Unité de Recherche de Physique Nucléaire et des Hautes Energies
Faculté des sciences de Tunis

Empirical study of three-body interactions in semi-magic nuclei

Ezzeddine Derbali

Joint ICTP-IAEA Workshop on Nuclear Structure and Decay Data: Experiment, Theory and Evaluation

Friday 19 October 2018

Three-body interactions

The Hamiltonian with up to three-body interactions, acting between identical nucleons in a single-j orbital:

$$\hat{H} = \hat{H}_1 + \hat{V}_2 + \hat{V}_3$$

The matrix elements of two and three-body interactions in the case of two and three nucleons in the j-orbital can be given by :

$$V_J = < j^2 JM \left| \hat{V}_2 \right| j^2 JM > \quad ; \quad W_{J_{\alpha\alpha'}} = < j^3 \alpha JM \left| \hat{V}_3 \right| j^3 \alpha' JM >$$

Matrix elements

The matrix elements of a scalar k-body interaction V_k between two states belonging to the j^n configuration of n nucleons in the j orbital can be calculated from the recurrence relation:

$$\left\langle j^n\ \alpha\ JM \middle| \hat{V}_k \middle| j^n\ \alpha' J\ M \right\rangle = \frac{n}{n-k} \sum_{\alpha_1 \alpha_1' J_1} [j^n \alpha J \{| j^{n-1}(\alpha_1 J_1) jJ]\, [j^{n-1}(\alpha_1' J_1) jJ |\} j^n \alpha' J]$$

$$\times \left\langle j^{n-1} \alpha_1 J_1 \middle| \hat{V}_k \middle| j^{n-1} \alpha_1' J_1 \right\rangle$$

The coefficient of fractional parentage (cfp) can be determined from the following expression :

$$n[j^{n-1}(\alpha_0 J_0) jJ |\} j^n [\alpha_0 J_0] J][j^{n-1}(\alpha_1 J_1) jJ |\} j^n [\alpha_0 J_0] J]$$
$$= \delta_{\alpha_1 \alpha_0} \delta_{J_1 J_0} + (n-1) \sum_{\alpha_2 J_2} (-1)^{J_0 + J_1} \sqrt{(2J_0+1)(2J_1+1)}$$
$$\times \begin{Bmatrix} J_2 & j & J_1 \\ J & j & J_0 \end{Bmatrix} [j^{n-2}(\alpha_2 J_2) jJ_0 |\} j^{n-1} \alpha_0 J_0][j^{n-2}(\alpha_2 J_2) jJ_1 |\} j^{n-1} \alpha_1 J_1]$$

Fit

W J Huang, G Audi, M Wang, F G Kondev, S Naimi and X Xu, Chin Phys C 41 (2017) 030002
+
ENSDF

$$\sigma = \sqrt{\frac{1}{N}\sum_{k}[E_{exp}(k) - E_{th}(k)]^2}$$

Root mean square deviation

- $j = 9/2, \; N = 50, \; 40 < Z < 50$

44 measured values of binding energies.

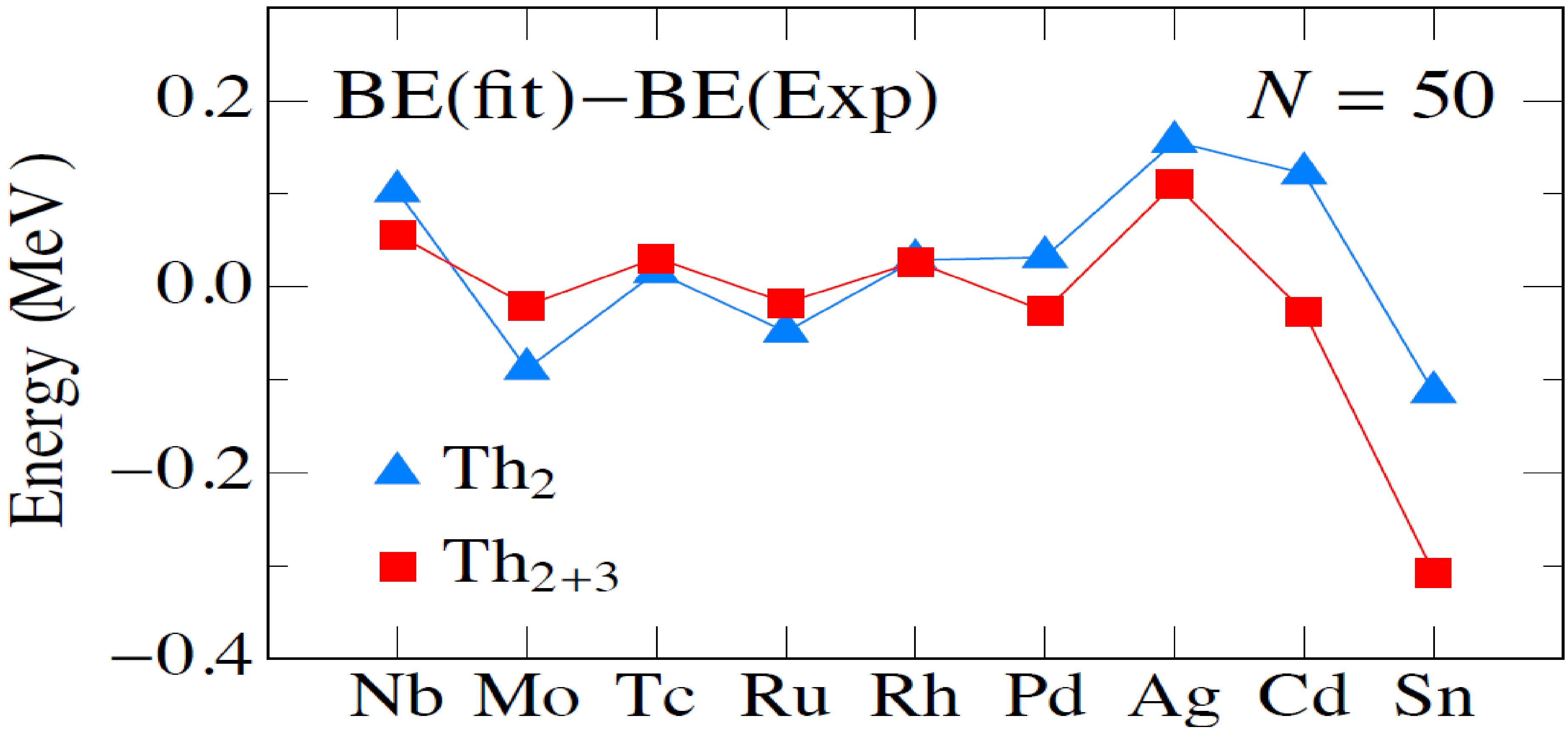
Two-body interaction : **σ = 63 keV**

+

Three-body interaction **σ = 46 keV**

The three-body interaction meaningfully improves the results.

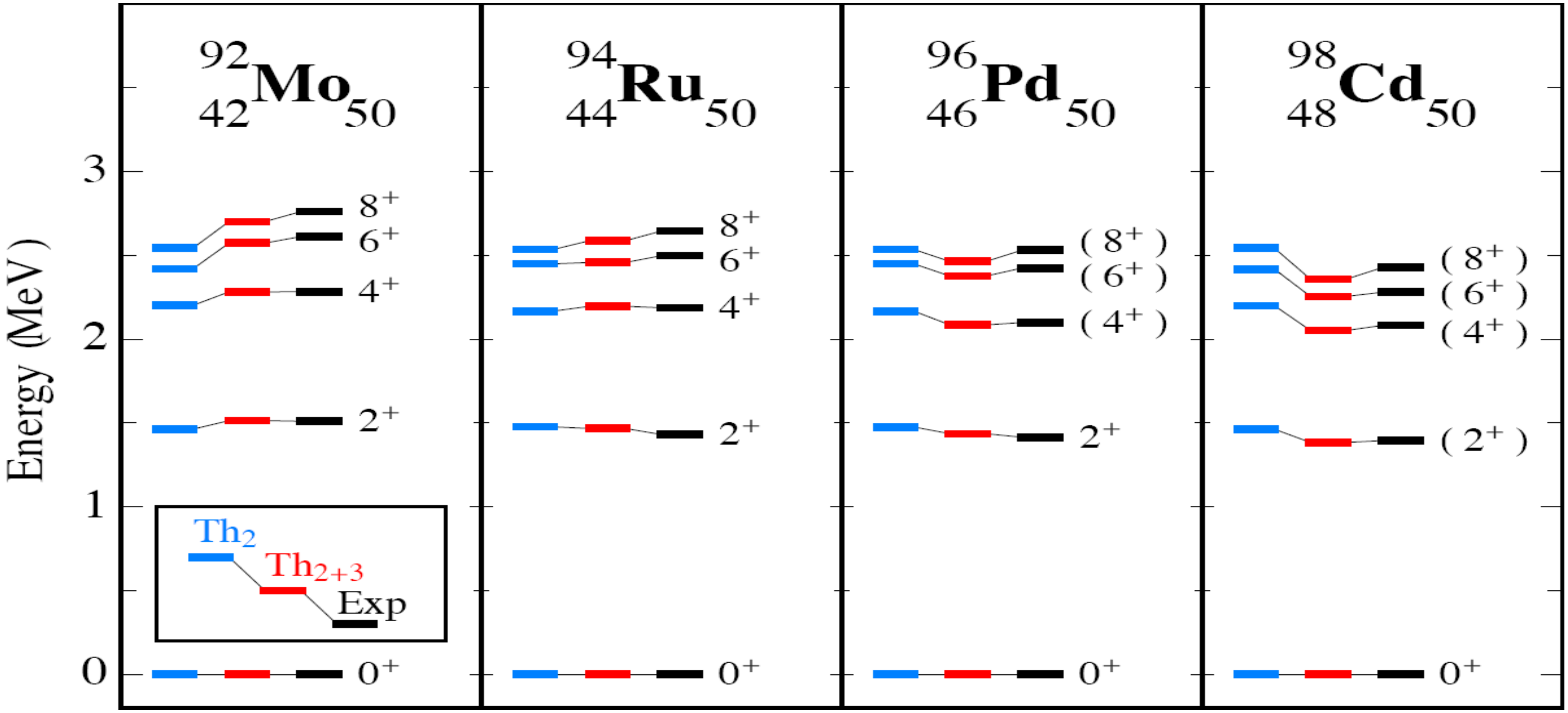
Ground states:



The difference between calculated and experimental ground-state binding energies of N = 50 isotones..

- The addition of a three-body interaction does not improve the description of the binding energies of ground states.

Excitation spectra of the even-mass



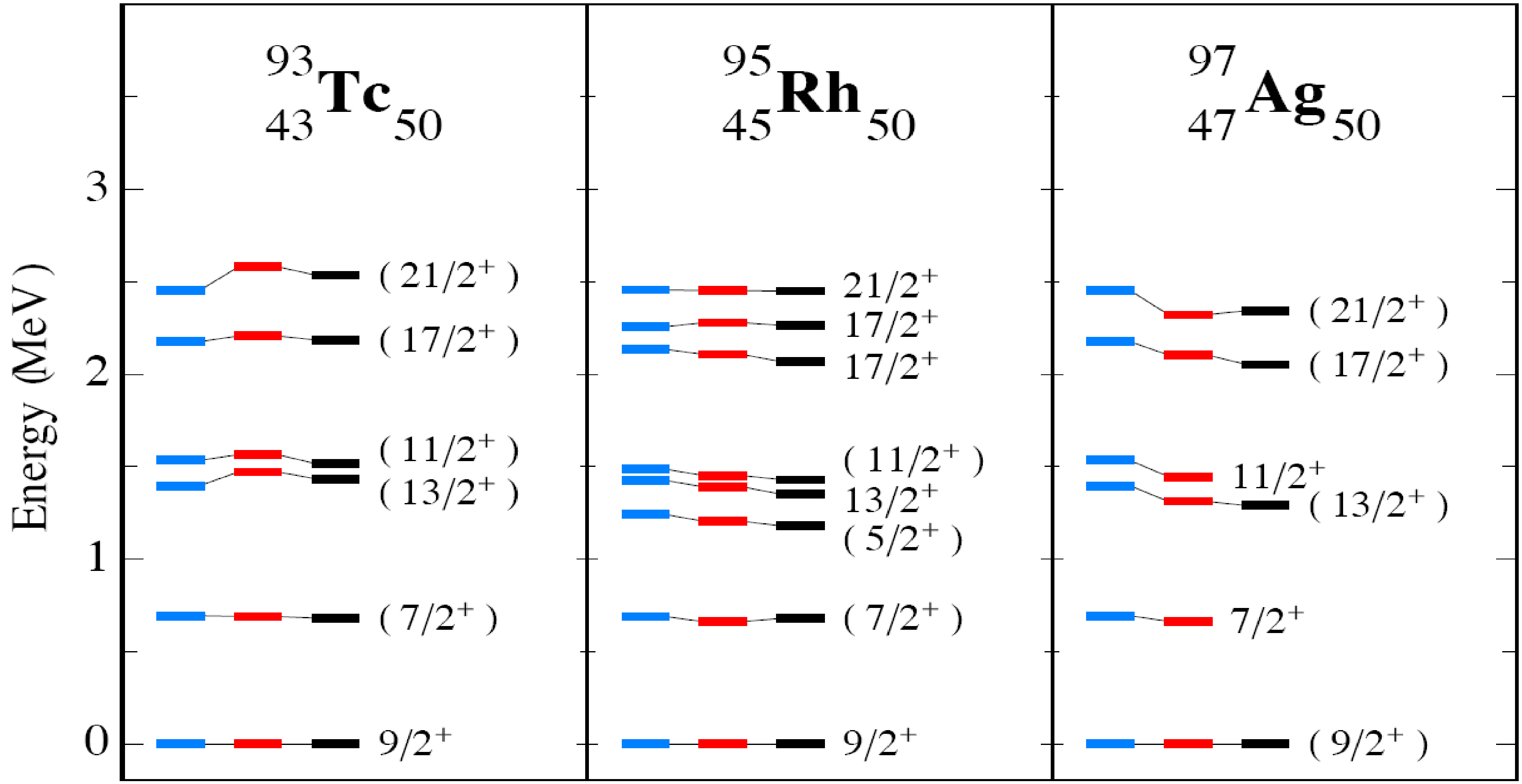
Excitation spectra of the even-mass N=50 isotones.

The three-body interaction meaningfully improves the results.

Conservation of the particle–hole symmetry using the two-body interactions.

This symmetry was beoken by adding the effect of the three-body interactions.

Excitation spectra of the odd-mass

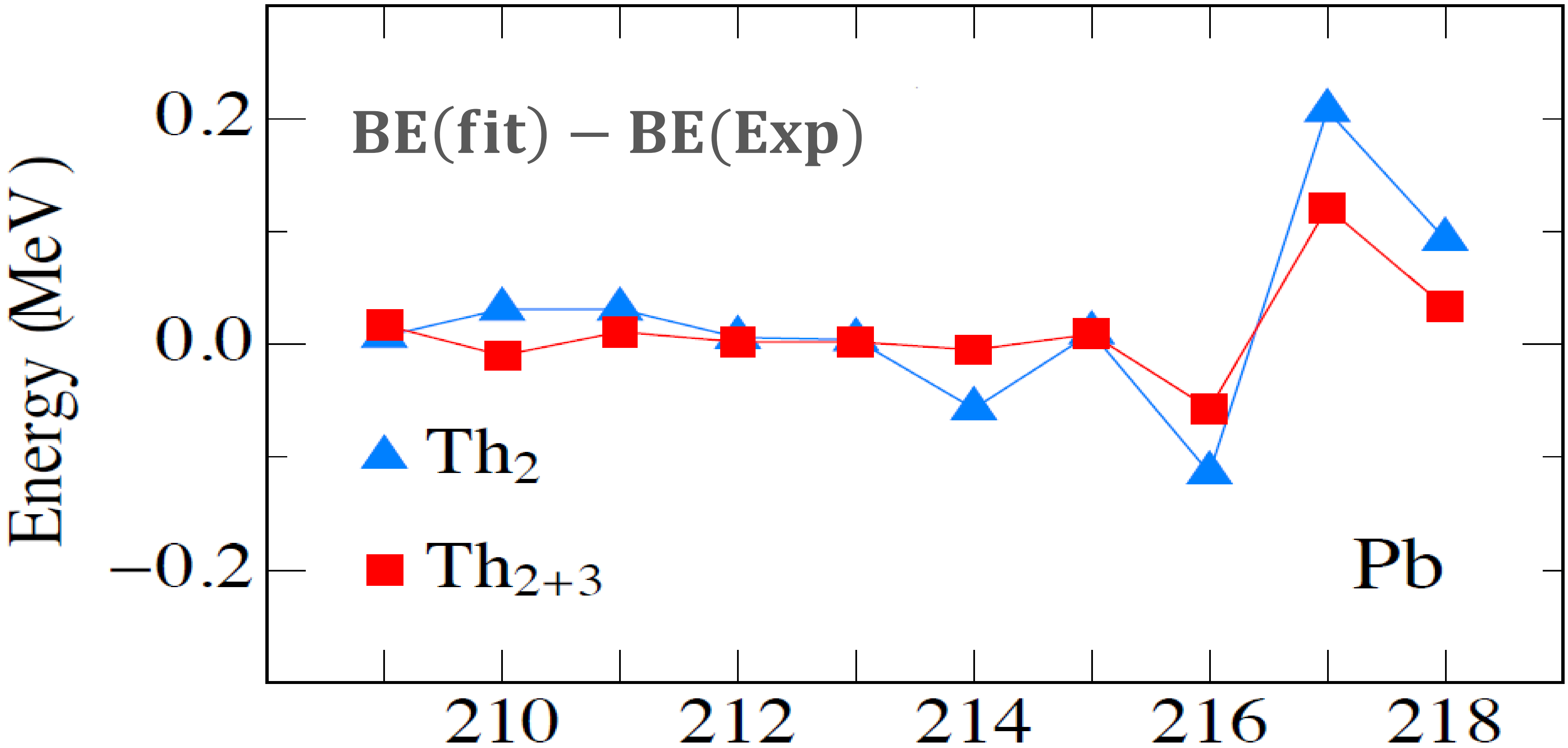


Excitation spectra of the odd-mass N=50 isotones.

- A good agreement between theory and experience is obtained by adding the effect of the three-body interaction.

Lead isotopes:

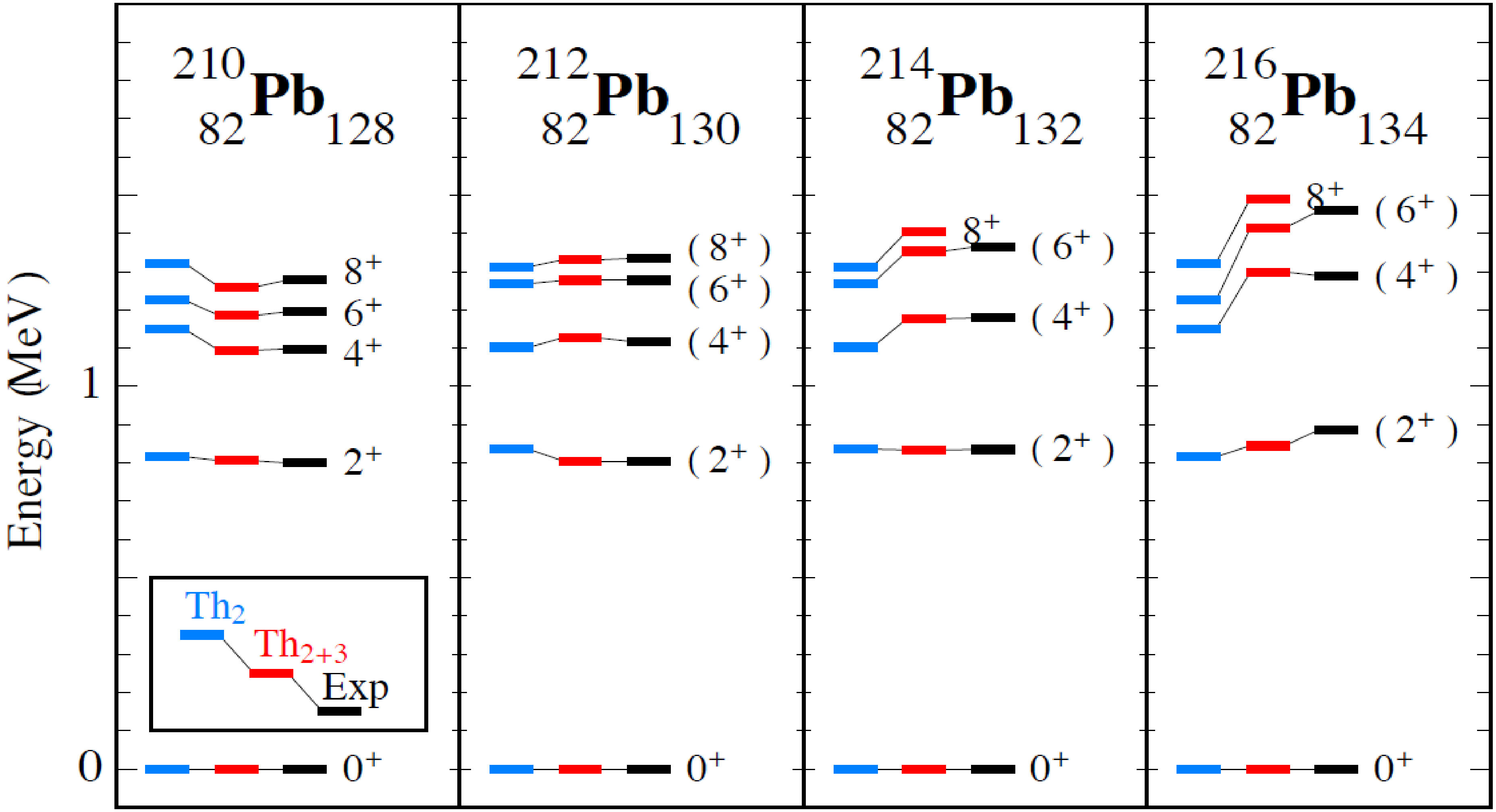
Binding energies of the ground states



The difference between calculated and experimental ground-state binding energies of Pb isotopes.

The addition of a three-body interaction does not notably improve the quality of the fit.

Excitation spectra of the even-mass



Excitation spectra of the even-mass Pb isotones.

Breaking of the particle–hole symmetry by adding the three-body component of the interactions.

Improved results following the inclusion of the effect of the 3-body interaction.

N = 50 isotones: 6^+ level ^{94}Ru

For the ^{211}Pb : two states $11/2^+$ → **643 keV**, → **894 keV**

$$\begin{cases} \sigma - minimum \\ \sigma_2 - minimum \end{cases}$$

- By adding a new level to the input data, one observes stability of the Th_2 and large fluctuations of the Th_{2+3} matrix elements.

The separate components of the three-body interaction could not be determined reliably from an empirical fit : A small variation in the input data leads to large fluctuations in the three-body matrix elements.

Thank you

Ezzeddine Derbali