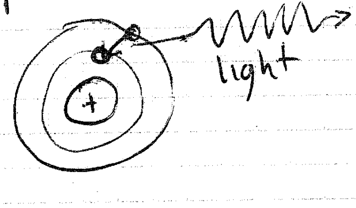
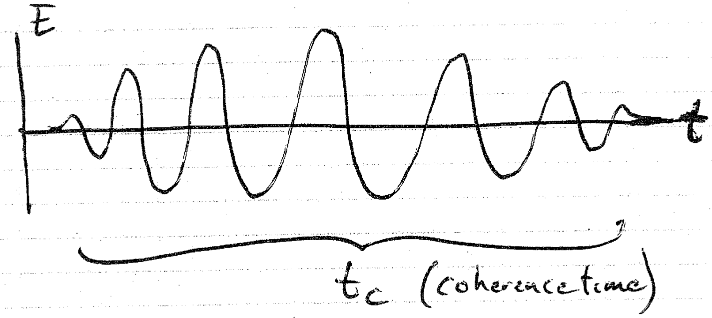
Coherence and partial coherence

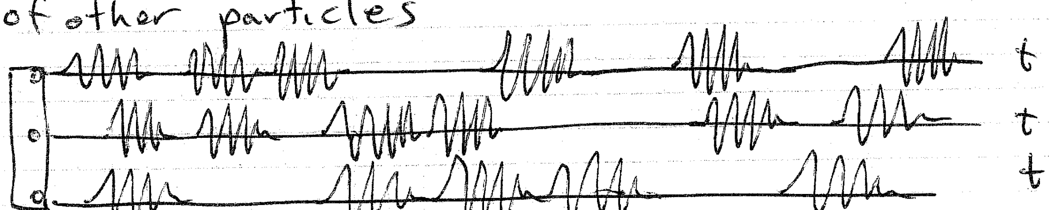
Temporal coherence

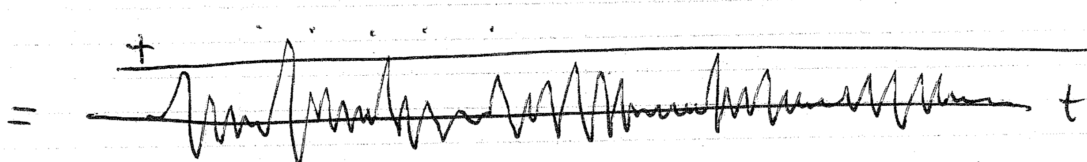
Pure monochromatic fields do not exist.
In practice, an atom or molecule emits a "photon" following a transition:





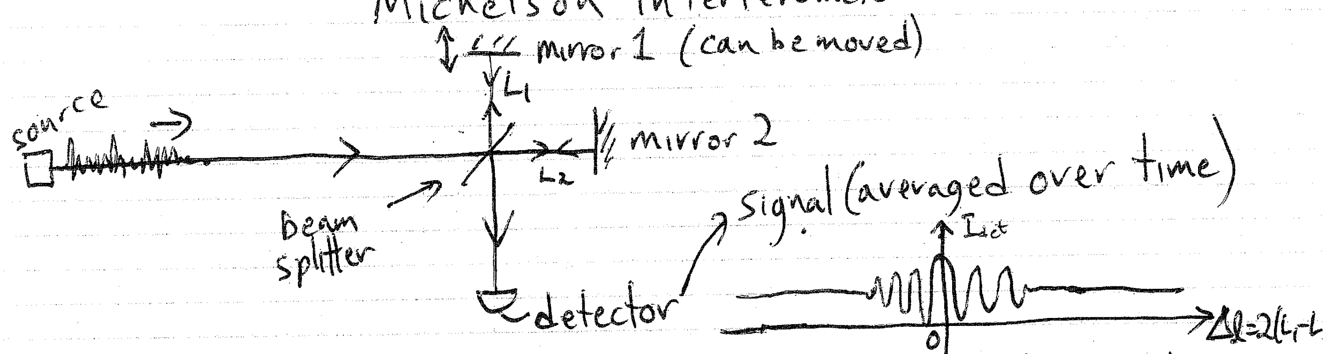
In an incoherent source, each particle emits light pulses randomly, uncorrelated from each other and from those of other particles





The oscillations are only correlated for time differences smaller than t_c.

Michelson interferometer



Where

$$I_{det} = \left\langle \left| \frac{E(t)}{2} + \frac{E(t+\overset{\frac{\Delta \ell}{c} = \tau}{\tau})}{2} \right|^2 \right\rangle_t = \frac{1}{4} \left\langle \left(E(t) + E(t+\tau) \right)^* \left(E(t) + E(t+\tau) \right) \right\rangle_t$$

$$= \underbrace{\frac{\langle |E(t)|^2 \rangle_t}{4}}_{\frac{I_0}{4}} + \underbrace{\frac{\langle |E(t+\tau)|^2 \rangle_t}{4}}_{\frac{I_0}{4}} + \frac{2}{4} \, \mathrm{Re} \underbrace{\left\langle E^*(t+\tau) E(t) \right\rangle_t}_{\mathcal{C}(\tau)}$$

$$= \frac{I_0 + \mathrm{Re}[\mathcal{C}(\tau)]}{2}$$

Notice: The correlation $\mathcal{C}(\tau)$ satisfies

$$\mathcal{C}(0) = I_0 \Rightarrow I_{det}(0) = I_0$$

$$\mathcal{C}(\tau \gg t_c) = 0 \Rightarrow I_{det}(\tau \gg t_c) = \frac{I_0}{2}$$

Wiener-Khinchin theorem (roughly)

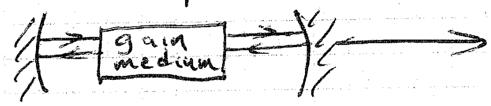
$$\mathcal{C}(\tau) = \langle E^*(t+\tau) E(t) \rangle_t \propto \int \underbrace{E^*(t+\tau)}_{\text{write as} \left[\frac{1}{\sqrt{2\pi}} \int \hat{E}(\omega) e^{-i\omega(t+\tau)} d\omega \right]^*} E(t) \, dt$$

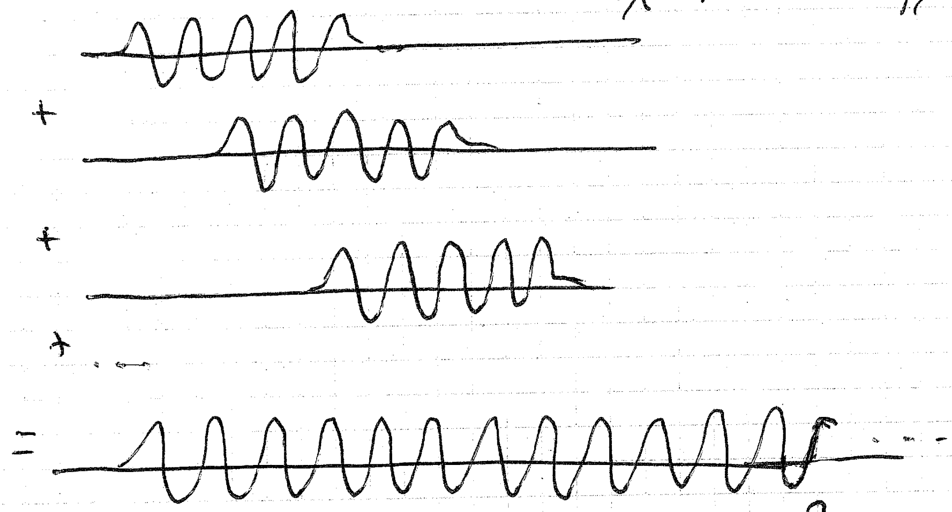
$$\mathcal{C}(\tau) \propto \frac{1}{\sqrt{2\pi}} \iint \hat{E}^*(\omega) \, e^{+i\omega(t+\tau)} E(t) \, dt \, d\omega$$

$$= \int \hat{E}^*(\omega) \underbrace{\frac{1}{\sqrt{2\pi}} \int E(t) e^{i\omega t} dt}_{\hat{E}(\omega)} \, e^{i\omega\tau} d\omega$$

$$= \int \underbrace{|\hat{E}(\omega)|^2}_{S(\omega) = \text{spectrum}} e^{i\omega\tau} d\omega, \quad \text{so } \mathcal{C}(\tau) = I_0 \frac{\hat{\mathcal{F}}^{-1}_{\omega \to \tau} S(\omega)}{\left[\hat{\mathcal{F}}^{-1}_{\omega \to \tau} S(\omega) \right] \Big|_{\tau = 0}}$$

In a <u>laser</u>, on the other hand, each photon triggers the coherent emission of another one (stimulated emission), so the pulses are not (entirely) random:

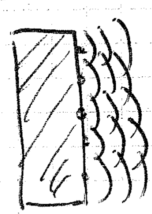




Very long correlation time & length!

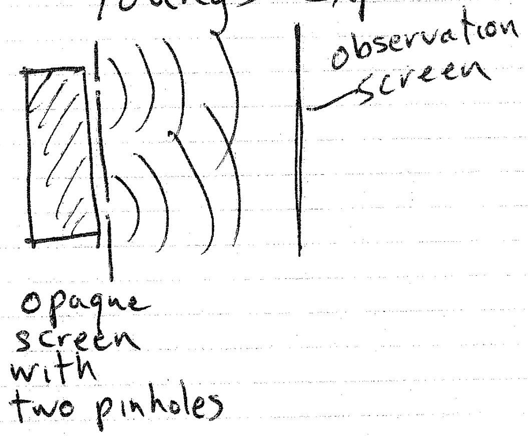
Spatial coherence

extended incoherent source



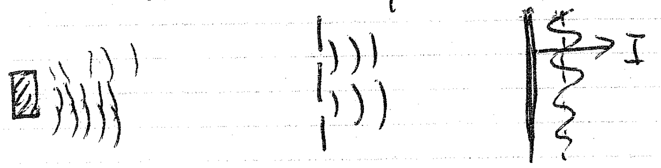
the emissions of each point are statistically uncorrelated

Young's experiment:

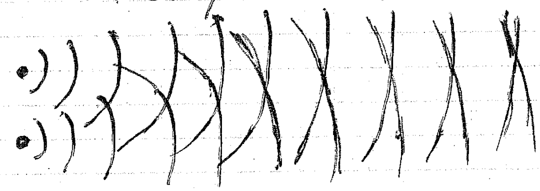


The interference between the light transmissions through the two pinholes washes out statistically over time, so the intensity at the observation screen looks uniform.

Upon propagation from the source, however, the field acquires some spatial coherence:



Explanation: consider a source that consists of only two incoherent emitters



as we move away, their wave fronts look more and more similar.