **universal**
  - two terminal value beyond e^2/h :
    correlated Andreev reflection

[Nayak *et al.*, PRB 1999]



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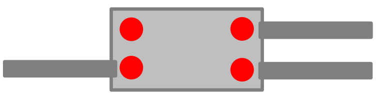
Kondo with more Majoranas

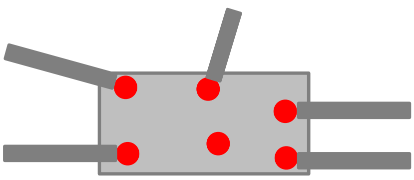
3 Majoranas/leads:



Kondo "impurity": $\gamma_j \gamma_k \to \sigma_\alpha$ $S = 1/2$

Cond. electrons: $J_3 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $S = 1$

M Majoranas/leads



M=5

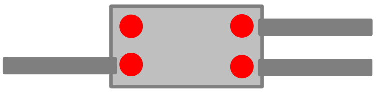
Kondo "impurity": $S = ?$

Cond. electrons: $S = ?$

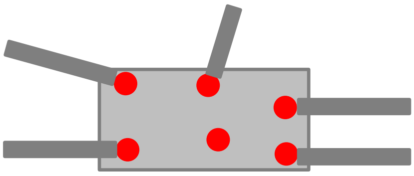
assumes SU(2)

Kondo with more Majoranas

3 Majoranas/leads:



M Majoranas/leads



M=5

Can also view as SO(3):

Kondo "impurity": $\gamma_j\gamma_k \to \sigma_\alpha$ SO(3) spinor

Cond. electrons: $J_3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ SO(3) def.

precisely how Clifford algebra gives spinors!

Kondo "impurity": $\gamma_j\gamma_k \to \Gamma_{jk}$ SO(M) spinor

Cond. electrons: $J_{12} = i\begin{pmatrix} 0 & -1 & 0 & \dots \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & 0 & 0 & \ddots \end{pmatrix}$ SO(M) def.

$$H_{\mathrm{K}} = \sum_{j<k} \lambda_{jk}\Gamma_{jk}\ (\psi^\dagger J_{jk}\psi)$$

SO(M) Kondo problem

Majorana Klein & QBM connection

SO(M) Kondo problem ($H_{\mathrm{eff}} =: H_{\mathrm{lead}} + H'_{\mathrm{eff}}$)

$$H'_{\mathrm{eff}} = \sum_{j\neq k} \lambda^+_{ij}\gamma_j\gamma_k\psi^\dagger_k\psi_j - \sum_j \lambda^-_{jj}\psi^\dagger_j\psi_j$$

General features:

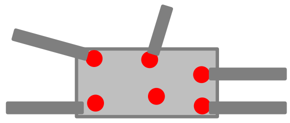
$$\gamma_j\psi_j \sim \gamma_j\Gamma_j\ e^{i\theta_j(0)}$$

$$H'_{\mathrm{eff}} = -\sum_{j\neq k}\lambda^+_{jk}\frac{e^{i(\theta_j-\theta_k)}}{a} - \sum_j \frac{\lambda^-_{jj}}{2\pi}\partial_x\varphi_j$$

Weak coupling RG gives:

$$\lambda^+(D) \sim \frac{1}{\ln(D/T_{\mathrm{K}})} \qquad \lambda^+_\alpha/\lambda^+_\beta \to 1$$

M Majoranas/leads



M=5

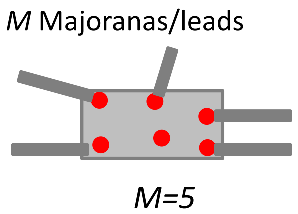
$T \gg T_{\mathrm{K}}$:

$$G_{kl}(T) \sim \frac{1}{\ln^2(T/T_{\mathrm{K}})}$$

$$G_{kl}/G_{mn} \to 1$$

Majorana Klein & QBM connection

SO(M) Kondo problem ($H_{\rm eff} =: H_{\rm lead} + H'_{\rm eff}$)



$$H'_{\rm eff} = \sum_{j\neq k} \lambda^{+}_{ij}\gamma_j\gamma_k\psi^\dagger_k\psi_j - \sum_j \lambda^{-}_{jj}\psi^\dagger_j\psi_j$$

General features:

$$\gamma_j\psi_j \;\sim\; \gamma_j\Gamma_j\, e^{i\theta_j(0)}$$

$$H'_{\rm eff} = -\sum_{j\neq k}\lambda^{+}_{jk}\frac{e^{i(\theta_j-\theta_k)}}{a} - \sum_j \frac{\lambda^{-}_{jj}}{2\pi}\partial_x\varphi_j$$

$$\theta_j = \mathbf{w}_j\cdot\mathbf{r} + \frac{R_0}{\sqrt{M}} \qquad \mathbf{w}_j\cdot\mathbf{w}_l = \delta_{jl} - \frac{1}{M}$$
$$\varphi_j = \mathbf{w}_j\cdot\mathbf{k} + \frac{K_0}{\sqrt{M}}$$

$$H'_{\rm eff} = -\sum_{j\neq k}\lambda^{+}_{jk}\frac{e^{i(\mathbf{w}_j-\mathbf{w}_k)\cdot\mathbf{r}}}{a} - \sum_j \frac{\lambda^{-}_{jj}}{2\pi}\mathbf{w}_j\cdot\partial_x\mathbf{k}$$

QBM action [H. Yi, C. L. Kane, PRB 1998]

$$S = S_0[\mathbf{r}(\tau)] - \int\frac{d\tau}{\tau_c}\sum_{\mathbf{G}} v_{\mathbf{G}} e^{i2\pi\mathbf{G}\cdot\mathbf{r}(\tau)},$$

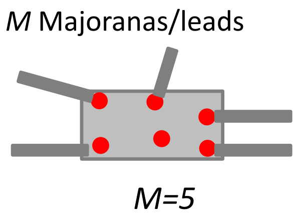
$$S_0[\mathbf{r}(\tau)] = \frac{1}{2}\int d\omega|\omega|e^{|\omega|\tau_c}|\mathbf{r}(\omega)|^2$$

$T \gg T_{\rm K}$:

$$G_{kl}(T) \sim \frac{1}{\ln^2(T/T_{\rm K})}$$
$$G_{kl}/G_{mn} \to 1$$

Majorana Klein & QBM connection

SO(M) Kondo problem ($H_{\rm eff} =: H_{\rm lead} + H'_{\rm eff}$)



$$H'_{\rm eff} = \sum_{j\neq k} \lambda^{+}_{ij}\gamma_j\gamma_k\psi^\dagger_k\psi_j - \sum_j \lambda^{-}_{jj}\psi^\dagger_j\psi_j$$

General features:

$$\gamma_j\psi_j \;\sim\; \gamma_j\Gamma_j\, e^{i\theta_j(0)}$$

$$H'_{\rm eff} = -\sum_{j\neq k}\lambda^{+}_{jk}\frac{e^{i(\theta_j-\theta_k)}}{a} - \sum_j \frac{\lambda^{-}_{jj}}{2\pi}\partial_x\varphi_j$$

$$\theta_j = \mathbf{w}_j\cdot\mathbf{r} + \frac{R_0}{\sqrt{M}} \qquad \mathbf{w}_j\cdot\mathbf{w}_l = \delta_{jl} - \frac{1}{M}$$
$$\varphi_j = \mathbf{w}_j\cdot\mathbf{k} + \frac{K_0}{\sqrt{M}}$$

$$H'_{\rm eff} = -\sum_{j\neq k}\lambda^{+}_{jk}\frac{e^{i(\mathbf{w}_j-\mathbf{w}_k)\cdot\mathbf{r}}}{a} - \sum_j \frac{\lambda^{-}_{jj}}{2\pi}\mathbf{w}_j\cdot\partial_x\mathbf{k}$$

Tunneling between minima; dimension:

$\Delta = \frac{2(M-1)}{M}$ ➡ robust NFL

$T \gg T_{\rm K}$:

$$G_{kl}(T) \sim \frac{1}{\ln^2(T/T_{\rm K})}$$
$$G_{kl}/G_{mn} \to 1$$

$T \ll T_{\rm K}$:

$$G_{kl}(T) = \frac{2e^2}{Mh} + c_{kl}T^{2\Delta-2}$$

$$G_{ll}(T) = \Delta\frac{e^2}{h} + c_{ll}T^{2\Delta-2}$$

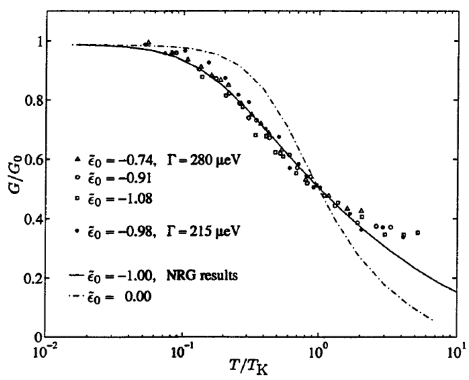
Outline

• Intro to Majorana fermions
  - what are they?
  - how do they emerge?
  - key features & potential uses
  - some of the experimental signatures

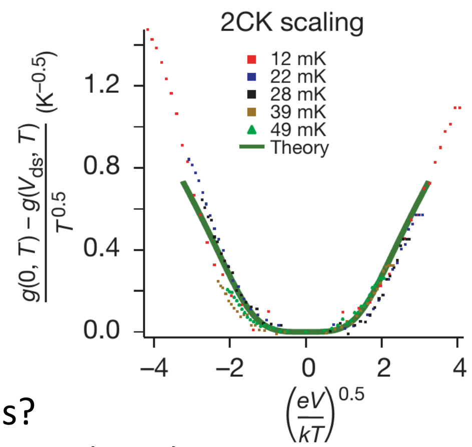
• Topological Kondo effect
  - from Majoranas to Kondo – the topological Kondo idea
  - transport signatures, incl. NFL features
  - topological Kondo beyond the minimal setup
  - (exact) scaling functions for nonequilibrium transport

Nanoscale Kondo experiments: universal scaling functions

Linear transport
[D. Goldhaber-Gordon *et al*. PRL, (1998)]



Non-equilibrium transport
[R. Potok *et al*. Nature, (2007)]



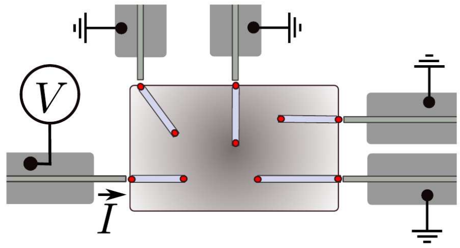
Theory for topological Kondo analogues?
Only for linear conductance, via numerics (NRG).
[M. R. Galpin *et al.*, Phys. Rev. B **89**, 045143 (2014)]

Goal: exact approach to the topological Kondo effect, able to access universal physics below T_K – both in and out of equilibrium.

Setup & strategy

Setup:
- consider M Majoranas/leads;
- leads taken as Fermi liquids;
- focus on local (lead M) observables

  e.g., $G = \frac{\partial I}{\partial V}$



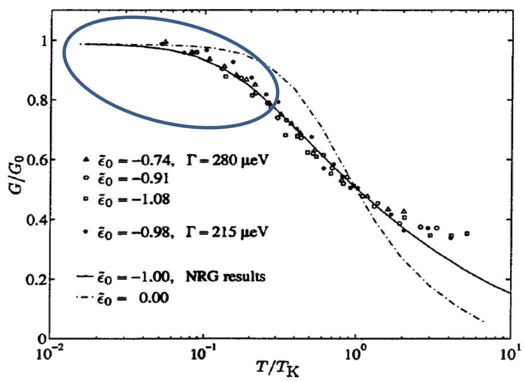
Strategy: **universality** for $k_B T, eV \lesssim T_K$

➡ identify an "easily solvable" limit.

"Toulouse limit" for topological Kondo

Note: top. Kondo is exactly solvable w/o Toulouse limit – in *equilibrium*.

[A. Altland, BB, R. Egger, A. Tsvelik, J. Phys. A. (2014)]

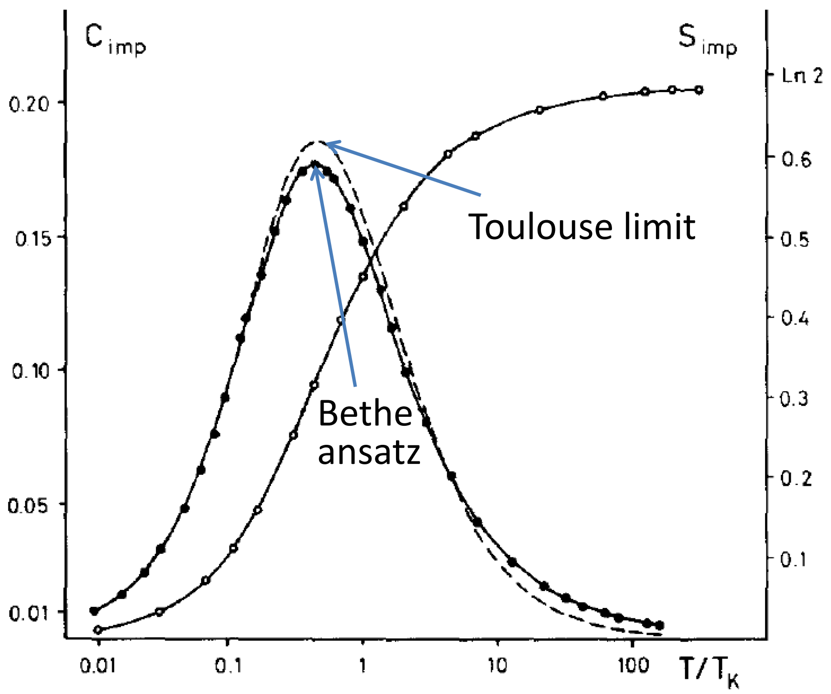


[D. Goldhaber-Gordon *et al*. PRL, (1998)]

Toulouse limit: expected performance

"Conventional" Toulouse limit vs Bethe ansatz for "conventional" Kondo:

[Desgranges & Schotte, Phys. Lett. A 1982]



Toulouse limit

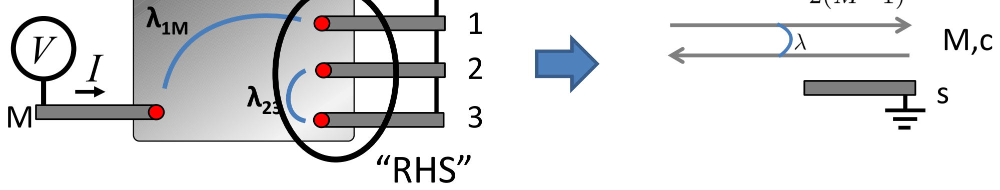
Main innovation:
topological Kondo ➡ backscattering in repulsive Luttinger liquid
(massless BSG)

Powerful exact BSG transport results ➡ exact top. Kondo transport in the Toulouse limit
[P. Fendley et al. PRL & PRB 1995; J. Stat Mech. 1996]

Framework: Bosonisation + Majorana Klein [B. Béri, PRL **110**, 216803 (2013); Altland&Egger PRL 2013]

Summary of the mapping:

$$g_b = \frac{M}{2(M-1)}$$



Anisotropic limit w/ "RHS" @ Kondo FP ➡ lead M tunneling to RHS charge mode ➡ backscatt. in LL w/ quantized g_b
[Chamon&Fradkin, PRB 1997]

Key feature: transparent relation b/w physical & mapped charges

Nanoscale Kondo experiments: universal scaling functions

Linear transport
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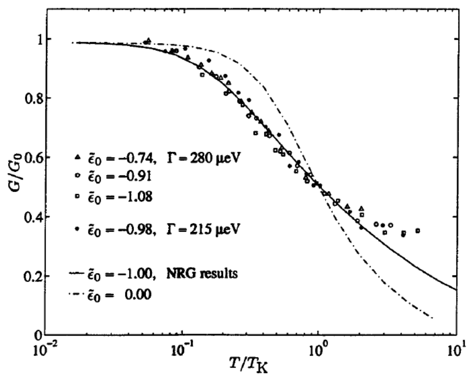

Non-equilibrium transport
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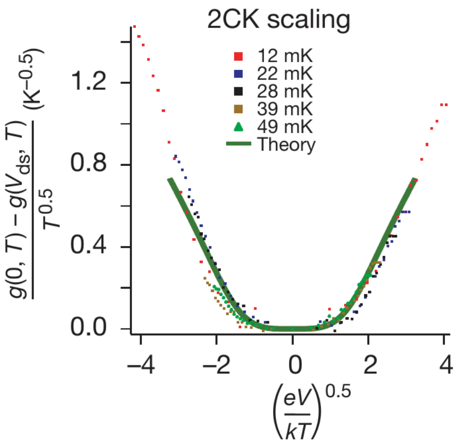
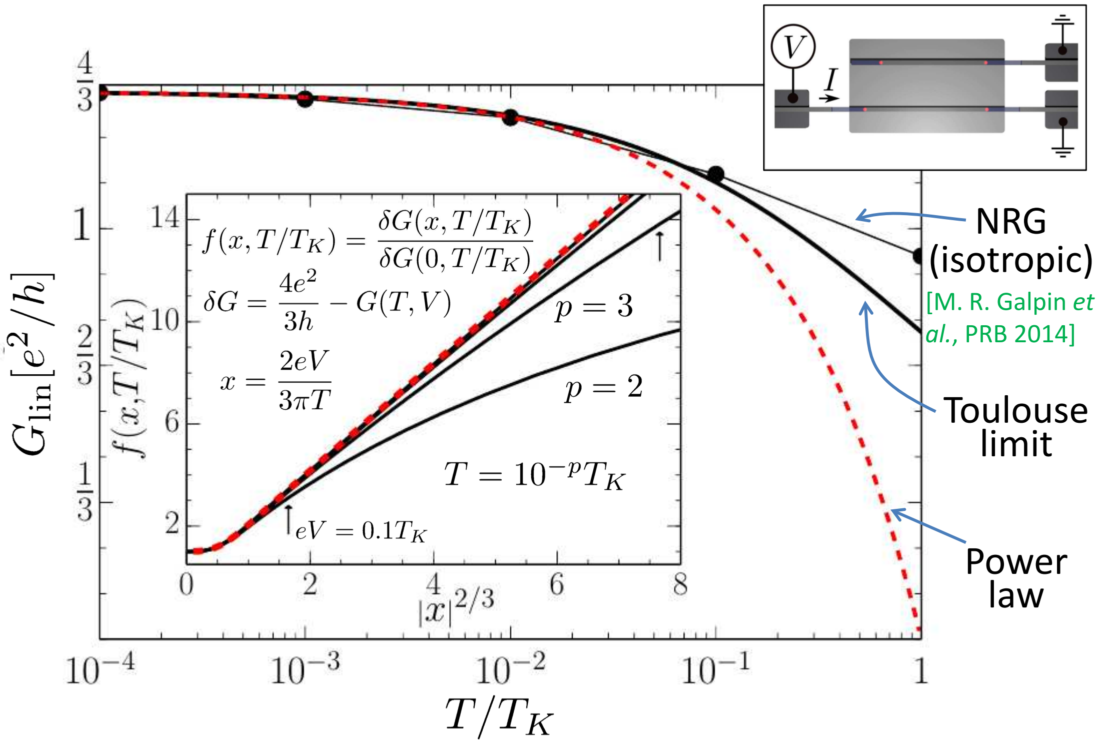


From BSG to topological Kondo transport: Exact conductance



Summary

Conduction electrons + Majoranas ➡ "**topological Kondo**"

- Demonstrates q-dynamics of top. qubits
  - ➡ Probes of the Majoranas' quantum computing potential
- Robust realisation of **NFL Kondo physics**
- "Smoking gun" signature