

03.07.2018, 10TP.

"Holographic renormalization and supersymmetry"

on

"The reduced phase space of AdS(super)gravity and
CFT on curved background."

Refs:

1. published lecture notes (2016) (check website later)
2. 1007.4892
3. 1703.04299
4. 1608.07018.

Overview:

(Super)gravity on manifold M
with codim. 1, conformal/asymptotic,
boundary ∂M

Hamiltonian formulation of (super)gravity



Reduced phase space and
local symmetries



CFTs on curved background
and Ward identities



AdS/CFT reform. I:
phase space
quantisation



Variational problem for AdS
(super)gravity and algorithm for
boundary terms



} →
Finite on-shell action
and conserved charges
→ Off-shell boundary sugra
→ AdS/CFT reform. II:
Hamiltonian path int,

Example:

- 9d $N=1$ and 3d $N=2$ theories on curved background
- AdS₂, dilaton gravity
- Wilson loops from AdS - OA

AdS₂

• normalisability of gauge fields:

appendix of 1510.08123

• boundary conditions and boundary terms!

1608.07018

□ Hamiltonian (Arnowitt-Deser-Misner) formulation of gravity.

• Einstein-Hilbert action with Gibbons-Hawking term:

$$S = \frac{1}{2\kappa^2} \left(\int d^d x \sqrt{-g} (R - 2\Lambda) + \int_{\partial M} d^{d-1} x \sqrt{-g} \gamma^{AB} \right)$$

• Radial ADM decomposition:

$$ds^2 = (N^2 + n_i n^i) dt^2 + 2n_i dx^i + g_{ij} dx^i dx^j$$

$$i, j = 1, \dots, d.$$

→ Ricci scalar becomes:

$$R[g] = R[\gamma] + \kappa^2 - \kappa_j^i \kappa^j_i + \gamma (-2\kappa u^A + n \nabla^A u)$$

where $\kappa_{ij} = \frac{1}{2\kappa} (\delta_{ij} - D_i N_j - D_j N_i)$, $u = \gamma^{ij} \kappa_{ij}$

and $u^A = \left(\frac{1}{\kappa}, -\frac{n^i}{\kappa} \right)$ unit normal vector to E_α .

• EH+GH action densities: $S = \int d^d x L$, where

$$L = \frac{1}{2\kappa^2} \int d^d x \sqrt{-g} (R[\gamma] - 2\Lambda + \kappa^2 - \kappa_j^i \kappa^j_i)$$

- Conserved momenta:

$$\pi^{\delta} = \frac{\delta L}{\delta \dot{x}_{ij}} = \frac{1}{2m} \sqrt{g} (x^j \dot{x}^i - x^i \dot{x}^j)$$

$$\pi_{\nu}^i = \frac{\delta L}{\delta \dot{n}_i} = 0, \quad \pi_n = \frac{\delta L}{\delta \dot{n}} = 0$$

$\rightarrow n_i, n$ are auxiliary fields!

- Hamiltonian:

$$H = \int d^d x \pi^i \dot{x}_i - L = \int d^d x (n \epsilon + n_i \dot{x}^i)$$

where

$$\mathcal{H} = -\frac{m^2}{\sqrt{g}} (\pi^{ij} \pi_{ij} - \frac{1}{m} \pi^2) - \frac{1}{m^2} \sqrt{g} (R\epsilon - m)$$

$$\dot{x}^i = -2D_i \sqrt{g}$$

\rightarrow Hamilton's equations for n, n_i imply
the constraints:

$$\mathcal{H} = 0, \quad \dot{x}^i = 0$$



 Hamiltonian constraints momentum constraints

- Hamilton-Jacobi equations:

$$\nabla^{\vec{p}} \vec{x} = \frac{\delta S[\vec{x}]}{\delta \dot{x}_i} \quad \left. \begin{array}{l} H_0, H_i = 0 \\ \downarrow \end{array} \right\}$$

Hamilton's Principal Eq.

$$\left\{ \begin{aligned} -\frac{e^2 k^2}{\sqrt{g}} \left(\gamma_{ik} \gamma_{jl} - \frac{1}{2m} \gamma_{ij} \gamma_{kl} \right) \frac{\delta S}{\delta x_i} \frac{\delta S}{\delta x_m} - \frac{1}{2m} \sqrt{g} (R_{ij} - g_{ij}) = 0 \\ - e D_j \left(\frac{\delta S}{\delta x_i} \right) = 0 \end{aligned} \right.$$

- Symplectic structure.

$$S = \int d^d x \, \delta \pi^i \wedge \delta x_i.$$

\leadsto Poisson brackets:

$$\{ \gamma_{ij}(r, x), \pi^{kl}(r, x') \}_{PB} = \delta_i^{(k)} \delta_j^{(l)} \delta^{(l)}(x - x')$$

can be realized as:

$$\{ A(r, \tau), B(r, \tau) \}_{PB} = \int d^d x \left(\frac{\partial A}{\partial x_i} \frac{\partial B}{\partial \pi_{ij}} - \frac{\partial B}{\partial x_i} \frac{\partial A}{\partial \pi_{ij}} \right)$$

$$\text{e.g. } \dot{x}_{ij} = \frac{\delta A}{\delta \pi_{ij}} = - \{ H, x_{ij} \}_{PB}$$

$$\dot{\pi}^{ij} = - \frac{\delta H}{\delta x_i} = - \{ H, \pi^{ij} \}_{PB}.$$

- o First order constraints and Diffr.

Define phase space function

$$C(\tilde{f}) = \int d^d x (\tilde{f}^a \dot{x} + \tilde{f}^i \dot{x}_i)$$

where $\tilde{f}^a(x), \tilde{f}^i(x)$ are arbitrary.

Exercise: $C(\tilde{f})$ generates infinitesimal diffr. with parameter

$$\tilde{g}^\mu = \left(\frac{\tilde{f}^a}{\tilde{n}}, \tilde{f}^i - \frac{\tilde{f}^a N^i}{\tilde{n}} \right)$$

i.e. $\{ C(\tilde{f}), \delta_{ij} \}_{PB} = \delta_{\tilde{f}} \delta_{ij}$

$$\{ C(\tilde{f}), \pi^j \}_{PB} = \delta_{\tilde{f}} \pi^j$$

where the RHS is a diffr.

- o Diffr. algebra:

$$\{ C(\tilde{f}), C(\tilde{f}') \}_{PB} = C(\tilde{f}'')$$

where

$$\tilde{f}''' = (\tilde{f}^0, \tilde{f}'' - \tilde{f}^{i0} \tilde{x}^i, \tilde{f}^{ij}, \tilde{f}^{i0}, \tilde{f}^{ij} - \tilde{f}^{i0} \tilde{x}^j - \underline{(\tilde{f}^0 \tilde{x}^i - \tilde{f}^i \tilde{x}^0)})$$

Structure constants
are field dependent!

D The Reduced Phase Space (RPS) of AdS gravity

- In the Fefferman-Graham gauge

$$ds^2 = dr^2 + \gamma_{ij}(r, x) dx^i dx^j \quad (\text{i.e. } n=1, n=0)$$

the most general assumption solution of Einstein's equation with $\Lambda < 0$ take the form ($l=1$)

$$\gamma_{ij}(r, x) = e^{2r} (g_{0,ij}(x) + e^{-2r} g_{1,ij}(x) + \dots + e^{-\alpha r} (-2\gamma_{0,ij}(x) + g_{1,ij}(x)) + \dots)$$

where $\gamma_{0,ij}(x)$ is arbitrary and correspond to a metric on the boundary of AdS.

* $g_{0,-}, h_{0i}$ are local curvatures of $g_{0,ij}$

* h_{0i}, f_0 off or even.

* g_{0i} satisfies the constraint:

$$D_{0j} \gamma_{ij}^1 = 0, \quad \gamma_{ij}^1 = A[g_{0,ij}]$$

where

$$\gamma_{ij}^1 = \frac{\partial}{\partial r} (g_{0,ij} - g_{0j}^{kl} g_{0,ik} g_{0,ij}) + X_{ij}[g_{0,ij}]$$

and $A[g_{0,ij}], X_{ij}[g_{0,ij}]$ are local and r -dependent expression.

→ This form of general assymt. solution follows from eqn., but can be determined more efficiently through H.J. theory

① The space of asymptotic solution via singular vector manifold:

→ In reality assymt expansion in Ω :

$$\Omega = \int d^d x \delta_{\text{ext}}^{(d)} \wedge \delta_{\text{gen}}^{(d)}, \quad \pi_{(d)}^{ij} = -\frac{i}{2} \sqrt{g_{\alpha\beta}} T^{ij}$$

→ Poisson bracket on Ω :

$$\{ g_{\alpha\beta}(x), \pi_{(d)}^{kl}(x') \}_{PB} = \delta_{\alpha}^{(k)} \delta_{\beta}^{(l)} \delta^{(d)}(x-x')$$

$$\{ A(g_{\alpha\beta}, \pi_{(d)}^{kl}), B(g_{\alpha\beta}, \pi_{(d)}^{kl}) \}_{PB} = \int d^d x \left(\frac{\delta A}{\delta g_{\alpha\beta}} \frac{\delta B}{\delta \pi_{(d)}^{kl}} - \frac{\delta B}{\delta g_{\alpha\beta}} \frac{\delta A}{\delta \pi_{(d)}^{kl}} \right)$$

• Restored gauge symmetries, preserving FG gauge.

$$\delta_{\tilde{f}} g_{\mu\nu} = \delta_{\tilde{f}} g_{\mu\nu} = \tilde{f}_v \tilde{f}_v + \tilde{f}_v \tilde{f}_v.$$

$$\delta_{\tilde{f}} g_{\mu\nu} = \delta_{\tilde{f}} \varphi_{\mu i} = \varrho \Rightarrow$$

$$\boxed{\tilde{f}^a = 0, \quad \tilde{f}^i + \delta^{ij} \partial_j \tilde{f}^a = 0}$$

gen. solution:

$$\begin{cases} \tilde{f}_{PBH}^a = \delta(x) \\ \tilde{f}_{PBH}^i = \tilde{f}^i \delta(x) - \frac{1}{2} e^{-2x} (g_{00}^{ij} - \frac{1}{2} e^{-2x} g_{00}^{ii} + \delta(e^{-4x})) \partial_j \delta \end{cases}$$

Penrose-Brown-Henneaux diffeomorphism.
(PBH).

Action on g.f. solution:

$$\delta_{PBH} g_{00,ij} = \underbrace{D_{00,i} \tilde{f}_{0j} + D_{00,j} \tilde{f}_{0i}}_{\text{boundary diff.}} + \underbrace{2\delta g_{00,ij}}_{\text{boundary Weyl.}}$$

$$\begin{aligned} \delta_{PBH} \tilde{\pi}_{(d)}^{ij} &= D_{00,k} (\tilde{\pi}_{(d)}^{ij} \tilde{f}_0^k) - \tilde{\pi}_{(d)}^{ik} D_{00,k} \tilde{f}_0^j - \tilde{\pi}_{(d)}^{jk} D_{00,k} \tilde{f}_0^i \\ &\quad - 2\delta \tilde{\pi}_{(d)}^{ij} + \frac{\delta}{\delta g_{00,ij}} \int d^d x \sqrt{g_{00}} L(g_{00}) \delta \alpha. \end{aligned}$$

Comments:

• For asymptotically flat gravity the analysis transformation are supertranslation and superrotation

• For bulk supergravity, these give off-shell boundary supergravity transformation

o Alternative derivation:

Functional on RPS:

$$C[\mathcal{F}_0, \sigma] = \int d^d x \sqrt{g_{00}} (\mathcal{F}_{0\alpha} P_{0\alpha}) T_{ij} + \sigma(x) (T_i^j - A[\mathcal{F}_0])$$

Exercise: Show that

$$\{ C[\mathcal{F}_0, \sigma], g_{00,ij} \}_{PB} = \delta_{PBH} g_{00,ij}$$

$$\{ C[\mathcal{F}_0, \sigma], T_{(ab)}^{(i)} \}_{PB} = \delta_{PBH} T_{(ab)}^{(i)}$$

o Algebra of PBH transformations:

$$\{ C[\mathcal{F}_0, \sigma], C[\mathcal{F}'_0, \sigma'] \}_{PB} = C[\mathcal{F}''_0, \sigma'']$$

where $\begin{cases} \mathcal{F}''_0{}^i = \mathcal{F}_0{}^j \partial_j \mathcal{F}'_0{}^i - \mathcal{F}_0{}^i \partial_j \mathcal{F}'_0{}^j \\ \sigma'' = \mathcal{F}_0{}^j \partial_j \sigma' - \mathcal{F}'_0{}^j \partial_j \sigma. \end{cases}$

→ Structure constants are
field independent!

→ Standard gauge algebra.

Example: 2d CFT on manifold with metric g_{ij} :

- Ward identities:

$$D_{\alpha i} T^i_j = 0 \quad , \quad T^i_i = A(g_{\alpha i}) = \frac{c}{2\pi\hbar} R(g_{\alpha i})$$

- Under local diffeos and Weyl transformation:

$$\delta g_{\alpha ij} = D_{\alpha i} \tau_j + D_{\alpha j} \tau_i + 2\sigma g_{\alpha ij}$$

$$\begin{aligned} \delta T_{ij} = & T_{ik} D_{\alpha j} \tau^k + T_{jk} D_{\alpha i} \tau^k + \tau_k D_{\alpha k} T_{ij} \\ & + \frac{c}{2\pi\hbar} V g_{\alpha i} (D_{\alpha j} D_{\alpha j} - g_{\alpha i} D_{\alpha i}) \sigma. \end{aligned}$$

anomalous term!

Precisely the transformation following from the Poisson Bracket

- Killing symmetries and the Virasoro algebra. (Rigid symmetries)

- pick $g_{\alpha ij} = \eta_{ij}$

- demand

$$\delta g_{\alpha ij} = D_{(0)i} \tau_{j)} + D_{(0)j} \tau_{i)} + 2\sigma g_{ij} = 0$$

conformal Killing
vector eq.

- Solution of CKV eq:

$$\sigma = - \frac{c}{2} D_{\rho i} \gamma_0^i$$

$$ds^2 = \gamma_{ij} dx^i dx^j = 2 dz d\bar{z},$$

$$\gamma_0^z = \epsilon(z), \quad \gamma_0^{\bar{z}} = \bar{\epsilon}(\bar{z})$$

- Specializing transf. of $\tau, \bar{\tau}$:

$$\left\{ \begin{array}{l} \delta_\epsilon \tau = 2 \bar{\tau} \partial \epsilon + \partial \bar{\tau} \epsilon - \frac{c}{2\pi i} \partial^3 \epsilon. \quad (\text{Schwarzian derivative}) \\ \delta_{\bar{\epsilon}} \bar{\tau} = 2 \bar{\tau} \bar{\partial} \bar{\epsilon} + \bar{\partial} \bar{\tau} \bar{\epsilon} - \frac{c}{2\pi i} \bar{\partial}^3 \bar{\epsilon}. \end{array} \right.$$

$$\text{where } \tau \equiv \tau_{zz}, \quad \bar{\tau} \equiv \bar{\tau}_{\bar{z}\bar{z}}.$$

- Virasoro algebra:

$$L_n = -2\pi \oint \frac{dz}{2\pi i} z^{n+1} \tau$$

$$\{L_m, L_n\} = (n-m)L_{m+n} + \frac{c}{12} m(m-1) \delta_{m+n,0}$$

\nearrow
Dirac bracket,

□ Symplectic structure of QFTs on curved background
 (see ch. 2 of lecture notes.)

- Local 1pt func:

$$\langle O_{\alpha_1} \rangle = \frac{\delta S[J]}{\delta J^{\alpha_1}}$$

- Renormalized version:

$$\langle O_{\text{ren.}\alpha_1} \rangle = \frac{\delta S[\text{Ren.}J]}{\delta J^{\alpha_1}}$$

- Since this holds for arbitrary time we can write an op. eq.

$$[O_{\text{ren.}\alpha_1} = \frac{\delta S[\text{Ren.}J]}{\delta J^{\alpha_1}}]$$

- This implies that the space of local source and operator admits a symplectic structure

$$\Omega = \int d^d x \delta O_\alpha(x) \wedge \delta J^\alpha$$

Sum over all local gauge-inv. operators.
 (renormalized)

→ AdS/CFT reformulation I:

- Reduced phase space of AdS.
(super)gravity (string theory.)

III

Symplectic space of local renorm.
gauge invariant operators of dual
CFT.

- First + class constraints.

II

Word identities.

□ Variational problem and boundary term.

[ch 4 and 5 of lecture notes]

• ALAdS manifolds have a conformal boundary

→ bulk fields induce only conformal esp. v. cheese or boundary field?

$$\text{e.g. } ds^2 = dr^2 + e^{2r}(g_{\theta\theta}(x) + \dots) dx^i dx^i$$

$$\text{PBH: } r \rightarrow r + \infty, \quad g_{\theta\theta} \rightarrow e^{2\alpha(x)} g_{\theta\theta}$$

→ Variational problem must be formulated in terms of conformal cheese.

→ $S_{\text{bulk}} + S_B$ is independent of $r \rightarrow$ finite!

- Can show in general that $S_B = -S$, where S is a solution of the HJ. eq. (recursive algorithm)
- S_B implements a canonical transformation.

Typically

$$\begin{pmatrix} \gamma_{ij} \\ \pi^{ij} \end{pmatrix} \mapsto \begin{pmatrix} \gamma_{ij} \\ \pi^{ij} = \pi^{ij} + \frac{\delta S_B}{\delta \dot{x}_i} \end{pmatrix}$$

$$\text{f.t. } \gamma_{ij} \sim e^{-r} g_{0ij}, \quad \pi^{ij} \sim e^{-r} \sqrt{-g_0} \cdot \tau^{ij}$$

- The solution $S = -S_B$ provides an efficient way to determine the space of asymptotic solutions.

$$\dot{\gamma}_{ij} = -\frac{4n^2}{\sqrt{g}} \left(\gamma_{ik} \gamma_{jl} - \frac{1}{dn} \gamma_{ij} \gamma_{kl} \right) \frac{\delta S}{\delta \dot{x}^{kl}},$$

D 4d: $W=1$ and 3d $W=2$ QFTs on curved background from AdS supergravity,
 [1703.04299].

- Minimal gauged supergravity in 5D and 4D ($d=4, 3$):

$$\begin{aligned} S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} & \left(R - F^2 - 2\Lambda + c_1 \epsilon^{\alpha\beta\rho\sigma} F_{\alpha\beta} F_{\rho\sigma} A_\gamma \right. \\ & - \bar{\Psi}_\nu \Gamma^{\alpha\beta} \left(\nabla_\nu + 2ig A_\nu \right) \bar{\mathcal{L}}_\rho \\ & \left. - \frac{d-1}{\ell} \bar{\Psi}_\nu \left(\Gamma^{\alpha\beta} + i\omega (\Gamma^{\alpha\beta\rho\sigma} F_{\rho\sigma} + 2F^{\alpha\beta}) \right) \Psi_\nu \right) \end{aligned}$$

where

$$c_1 = -\frac{2\ell}{3\sqrt{3}} S_{d,4}, \quad c_2 = \frac{\ell}{\sqrt{2(d-2)(d-1)}}.$$

$$\Lambda = -\frac{d(d-1)}{2\ell^2}, \quad g = \frac{1}{\ell} \sqrt{(d-1)(d-2)/2}.$$

* $\bar{\Psi}_\mu$ in a Poincaré sphere.

$$\bar{\nabla}_\mu \bar{\Psi}_\nu = \bar{\gamma}_\mu \bar{\Psi}_\nu + \frac{i}{4} \omega_{\mu\alpha\beta} \Gamma^{\alpha\beta} \bar{\mathcal{L}}_\nu - \bar{\Gamma}_{\mu\nu}^\rho \bar{\Psi}_\rho.$$

* GH term.

$$S_{GH} = \frac{1}{2\kappa^2} \int d^d x \sqrt{-g} \left(2\Lambda + \bar{\Psi}^i \hat{\Gamma}^{ij} \Psi_j \right)$$

* SUSY transformation

$$\delta E_\mu^\alpha = \frac{1}{2} (\bar{\epsilon} \Gamma^\alpha \not{E}_\mu - \not{E}_\mu \Gamma^\alpha \epsilon), \quad \delta A_\mu = i c_3 (\not{E}^\nu \epsilon - \bar{\epsilon} \not{E}^\nu),$$

$$\delta \not{E}_\mu = \not{E}_\mu \epsilon + i c_3 (\Gamma_\mu^{\nu\rho} - 2(\alpha-2) \not{E}^\nu \Gamma^\rho) F_{\nu\rho} \epsilon - f \frac{1}{2} (\Gamma_\mu - 2ig A_\mu) \epsilon$$

$$c_3 = \sqrt{\frac{\alpha-1}{8(\alpha-3)}} \rightarrow \alpha = \frac{1}{\sqrt{8(\alpha-1)(\alpha-2)}}.$$

$$g_{\mu\nu} = E_\mu^\alpha E_\nu^\beta \eta_{\alpha\beta}.$$

□ Hamiltonian formulation:

* \not{E}_μ must be decomposed using
realistic projections

$$P_\pm = \frac{1}{2} (1 \pm \Gamma^z)$$

$$\rightarrow \not{E}_{+\mu}, \not{E}_{-\mu}$$

* $n, n_i, A_\mu, \not{E}_{+\mu}, \not{E}_{-\mu}$ are auxiliary
fields \rightarrow from clear constraint
 \rightarrow Ward identities on
reduced phase space.

Ω Space of asymptotic solutions.

d=4 (AdS₅)

$$e_i^a = e^{n\ell} e_{i(0)}^a(x) + e^{-n\ell} e_{i(0)}^a(x) + e^{-3n\ell} (\log e^{-2\pi\ell} \tilde{e}_{i(0)}^a(x) + e_{i(0)}^a(x)) + \dots$$

$$A_i = A_{0,i}(x) + e^{-2n\ell} (\log e^{-2\pi\ell} \tilde{A}_{0,i}(x) + A_{0,i}(x)) + \dots$$

$$\Phi_{+i} = e^{\frac{n}{2}\ell} \Phi_{(0)+i}(x) + e^{-\frac{3n}{2}\ell} \tilde{\Phi}_{(0)+i}(x) + \dots$$

$$\Phi_{-i} = e^{-\frac{n}{2}\ell} \Phi_{(0)-i}(x) + e^{-\frac{3n}{2}\ell} (\log e^{-2\pi\ell} \tilde{\Phi}_{(0)-i}(x) + \tilde{\Phi}_{(0)-i}(x)) + \dots$$

d=3 (AdS₄)

$$e_i^a = e^{n\ell} e_{i(0)}^a(x) + e^{-n\ell} e_{i(0)}^a(x) + e^{-2n\ell} e_{i(0)}^a(x) + \dots$$

$$A_i = A_{0,i}(x) + e^{-n\ell} A_{1,i}(x) + \dots$$

$$\Phi_{+i} = e^{\frac{n}{2}\ell} \Phi_{(0)+i}(x) + e^{-\frac{3n}{2}\ell} \tilde{\Phi}_{(0)+i}(x) + \dots$$

$$\Phi_{-i} = e^{\frac{n}{2}\ell} \Phi_{(0)-i}(x) + e^{-\frac{3n}{2}\ell} \tilde{\Phi}_{(0)-i}(x) + \dots$$

- Using S_{ct} . (\equiv off shell const. source on marked slice)

we define the operators at the symplectic conjugate of the sources:

Current multiplied.

$$\left\{ \begin{array}{l} \gamma_a^i = - \lim_{m \rightarrow \infty} \left(\frac{e^{S_{\text{marked}}}}{\sqrt{\gamma}} \left(\pi_a^i + \frac{\delta S_{ct}}{\delta \phi_a^i} \right) \right) \\ J^i = \lim_{m \rightarrow \infty} \left(\frac{e^{S_{\text{marked}}}}{\sqrt{\gamma}} \left(\pi^i + \frac{\delta S_{ct}}{\delta A_i} \right) \right) \\ \Sigma^i = \lim_{m \rightarrow \infty} \left(\frac{e^{(d+k_0)m}}{\sqrt{\gamma}} \left(\pi_{\bar{E}_i}^i + \frac{\delta S_{ct}}{\delta \bar{E}_i} \right) \right) \end{array} \right.$$

- First class constraints reduce to Ward identities in RPS.

- $D_{0ij} (\epsilon_{0ij}^a \pi_a^j - \bar{\epsilon}_{0ij}^j \Sigma^i) + \bar{\epsilon}_{0ij}^j D_{0ij} \bar{\epsilon}_{0ij}^i + \bar{\epsilon}_{0ij}^i D_{0ij} \bar{\epsilon}_{0ij}^j + \bar{\epsilon}_{0ij}^i J^i = U_{R0ij}$
- $D_{0ii} \bar{\epsilon}_{0ii}^i + ig(\bar{\epsilon}_{0ii}^i \bar{\epsilon}_{0ii}^i - \bar{\epsilon}_{0ii}^i \Sigma^i) = U_{R0i}$
- $\epsilon_{00i}^a \bar{\epsilon}_{00i}^i - \frac{1}{2} \bar{\epsilon}_{00i}^i \Sigma^i - \frac{1}{2} \bar{\epsilon}_{00i}^i \bar{\epsilon}_{00i}^i = U_{W(0)}$
- $D_{00i} \Sigma^i + \frac{1}{2} \bar{\Gamma}_a^i \Gamma^a \bar{\epsilon}_{00i}^i - \frac{i\alpha_2}{2(d-2)} \bar{\Gamma}^i (\Gamma_{00j}^{(d-1)} \Gamma_{00j}^{(d-2)} D_{00j} \bar{\epsilon}_{00i}^i) = U_{S0i}$
- $\bar{\epsilon}_{00i}^i \Sigma^i - \frac{i(d-1)\alpha_2}{2} \bar{\Gamma}^i \bar{\epsilon}_{00i}^i = U_{S00i}$

□ Residual local symmetries:

- Bulk offshell, Lorentz, U(1) and SU(2) transformations preserving the gauge:

$$\bar{E}_\mu^a = 1, \quad \bar{F}_{\mu\nu}^a = 0, \quad \bar{F}_\mu^i = 0, \quad a=0, \quad \bar{L}_a = 0$$

$$\left\{ \begin{array}{l} \bar{\mathcal{F}}^r = \sigma(x) + \dots \\ \bar{\mathcal{F}}^i = \bar{\mathcal{F}}_0^i(x) + \dots \\ \bar{\lambda}_b^a = \bar{\lambda}_{0,b}^a(x) + \dots \\ \bar{\Theta} = \bar{\Theta}(x) + \dots \\ \bar{e}_t = e^{+ \bar{\gamma}_{ab}^r} e_{0,t}^a(x) + \dots \end{array} \right.$$

- Transformations of the sources (conservation laws)

$$\delta_{\epsilon_x, \epsilon_-} \bar{e}_{0,i}^a = \frac{i}{2} (\bar{e}_{0x} \Gamma^a \bar{e}_{0,-i} + h.c.)$$

$$\delta_{\epsilon_x, \epsilon_-} A_{0,i}^a = i c_3 (\bar{e}_{0+i} \epsilon_0 + \bar{e}_{0,-i} \epsilon_{0+} + h.c.)$$

$$\delta_{\epsilon_x, \epsilon_-} \bar{F}_{0,i}^a = D_{0,i} \epsilon_{0+} - \frac{i}{2} \Gamma_{0,i} \epsilon_{0-}$$

$\mathcal{N}=1$ off-shell conf. supergravity
transformations.

0 Transformations of the currents.

→ use symplectic structure!

→ Ward identities are first class constraints

→ compute the Poisson bracket of the Ward identities with the currents

For the susy transformation of the supercurrents we get:

$$\delta_{E_0+} S^i = -\frac{1}{2} T^i_a \Gamma^a \epsilon_+$$

$$+ \frac{i c_2}{2(d-2)} \Gamma^{ijk}_{(0)} (\Gamma_{(0)lk} - (d-2) g_{(0)lk}) D_{0ij} \left[\left(\gamma^l - \frac{2c_1}{\omega^2} \epsilon^{lprq} F_{(0)pq} A_{(0)r} \right) \epsilon_+ \right]$$

$$\delta_{E_0-} S^i = -\frac{i(d-1)c_2}{2\ell} \left(\gamma^i - \frac{2c_1}{\omega^2} \epsilon^{lprq} F_{(0)pq} A_{(0)r} \right) \epsilon_-$$

$$- \frac{i}{2(d-2)\omega^2} \Gamma^{ijk}_{(0)} \Gamma^l_{(0)} D_{0ij} \left[\left(R_{lk} - \frac{1}{2(d-2)} R^2 g_{(0)lk} \right) \epsilon_- \right]$$

$$- \frac{ic_2}{2(d-2)\omega^2} \Gamma^{ijk}_{(0)} \Gamma^l_{(0)k} \left((d-2) \Gamma^k_{(0)} \Gamma^{pq}_{(0)} - (d-1) \Gamma^{kpq}_{(0)} \right) D_{0ij} (F_{(0)pq} \epsilon_-)$$