

ICTP Lectures on

Quantum Black Hole entropy
& Localization in supergravity.



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Plan of lectures

Part 1 : Why ?

The idea of quantum black hole entropy

1.1. Black hole entropy

BH Thermodynamics, Temperature, Entropy,
 Boltzmann eqn, finite size effects,
 Wald entropy, Non-local terms.

1.2. Supersymmetric BHs

Isolated quantum systems,
 near-horizon limit

1.3 Classical entropy function

$AdS_2 \times S^2$ w/ fixed \vec{E} & \vec{B} ,
 entropy as extremization

1.4 Quantum entropy

Fall-off conditions in quantum problem,
 Definition of Z_{AdS_2} , Renormalized action Sren.

Part 2 : What? / Context

BPS BHs in 4d $N=2$ sugra

2.1 Quick review of $N=2$ sugra

Fields : Weyl multiplet, vector multiplet -
Action, Prepotential F

2.2 Dyonic BPS BH solutions

Near-horizon configuration, [attractor mechanism]
classical entropy

2.3 Quantum entropy

B.C.S, Z_{AdS_2} , Wilson line ensemble,
 AdS_2/CFT_1 , Action: Massive & massless modes
UV & IR divergences.

Part 3 How?

Localization in supergravity & computation of exact quantum entropy.

3.1 Review of localization

$SU(2) \times SU(2)$ susy transformations,

3.2 BPS basis
2 different methods $\rightarrow S = \int \square + \square^+ \dots - \dots + \square$
Solutions \rightarrow Fermion bilinears

3.3 Renormalized action

Bulk & bdry terms

F-terms & D-terms

3.4 One-Loop determinants : results

3.5 Quantum entropy formula

$N=2$ formula,

Application to $N=1$ theory,

Bessel function, (connection to modular forms).

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Part 4

Loose ends , looking ahead

4.1 One loop determinants

Set up , show complex ,

Determinant from index ,

Miyah- Bott fixed-pt. formula .

4.2 What is \hat{Q} in gravity ?

\hat{Q} vs $\hat{\bar{Q}}$ in gauge theory ,
 what is the problem in gravity ?
 Sketch of solution .

