Finite determinacy of matrices of power series

Pham Thuy Huong

Abstract

Let $R = K[[x_1, x_2, ..., x_s]]$ be the ring of formal power series over an algebraically closed field K of arbitrary characteristic with maximal ideal $\mathfrak{m}$. On the ring $M_{m,n}$ of $m \times n$ matrices A with entries in R we consider several equivalence relations given by the action on $M_{m,n}$ of a group G. G can be the group of automorphisms of R, combined with the multiplication of invertible matrices from the left, from the right, or from both sides, respectively. We call A finitely G-determined if A is G-equivalent to any matrix B with $A - B \in \mathfrak{m}^k M_{m,n}$ for some positive integer k, which implies in particular that A is G–equivalent to a matrix with polynomial entries.

Over the complex numbers, the classical criterion says that $A \in M_{m,1}$ is finitely determined (with respect to various group actions) if and only if the tangent space to the orbit of A has finite codimension in $M_{m,1}$. In arbitrary characteristic, we give a general sufficient criterion for finite G-determinacy in $M_{m,n}$, which is also necessary if the characteristic of K is 0. This criterion provides a computable bound for the G-determinacy of a matrix A in $M_{m,n}$. This is a joint work with Gert-Martin Greuel.