An Intranuclear Cascade Model for Cluster-Induced Reactions

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Overview

- ✓ Background and motivation
- ✓ Model description
- ✓ Extension of model for cluster-induced reactions
- ✓ Conclusions

Particle transport codes

- **Particle transport codes** deal with transport and collision of various kinds of particles and heavy ions over wide energy ranges.
  - Nuclear physics, material sciences, space and geosciences, medical sciences.
- Nuclear reaction model is an essential part of transport code.
- The model I have been working with is to simulate the cascade stage of nuclear reactions. And it is incorporated in a particle transport code PHITS.

Application (1)

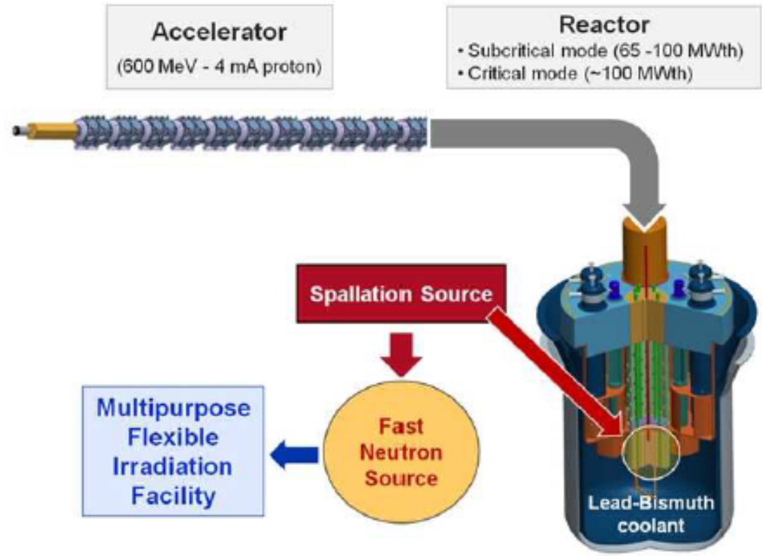
Accelerator Driven System (ADS)

✓ Transmutation of nuclear waste

Tens of thousands years

↓

Several hundred years



Source: Pedoux, S (2012) PhD Thesis

✧ To optimize ADS, particle transport code is essential.

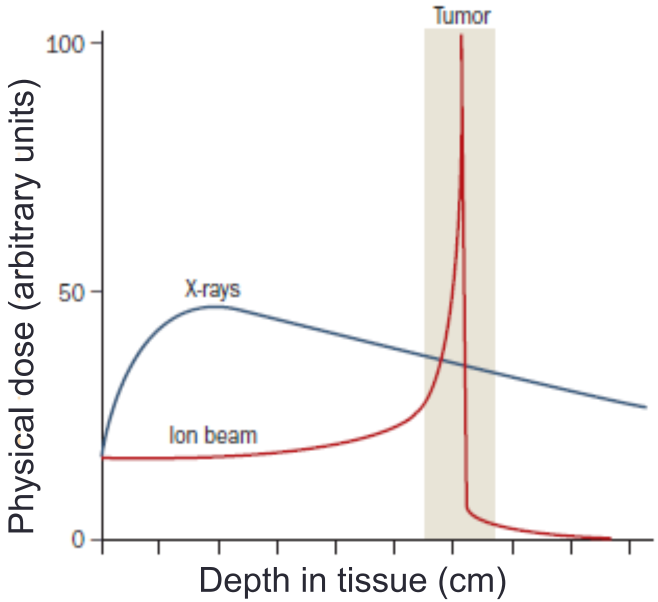
✧ The nuclear reaction models in the transport code need to simulate secondary particles like neutron, deuteron, alpha etc. initiated reactions besides proton induced reactions.

Application (2)

Heavy ion cancer therapy

Charged particle therapy (proton, ^{4}He, ^{12}C)

- Sharp increase of dose at well defined region
- RBE ratio is highest for Carbon therapy



Source: Durante, M. & Loeffler, J. S. *Nat. Rev. Clin. Oncol.* **7,** 37–43 (2010).

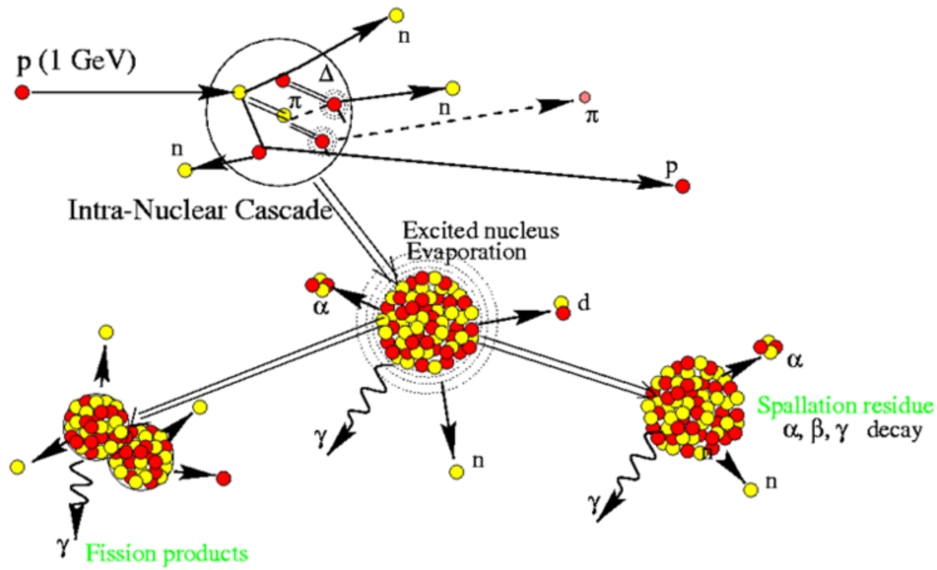
✓ Fragments (e.g. deuteron, alpha) produced in carbon therapy at large angle causes dose deposition in normal tissues.

✓ The model in transport code need to capable of handling the cluster-induced reactions for accurate dose estimation.

Nuclear reaction

High energy reactions are two stage process proposed by Serber*.

- **First stage**
  - Cascade stage, 10^{-22} sec.
  - Bertini, JAM, VEGAS, INCL, JQMD.



https://www-nds.iaea.org/spallations/

- **Second stage**
  - De-excitation of residual nucleus, 10^{-16} sec.
  - Evaporation/Fission model.

Nuclear Reactions at High Energies

R. Serber

Phys. Rev. **72**, 1114 – Published 1 December 1947

INC model overview

- Interactions between high-energy incident particle and target nucleons are approximated as individual nucleon-nucleon (NN) collision.
- The scattered nucleon follows a straight-line trajectory and repeats the collision one after another.
- The two-body collision is approximated as Quasi-Free scattering (QFS) with two-body collision cross-section.
- The nucleons that acquire enough momentum will emit the nucleus.

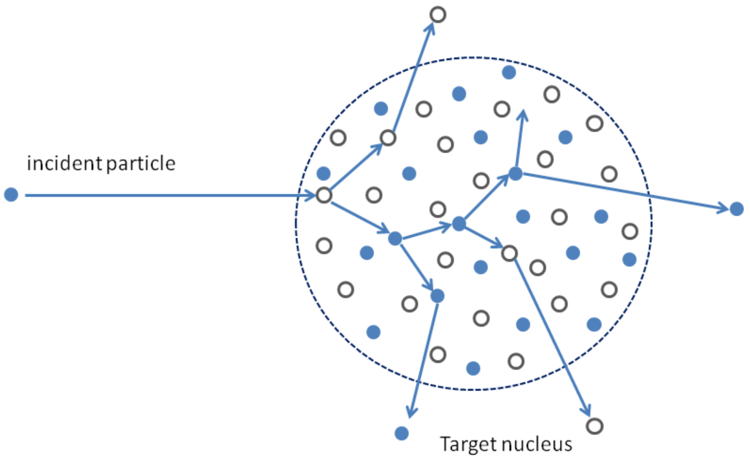


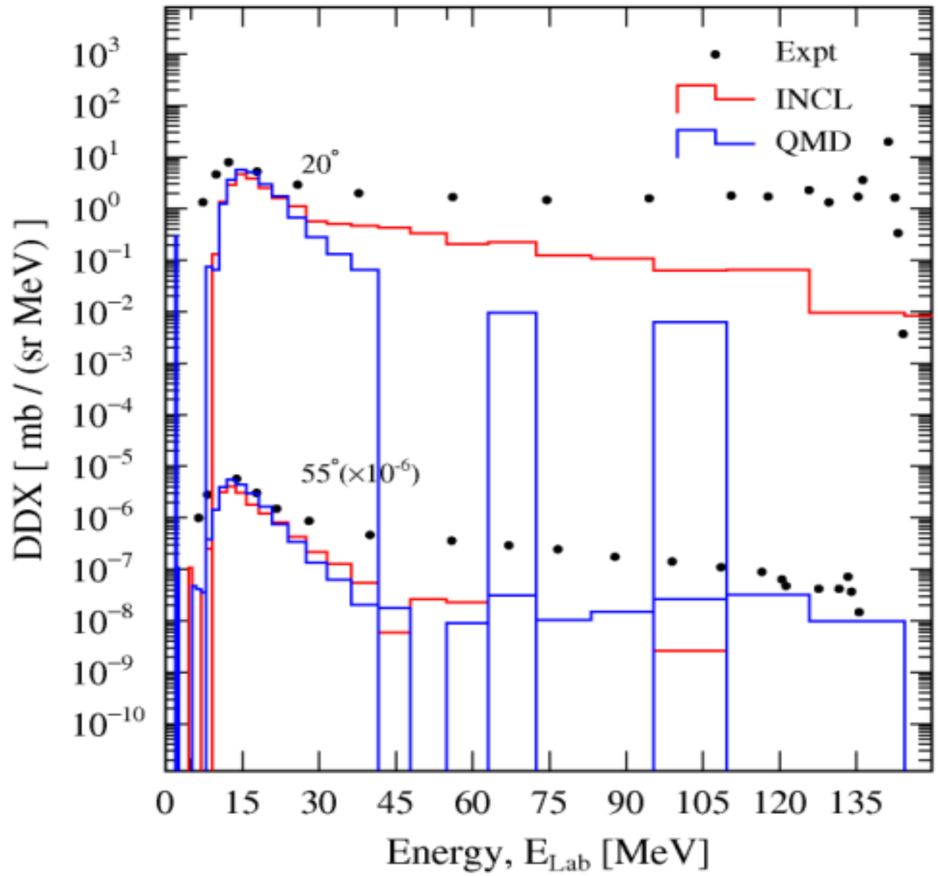
Fig. Schematic diagram of INC model.

Problems of nuclear models

For cluster incident reactions

- Bertini, JAM can not work
- INC and QMD show large discrepancies

$^{58}Ni(\alpha, \alpha'x)$, E_α = 140 MeV ; INCL, QMD



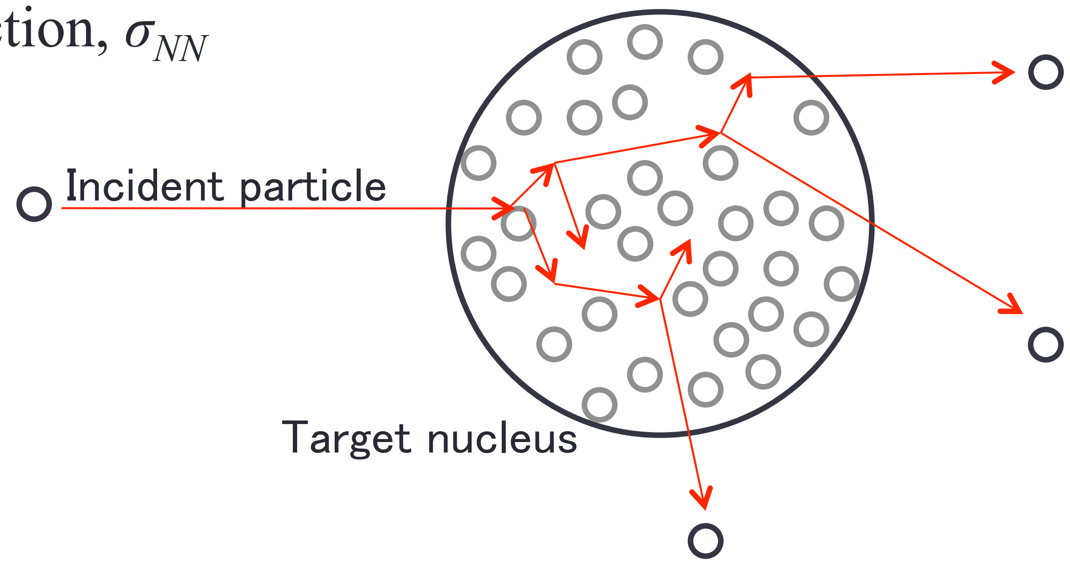
Purpose

- The purpose of this work is to introduce into the INC framework an idea of virtual excited state of cluster projectile, whose wave function is expressed as a superposition of different cluster units.
- To widen the applicable range of INC model for cluster-induced reactions.

INC Model for proton-induced reactions

1. Position and momenta of nucleons in target
   - Density $dist^{n}$: Woods-Saxon type
   - Momentum $dist^{n}$: Fermi-Dirac Distribution
2. Projectile sent to target with random impact parameter
3. Two nucleon undergo collision when the distance is smaller than NN cross-section, σ_{NN}

$$r \leq \sqrt{\frac{\sigma_{NN}}{\pi}}$$



INC model for cluster-induced reactions

Projectile ground state

- Position of nucleons ⇨ Wood-Saxon distribution.

$$\rho_{ws} = \begin{cases} \dfrac{\rho_0}{1 + \exp\left(\dfrac{r - R_{inc}}{a}\right)} & (r \le R_{max}) \\ 0 & (R_{max} \le r) \end{cases}$$

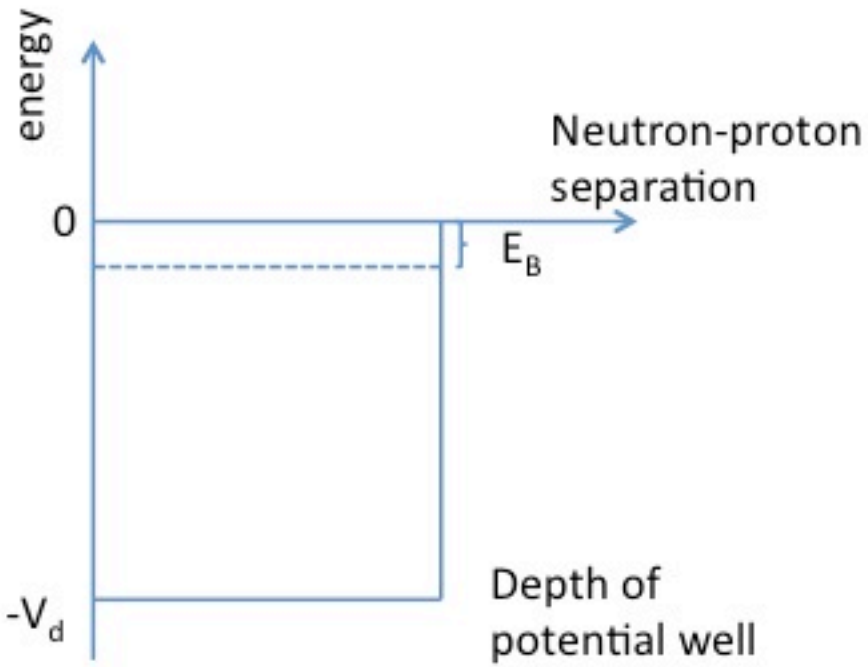
$$\mathrm{R_{max}} = \mathrm{R_{inc}} + 5a$$

projectile average radius, R_{inc}

- Nucleon momenta ⇨ Fermi-Dirac distribution.

Projectile potential depth

- Potential depth is chosen
- To fit the experimental data.
- V_d= 15 MeV, V_α = 40 MeV

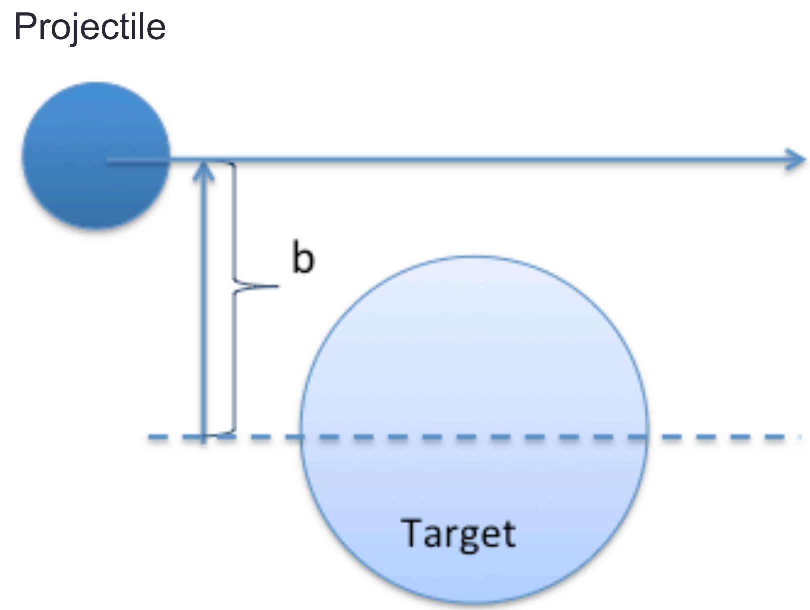


Maximum impact parameter

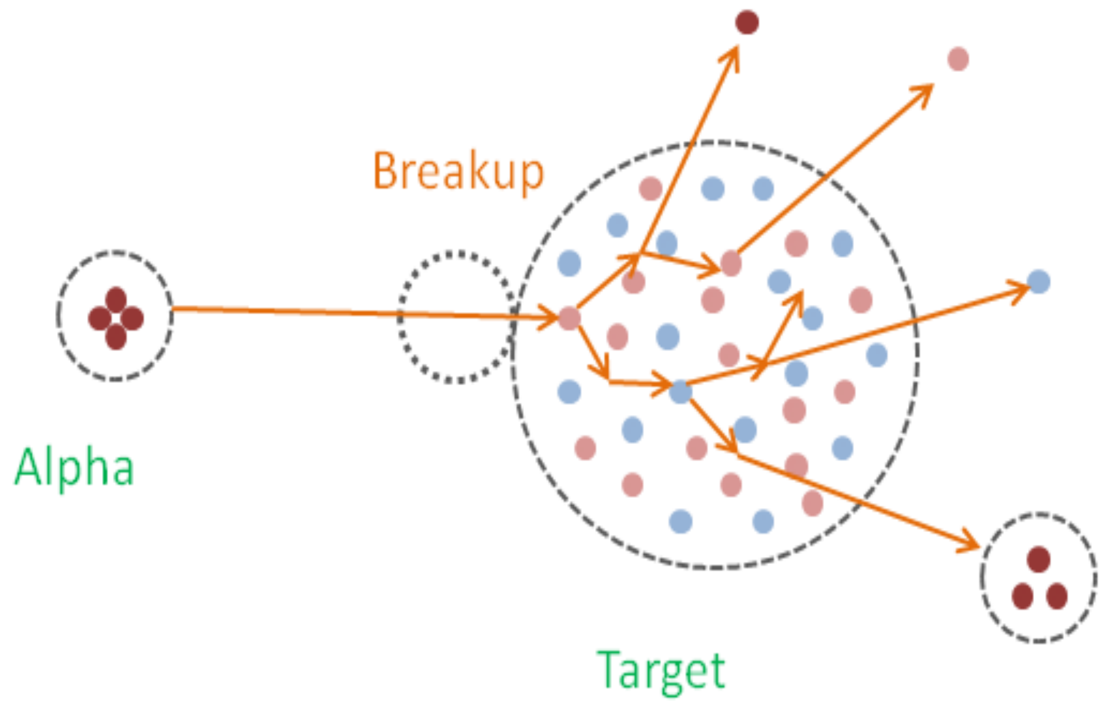
- Maximum impact parameter

$$b_{max} = R_P + R_T + 5a$$

- To fit the experimental data.



Projectile breakup



- Incident cluster may break up due to nuclear potential while entering the target nucleus.
- The breakup reaction is assumed to occur at the initial-state interaction.

Projectile breakup (alpha, deuteron)

- The initial alpha is considered as superposition of the different states that consists of cluster units. The wave function is

$$|\alpha_{init}\rangle = c_{\alpha 0}|\alpha\rangle + c_{\alpha 1}|{}^3\mathrm{He}n\rangle + c_{\alpha 2}|tp\rangle + c_{\alpha 3}|dd\rangle + c_{\alpha 3}|nnpp\rangle$$

with normalization of $\sum_{i=all} c_i^2 = 1$

- The deuteron wave function,

$$|d_{init}\rangle = c_{d0}|d\rangle + c_{d1}|pn\rangle$$

Breakup fragments	C
d	√70
p+n	√30

Cluster unit	C_α
α	√58
^{3}He+n	√5
t + p	√11
d + d	√16
2p + 2n	√10

Projectile break-up

The momentum of fragment,

$$\vec{P}_F = \sum_{N_i=1}^{A_F} \vec{P}_{N_i} + \frac{A_F}{A_\alpha} \vec{P}_\alpha$$

A_F fragment mass

A_α is alpha particle mass

$\vec{P}_F$ is the fragment momentum.

$\vec{P}_{N_i}$ is the momentum of the i-th nucleon in the fragment.

As example, the ^{3}He momentum is

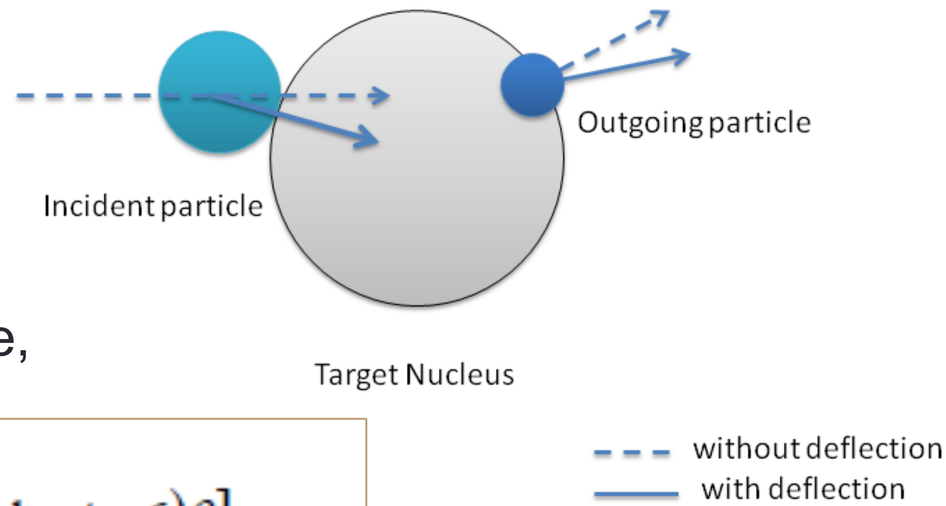
$$\vec{P}_{^3He} = \sum_{N_i=1}^{3} \vec{P}_{N_i} + \frac{3}{4} \vec{P}_\alpha$$

$\vec{P}_{N_i}$ is the momentum of ith nucleon of ^{3}He.

$\vec{P}_\alpha$ is the momentum of projectile alpha.

Probability of deflection angle

- The trajectory of incoming and outgoing particle get deflected due to nuclear potential.



The probability of deflection angle,

$$W_{def,d}(\varepsilon,\theta,A)=\exp[-0.001(1.3\varepsilon+\ln A+6)\theta]$$

$$W_{def,t}(\varepsilon,\theta,A)=\exp[-0.001(1.2\varepsilon+6\ln A-5)\theta]$$

$$W_{def,3He}(\varepsilon,\theta,A)=\exp[-0.001(1.2\varepsilon+6\ln A-5)\theta]$$

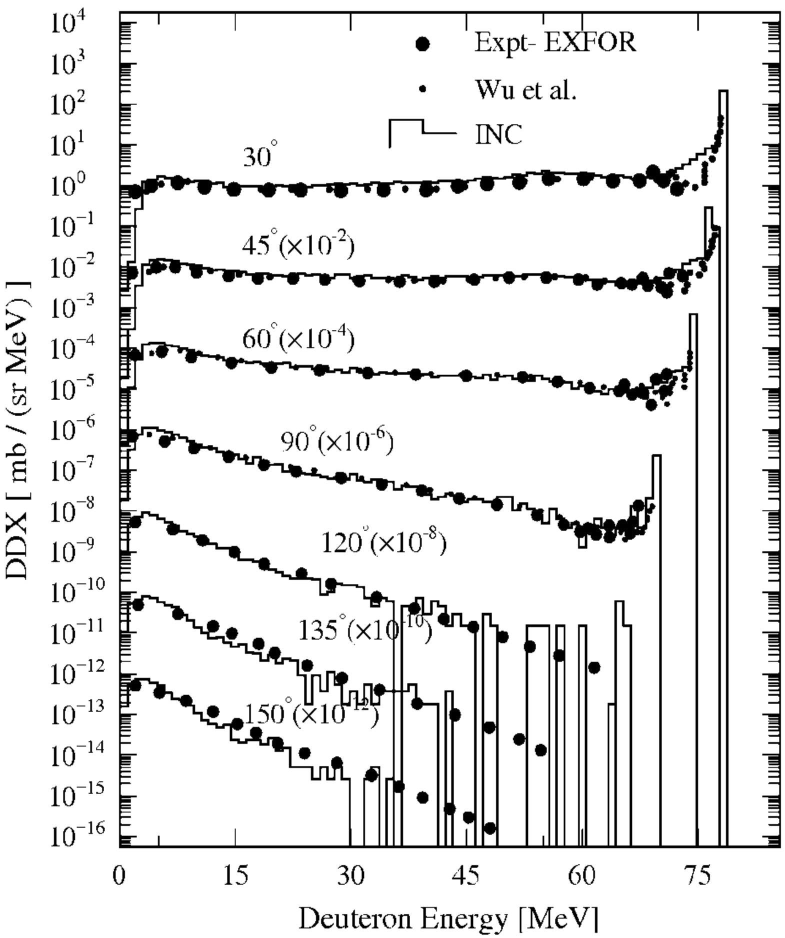
$$W_{def,\alpha}(\varepsilon,\theta,A)=\exp[-0.001(1.2\varepsilon-10\ln A+40)\theta]$$

✓ The angular distribution for elastic scattering experimental data were used to find these parameters for trajectory-deflection angular distribution.

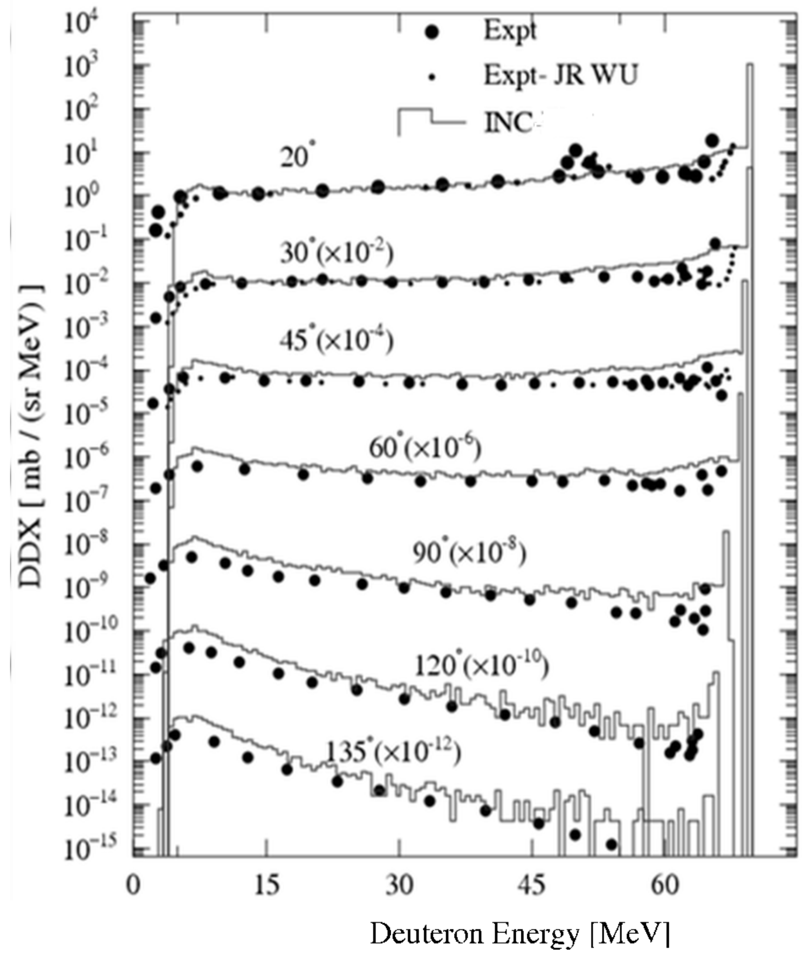
Calculation results and discussions

DDX spectra: comparison of the model calculations with experimental data.

^{27}Al(d,d'x), E_d = 80 MeV

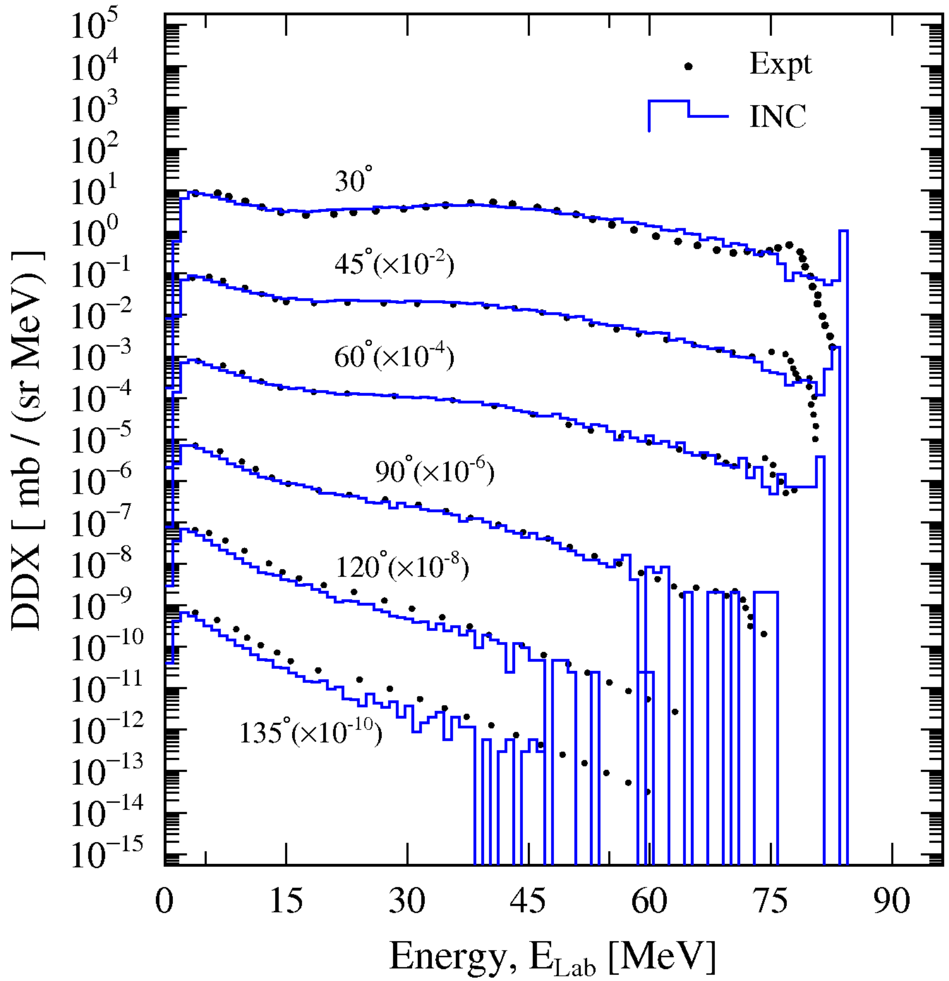


^{90}Zr(d, d'x), E_α = 70 MeV

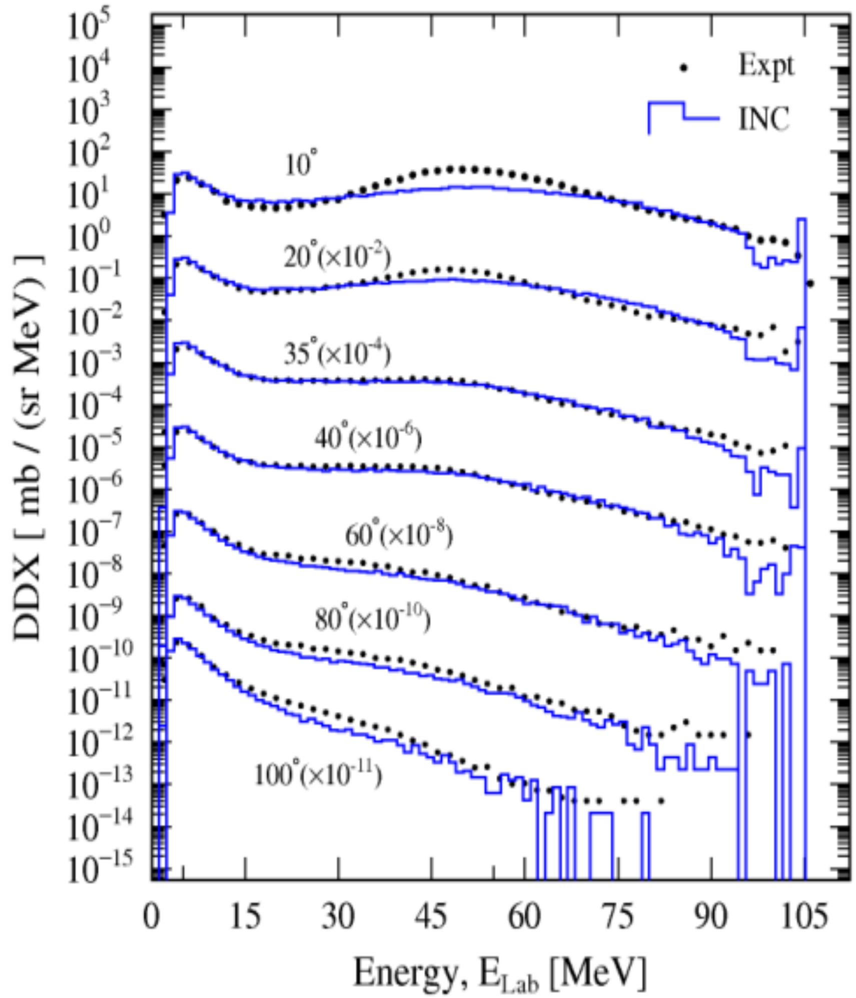


Calculation results and discussions

^{27}Al(d,px), E_d = 80 MeV



^{58}Ni(d, px), E_d = 99.6 MeV

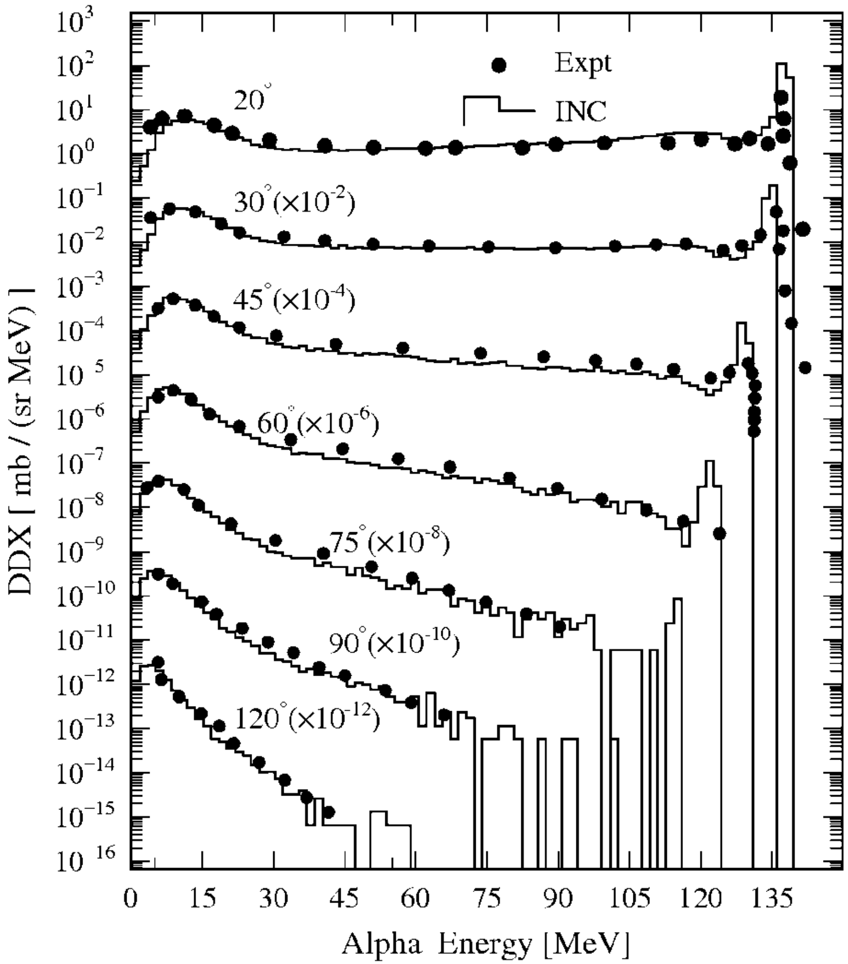


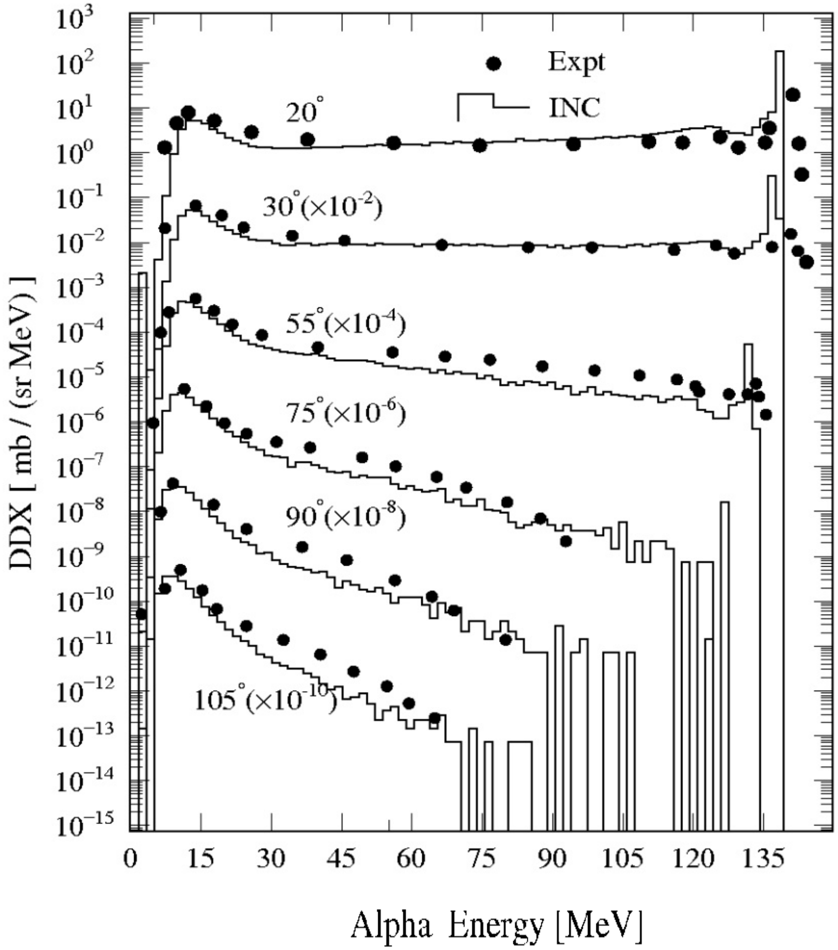
Calculations results and discussions

$^{27}Al(\alpha, \alpha'x)$

140 MeV

$^{58}Ni(\alpha, \alpha'x)$

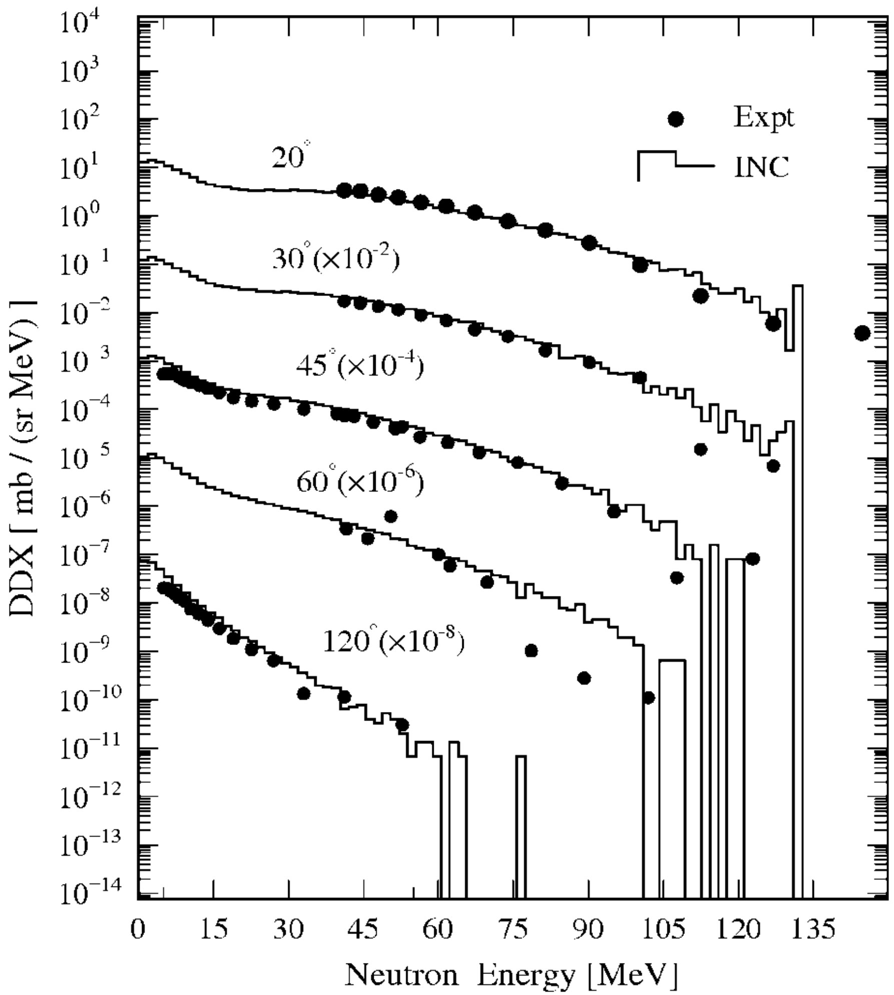




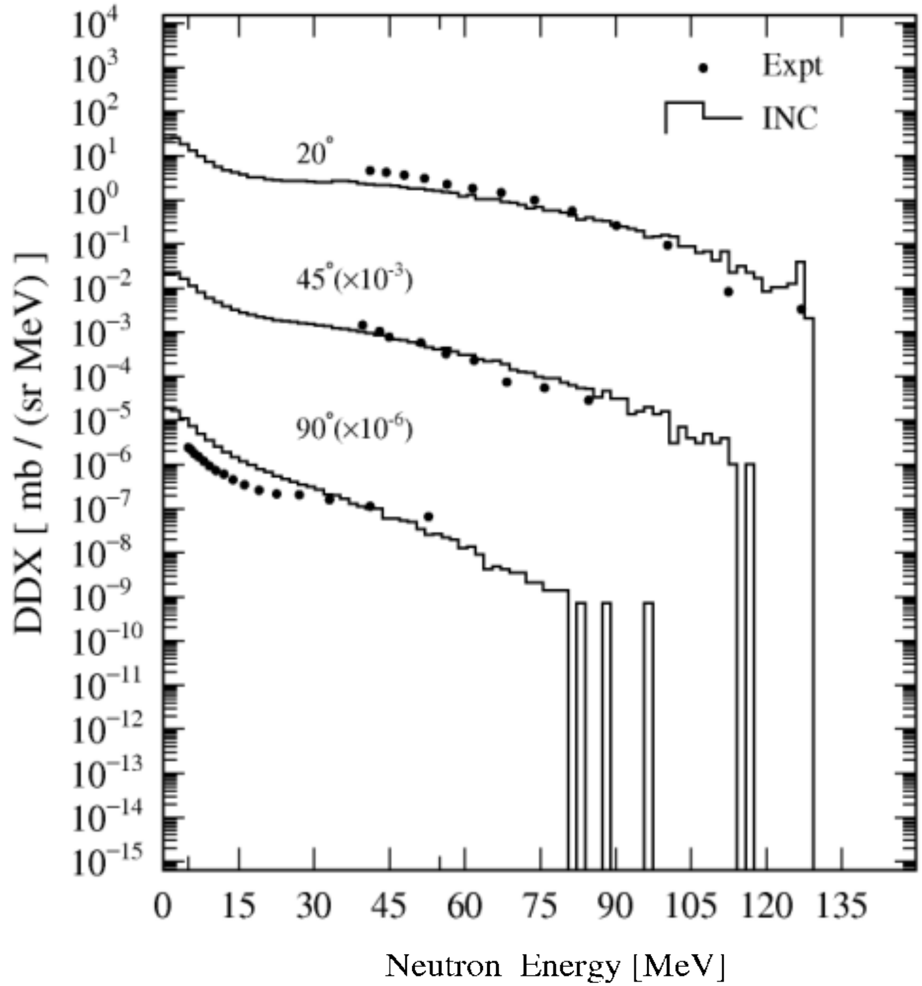
Comparison of INC results with experimental data.

140 MeV

$^{27}Al(\alpha, nx)$



$^{58}Ni(\alpha, nx)$

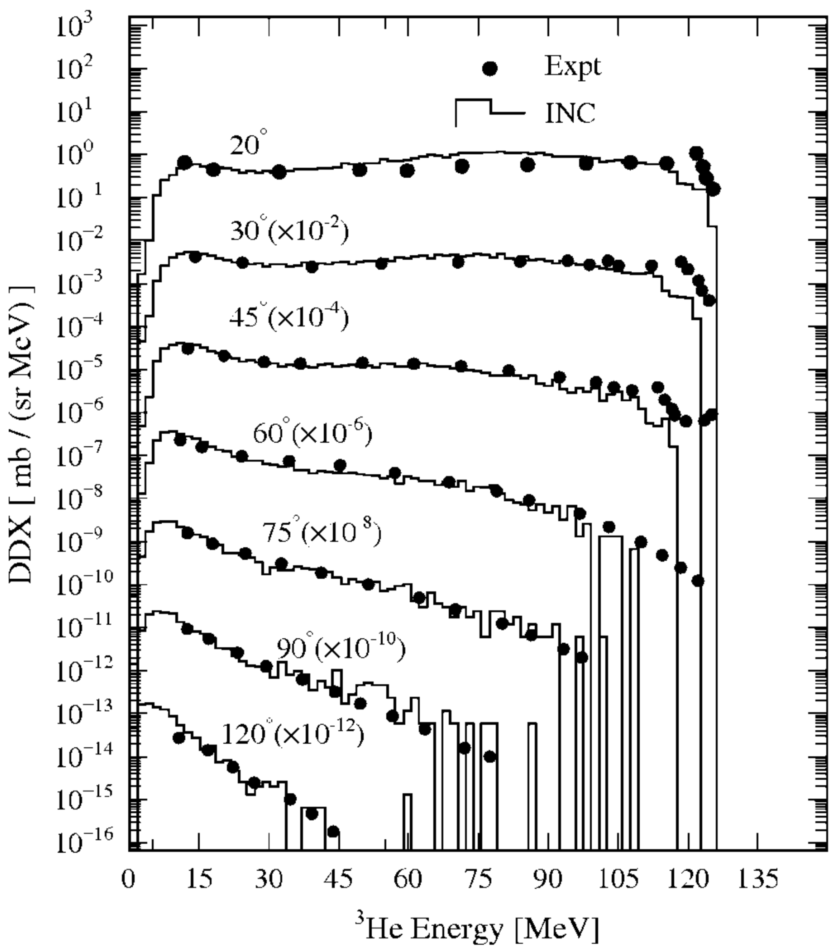


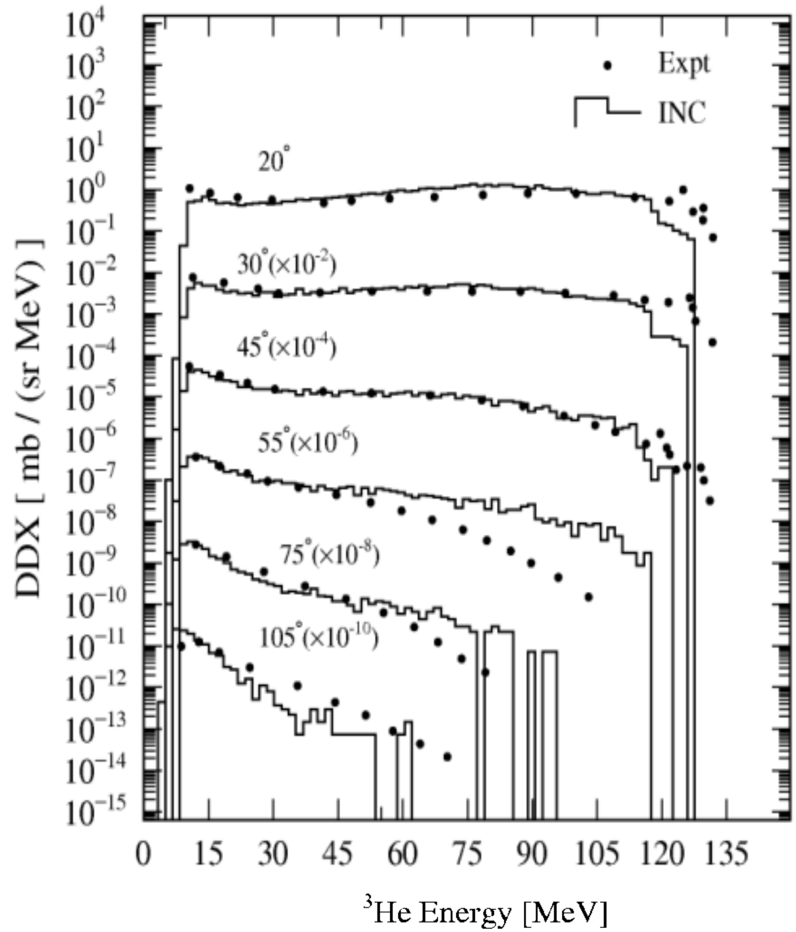
Comparison of INC results with experimental data.

$^{27}Al(\alpha, {}^{3}He x)$

140 MeV

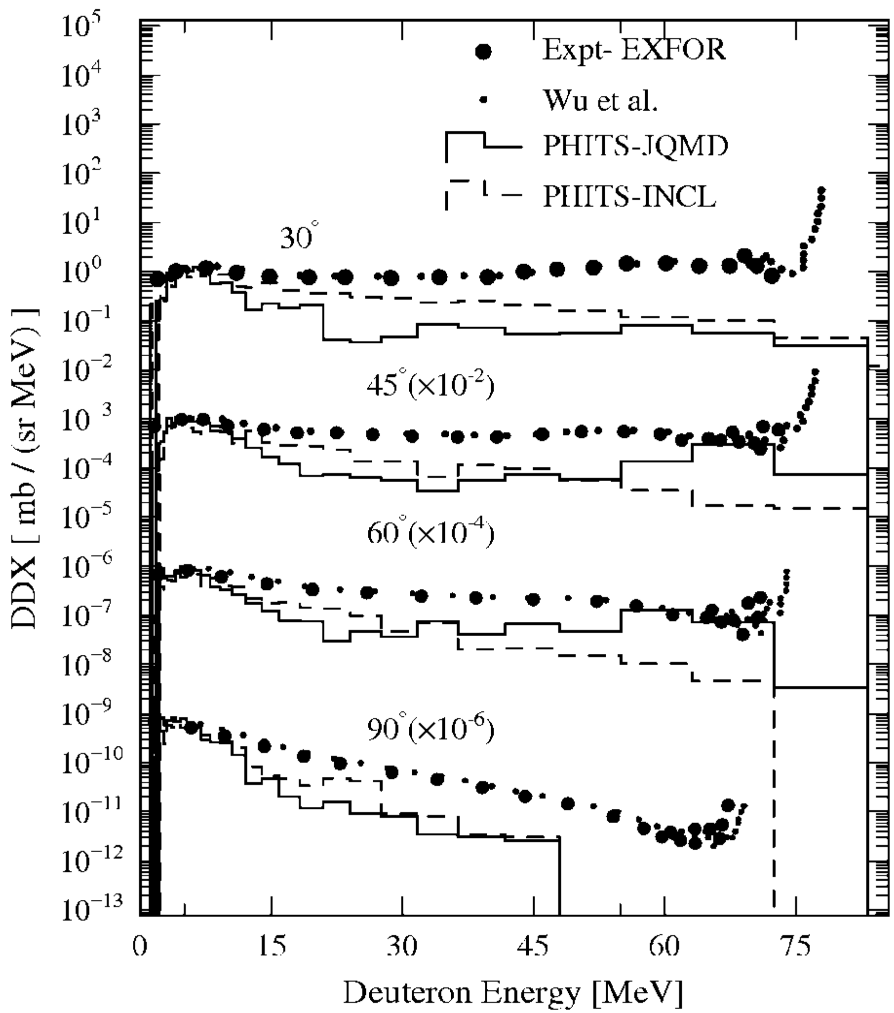
$^{58}Ni(\alpha, {}^{3}He x)$



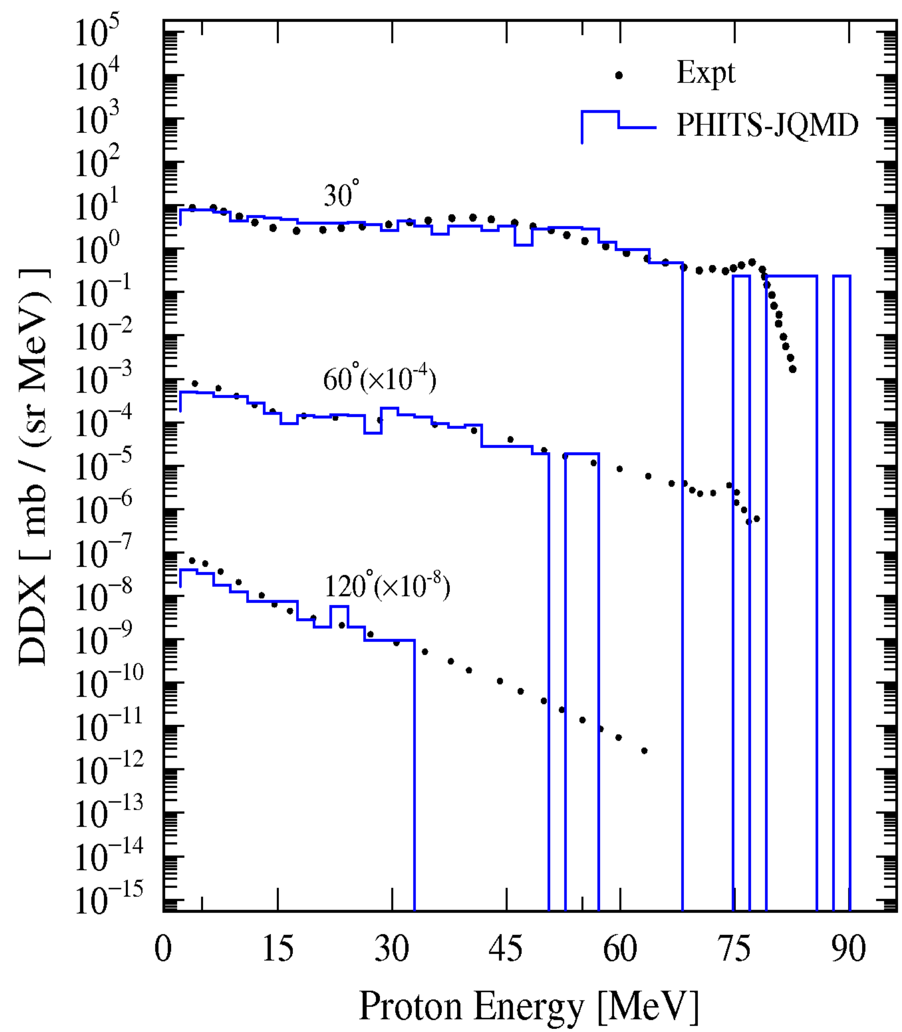


Other model results: INCL and JQMD model

$^{27}Al(d, d'x)$, E_d = 80.0 MeV



$^{27}Al(d, px)$, E_d = 80.0 MeV



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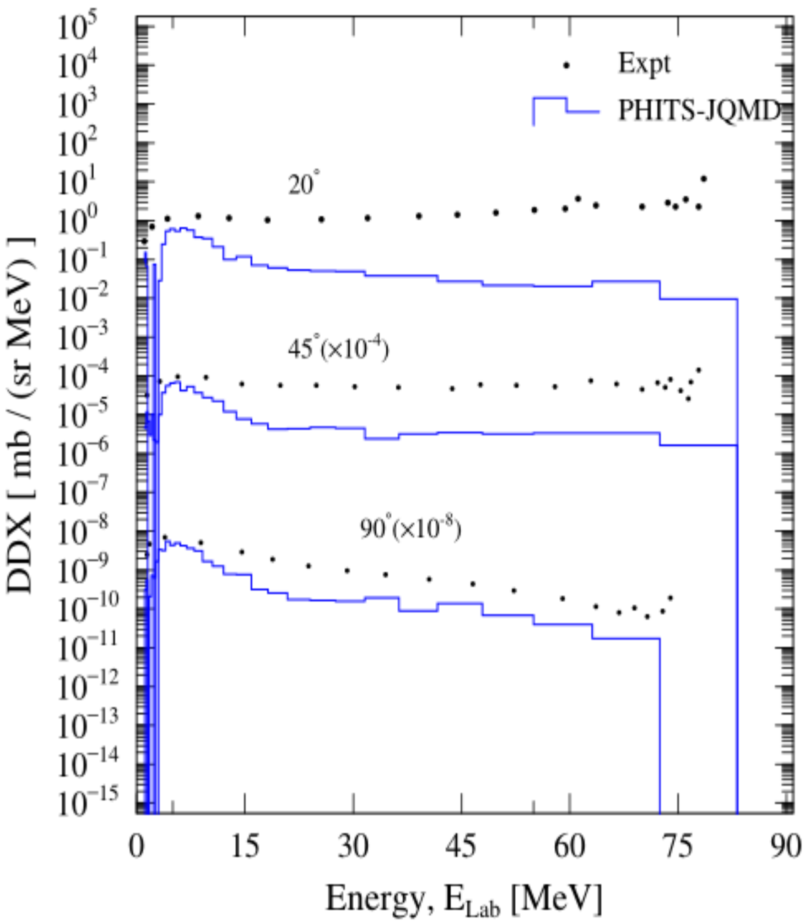
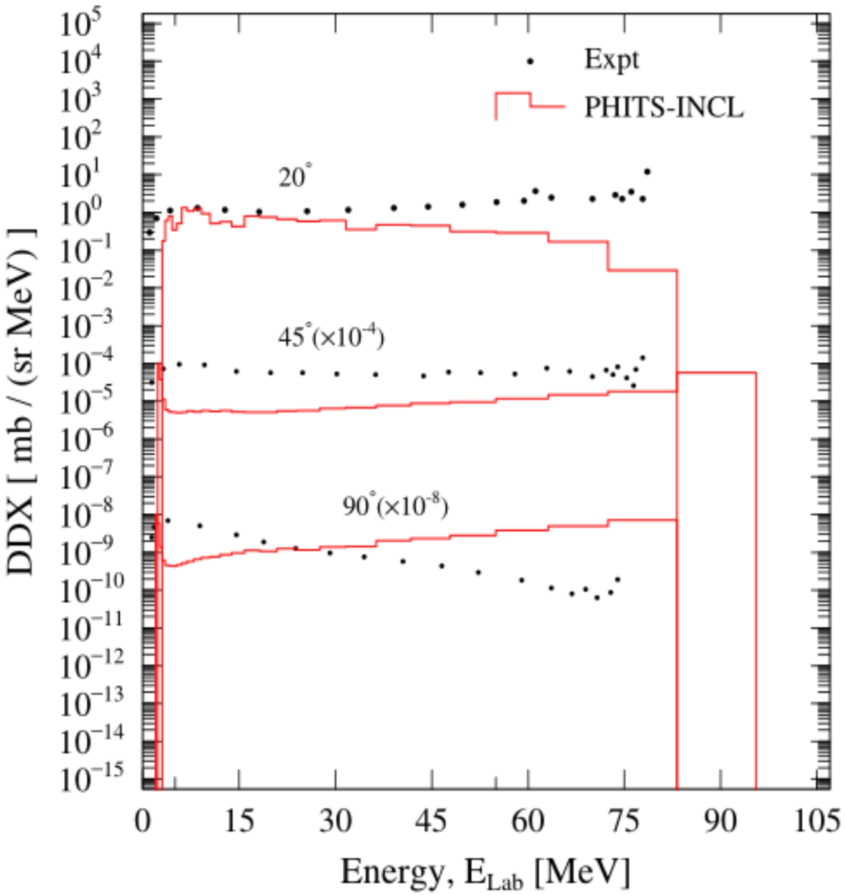
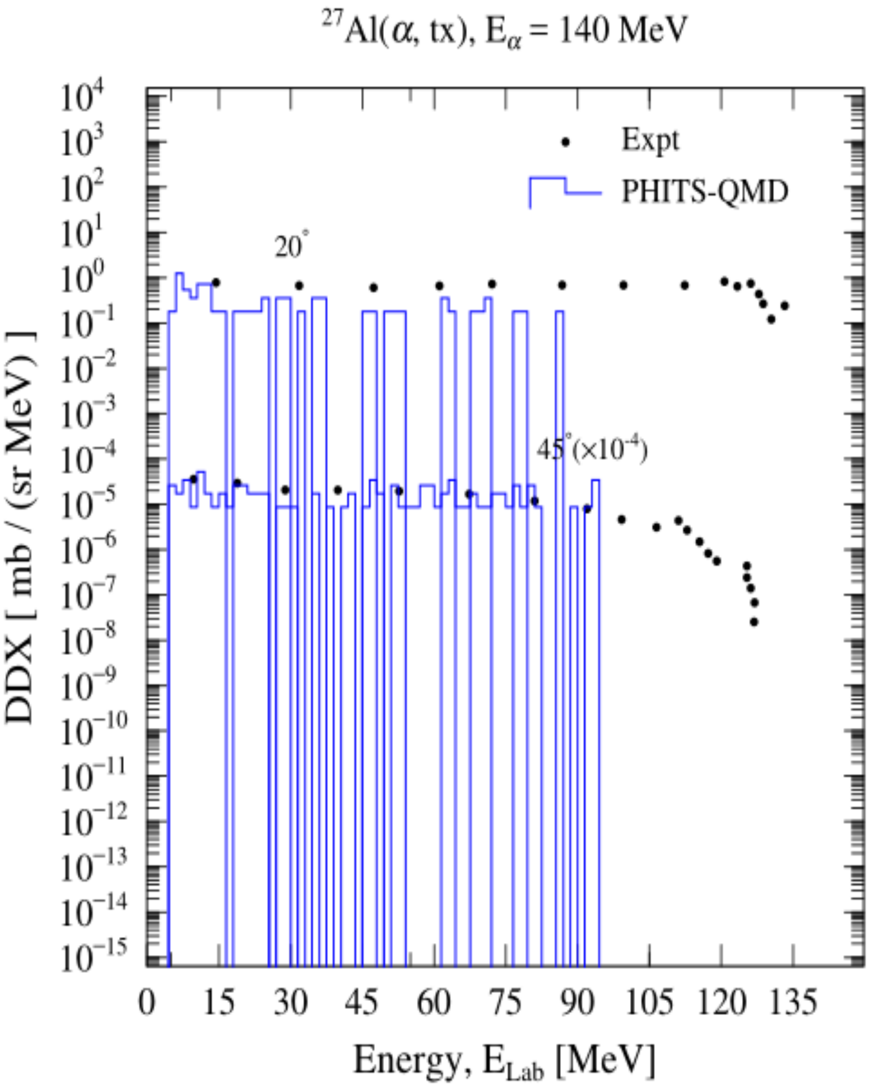


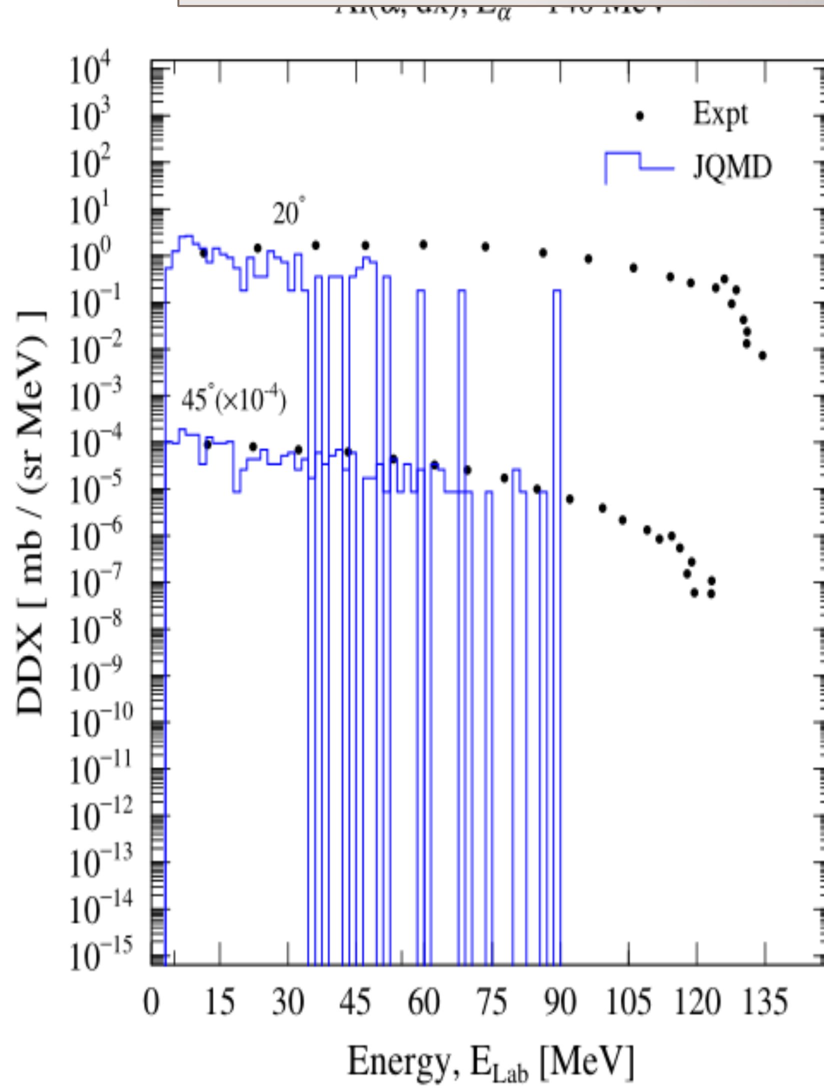


Comparison of **JQMD** model with experimental data

^{27}Al

Incident energy: 140 MeV
20°, 45° and 75°

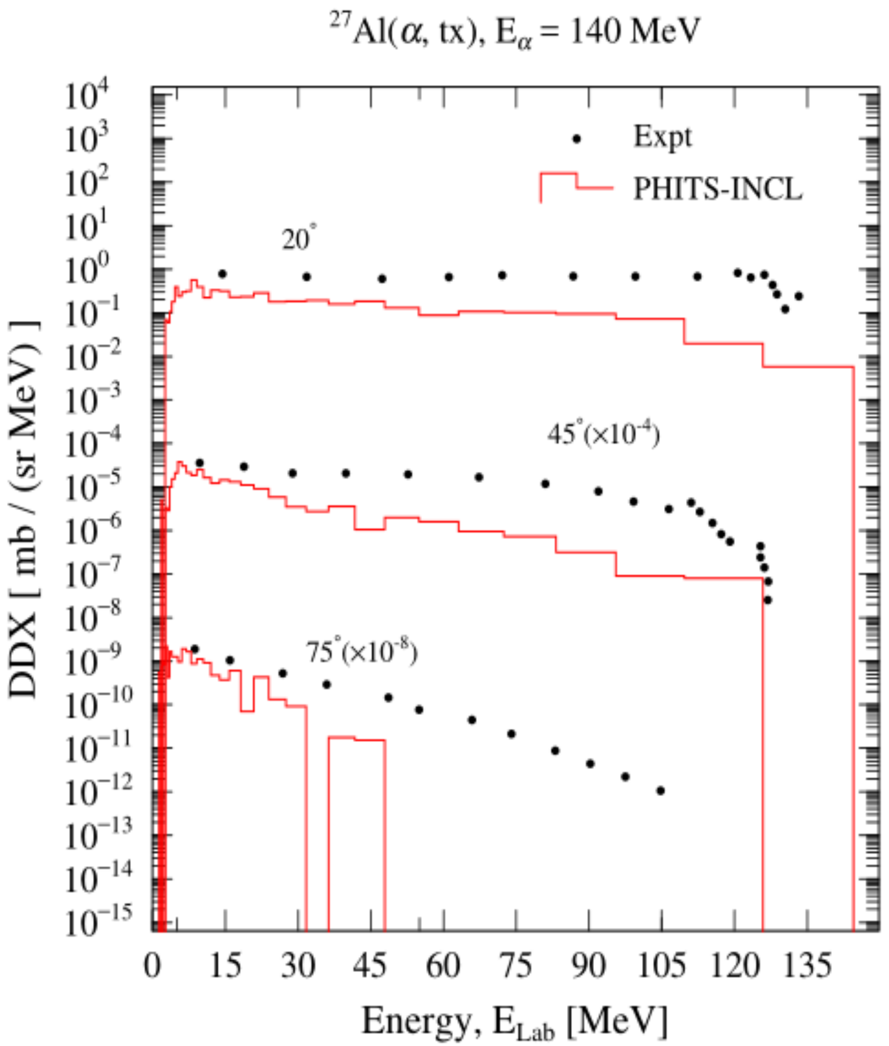


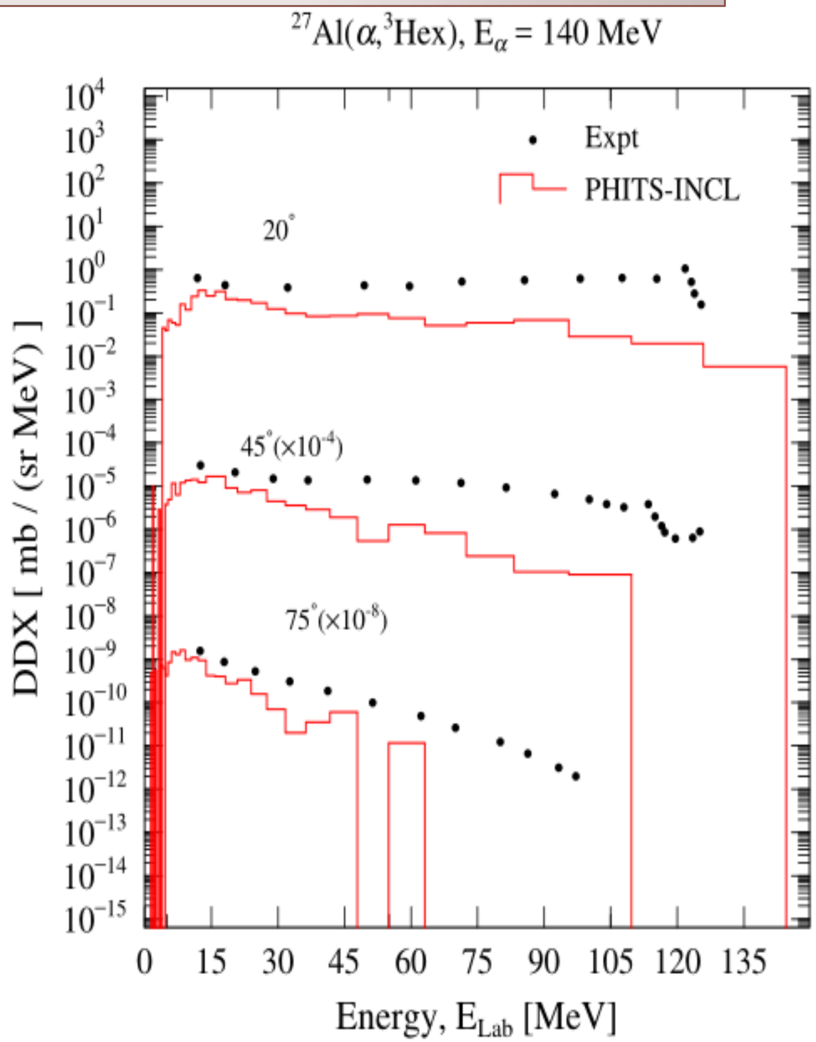


Comparison of experimental data with **INCL** model.

^{27}Al

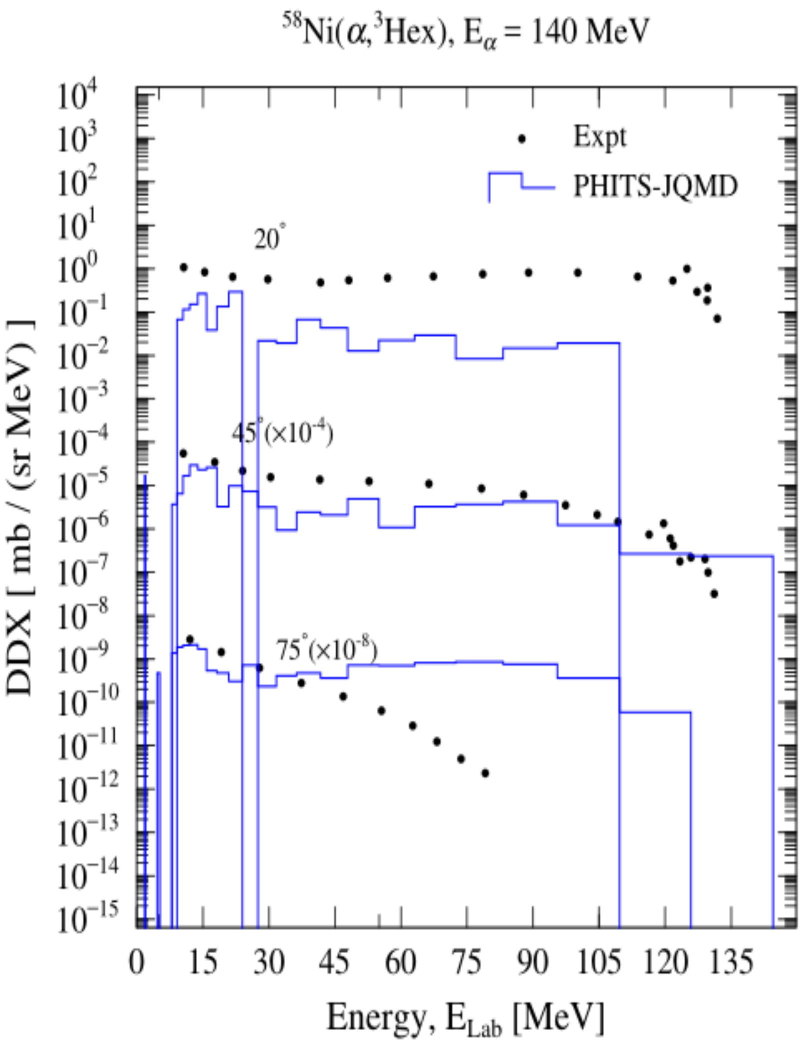
Incident energy: 140 MeV
20°, 45° and 75°



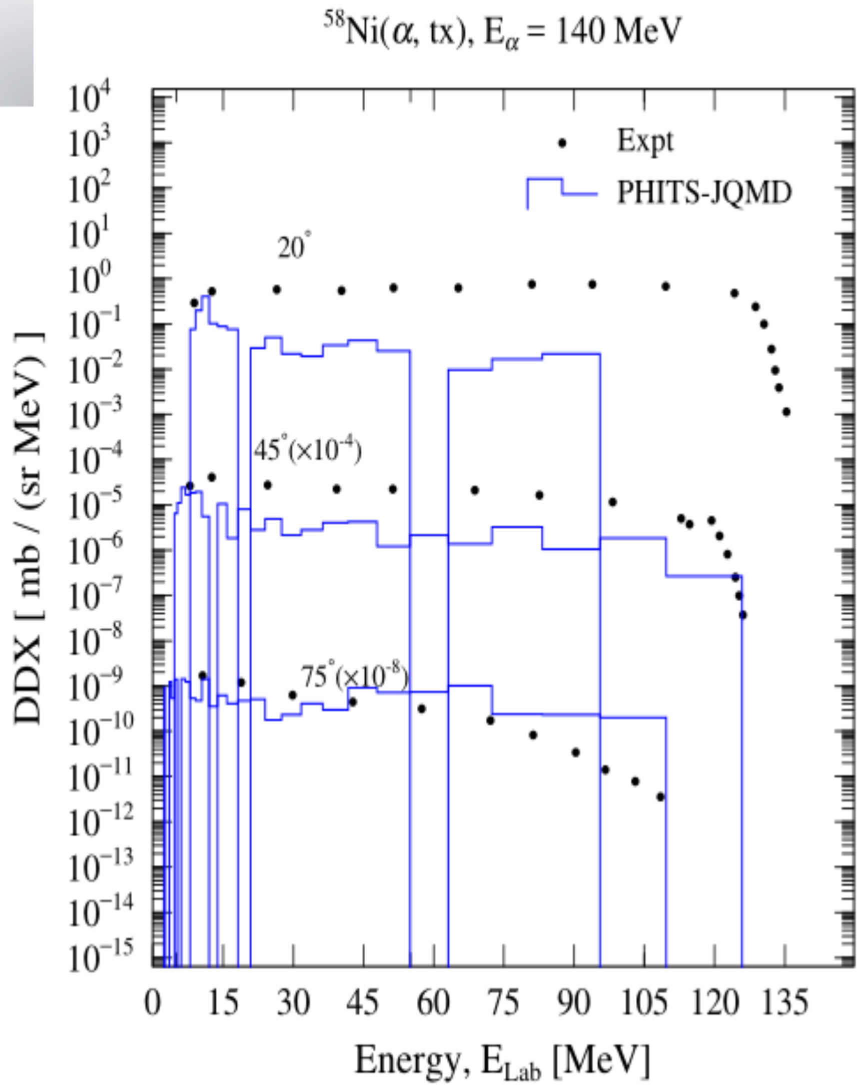


Comparison of **JQMD** model with experimental data

Incident energy: 140 MeV 20°, 45° and 75°



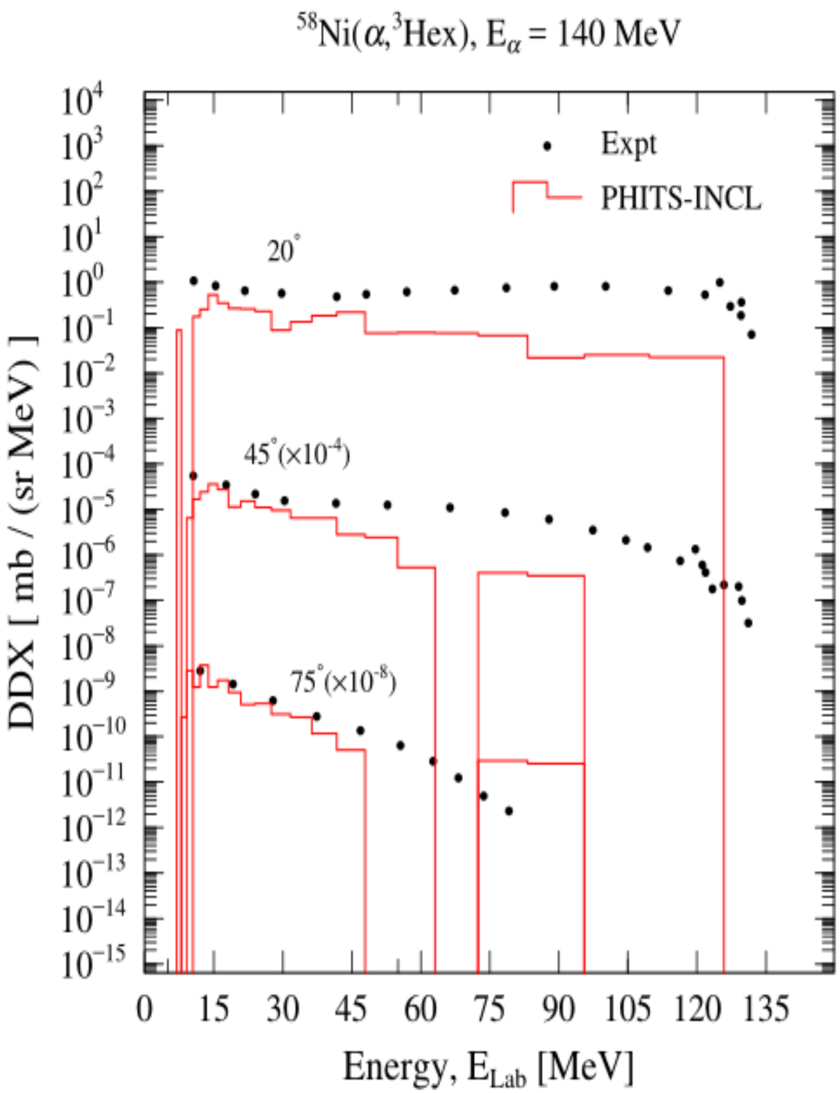
^{58}Ni

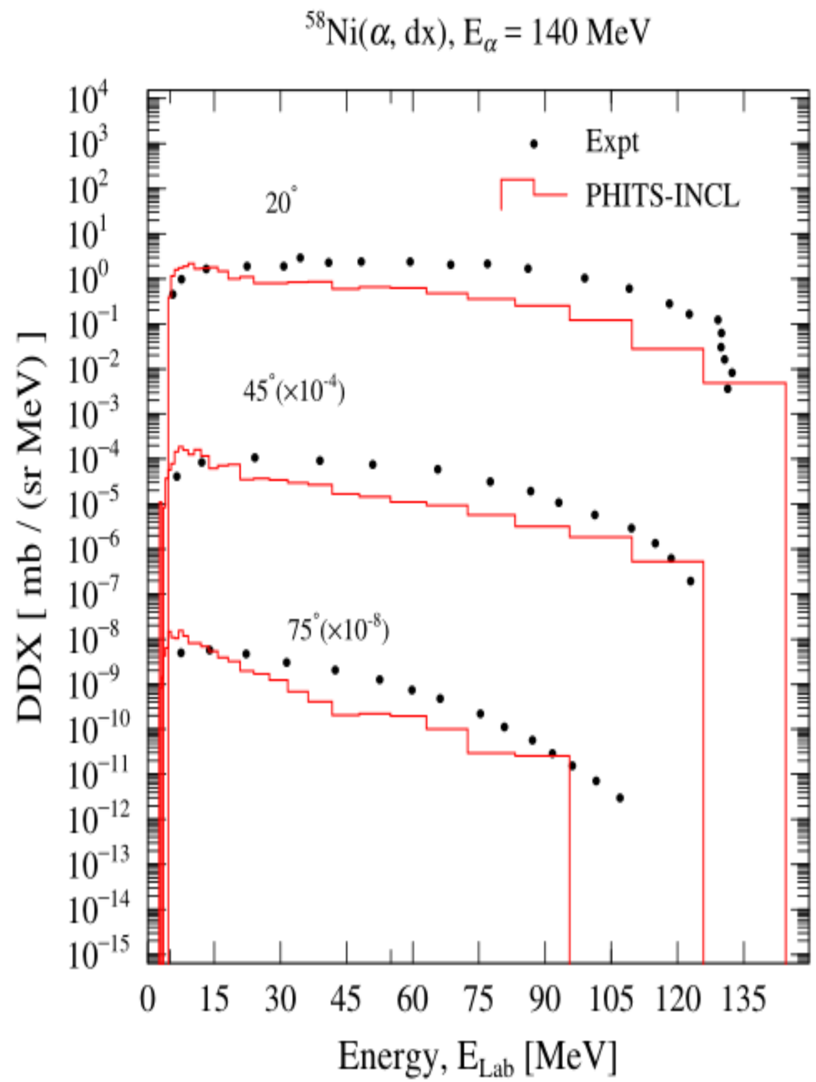


Comparison of experimental data with **INCL** model.

Incident energy: 140 MeV 20°, 45° and 75°

^{58}Ni





Conclusions

- The INC model was investigated to widen its application range for cluster (deuteron and alpha) induced reactions.
- We introduced the idea of virtual excited states of incoming cluster in the INC framework where the projectile ground state is expressed as superposition of wave functions of its different states.
- As the angular distributions are sensitive to the deflection of fragments, trajectory deflection for both the cluster projectile and the outgoing particles were incorporated.
- The extended model was verified comparing with the experimental data for deuteron and alpha induced reactions at incident energies 22.3 – 160 MeV.
- The extended model shows high predictive power for deuteron induced (*d, d'x*), (d,px), (d,nx) reactions and all channels of alpha induced reactions.
- The inclusion of cluster induced reactions to the INC model will open the pathway to carbon–induced induced reactions for accurate dose calculations in cancer therapy.

Future Work

- Stripping Reactions
- Widen applicability for ^{12}C-induced reactions

Thank you


Questions?