Dirac Medallists 2017

Charles H. Bennett

David Deutsch

Peter W. Shor

Citation:

for pioneering work in applying fundamental concepts of quantum mechanics to solving basic problems in computation and communication, and therefore bringing together the fields of quantum mechanics, computer science and information.

When it all began …

Villa Gualino meeting in Fall, 1994 (ISI Torino)

First presentation of ion trap quantum computer

Source: DiVincenzo Blog: http://blog.qutech.nl/index.php/2018/02/22/looking-b … divincenzo-criteria/

When it all began …

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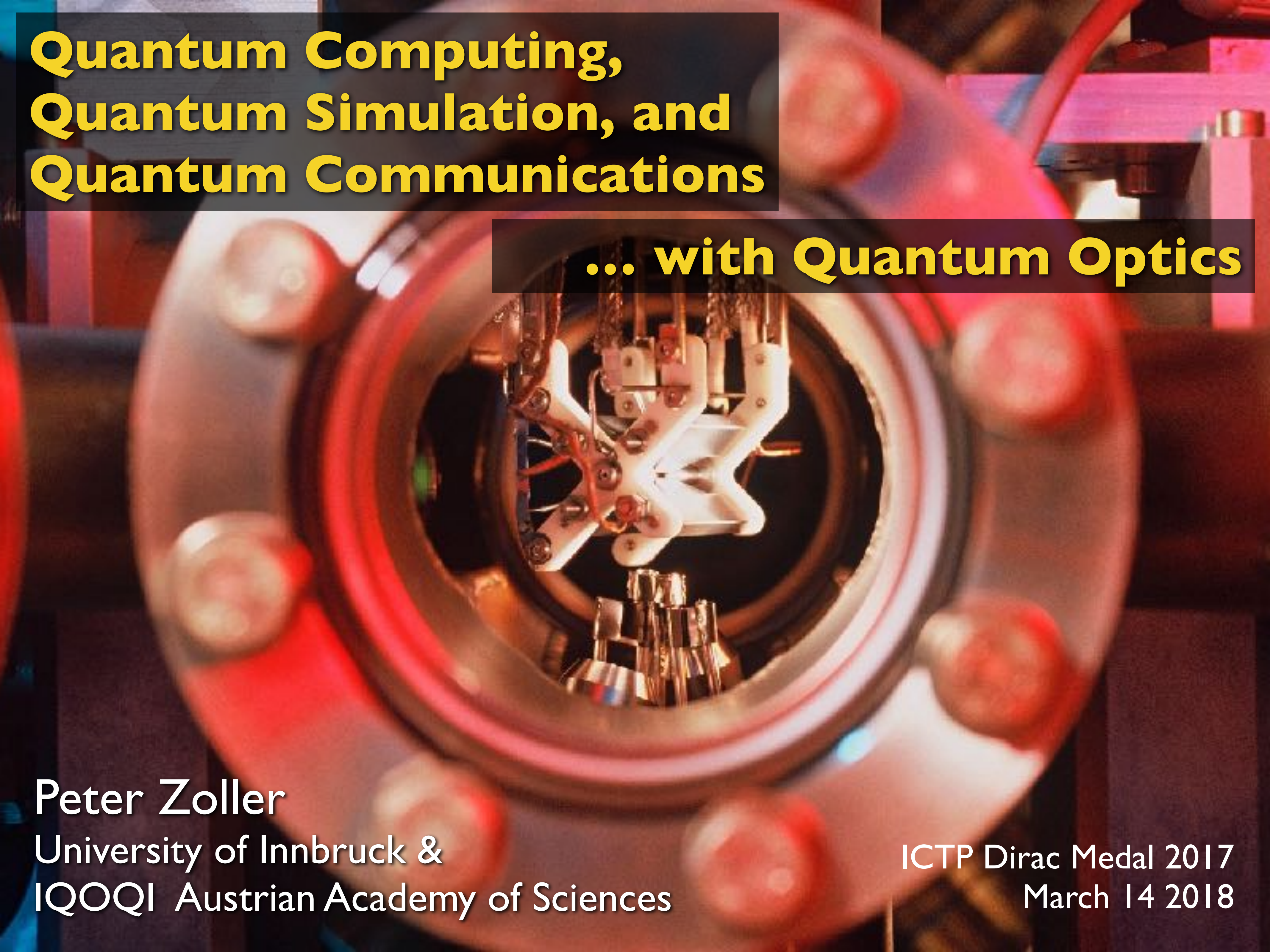
Source: DiVincenzo Blog: http://blog.qutech.nl/index.php/2018/02/22/looking-back-at-the-divincenzo-criteria/

Quantum Computing, Quantum Simulation, and Quantum Communications

... with Quantum Optics

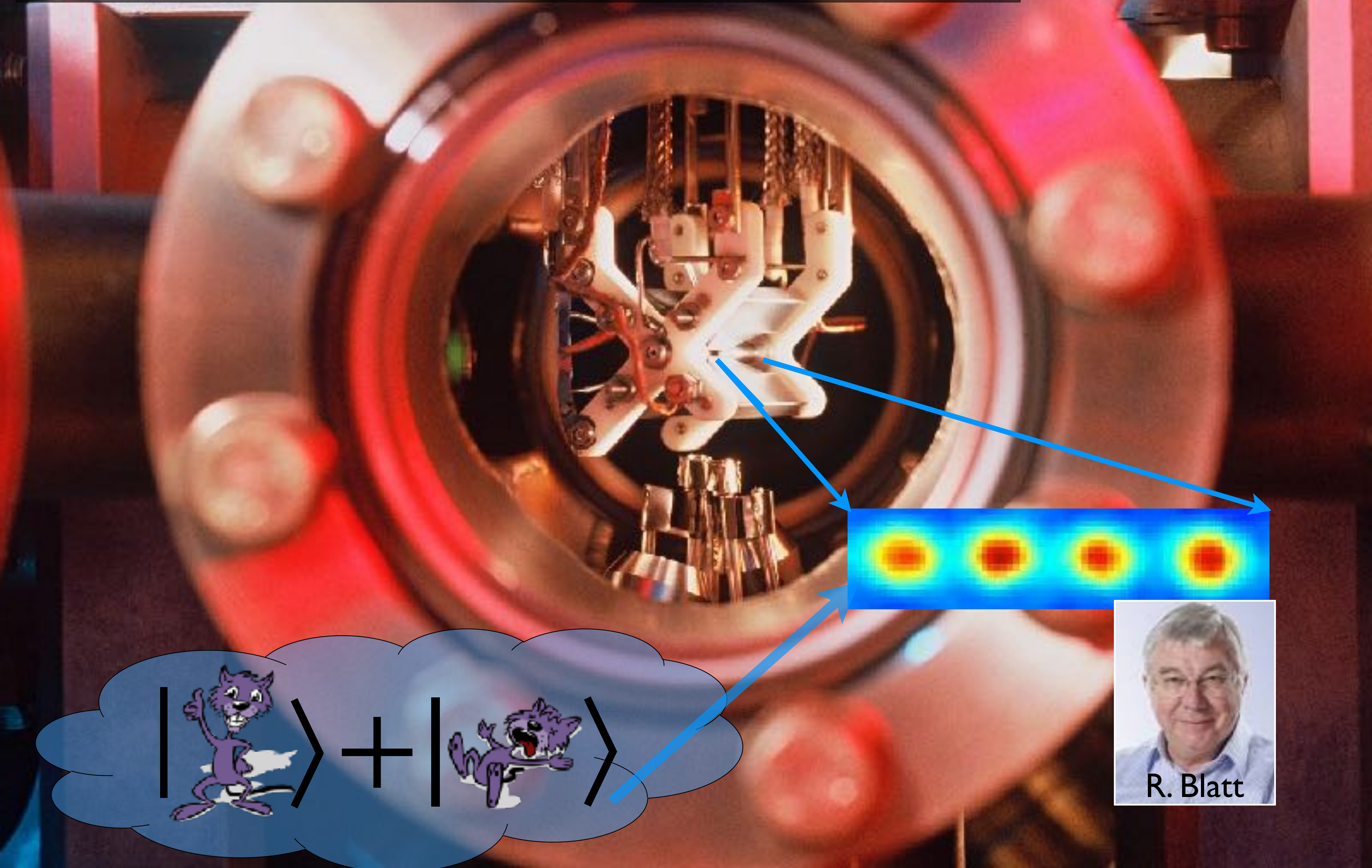


Peter Zoller
University of Innbruck &
IQOQI Austrian Academy of Sciences

ICTP Dirac Medal 2017
March 14 2018

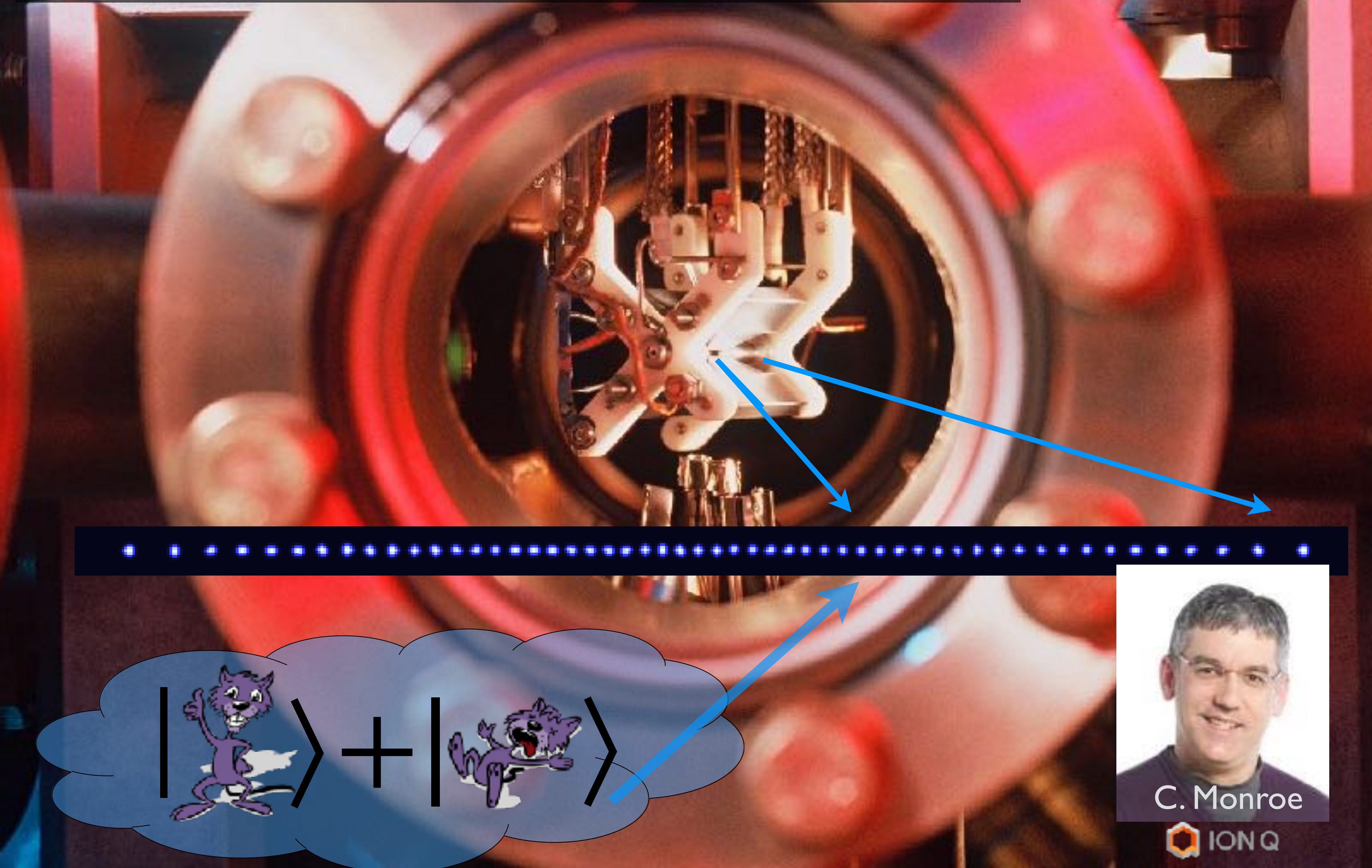
Engineered Quantum Many-Body Systems

quantum computing with trapped ions



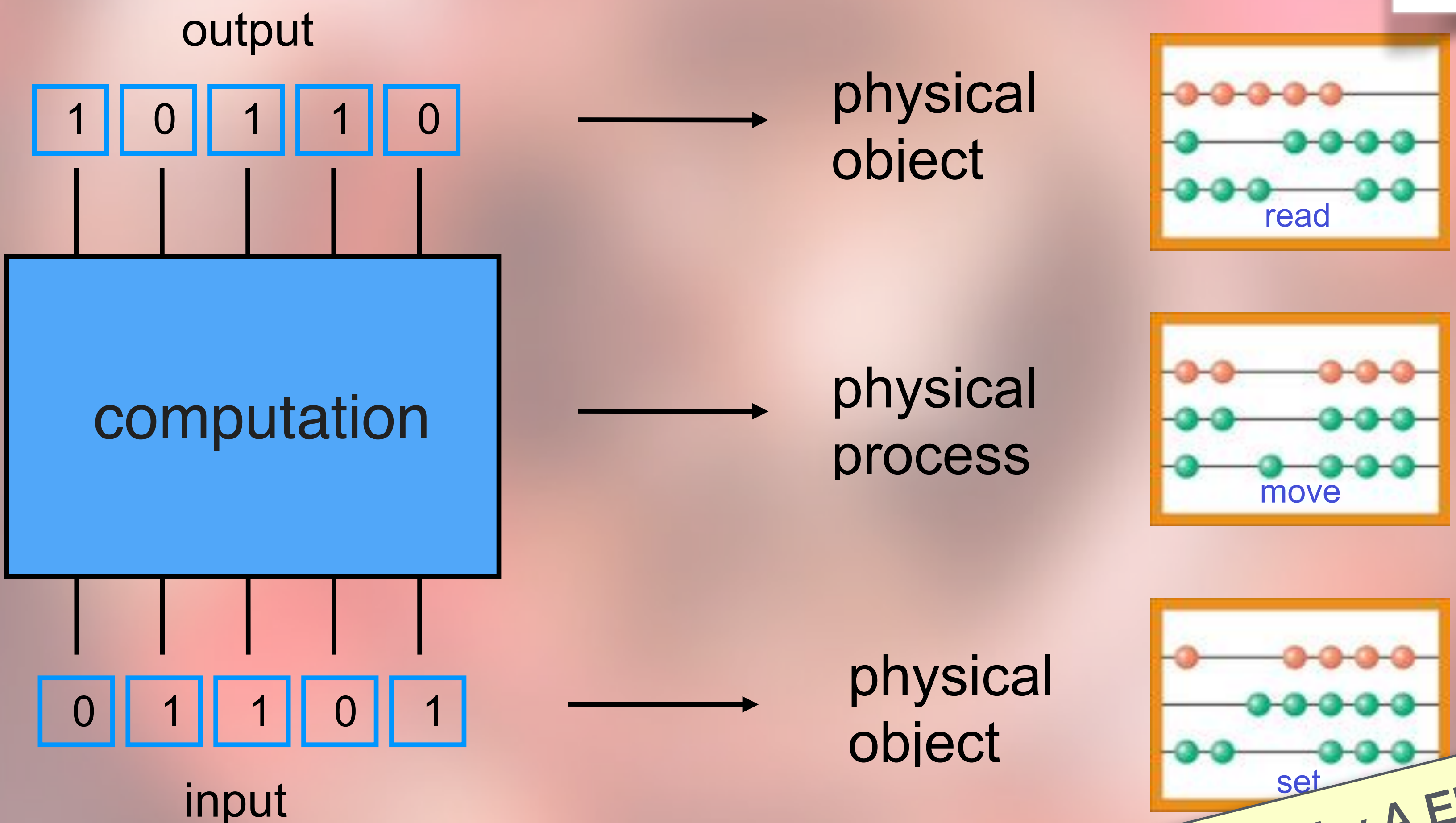
Engineered Quantum Many-Body Systems

quantum computing with trapped ions



Information and physics

computing as a physical process

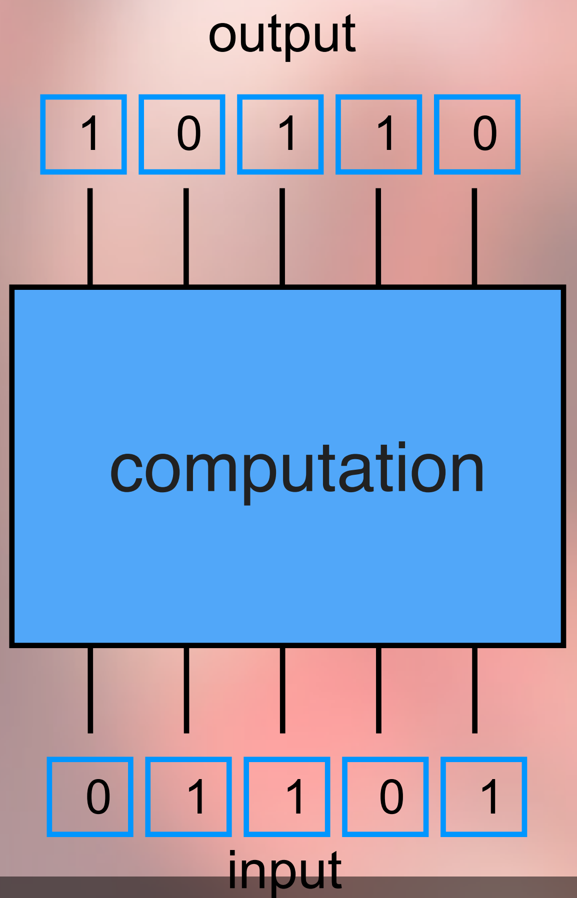


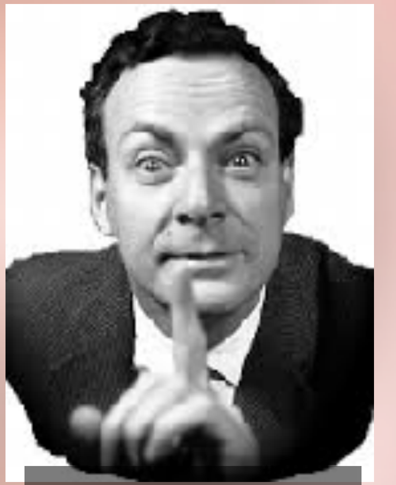
Our present computers process informat
laws of *classical* physics!

Presentation by A Ekert at ICAP
Aug 1994, Boulder
(C. Wieman & D. Wineland)

Information and physics

computing as a physical process



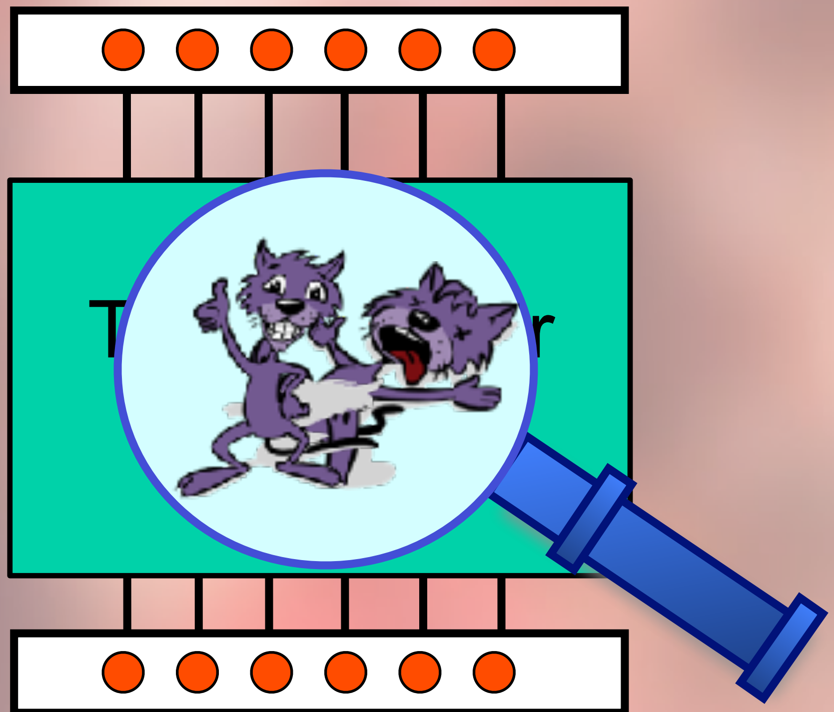

R. Feynman


D. Deutsch

“At a fundamental level nature obeys the laws of *quantum physics*. At a fundamental level information science must be a *quantum information science*.”

Information and physics

Quantum Processor



Why Quantum Computing?

Technology: to beat Moore's law

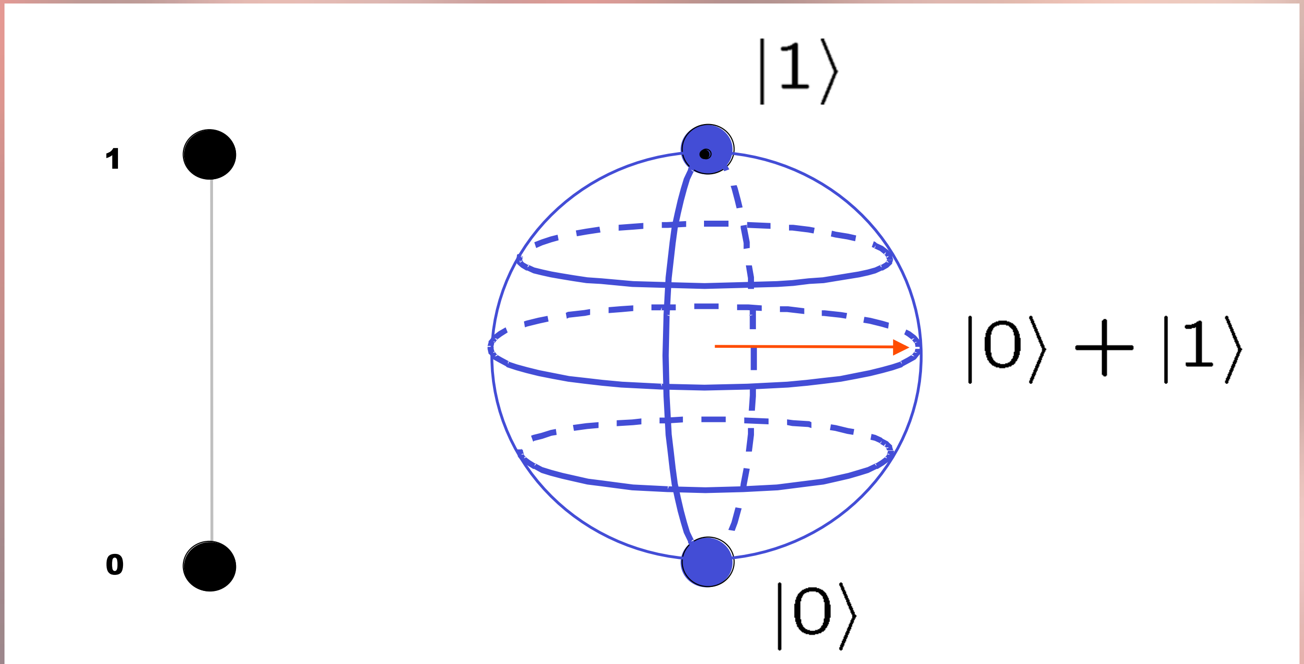
Computer Science: new complexity classes

Physics: to learn about quantum theory

… perform tasks beyond classical computing?

Qubit ...

classical bit	quantum-bit or qubit
0 or 1	0 or 1 or 01



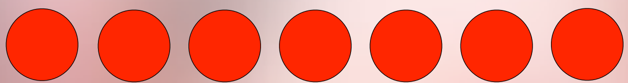
... and quantum registers

classical bit	quantum-bit or qubit
0 or 1	0 or 1 or 01

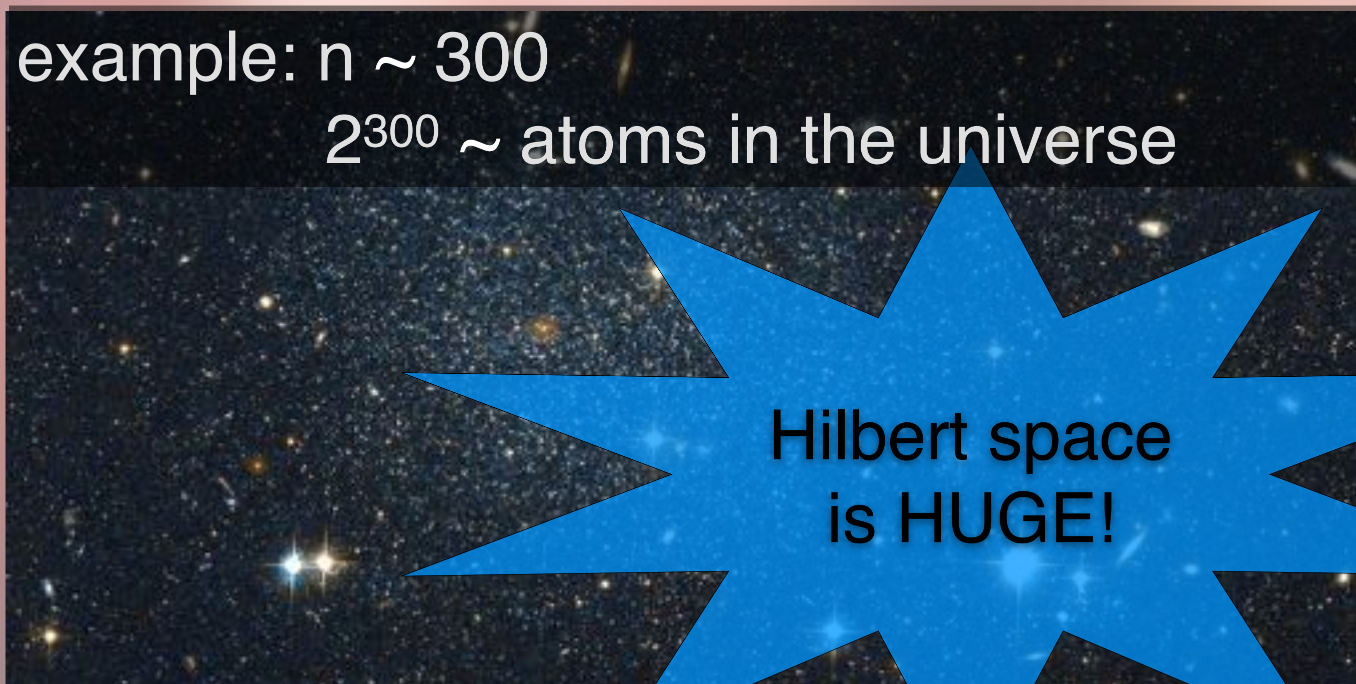
classical registers	quantum registers
010	000 001 010 011 100 101 110 111

Erwin Schrödinger:
Entanglement

How big is quantum memory?



n qubits



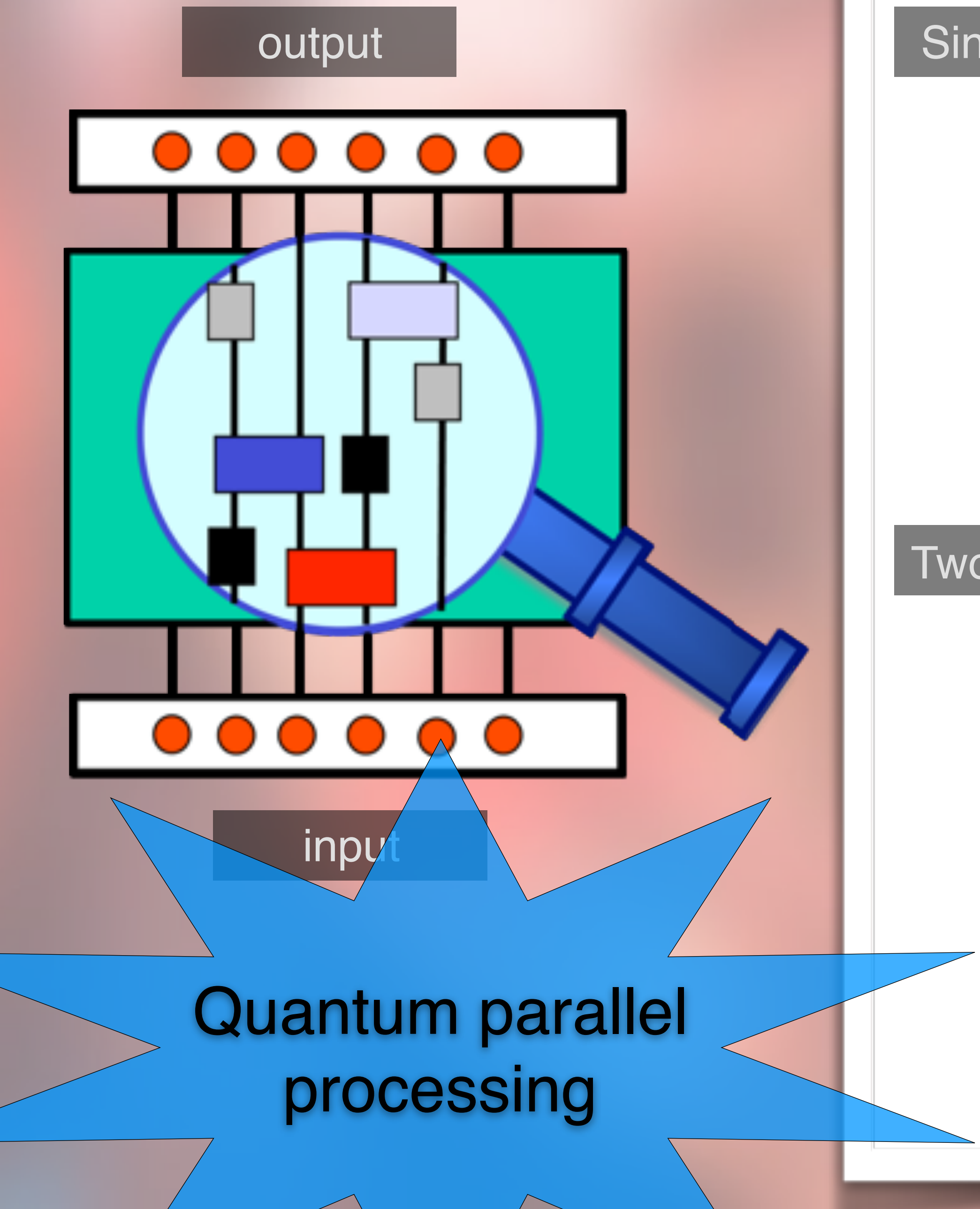
example: n ~ 300

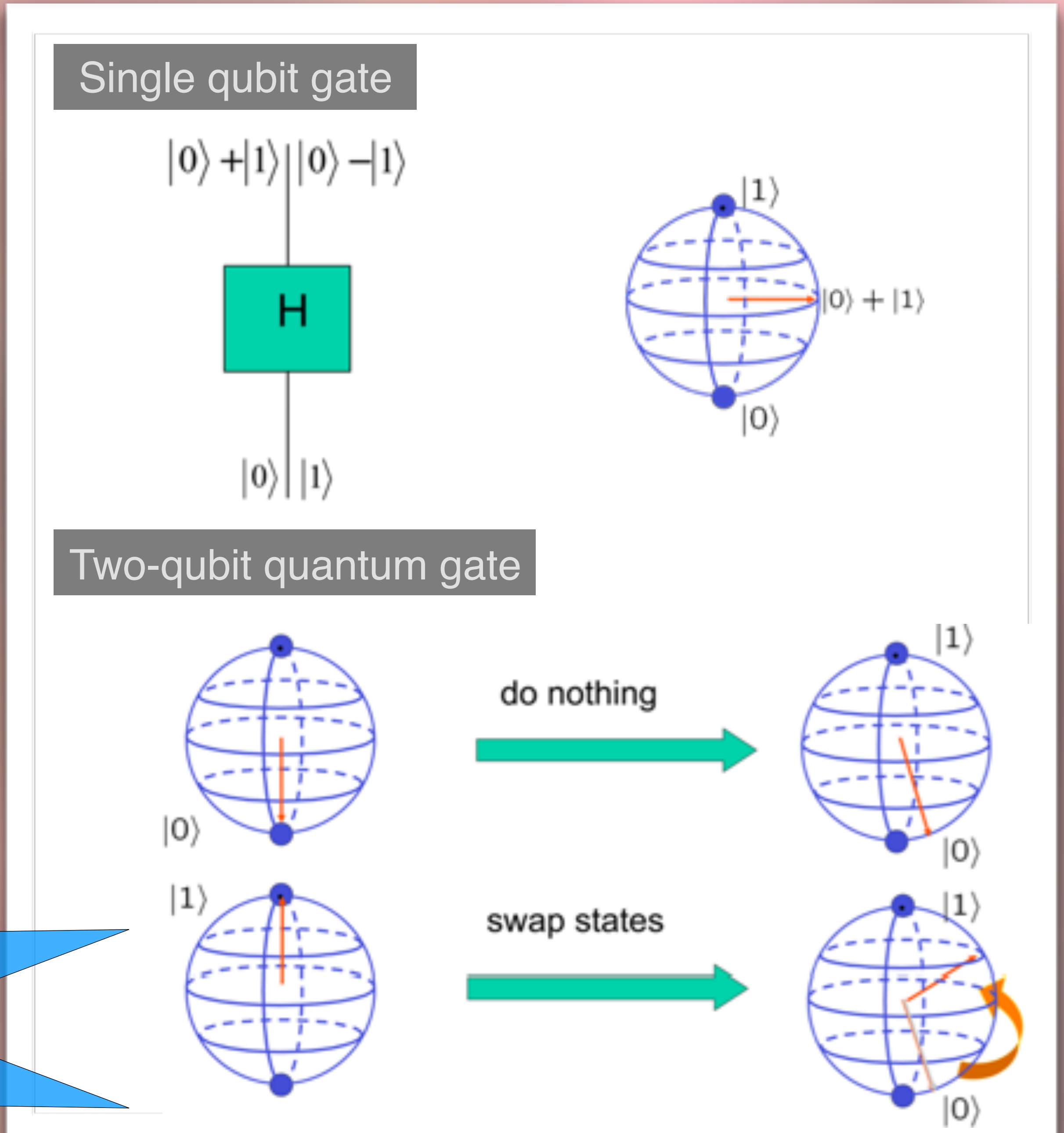
2^{300} ~ atoms in the universe

Hilbert space is HUGE!

„...it is difficult to simulate quantum mechanics on a classical computer.“ (Richard Feynman, 1986)

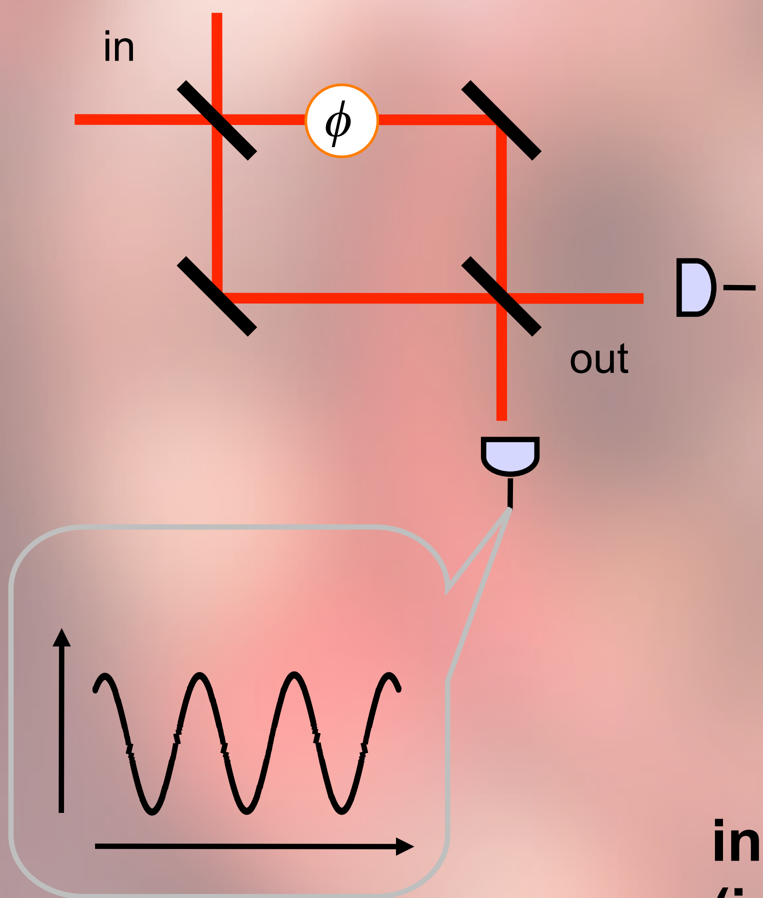
How a quantum computer works ...





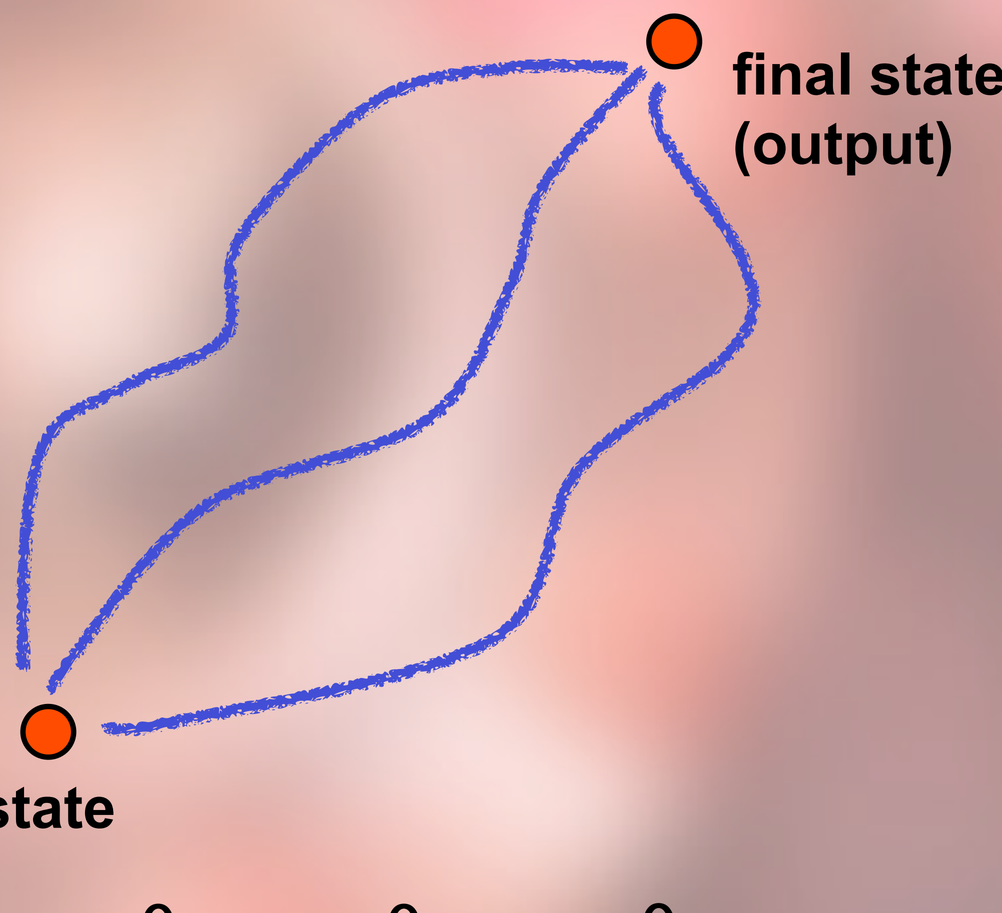
Quantum Parallelism & Algorithms

interferometer



interference pattern

computational paths can interfere in Hilbert space



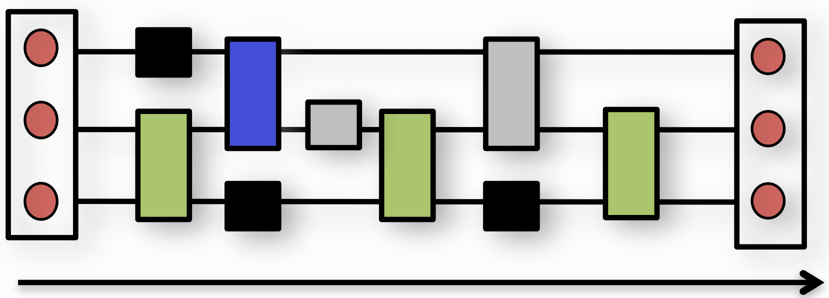
$$e^{i\theta_1} + e^{i\theta_2} + e^{i\theta_3} + \ldots$$

Quantum Computing with Trapped Ions

- **general purpose quantum computing**

quantum logic network model

qubits quantum gates read out



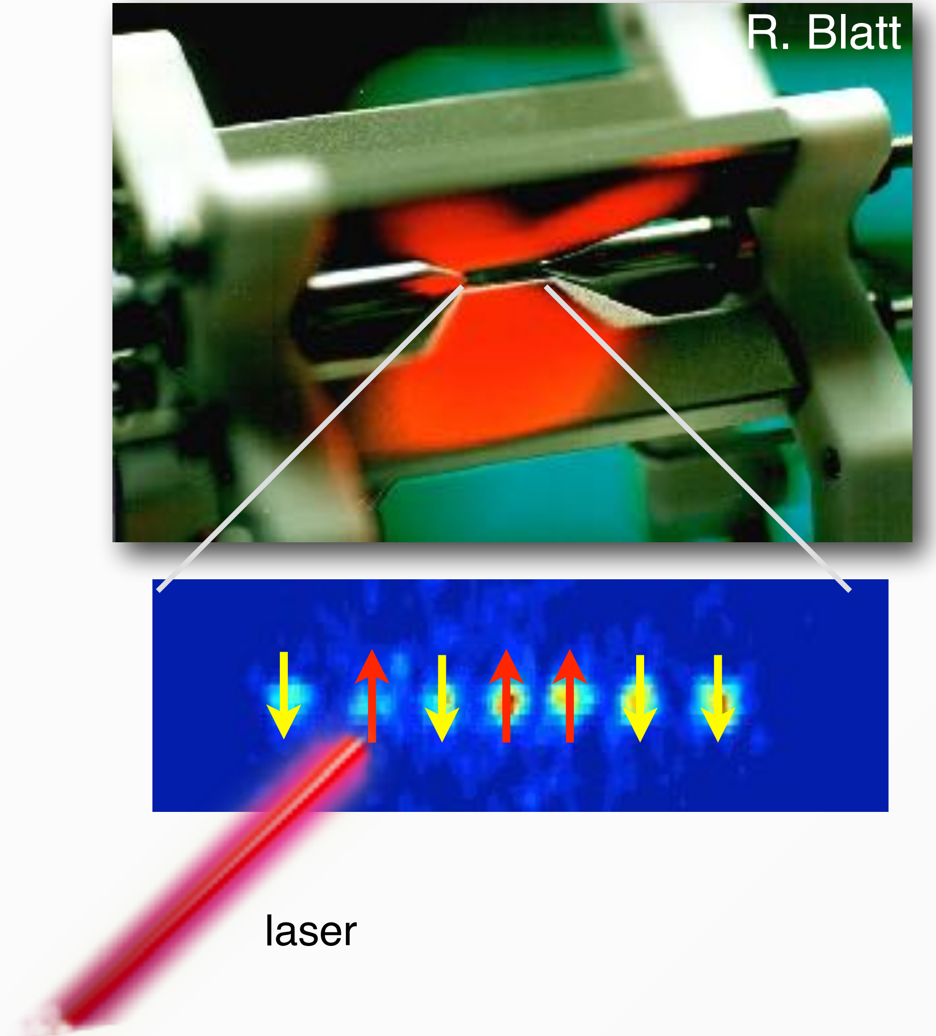
time

$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

coherent Hamiltonian evolution

- quantum gates
- deterministic

- **atomic physics: trapped ions**



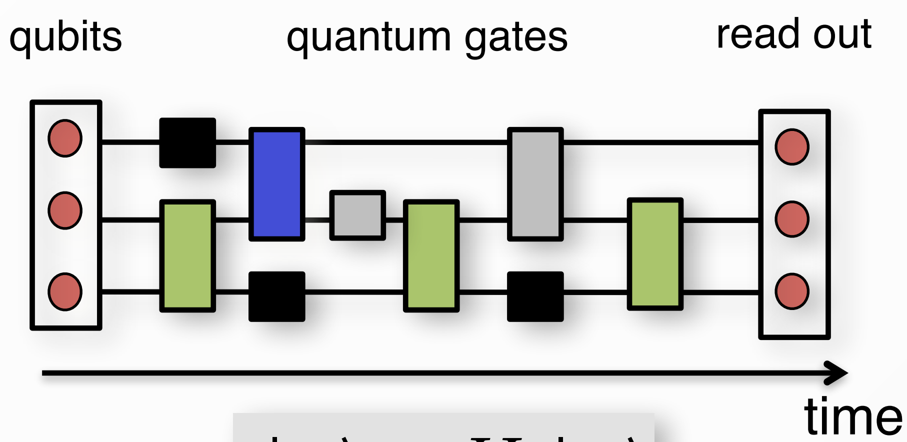
Exp.: Innsbruck, NIST, JQI, MIT, Mainz, MPQ ...

Quantum Computing with Trapped Ions

uibk

- **general purpose quantum computing**

quantum logic network model



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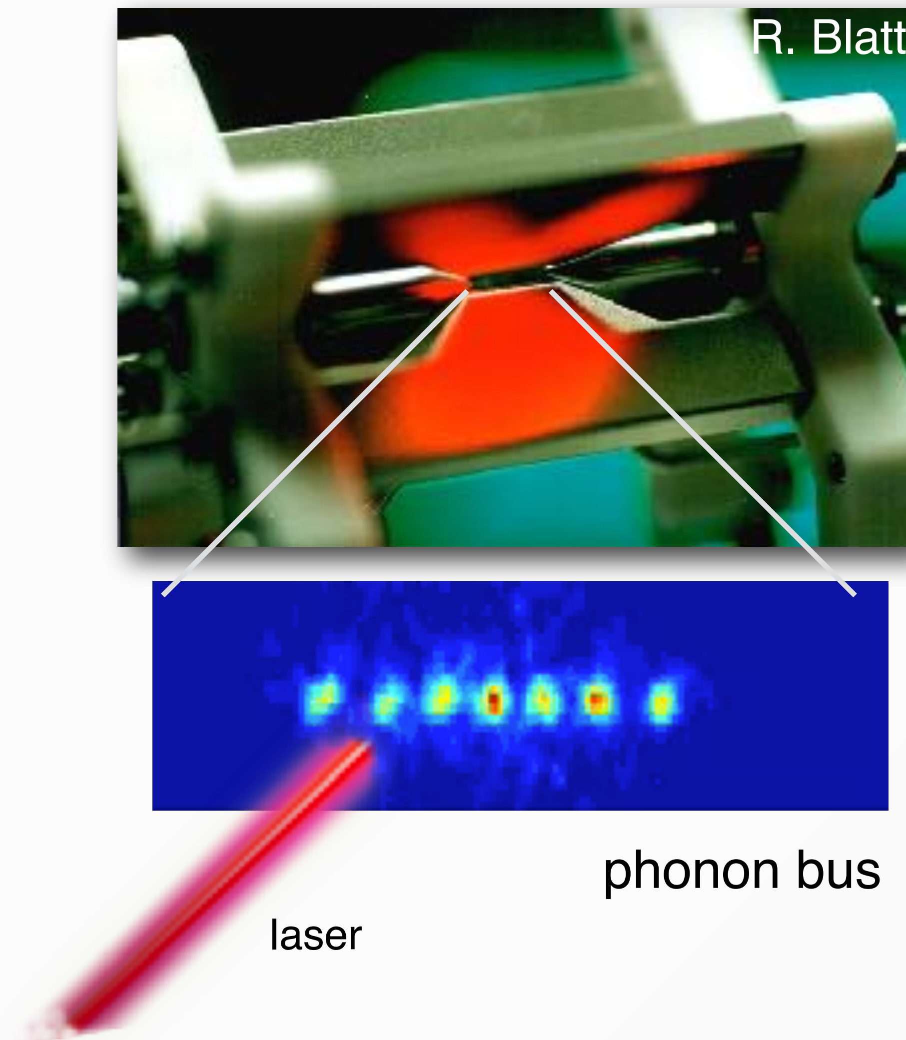


Exp.: Innsbruck, NIST, JQI, MIT, Mainz, MPQ ...

Innsbruck Ion Trap Quantum Computer

uibk

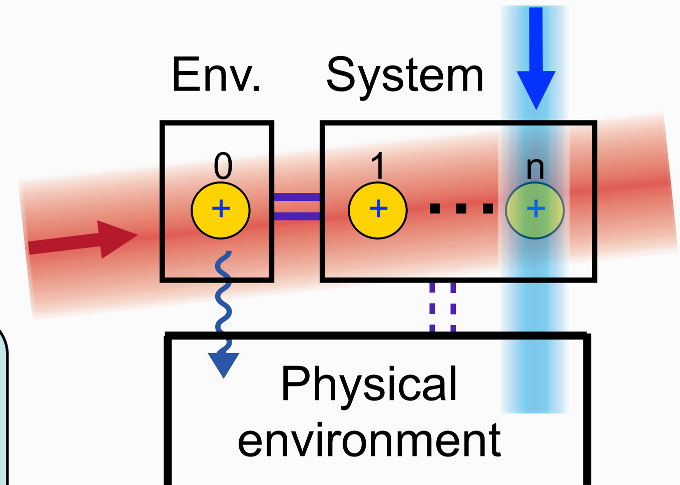
Quantum operations & compiler:

Individual light-shift gates

$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$

Collective spin flips

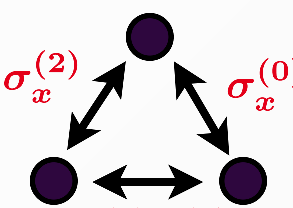
S_x, S_y

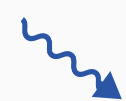


Mølmer-Sørensen gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$

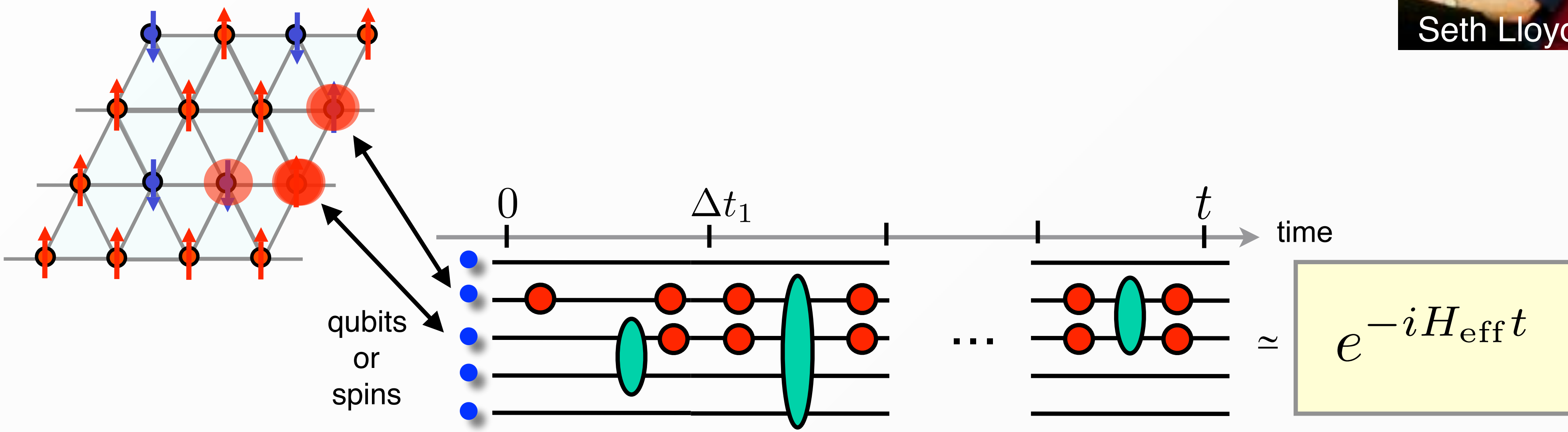
Coupling of environment with physical environment



Optical pumping of „environmental“ ion 

J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Digital Quantum Simulation



idea: approximate time evolution by a stroboscopic sequence of gates

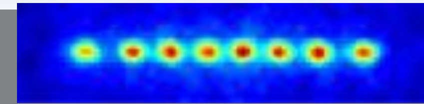
$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots e^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

$$H = \boxed{-J\sigma_1^z\sigma_2^z} + \boxed{B(\sigma_1^x + \sigma_2^x)}$$

SCIENCE VOL 334 7 OCTOBER 2011



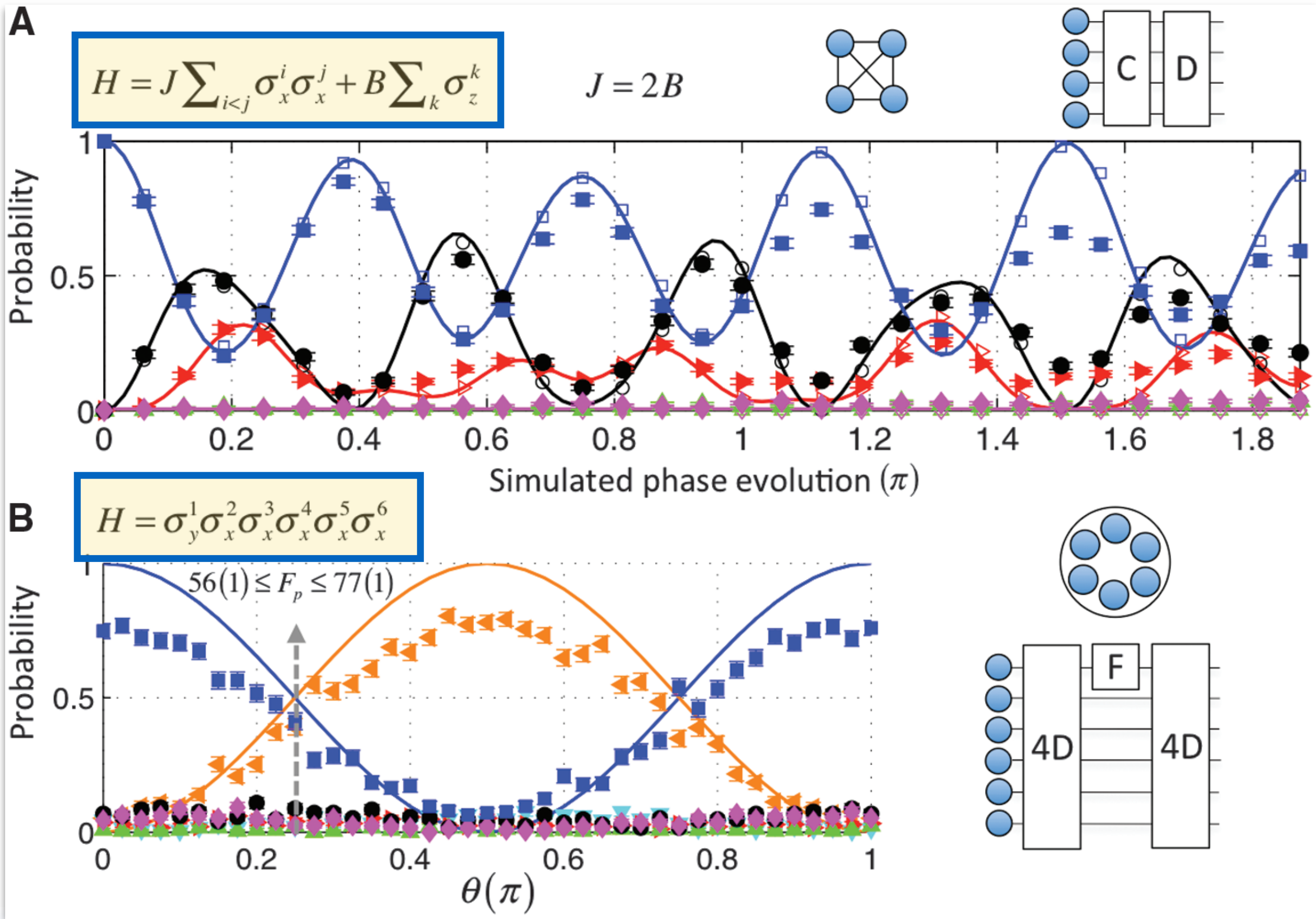
REPORTS

Universal Digital Quantum Simulation with Trapped Ions

B. P. Lanyon,[1,2*] C. Hempel,[1,2] D. Nigg,[2] M. Müller,[1,3] R. Gerritsma,[1,2] F. Zähringer,[1,2] P. Schindler,[2] J. T. Barreiro,[2] M. Rambach,[1,2] G. Kirchmair,[1,2] M. Hennrich,[2] P. Zoller,[1,3] R. Blatt,[1,2] C. F. Roos[1,2]

B. Lanyon C. Roos

4 & 6 Spins



remarks:

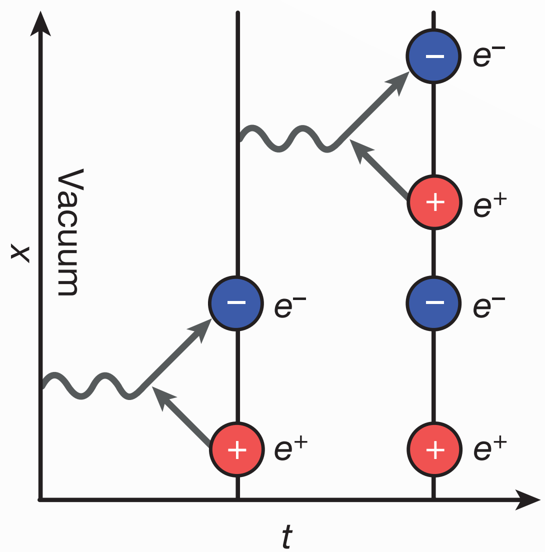
- scalability (?)
- error correction (?)

Real-time dynamics of lattice gauge the… few-qubit quantum computer

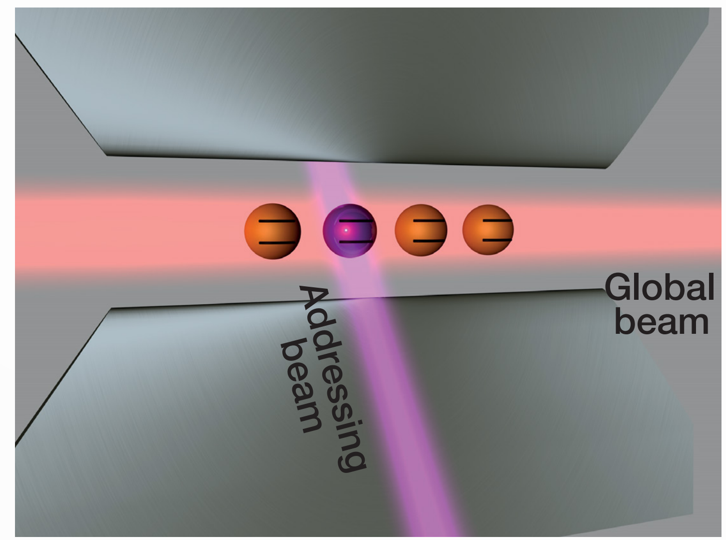
doi:10.1038/nat

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erh
Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

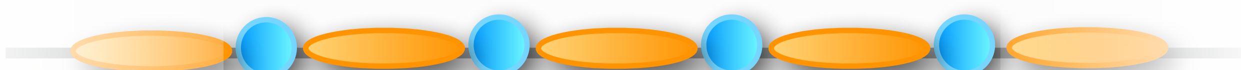
Schwinger pair production



ion trap quantum computer



Schwinger Model: 1+1D QED

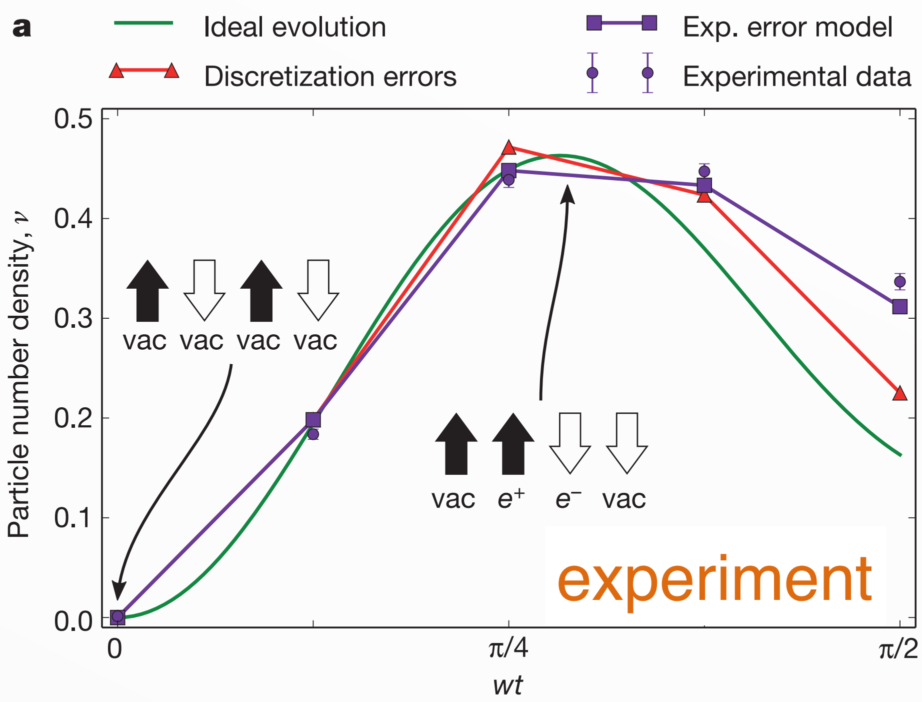


$$\hat{H}_{\text{lat}} = -i\,w \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

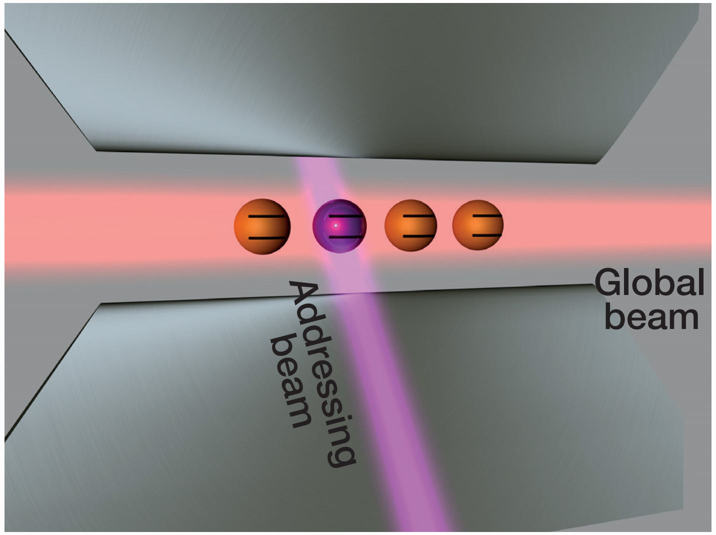
Kogut-Susskind Hamiltonian (Wilson LGT)

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Schwinger pair production



ion trap quantum computer



Digital Quantum Simulation of an Exotic Spin Model

- obtained after integrating … gauge field

220 quantum gates

Analog Quantum Simulation

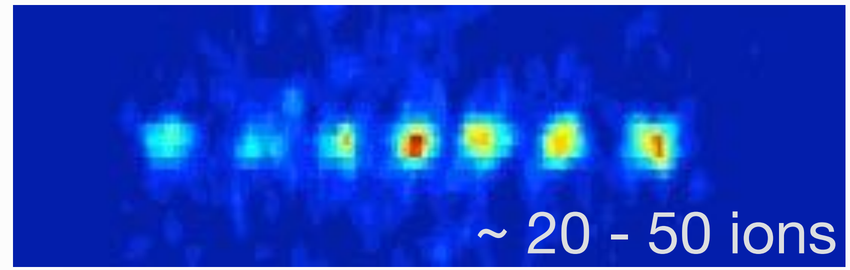
- **spin models**

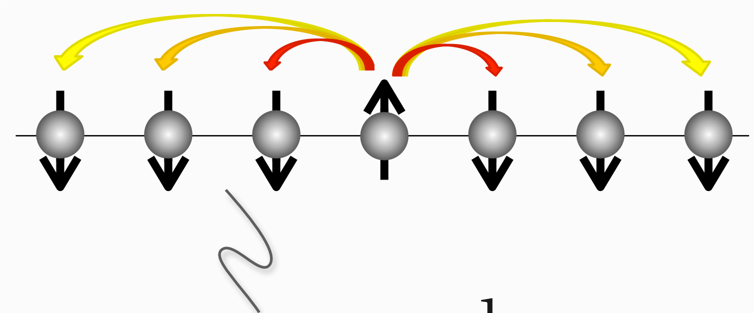
$$H = \hbar \sum_{i,j} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + \hbar B \sum_i \sigma_z^{(i)}$$

K. Kim, C. Monroe et al., Nature (2010)

P Jurcevic, BP Lanyon, R Blatt, C. Roos et al. (2014)

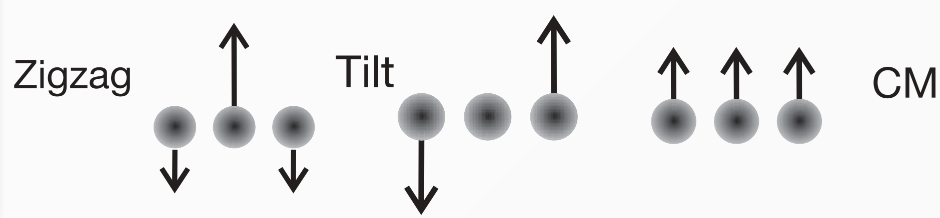
- **trapped ions**



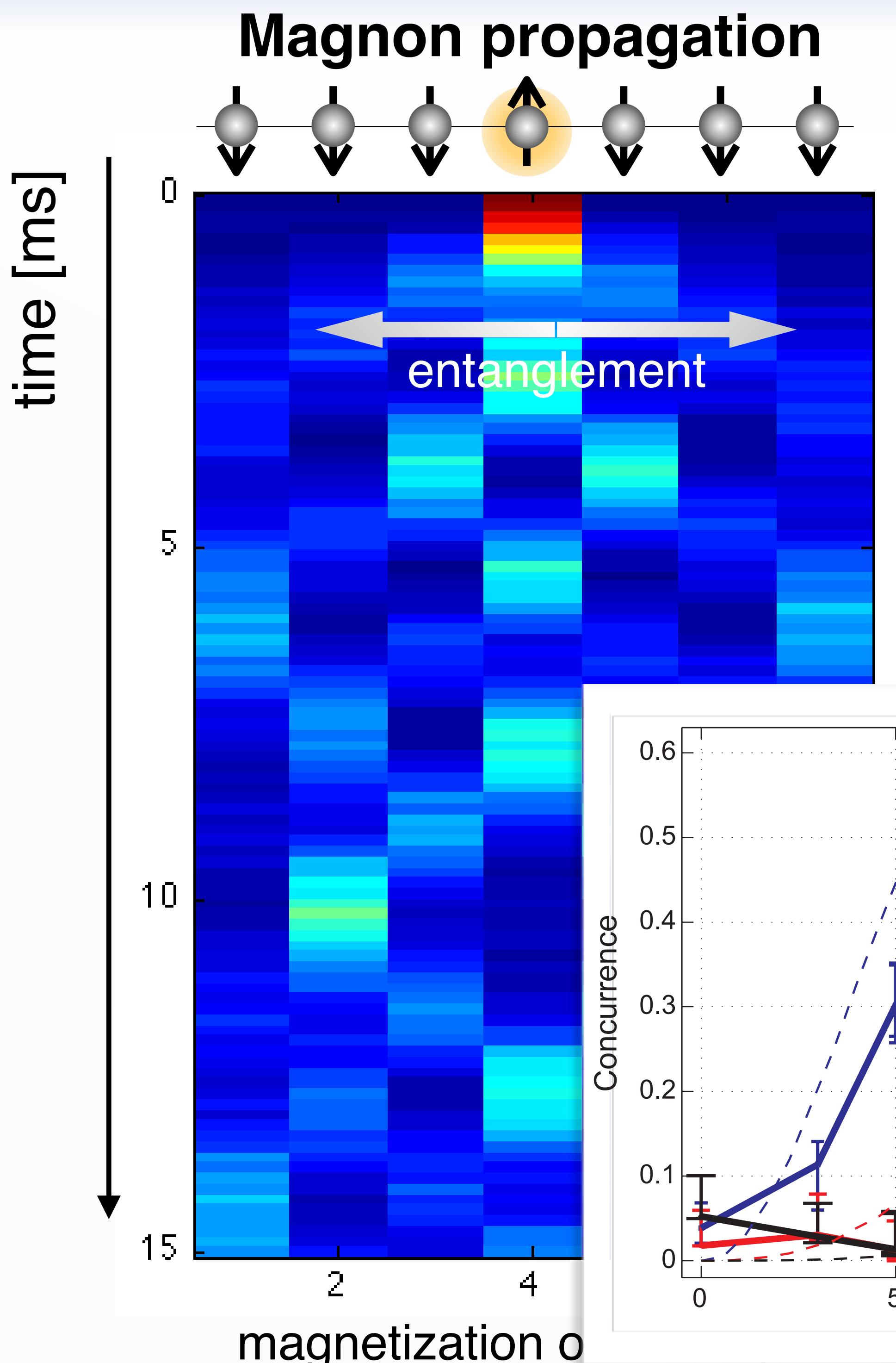


$$J_{ij} \sim \frac{1}{|i-j|^{\alpha}} \qquad 0 \ldots 3$$

tunable range interaction



- We “build” a quantum system with desired Hamiltonian & *controllable parameters*, e.g. Hubbard models of atoms in optical lattices

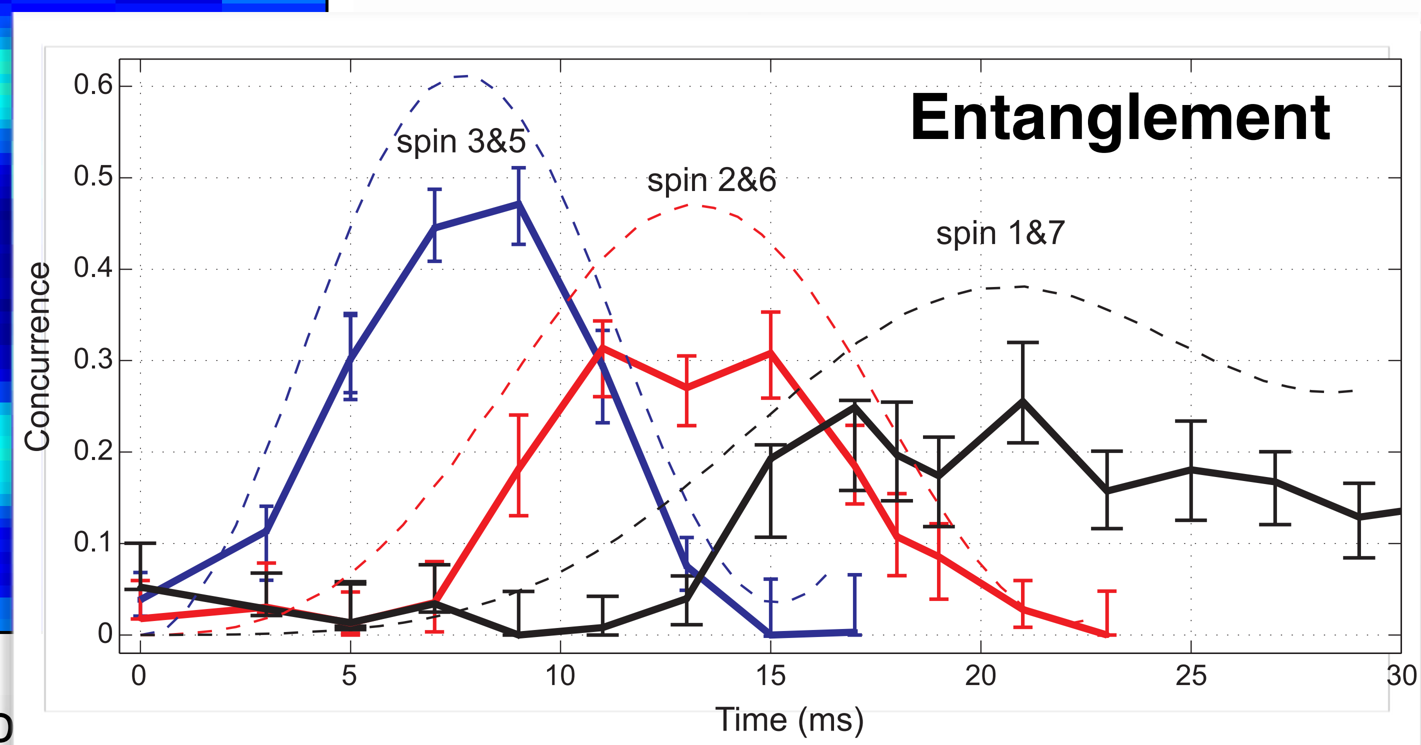


$$\breve{H}_{XY} = \sum_{i<j} J_{ij}(\sigma^+\sigma^- + \sigma^-\sigma^+).$$

✓ Light-cone-like spreading of entanglement

✓ breakdown of the quantum speed-limit due to long range interactions

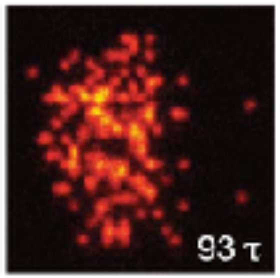
[exp & theory indistinguishable]



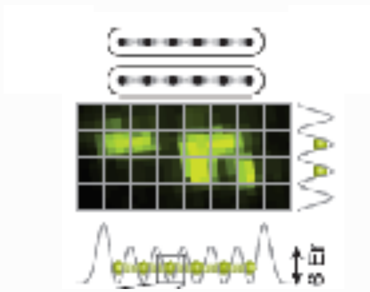
P Jurcevic, et al., PZ, R Blatt & CF Roos, Nature (2014); related work by C. Monroe group

Progress in Engineered Atomic Many-Body Systems

Hubbard models (MPQ, CUA, JQI, …)



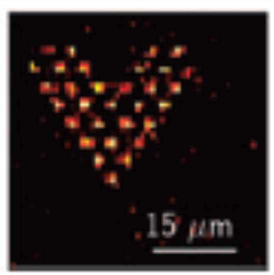
Choi et al., Science (2016)



Kaufman et al., Science (2016)

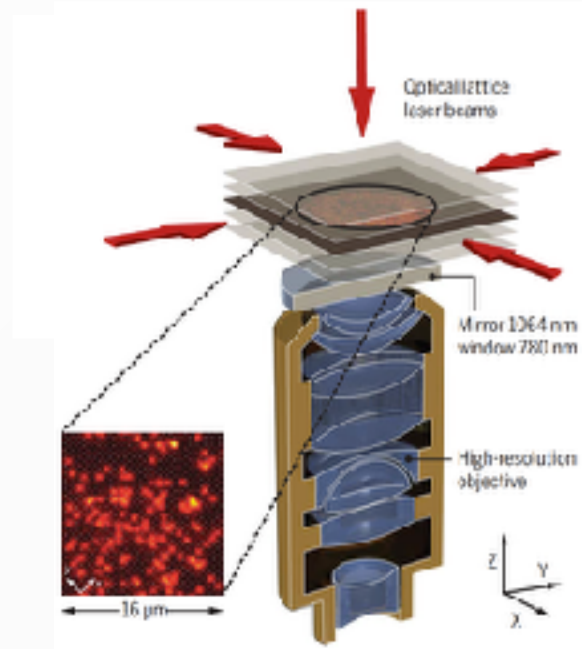
Rydberg Atoms (MPQ, CUA, IOGS,…)

Endres et al., Science (2016)



Barredo et al., Science (2016)

New tools: quantum gas microscope

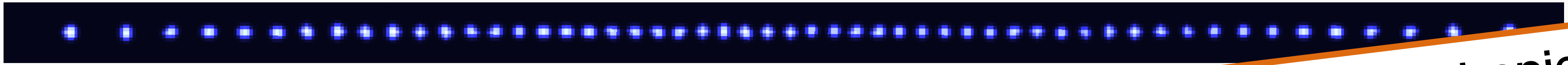


Gross, Bloch, Science (2017)

→ **single site** quantum **control** and **measurement**

Ion Traps (IBK, NIST, JQI, Oxford,…)

C. Monroe, JQI, Nature 2017



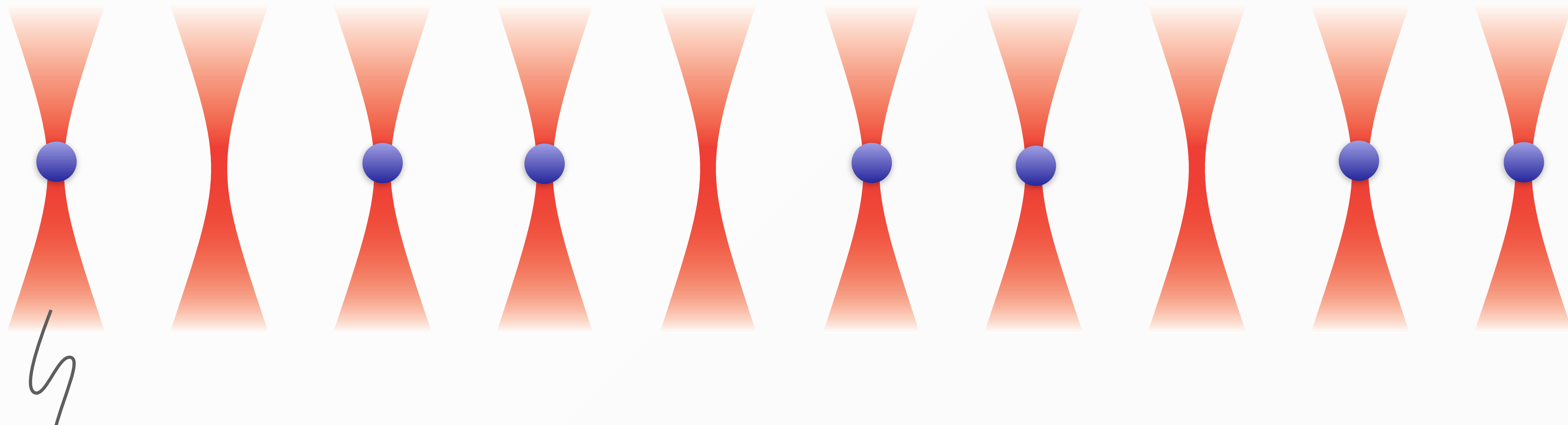
New theory protocols enabled / motiva

…gh repetition rate

How to measure Renyi entropies demonstrating entanglement?

Spin Models with Arrays of Rydberg Atoms

Preparation of a *low entropy* state



Removing entropy by observation

Challenges:

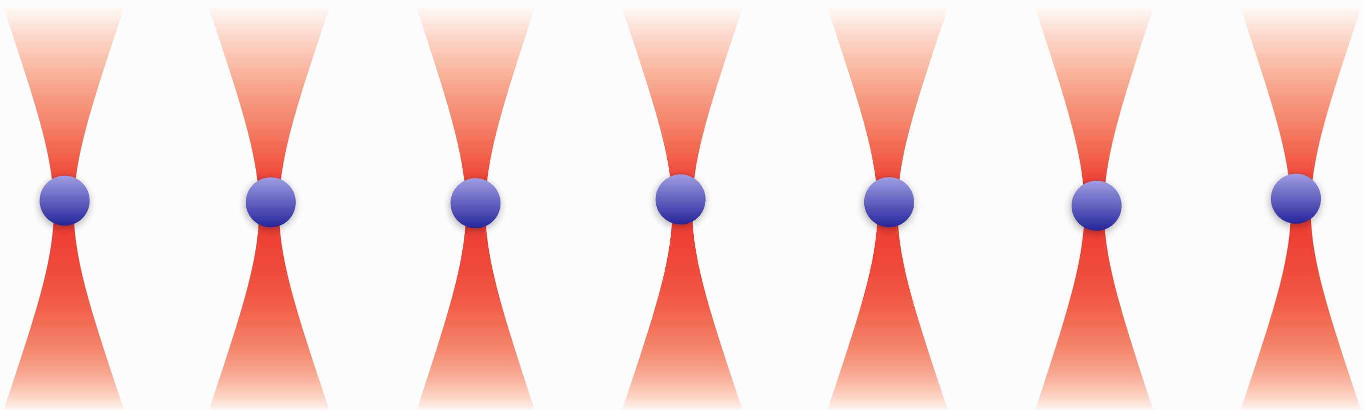
✓ controllability
✓ scalability

… or Quantum Computer

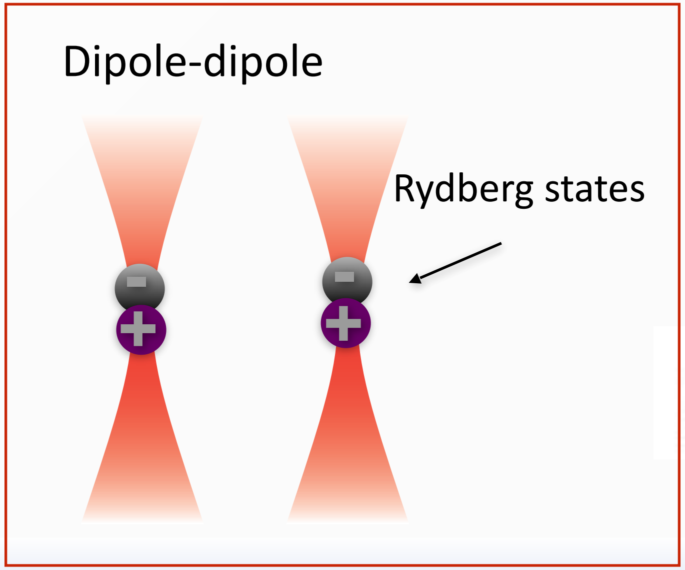
Lukin-Greiner-Vuletic groups (Harvard - MIT), Browaeys (Palaiseau), Saffman (Wisconsin), Biedermann (Sandia)
see also Hubbard-type Models: Regal (JILA), Jochim (Heidelberg), M. Andersen (Otago)

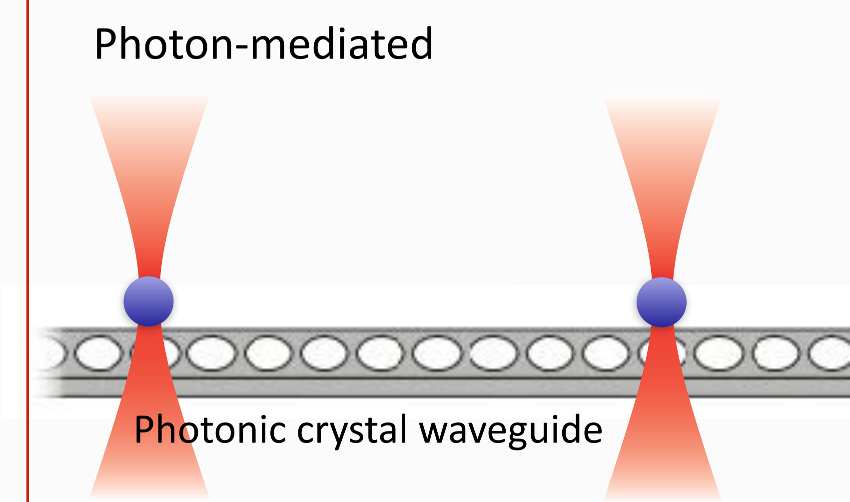
Spin Models with Arrays of Rydberg Atoms

Hamiltonian Engineering



turning on
interactions

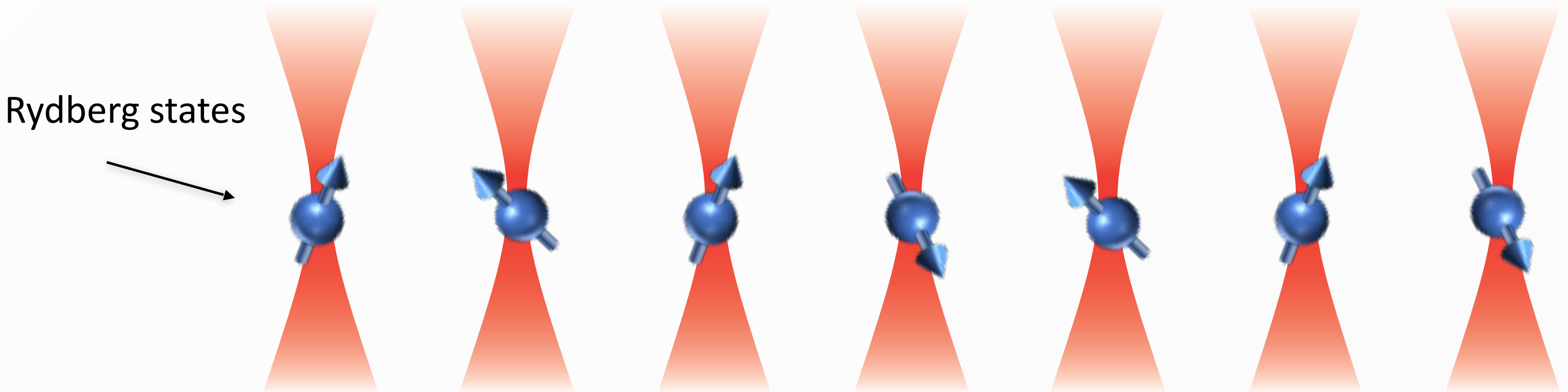


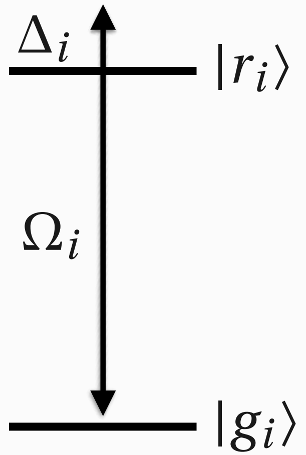


IQI uibk

Rydberg Spin-Models [or Quantum Computer]

Hamiltonian Engineering





Hamiltonian

$$H = \sum_i \frac{1}{2}\Omega_i \sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

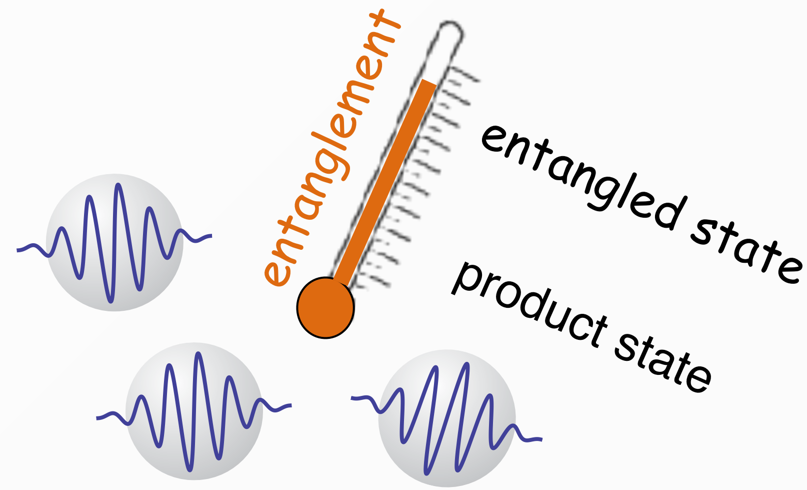
$$\sigma_x^i = |g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|$$

$$V_{ij} = C_6 / r_{ij}^6$$

$$n_i = |r_i\rangle\langle r_i| \equiv \frac{1}{2}(1+\sigma_z^i)$$

Exp: Lukin-Greiner-Vuletic groups (Harvard - MIT), Browaeys (Palaiseau), Saffman (Wisconsin), Biedermann (Sandia)

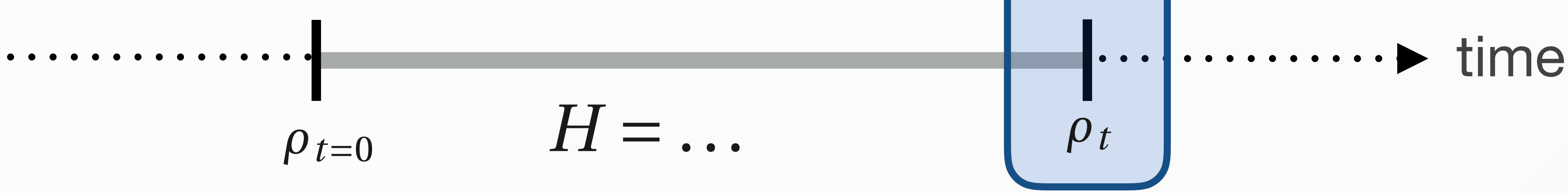
Can we measure Entropy/Entanglement?



atomic quantum system

- Cold Atom Experiment

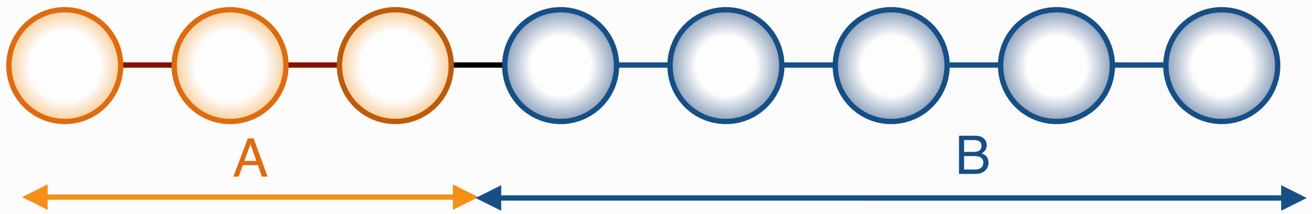
1. quantum state preparation
2. (non-) equilibrium quantum phases
3. measurement



Entanglement Entropy & Entanglement Spectrum

Entanglement & Quantum Many-Body

Quantum many-body system (isolated)



$$H = H_A + H_B + H_{AB}$$

Reduced density matrix & entanglement spectrum

$$\rho_A \equiv \mathrm{Tr}_B \, |\Psi\rangle \langle\Psi|$$

e.g. ground state

Entanglement entropy

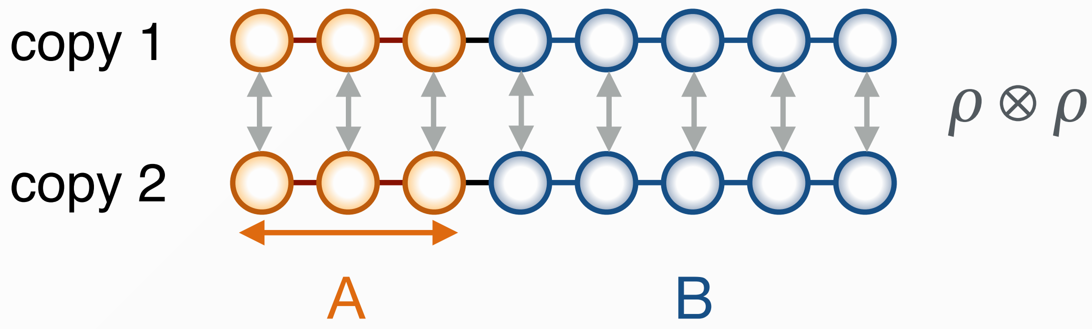
$$S_A = -\mathrm{Tr}_A\left(\rho_A \log \rho_A\right)$$ Von Neumann

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n \quad (\leq S_A)$$ Renyi

...characterize topological/strongly correlated quantum phases

Question: Can we measure entanglement?

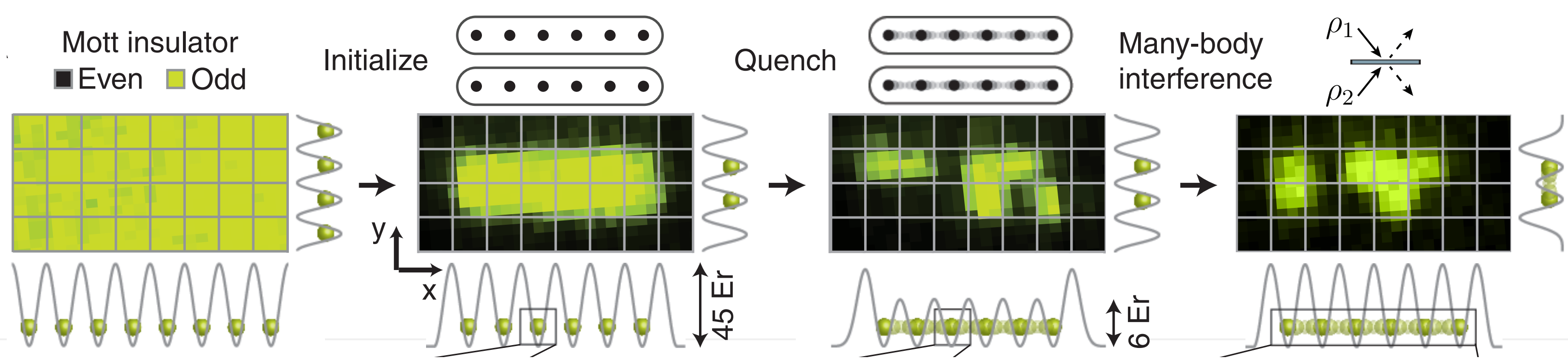
1. Measuring Renyi Entropies *with Copies*



$\mathrm{Tr}\{\rho_A^2\}$ Renyi entropy n=2 ~ purity

1. Measuring Renyi Entropies *with Copies*

Controlled few-atom systems & quantum gas microscope



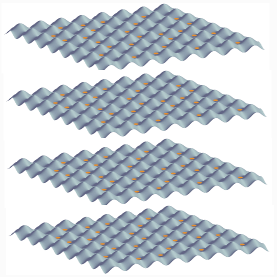
R. Islam, M. Greiner et al., Nature (2015)
A.M. Kaufmann, M. Greiner et al., Science (2016)

A Quantum Information Perspective

$\mathrm{Tr}\{\rho^n\}$

$\rho \otimes \rho \otimes \rho \otimes \rho \otimes \rho$

n copies



A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

A Quantum Information Perspective

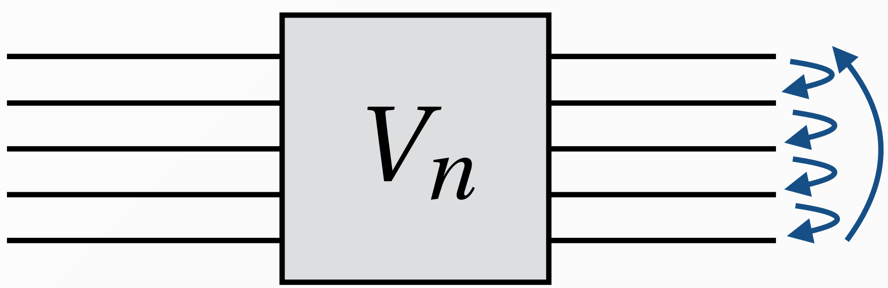
$$\mathrm{Tr}\{\rho^n\} = \mathrm{Tr}\{V^{(n)} \rho \otimes \rho \otimes \ldots \otimes \rho \otimes \rho\} \equiv \langle V^{(n)} \rangle$$ expectation value

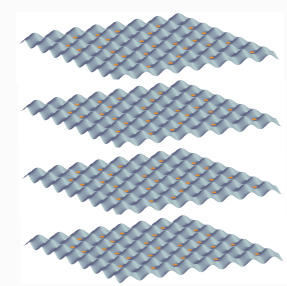
$$V^{(n)} |\psi_1\rangle \ldots |\psi_n\rangle = |\psi_n\rangle |\psi_1\rangle \ldots |\psi_{n-1}\rangle$$

shift (or swap) operator

$\rho \otimes \rho \otimes \rho \otimes \rho \otimes \rho$

n copies





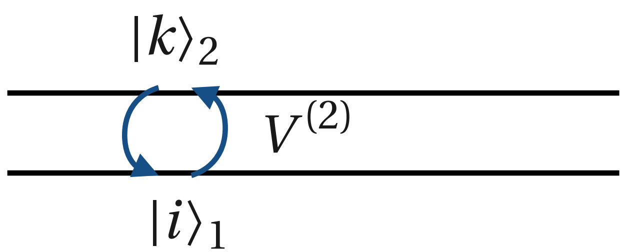
A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

A Quantum Information Perspective

$$\mathrm{Tr}\{\rho^n\} = \mathrm{Tr}\{V^{(n)} \rho \otimes \rho \otimes \ldots \otimes \rho \otimes \rho\} \equiv \langle V^{(n)} \rangle$$ expectation value

Example: n=2

$\rho_1 \otimes \rho_2$



$$V^{(2)} |i\rangle_1 \otimes |k\rangle_2 = |k\rangle_1 \otimes |i\rangle_2$$

$$\mathrm{Tr}\{V^{(2)} \rho_1 \otimes \rho_2\} = \mathrm{Tr}\left\{ V^{(2)} \sum_{ijkl} \rho^{(1)}_{ij} \rho^{(2)}_{kl} |i\rangle\langle j| \otimes |k\rangle\langle \ell| \right\}$$

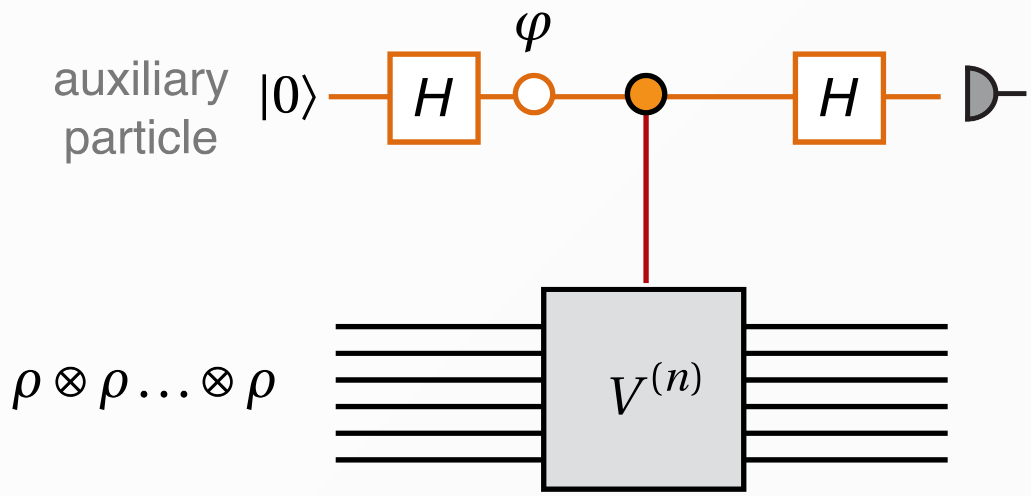
$$= \mathrm{Tr}\{\rho_1 \rho_2\}$$

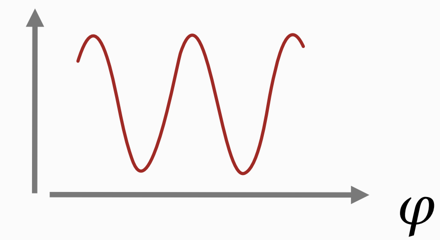
A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

Quantum Circuit

$$\mathrm{Tr}\{\rho^n\} = \mathrm{Tr}\{V^{(n)}\rho \otimes \rho \otimes \rho \otimes \rho \otimes \rho\} \equiv \langle V^{(n)} \rangle$$ expectation value

Ramsey interferometer:





visibility $\sim \langle V^{(n)} \rangle$

… we need a quantum computer (?)

A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

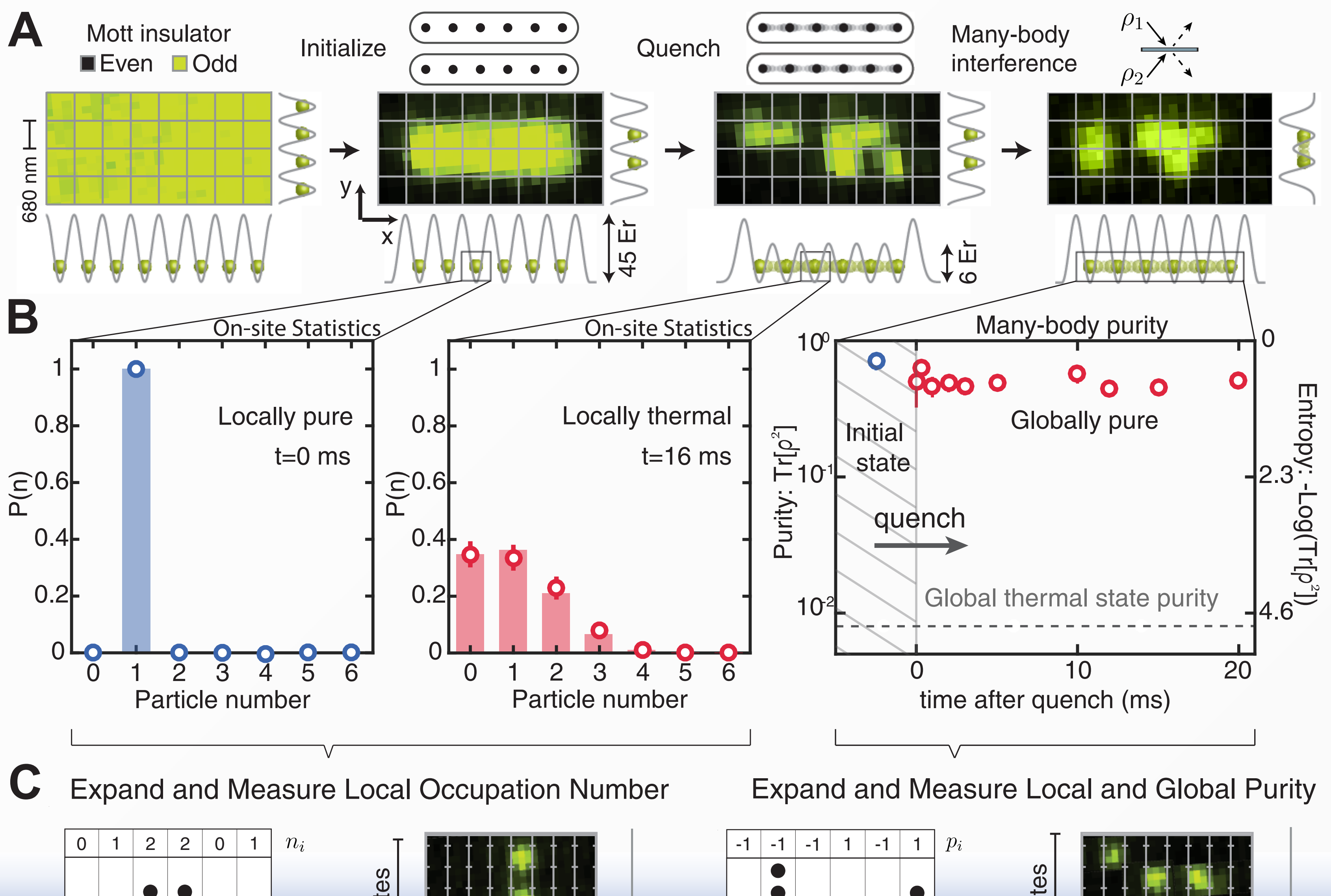
Quantum thermalization through entanglement in an isolated many-body system

Adam M. Kaufman, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Robert Schittko, Philipp M. Preiss, Markus Greiner*

794 19 AUGUST 2016 • VOL 353 ISSUE 6301

A.J. Daley, H. Pichler, J. Schachenmayer, PZ, PRL (2012).

see also:
C. Moura Alves and D. Jaksch, PRL (2004)

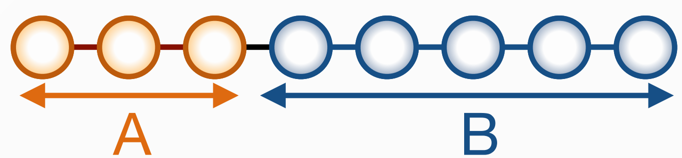


A. Elben

B. Vermersch

2. Measuring Renyi Entropies *via Random Measurements*

single system



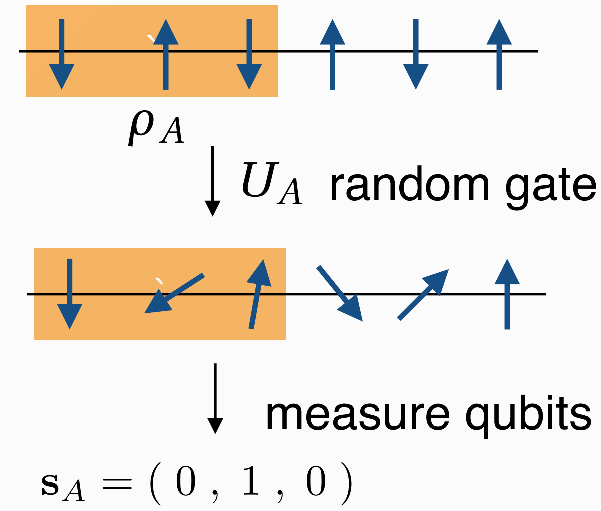
$\rho \otimes \rho$

virtual copy
(replica trick)

A. Elben, B. Vermersch, M. Dalmonte, J.I. Cirac & P.Z., PRL 2018; PRA 2018

Random Measurements & Quantum Information

Protocol for chain of qubits:



Random measurement

$$P(\mathbf{s}_A) = \mathrm{Tr}_A\left(|\mathbf{s}_A\rangle\langle\mathbf{s}_A|\, U_A \rho_A U_A^\dagger\right)$$

Average over Circular Unitary Ensemble (CUE)

$$\langle P(\mathbf{s}_A)\rangle = \frac{1}{N_{H_A}}$$

$$\langle P(\mathbf{s}_A)^2\rangle = \frac{1 + \mathrm{Tr}\rho_A^2}{N_{H_A}(N_{H_A}+1)}$$

dimension Hilbert space A

2-design / t-design

$$\langle P(\mathbf{s}_A)^2\rangle = \langle \mathrm{Tr}_{1+2}\left[\ldots U_A \rho_A U_A{}^\dagger \otimes U_A \rho_A U_A^\dagger\right]\rangle = \ldots$$

2 virtual copies

CUE

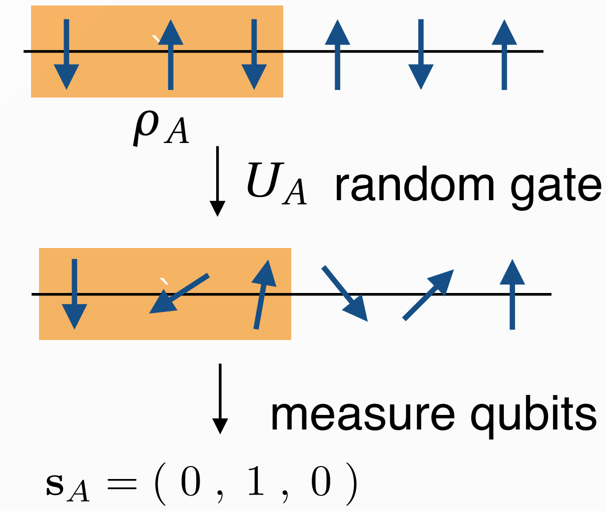
$$\langle U_{ij} U^*_{kl} U_{mn} U^*_{op}\rangle \approx \frac{\delta_{ik}\delta_{jl}\delta_{mo}\delta_{np} + \delta_{io}\delta_{jp}\delta_{mk}\delta_{nl}}{N_H^2}$$

~ Gaussian

van Enk, Beenakker (PRL 2012)

Random Measurements & Quantum Information

Protocol for chain of qubits:



Random measurement

$$P(\mathbf{s}_A) = \mathrm{Tr}_A \left(|\mathbf{s}_A\rangle \langle \mathbf{s}_A| \, U_A \rho_A U_A^\dagger \right)$$

Average over Circular Unitary Ensemble (CUE)

$$\langle P(\mathbf{s}_A) \rangle = \frac{1}{N_{H_A}} \qquad \langle P(\mathbf{s}_A)^2 \rangle = \frac{1 + \mathrm{Tr}\rho_A^2}{N_{H_A}(N_{H_A}+1)}$$

dimension Hilbert space A

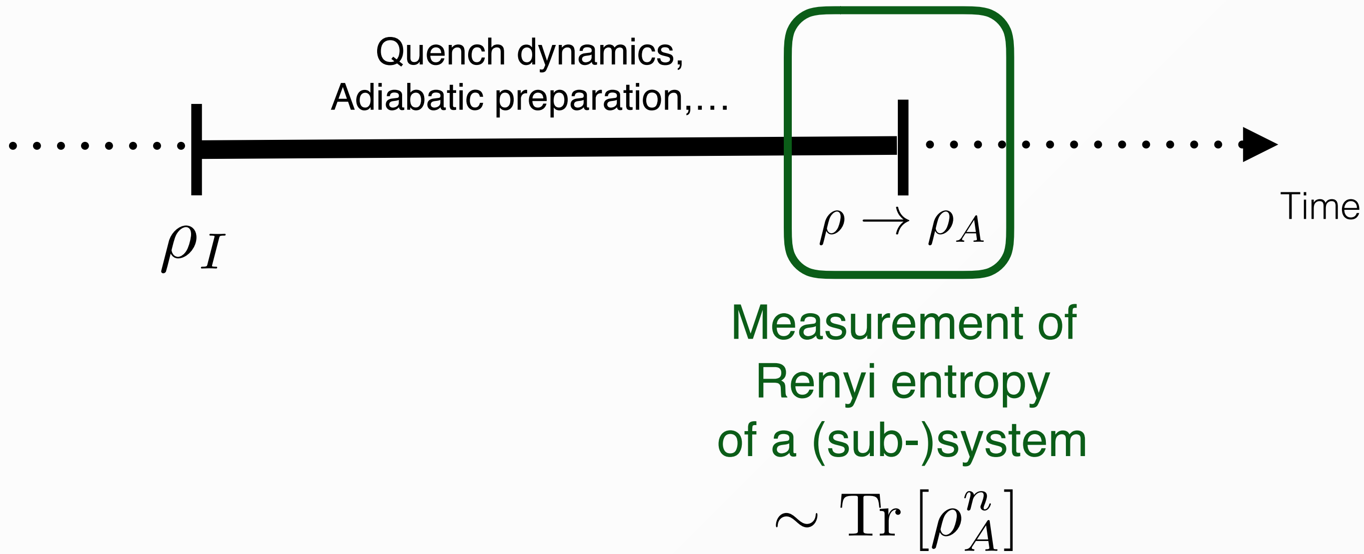
✓ Random measurement gives purity, and higher order Renyi entropies

✓ Required resources:
- how realize random unitaries
- # measurements, # unitaries

? "the signal is the noise"

Realization in Hubbard or Spin Model

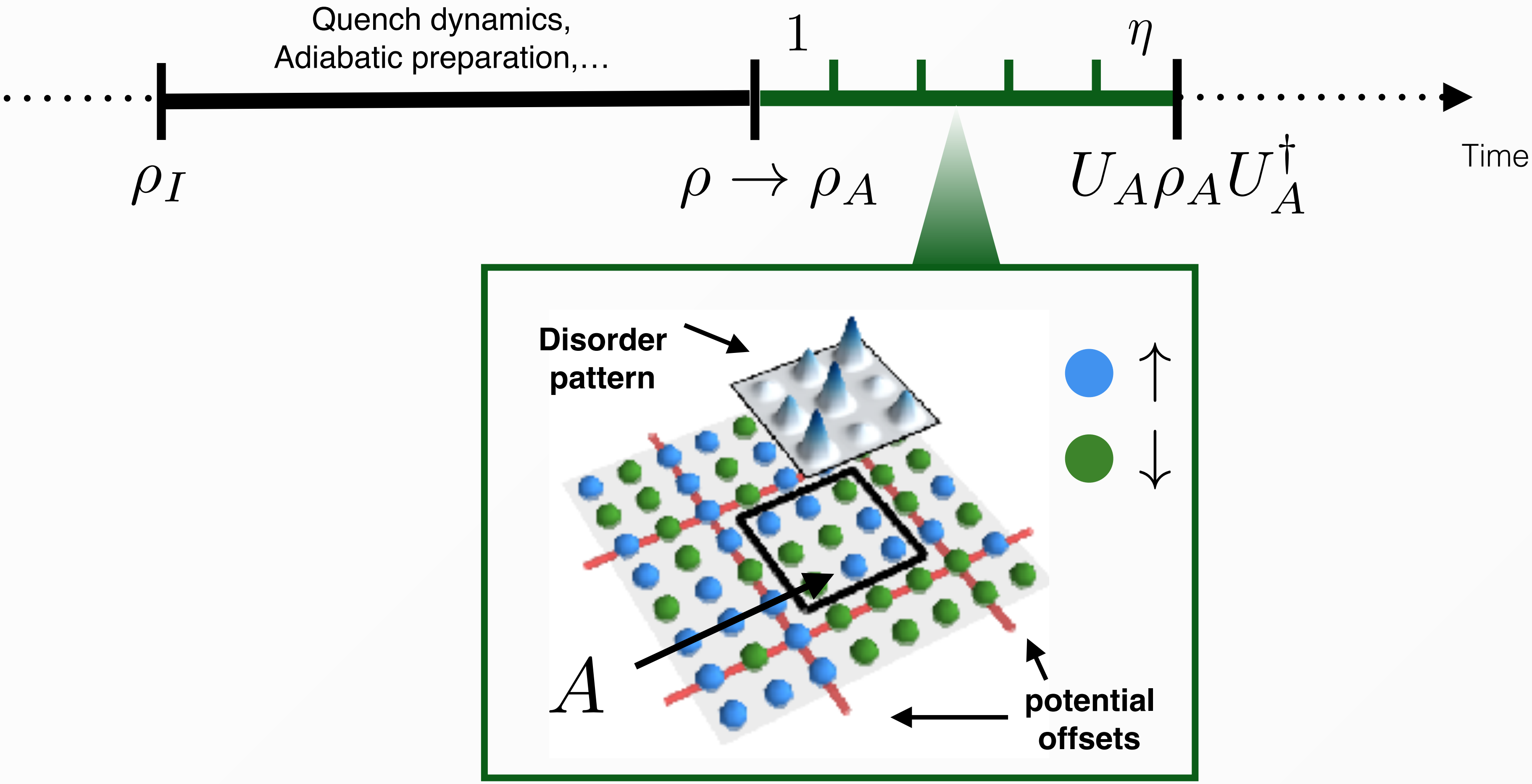
Measurement Protocol



Realization in Hubbard or Spin Model

Random unitary
as time evolution operator under
random quenches

$$U_A = e^{-iH_\eta T} \cdots e^{-iH_1 T}$$



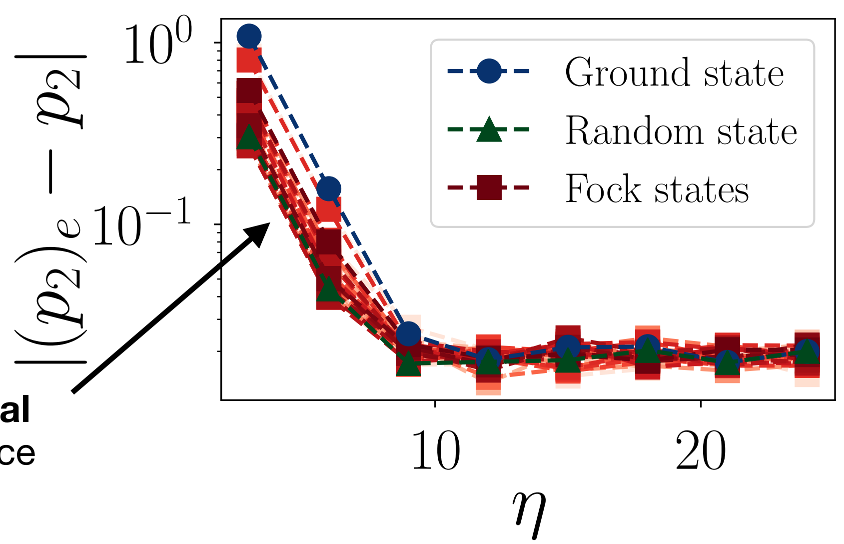
See also: M Ohliger, V Nesme, J Eisert - NJP 2013

Generation of Random Unitaries

Question: Is $U_A = e^{-iH_\eta T} \cdots e^{-iH_1 T}$ a random unitary?

⟶ **Apply the protocol to a known input state and compare estimated to true purity to test the ensemble**

Ising model (here: 1D, 8 sites)



exponential convergence

$$H_{j} = \Omega \sum_i \sigma_i^x + \sum_{i<j} \frac{C_6}{|r_i - r_j|^6} \sigma_i^z \sigma_j^z + \sum_i \Delta_i^{j} \sigma_i^z$$

Δ_i^j from gaussian distribution with standard deviation Δ

$$\Omega = C_6/a^6 = \Delta = 1/T$$

Random unitaries using
- ✓ generic interactions
- ✓ engineered disorder

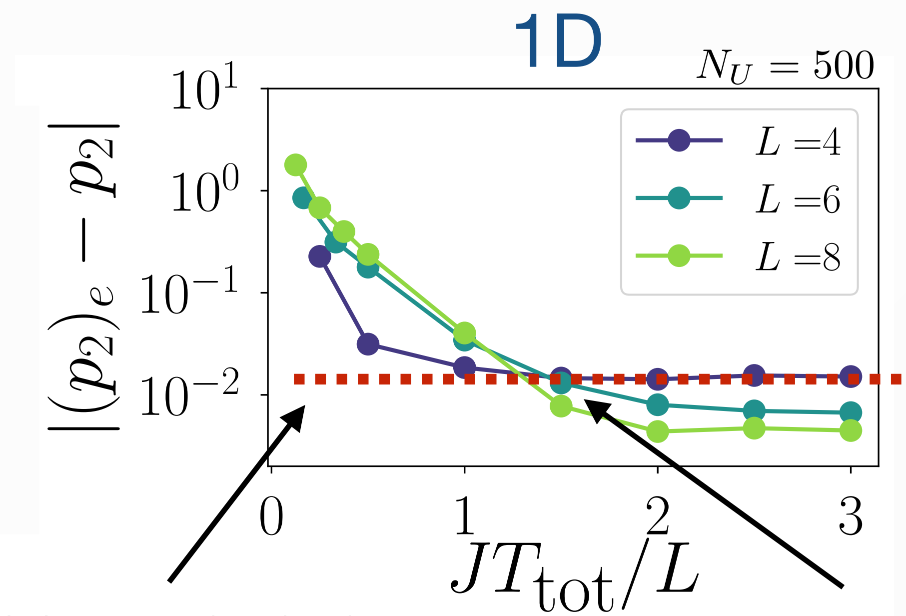
Ising model with L sites

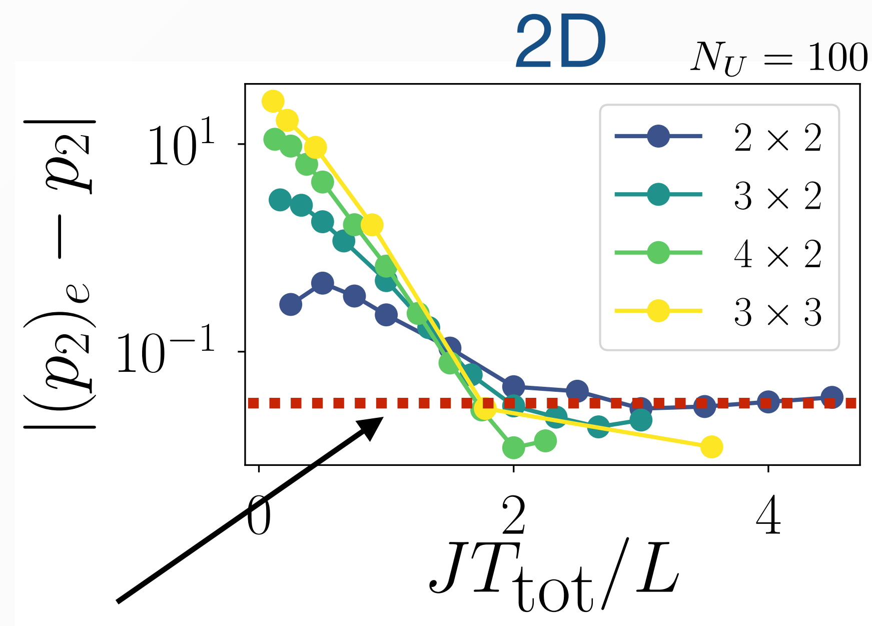
$$U_A = e^{-iH_\eta T} \cdots e^{-iH_1 T}$$

$$H_j = \Omega \sum_i \sigma_i^x + \sum_{i<j} \frac{C_6}{|r_i - r_j|^6} \sigma_i^z \sigma_j^z + \sum_i \Delta_i^j \sigma_i^z$$

Δ_i^j from gaussian distribution with standard deviation Δ

Error for estimated purity





statistical error threshold due to finite number (500) random unitaries

Number of necessary random quenches $\eta \sim L$

Efficient generation of random unitaries for purity measurements

Certification of unitary n-designs

IQI uibk

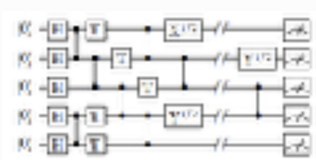
S : set of generated random unitaries

$$\langle P(\mathbf{s}_A)^n\rangle_S \overset{!}{\sim} \mathrm{Tr}\left[\rho_A^n\right] \longleftrightarrow \langle \underbrace{U_{ij}U_{kl}^* \ldots U_{mn}U_{op}^*}_{\text{up to } 2n \text{ matrix elements}} \rangle_S \overset{!}{=} \langle U_{ij}U_{kl}^* \ldots U_{mn}U_{op}^* \rangle_{\mathrm{CUE}}$$

⟶ S is required to form a **unitary n-design** (not full CUE)

Random quantum circuits: Efficient generation of n-designs

Ohliger et al., New J. Phys., 2014
Brandão et al., Comm. Mat. Phys, 2016



S. Boixo, et al., arXiv:1608.00263

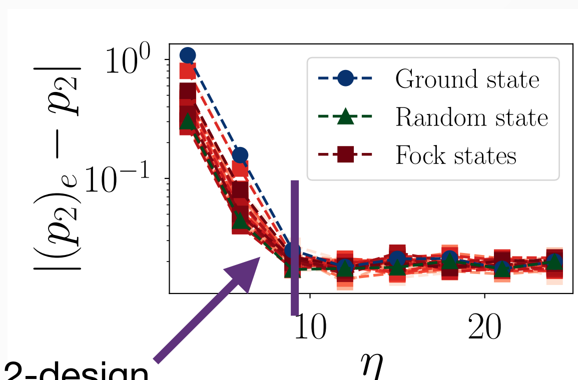
Number of random quenches η ⟷ depth of circuit

Certification hierarchy of random unitaries:

$$\langle \mathrm{P}(\mathbf{s}_A)\rangle_S \overset{!}{=} \frac{1}{N_{\mathcal{H}_A}}$$ holds if S forms 1-design

$$\langle \mathrm{P}(\mathbf{s}_A)^2\rangle_S \overset{!}{=} \frac{1+\mathrm{Tr}\left[\rho_A^2\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$ holds if S forms 2-design

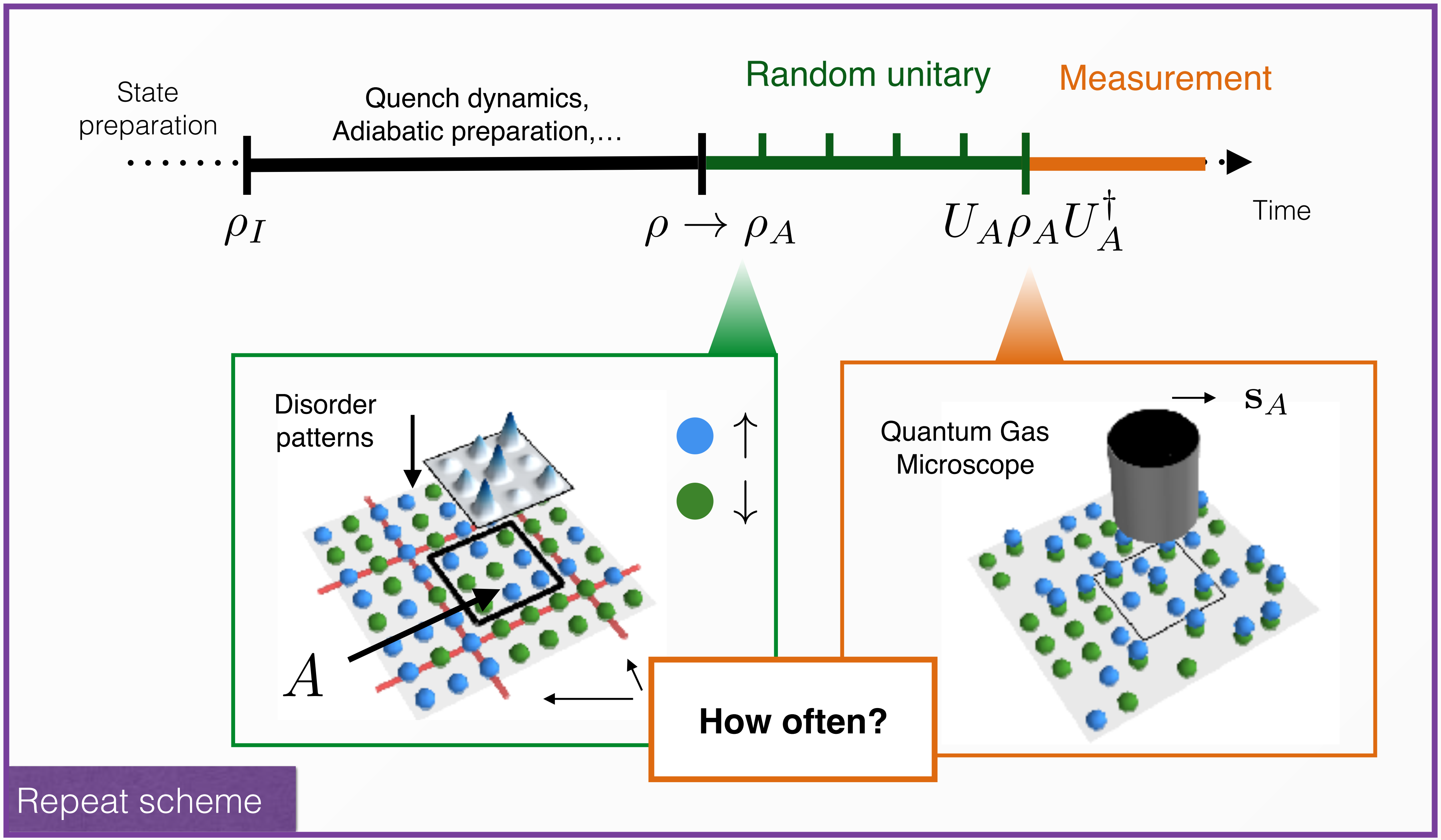
$\vdots$



approximate 2-design

'Chaos by design' Roberts, Yoshida. JHEP, 2017

Measurement Protocol in Hubbard & Spin Models



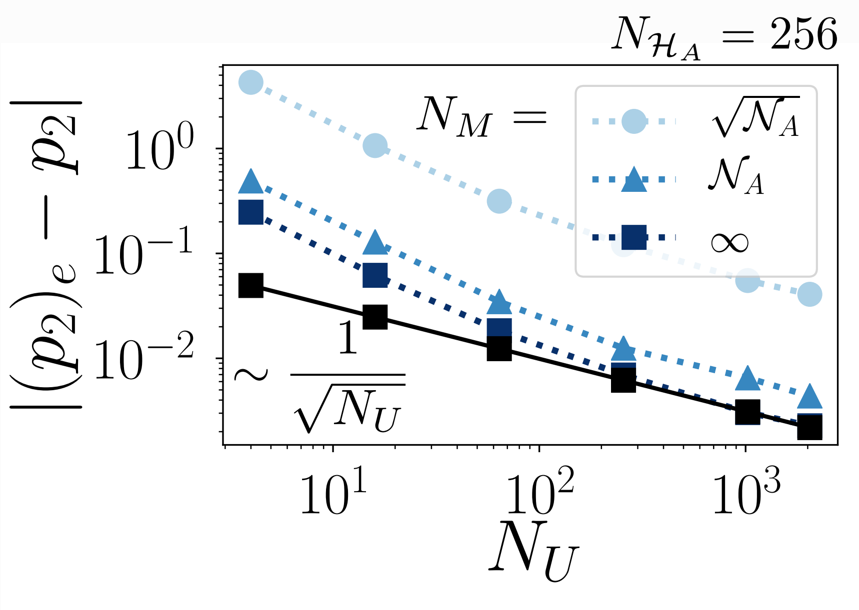
For the same random unitary $\longrightarrow$ probabilities $\mathrm{P}(\mathbf{s}_A)$ for all measurement outcomes $\mathbf{s}_A$

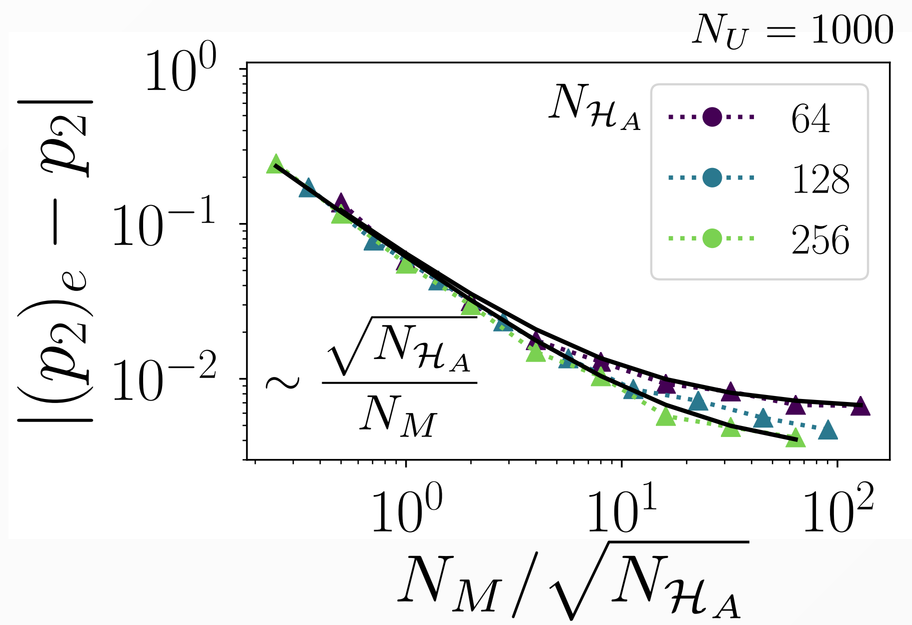
For many random unitaries $\longrightarrow$ correlations $\langle \mathrm{P}(\mathbf{s}_A)^2 \rangle \sim \mathrm{Tr}\left[\rho_A^2\right]$

Scaling of Statistical Error

N_M: number of measurements per unitary
N_U: number of unitaries
$N_{\mathcal{H}_A}$: Hilbert space dimension of A

Error for estimated purity (averaged over all outcomes)





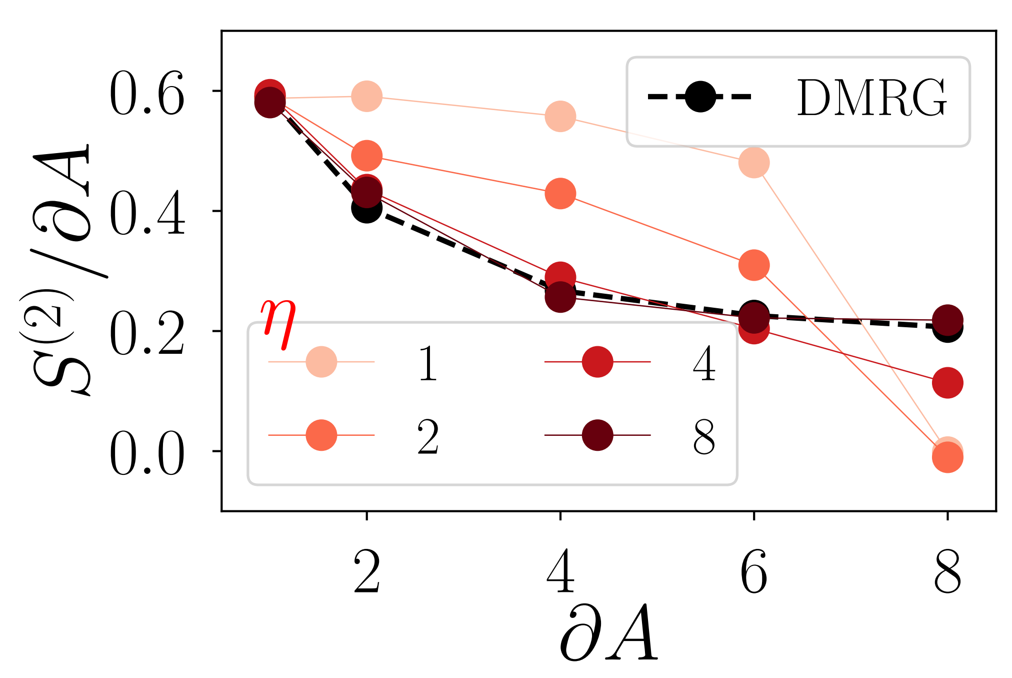
Number of measurements to determine p_2
up to error $\sim 1/\sqrt{N_U}$
$N_M \sim \sqrt{N_{\mathcal{H}_A}}$

Illustrations of Renyi Entropy Measurements

IQI uibk

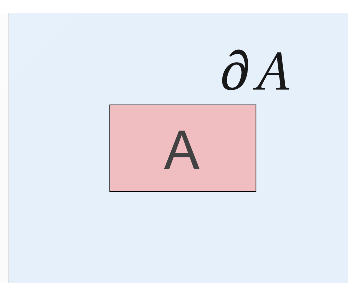
Our protocol allows to *measure* ...

area law in 2D Heisenberg model

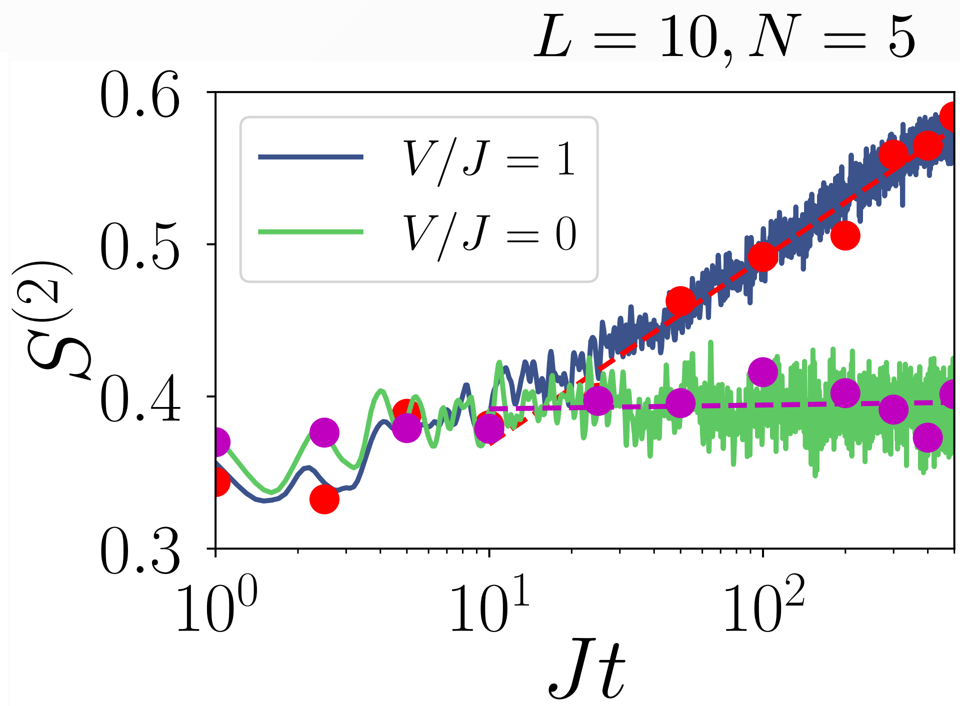


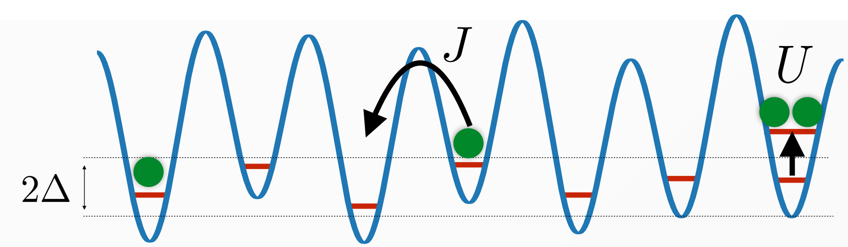
area law

$S_A \simeq \alpha\, \partial A$



entropy growth in MBL phase





1D Bose Hubbard with disorder

$N_M = 100 \quad N_U = 100$

Dirac Medallists 2017

Charles H. Bennett

David Deutsch

Peter W. Shor

Citation:

for pioneering work in applying fundamental concepts of quantum mechanics to solving basic problems in computation and communication, and therefore bringing together the fields of quantum mechanics, computer science and information.

quantum science as fundamental research & quantum technologies