Core turbulence transport modelling in tokamaks

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Appreciation to the excellent lectures by Greg Hammett, Frank Jenko, Xavier Garbet, Steve Cowley, Ozgur Gurcan, Bruce Scott, Jan Weiland

Any mistakes are my own

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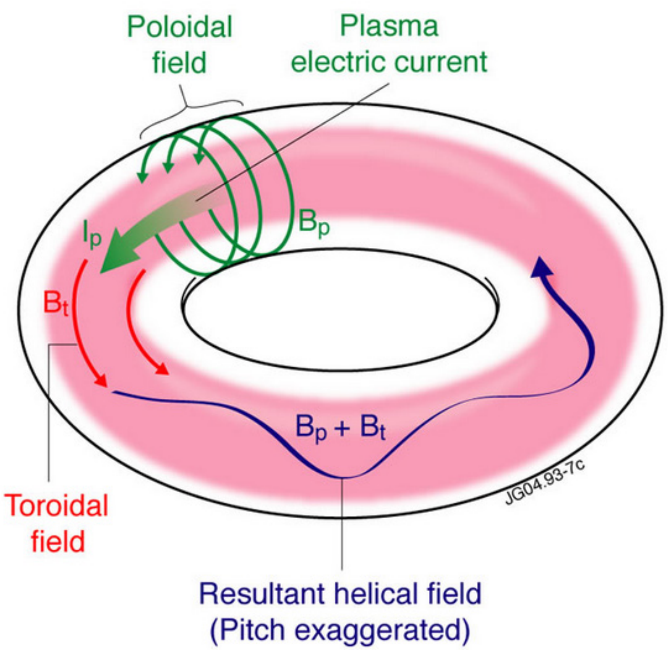
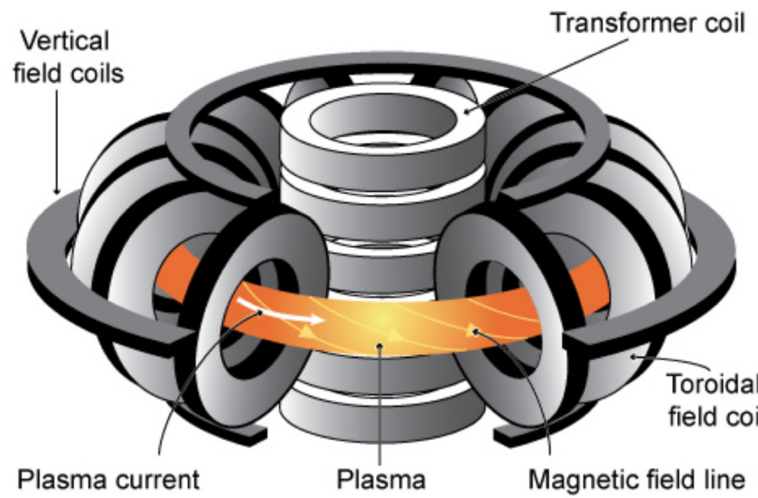
The "tokamak" is the leading toroidal confinement design to achieved controlled fusion

Plasma configuration



Magnetic coil configuration



- Helical magnetic field needed to achieve confinement.
- Typical parameters: $R = 2$ m, $B = 5$ T, $T = 10$ keV, $n = 10^{20}$ m^{-3}
- Plasma heated to fusion conditions by radiofrequency waves and neutral beams

Energy losses due to perturbations from large-scale instabilities (Kim lectures) and small-scale turbulence (these lectures)!

Contents

Today

1. Fluctuations: experimental evidence of "anomalous" core transport. Spatiotemporal characteristics and transport mechanisms

2. Nonlinear gyrokinetics in plasma core: phenomenological overview of core turbulent transport. Successes and future challenges

Tomorrow

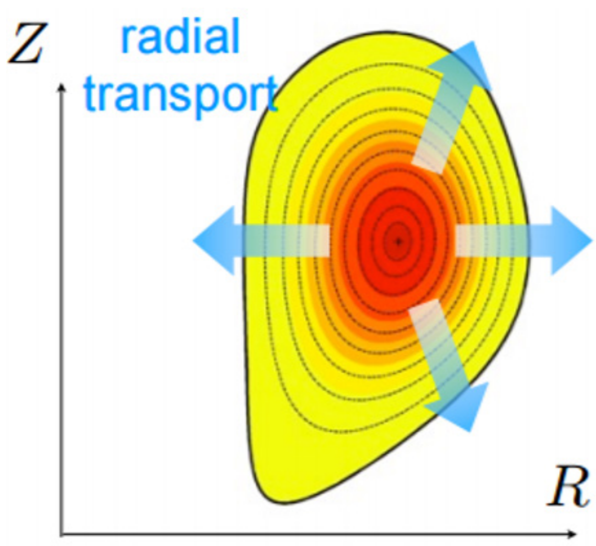
3. Microinstabilities: the driver of tokamak turbulence. Physical insights from the linear gyrokinetic equation. Reduced transport models

4. Improved confinement regimes: Ingredients and physical background

Will focus on fundamental concepts and intuition

Contents

By “transport” we mean the leakage of particles + energy across the nested magnetic surfaces



- High pressure in core → more fusion power.
  What limits the core pressure? Turbulence
- Nested magnetic surfaces.
  Transport parallel to magnetic field ≫ cross field
- On transport timescales, n, T, U (density, temperature, rotation) approximated as flux functions (constant on flux surface)
  → sufficient to describe only radial transport

Energy balance:
(cylindrical geometry here for simplicity)

$$\frac{3}{2}\frac{\partial P_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rq_i) = Q_{heating} - Q_{ei}$$

(and also particle + momentum balance equations)

Heat flux
W/m^2

Ohmic,
beams,
RF waves…
W/m^3

Collisional ion-electron energy exchange if $T_i - T_e \neq 0$

Transport estimated from power balance and compared to theory. Transport higher than neoclassical (collisions)! (1/2)

Energy balance:

$$\frac{3}{2}\frac{\partial P_i}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rq_i)=Q_{heating}-Q_{ei}$$

Heat flux:

$$q_i \approx -n_i\chi_i\frac{\partial T_i}{\partial r}$$

Ion heat conductivity χ_i not a constant: complicated function of plasma parameters!

In stationary state:

$$\chi_i(r)=-\frac{1}{n_i r\frac{\partial T}{\partial r}}\int_0^r\left(Q_{heating}-Q_{ei}\right)r'dr'=$$

$$=-\frac{1}{n_i r\frac{\partial T}{\partial r}}\tilde{Q}(r)$$

Transport estimated from power balance and compared to theory. Transport higher than neoclassical! (2/2)

Power balance: from measurements of T_i, T_e and n, and auxilliary calculations of the heating sources, ${\chi_{i,e}}_{PB}$ can be estimated

$$\chi_i(r) = -\frac{1}{n_i r \frac{\partial T}{\partial r}} \tilde{Q}(r)$$

Neoclassical estimate:
banana-width × collision-frequency

$$\chi_{i,e_{neo}} \approx \frac{q^2}{\epsilon^{3/2}} \frac{T_{i,e}}{m_{i,e}\Omega_{i,e}^2 \tau_{i,e}}$$

For typical JET at mid-radius:

$$\chi_{i_{neo}} \approx 0.3\, m^2/s$$

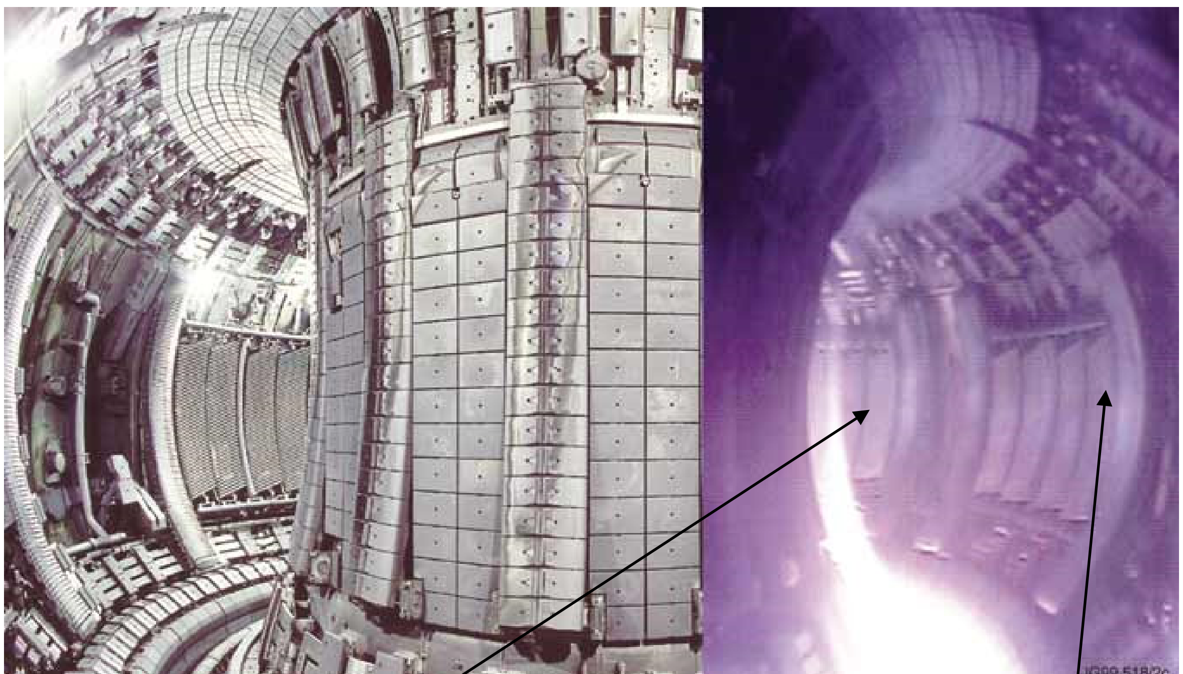
$$\chi_{e_{neo}} \approx 0.005\, m^2/s$$

But $\chi_{i_{PB}}$ typically measured 1 order of magnitude higher, and 2-3 orders for electrons!
Anomalous transport (turbulence)

Tokamak plasma turbulence due to gradient driven instabilities

Plasma pressure limited by small-scale plasma instabilities which lead to turbulent plasma transport and energy losses

Driven by significant plasma temperature and density gradients in the system
Known and studied since the 1960s, but quantifiable only in last ~25 years



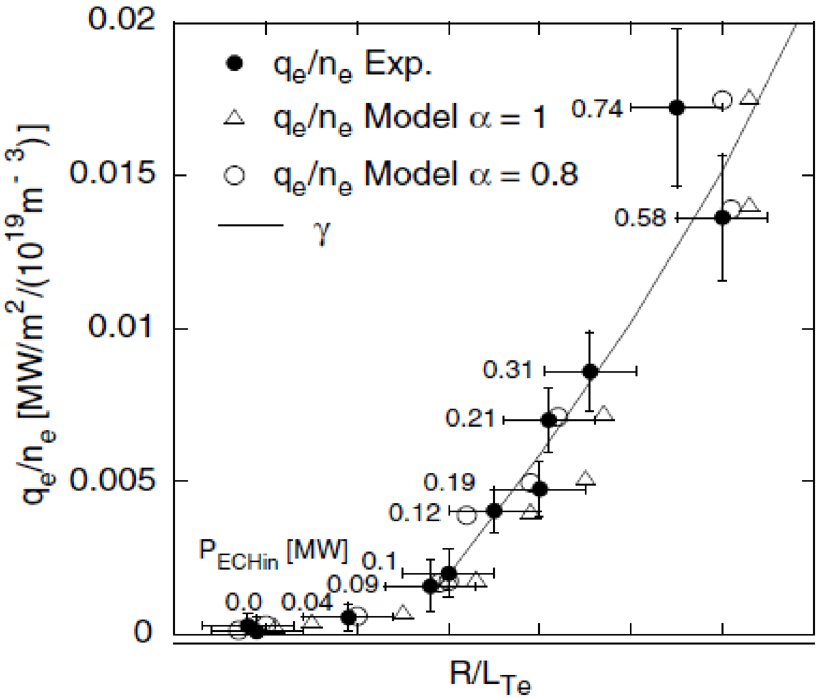
JET tokamak

Plasma center: $\sim 10^8$ K

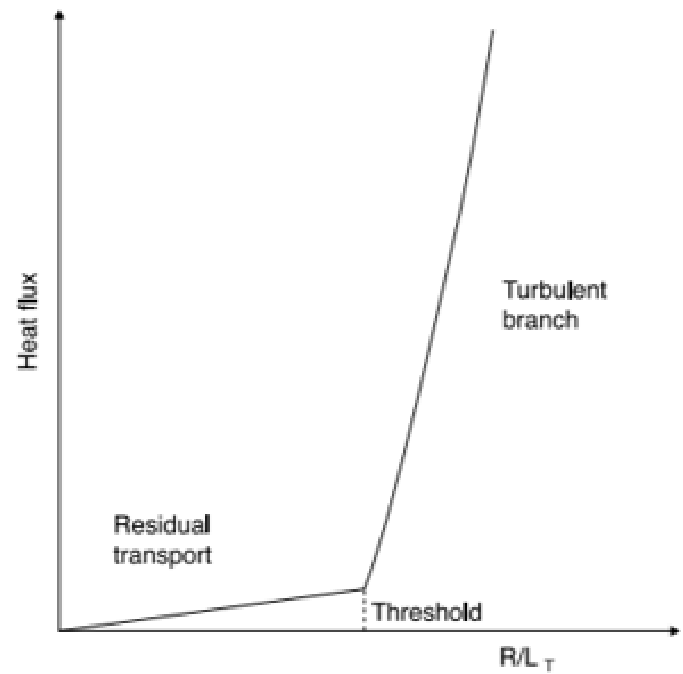
Tokamak wall 1m away: $\sim 10^3$ K

Measurements show critical logarithmic gradient thresholds for turbulent transport

Electron heat flux q_e plotted against $\frac{R}{L_{Te}} \equiv -R\frac{\nabla T_e}{T}$



F. Ryter et al, Phys. Rev. Lett. **95** 085001 (2005).
ASDEX Upgrade experiments

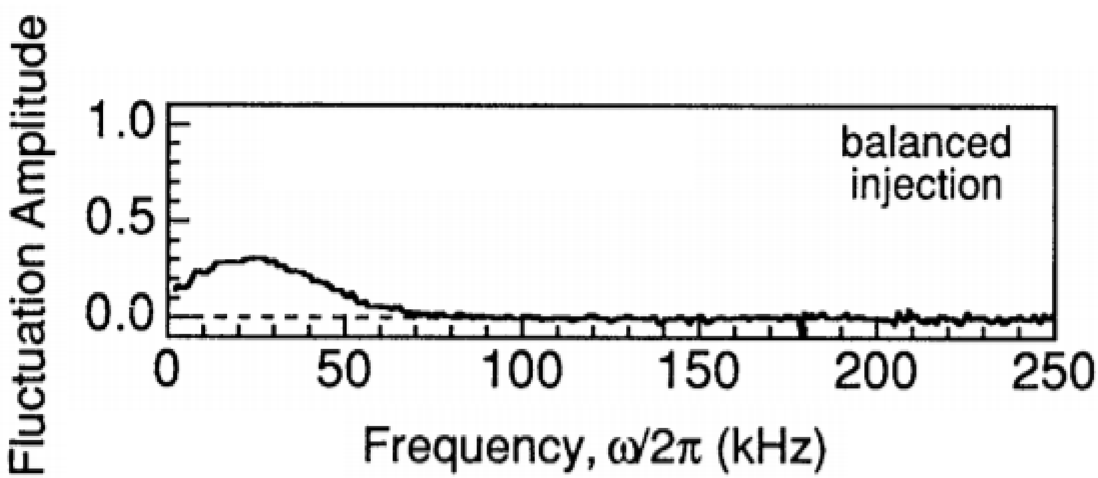


P. Mantica *et al*, Fus. Sci. Tech. **53** 2008

- Sharp discontinuity in heat conductivity above a critical gradient.
- Related to destabilization of underlying linear modes
- Heating less effective above critical threshold. Stiff profiles

Experiments have uncovered the spatiotemporal scales and fluctuation levels of the turbulence (1/3)

- Seminal Beam Emission Spectroscopy (BES) density fluctuation measurements on TFTR tokamak (Princeton 1980s-1990s)
- BES analyzes Dopper shifted line radiation from injected neutral beams. Fluctuations in emissivity related to underlying δn fluctuations from turbulence. Sensitive down to ion-Larmor-radius scales
- Can measure fluctuations frequencies, correlation lengths, decorrelation times



Broadband turbulence:

$$\omega_{peak} \approx \Delta\omega \approx 100\ kHz$$

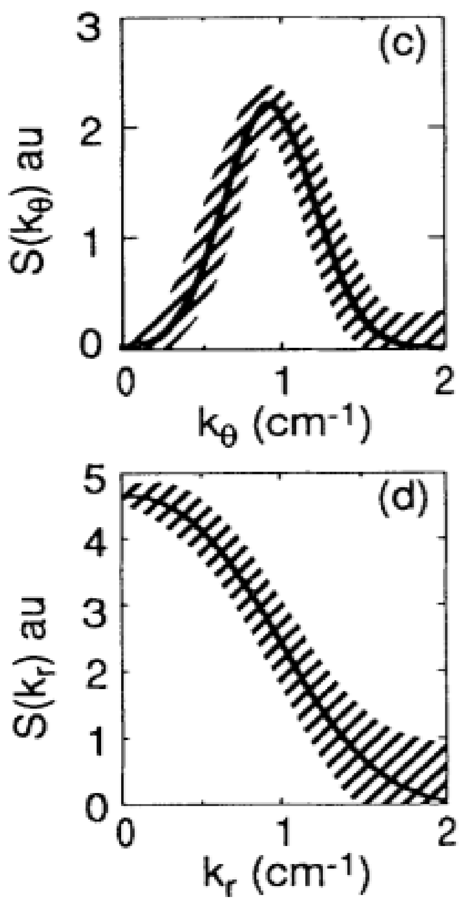
Much slower than cyclo-motion:

$$\Omega_i = \frac{eB}{m_i} \approx 100\ MHz$$

(values calculated with TFTR parameters)

R J Fonck *et al.,* Phys. Rev. Lett **70** 1993

Experiments have shown the spatiotemporal scales and fluctuation levels of the turbulence (2/3)



- Correlation lengths of δn eddies ~ 1 cm
  Several ion Larmor radii: "microturbulence"
  $L_{turb} \ll L_{device}$. Different from "macro" MHD modes

- Spatial anisotropy: $k_r \sim k_\theta \gg k_\parallel$ (more on this later)

- From measured ω and k of the turbulence, can estimate phase velocity of underlying fluctuations

$$v_{phase} \sim \frac{100kHZ}{1cm^{-1}} \approx 1 \text{ km/s}$$

- Comparable to diamagnetic drift velocity

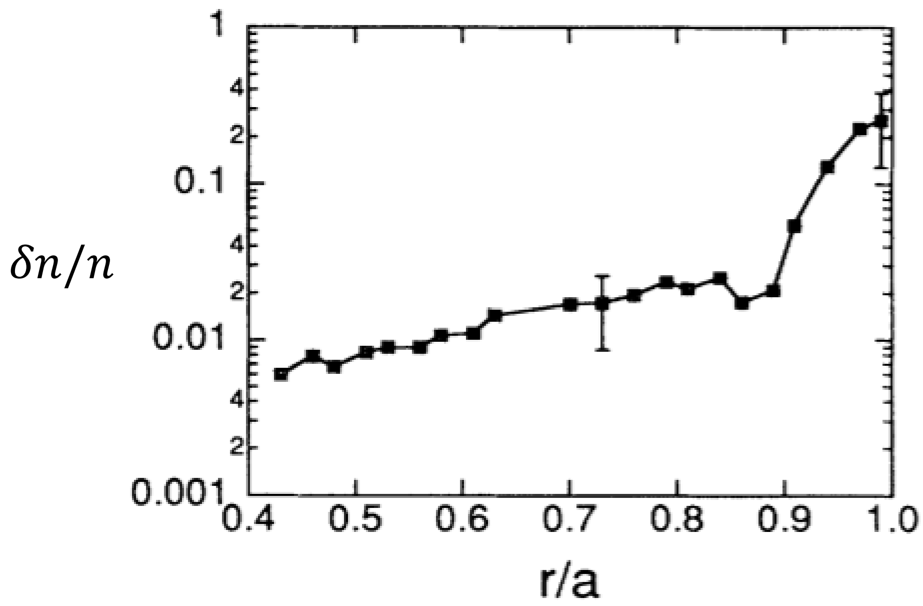
$$v_{*i} = \frac{\nabla p_i}{enB} \approx 0.5 - 2\ km/s$$

Already shows a link between turbulence and driving gradients! Turbulence is driven by unstable drift waves driven by $\nabla T_i/T_i$, $\nabla T_e/T_e$, $\nabla n_e/n_e$ (more on this later)

R J Fonck *et al.,* Phys. Rev. Lett **70** 1993

Experiments have shown the spatiotemporal scales and fluctuation levels of the turbulence (3/3)

Measured relative fluctuation level as a function of tokamak radius



- In the core plasma, level of fluctuations only ~1% amplitude!
- The tokamak plasma core is simmering rather than boiling
- Linear physics can be relevant → reduced transport modelling (major topic for tomorrow)

Towards the edge and SOL at $\frac{r}{a} > 0.9$, fluctuations become much larger.
Cannot use small fluctuation approximations there. We focus on core turbulence

R J Fonck *et al.,* Phys. Rev. Lett **70** 1993

δn fluctuations must mean $\delta\phi$ (electric potential) flucuations, leading to "electrostatic transport"

Electron momentum balance, from moment of kinetic equation

$$\frac{\partial}{\partial t}(m_e n_e \mathbf{V}_e) + \nabla \cdot (m_e n_e \boldsymbol{V_e} \boldsymbol{V_e}) = e n_e(\boldsymbol{E} + \boldsymbol{V_e} \times \boldsymbol{B}) - \nabla \mathrm{p_e} - \nabla \cdot \boldsymbol{\pi_e} + friction + sources$$

$$\boldsymbol{E} = \nabla \delta\phi, \qquad p_e = \overline{p_e} + \delta p_e \approx \overline{p_e} + T_e\, \delta n_e$$

Due to small m_e, assume instantaneous force balance.
To leading order for fluctuating terms:

$e n_e \nabla \delta\phi = \nabla(T_e \delta n_e)$. $\quad \nabla T_e \approx \frac{T_e}{a}$, and $\nabla(\delta n_e) \approx \frac{\delta n_e}{\rho_i}$, with $\frac{\rho_i}{a} \ll 1$, therefore

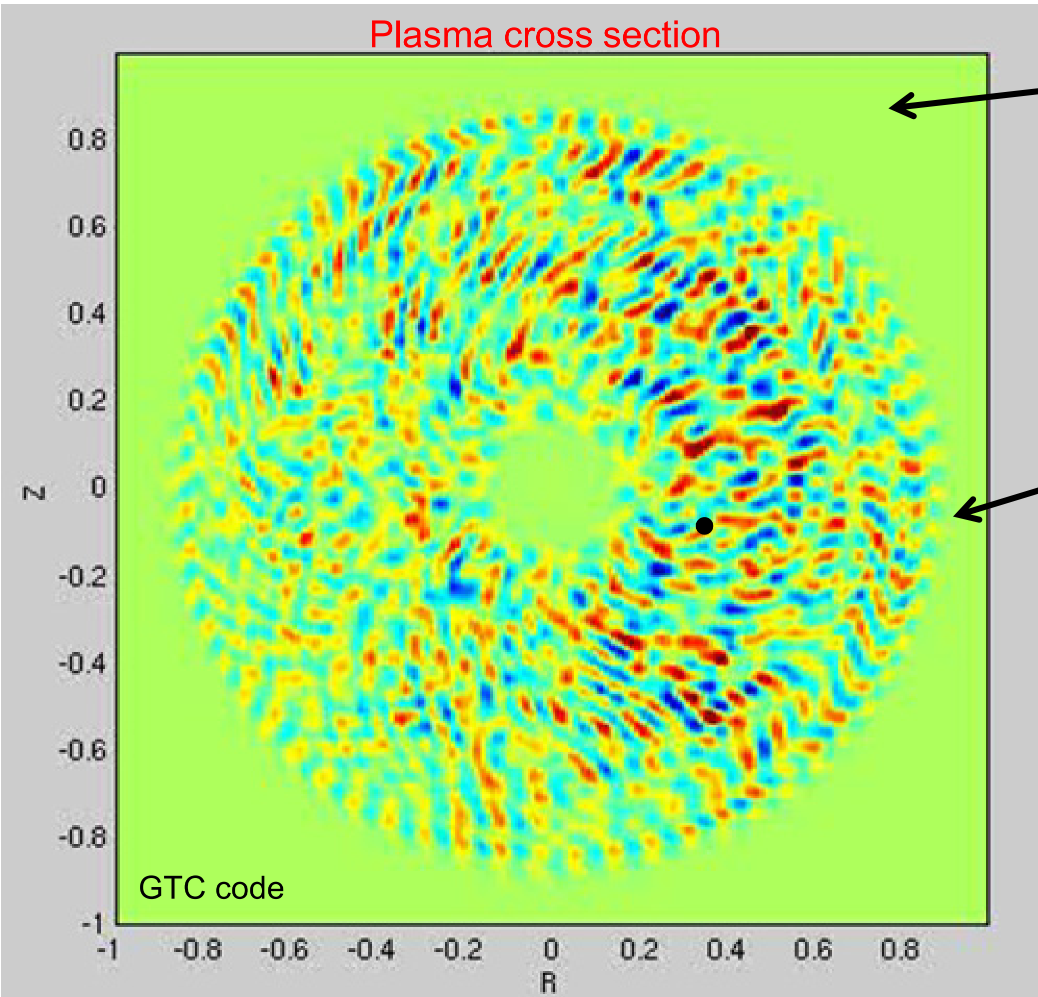
$$\frac{e}{T}\delta\phi \sim \frac{\delta n}{n}$$

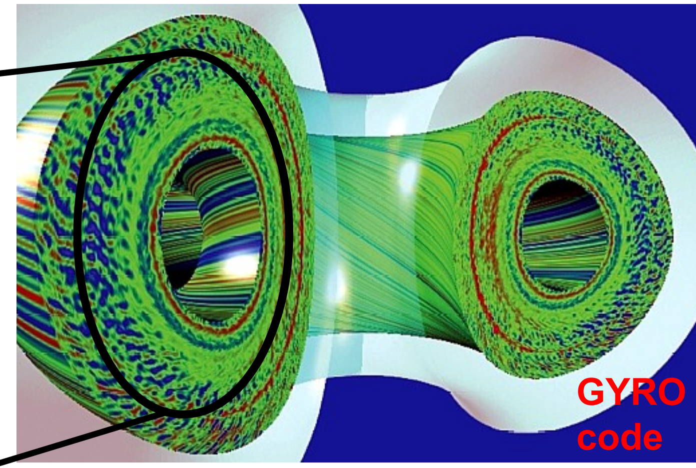
Electrostatic potential and density fluctuations intimately linked

So measurements of $\frac{\delta n}{n}$ provide estimate of $\delta\phi$. Why does this matter?

Electrostatic transport is a random walk in a fluctuating $E \times B$ drift

Simulated tokamak turbulent electrostatic potential structure ($\delta\phi$)





- Chaotic fluctuations (turbulence) of $\delta\phi$ kick around the plasma
- Particles undergo a random walk with velocity $v_r = \frac{1}{B^2}(\boldsymbol{E} \times \boldsymbol{B})_r \propto \frac{k_\theta}{B}\delta\phi$ and escape the system
- Leads to energy and particle loss

Estimate the turbulent transport coefficients using previously found fluctuation spatiotemporal info

Random walk diffusion dimensional analysis. Two separate estimates

$\chi_1 \sim \left[\frac{L^2}{T}\right] \sim \frac{l^2}{\tau}$ or: “step-size^2 / time between steps”

$\chi_2 \sim \left[\frac{L^2}{T}\right] \sim v^2\tau$ or: “walk-velocity^2 × time between random direction shift”

v is typical velocity of outward $E \times B$ drifts:

$$v_r = \frac{k_\theta}{B}\delta\phi \sim \frac{k_\theta T}{eB}\frac{\delta n}{n} \sim \frac{(0.5-2)cm^{-1}\cdot 5keV}{e\cdot 3T} \cdot (0.005 - 0.01) \approx 1\ km/s$$

l is radial length scale of turbulent eddies $\sim 1 - 10\ \rho_i \approx 1cm$

τ is the eddy decorrelation time: $\tau \sim \frac{1}{\Delta\omega} \approx 10\mu s$

$$\chi_1 \sim \chi_2 \sim O(10)$$

Two estimates are consistent and can explain the observed amount of anomalous transport!

In general, turbulence can be electromagnetic, with electrons following magnetic field line perturbations

Perturbed radial velocity due to fluctuating electromagnetic field

$$\delta v_r = \frac{k}{B}\delta\phi + v_{\parallel}\frac{\delta B_r}{B_0}$$

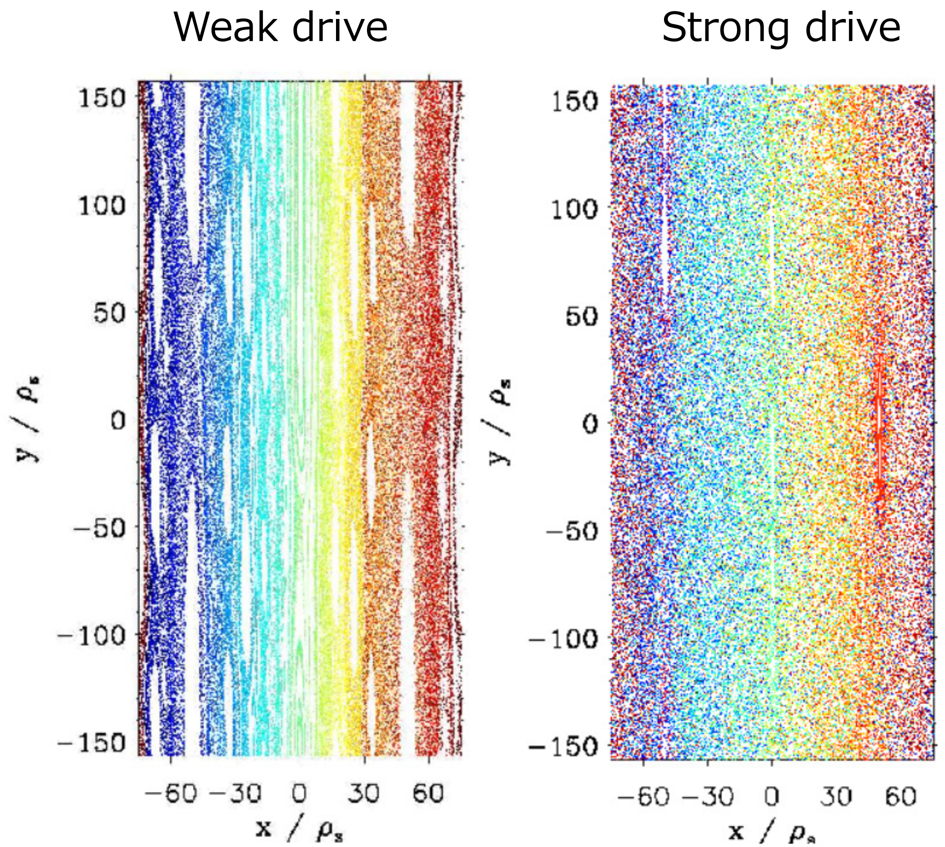
- $\frac{v_{\parallel e}}{v_{\parallel i}} \propto \sqrt{\frac{m_e}{m_i}}$

  "Magnetic flutter" can be important for electrons already for

  $$\frac{\delta B_r}{B_0} \sim O(10^{-4} - 10^{-5})$$

- No net particle transport from δB_r, due to ambipolarity. But can lead to electron heat transport in some regimes where "microtearing" modes unstable (small magnetic islands). See papers by Hatch et al

Poincare map of electron trajectories with microtearing turbulence



H. Doerk et al., Phys. Rev. Lett **106** 2011

Transport from fluctuations: the phase shifts matter

Transport fluxes defined from the spatiotemporal average of the transported quantity and the radial drift velocity

$$\mathrm{Q} = \langle \delta T \delta v_r \rangle_{t,V} = \langle \delta T \frac{\nabla \delta \phi}{B} \rangle_{t,V}$$

t=time, V=flux surface average + eddy radius

Assume harmonic oscillations

$$\delta T = \sum_k \delta T_k \, e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta T_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

$$\delta \phi = \sum_k \delta \phi_k \, e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta \phi_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

Complex coefficients

$$\delta T_k = |\delta T_k|$$
$$\delta \phi_k = |\delta \phi_k| e^{i\varphi_k}$$

φ_k is the phase shift between δT_k and $\delta \phi_k$

$$Q = \sum_k \frac{ik_\theta}{B} \langle \left(\delta T_k e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta T_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}\right) \cdot \left(\delta \phi_k e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta \phi_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}\right) \rangle$$

Transport from fluctuations: the phase shifts matter

$$Q = \sum_k \frac{ik_\theta}{B} \langle \left(\delta T_k e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta T_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}\right) \cdot (\delta\phi_k e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} + \delta\phi_k^* e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}) \rangle$$

All terms like $\langle e^{\pm 2\omega t}\rangle_t$ average to zero!

$$Q = \sum_k \frac{ik_\theta}{B} \langle \delta T_k \delta\phi_k^* + \delta T_k^* \delta\phi_k \rangle = Re \sum_k \frac{ik_\theta}{B} \langle \delta T_k \delta\phi_k^* \rangle = Re \sum_k \frac{ik_\theta}{B} \langle |\delta T_k||\delta\phi_k^*| e^{-i\varphi_k} \rangle$$

And finally: $Q = \sum_k \frac{k_\theta}{B} \langle |\delta T_k||\delta\phi_k^*| sin\varphi_k \rangle$

Similar expression for particle and momentum transport

Main result: if the transported quantity and $\delta\phi$ have no phase shift then there is no transport!

Do we have a theoretical framework to model quantities like $\delta T, \delta\phi,$ phase-shifts, and fundamentally understand the transport physics? Yes!

Contents

Gyrokinetics: solving the Vlasov-Maxwell system with simplifications allowed by the observed ordering

Kinetic model necessary due to wave-particle interactions, magnetic trapping

Kinetic approach: solve Vlasov-Maxwell system

Vlasov equation: Liouville theorem $\frac{df}{dt} = 0$

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \frac{\partial}{\partial \boldsymbol{v}}\right] f_{j}(\boldsymbol{x}, \boldsymbol{v}, t) = 0$$

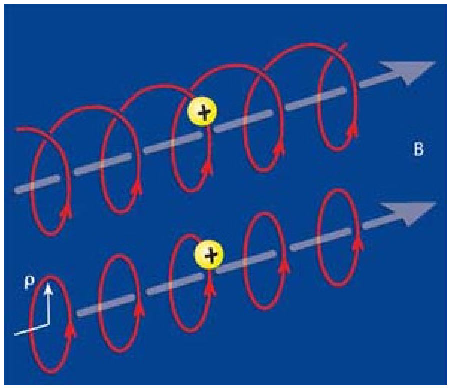
(we ignore collisions here: in the codes they are taken into account)

Coupled to field equations:

$$\begin{aligned}
\nabla \times E &= -\frac{1}{c}\frac{\partial B}{\partial t} \\
\nabla \times B &= \frac{4\pi}{c}\sum_j q_j \int v f_j \mathrm{d}v + \frac{1}{c}\frac{\partial E}{\partial t} \\
\nabla \cdot E &= 4\pi \sum_j q_j \int f_j \mathrm{d}v
\end{aligned}$$

- Nonlinear equation (fields depend on f_j): no analytical solution. Need (super)computer simulations
- 6D kinetic equation including fast gyromotion dynamics: not tractable even with modern supercomputers

Major simplifications: we are interested in "slow" dynamics and "small" spatial scales



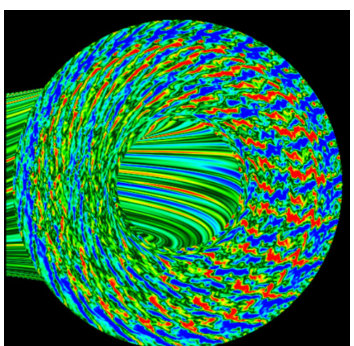
(Plot from SCIDAC review 2005)

Time: $\frac{\omega}{\Omega_i} \ll 1$:

- Major computational speedup by averaging over fast gyromotion around field lines. 6D→5D, larger timesteps
- Justified since instability frequency (10-100 kHz) lower than gyrofrequency (10-100 MHz for ions)
- Kinetic theory of charged particle rings → "gyrokinetics"

Space: $\frac{\rho_i}{L_{T,n}} \equiv \rho_i \frac{(T,n)}{\nabla(T,n)} \ll 1$

- Turbulence size smaller than system and gradient lengths
- Simulate single radius (flux tube): "Local" transport
- Periodic boundary conditions, work in Fourier space


courtesy G. Kerbel of the Numerical Tokamak Project:

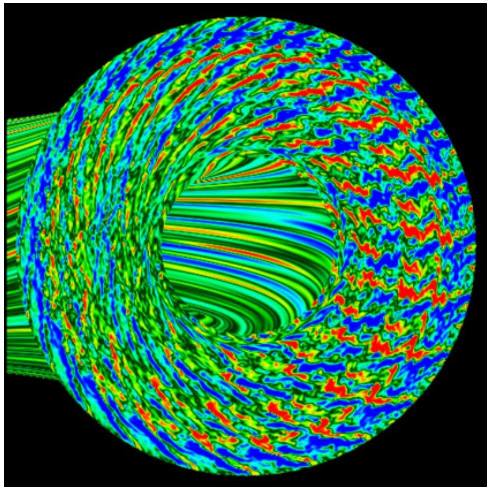
"Global" simulations are more expensive but can be important! e.g. in edge region

Additional consequences of gyrokinetic ordering which allows computational speedup

$$\frac{k_\parallel}{k_\perp} \ll 1$$

Instability anisotropy – modes aligned along field lines.

In simulation codes, can align the computional grid with the magnetic field, and use few points along the field line



courtesy G. Kerbel of the Numerical Tokamak Project

$f \approx F_M + \delta f$. Small fluctuations. Plasma distribution function dominated by background Maxwellian. Can evolve only δf, significantly simplifying equations

Summary: can simulate core turbulence by evolving distribution function under simplifying assumptions

Vlasov-Maxwell system

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}} + \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right)\cdot\frac{\partial}{\partial \boldsymbol{v}}\right] f_{j(\boldsymbol{x},\boldsymbol{v},t)} = 0$$

$$\begin{aligned}
\nabla \times \boldsymbol{E} &= -\frac{1}{c}\frac{\partial B}{\partial t} \\
\nabla \times B &= \frac{4\pi}{c}\sum_j q_j \int v f_j \mathrm{d}v + \frac{1}{c}\frac{\partial E}{\partial t} \\
\nabla \cdot E &= 4\pi \sum_j q_j \int f_j \mathrm{d}v
\end{aligned}$$

$\frac{\omega}{\Omega_i} \ll 1$ Gyroaverage. 6D→5D, avoid short timesteps

$\frac{\rho_i}{L_{T,n}} \ll 1$ Assume locality. Can solve in Fourier space: $\frac{\partial}{\partial \boldsymbol{x}} = i\boldsymbol{k}$

$f \approx F_M + \delta f$ Can simplify Vlasov equation and only solve for δf

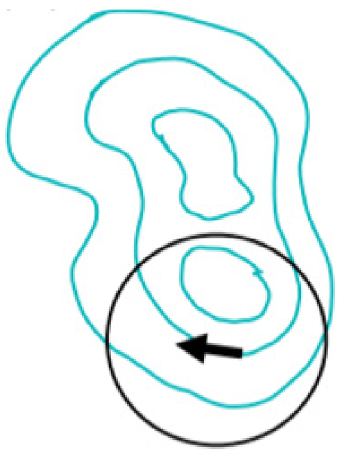
$\frac{k_\parallel}{k_\perp} \ll 1$ Parallel direction easier, only few (10-20) grid points in code

Gyroaveraging leads to screening when turbulence is at Larmor radius scale.

$\boldsymbol{R_G}$ = guiding center
$\boldsymbol{\rho}$ = particle position in gyro-orbit

$$\delta\phi(\boldsymbol{r}) = \delta\phi(\boldsymbol{R_G} + \boldsymbol{\rho})$$

$\delta\phi(\boldsymbol{x})$



$$\delta\phi(\boldsymbol{r}) = \sum_k \delta\phi_k e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_k \delta\phi_k e^{i\boldsymbol{k}\cdot\boldsymbol{R}_g} e^{ik_\perp \rho cos\varphi}$$

$$k_\perp = \left|\mathbf{k} - \mathbf{k}\cdot\hat{\mathbf{b}}\right| \approx \sqrt{k_r^2 + k_\theta^2}.$$

φ is the gyroangle

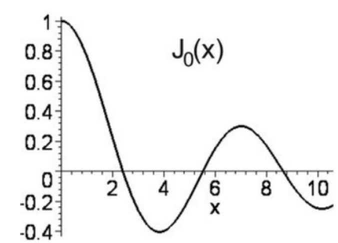
$\langle\ \rangle$ is gyroaveraging, so

$$\boldsymbol{E}\times\boldsymbol{B} = -\boldsymbol{\nabla}\langle\delta\phi\rangle \times \boldsymbol{B}$$

$$\langle\delta\phi\rangle = \sum_k \delta\phi_k e^{i\boldsymbol{k}\cdot\boldsymbol{R}_g} \frac{1}{2\pi}\int_0^{2\pi} e^{ik_\perp \rho cos\varphi}\, d\varphi =$$

$$= \sum_k \delta\phi_k J_0(k_\perp \rho) e^{i\boldsymbol{k}\cdot\boldsymbol{R}_g}$$

Shielding: gyroaveraging over the electrostatic potential reduces effective field



In Fourier space gyroaveraging is “just” multiplication by Bessel function J_0

Based on B. Scott, G. Hammett lectures

The gyrokinetic equation: physical content

δf gyrokinetic equation, electrostatic only. Most dominant terms included

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}} - \frac{q_j}{m_j}\left(\mathbf{v}\times\boldsymbol{B} + \boldsymbol{\nabla}\langle\delta\phi\rangle\right)\cdot\frac{\partial}{\partial \boldsymbol{v}}\right](F_m + \delta f) = 0$$

+ gyrokinetic Poisson equation for $\delta\phi$.

$\langle\,\rangle$ is the gyroaverage

$$F_M = \frac{n(x)}{\left(\frac{2\pi T(x)}{m}\right)^{3/2}} e^{-m\frac{v_\perp^2+v_\parallel^2}{2T(x)}}$$

Maxwellian distribution with radial inhomogeneity. Very important! Radial gradients are the source of free energy and turbulence drive

$$\frac{\partial}{\partial \mathbf{v}} = \hat{\mathrm{e}}_\perp\frac{\partial}{\partial v_\perp} + \hat{\boldsymbol{\varphi}}\frac{1}{v_\perp}\frac{\partial}{\partial\varphi} + \hat{\mathrm{e}}_\parallel\frac{\partial}{\partial v_\parallel}$$

Velocity space derivative. Cylindrical coordinates. φ is the gyroangle

$$\boldsymbol{v}\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x}}F_m = \omega_c\frac{\partial F_m}{\partial\varphi}$$

Zero order solution.
Derivative with respect to gyrophase directly relates to radial derivative (which is intuitive).
This is how turbulence drive enters the system for δf

The gyrokinetic equation: physical content

Putting everything together: derive δf gyrokinetic equation

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}} - \frac{q_j}{m_j}\,\boldsymbol{\nabla}\langle\delta\phi\rangle\cdot\frac{\partial}{\partial \boldsymbol{v}}\right](F_m + \delta f) = 0$$

Recall: $\boldsymbol{v}\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x}}F_m = \omega_c\frac{\partial F_m}{\partial\varphi}$

$$\left(\frac{\partial}{\partial t} + (v_\parallel\widehat{\boldsymbol{b}}\cdot\ + (\boldsymbol{v}_{\boldsymbol{\nabla B}} + \boldsymbol{v}_{\boldsymbol{E\times B}})\cdot)\boldsymbol{\nabla}\right)\delta f + \left(\frac{qF_M}{T}\,\boldsymbol{v}\cdot + \frac{1}{B}\boldsymbol{\nabla}F_m\cdot\right)\boldsymbol{\nabla}\langle\delta\phi\rangle = 0$$

1st order

$$v_{E\times B} = \frac{\nabla\delta\phi}{B}$$

This is the only nonlinear term kept here, since it's dominant: it's the "$E\times B$" nonlinearity, leading to $\delta\phi\delta f$ terms

Let's write this out in perpendicular Fourier space (coordinate z along field line is kept in real space), and interpret all the terms

$$\delta f = \textstyle\sum_k \delta f_k(z)\,e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})}\,,\quad \delta\phi = \textstyle\sum_k \delta\phi_k\,(z)e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})}$$

The gyrokinetic equation in Fourier space

Warning: sloppy notation and likely mistakes here. Treat this as a "rough sketch" for pedagogical purposes on the physical content of the various terms

$$\left(\frac{\partial}{\partial t} + (v_\parallel \hat{\boldsymbol{b}} \cdot \ + (\boldsymbol{v}_{\boldsymbol{\nabla B}} + \boldsymbol{v}_{\boldsymbol{E\times B}}) \cdot) \boldsymbol{\nabla}\right) \delta f + \left(\frac{qF_M}{T} \boldsymbol{v} \cdot + \frac{1}{B} \boldsymbol{\nabla} F_M \cdot\right) \boldsymbol{\nabla} \langle \delta\phi \rangle = 0$$

Diamagnetic frequency pops out from Maxwellian radial derivatives

$$\nabla F_m = -F_m \left(\frac{\nabla n}{n} + \left(\epsilon - \frac{3}{2}\right) \frac{\nabla T}{T}\right) \qquad \omega^* = -\frac{kT}{q}\left(\frac{\nabla n}{n} + \left(\epsilon - \frac{3}{2}\right) \frac{\nabla T}{T}\right) \qquad \epsilon \equiv v_\parallel^2 + v_\perp^2$$

Landau damping (hence low $k_\parallel$)

Wave-particle resonances

Nonlinear mode coupling! (3 wave coupling here)

$$\frac{\partial}{\partial t}\delta f_k = (v_\parallel \nabla_\parallel + k_\perp v_{\nabla B})\delta f_k - \frac{1}{B}\sum_{k'} \hat{\boldsymbol{b}} \cdot (\boldsymbol{k} \times \boldsymbol{k}')\, \delta f_{k'} \delta\phi_{k-k'}$$

$$- \frac{\mathrm{qF_M}}{\mathrm{T}} (v_\parallel \nabla_\parallel + k_\perp v_{\nabla B} - \omega^*)\, J_0(k_\perp \rho) \delta\phi_k$$

Driving gradients

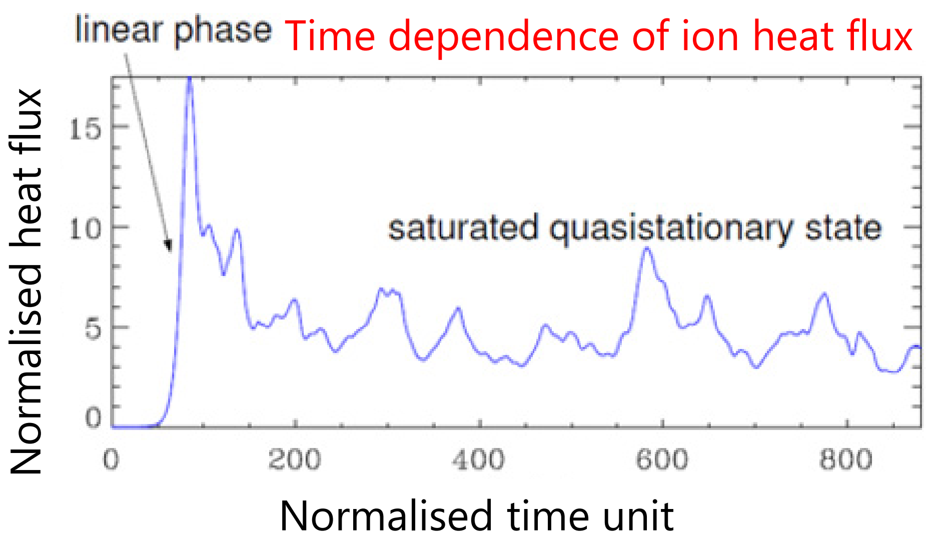
+ gyrokinetic Poisson equation for $\delta\phi_k$

What does a typical nonlinear simulation look like?

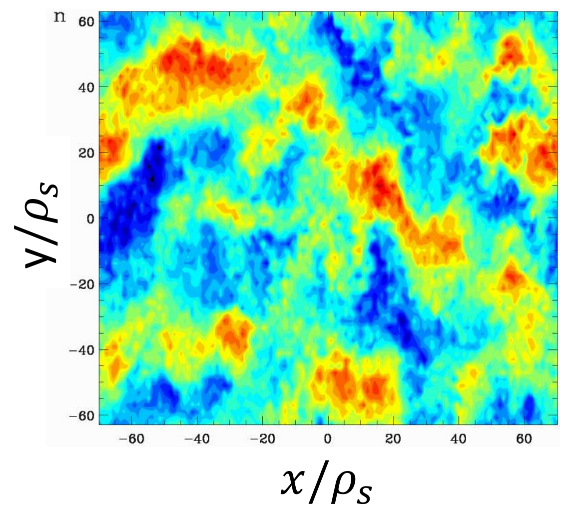
Inputs: radius, mode numbers, T and n gradients, magnetic geometry, beta, etc...

Outputs: time dependent $\delta f \rightarrow \delta n = \int \delta f$, $\delta T = \int v^2 \delta f$. EM fields, phase-shifts, fluxes...

Time dependence of ion heat flux



Density fluctuations

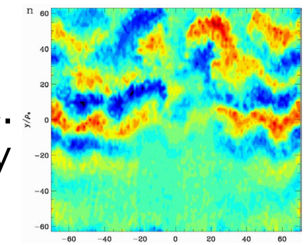


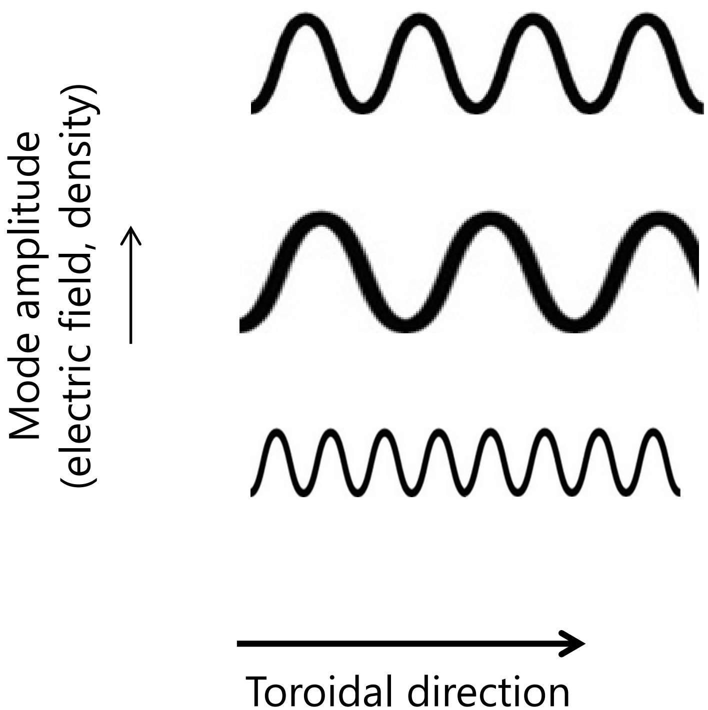
- Initially, linearly unstable modes grow, until non-linear terms mix the modes and lead to saturation of fluctuation growth
- Depending on case, between $10^4 - 10^6$ CPUh for a single simulation!
- Simulation provides wealth of information that can be compared to experiments. Very active field

Linear vs nonlinear stages

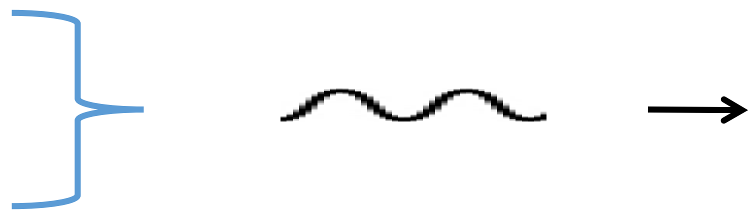
Evolution of
unstable plasma waves

1. Linear phase: fluctuations at different spatial scales grow.
Low interaction between the modes.
Note that leading modes are radially elongated! (ostensibly bad for confinement, mixes regions)





2. Nonlinear phase: fluctuation amplitudes are large enough such that modes interact and spawn new modes



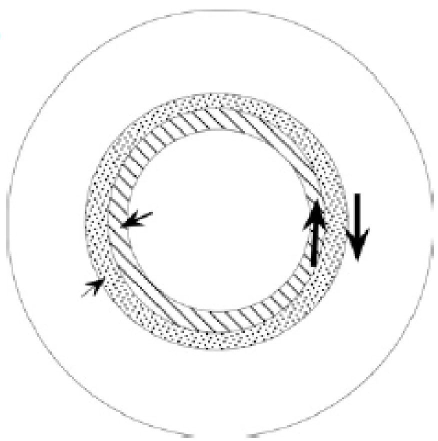
Damping
(heat)

3. Steady-state: some new modes are stable modes, and damp the system leading to saturation

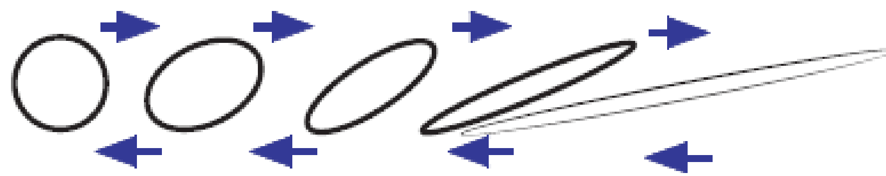
Zonal flows are a class of important nonlinearly excited modes

Through non-linear mode-coupling, a special class of linearly stable modes arise: “zonal flows”. They help tear apart the turbulent eddies

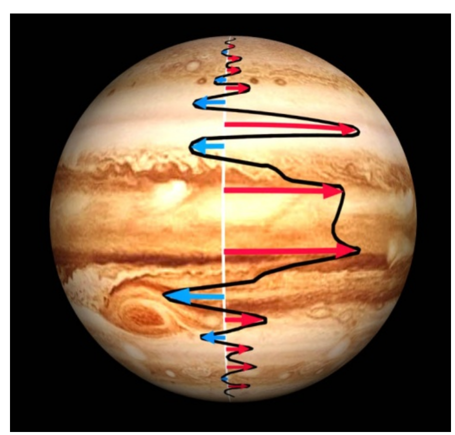
Radial mode with flow shear

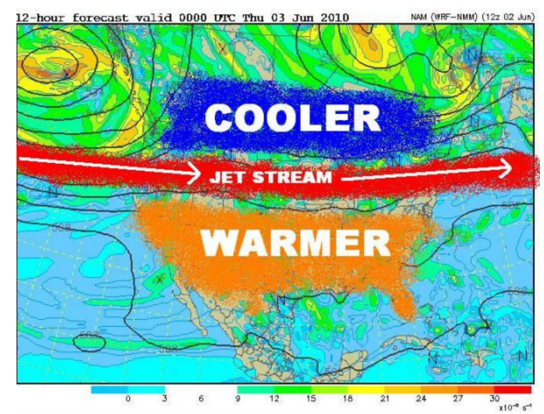


Turbulent eddies are stretched, reducing radial correlation length



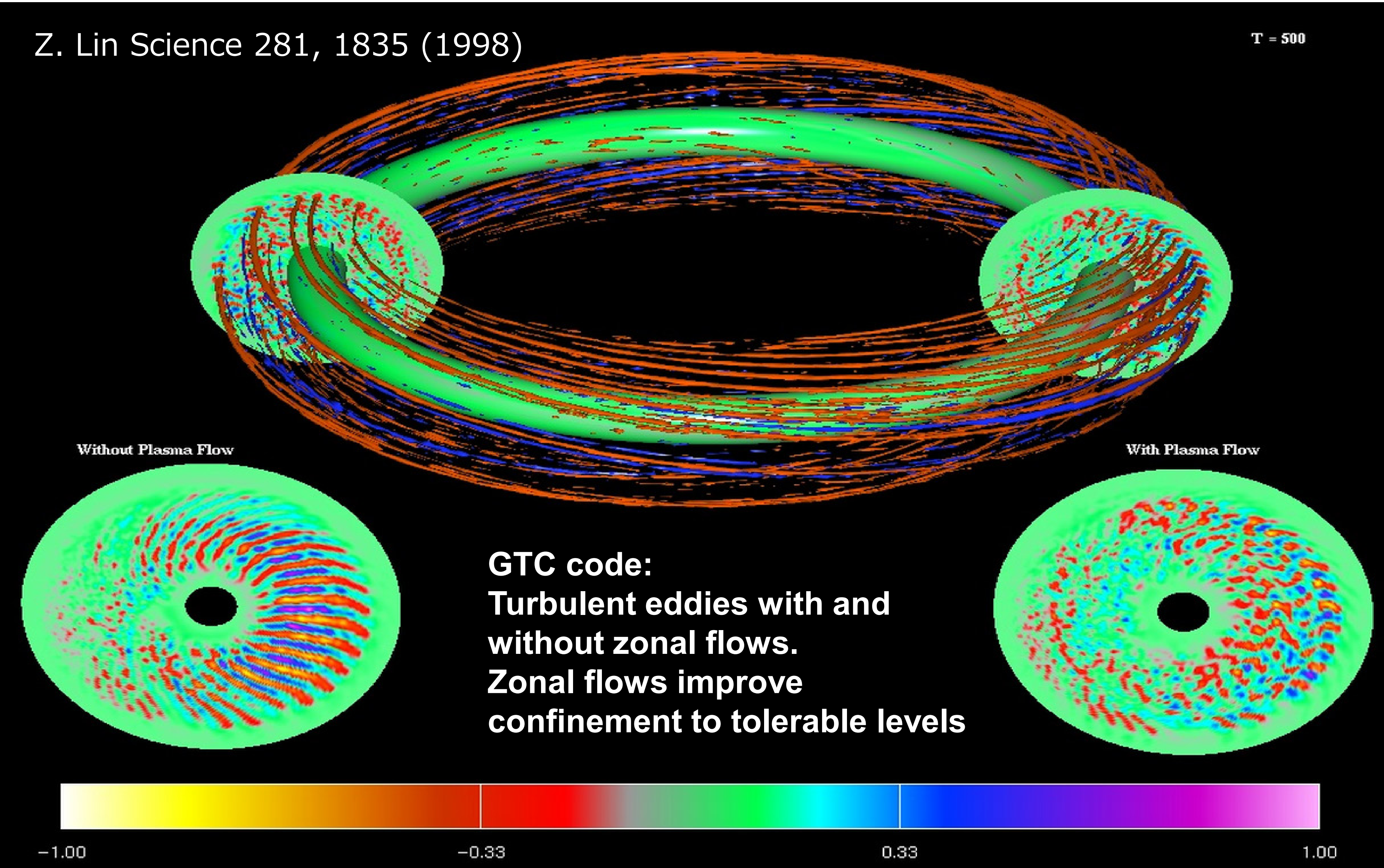
Also observable in Jupiter, Earth weather:





Zonal flows clearly reduce the radial correlation lengths of the turbulence

Z. Lin Science 281, 1835 (1998)

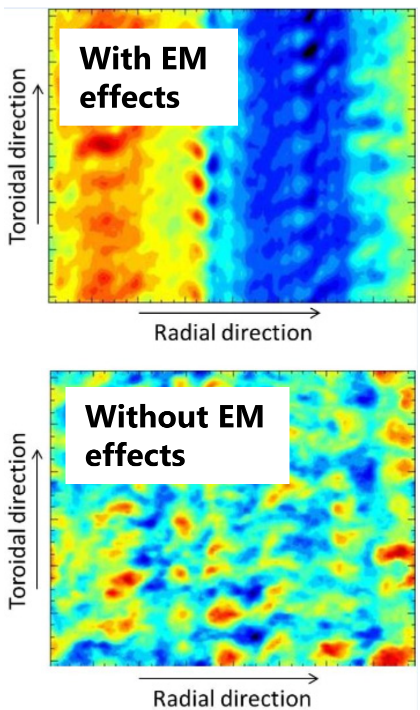


GTC code:
Turbulent eddies with and without zonal flows.
Zonal flows improve confinement to tolerable levels

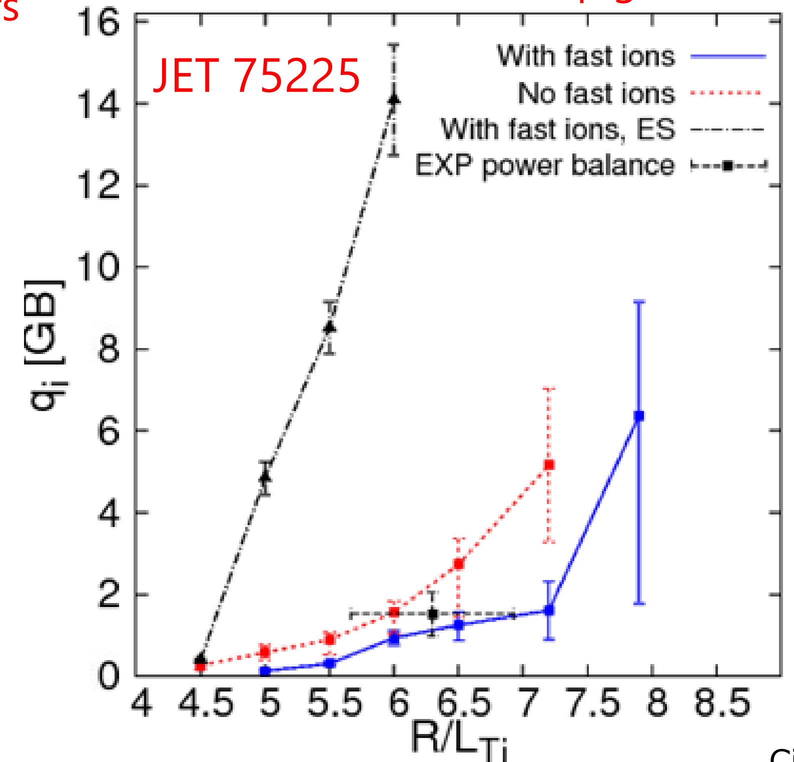
Example of recent experimental validation of advanced nonlinear gyrokinetics

Fluctuating B-field (high-β scenario) can stabilize electrostatic turbulence without onset of magnetic chaos. Example from JET

Electric potential contours



Ion heat flux vs ion temp gradient



"Power balance" means calculated heat flux from experiment based on sources (e.g. NBI)

Each simulation takes $\sim 10^5\ CPUh$

Related to increased stratification of turbulent structures: "zonal flows"

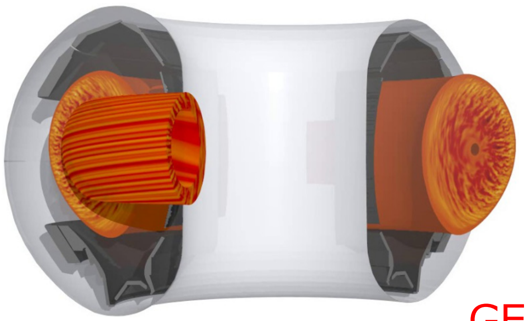
Citrin et al., PRL 2013, Nucl. Fusion 2014, Plasma Phys. Control. Fusion 2015
Using GENE code: IPP Garching, F. Jenko *et al*

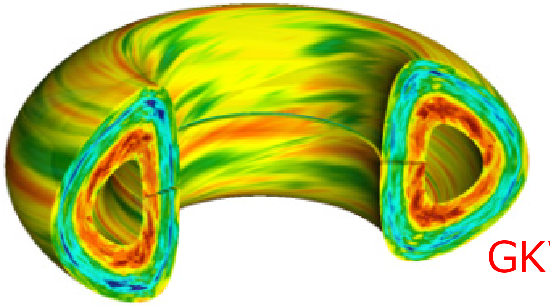
Multiple groups and codes worldwide working on simulations and interpretation of gyrokinetic turbulence and comparison with experiment

For more info:

GENE (www.genecode.org), IPP Garching
GKW (http://www.gkw.org.uk)

Many others, e.g. GYRO (USA, General Atomics), GS2 (USA), XGC1 (USA), ORB5 (Switzerland), GKV (Japan)


GENE


GKW

Theory breakthroughs and advent of high-performance-supercomputing led to amazing advances in past 20 years, with quantitative agreement with experiments in plasma core

Current horizons: global "full-f" self-consistent evolution of turbulence and background on meso-scale (see e.g. Dif-Pradlier PRL 2015).
Edge turbulence, also with global "full-f" (see e.g. Chang PRL 2017)

Contents

The gyrokinetic equation in Fourier space

Warning: sloppy notation and likely mistakes here. Treat this as a “rough sketch” for pedagogical purposes on the physical content of the various terms

$$\left(\frac{\partial}{\partial t} + (v_\parallel \hat{\boldsymbol{b}}\cdot\ + (\boldsymbol{v}_{\nabla \boldsymbol{B}} + \boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}})\cdot)\boldsymbol{\nabla}\right)\delta f + \left(\frac{qF_M}{T}\,\boldsymbol{v}\cdot + \frac{1}{B}\boldsymbol{\nabla}F_M\cdot\right)\boldsymbol{\nabla}\langle\delta\phi\rangle = 0$$

Diamagnetic frequency pops out from Maxwellian radial derivatives

$$\nabla F_m = -F_m\left(\frac{\nabla n}{n} + \left(\epsilon - \frac{3}{2}\right)\frac{\nabla T}{T}\right) \qquad \omega^* = -\frac{kT}{q}\left(\frac{\nabla n}{n} + \left(\epsilon - \frac{3}{2}\right)\frac{\nabla T}{T}\right) \qquad \epsilon \equiv v_\parallel^2 + v_\perp^2$$

Landau damping (hence low $k_\parallel$)

Wave-particle resonances

Nonlinear mode coupling! (3 wave coupling here)

$$\frac{\partial}{\partial t}\delta f_k = (v_\parallel \nabla_\parallel + k_\perp v_{\nabla B})\delta f_k - \frac{1}{B}\sum_{k'} \hat{\boldsymbol{b}}\cdot(\boldsymbol{k}\times\boldsymbol{k}')\,\delta f_{k'}\delta\phi_{k-k'}$$

$$-\frac{\mathrm{qF_M}}{\mathrm{T}}(v_\parallel \nabla_\parallel + k_\perp v_{\nabla B} - \omega^*)\,J_0(k_\perp\rho)\delta\phi_k$$

Driving gradients

+ gyrokinetic Poisson equation for $\delta\phi_k$

Linearization of gyrokinetic equation

Neglect nonlinear term

$$\frac{\partial}{\partial t}\delta f_k = (v_\parallel \nabla_\parallel + k_\perp v_{\nabla B})\delta f_k - \cancel{\frac{1}{B}\sum_{k'} \hat{\boldsymbol{b}}\cdot(\boldsymbol{k}\times\boldsymbol{k}')\,\delta f_{k'}\delta\phi_{k-k'}}$$

$$-\frac{\mathrm{qF_M}}{\mathrm{T}}(v_\parallel \nabla_\parallel + k_\perp v_{\nabla B} - \omega^*)\,J_0(k_\perp \rho)\delta\phi_k$$

$\delta f \propto e^{-i\omega t}$ Introduce harmonic oscillation of mode: → $\frac{\partial}{\partial t} = -i\omega$

$\omega_s^* = \frac{k_\theta T_s}{q_s B R}\left(\frac{R}{L_{ns}} + \frac{R}{L_{Ts}}\left(\epsilon - \frac{3}{2}\right)\right)$ Recall diamagnetic frequency

$\omega_D \equiv k v_{\nabla B}$ Drift frequency

$\nabla_\parallel \rightarrow i k_\parallel$ parallel wavenumber of linear mode (defined later)

Rearranging a bit… we get the linear response for species s, mode number k

$$\delta f_{sk} = \frac{F_{Ms} Z_s}{T_s}\left(1 - \frac{\omega_k - \omega_s^*}{\omega_k - k_\parallel v_{s\parallel} - \omega_{Ds}} J_0(k_\perp \rho_s)\right)\delta\phi_k$$

Resonant denominator

Linearization of gyrokinetic equation

Linear response for species s, mode number k

$$\delta f_{sk} = \frac{F_{Ms} Z_s}{T_s} \left(1 - \frac{\omega_k - \omega_s^*}{\omega_k - k_\parallel v_{s\parallel} - \omega_{Ds}} J_0(k_\perp \rho_s) \right) \delta\phi_k$$

Resonant denominator

The dispersion relation is set by the quasineutrality constraint, summing over all species

$$\sum_s Z_s n_s = \sum_s \int dv_\parallel dv_\perp \frac{F_{Ms} Z_s^2}{T_s} \left(1 - \frac{\omega_k - \omega_s^*}{\omega_k - k_\parallel v_{s\parallel} - \omega_{Ds}} J_0(k_\perp \rho_s) \right) \delta\phi_k = 0$$

$\delta\phi_k$ is a function of space $\delta\phi_k(r, z)$. (2D due to axisymmetry. r=radial, z=parallel)

Solving the dispersion relation to have the same ω_k for all (r,z) is the “eigenvalue solution” of the equation: mode structure $\delta\phi_k(r, z)$ for the eigenfrequency ω_k

$\omega_k = \omega_{rk} + i\gamma_k$, and recall that $\delta f_k \propto e^{-i\omega_k t} \propto e^{-i\omega_{rk} t} e^{\gamma t}$

Solutions with positive γ_k are the unstable modes with exponential growth!

Linearization of gyrokinetic equation

The dispersion relation is set by the quasineutrality constraint

$$\sum_s \int dv_\parallel dv_\perp \frac{F_{Ms} Z_s^2}{T_s}\left(1 - \frac{\omega_k - \omega_s^*}{\omega_k - k_\parallel v_{s\parallel} - \omega_{Ds}} J_0(k_\perp \rho_s)\right)\delta\phi_k = 0$$

Notation: $\langle \quad \rangle_{p,t} \equiv \int_{passing,trapped} dv_\parallel dv_\perp$ $\qquad f_{p,t}$ is the passing/trapped fraction = $\langle F_M \rangle_{p,t}/n$

Trapped electrons $\qquad$ Trapped ions $\qquad$ Passing ions

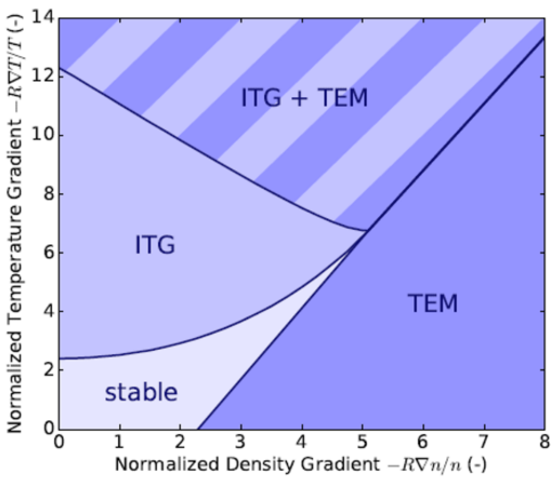
$$\sum_s \left(1 + Z_{eff}\frac{T_e}{T_i} - f_t \left\langle \frac{\omega_k - \omega_e^*}{\omega_k - \omega_{De}} J_0(k_\perp \delta_e)\right\rangle_t - \frac{n_i Z_i^2}{n_e}\frac{T_e}{T_i}\left(f_t \left\langle \frac{\omega_k - \omega_e^*}{\omega_k - \omega_{Di}} J_0(k_\perp \delta_i)\right\rangle_t + f_p \left\langle \frac{\omega_k - \omega_i^*}{\omega_k - k_\parallel v_{\parallel i} - \omega_{Di}} J_0(k_\perp \rho_i)\right\rangle_p\right)\right)\delta\phi_k = 0$$

- For ion-scales $k\rho_i < 1$ (which we discuss here), passing electrons have small response
- Bounce average for trapped species, average $v_\parallel = 0$. Gyroscreening → banana-screening
- Drift frequency have opposite sign for ions/electrons. Critical role of wave-particle resonance with drift frequency. Sets different classes of modes

Different classes of instability depending on resonance and driving gradient

Trapped electrons — Trapped ions — Passing ions

$$\sum_s \left(1 + Z_{eff}\frac{T_e}{T_i} - f_t\left\langle\frac{\omega_k-\omega_e^*}{\omega_k-\omega_{De}}J_0(k_\perp\delta_e)\right\rangle_t - \frac{n_iZ_i^2}{n_e}\frac{T_e}{T_i}\left(f_t\left\langle\frac{\omega_k-\omega_e^*}{\omega_k-\omega_{Di}}J_0(k_\perp\delta_i)\right\rangle_t + f_p\left\langle\frac{\omega_k-\omega_i^*}{\omega_k-k_\parallel v_{\parallel i}-\omega_{Di}}J_0(k_\perp\rho_i)\right\rangle_p\right)\right)\delta\phi_k = 0$$



$$\omega_s^* = \frac{k_\theta T_s}{q_s BR}\left(\frac{R}{L_{ns}} + \frac{R}{L_{Ts}}\left(\epsilon - \frac{3}{2}\right)\right)$$

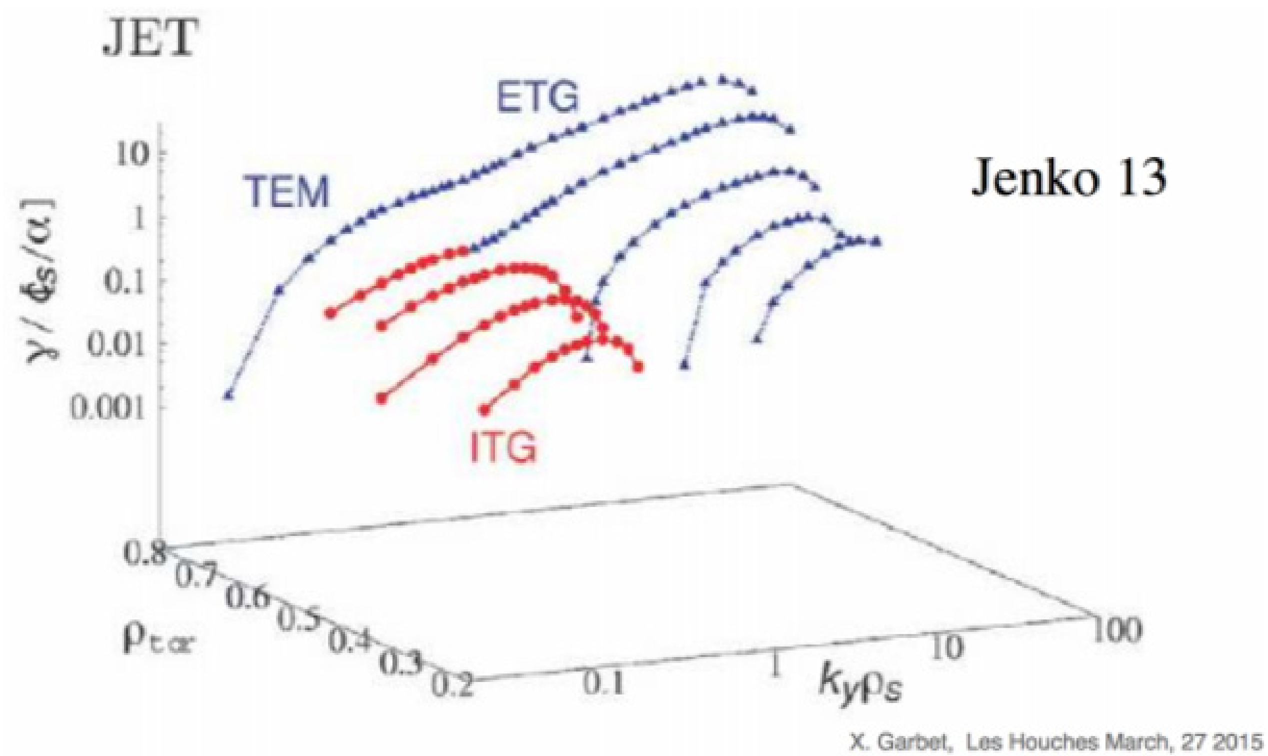
$$1/L_{n,T} \equiv -\nabla(T,n)/(T,n)$$

$$\epsilon \equiv v_\perp^2 + v_\parallel^2$$

$$\omega_{Ds} \approx \frac{k_\theta T_s}{q_s B}\frac{(\boldsymbol{B}\times\boldsymbol{\nabla}\mathrm{B})}{\mathrm{B}^2}$$

- Trapped Electron Modes (TEM) resonate with electron drift frequency, propagate in electron diamagnetic direction ($sign(\omega_k) = sign(\omega_e^*)$). Driven by $\frac{R}{L_{ne}}, \frac{R}{L_{Te}}$
- Ion Temperature Gradient (ITG) resonate with ion drift frequency, propagate in ion diamagnetic direction ($sign(\omega_k) = sign(\omega_i^*)$). Driven mostly by $\frac{R}{L_{Ti}}$. Most ubiquitous mode in tokamak
- ETG driven by R/L_{Te} can arise at electron scales $k\rho_e \sim 1$, not discussed here.

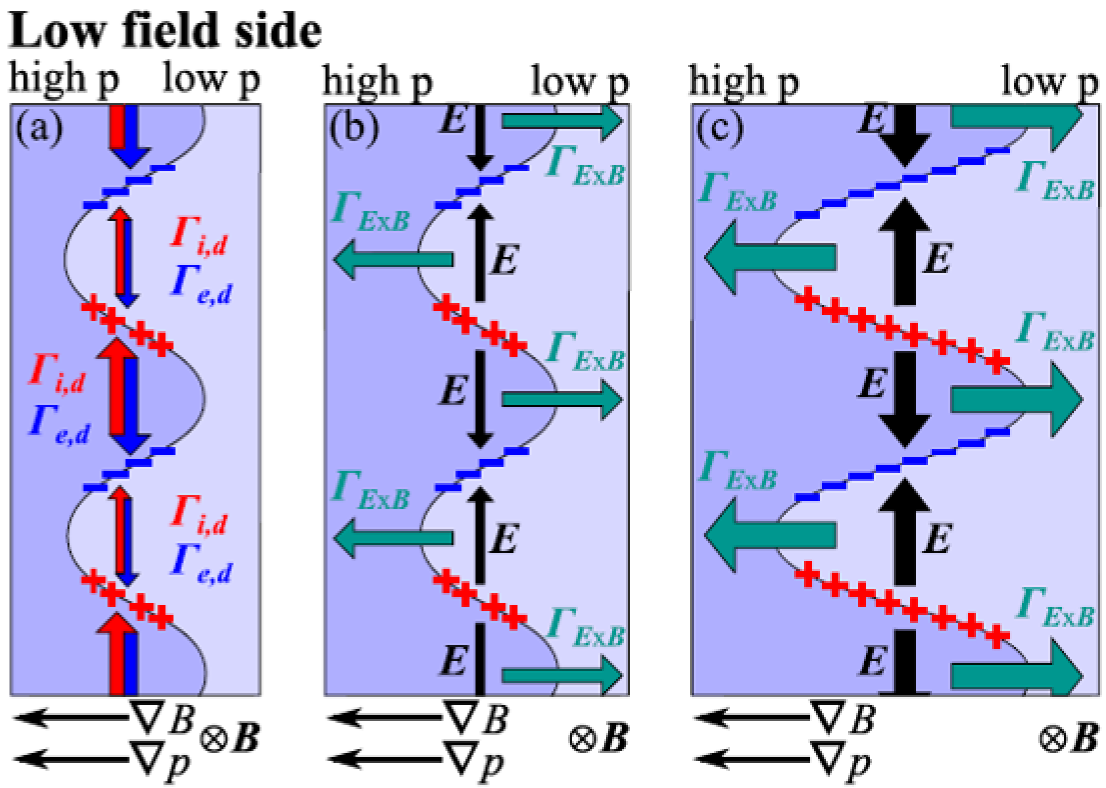
Zoo of linear instabilities predicted at various spatial scales



In addition, there can be electromagnetically driven instabilities, driven unstable at higher $\beta \propto \frac{p}{B^2}$: microtearing modes (MTM), kinetic ballooning modes (KBM)

Non-trivial nonlinear interactions between electron and ion scales currently a hot topic (Maeyama PRL 2015, Howard NF 2016)

Physical intuition of destabilising mechanism of ∇B drift. Rosenbluth-Longmire picture



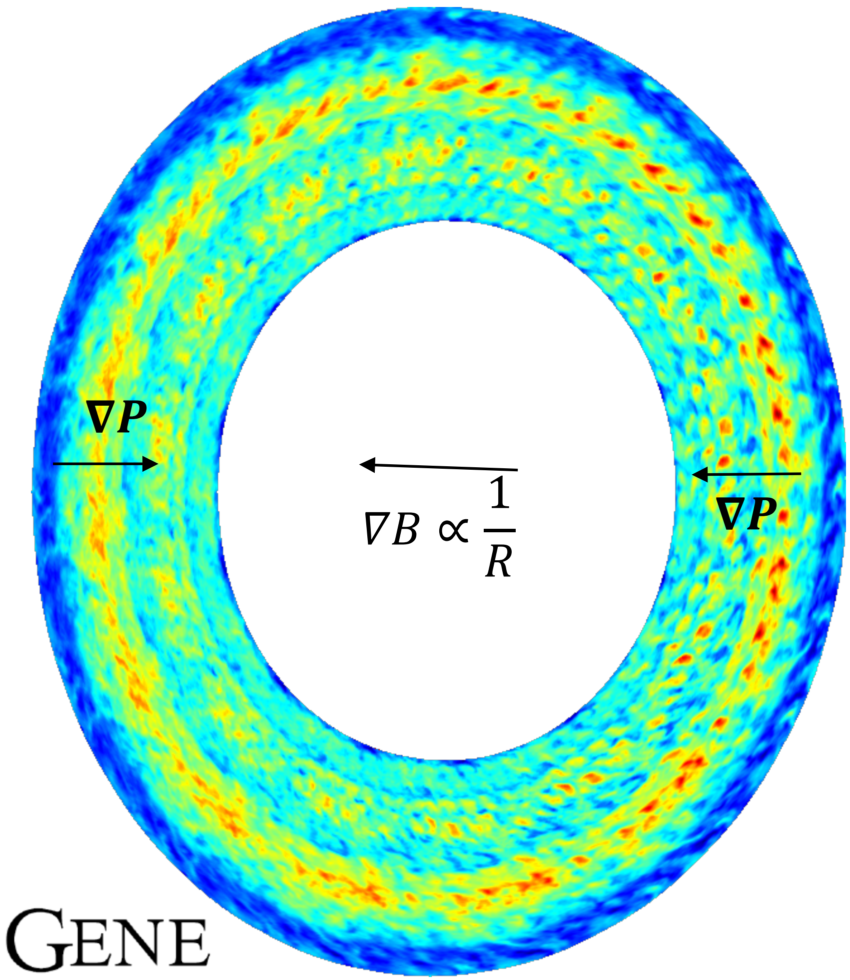
$$v_{Ds} \approx \frac{T_s}{q_s B} \frac{(\boldsymbol{B} \times \boldsymbol{\nabla}\mathrm{B})}{\mathrm{B}^2}$$

$$\Gamma_{sd} = n v_{Ds} = \frac{p_s}{q_s B} \frac{(\boldsymbol{B} \times \boldsymbol{\nabla}\mathrm{B})}{\mathrm{B}^2}$$

- ∇B drift → charge separation. Different fluxes due to pressure gradient → charge buildup
- When $\nabla B \nabla p > 0$. Leads to $E \times B$ drift in a direction that amplifies the fluctuation → instability
- This occurs in the tokamak low-field-side. On high-field-side fluctuations dampen

Acknowledgements to Oliver Linder

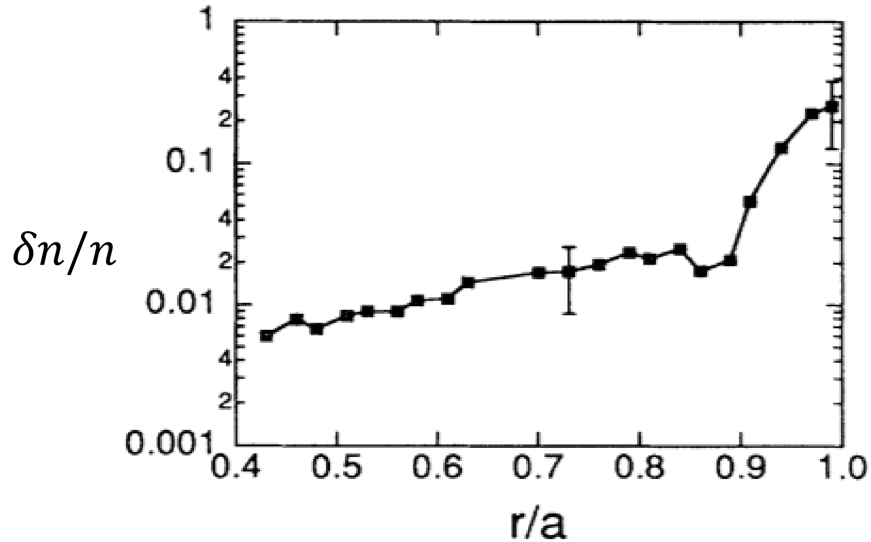
Example of "ballooned" turbulence at low-field-side, a consequence of "bad-curvature"



- Where ∇B and ∇P are in same direction ("bad curvature"), fluctuations much larger
- Similar mechanism for MHD "ballooning modes"
- Minimizing the region of bad-curvature is a method to optimize confinement: magnetic surface shaping
- Stellarators have increased freedom for minimizing bad curvature – an additional new component in stellarator optimisation

Orders of magnitude calculational speedup can be gained with reduced turbulence modelling

- Nonlinear simulations need massive HPC resources. Validation by experiment has proven the veracity of underlying model
- However, not fast enough for routine prediction of tokamak temperatures, densities, rotation. This is a prerequisite for confinement optimisation
- We have already seen that fluctuations in core are small: $\frac{\delta n}{n} \sim 1\%$. Suggests that linear physics could be relevant! Can we develop fast reduced models based on linear instabilities directly?

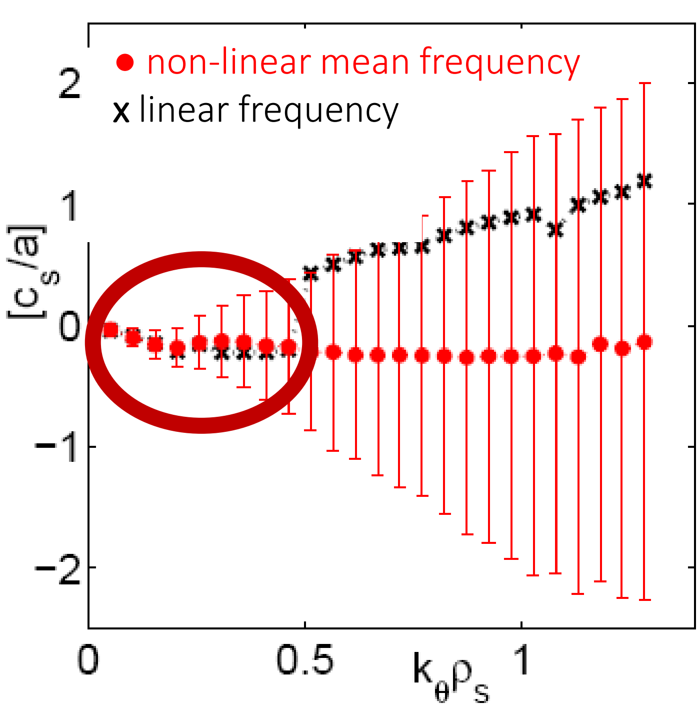


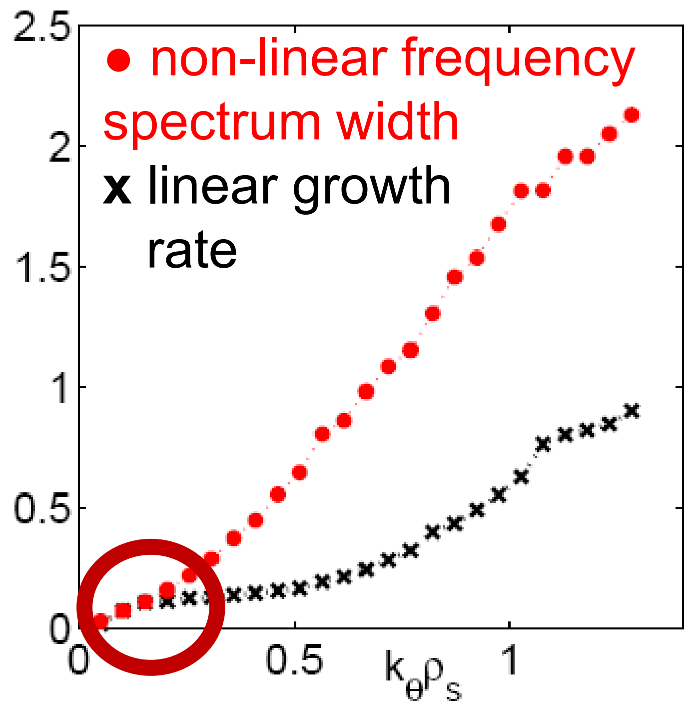
Fonck PRL 1993

In wavenumber range that drives most transport (<0.5 $k_\theta \rho_s$), nonlinear simulations shown strong signatures of underlying linear modes

- Mean frequencies in nonlinear simuations match linear eigenfrequencies
- Nonlinear frequency broadening matches linear growth rate: nonlinear decorrelation balances linear growth

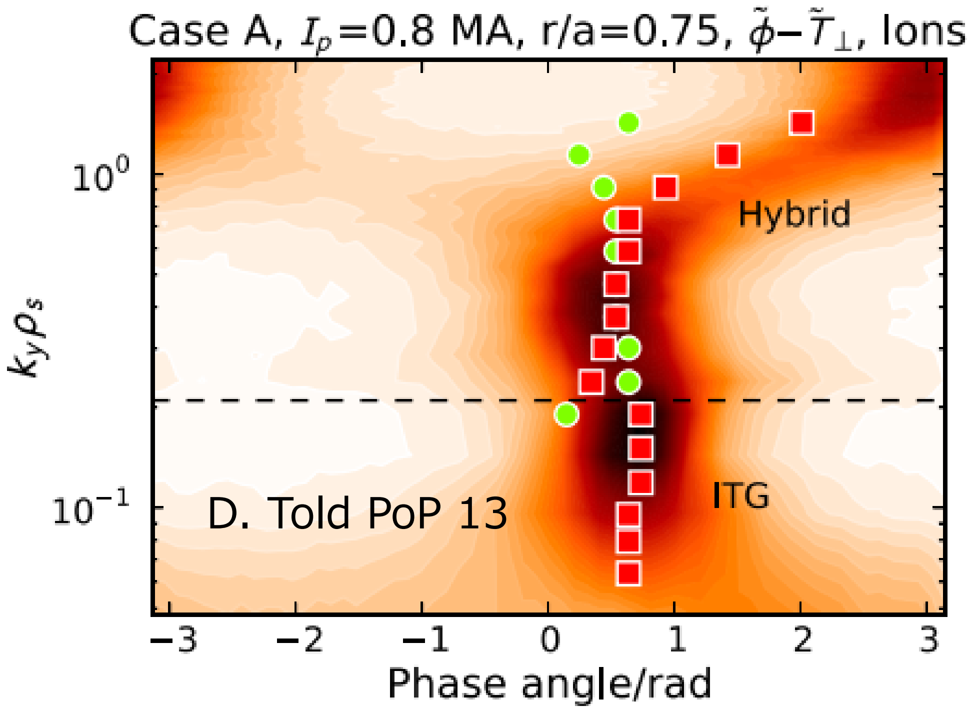
GYRO linear and non-linear of Tore-Supra #39596 at r/a = 0.7 [Casati 2009 PhD]





[More examples: Dannert PoP05, Lin PRL07, Merz PRL08, Casati NF09]

Phase shifts from nonlinear simulations in transport driving range (<0.5 $k_\theta \rho_s$) matches linear phase-shifts



Relative phase comparison (cross-phase) between $T_\perp$ and electrostatic potential fluctuations

GENE nonlinear simulation of AUG discharge

Can consider tokamak turbulence in transport driving ranges:
"Bath of linear-like fluctuations whose amplitude and exact wavenumber spectra are set by nonlinear physics"

Linear fluctuation characteristics are not washed out of nonlinear system

Sketch of how to construct a "quasilinear" transport model

1. Solve dispersion relation. Get set of ω_k
   (typically the bottleneck)

Quasineutrality

$$\sum_s q_s n_s = \sum_s q_s \int d^3 v \delta f_{s,\omega,k} = 0$$

The linear response

$$\delta f_{s,\omega,k} = \frac{F_M Z_s}{T_s} \frac{\omega_s^* - k_\parallel v_{\parallel s} - \omega_{Ds}}{\omega_k - k_\parallel v_{\parallel s} - \omega_{Ds}} \delta\phi_k$$

Solution for instabilities

$$\omega_k = \omega_r + i\gamma$$

γ is growth rate since all $\propto e^{-i\omega t}$

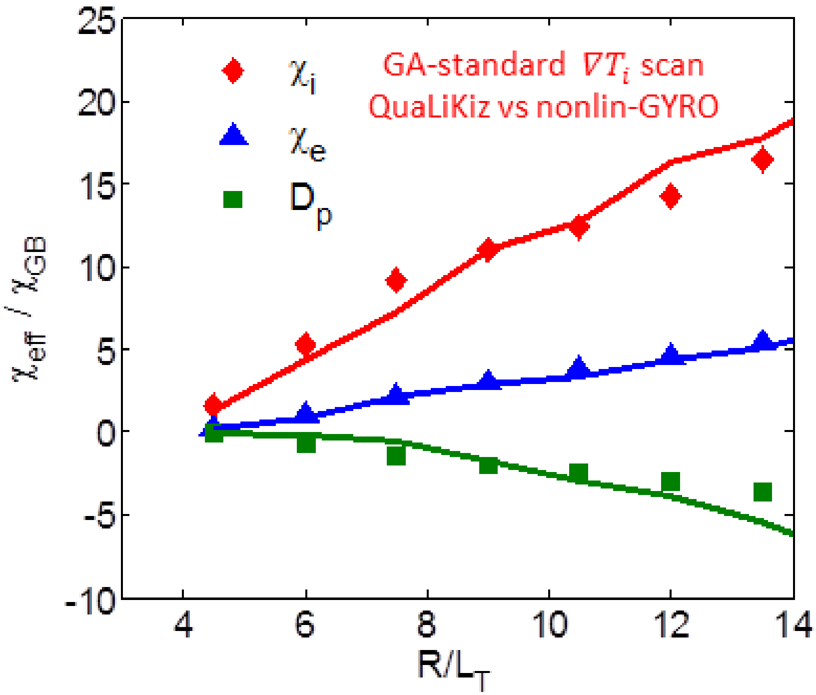
2. For those ω_k, calculate quasilinear fluxes. Phase shifts from linear response

$$\mathrm{Q}_s = \sum_k \langle \delta T_s \delta \mathrm{v}_{rk} \rangle_{t,\theta} \quad \longrightarrow \quad \mathrm{Q} = \sum_k \left\langle \frac{v^2 k F_M Z_s}{B T_s} \frac{\omega_s^* - k_\parallel v_{\parallel s} - \omega_{Ds}}{\omega_k - k_\parallel v_{\parallel s} - \omega_{Ds}} \right\rangle_v \left|\delta\phi_k\right|^2$$

Key point: linear theory provides no information on $\delta\phi_k$ spectral form and nonlinear saturated amplitude! Needed to evaluate transport level

$|\delta\phi|^2$ prescribed based on physical motivations and fits to nonlinear simulations (the dark arts of quasilinear transport models)

Quaslinear transport models have been developed and are in use by community



The "QuaLiKiz" quasilinear gyrokinetic reduced turbulence model
(C. Bourdelle, *et al.* 2016 PPCF
J. Citrin *et al.*, 2017 PPCF)

10 CPU seconds to calculate fluxes at one radius. $\times 10^6$ faster than nonlinear!

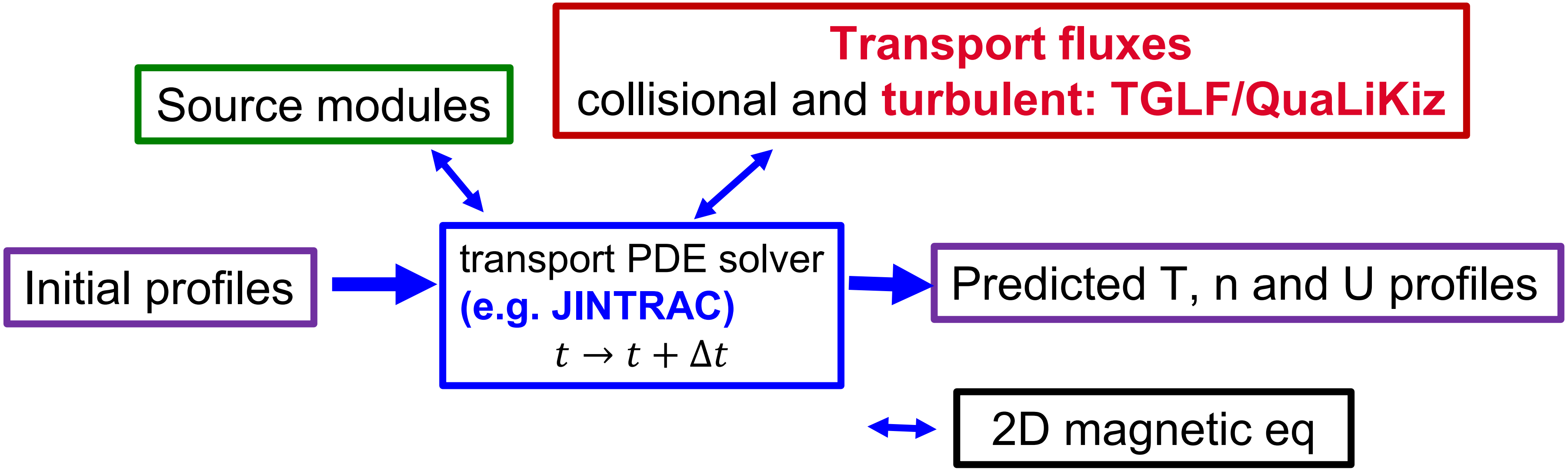
Allows for dynamic simulations of the tokamak discharge in reasonable times

Simulates turbulent transport of electron and ion heat, particle, impurity, and rotation

Other example is widely used model TGLF [Staebler PoP '08]. Gyrofluid model (with closures to approximate gyrokinetic)

Quasilinear transport models coupled to tokamak simulation suites for discharge prediction and optimisation

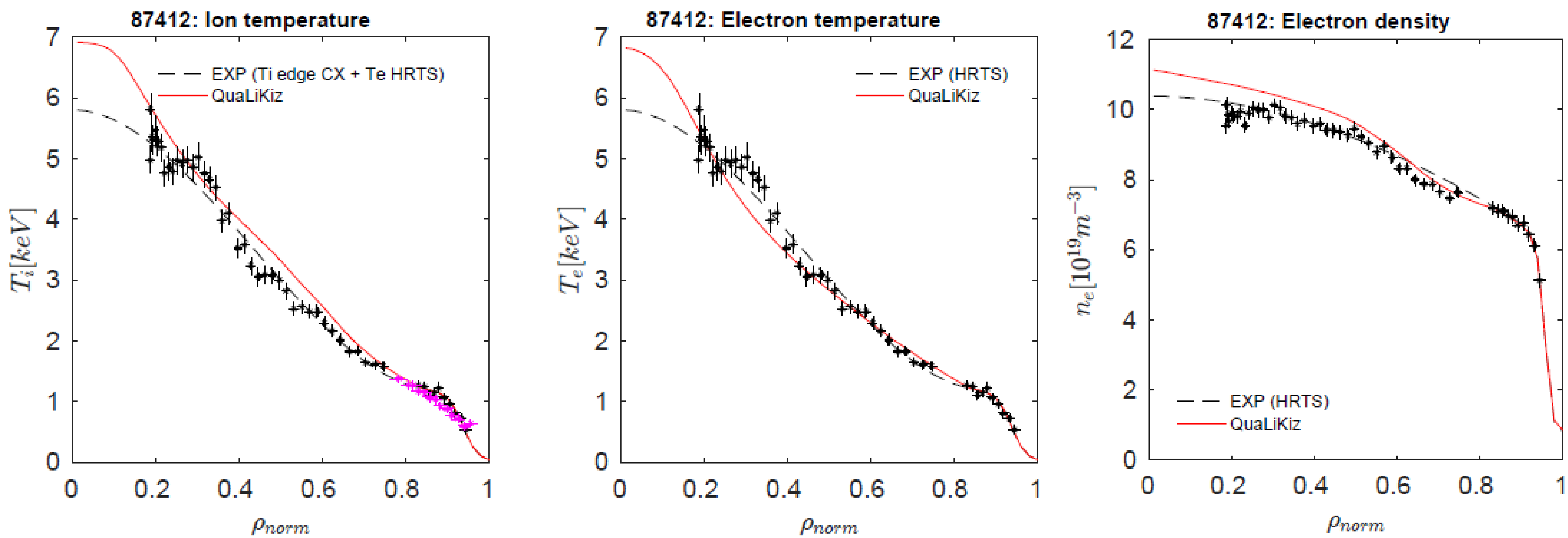
- First-principle-based tokamak core confinement predictions with tokamak simulators. Multiple tokamak scenario suites in community, e.g. JINTRAC*/ASTRA/CRONOS/ETS/RAPTOR.
- Computation time is ~100CPUh for 1s of JET plasma. Efficient parallelization up to ~20 cores (bottleneck is turbulent transport)
- Example of "integrated modelling". In principle can include pedestal, edge, SOL, neoclassical transport, MHD... towards full device modelling



* G. Cenacchi, A. Taroni, JETTO: A free-boundary plasma transport code, JET-IR (1988)
M. Romanelli *et al.*, 2003, 23rd International Toki Conference

JINTRAC-QuaLiKiz can reproduce actual core confinement observaitons in actual discharges

JET-ILW baseline scenario #87412 (3.5MA/3.35T)

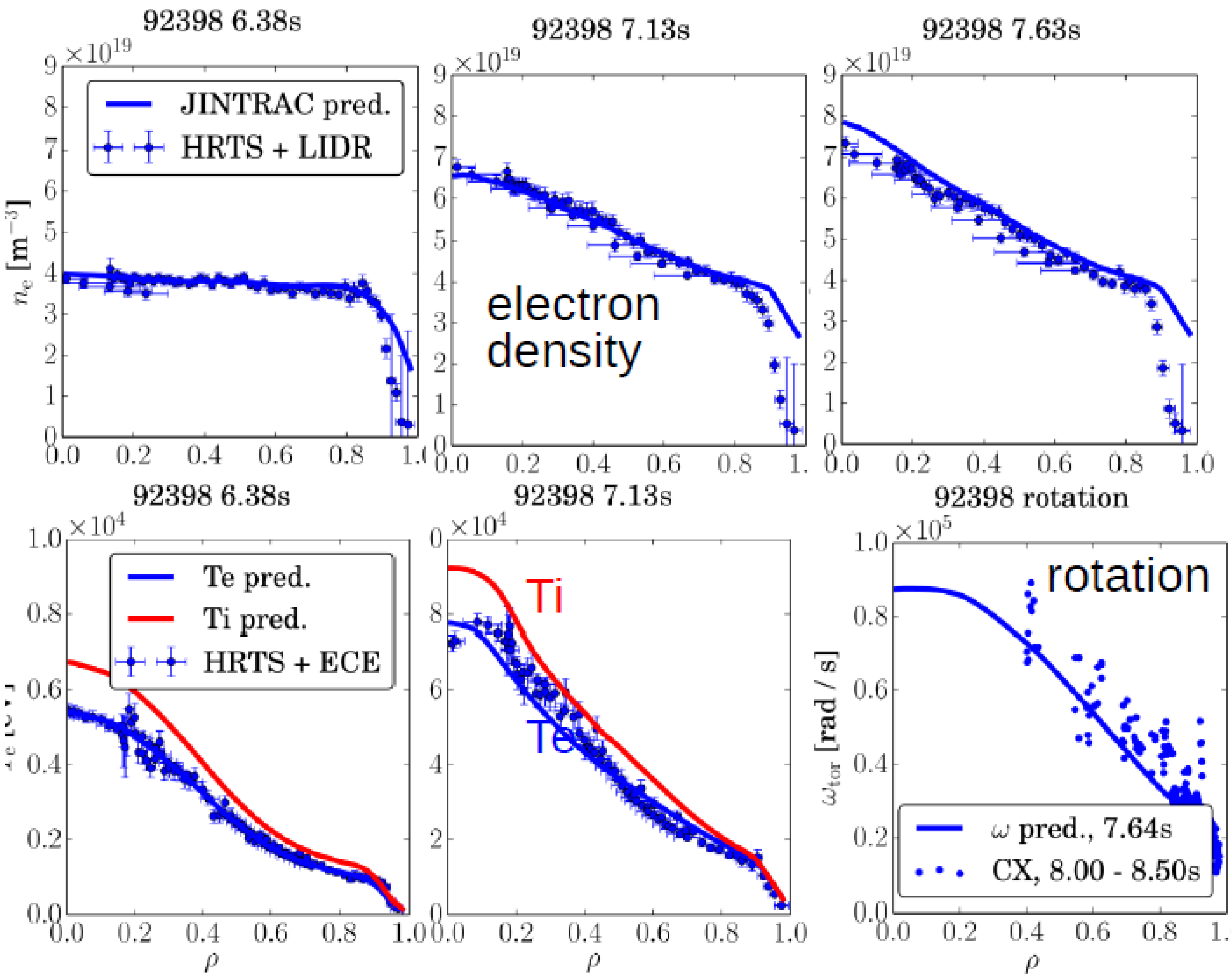


- Core boundary condition at $\rho = 0.85$
- Includes impact of turbulence on ion and electron scales, and a model of how rotation shear stabilises turbulence (see Hatch talks)

J.Citrin et al., PPCF 2017

JETTO-QuaLiKiz dynamic validation of JET discharge including rotation prediction and impurities

Among highest performance JET-ILW scenario: $B_T/I_p = 2.8T/2.2MA$, $\tau_E = 0.17s$



- Heat, particle, momentum, and impurity transport
- Dynamic density evolution well captured
- Captures W-accumulation timescale due to density gradient buildup. Neoclassical transport by NEO code [Belli and Candy 2016]
- Basis for predict first simulations of JET-DT, ITER, DEMO

F Casson IAEA 2018, *to be submitted to Nucl. Fusion*

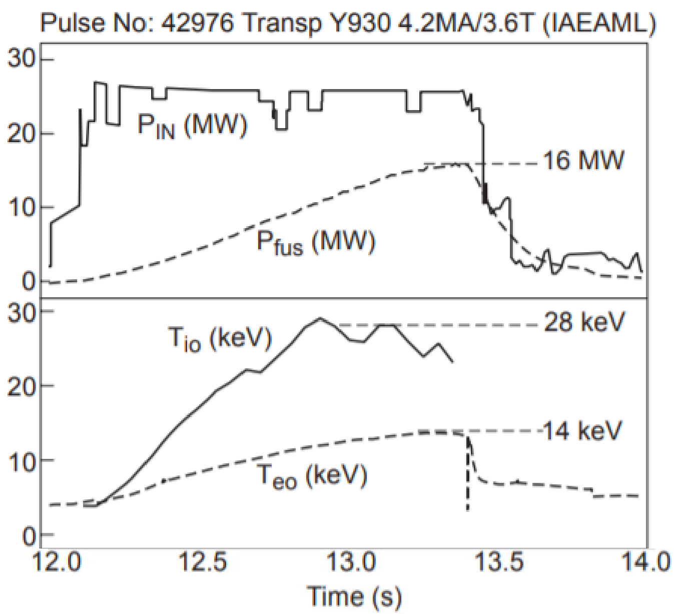
Contents

Understanding core turbulence opens the road to optimizing core confinement

Let's look closer at the gyrokinetic dispersion and get a feeling for the trends

Trapped electrons — Trapped ions — Passing ions

$$\sum_s \left(1 + Z_{eff}\frac{T_e}{T_i} - f_t \langle \frac{\omega_k - \omega_e^*}{\omega_k - \omega_{De}} J_0(k_\perp \delta_e)\rangle_t - \frac{n_i Z_i^2}{n_e}\frac{T_e}{T_i}\left(f_t \langle \frac{\omega_k - \omega_e^*}{\omega_k - \omega_{Di}} J_0(k_\perp \delta_i)\rangle_t + f_p \langle \frac{\omega_k - \omega_i^*}{\omega_k - k_\parallel v_{\parallel i} - \omega_{Di}} J_0(k_\perp \rho_i)\rangle_p\right)\right)\delta\phi_k = 0$$



JET team, Nucl. Fusion 1999

- Ion response reduced at increased $\frac{T_i}{T_e}$, expect higher critical threshold for ITG turbulence
- Indeed, most performant fusion discharges, including JET DT record of 16.1MW in 1997, achieved in "hot ion mode" with $\frac{T_i}{T_e} \approx 2$

Picking apart the stabilising and destabilising terms in the resonant denominator of the response function

Passing ion response (key for ITG)

$$\langle \frac{\omega_k - \omega_i^*}{\omega_k - k_\parallel v_{\parallel i} - \omega_{Di}} J_0(k_\perp \rho_i) \rangle_p$$

$k_\parallel v_\parallel$ term leads to stabilising Landau damping.
Increasing this term is stabilising.
How to approximate $k_\parallel$?

We know that $k_\parallel \ll k_\perp$, so let's assume the following ansatz for the mode structure

$$\delta f \propto \delta\phi = A(r,\theta) e^{in(\varphi - q(r)\theta)}$$

Safety factor: $q = \frac{rB_t}{RB_p}$ circular geometry approximation

Lower q → more current inside flux surface

φ is toroidal direction (so n is toroidal mode number)
θ is poloidal direction
Since $q = \frac{d\varphi}{d\theta}$, eikonal phase constant along the field line of reference flux surface

Split into slow varying part A, and fast eikonal
$k_\theta, k_\parallel$ defined only to operate on eikonal

Picking apart the stabilising and destabilising terms in the resonant denominator of the response function

Passing ion response (key for ITG)

$$\langle \frac{\omega_k - \omega_i^*}{\omega_k - k_\parallel v_{\parallel i} - \omega_{Di}} J_0(k_\perp \rho_i) \rangle_p \qquad \delta f \propto \delta\phi = A(r,\theta) e^{in(\varphi - q(r)\theta)}$$

$$k_\theta = -\frac{i}{r}\frac{\partial}{\partial\theta} \rightarrow k_\theta = \frac{nq}{r}$$

Relation between toroidal and poloidal mode number

$$k_\parallel = i\frac{\partial}{\partial s}$$

s is path in parallel direction on reference flux surface:
$\frac{ds}{d\varphi} = R, \; \frac{ds}{d\theta} = q_0 R$ (circular geometry approximations)

$$k_\parallel = n\left(\frac{d\varphi}{ds} - q(r)\frac{d\theta}{ds}\right) = n\left(\frac{1}{R} - \frac{q(r)}{Rq_0}\right) \approx \frac{nq'}{qR}x = \frac{k_\theta}{R}\frac{s}{q}x$$

- Taylor expand q-profile, and magnetic shear $s \equiv r\frac{q'}{q}$
- x is distance from reference flux surface. More Landau damping at higher x, important for setting radial structure of mode
- Increased magnetic shear, and lower q, should increase mode stability!

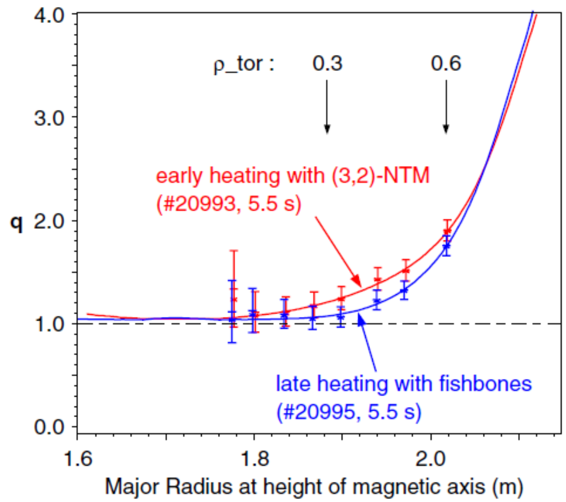
Indeed, from more detailed instability analysis for ITG (Guo-Romanelli PFB 1993):

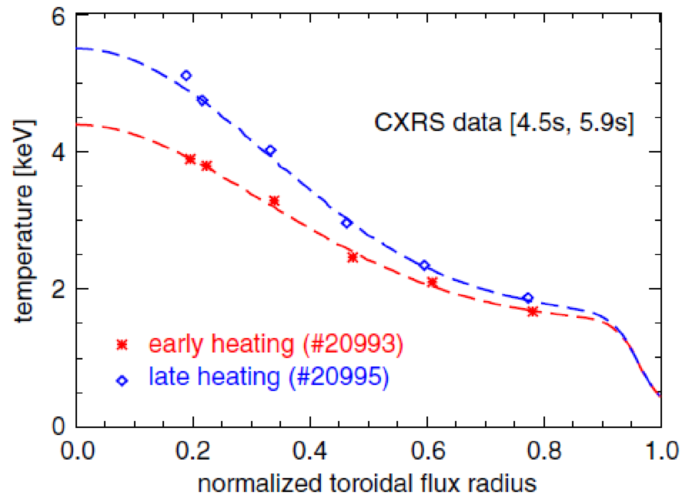
$$\frac{R}{L_{Ti}}\bigg|_{crit} \approx \frac{4}{3}\left(1 + \frac{T_i}{T_e}\right)\left(1 + 2\frac{s}{q}\right)$$

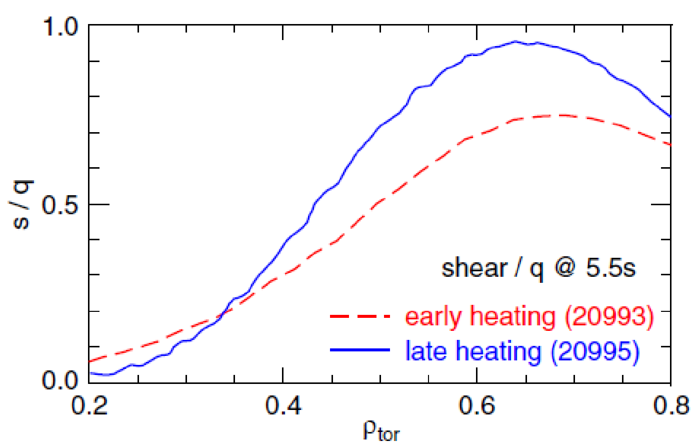
(with adiabatic electron response and flat density)

Tailoring the q-profile is one method to optimise confinement

Example from ASDEX-U advanced scenarios: changing the heating time impacts the q-profile evolution, with a clear impact on confinement in the expected direction







ITG critical threshold approximation

$$\left.\frac{R}{L_{Ti}}\right|_{crit} \approx \frac{4}{3}\left(1+\frac{T_i}{T_e}\right)\left(1+2\frac{s}{q}\right)$$

J Stober et al., Nucl. Fusion **47** (2007)

Picking apart the stabilising and destabilising terms in the resonant denominator of the response function

Passing ion response (key for ITG)

$$\langle \frac{\omega_k - \omega_i^*}{\omega_k - k_\parallel v_{\parallel i} - \omega_{Di}} J_0(k_\perp \rho_i) \rangle_p$$

ω_D term (∇B drift) impacts the degree of bad curvature. Destabilizing when in same direction as ω^* (diamagnetic drift)

$$\omega_D = k_\theta \frac{mT}{ZB} \frac{\nabla B}{B}$$

Sketch of ∇B and ω_D calculation in shifted circle geometry in a small inverse aspect ratio expansion

$$R = R_0 + rcos\theta + \Delta(r)$$
$$Z = sin\theta$$

$\Delta(r) \equiv$ Shafranov shift

$\Delta''(r) \approx -\frac{\alpha}{r}$, with $\alpha \equiv -q^2 R \beta'$

q is q-profile
β is plasma beta, so
β' related to pressure gradients

In this coordinate system (sorry, skipping steps):

$$\boldsymbol{\nabla} = (1 + 2\Delta' \cos\theta)^{-1/2} \partial_r \hat{\boldsymbol{r}} + \frac{1}{r} \partial_\theta \hat{\boldsymbol{\theta}} \approx (1 + \alpha cos\theta) \partial_r \hat{\boldsymbol{r}} + \frac{1}{r} \partial_\theta \hat{\boldsymbol{\theta}}$$

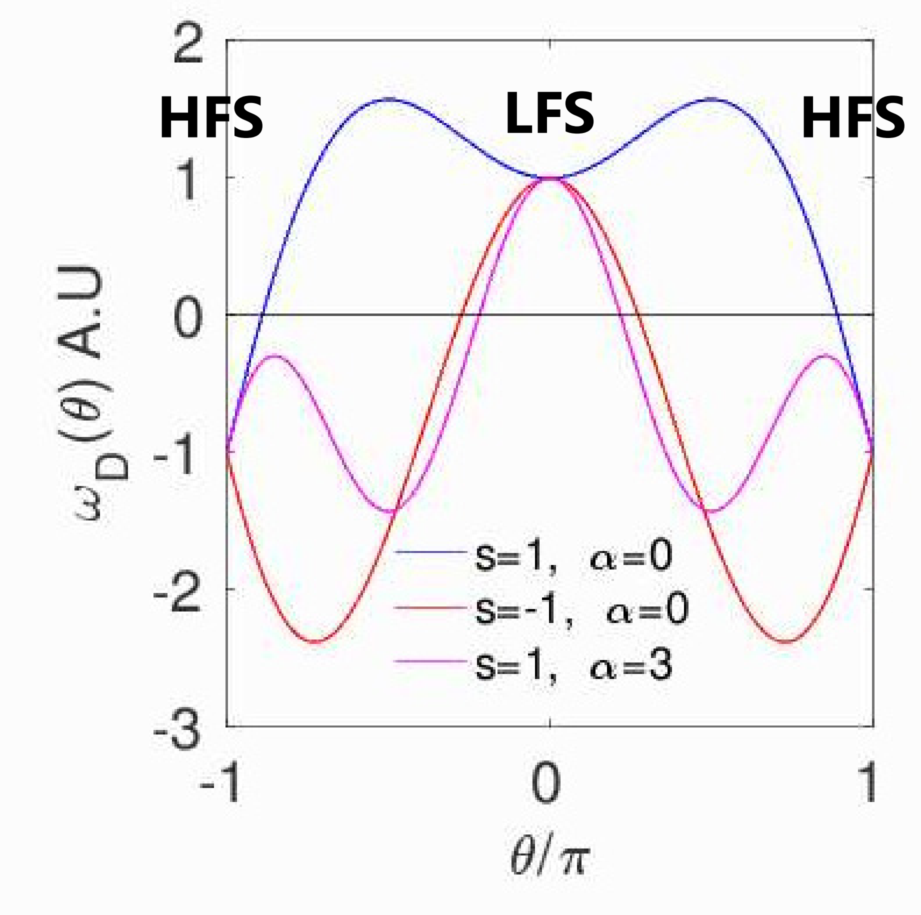
$$B = \frac{B_0}{1 + \frac{r}{R} cos\theta + \frac{\Delta(r)}{R}} \approx B_0 (1 - \frac{r}{R} cos\theta - \frac{r}{R} \Delta')$$

$$\omega_d = \frac{k_\theta m T}{ZBR} (cos\theta + (s\theta - \alpha sin\theta) sin\theta)$$

Acknowledgements to Oliver Linder

Negative magnetic shear and/or high pressure gradients can reverse the sign of the drift frequency and stabilise the mode! This is relevant for ITG and TEM instabilities

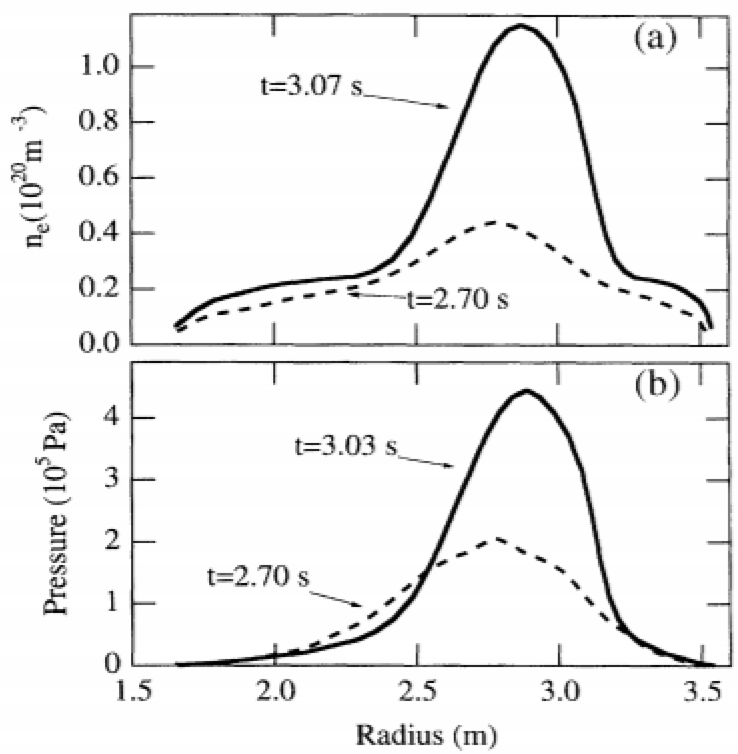
$\omega_d \propto \ (cos\theta + (s\theta - \alpha sin\theta)sin\theta)$ Parallel structure of the ∇B drift frequency

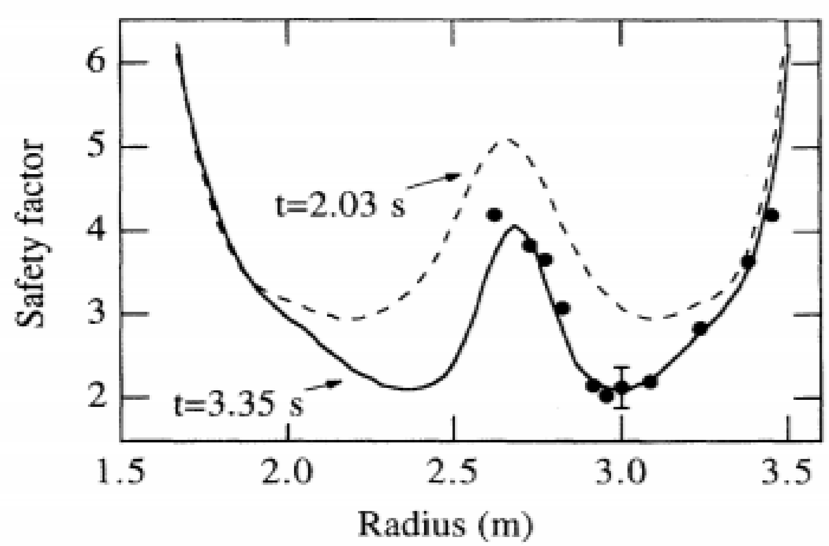


At negative magnetic shear, and for high Shafranov shift, a significant portion of the drift frequency can flip direction, which is highly stabilizing

Current distributions leading to inverse magnetic shear can show improved confinement and “internal transport barriers”

Decreased s reduces (and can even reverse) drift resonance and is stabilizing!
Internal transport barriers can be found with reverse magnetic shear

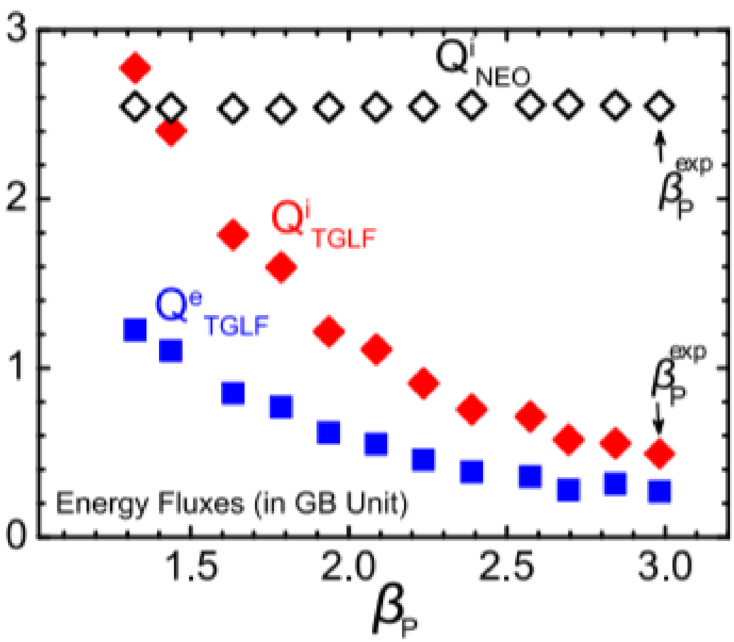


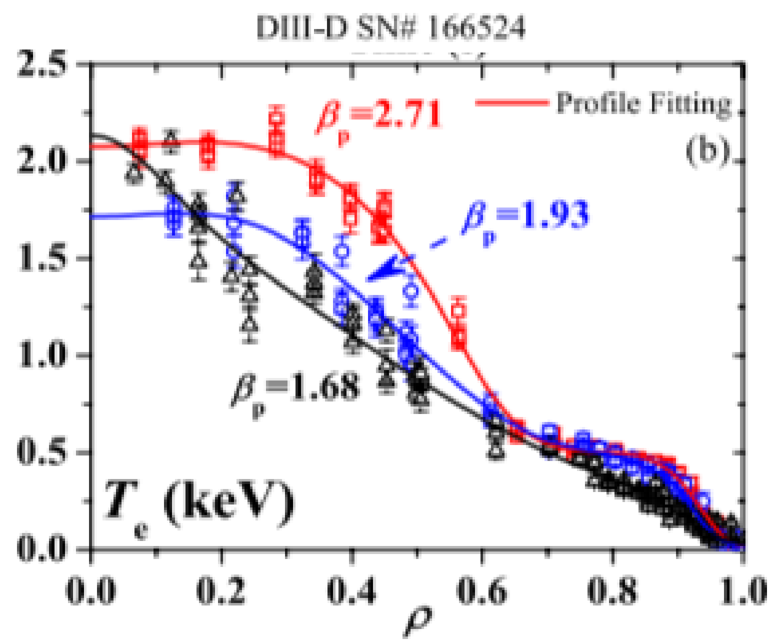


Levinton PRL 1995, at TFTR

Improved confinement observed with increased Shafranov shift in advanced scenarios, e.g. in DIII-D and EAST

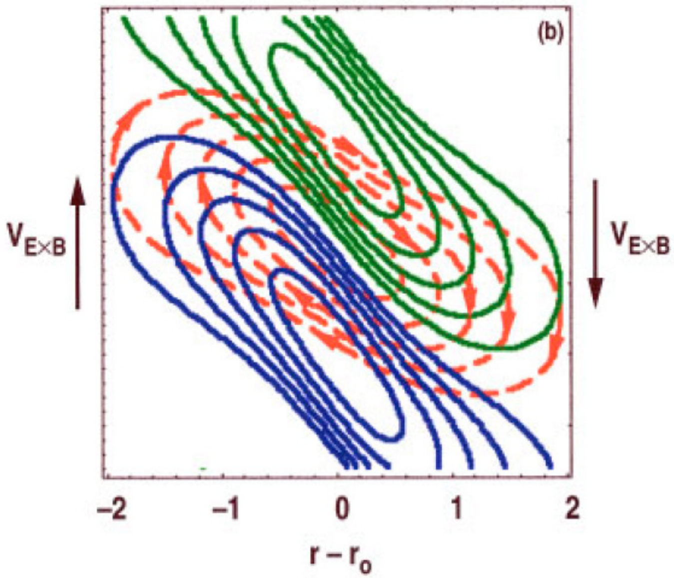
- High beta-poloidal scenarios DIII-D, where $\alpha \propto \beta_p$
- Internal transport barrier (ITB) arises with increasing β_p (done by increasing q)
- Predicted ion-scale turbulent transport level in ITB drops beneath neoclassical values





Ding et al., Phys. Plasmas **24** (2017)

Perpendicular rotation shear quenches ion scale turbulence

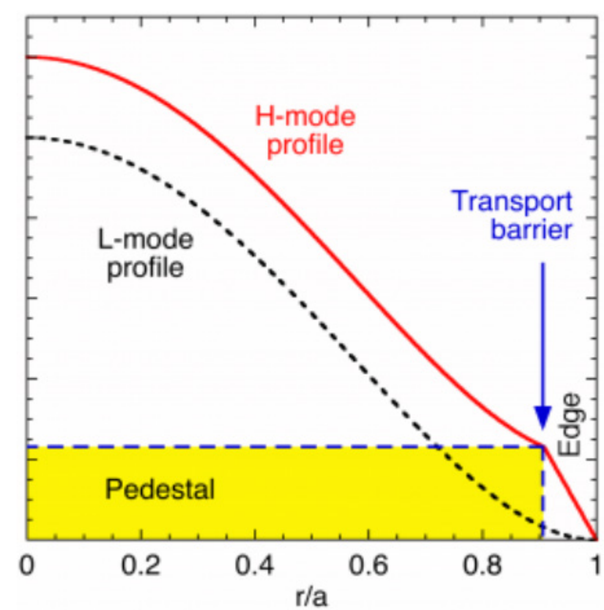


K Burrell Phys. Plasmas **4** (1997)

- Direct stabilisation of linear modes
- Reduces radial correlation length in nonlinear system
- In core, mostly due to torque injection by NBI heating. Increases critical gradients

- Towards edge, transport bifurcation past heating threshold, associated with rise of large $E \times B$ shear. Formation of pressure "pedestal" and transition from Low to High confinement mode

fusionwiki



Summary of key points

- Core confinement degradation compared to neoclassical expectations due to turbulence driven by linear instabilities arising from $\frac{\nabla T}{T}, \frac{\nabla n}{n}$
- Enormous strides in gyrokinetic theory and nonlinear simulation. We have a “standard model” for tokamak core turbulence that agrees with experiments
- Weak turbulence approximation means that reduced modelling based on quasilinear theory can provide relatively fast (24h on small cluster) tokamak scenario predictions, 1 million times faster than nonlinear.
- Many known mechanisms for confinement improvement: negative magnetic shear, Shafranov shift, rotation shear, increased β, magnetic geometry shaping. Pragmatic scenario optimisation feasible with tokamak simulators using reduced quasilinear models tuned to nonlinear simulations
- Open questions: interaction between ion and electron turbulence, exact mechanism of L-H transition, turbulence in edge and SOL (no reduced models in routine use), scaling of confinement with changing isotopes

Further reading

Topical reviews:
X. Garbet, "Gyrokinetic simulations of turbulent transport" Nucl. Fusion **50** (2010)
W Horton "Drift waves and transport" Rev. Mod. Phys. (1999)

Classical approaches to nonlinear gyrokinetics
Frieman and Chen Physics of Fluids **25**, 502 (1982);
Lee Phys. Fluids **26**, 556 (1983)
Abel, Rep. Prog. Phys. 76 (2013)

Gyrokinetic field theory
Sugama Phys. Plasmas **7**, 466 (2000);
Brizard and Hahm, Rev. Mod. Physics **79** (2007)

Gyrokinetic simulations
References in www.genecode.org

Reduced transport models
C Bourdelle, Plasma Phys. Control. Fusion **58** (2016); www.qualikiz.com
G Staebler, Phys. of Plasmas **14** (2007);