The Interacting shell model

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The shell model potential

Nuclei are made up of protons and neutrons held together by the strong interaction inside of a volume with a radius of a few Fermi.

One might expect that the motions of these nucleons in this closely packed system should be very complex because of the large number of frequent collisions. Due to these collisions nucleons could not maintain a single-particle orbit.

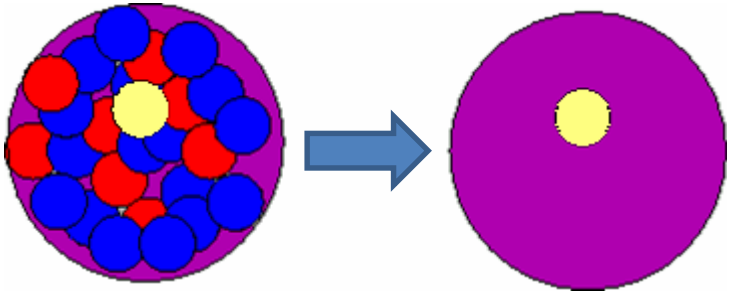
But, because of Pauli exclusion the nucleons are restricted to only a limited number of allowed orbits.

Nuclear Potentials

There are two approaches:

1. An empirical form of the potential is assumed, e.g. square well, harmonic oscillator, Woods-Saxon
2. The mean field is generated self-consistently from the nucleon-nucleon interaction

N nucleons in a nucleus



A nucleon in the Mean Field of N-1 nucleons

- Assumption – ignore detailed two-body interactions
- Each particle moves in a state independent of the other particles
- The Mean Field is the average smoothed-out interaction with all the other particles
- An individual nucleon only experiences a central force

The one-body potential

This is an independent particle model where the nucleus is described in terms of non-interacting particles in the orbits of a spherical symmetric (central) potential U(r) which is itself produced by all the nucleons.

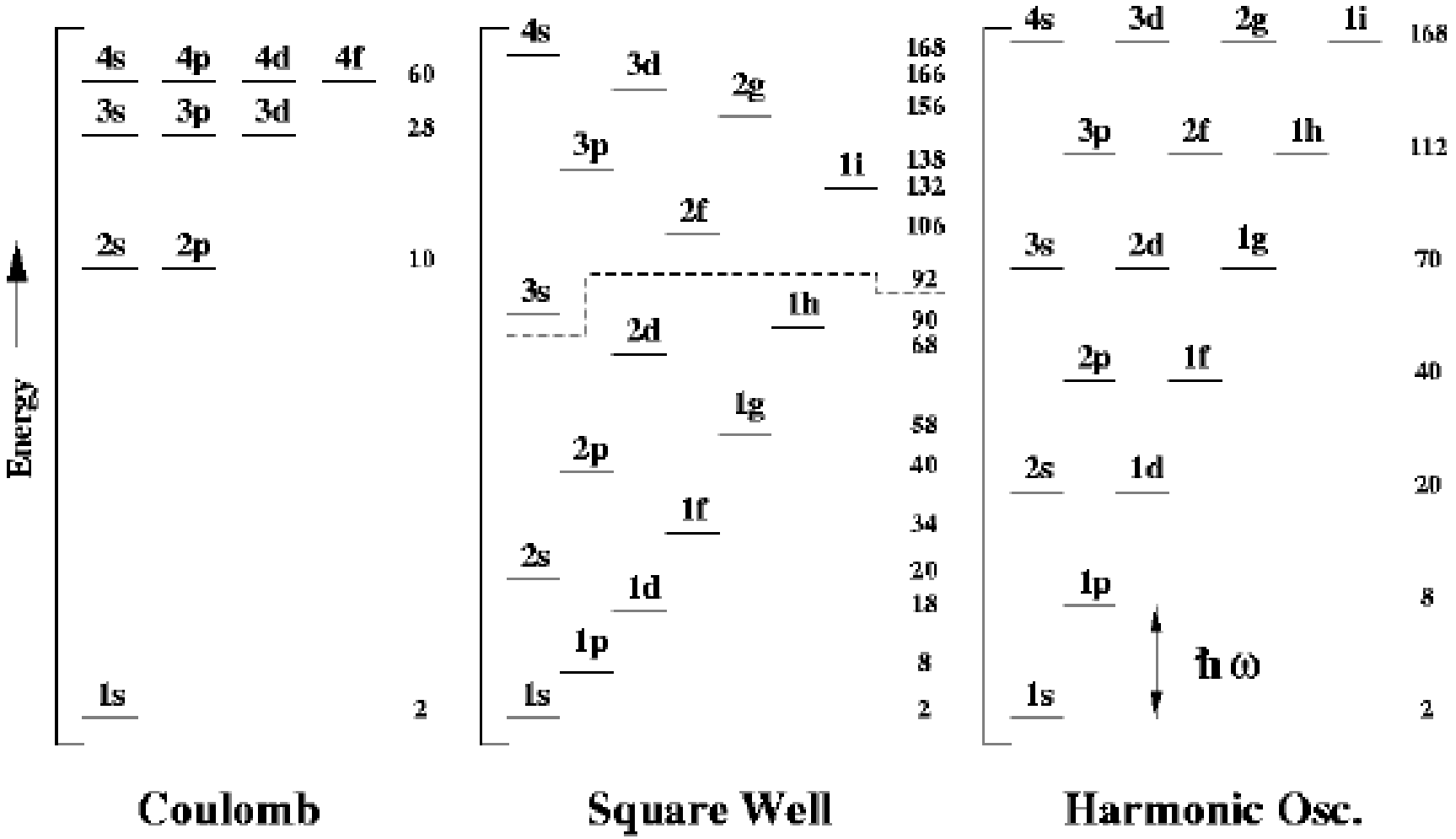
Then, the resulting orbit energies are mass dependent.

This model is applicable to nuclei with one single nucleon outside closed shell.

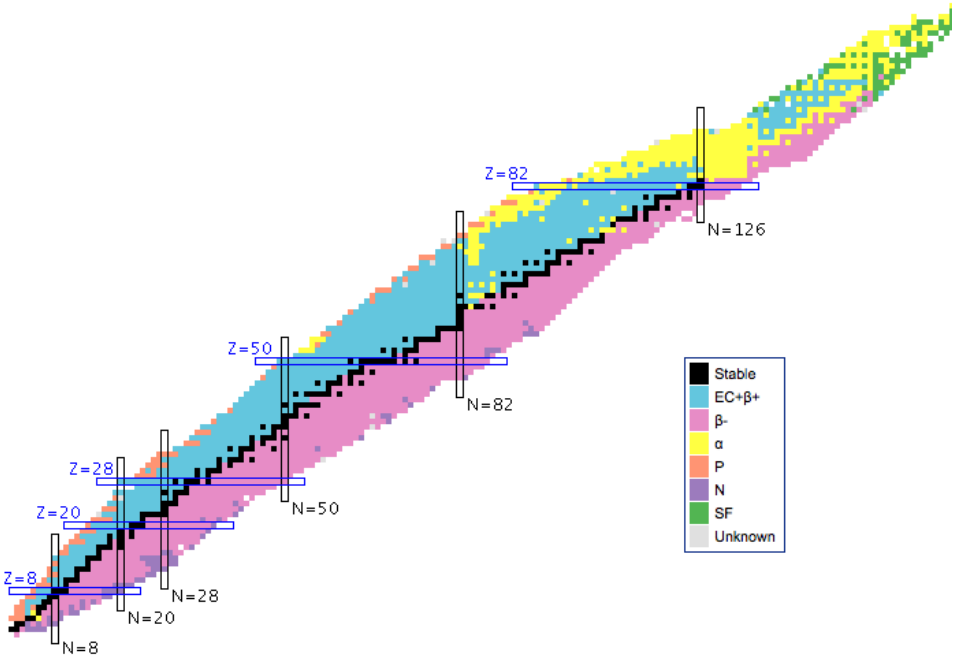
When more valence nucleons are considered, we have to include the residual interaction between these nucleons.

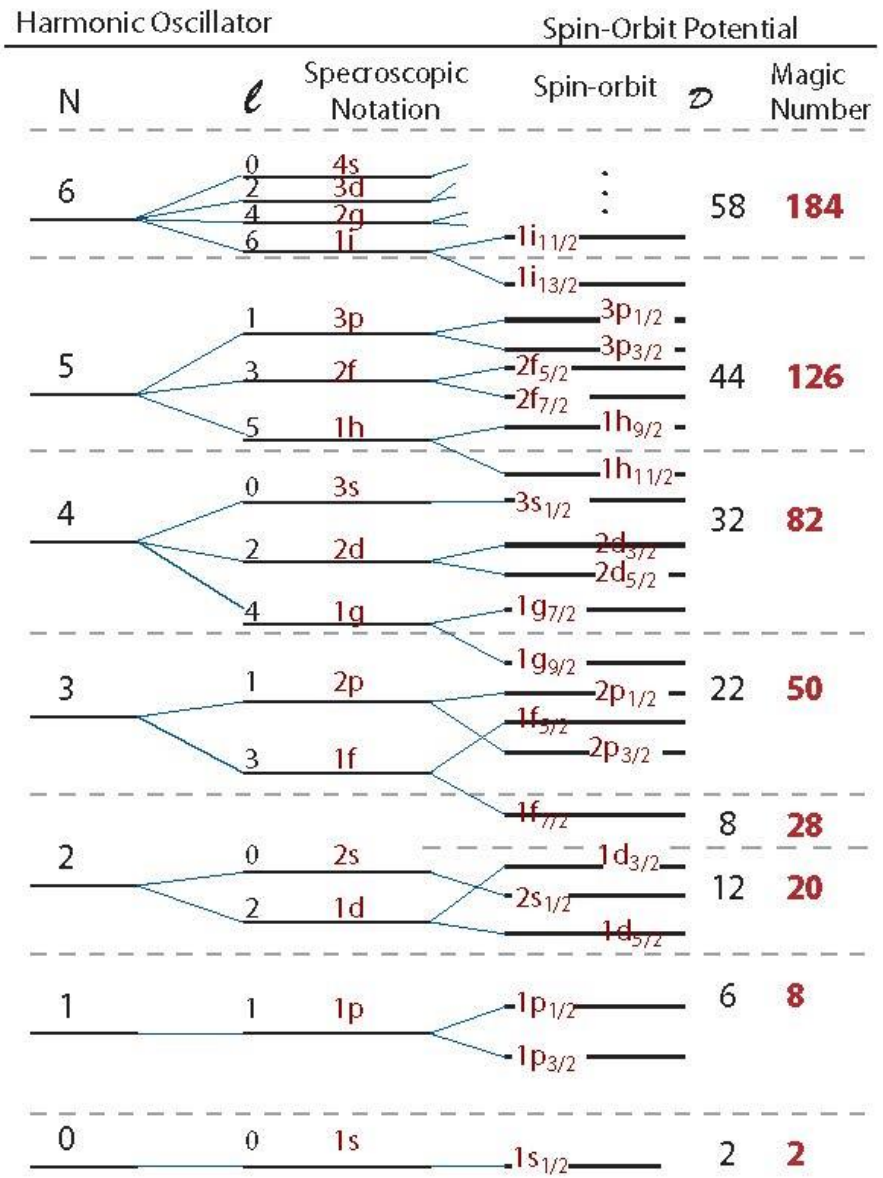
The simplest potentials are the square well and the harmonic oscillator.

Wrong magic numbers



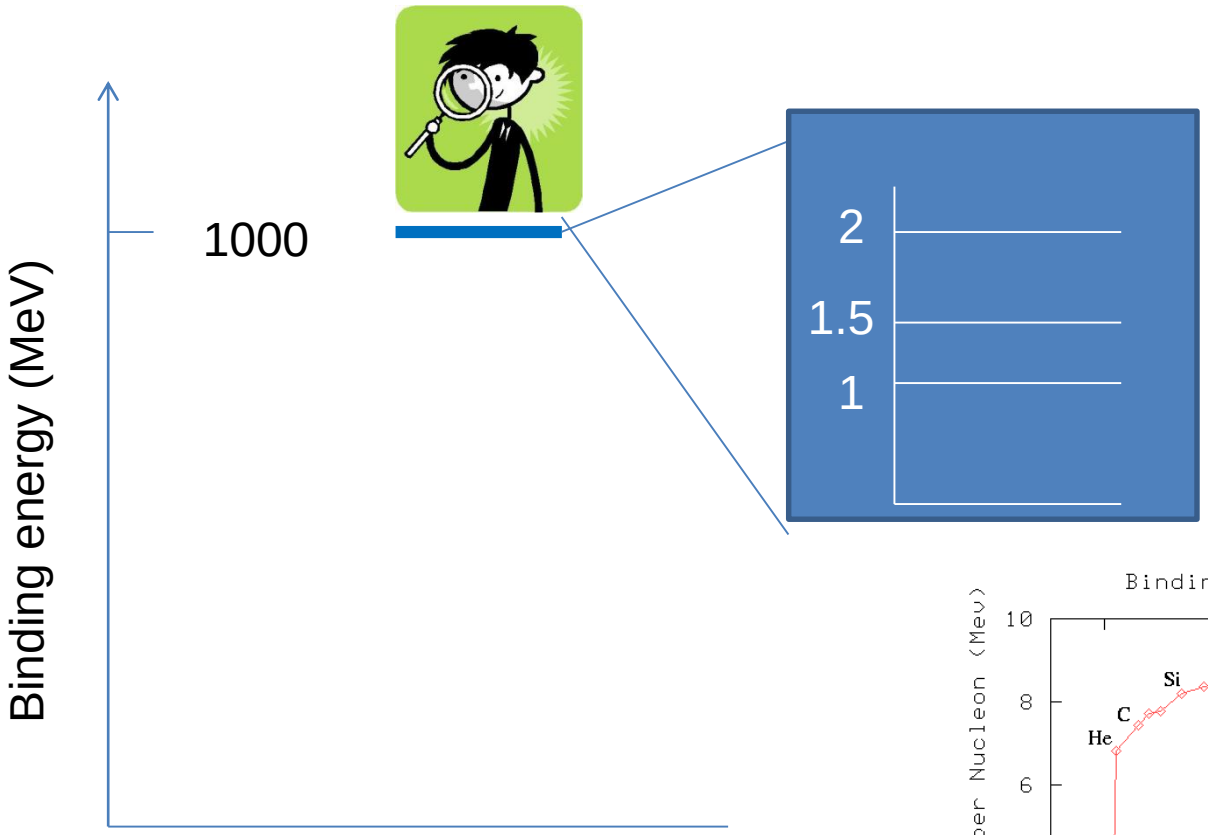
The magic numbers: H.O. + $\ell.\ell$ + $\ell.s$



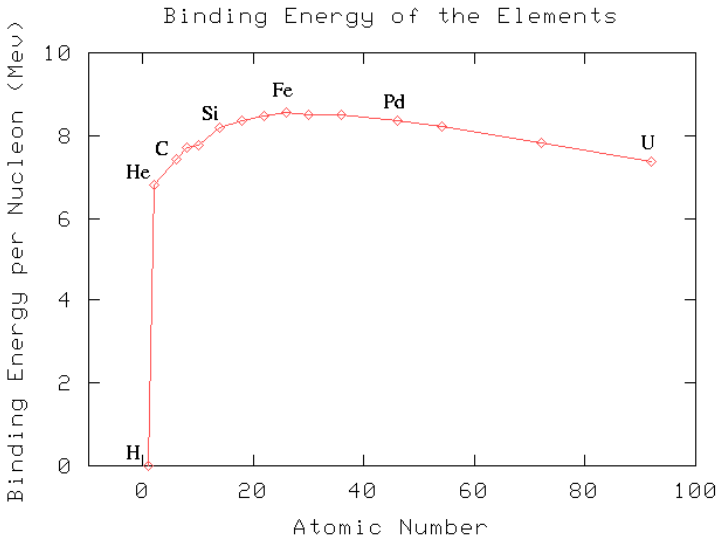


The nuclear shell model with residual interactions

Global and local properties



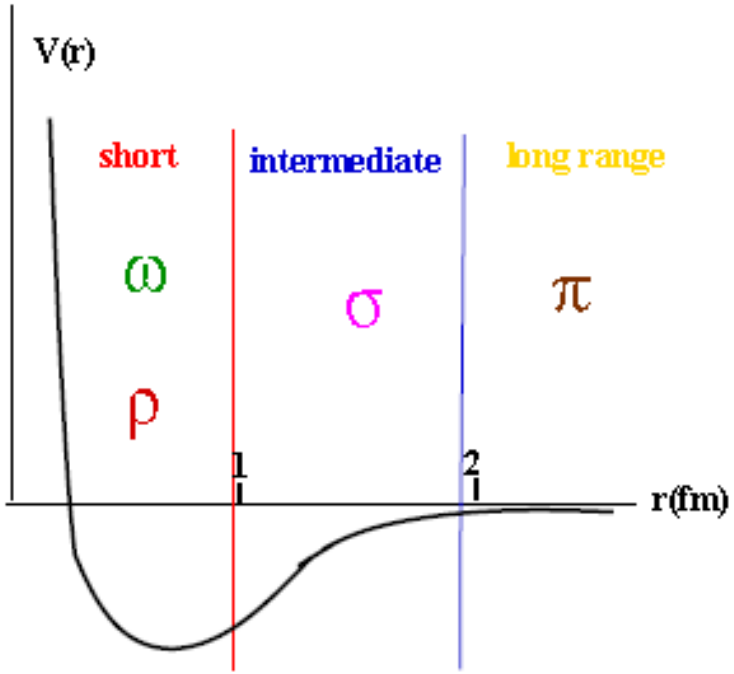
3 orders of magnitude of difference between the two scales!



Are we able to describe both global and local properties using the same, original nucleon-nucleon interaction?

The nucleon-nucleon potential

The bare nucleon-nucleon (or nucleon-nucleon-nucleon) interactions are inspired by meson exchange theories or more recently by chiral perturbation theory, and must reproduce the NN phase shifts, and the properties of the deuteron and other few body systems



Independent Particle Motion and correlations

In the nucleus, due to the very **strong short range repulsion** and the **tensor force**, the **independent particle** motion or Hartree-Fock approximation, based upon the **bare nucleon-nucleon force,** are **impracticable**.

However, at low energy, the nucleus do manifest itself as a system of independent particles in many cases, and when it does not, it is due to the medium range correlations that produce strong configuration mixing and not to the short range repulsion.

The unique interaction

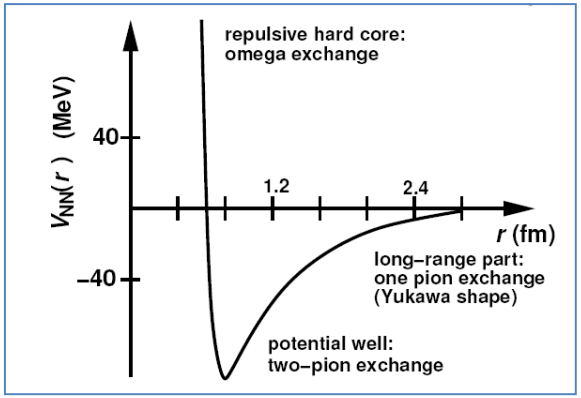
To have a tractable problem, the critical point is the choice of the model space and the “effective” nucleon-nucleon interaction.

The starting point should be a realistic interaction that reproduces the nucleon-nucleon scattering properties in the energy region 0-500 MeV

Deriving a realistic effective interaction

All modern NN potentials fit equally well ($\chi^2/N_{data} \sim 1$) the deuteron properties and the NN scattering data up to the inelastic threshold: CD-Bonn, Argonne V18 , Nijm I, Nijm II, N^{3}LO potentials,…

However, these potentials cannot be used directly in the derivation of V_{eff} due to the strong short-range repulsion, but a renormalization procedure is needed.



Many-body methods:

- Brueckner G-matrix
- $V_{low\text{-}K}$: smooth low-momentum potential that is used in a perturbative approach to derive V_{eff}

The full problem

The challenge is to find $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_A)$ such that

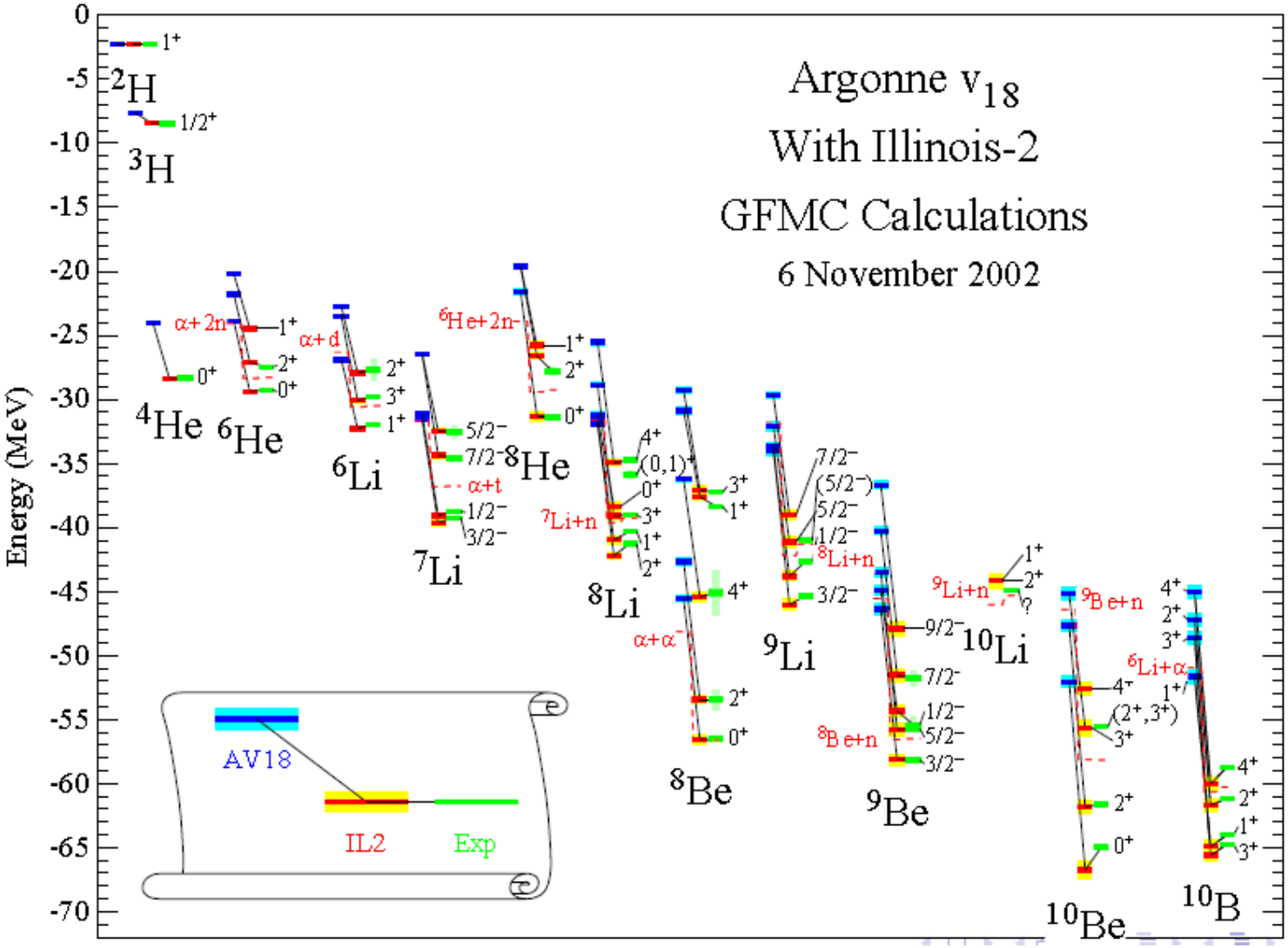
$H\Psi = E\Psi$, with

$$H = \sum_{i}^{A} T_i + \sum_{i,j}^{A} V_{2b}(\vec{r}_i, \vec{r}_j) + \sum_{i,j,k}^{A} V_{3b}(\vec{r}_i, \vec{r}_j, \vec{r}_k)$$

The knowledge of the eigenvectors Ψ and the eigenvalues E make it possible to obtain electromagnetic moments, transition rates, weak decays, cross sections, spectroscopic factors, etc.

Only very recently and only for very light nuclei $A \leq 10$ the problem has been solved "exactly",

Ab initio and 3-body forces



Approaches for heavy systems

For medium-heavy systems, ab initio calculations are not possible and one is obliged to resort to an effective force

We are simply forced to simplify the force (B.R. Mottelson)

Two main approaches:

Shell Model based on the bare forces, introduces correlations in the many-body states, or in the associated matrix elements. Spherically symmetric average potential + residual interaction in a subspace of the Hilbert space.

Mean-field methods devise a complementary strategy: defines an energy-density functional to produce directly the appropriate single particle potential. Search for the 'best' mean-field potential starting from a phenomenological energy functional + correlations. Self-consistent potentials.

The $V_{low\text{-}k}$

Inspired by the effective field theory and renormalization group for low-energy systems

Realistic V_{NN} potential → Construct $V_{low\text{-}k}$ integrating out the high-momentum part of V_{NN} → preserves the physics of V_{NN} up to a cutoff momentum Λ

S. Bogner et al. Phys Rev. C 65 (2002) 05130(R)
T.T.S. Kuo and E. Osnes, Lecture Notes in Physics, vol 364 (1990)
L. Coraggio et al. Prog. Part. Nucl. Phys 62 (2009) 135

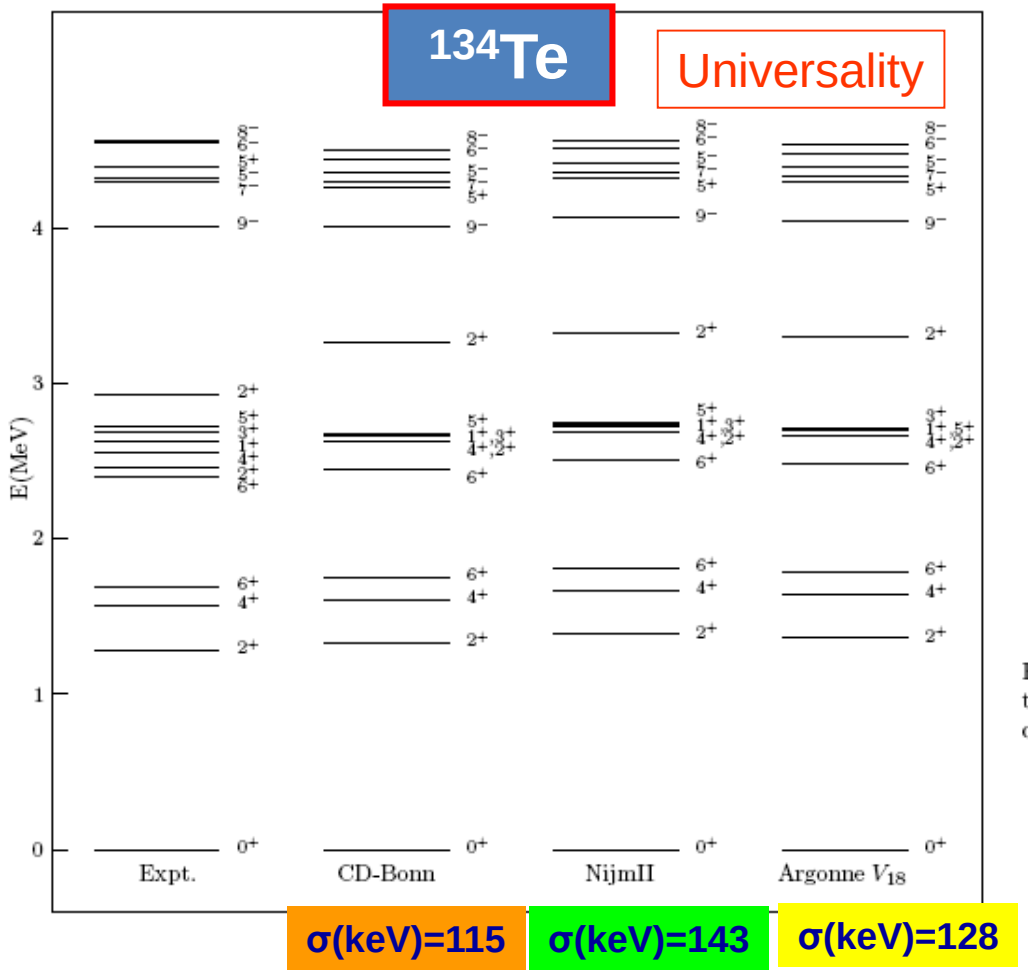
Features of $V_{low\text{-}k}$

- eliminates sources of non-perturbative behavior
- real potential in the *k*-space
- gives an approximately unique representation of the NN potentials for $\Lambda \cong 2$ fm^{-1} ⇨ $E_{Lab} \cong 350$ MeV

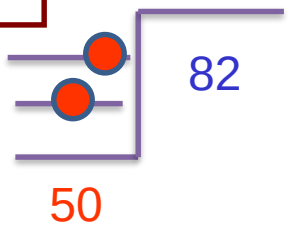
- $V_{low\text{-}k}$ is used to derive V_{eff} in a perturbative approach: folded diagrams expansion, in the framework of the Q-box formalism

To what extent the nuclear structure results depend on the choice of the starting potential?

V_{low-k} in the ^{132}Sn region: the 2p case



2 protons above ^{132}Sn



model space:
50≤Z≤82, 82≤N≤126

B(E2) values (in W.u.)

	CD-Bonn	Calc.	Expt.
0+ → 2+		20	24±3
4+ → 2+		4.3	4.3±0.30
6+ → 4+		1.9	2.05±0.03

A. Covello, L. Coraggio, A. Gargano, N. Itaco
PPNP 59 (2007) 401

The Interacting Shell Model

Is an approximation to the exact solution of the nuclear A-body problem using effective interactions in restricted spaces.

The effective interactions are obtained from the bare nucleon-nucleon interaction by means of a regularization procedure aimed to soften the short range repulsion.

The only way to obtain a tractable problem is to define a new reference “vacuum”.

Some hypothesis

The microscopic description of the nucleus we adopt is that of a non-(explicitly)-relativistic quantum many body system.

Therefore we assume:

- nucleon velocities small enough to justify the use of non-relativistic kinematics
- hidden meson and quark-gluon degrees of freedom
- two body interactions

The many-body hamiltonian

We want to solve the Schrödinger equation

$$H|\psi\rangle = \left(\sum_i T_i + \sum_{i<j} V_{ij} \right) |\psi\rangle = E|\psi\rangle$$

To treat this perturbatively, we express the Hamiltonian as

$$H|\psi\rangle = (H_0 + H_1)|\psi\rangle = E|\psi\rangle$$

with the unperturbed Hamiltonian

$$H_0 = \sum_i h_{0i} \qquad h_0|\varphi_i\rangle = (T_i + U_i)|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$$

and the perturbation

$$H_1 = \sum_{i<j} V_{ij} - \sum_i U_i$$

Where the auxiliary one-body potential *U* is chosen to make H_1 small

The interacting shell model

We want to solve the eigenvalue problem

$$H|\psi\rangle = (H_0 + H_1)|\psi\rangle = E|\psi\rangle$$

and E is the true energy of the system.

However we work in the model space, not in the full Hilbert space. We thus need to construct an effective Hamiltonian H_{eff} acting in the model space, such as

$$H_{eff}|\psi'\rangle = E|\psi'\rangle$$

Reducing the model space (2)

To reduce the Schrödinger equation of the A-body system

to a secular equation acting only in the selected subspace, with the condition

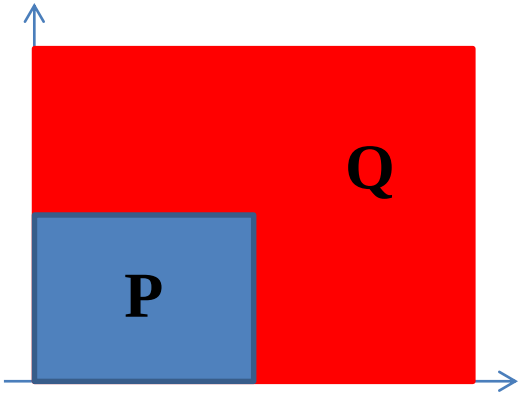
$$\langle \psi' | H_{eff} | \psi' \rangle = E$$

We divide the full Hilbert space into a model space P and an excluded space Q which is achieved using projection operators

$$P = \sum_{i=1}^{d} |\psi_i\rangle\langle\psi_i|$$

$$Q = \sum_{i=d+1}^{\infty} |\psi_i\rangle\langle\psi_i|$$

$$P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0$$



The model space

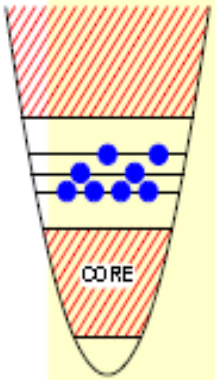
The success of the independent particle model strongly suggests that the very singular free NN interaction can be regularized in the nuclear medium.
For a given number of protons and neutrons the mean field orbitals can be grouped in three blocks

- **Inert core**: orbits that are always fully occupied
- **Valence space**: orbits that contain the physical degrees of freedom relevant to a given property. The distribution of the valence particles among these orbitals is governed by the interaction
- **External space**: all the remaining orbits that are always empty

Shell model approach

Calculations Ab Initio ☞QCD

- Realistic NN interactions
- 3-body forces



- define valence space
- $H_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$
- ⇝ INTERACTIONS (monopole corrections)
- build and diagonalize Hamiltonian matrix
- ⇝ CODES

Weak processes:

- β decays
- $\beta\beta$ decays

$$[T^{0\nu}_{1/2}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$$

☞ASTROPHYSICS

☞PARTICLE PHYSICS

Collective excitations:

- deformation, superdeformation
- superfluidity
- symmetries

Shell evolution far from stability:

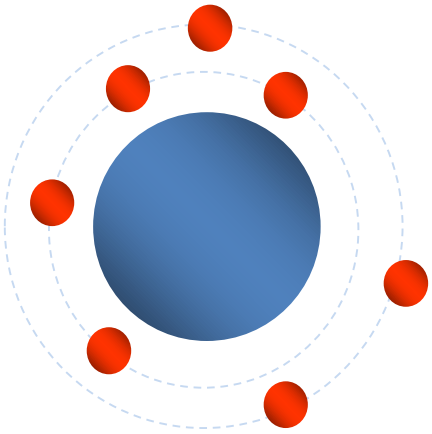
- Shell quenching
- New magic numbers

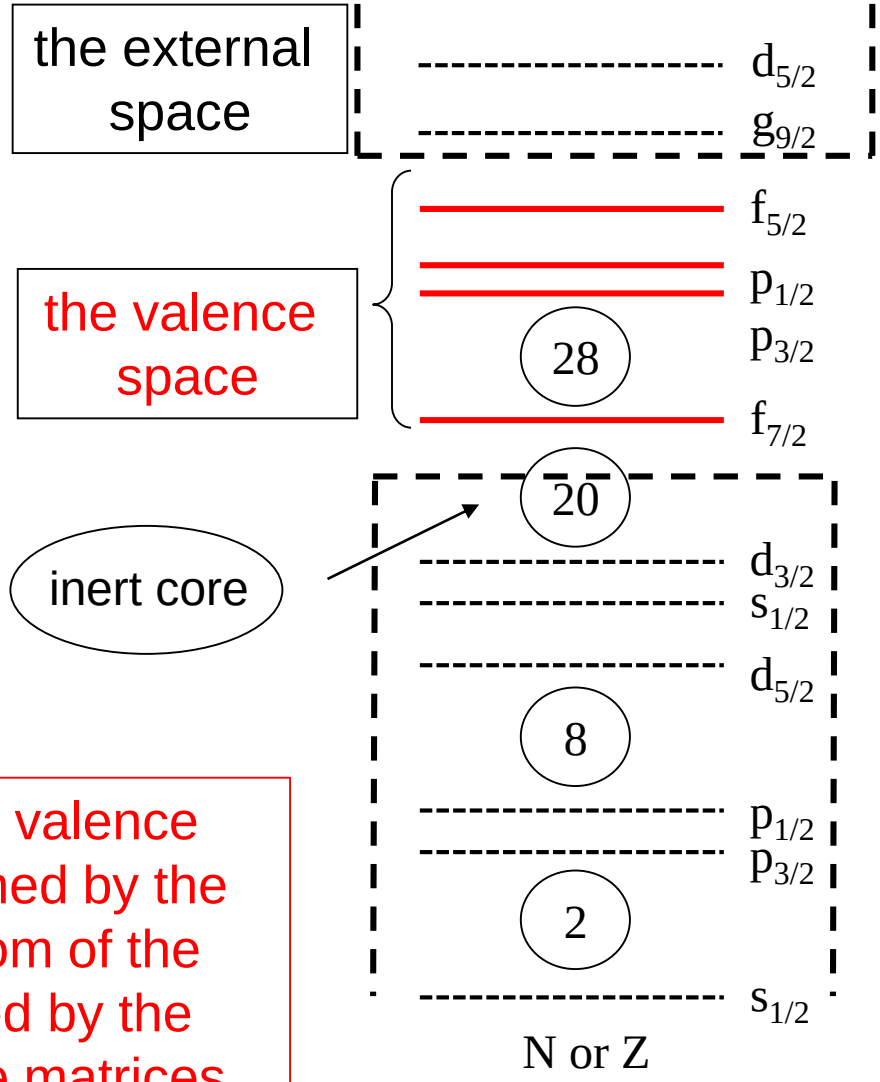
☞ASTROPHYSICS

Ingredients for the Shell Model calculations

1) an inert core
2) a valence space
3) an effective interaction that mocks up the general Hamiltonian in the restricted basis

$$H_{\text{eff}}\psi_\alpha = H_0 + V_{\text{eff}}\psi_\alpha = E_\alpha\psi_\alpha$$
$$\text{with } H_0 = T + U$$





The choice of the valence space is determined by the degrees of freedom of the system and limited by the dimensions of the matrices to be diagonalized

The effective interaction

Perturbation theory

Hilbert space → Valence space

$$V_{NN} \Rightarrow V^{eff}$$

$$\hat{H}\left|\Psi\right\rangle = E\left|\Psi\right\rangle \Rightarrow \hat{H}^{eff}\left|\Psi_{eff}\right\rangle = E\left|\Psi_{eff}\right\rangle$$

$$\left\langle\Psi\middle|\hat{O}\middle|\Psi\right\rangle \Rightarrow \left\langle\Psi_{eff}\middle|\hat{O}_{eff}\middle|\Psi_{eff}\right\rangle$$

- *microscopic* effective interaction (*realistic* interaction)

$$V_{NN} \Rightarrow \mathrm{G} \Rightarrow V^{eff}$$

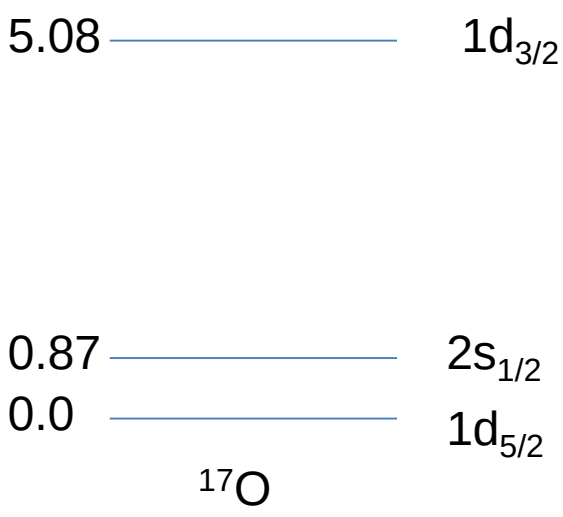
M. Hjorth-Jensen et al, Phys.Rep.261 (1995)

- *empirical* interaction (fitted to the data)

B.A.Brown, B.H.Wildenthal, Ann. Rev. Nucl.Part.Sci. 38 (1988)

- *schematic* interaction (delta-force, etc)

A simple case: ^{18}O



We will calculate some states of ^{18}O as an example of ISM calculation

Two neutrons above a core of ^{16}O

The lowest states have nucleons in the $1d_{5/2}$ orbit:

with $(1d_{5/2})^2$: we can construct only 3 states (J^π = 0+, 2+ or 4+)

To construct other states, we have to consider a wider valence space: we thus include the $2s_{1/2}$ level.

The strength of the effective interaction V(1,2) depends on the valence space

The states in the model space

We can then construct the following states:

$$J^{\pi} = 0^{+} \rightarrow (1d_{5/2})^{2}_{0^{+}}; \quad (2s_{1/2})^{2}_{0^{+}}$$

$$J^{\pi} = 2^{+} \rightarrow (1d_{5/2})^{2}_{2^{+}}; \quad (1d_{5/2}2s_{1/2})_{2^{+}}$$

$$J^{\pi} = 3^{+} \rightarrow (1d_{5/2}2s_{1/2})_{3^{+}}$$

$$J^{\pi} = 4^{+} \rightarrow (1d_{5/2})^{2}_{4^{+}}$$

The energy values for these states will be the corresponding of the eigenstates of the hamiltonian:

$$H = H_0 + H_{res} = \sum_{i=1}^{2} h_0(i) + V_{12}$$

where the core energy corresponding to the closed shell system (16O) E_0 is taken as a reference value

eigenfunctions

The wave functions will be linear combinations of the possible basis functions.
For J=0+ we obtain two eigenfunctions

$$\left|\Psi_{0^+;1}\right\rangle = \sum_{k=1}^{n} a_{k,1}\left|\psi_k^0;0^+\right\rangle$$

$$\left|\Psi_{0^+;2}\right\rangle = \sum_{k=1}^{n} a_{k,2}\left|\psi_k^0;0^+\right\rangle$$

For the particular case of ^{18}O we define

$$\left|\psi_1^0;0^+\right\rangle = \left|(1d_{5/2})^2;0^+\right\rangle$$

$$\left|\psi_2^0;0^+\right\rangle = \left|(2s_{1/2})^2;0^+\right\rangle$$

The general case

If the basis set is defined as $\left|\psi_k^0\right\rangle (k=1,...,n)$

$$\psi_k^0(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)=\frac{1}{\sqrt{A!}}\det\begin{pmatrix}\varphi_1(\vec{r}_1) & \varphi_1(\vec{r}_1) & \cdots & \varphi_1(\vec{r}_A)\\ \vdots & \ddots & & \vdots\\ \varphi_A(\vec{r}_1) & \varphi_A(\vec{r}_1) & \cdots & \varphi_A(\vec{r}_A)\end{pmatrix}$$

The total wave function can be expanded as: $\left|\Psi_p\right\rangle=\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle$

The coefficients a_{kp} have to be determined by solving the Schrödinger equation

$$H\left|\Psi_p\right\rangle=E_p\left|\Psi_p\right\rangle$$

$$(H_0+H_{res})\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle=E_p\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle$$

or

$$\sum_{k=1}^{n}\left\langle\psi_l^0\left|H_0+H_{res}\right|\psi_k^0\right\rangle a_{kp}=E_p a_{lp}$$

the matrix to be diagonalized

Since $\left|\psi_k^0\right\rangle$ corresponds to eigenfunctions of H_0 with eigenvalues (unperturbed energies) $E_{0,}$, calling

$$\left\langle \psi_\ell^0 \left| H_0 + H_{res} \right| \psi_k^0 \right\rangle = H_{\ell k}$$

we obtain:

$$\sum_{k=1}^{n} H_{lk} a_{kp} = E_p a_{lp}$$

$$H_{lk} = E_k^0 \delta_{lk} + \left\langle \psi_l^0 \left| H_{res} \right| \psi_k^0 \right\rangle$$

The eigenvalue equation becomes a matrix equation:

$$[H][A] = [E][A]$$

The matrix equation

The eigenvalue equation becomes a matrix equation:

$$[H][A] = [E][A]$$

This forms a secular equation for the eigenvalues E_p:

$$\begin{vmatrix} H_{11} - E_p & H_{12} & \dots H_{1n} \\ H_{21} & H_{22} - E_p & \dots H_{2n} \\ \vdots & \vdots & \vdots \\ H_{n1} & \dots \quad \dots & H_{22} - E_p \end{vmatrix} = 0$$

This is a nth degree equation for the n-roots E_p (p=1,2,…n)

The solutions

Once we have the energies E_p, we can use

$$\sum_{k=1}^{n} E_k^0 \delta_{lk} a_{kp} + \sum_{k=1}^{n} \left\langle \psi_l^0 \middle| H_{res} \middle| \psi_k^0 \right\rangle a_{kp} = E_p a_{lp}$$

to obtain the coefficients a_{kp}. Using the orthonormalization: $\sum_{k=1}^{n} a_{kp} a_{kp'} = \delta_{pp'}$

We can then write

$$\sum_{l,k=1}^{n} a_{lp'} E_k^0 \delta_{lk} a_{kp} + \sum_{l,k=1}^{n} a_{lp'} \left\langle \psi_l^0 \middle| H_{res} \middle| \psi_k^0 \right\rangle a_{kp} = E_p \delta_{pp'}$$

Which is a matrix equation of the form $[A]^{-1}[H][A] = [E]$

This equation indicates a similarity transformation to a new basis that makes [H] diagonal

If n is large this process needs a high-speed computer

Configuration mixing

If the non-diagonal matrix elements are of the order of the unperturbed energy differences

$$|H_{ij}| \approx |E_i^0 - E_j^0|$$

large configuration mixing will result and the final energy eigenvalues E_p will be very different from the unperturbed ones.

On the contrary, if the non-diagonal matrix elements are small, these energy shifts will be small and we can use perturbation theory to solve the problem

$$|H_{ij}| << |E_i^0 - E_j^0|$$

Back to the ^{18}O problem

We now consider the case of the $J=0^+$ states in ^{18}O in the $1d_{5/2}$, $2s_{1/2}$ model space.

$$\left|\psi_1^0;0^+\right\rangle = \left|(1d_{5/2})^2;0^+\right\rangle$$

$$\left|\psi_2^0;0^+\right\rangle = \left|(2s_{1/2})^2;0^+\right\rangle$$

The Hamiltonian matrix is now

$$H = \begin{bmatrix} 2\varepsilon_{d_{5/2}} + \left\langle (d_{5/2})^2;0^+ \left| V_{12} \right| (d_{5/2})^2;0^+ \right\rangle & \left\langle (d_{5/2})^2;0^+ \left| V_{12} \right| (s_{1/2})^2;0^+ \right\rangle \\ \left\langle (s_{1/2})^2;0^+ \left| V_{12} \right| (d_{5/2})^2;0^+ \right\rangle & 2\varepsilon_{s_{1/2}} + \left\langle (s_{1/2})^2;0^+ \left| V_{12} \right| (s_{1/2})^2;0^+ \right\rangle \end{bmatrix}$$

These diagonal matrix elements yield the first correction to the unperturbed single-particle energies $2\varepsilon_{s_{1/2}}$ e $2\varepsilon_{d_{5/2}}$

The diagonalization

The secular equation can thus be written:

$$\begin{vmatrix} H_{11}-\lambda & H_{12} \\ H_{12} & H_{22}-\lambda \end{vmatrix} = 0$$

We then get the quadratic equation:

$$\lambda^2 - \lambda(H_{11}+H_{22}) - H_{12}^2 + H_{11}H_{22} = 0$$

With the roots:

$$\lambda_{\pm} = \frac{H_{11}+H_{22}}{2} \pm \tfrac{1}{2}[(H_{11}-H_{22})^2 + 4H_{12}^2]^{1/2}$$

$$\Delta\lambda = \lambda_+ - \lambda_- = [(H_{11}-H_{22})^2 + 4H_{12}^2]^{1/2}$$

Even if the **unperturbed states are degenerate** there is a repulsion between them that separates the two solutions.

in perturbation theory…

$$\lambda_{\pm} = \frac{H_{11} + H_{22}}{2} \pm \tfrac{1}{2}[(H_{11} - H_{22})^2 + 4H_{12}^2]^{1/2}$$

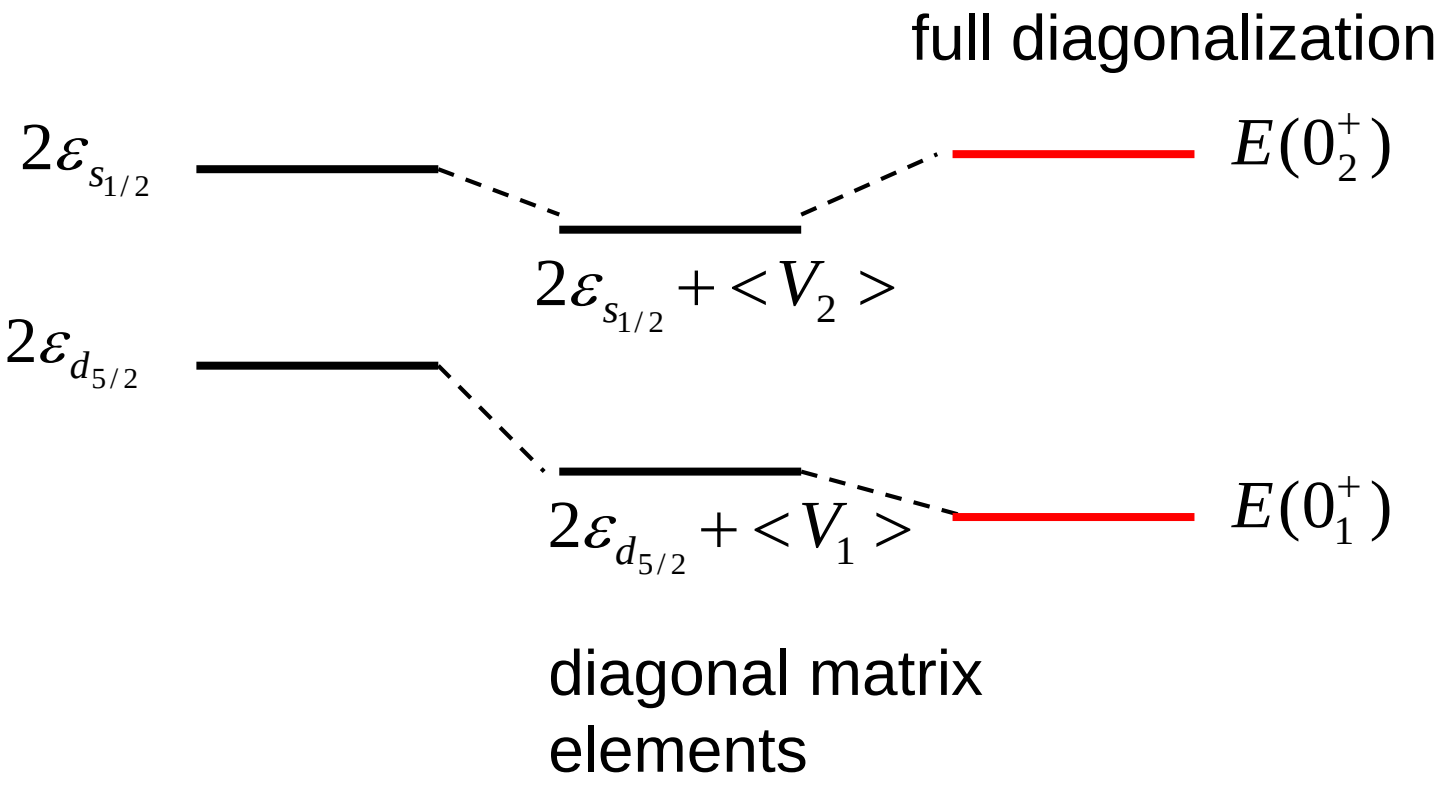
If: $|H_{12}| << |H_{11} - H_{22}|$ ⇒ $\Delta\lambda \to |H_{11} - H_{22}|$

We can write the series,

$$\lambda_+ = \lambda_1 = H_{11} + \frac{H_{12}^2}{H_{11} - H_{22}} + \ldots$$

$$\lambda_- = \lambda_2 = H_{22} + \frac{H_{12}^2}{H_{11} - H_{22}} + \ldots$$

Final spectrum for the 0^+ in ^{18}O

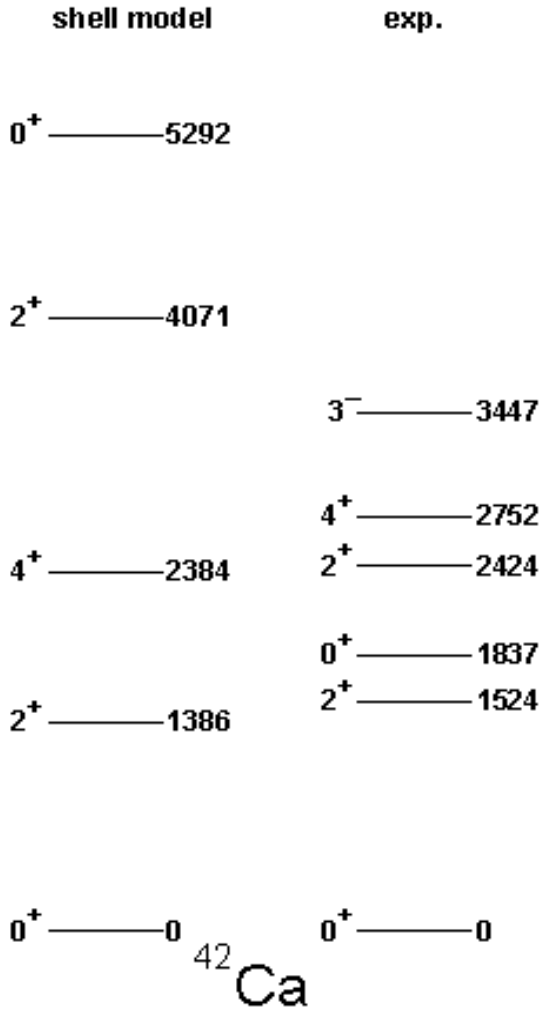


Eigenvalues and eigenvectors

The problem consists on diagonalizing a matrix in the model space. The basis is formed by the eigenfunctions of the mean field

For "normal" 0^+ states, one needs to diagonalize the 4×4 matrix:

$$
\begin{array}{cccc}
(f_{\frac{7}{2}})^2 & (p_{\frac{3}{2}})^2 & (p_{\frac{1}{2}})^2 & (f_{\frac{5}{2}})^2 \\
\left(\begin{array}{c} 0.0\times2 \\ +(-1.920) \end{array}\right. & -0.783 & -0.714 & -2.788 \\
 & \begin{array}{c} 2.0\times2 \\ +(-1.206) \end{array} & -1.465 & -0.777 \\
 & & \begin{array}{c} 4.0\times2 \\ +(-0.249) \end{array} & -0.392 \\
 & & & \left.\begin{array}{c} 6.5\times2 \\ +(-1.687) \end{array}\right)
\end{array}
$$



Basic Shell Model

The hamiltonian (only two-body forces)

$$H = \sum_{i=1}^{A} \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} \sum_{i,j=1}^{A} V_{ij}(\vec{r})$$

$$H = \sum_{i=1}^{A} \left(\frac{\vec{p}_i^{\,2}}{2m} + U(r_i) \right) + \sum_{i,j=1}^{A} V_{ij}(|\, r_i - r_j \,|) - \sum_{i=1}^{A} U(r_i) = H_0 + H_{res}$$

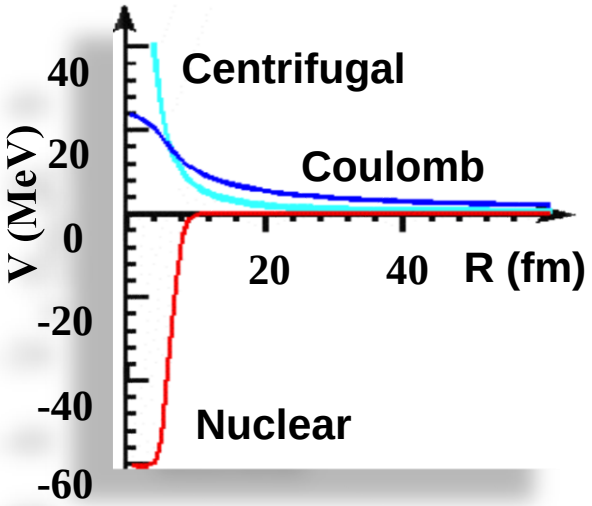
$$H_0 \phi(\mathrm{r}_1, \ldots, \mathrm{r}_A) = E_0 \phi(\mathrm{r}_1, \ldots, \mathrm{r}_A)$$

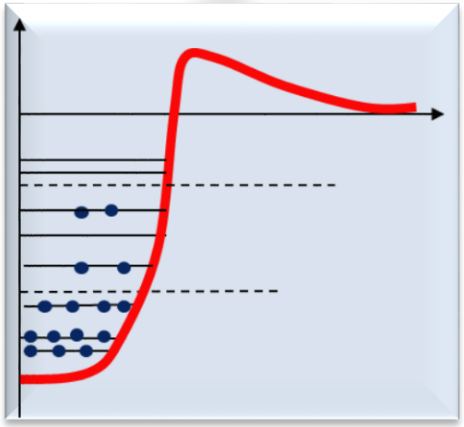
$$E_0 = \sum_i \varepsilon_i^{(0)}$$

$$\phi = \frac{1}{\sqrt{A!}} \det \begin{pmatrix} \psi_1(r_1) & \cdots & \psi_1(r_A) \\ \vdots & \ddots & \vdots \\ \psi_A(r_1) & \cdots & \psi_A(r_A) \end{pmatrix}$$

spherical mean field

U(r) is a central (1-body) potential





Shell model basis

Shell-model basis states

The basis states have good angular momentum (coupling all j values to J), good parity and good isospin.

The slater determinants can also be constructed with good M and Tz
e.g., 4 particles in the sd shell with M=0:

$$\left|1d_{5/2},-\tfrac{1}{2};1d_{5/2},-\tfrac{3}{2};1d_{5/2},+\tfrac{3}{2};1d_{5/2}+\tfrac{1}{2}\right\rangle$$

The number of basis states can be estimated approximately as (Ω: shell degeneracy, n: valence particles)

$$N_{basis}=\binom{\Omega_p}{n_p}\binom{\Omega_n}{n_n}$$

For example for ^{60}Zn with 10 valence protons and 10 valence neutrons in the *fp* shell

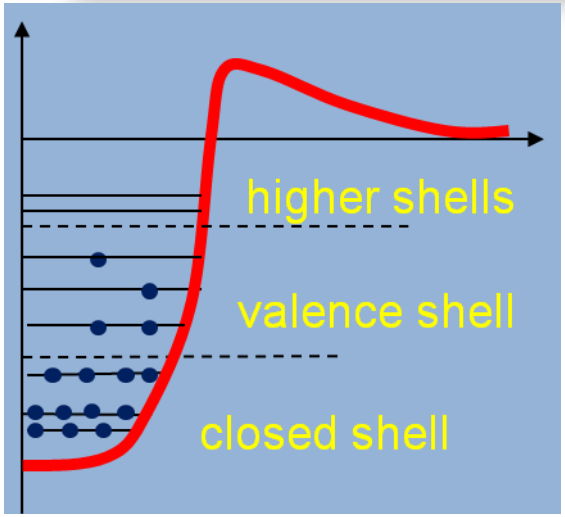
$$N_{basis}=\binom{20}{10}\binom{20}{10}\approx 3.4\cdot 10^{10}$$

Configuration mixing

$$\phi = \frac{1}{\sqrt{A!}} \det \begin{pmatrix} \psi_1(r_1) & \cdots & \psi_1(r_A) \\ \vdots & \ddots & \vdots \\ \psi_A(r_1) & \cdots & \psi_A(r_A) \end{pmatrix}$$

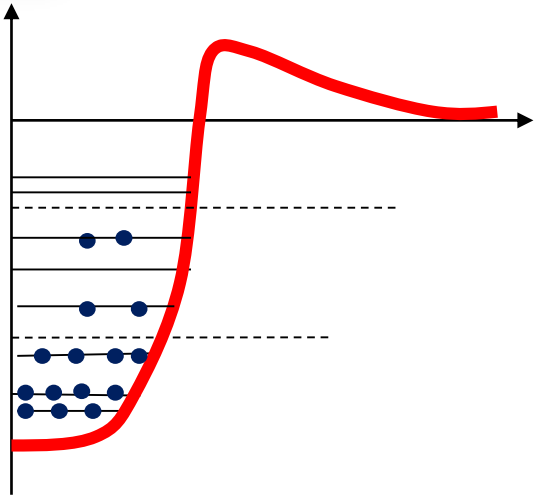
$$\mathcal{H} = \begin{pmatrix} \langle\phi_1|H|\phi_1\rangle & \langle\phi_1|H|\phi_2\rangle & \cdots \\ \langle\phi_2|H|\phi_1\rangle & \langle\phi_2|H|\phi_2\rangle & \cdots \\ \langle\phi_3|H|\phi_1\rangle & \vdots & \end{pmatrix} = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & \ddots \end{pmatrix}$$

$$\Psi = \sum_i^{\infty} c_i \phi_i$$

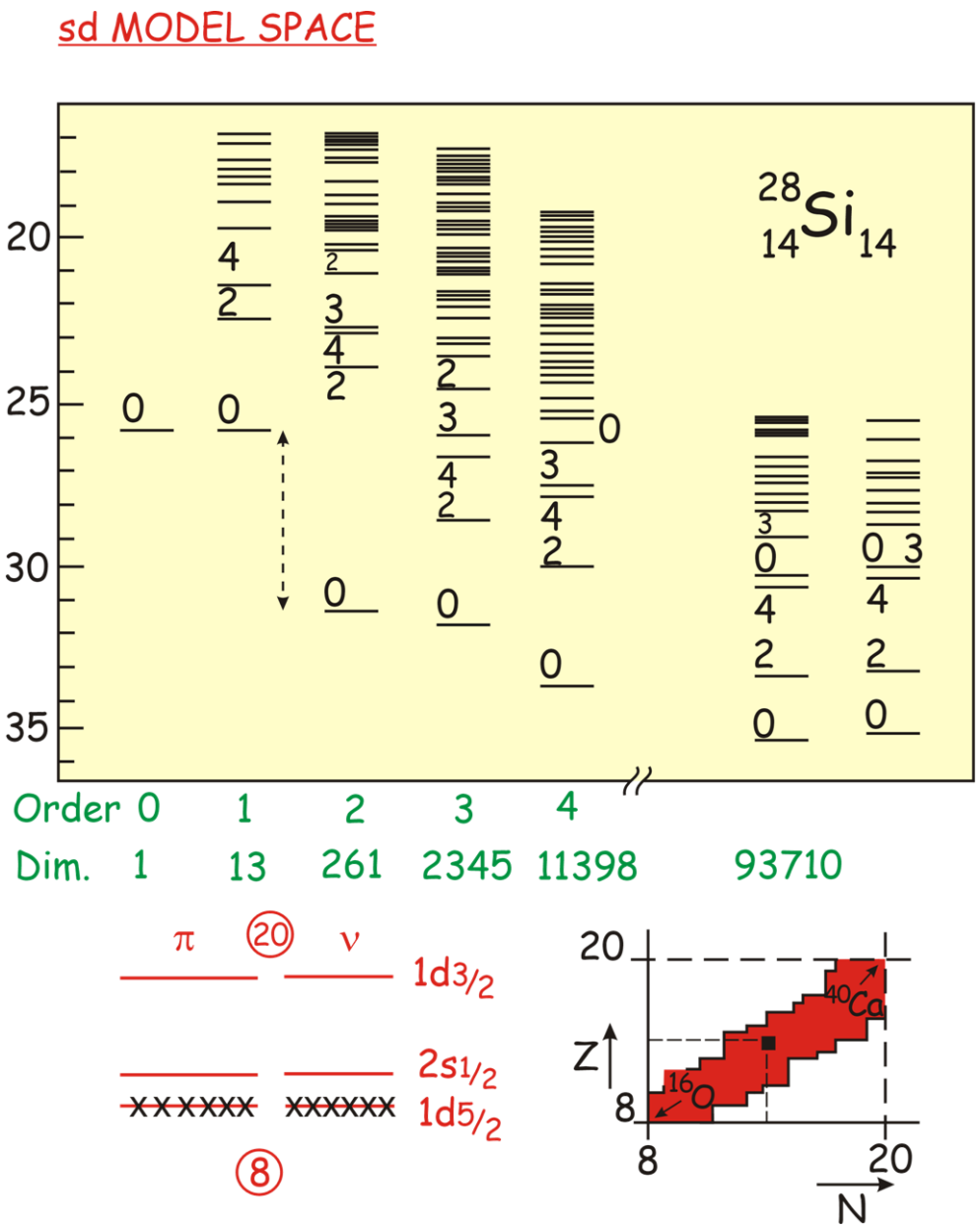


Mixing of configurations

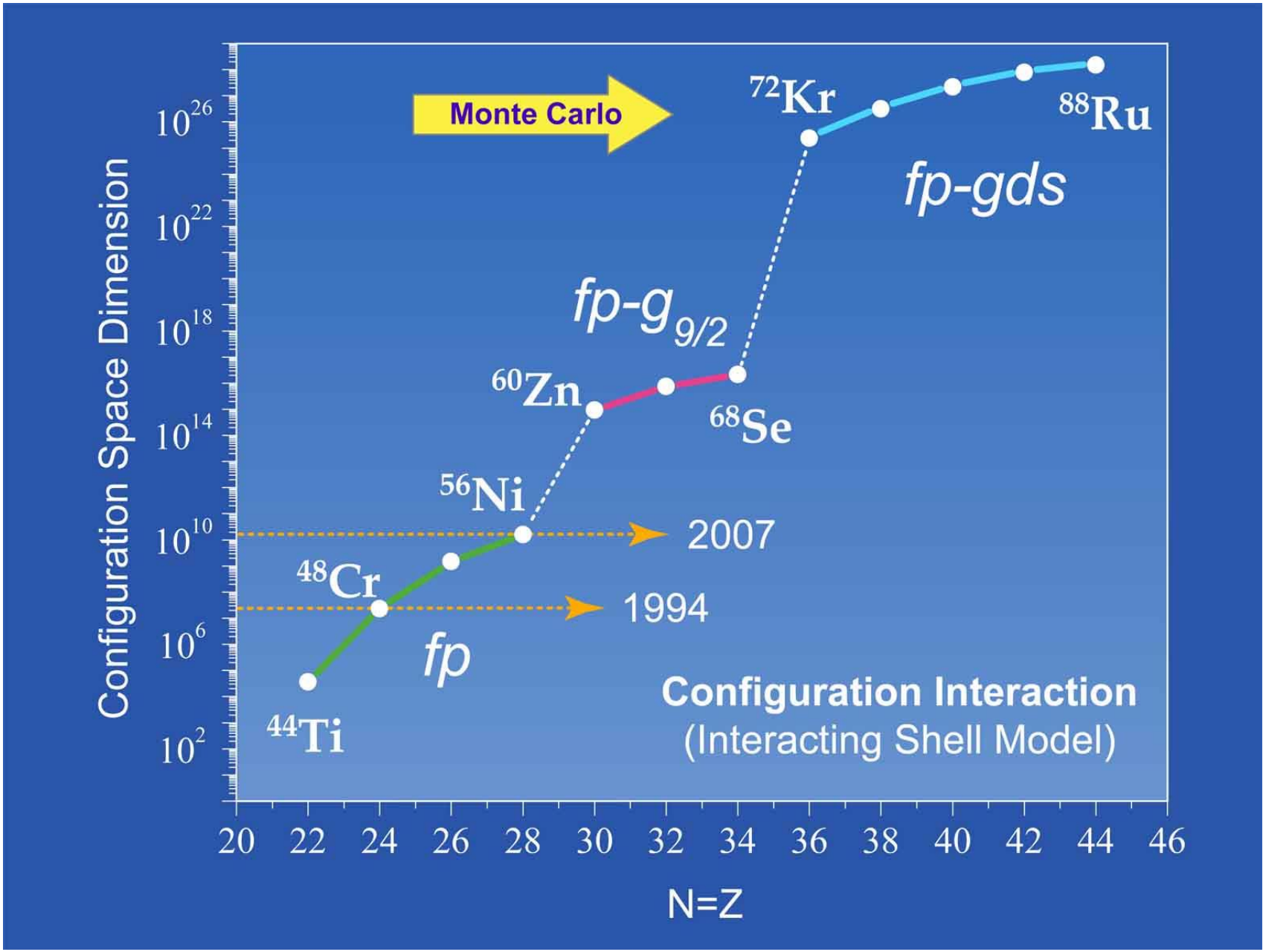
due to the residual interaction



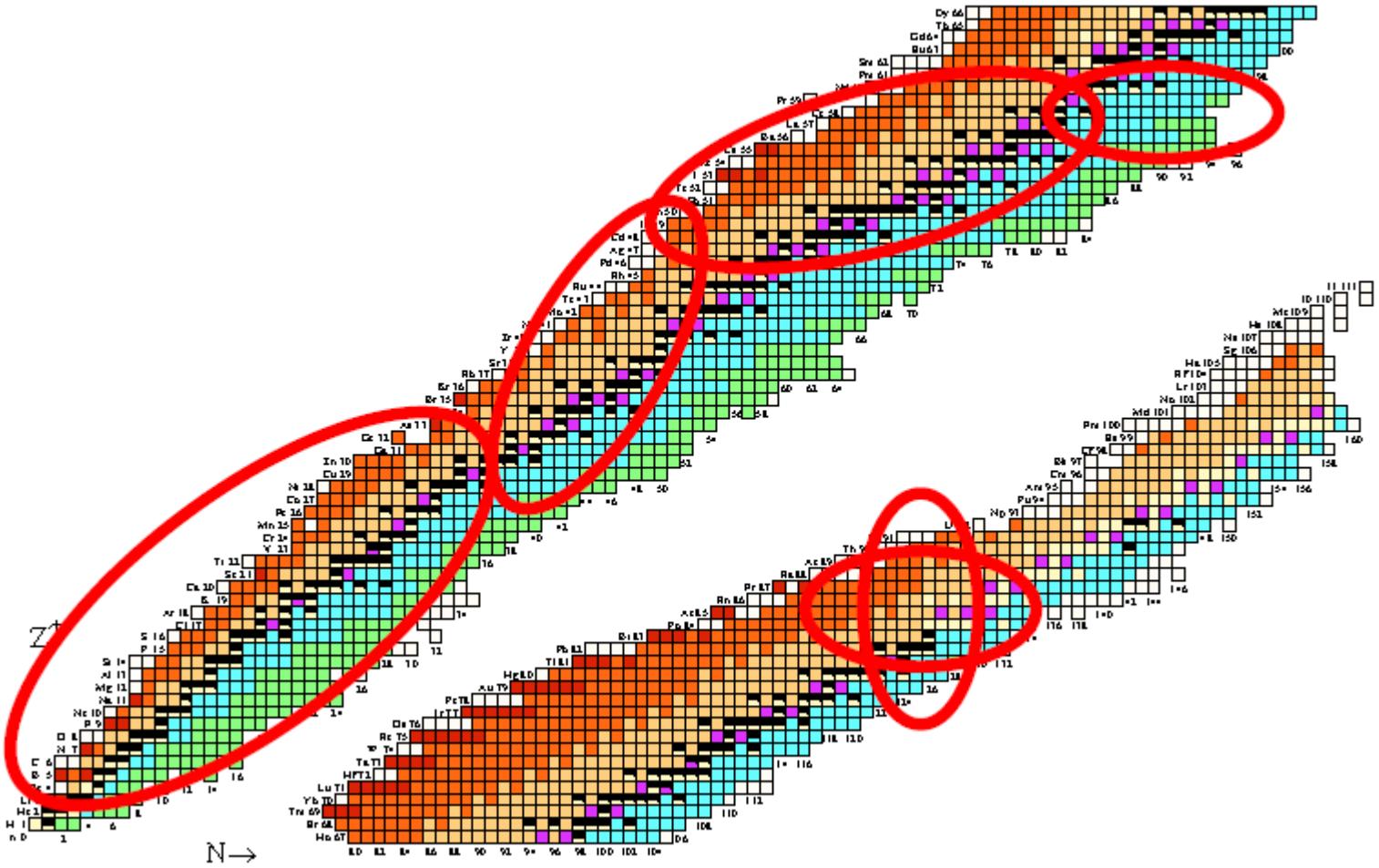
An example


sd MODEL SPACE
20
25
30
35
$^{28}_{14}Si_{14}$
Order 0 1 2 3 4
Dim. 1 13 261 2345 11398 93710
π
20
ν
1d3/2
2s1/2
1d5/2
8
Z
N
8
20
^{40}Ca
^{16}O

What can be achieved



Accessible regions



Algorithms and codes

Algorithms include Direct Diagonalisation, Lanczos, Monte Carlo Shell Model, Quantum Monte Carlo Diagonalization, DMRG etc. There are also a number of different extrapolation ansatzs

The Strasbourg-Madrid codes (Antoine, Nathan), can deal with problems involving basis of 10^{10} Slater determinants, using relatively modest computational resources. Other competitive ones in the market are OXBACH, NUSHELL and MSHELL

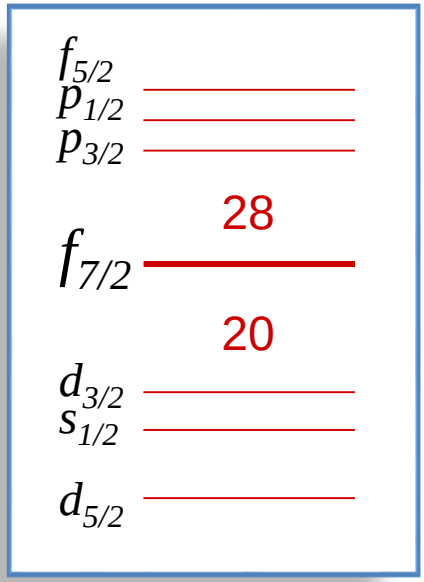
Can shell model describe collective states?

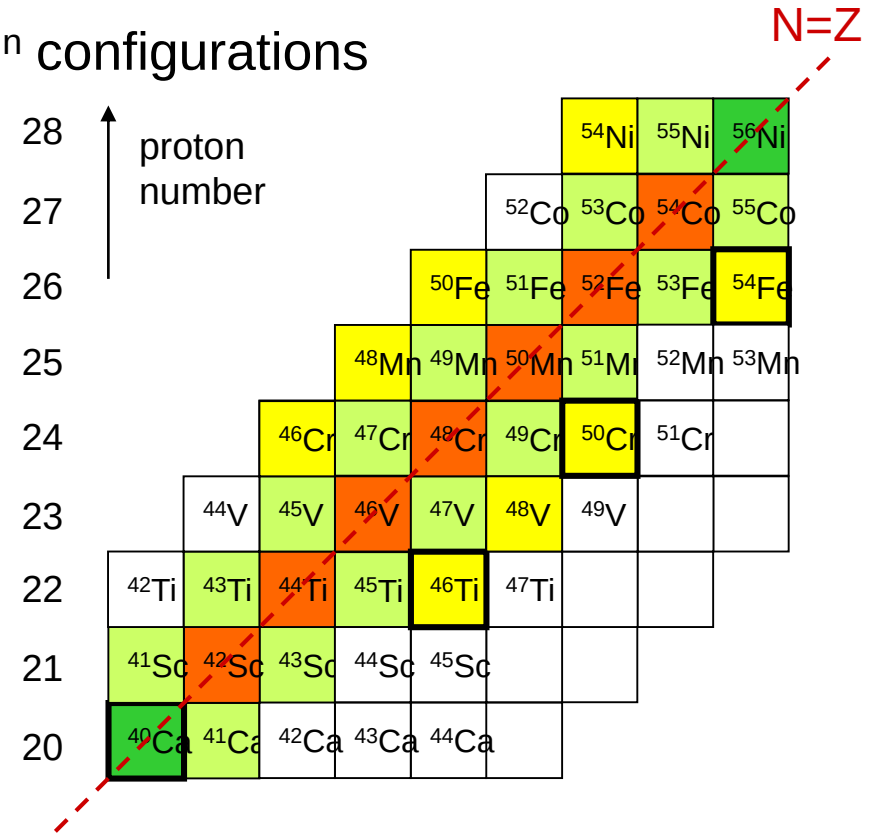
Example: the $f_{7/2}$ shell

The $1f_{7/2}$ shell is isolated in energy from the rest of fp orbitals

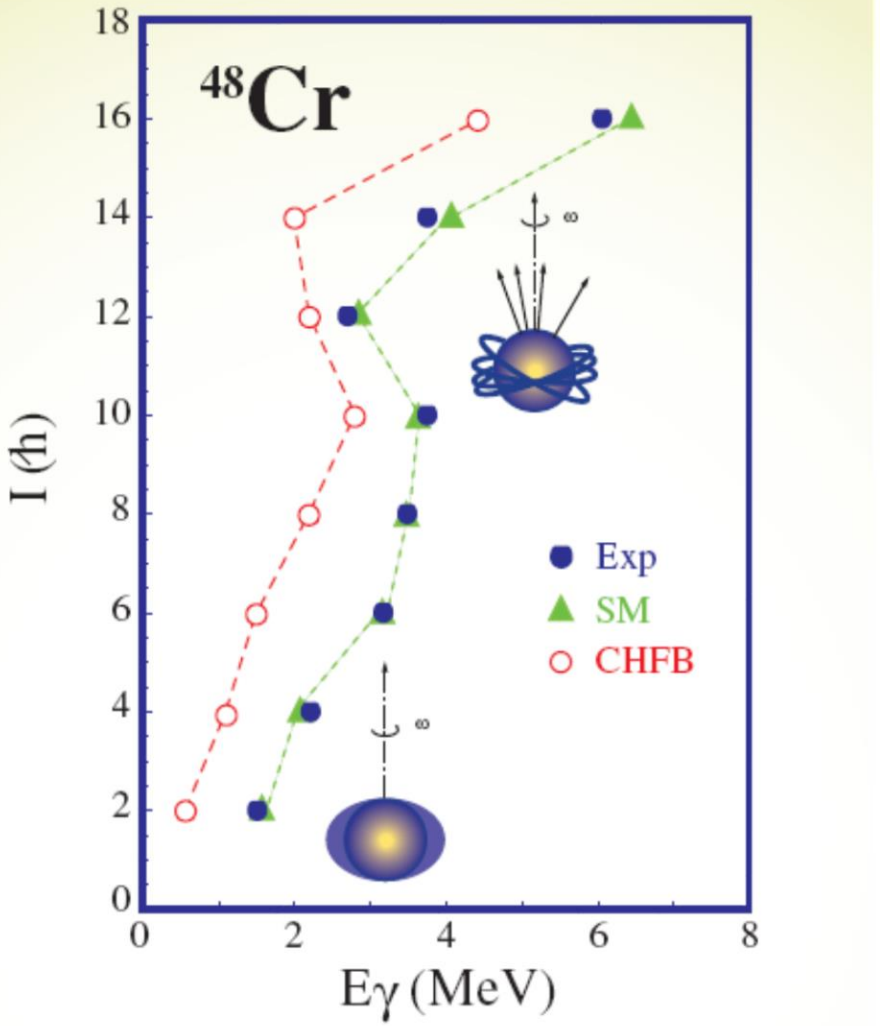
Wave functions are dominated by $(1f_{7/2})^n$ configurations

High-spin states experimentally reachable

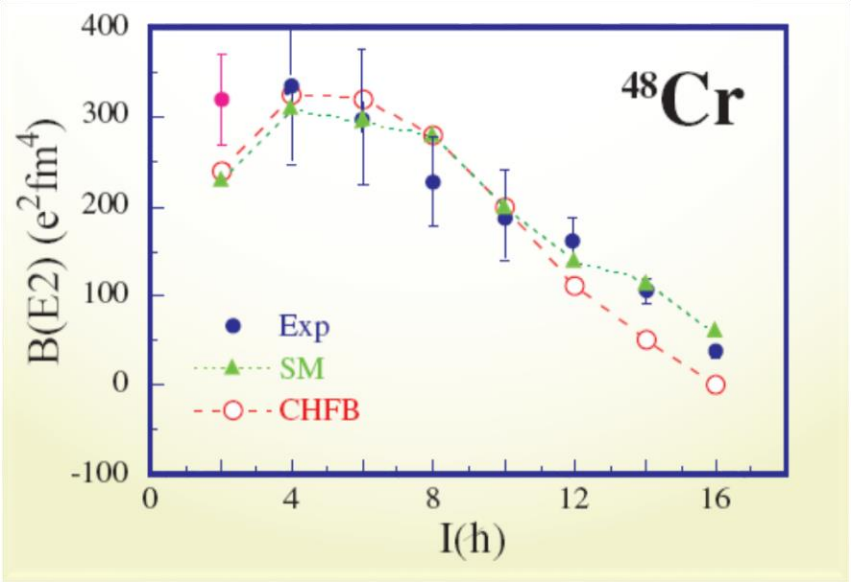




Shell model and collective phenomena

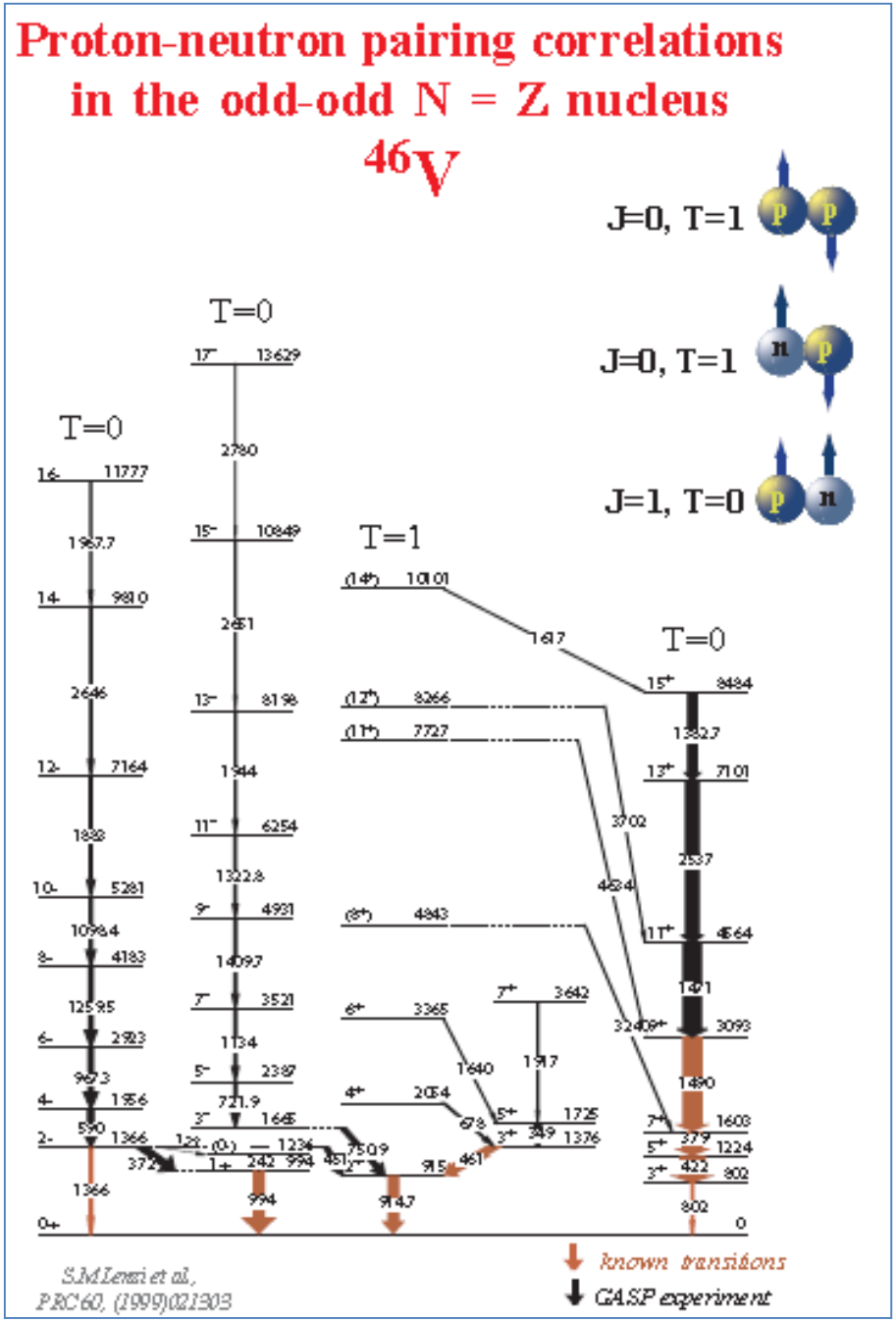


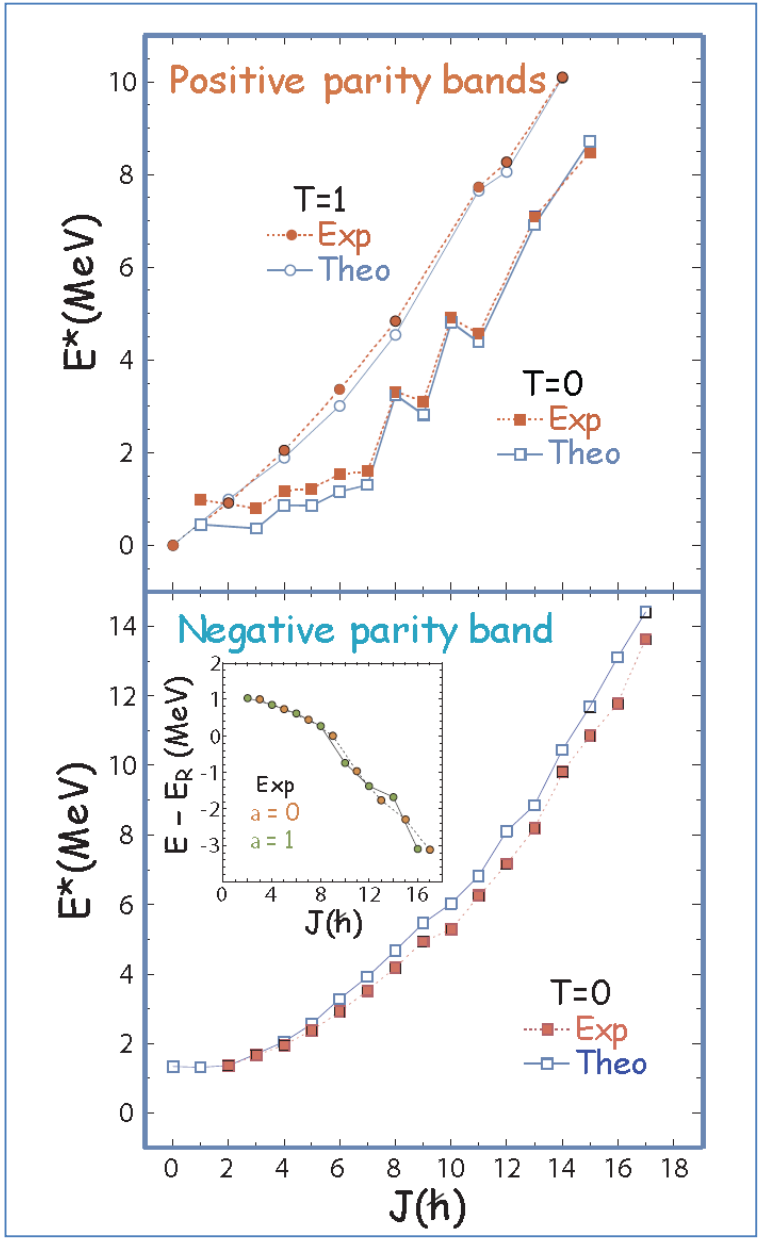
Shell model calculations in the full fp shell give an excellent description of the structure of collective rotations in nuclei of the $f_{7/2}$ shell



Theory: *E. Caurier et al.*, Phys.Rev.Lett.75(1995)225
Experiments: *S. M. Lenzi et al.*, Z.Phys.**A354**(1996)117 - *F. Brandolini et al.*, Nucl.Phys.**A642**(1998)387

Proton-neutron pairing correlations in the odd-odd N = Z nucleus ^{46}V





How well shell model describes the nuclear structure far from stability?

Bibliography

- P. J. Brussaard, P. W. M. Glaudemans, Shell-Model Applications in Nuclear Spectroscopy (North-Holland, Amsterdam, 1977).
- K. Heyde, The Nuclear Shell Model (Springer-Verlag, Heidelberg, 2004)
- A. DeShalit, I. Talmi, Nuclear Shell Theory (Academic Press, New York and
- London, 1963)
- I. Talmi, Simple Models of Complex Nuclei, The Shell Model and the Interacting Boson Model (Harwood Academic Publishers, New York, 1993)
- R. D. Lawson, Theory of the Nuclear Shell Model (Clarendon Press, 1980).
- Review article: E. Caurier et al, The shell model as a unified view of nuclear structure, Rev. Mod. Phys. 77 (2005) 427.