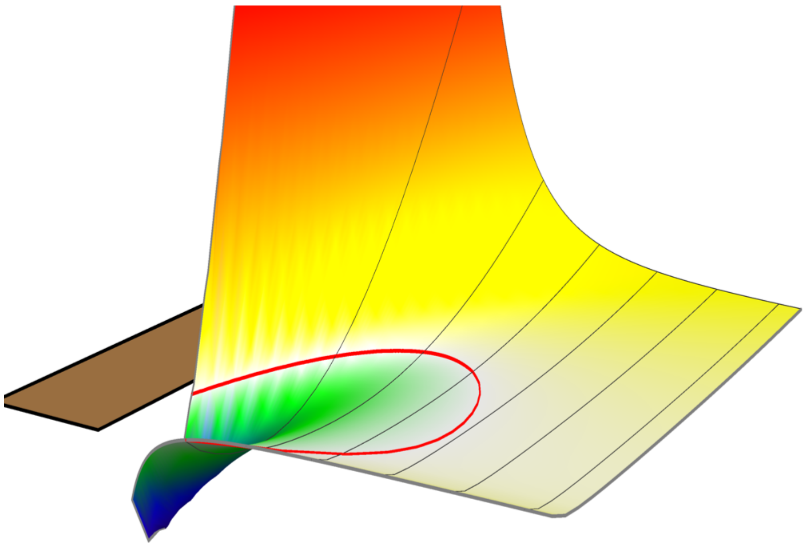
FLUCTUATION SPECTROSCOPY OF THE NERNST COEFFICIENT



"Natura non facit saltus"
Gottfried Leibniz, **New Essays** (1704).

ANDREAS GLATZ
Materials Science Division

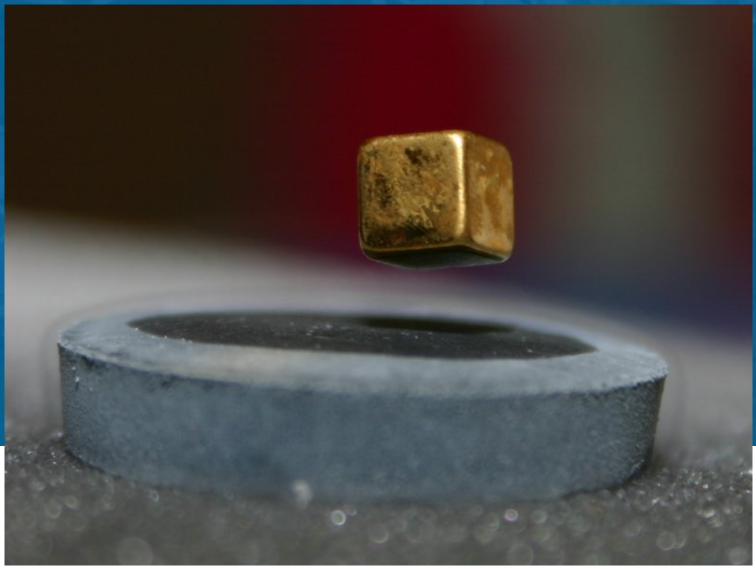
A. Varlamov, A. Galda, and A. Glatz
Fluctuation Spectroscopy: from Rayleigh-Jeans Waves to Abrikosov Vortex Clusters
Rev. Mod. Phys. **90**, 015009 (2018)

2019-03-14: Conference on Modern Concepts and New Materials for Thermoelectricity

OUTLINE

- **Introduction to fluctuation corrections, qualitative picture**
- **Diagrammatic representation**
- **Complete *exact* result for the Nernst coefficient**
- ***Fluctuoscopy* of the Nernst coefficient**
  - Numerical Fluctuoscopy
  - Application to experiments
  - **Ghost field**
- Summary

SUPERCONDUCTING FLUCTUATIONS
QUALITATIVE PICTURE



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SUPERCONDUCTING FLUCTUATIONS AND WHY THEY ARE USEFUL

Superconducting Fluctuations (SF)

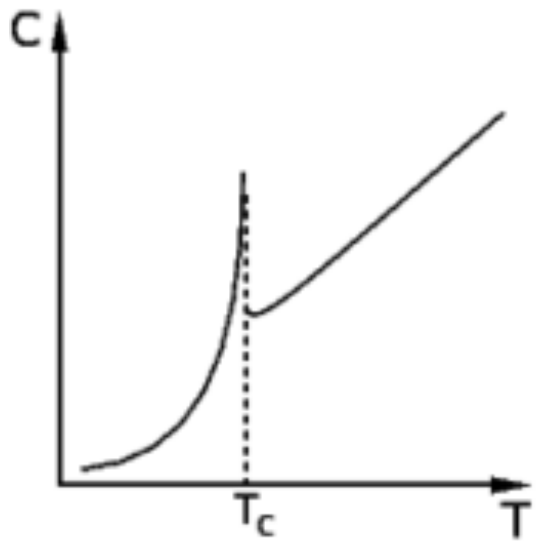
- ✧ Are due to Cooper pairs with finite lifetime above the transition (FCPs)
- ✧ do not form a stable condensate yet
- ✧ affect thermodynamic and transport properties of the normal state directly and through the changes in the normal quasiparticle subsystem, ***sometimes up to room temperature***

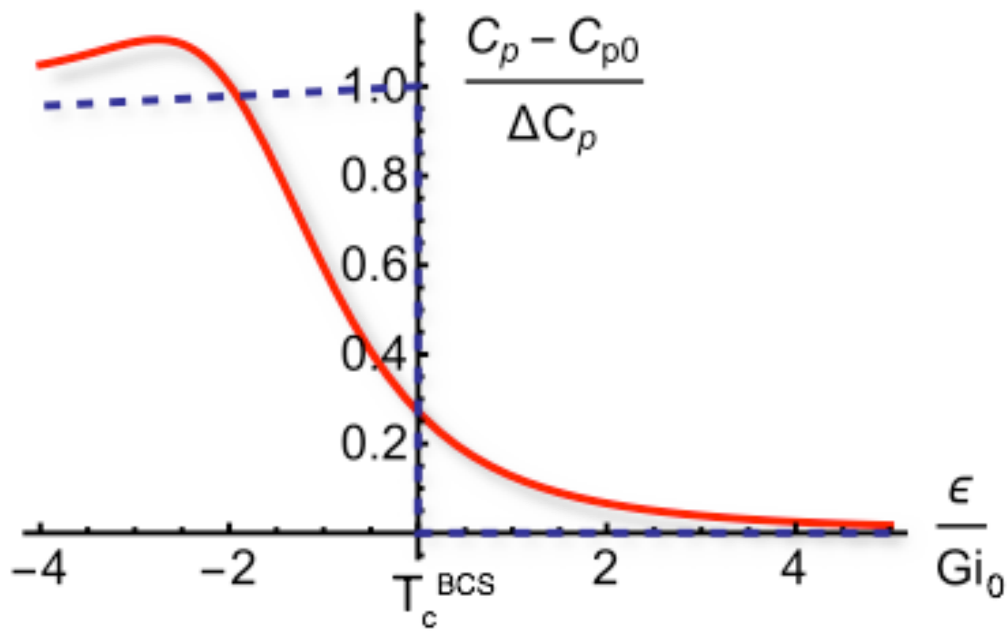
→ ***Complex behavior and multitude of interesting physical phenomena***

→ *Provide valuable information about microscopic properties of normal and superconducting state:* ***Fluctuation Spectroscopy*** *or* ***Fluctuoscopy***

SMEARING OF THE SUPERCONDUCTING TRANSITION

Heat capacity in small, 0D superconducting granules (Schmidt, 1966)





$\varepsilon = \ln(T/T_c)$

Critical temperature region of "smearing" determined the Ginzburg-Levanyuk number Gi, here by Gi_0T_c with:

$$Gi_{(0)} = \frac{\sqrt{7\zeta(3)}}{2\pi}\frac{1}{\sqrt{\nu T_{c0}V}} \approx 13.3\left(\frac{T_{c0}}{E_F}\right)\sqrt{\frac{\xi_0^3}{V}}.$$

$Gi \sim (10^{-14}-10^{-6})T_{c0}$ in 3D
$Gi \sim (10^{-2}-10^{-3})T_{c0}$ in 1D, 2D

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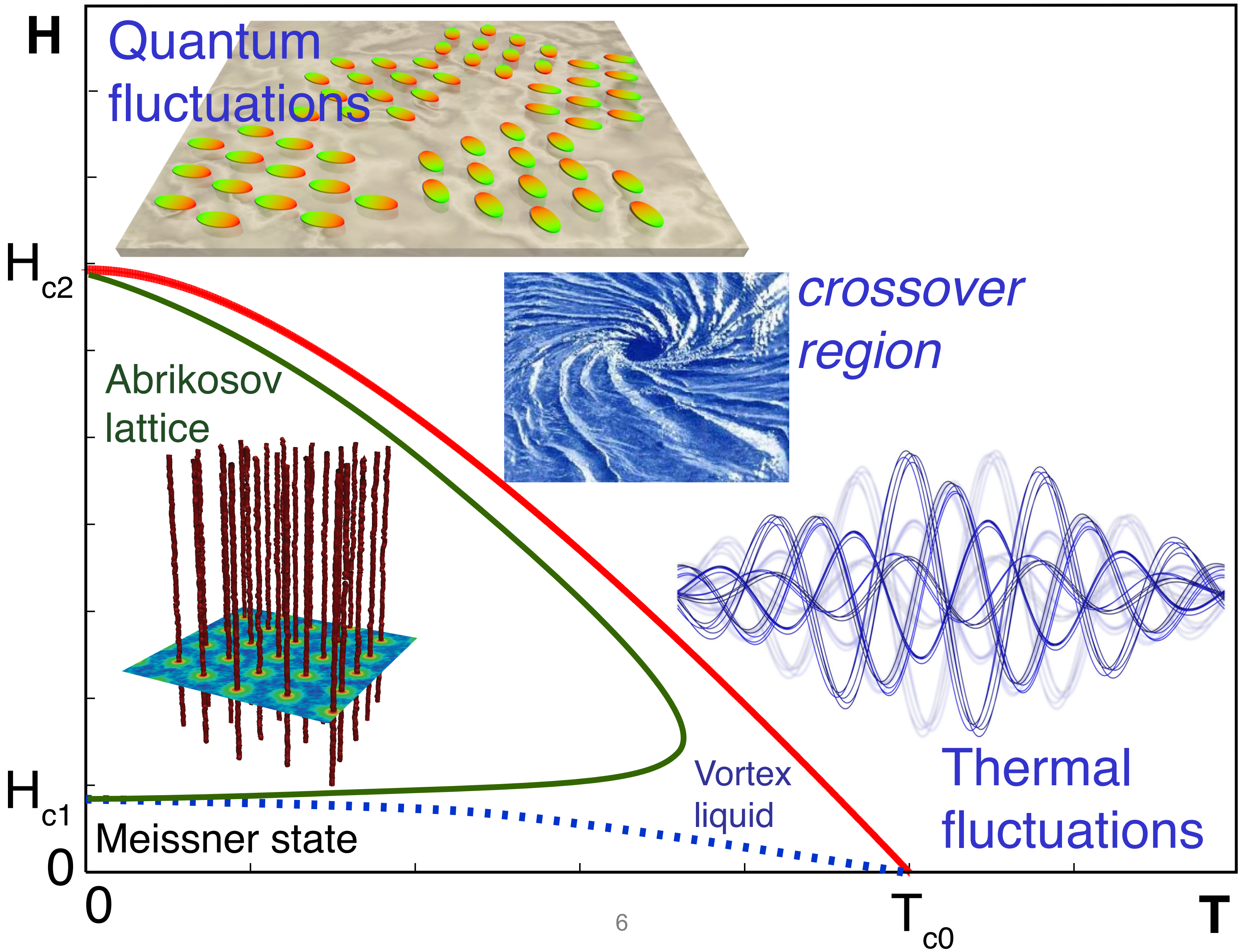
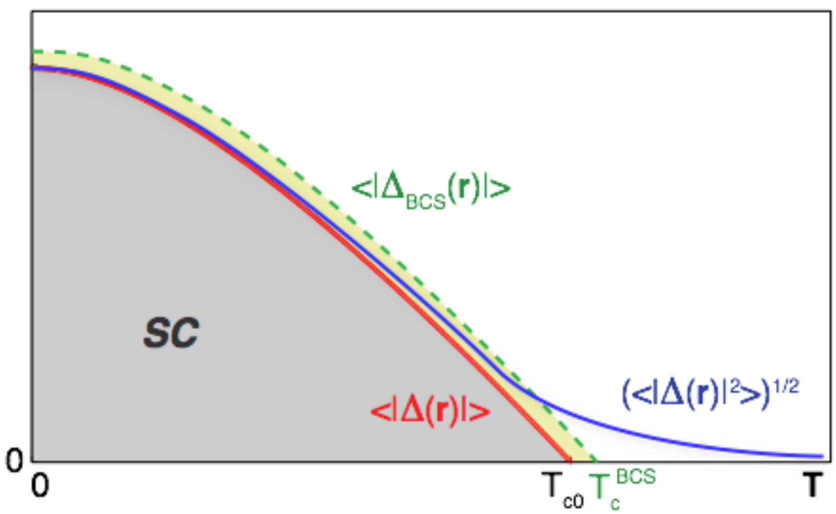
SCHEMATIC PHASE DIAGRAM



QUALITATIVE PICTURE: SUPERCONDUCTING FLUCTUATIONS NEAR T_C



Only small energies are involved:

$$n(\mathbf{q}) = \frac{1}{\exp[\varepsilon(\mathbf{q})/k_B T] - 1} \to \frac{k_B T}{\varepsilon(\mathbf{q})}$$

Rayleigh-Jeans waves rather than Boltzmann particles

In normal state, above T_{c0}, no persistent Cooper pairs, but FCPs with finite life time:

$$\tau_{GL} = \frac{\pi\hbar}{8k_B(T - T_{c0})}$$

Characteristic size, in dirty SC:

$$\xi_{\mathcal{D}}(T) = \sqrt{\mathcal{D}\tau_{GL}} \sim v_F\sqrt{\tau\tau_{GL}}$$

Characteristic size, in clean SC:

$$\xi_{cl}(T) \sim v_F\sqrt{\hbar\tau_{GL}/k_B T}$$

In both cases for Gi<ε<1:

$$\xi_{GL}(\epsilon) = \xi/\sqrt{\epsilon}.$$

$$\epsilon \equiv \ln\frac{T}{T_{c0}} \approx \frac{T - T_{c0}}{T_{c0}} \ll 1$$

SF CLOSE TO T_C

FCP concentration

$$N_{(d)} = \int_{|\mathbf{q}| \lesssim \hbar/\xi} n(\mathbf{q}) \frac{d^d \mathbf{q}}{(2\pi\hbar)^d}$$

$$= \frac{m^* k_B T_{c0}}{2\pi\hbar^2} \begin{cases} 2\pi\xi_{GL}(\epsilon) & d = 1, \\ \ln(1/\epsilon) & d = 2, \\ \mathrm{const} - \xi_{GL}^{-1}(\epsilon) & d = 3. \end{cases}$$

Formation of FCPs depletes quasiparticle subsystem by $2N_{(d)}$ → correction to heat capacity:

$$C_V^{(fl)} = \left(\frac{dE^{(fl)}}{dT}\right)_V \sim -2k_B \frac{dN_{(d)}(\epsilon)}{d\epsilon} \sim \frac{k_B}{\xi^d} \epsilon^{d/2-2}$$

Direct contribution to conductivity (Azlamasov-Larkin paraconductivity):

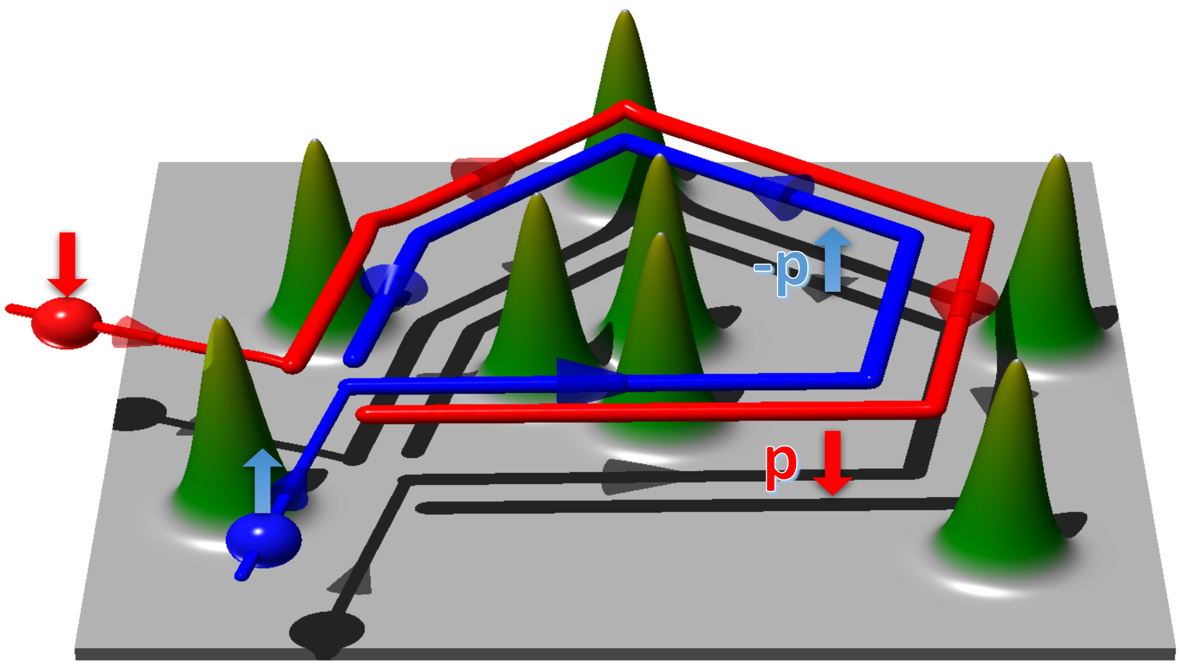
$$\sigma_{xx}^{(AL)} = \frac{4\tilde{N}_{(3)}(\epsilon) e^2 \tau_{GL}(\epsilon)}{m^*} \sim \frac{e^2}{\hbar s^{3-d}} \xi^{2-d} \epsilon^{d/2-2}$$

Using Drude formula (here m* determined by ξ^2, scattering time replaced by τ_{GL}, charge by 2e)

INDIRECT MANIFESTATIONS OF SFS

Besides the direct AL contribution there are indirect, purely quantum manifestations:

- **Maki-Thompson: relevant for transport coefficients near T_{c0} in dirty SCs**



In 2D:

$$\sigma_{(2)}^{(\mathrm{MT,an})} \sim \frac{e^2}{8\epsilon\hbar} \ln \frac{\tau_\varphi}{\tau_{\mathrm{GL}}}.$$

Has same, positive singularity near T_{c0} as AL

Electron moves along self-intersecting diffusive trajectory and returns to origin after some time → decrease of Drude conductivity [weak localization]

Minimal time: τ_{GL}

Maximum time: τ_φ (phase-breaking time)

…

- **DOS contribution:** due to depletion of qp density of states → **opening of pseudogap** in one-electron spectrum, leading to **decrease of decrease of Drude-like conductivity**

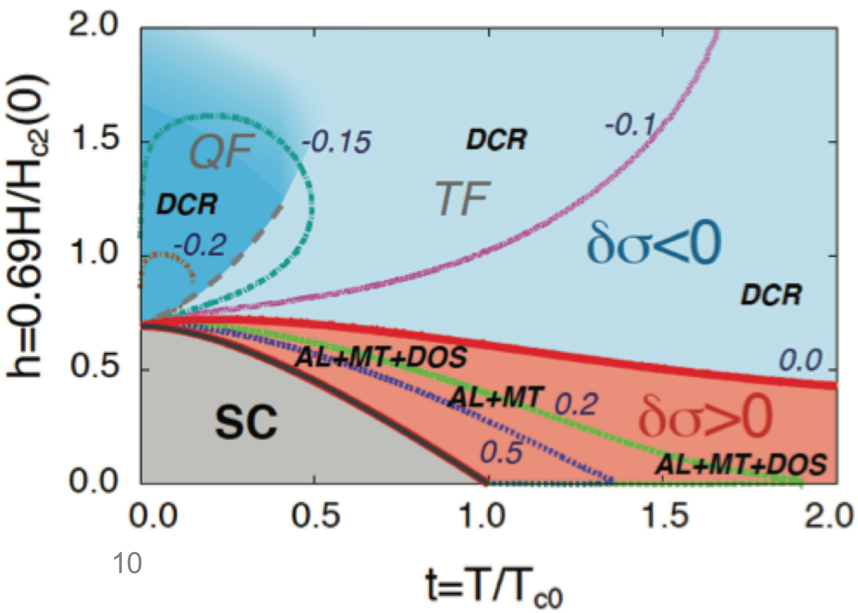
$$\sigma^{(\mathrm{DOS})}_{\mathrm{xx}(2)} \sim -\frac{2N_{(2)}e^2\tau}{m_{\mathrm{e}}} \sim -\frac{e^2}{\hbar}\ln\frac{1}{\epsilon}.$$

Negative, but has weaker temperature dependence as AL & MT, since opening of new transport channel dominates

- **Renormalization of the one-electron diffusion coefficient (DCR)**

$$\sigma^{(\mathrm{DCR})}_{\mathrm{xx}} \sim \frac{e^2}{\hbar}\ln\ln\frac{1}{T_{\mathrm{c0}}\tau} + O(\epsilon).$$

Not singular in ε close to T_{c0}, was neglected till recently, but becomes important far from T_{c0} at low temperatures



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GIANT NERNST-ETTINGSHAUSEN EFFECT AND CHEMICAL POTENTIAL: QUALITATIVE PICTURE

- Important: Uncompensated magnetization currents in non-homogeneously heated samples can play crucial role (Obraztsov, 1964).
- Thermodynamic part:

$$\nu_{(d)}^{(\mathrm{th})} = \frac{\sigma_{(d)}}{N_{(d)} c e^2} \left(\frac{d\mu_{(d)}}{dT} \right)$$

- For electron gas in normal metal with elastic electron scattering and $\mu(T) \approx \mu(0) - \pi k^2 T^2 / [12\mu(0)]$ gives

$$\nu_e = -\frac{\pi\tau}{6 m_e c} \left(\frac{k_B T}{\mu(0)} \right)$$

Sondheimer 1948

- Now use AL result and $N_{(d)}$, but need to define chemical potential of FCPs - since usual (Bose) systems with variable particle numbers have zero μ

CHEMICAL POTENTIAL OF FCPs

Is finite since Cooper pairs do not form an isolated system

- Creation of a FCP removes two quasiparticles from the fermionic subsystem
- μ for multicomponent system:

$$\mu_i = (\partial F^{(\mathrm{fl})}/\partial N_i)_{V,T,N_j}$$

j≠i, here i,j={fl,qp}

- Therefore $\mu^{(fl)}-2\mu^{(qp)}=0$ and not $\mu^{(fl)}=\mu^{(qp)}=0$
- $\mu^{(fl)}$ can be estimated as negative binding energy of FCPs: $\mu^{(fl)}=T_{c0}-T$ or using more accurately the fluctuation free energy near T_{c0} (in 2D)

$$F_{(2)}^{(\mathrm{fl})}(\epsilon) = -\frac{T_{\mathrm{c}0}}{4\pi\xi^2}\epsilon\ln\frac{1}{\epsilon} \quad \text{and} \quad N_{(2)}^{(\mathrm{fl})}(\epsilon) = \frac{1}{4\pi\xi^2}\ln\frac{1}{\epsilon}$$

$$\mu_{(2)}^{(\mathrm{fl})} = \left(\frac{\partial F_{(2)}^{(\mathrm{fl})}}{\partial N_{(2)}^{(\mathrm{fl})}}\right)_{V,T} = \frac{\partial F_{(2)}^{(\mathrm{fl})}/\partial\xi}{\partial N_{(2)}^{(\mathrm{fl})}/\partial\xi} = -T_{\mathrm{c}0}\epsilon$$

$$\nu_{(2)}^{(\mathrm{th}),(\mathrm{fl})} = -\frac{\sigma_{\mathrm{xx}(2)}^{(\mathrm{AL})}}{(2e)^2 N_{(2)}^{(\mathrm{fl})} c} = -\frac{\tau_{\mathrm{GL}}(\epsilon)}{m^* c} \sim -\frac{k_{\mathrm{B}}\xi^2}{c\hbar}\frac{1}{\epsilon}$$

Exceeding Sondheimer!

REFINEMENT OF NERNST COEFFICIENT IN GINZBURG-LANDAU REGION

Sondheimer, 1948

$$\nu_N = -\frac{\pi^2}{3}\frac{T}{mc}\frac{\partial\tau(\mu)}{\partial\mu} \sim \frac{\tau}{mc}\frac{T}{\mu}$$

Ullah, Dorsey, 1991

$$\nu_{\text{fl}} = -\frac{\pi\hbar\xi^2}{4c}\frac{T_c}{T-T_c}$$

Ussishkin, Sondhi, Huse, 2002

$$\nu_{\text{fl}} = -\frac{\pi\hbar\xi^2}{12c}\frac{T_c}{T-T_c}$$

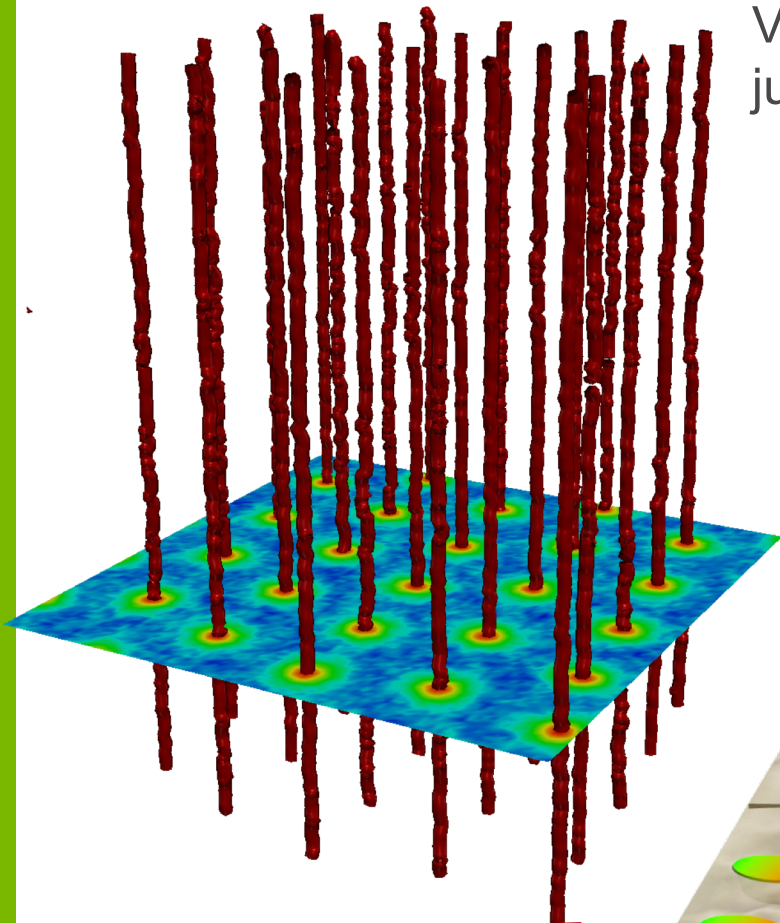
Serbyn, Skvortsov, Varlamov, Galitski, 2009

$$\nu_{\text{fl}} = -\frac{\pi\hbar\xi^2}{12c}\frac{T_c}{T\ln T/T_c}$$

QUANTUM FLUCTUATIONS NEAR $H_{C2}(0)$: QUALITATIVE PICTURE

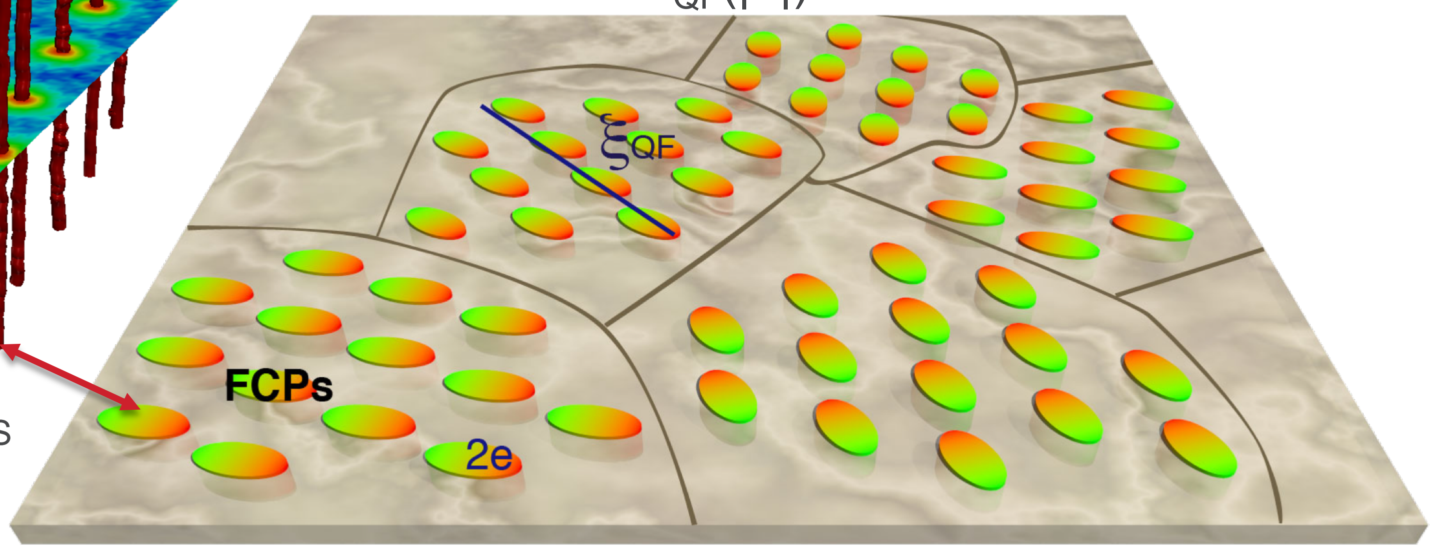
Vicinity of T_c	Vicinity of $H_{c2}(0)$
$\tau_{GL} = \hbar / \Delta E_{GL}$	$\tau_{QF} = \hbar / \Delta E_{QF}$
$\Delta E_{GL} \sim k_B \left(T - T_c\right) = k_B T_c \varepsilon$	$\Delta E_{QF} \sim \hbar\omega\left(H\right) - \hbar\omega\left(H_{c2}\right) \sim \Delta_{BCS} \bar{h}$
$\varepsilon = \left(T - T_c\right) / T_c << 1$	$\bar{h} = \left(H - H_{c2}(0)\right) / H_{c2}(0)$
$\tau_{GL} = \pi\hbar T_c^{-1} / 8 k_B \varepsilon$	$\tau_{QF} = \hbar \Delta_{BCS}^{-1} / \bar{h}$
$\xi_{GL} = \sqrt{D\tau_{GL}} = \xi_{BCS} / \sqrt{\varepsilon}$	$\xi_{QF} = \sqrt{D\tau_{QF}} = \xi_{BCS} / \sqrt{\bar{h}}$

FCPs vs VORTICES – ROTATING FCP CLUSTERS



Vortices in the SC phase, just below $H_{c2}(0)$

FCP clusters above $H_{c2}(0)$
size: $\xi_{QF}(|\tilde{h}|)$
life time: $\tau_{QF}(|\tilde{h}|)$



FCP & vortex size: ξ_{BCS}

QF ABOVE $H_{C2}(0)$

At T=0 and $H>H_{c2}(0)$, fluctuation corrections change considerably:

- Free rotation of FCP → no direct contribution to transverse transport: AL contribution vanishes [probability for hopping ~ T^2]

$$\sigma_{xx}^{(AL)} \sim \frac{e^2}{\hbar}\left(\frac{T}{T_{c0}}\right)^2 \frac{1}{\tilde{h}^2}.$$

- Anomalous MT [no phase coherence] and
- DOS [field suppresses fluctuation gap in one-electron spectrum] contributions become zero as well
- All corrections come from DCR:

$$\sigma_{xx}^{(DCR)} \sim -\frac{e^2}{\Delta_{BCS}}\int_0^{\Delta_{BCS}} \frac{d\omega}{\tilde{h}+\hbar\omega/\Delta_{BCS}} \sim -\frac{e^2}{\hbar}\ln\frac{1}{\tilde{h}}.$$

- Contribution of QFs to Nernst follows from $\mu^{(QF)}=-\Delta_{BCS}\tilde{h}(t)$

$$\nu^{(QF)} \sim \frac{\tau_{QF}}{m^* c}\left(\frac{d\mu^{(QF)}}{dT}\right) \sim -\frac{k_B\xi_{BCS}^2}{c\hbar}\left(\frac{k_B T}{\Delta_{BCS}}\right)\frac{1}{\tilde{h}}.$$

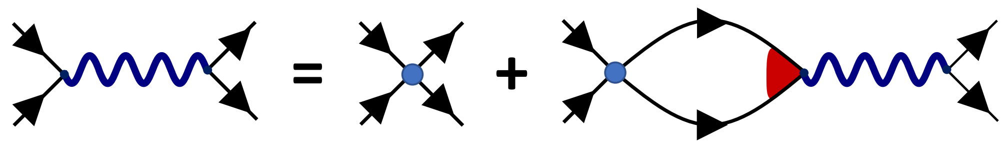
BASIC ELEMENTS OF THE MICROSCOPIC DESCRIPTION

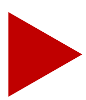
Staring point: BCS model in diffusive electron scattering regime (diffusion constant $\mathcal{D}$)

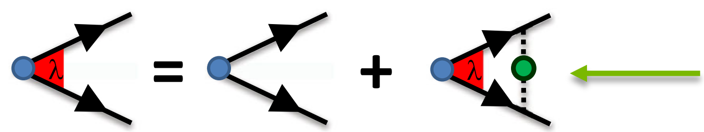
Using Matsubara diagrammatic technique

- fluctuation propagator $L(q,\Omega_k)$ [e-e interaction in Cooper channel] described by Dyson equation:

$$L^{-1}(\mathbf{q},\Omega_k) = -g^{-1} + \langle\!\langle \Pi(\mathbf{q},\Omega_k)\rangle\!\rangle_{\text{imp}}$$



- With single electron Green's function 
- e-e interaction 
- Cooperon  defined by



impurity

given in Landau representation:

$$\lambda_n(\varepsilon_1,\varepsilon_2) = \frac{\tau^{-1}\theta(-\varepsilon_1\varepsilon_2)}{|\varepsilon_1-\varepsilon_2| + \Omega_{\text{H}}(n+1/2) + \tau_\varphi^{-1}}$$

$$\Omega_{\text{H}} = 4\mathcal{D}eH/c$$

FLUCTUATION PROPAGATOR

This finally results in

$$L_n^{-1}(\Omega_k) = -\rho_e \left[\ln \frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{|\Omega_k| + \Omega_H (n+1/2)}{4\pi T} \right) - \psi\left(\frac{1}{2}\right) \right]$$

valid for $H/H_{c2}(0) \ll \min\left\{ (T_{c0}\tau)^{-1}, \varepsilon_F \tau \right\}$ and $T \ll \min\{\tau^{-1}, \omega_D\}$

In dimensionless units

$$t = \frac{T}{T_{c0}} \qquad h = \frac{H}{\tilde{H}_{c2}(0)} = \frac{\pi^2}{8\gamma_E} \frac{H}{H_{c2}(0)} = 0.69 \frac{H}{H_{c2}(0)} \qquad \tilde{H}_{c2}(0) = \frac{\Phi_0}{2\pi\xi^2}$$

$$L_n^{-1}(\Omega_k) = -\rho_e \mathcal{E}_n(t, h, |k|)$$

with

$$\mathcal{E}_n(t,h,x) \equiv \ln t + \psi\left[\frac{x+1}{2} + \frac{4h}{\pi^2 t}\left(n + \frac{1}{2}\right)\right] - \psi\left(\frac{1}{2}\right)$$

TRANSPORT COEFFICIENTS

Standard Kubo formalism

- Current:

$$j_\alpha = -\int Q_{\alpha\gamma}(\mathbf{r}, \mathbf{r}', t, t')\mathbf{A}_\gamma(\mathbf{r}', t')d\mathbf{r}'dt'.$$

- Conductivity:

$$\sigma^{(\mathrm{fl})}(T, H) = -\lim_{\omega\to 0} \frac{\mathrm{Im}Q^{(\mathrm{fl})}(\omega, T, H)}{\omega}.$$

- Nernst (w/o magnetization):

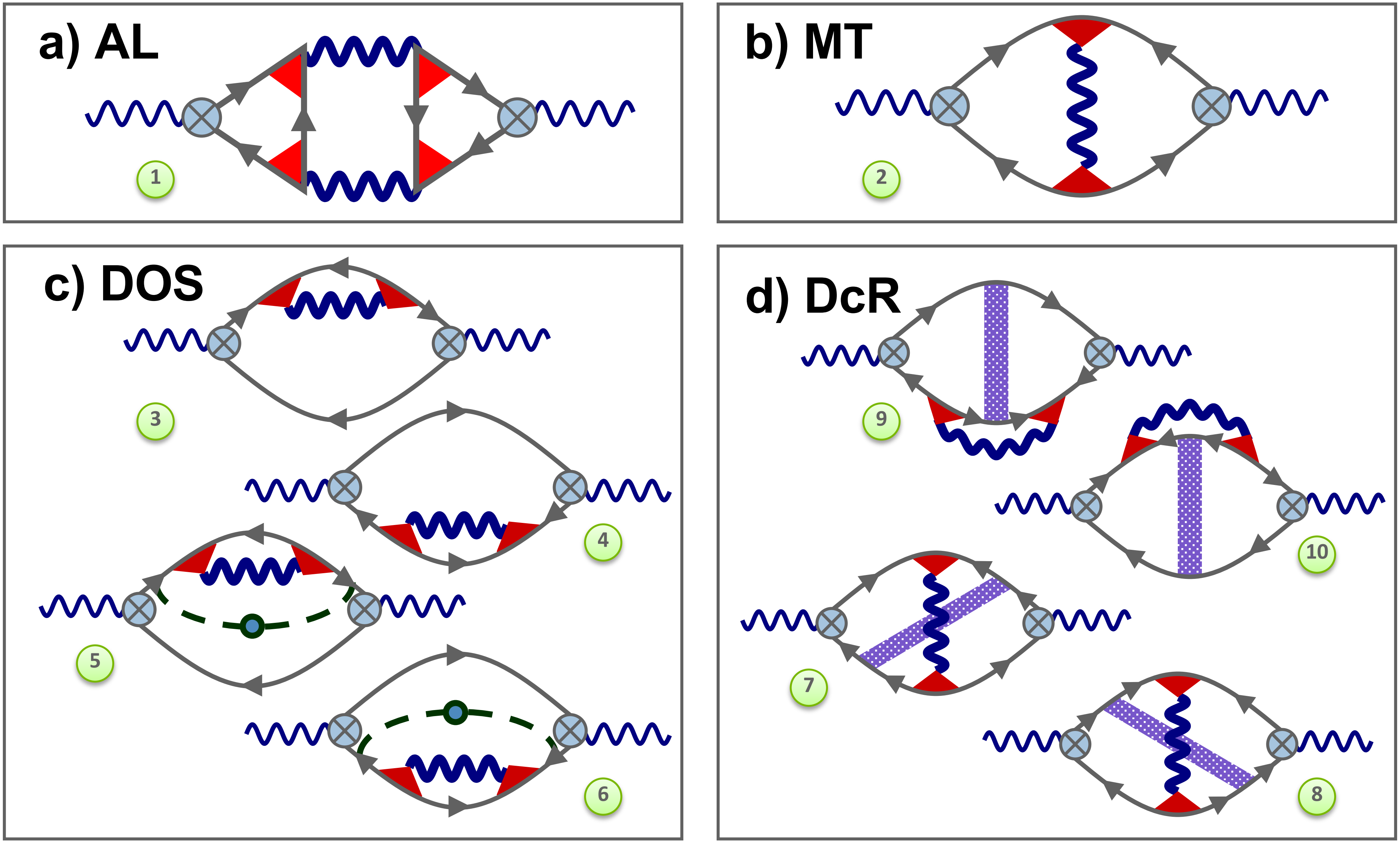
$$\tilde{\gamma}^{\alpha\delta} = -\lim_{\omega\to 0} \frac{\mathrm{Im}\tilde{Q}^{R}_{\alpha\delta}(-i\omega + 0)}{\omega}$$

- Tunnel current:

$$I^{(\mathrm{fl})}(V) = -e\mathrm{Im}K^R(\omega_k \to -ieV)$$

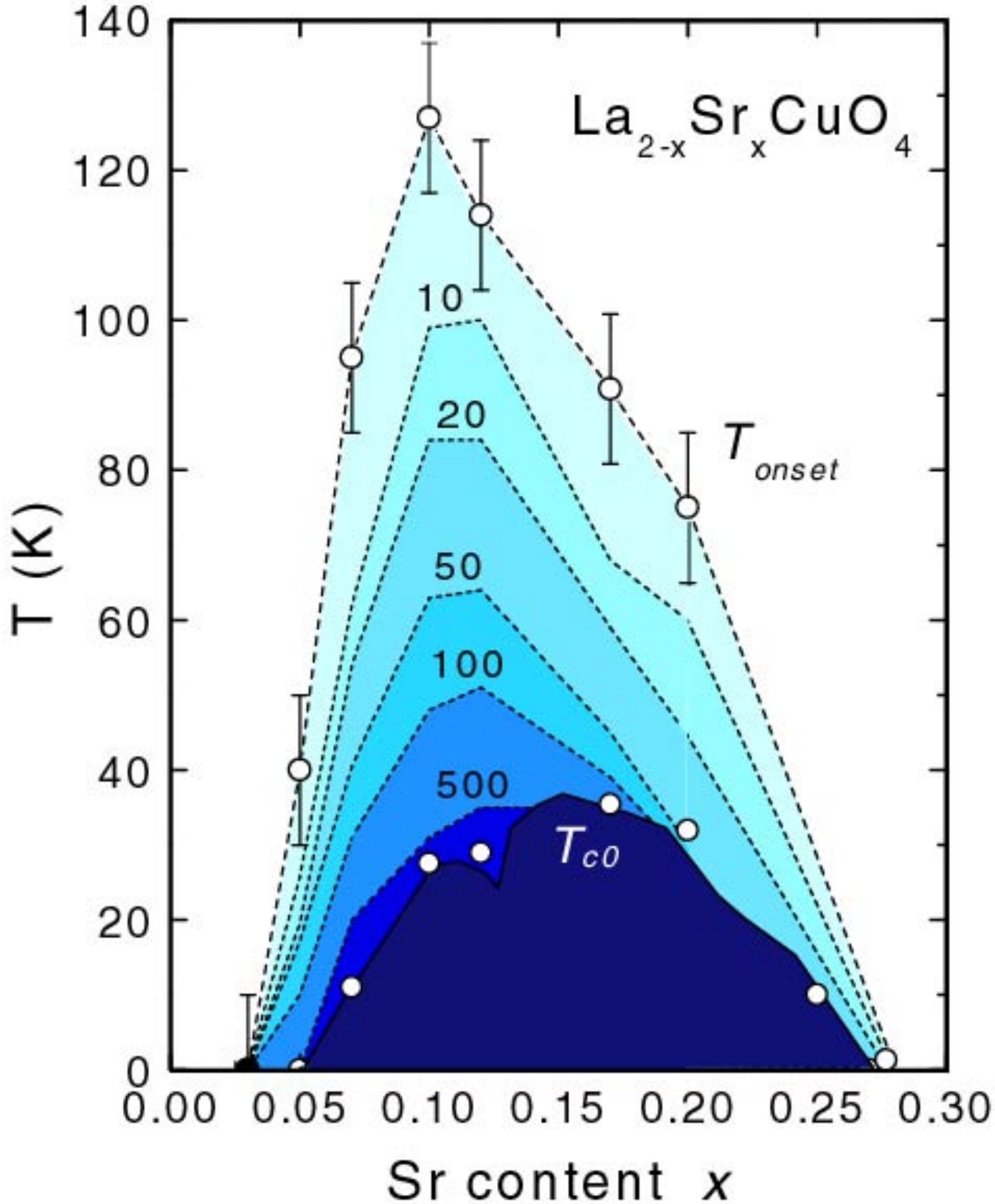
$$K(\omega_k) = 4T\sum_{\varepsilon_n}\sum_{\mathbf{q},\mathbf{p}} |T_{\mathbf{p},\mathbf{q}}|^2 G_{\mathrm{L}}(\mathbf{p}, \varepsilon_n + \omega_k)G_{\mathrm{R}}(\mathbf{q}, \varepsilon_n)$$

DIAGRAMMATIC DESCRIPTION OF SF CONTRIBUTIONS (HERE CONDUCTIVITY)



NERNST-ETTINGSHAUSEN EFFECT: EXPERIMENTS

GIANT NERNST SIGNAL IN CUPRATES



Phase diagram of LSCO showing Nernst coefficient (in nV/KT)

- between T_{c0} and T_{onset} (from x=0.03 to 0.26)
- peak at x=0.1
- T_{c0}~30K

Wang, Li, Ong, PRB **73,** 024510 (2006)

GAUSSIAN THEORY FOR UNDERDOPED CUPRATES

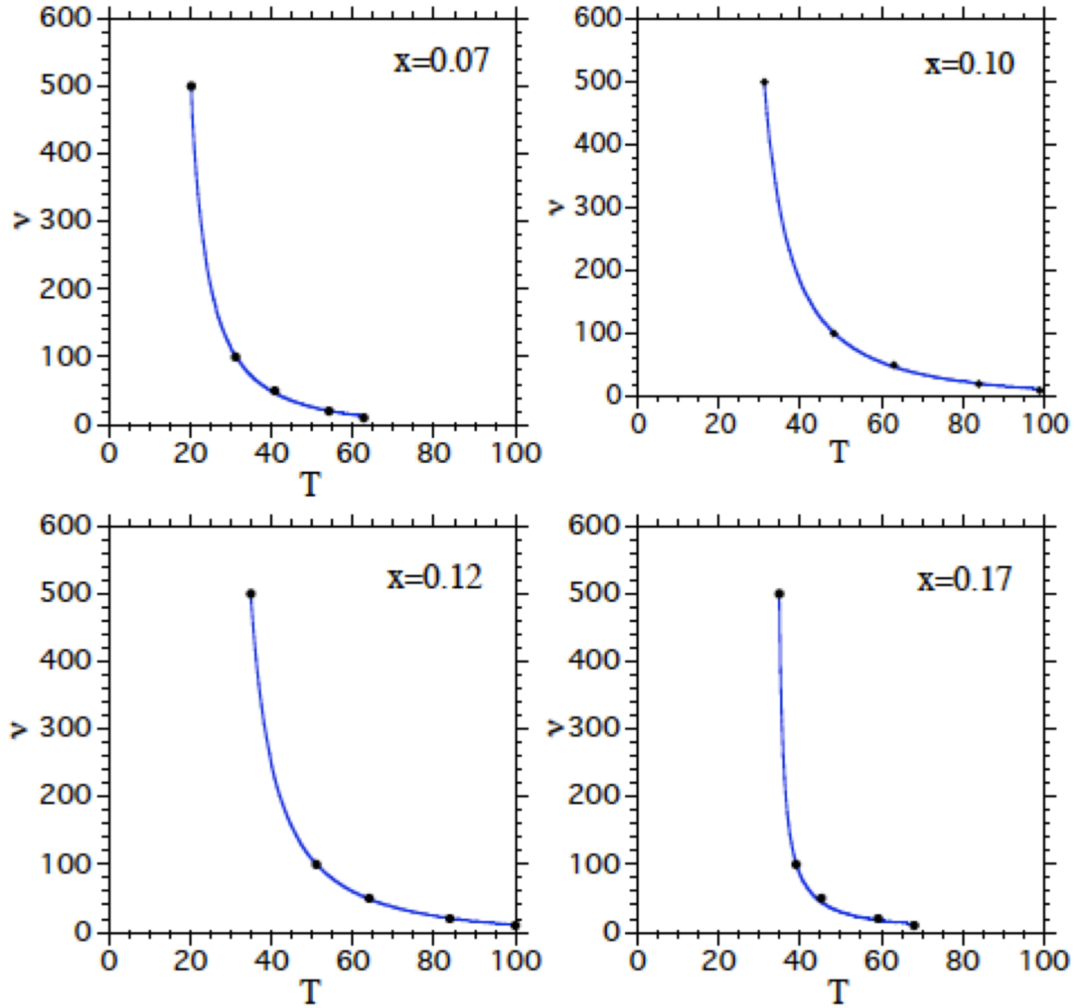
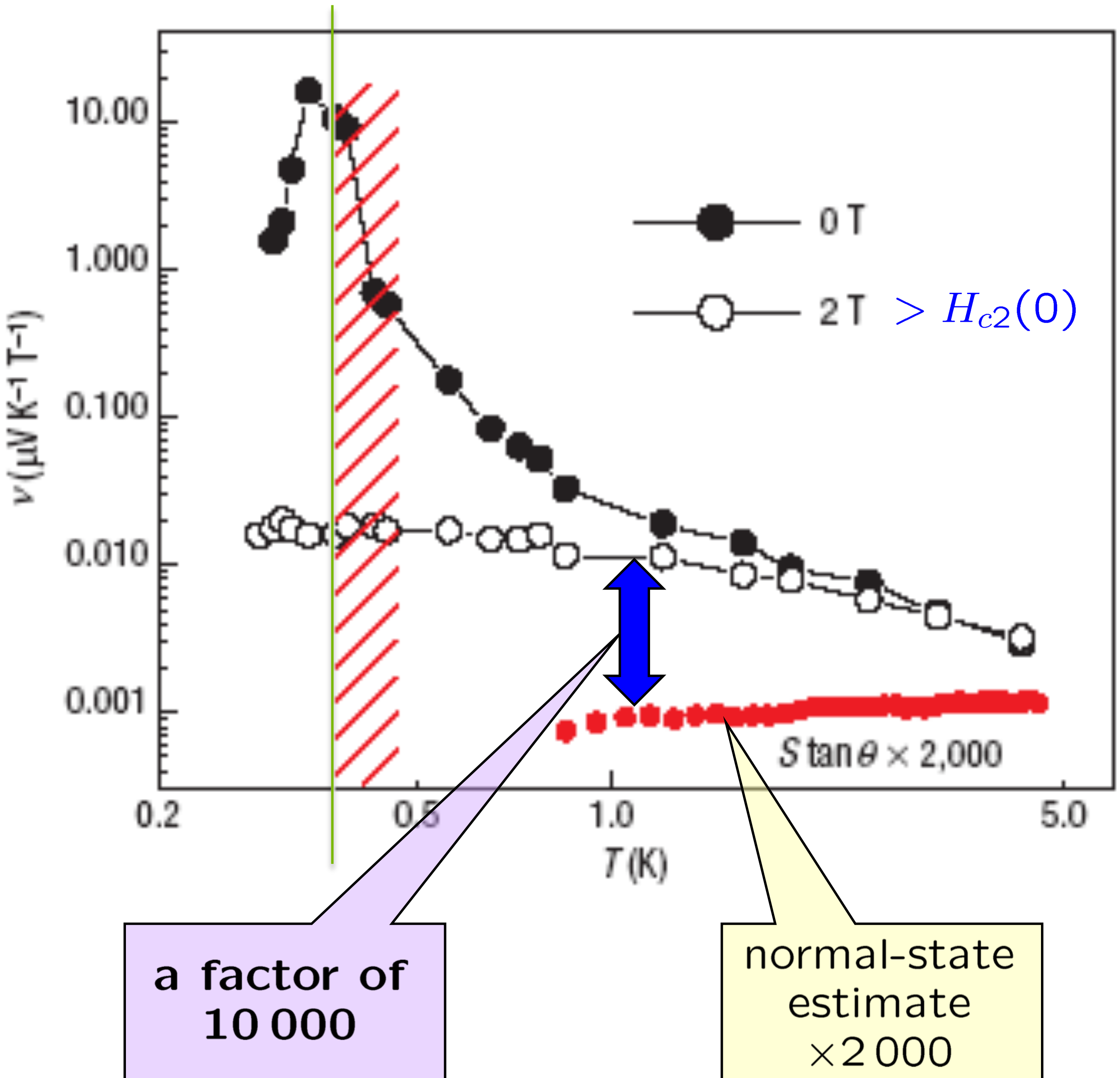


FIG. 2: Nernst signal, ν, versus T for $La_{2-x}Sr_xCuO_4$ for four different values of the doping, x.[4] The curve for x=0.17 is a fit to the gaussian expression for $\bar{\alpha}_{xy}$. The other curves include the vertex correction as described in this paper.

NERNST EFFECT IN CONVENTIONAL SUPERCONDUCTOR



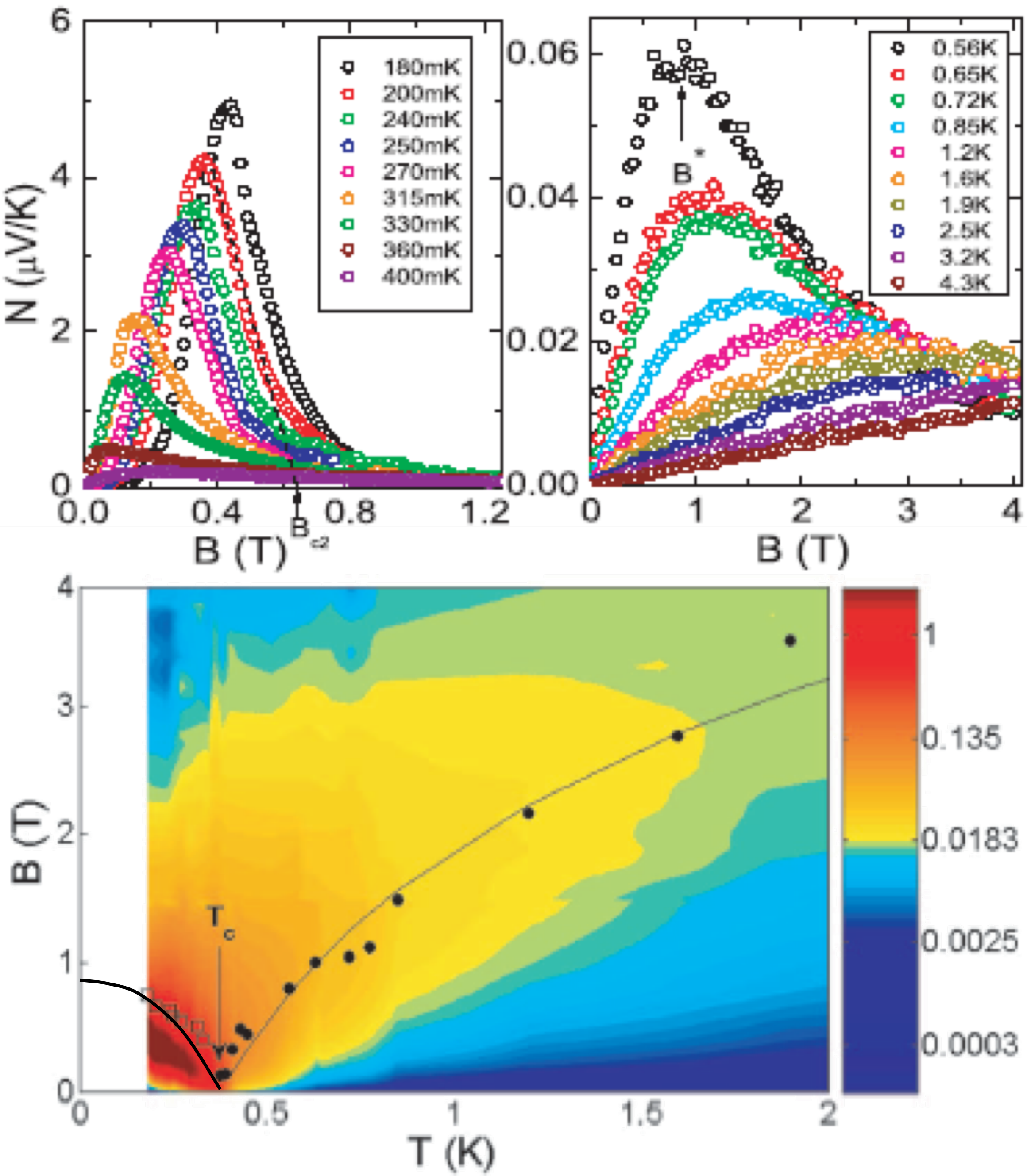
A. Pourret et al., Nature Phys. (2006); Phys. Rev. B (2007)

$Nb_{0.15}Si_{0.85}$ film

thickness
$d = 35$ nm

$T_c = 0.38$ K

$R_\square = 350$ Ohm



Nernst signal

$$\mathfrak{N} = \nu H$$

$Nb_{0.15}Si_{0.85}$ film

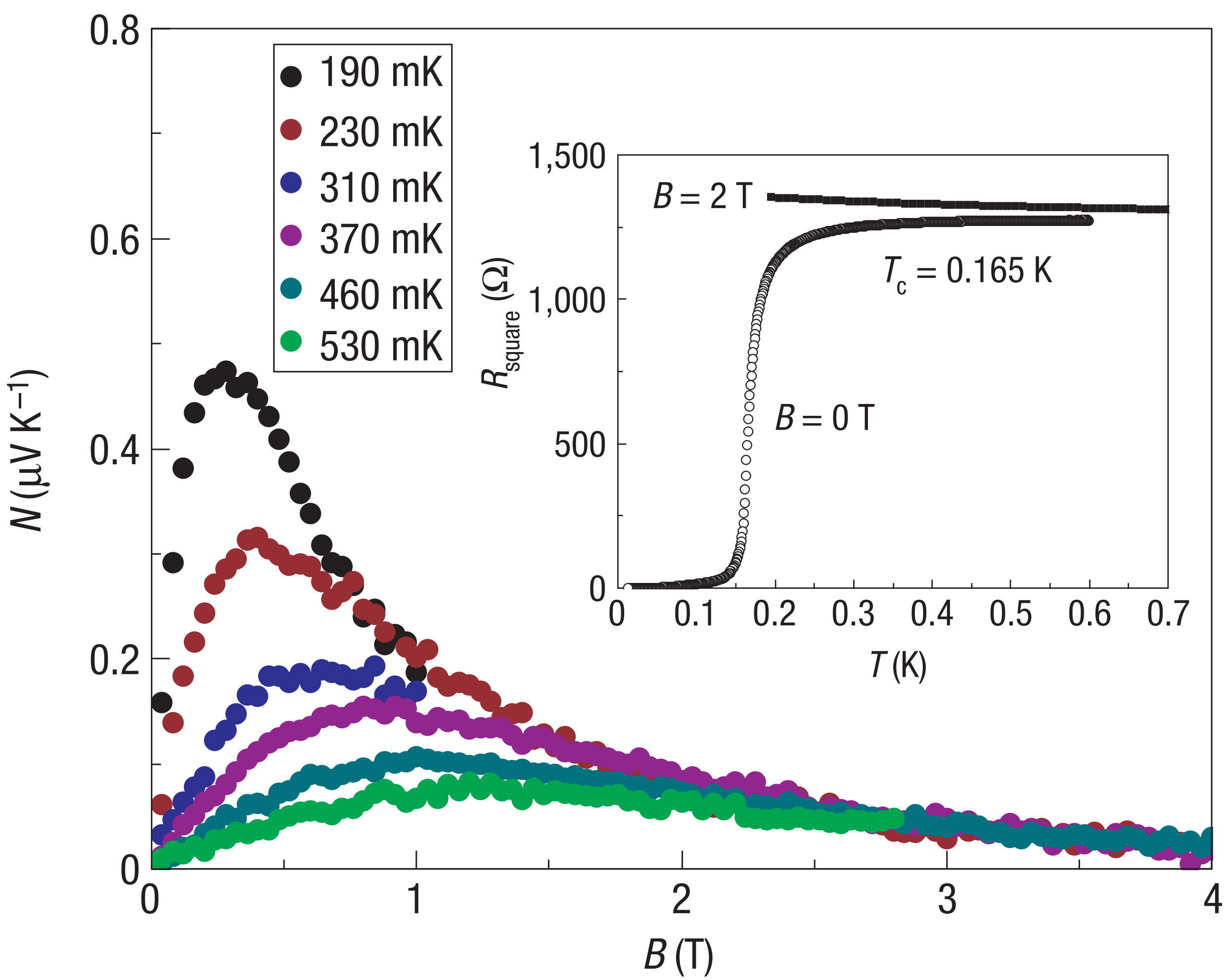
A. Pourret et al., Nature Phys. (2006); Phys. Rev. B (2007)

thickness
$d = 35$ nm

$T_c = 0.38$ K

$R_\square = 350$ Ohm

NERNST SIGNAL



A. Pourret et al., Nature Phys. (2006); Phys. Rev. B (2007)

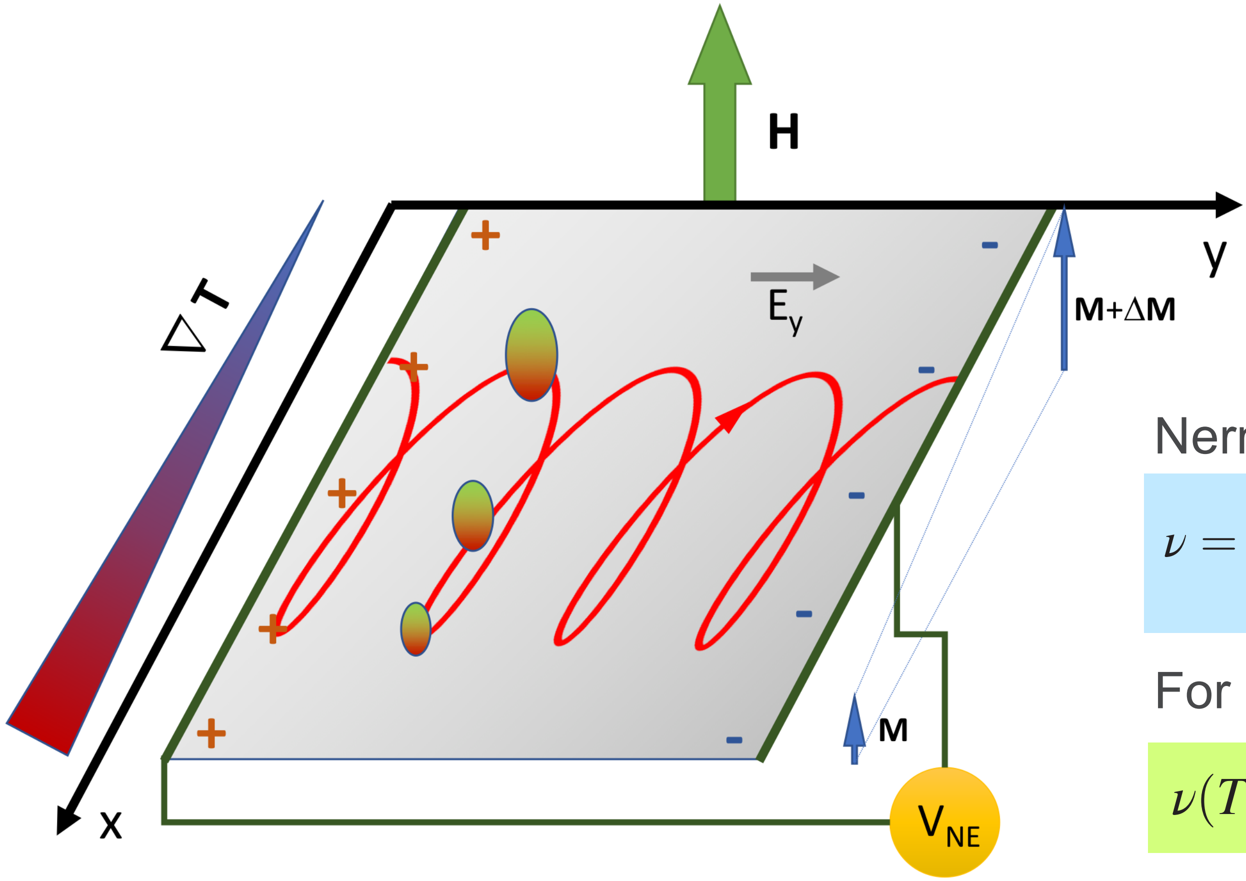
$Nb_{0.15}Si_{0.85}$ film

Sample 1:
d=12.5nm
T_c=0.165K

FLUCTUATION NERNST-ETTINGSHAUSEN EFFECT ABOVE THE TRANSITION LINE $H_{C2}(T)$

FLUCTUATION NERNST-ETTINGSHAUSEN (NE) EFFECT

Superconductor placed in magnetic field and temperature gradient



Potential difference V_{NE}

Nernst coefficient:

$$\nu = \frac{E_y}{(-\nabla_x T)H} = \frac{1}{H}\frac{\beta^{xy}\sigma_{xx} - \beta^{xx}\sigma_{xy}}{(\sigma_{xx})^2 + (\sigma_{xy})^2}.$$

For small Hall conductivity:

$$\nu(T,H) = R_{\square}\beta^{xy}(T,H)/H$$

Sheet resistance of the film, $(\sigma_{xx})^{-1}$

Transport currents:

$$\mathbf{j}^{(e)}_{\mathrm{tr},\alpha} = \sigma^{\alpha\delta}(\mathbf{H})\mathbf{E}_\delta + \beta^{\alpha\delta}(\mathbf{H})\nabla_\delta T,$$

$$\mathbf{j}^{(h)}_{\mathrm{tr},\alpha} = \gamma^{\alpha\delta}(\mathbf{H})\mathbf{E}_\delta - \kappa^{\alpha\delta}(\mathbf{H})\nabla_\delta T,$$

MAGNETIZATION CURRENTS AND ONSAGER RELATIONS

Magnetization current: from $\mathbf{j}^{\mathrm{mag}} = \frac{c}{4\pi}\nabla \times \mathbf{B}$ with $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$ and T-gradient follows:

$$j_{\mathrm{y}}^{\mathrm{mag}} = -c(dM_{\mathrm{z}}/dT)\nabla_{\mathrm{x}}T$$

and we can write

$$\beta^{\alpha\delta}(\mathbf{H}) = \tilde{\beta}^{\alpha\delta}(\mathbf{H}) + \epsilon^{\alpha\beta\zeta} c \frac{dM_\zeta}{dT}$$

for the open circuit (no y-transport current) this is compensated by $E_{\mathrm{y}}^{\mathrm{NE}} = R_\Box j_{\mathrm{y}}^{\mathrm{mag}}$

the transport heat current is also affected by $\mathbf{M}$: $\mathbf{j}_{\mathrm{M}}^{(h)} = c\mathbf{M} \times \mathbf{E}$

$$\gamma^{\alpha\delta} = \tilde{\gamma}^{\alpha\delta} + \epsilon^{\alpha\delta\zeta} c M_\zeta / T.$$

(reciprocal) Onsager relations fulfilled

$$\tilde{\gamma}^{\alpha\delta}(H) + \epsilon^{\alpha\delta\zeta}\frac{cM_\zeta(H)}{T} = -T\left[\tilde{\beta}^{\alpha\delta}(-\mathbf{H}) + \epsilon^{\alpha\delta\zeta} c \frac{dM_\zeta(-\mathbf{H})}{dT}\right]$$

MICROSCOPIC EXPRESSION FOR THE NE COEFFICIENT

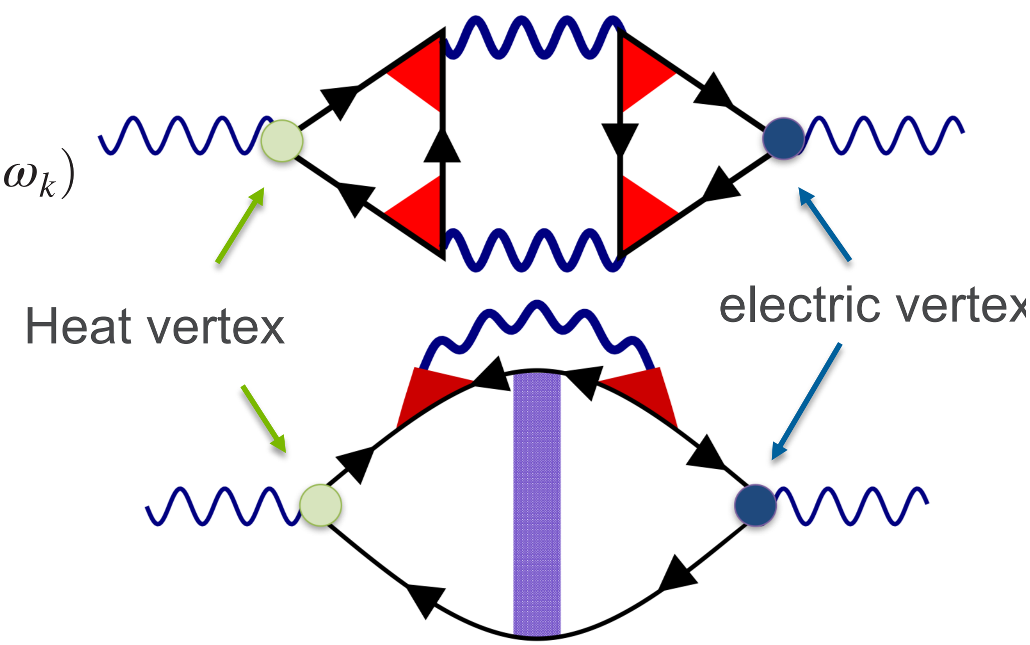
It turns out to be more straightforward to calculate:

$$\nu(T,H) = -R_{\square}\frac{\tilde{\gamma}^{\mathrm{xy}}(H) + cM_z(H)/T}{TH}$$

And the magnetization current free tensor follows from Kubo formalism:

$$\tilde{\gamma}^{\alpha\delta} = -\lim_{\omega\to 0}\frac{\mathrm{Im}\tilde{Q}^{R}_{\alpha\delta}(-i\omega+0)}{\omega}$$

The electric-heat-current correlator $\tilde{Q}_{\alpha\delta}(\omega_k)$ is calculated at Bosonic Matsubara frequencies $\omega_k=2\pi k T$ and then analytically continued to real frequencies.
Diagrams are the same as for current, but with heat and electric vertices: MT contribution vanishes



COMPLETE FLUCTUATION NERNST COEFFICIENT

Calculation of all diagrams yields

$$
\begin{aligned}
\nu^{(\mathrm{fl})} = \frac{\beta_0 R_\square}{8H} \Bigg[\eta \sum_{m=0}^{M_t} (m+1) \sum_{k=0}^{\infty} \Bigg\{ \left(\frac{3}{\mathcal{E}_m} + \frac{1}{\mathcal{E}_{m+1}} \right) (\mathcal{E}'_m - \mathcal{E}'_{m+1}) + [\eta(2m+1)+k] \frac{\mathcal{E}''_m}{\mathcal{E}_m} + [\eta(2m+3)+k] \frac{\mathcal{E}''_{m+1}}{\mathcal{E}_{m+1}} \Bigg\} \\
+ 4\pi^2 \sum_{m=0}^{M_t} (m+1) \int_{-\infty}^{\infty} \frac{dx}{\sinh^2 \pi x} \Bigg\{ \frac{\eta \mathrm{Im}\mathcal{E}_m \mathrm{Im}(\mathcal{E}_m + \mathcal{E}_{m+1}) + [\eta(m+1/2)\mathrm{Im}\mathcal{E}_m + x\mathrm{Re}\mathcal{E}_m] \mathrm{Im}(\mathcal{E}_{m+1} + \eta\mathcal{E}'_m - \mathcal{E}_m)}{|\mathcal{E}_m|^2} \\
+ \frac{\eta \mathrm{Im}\mathcal{E}_{m+1} \mathrm{Im}(\mathcal{E}_m + \mathcal{E}_{m+1}) + [\eta(m+3/2)\mathrm{Im}\mathcal{E}_{m+1} + x\mathrm{Re}\mathcal{E}_{m+1}] \mathrm{Im}(\mathcal{E}_{m+1} + \eta\mathcal{E}'_{m+1} - \mathcal{E}_m)}{|\mathcal{E}_{m+1}|^2} + 4x \mathrm{Im} \ln \frac{\mathcal{E}_m}{\mathcal{E}_{m+1}} \\
- 2 \frac{\mathrm{Im}(\mathcal{E}_m + \mathcal{E}_{m+1})(\mathrm{Im}\mathcal{E}_m \mathrm{Im}\mathcal{E}_{m+1} + \mathrm{Re}\mathcal{E}_m \mathrm{Re}\mathcal{E}_{m+1})}{|\mathcal{E}_{m+1}|^2 |\mathcal{E}_m|^2} \left[\eta \left(m + \frac{3}{2} \right) \mathrm{Im}\mathcal{E}_{m+1} - \eta \left(m + \frac{1}{2} \right) \mathrm{Im}\mathcal{E}_m + x\mathrm{Re}(\mathcal{E}_{m+1} - \mathcal{E}_m) \right] \Bigg\} \Bigg]
\end{aligned}
$$

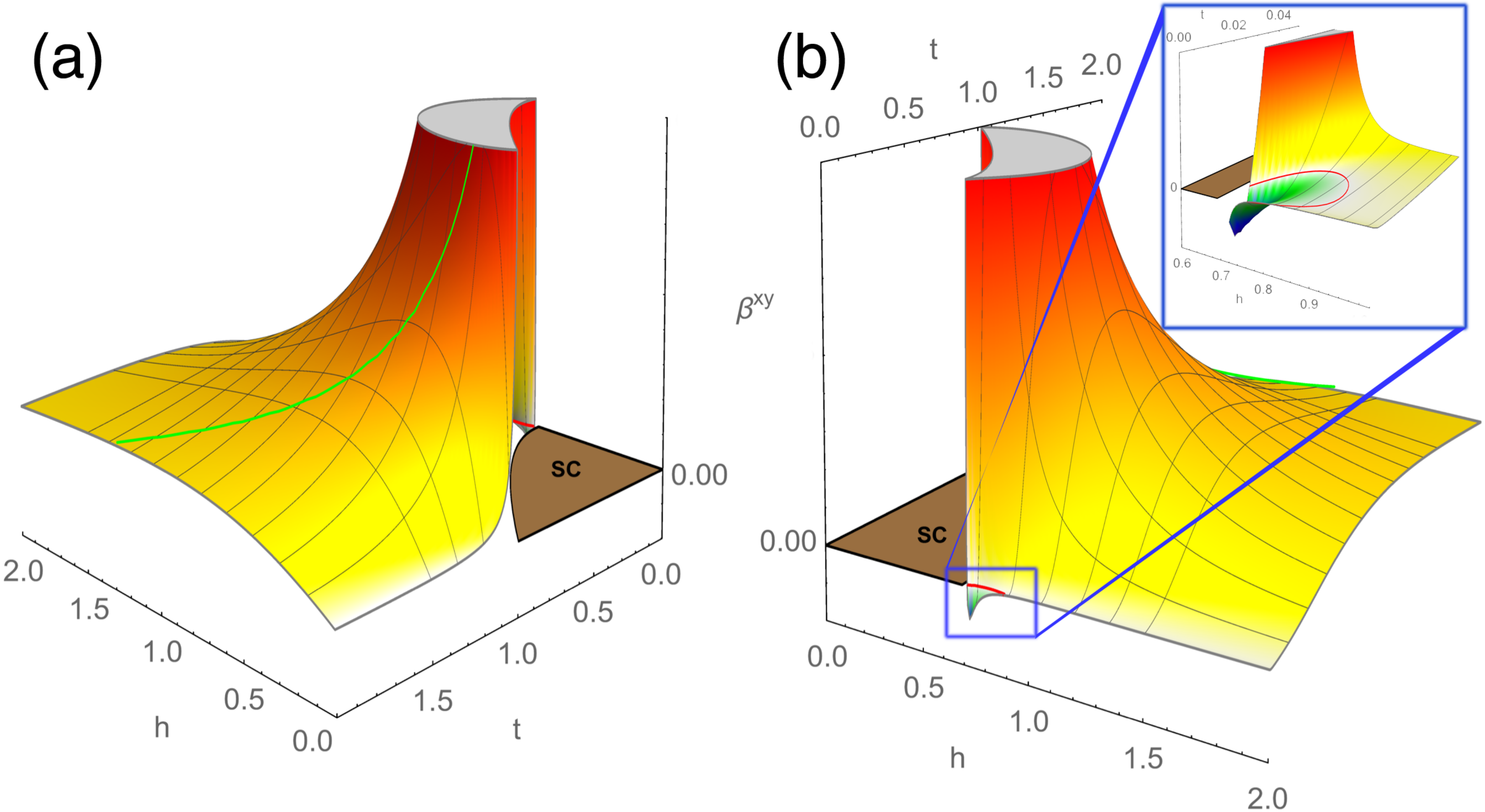
with $\eta = 4h/(\pi^2\ t)$

$$\beta_0 = k_{\mathrm{B}} e/\pi\hbar = 6.68\ \mathrm{nA/K}$$ quantum of thermoelectric conductance

$$M_t = (tT_{c0}\tau)^{-1}$$

$$\mathcal{E}_n(t,h,x) \equiv \ln t + \psi\left[\frac{x+1}{2} + \frac{4h}{\pi^2\ t}\left(n+\frac{1}{2}\right)\right] - \psi\left(\frac{1}{2}\right)$$

SURFACE PLOT OF THE NERNST COEFFICIENT



Green line: Ghost field line

Red line: $\nu^{(fl)}=0$

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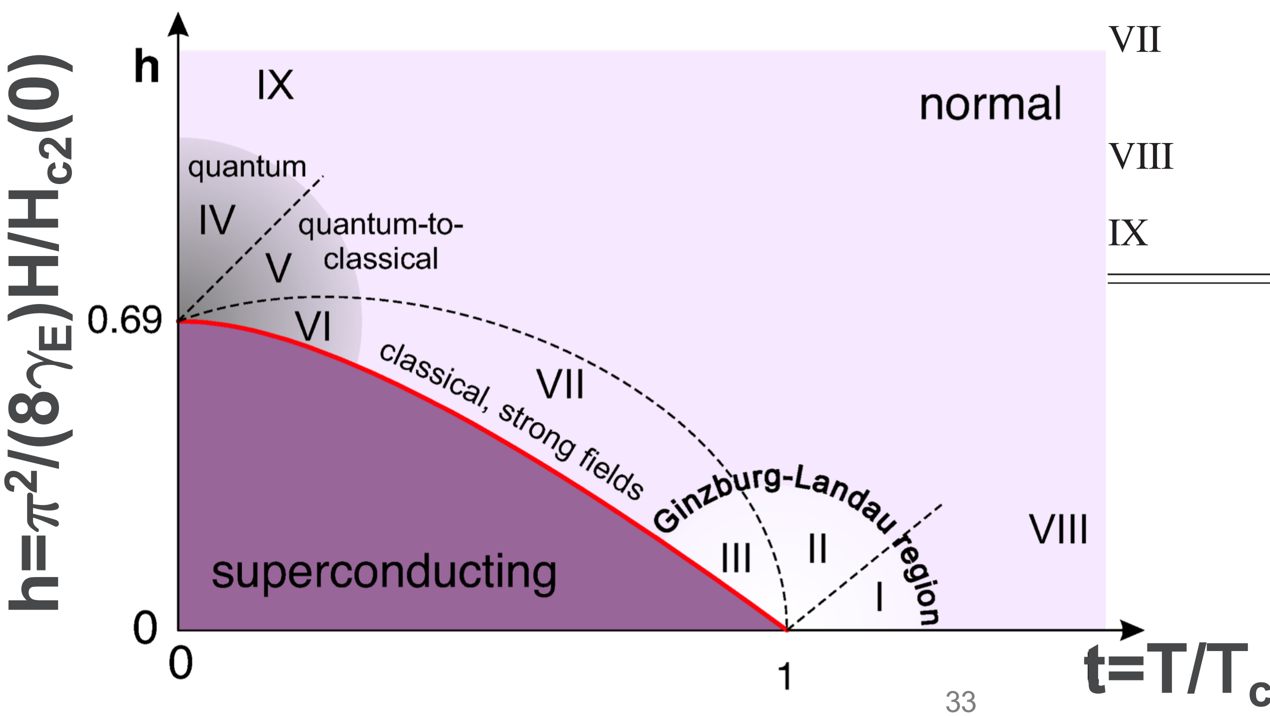
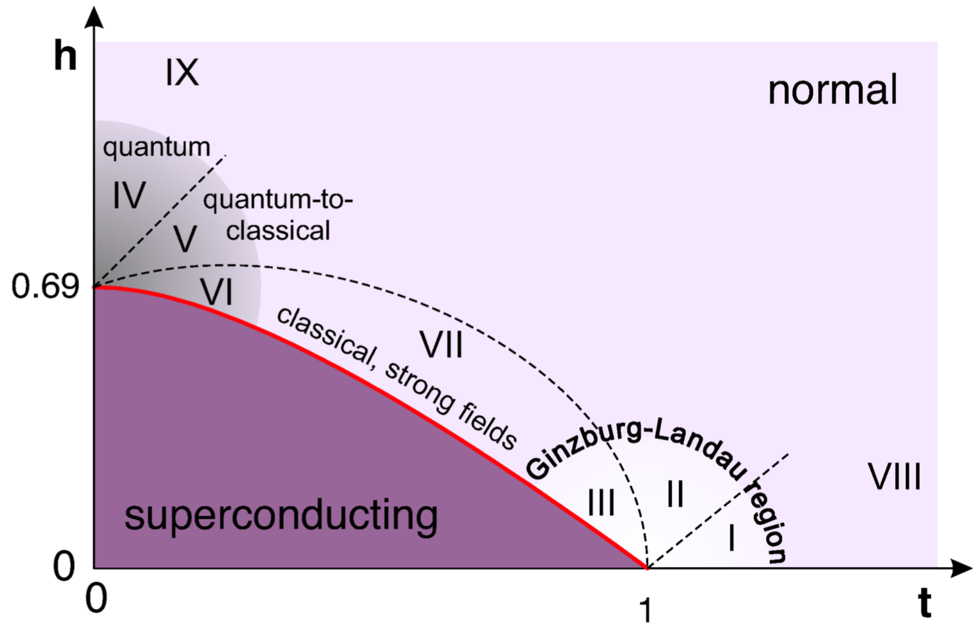
ASYMPTOTIC BEHAVIOR: DIFFERENT REGIONS AND LIMITING CASES

domain	t and h range	description
I	$h=0,\ \epsilon \ll 1$	zero field, near T_{c0}
II	$\epsilon \ll h \ll 1$	near T_{c0}, above the mirror reflected h_{c2}-line
III	$h-h_{c2}(t) \ll 1,\ \epsilon \ll 1$	near h_{c2}-line
I − III	$h \ll 1,\ \epsilon \ll 1$	GL region
IV	$t \ll \tilde{h}$	region of quantum fluctuations
V	$t^2/\ln(1/t) \ll \tilde{h} \ll t \ll 1$	quantum-to-classical
VI	$\tilde{h} \ll t^2/\ln(1/t) \ll 1$	classical, near $h_{c2}(t \ll 1)$
VII	$\tilde{h} \lesssim t \ll h_{c2}(t)$	classical, strong fields
VIII	$\ln t \gtrsim 1, h \ll t$	high temperatures
IX	$h \gg \max\{1,t\}$	high magnetic fields



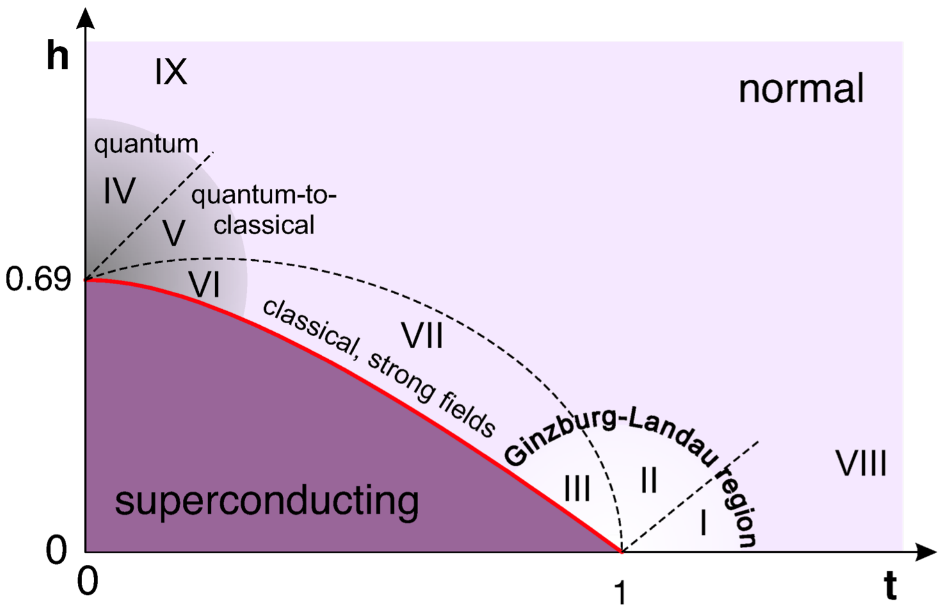
Domain	$\frac{H}{\beta_0 R_\square}\nu^{(\mathrm{fl})}$
I	$\frac{2eH\xi_{\mathrm{GL}}^2(T)}{3c}=\frac{2eH\xi^2}{3c}\frac{1}{\epsilon}$
II	$1-(\ln 2)/2$
III	$\frac{1}{\epsilon+h}$
IV	$-\frac{2\gamma_{\mathrm{E}}}{9}\frac{t}{\tilde{h}}$
V	$\ln\frac{t}{\tilde{h}}$
VI	$\frac{8\gamma_{\mathrm{E}}^2}{3}\frac{t^2}{\tilde{h}(t)}$
VII	$\frac{1}{\tilde{h}(t)}\left[1+\frac{2h_{c2}(t)}{\pi^2 t}\frac{\psi''\left(\frac{1}{2}+\frac{2h_{c2}(t)}{\pi^2 t}\right)}{\psi'\left(\frac{1}{2}+\frac{2h_{c2}(t)}{\pi^2 t}\right)}\right]$
VIII	$\frac{4e\xi^2}{3\pi^2}\frac{H}{c}\frac{1}{t\ln t}$
IX	$\frac{\pi^2}{48}\frac{t}{h\ln h}$

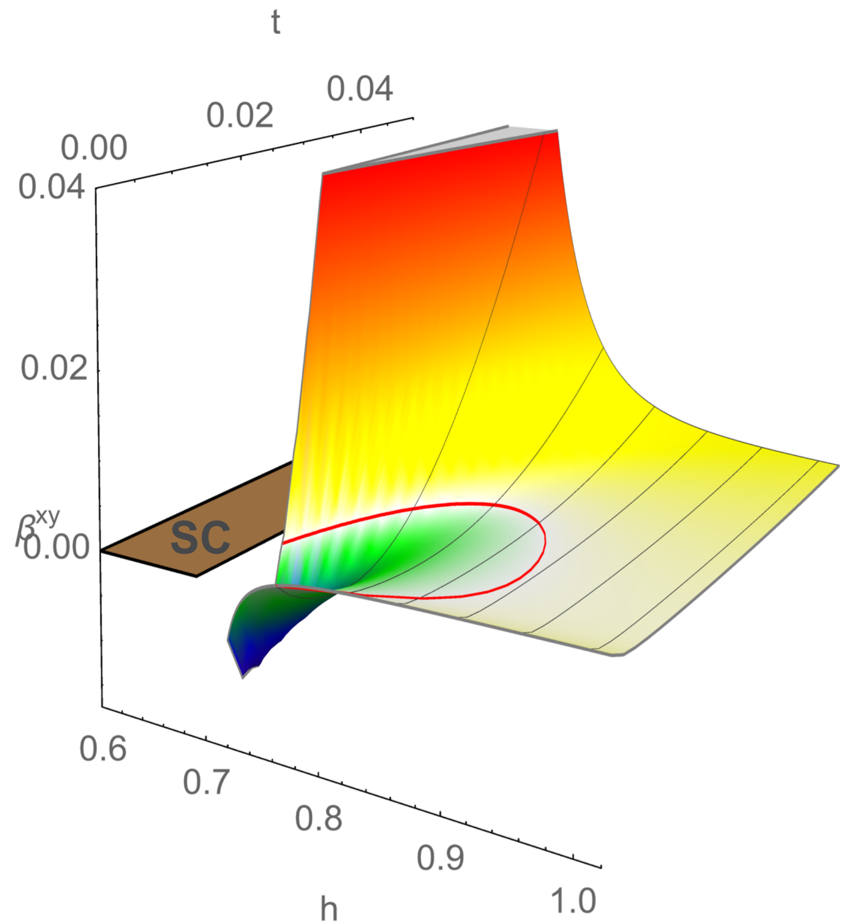
GINZBURG-LANDAU REGION NEAR T_{C0}



- Domains I-III: Thermal fluctuations: Only the AL contributions is essential taking magnetization currents into account
- For small fields, $h \ll \varepsilon$, (domain I) the numerical prefactor differs between GL and microscopic theory – origin still unclear
- In domains II ($h \gg \varepsilon$) & III [near $H_{c2}(T)$], the NE signal diverges

NEAR $H_{C2}(0)$

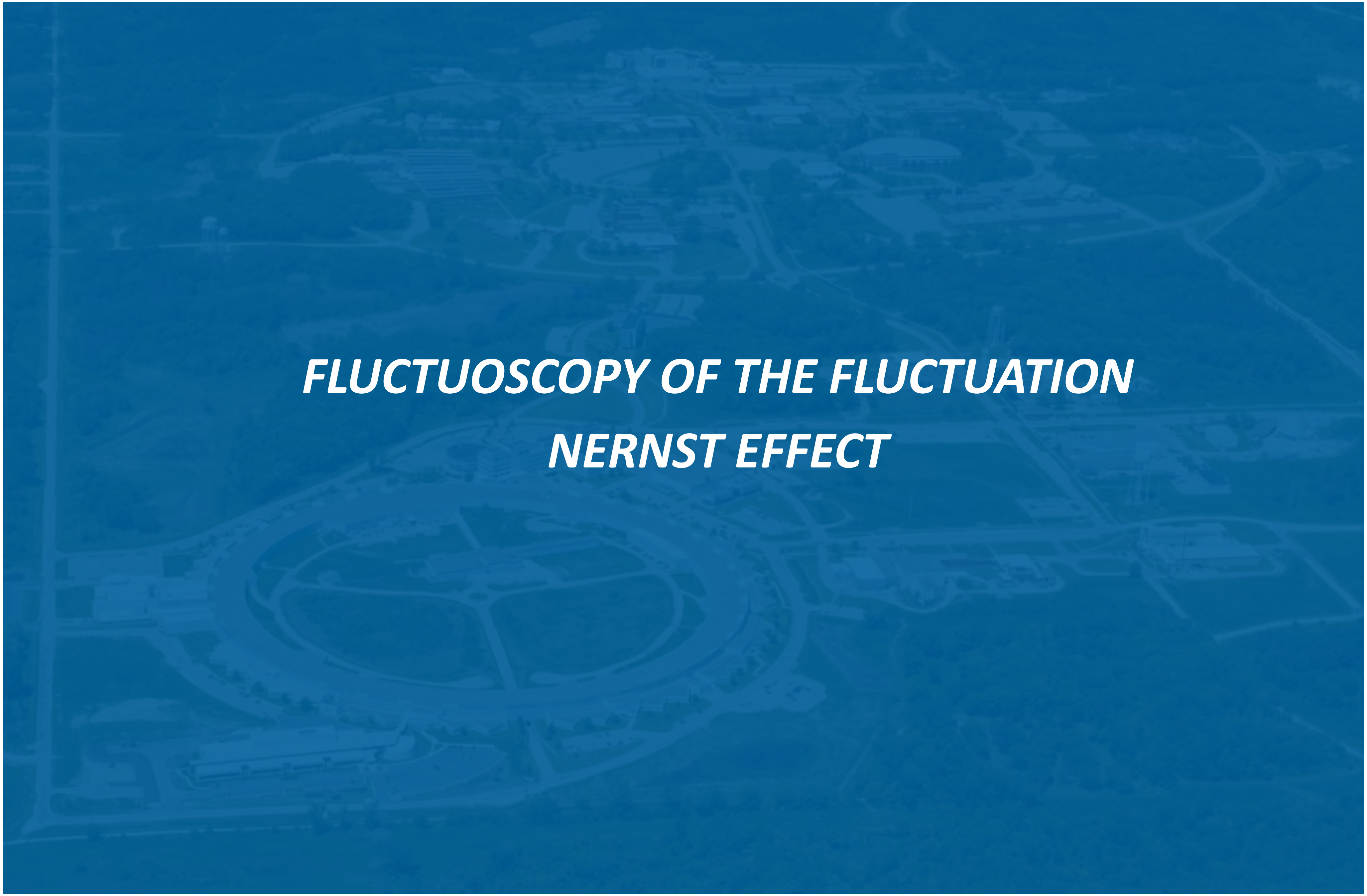




In domains IV-VI, magnetization term becomes crucial:

- Its 1/T divergence ensures the validity of the third law of thermodynamics
- The total NE coefficient remains finite for T→0

- In the quantum domain IV, $\nu^{(fl)}$ is negative! $-\frac{2\gamma_E}{9}\frac{t}{\tilde{h}}$ *agrees with qualitative estimate*
  → The DCR contribution wins over AL
- In domain V, $\nu^{(fl)}$ become positive and less singular: $\ln(t/\tilde{h})$
- Moving further along $H_{c2}(T)$, $\nu^{(fl)}$ grows as t^2 in domain VI when fixing distance to $H_{c2}(T)$



FLUCTUOSCOPY OF THE FLUCTUATION NERNST EFFECT

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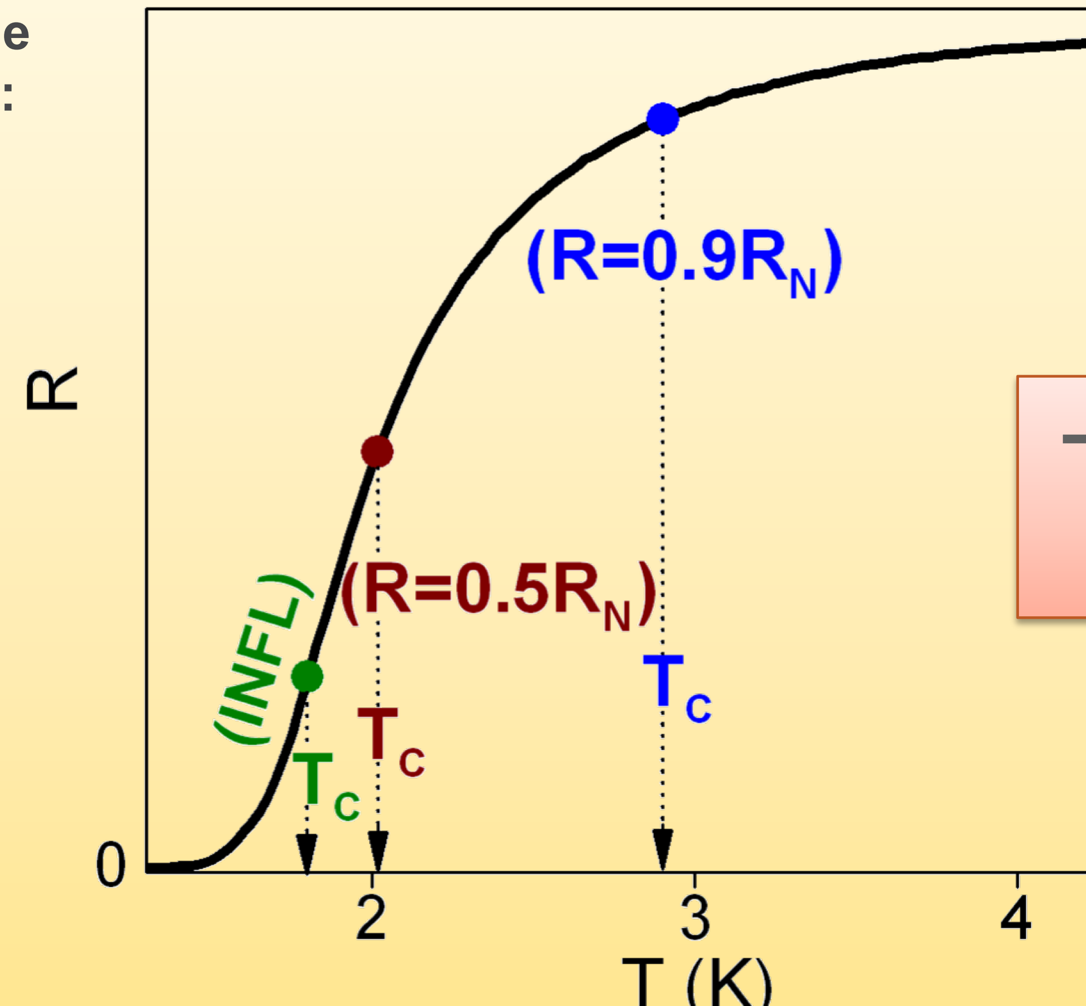
WHAT IS FLUCTUATION SPECTROSCOPY (FLUCTUOSCOPY)?

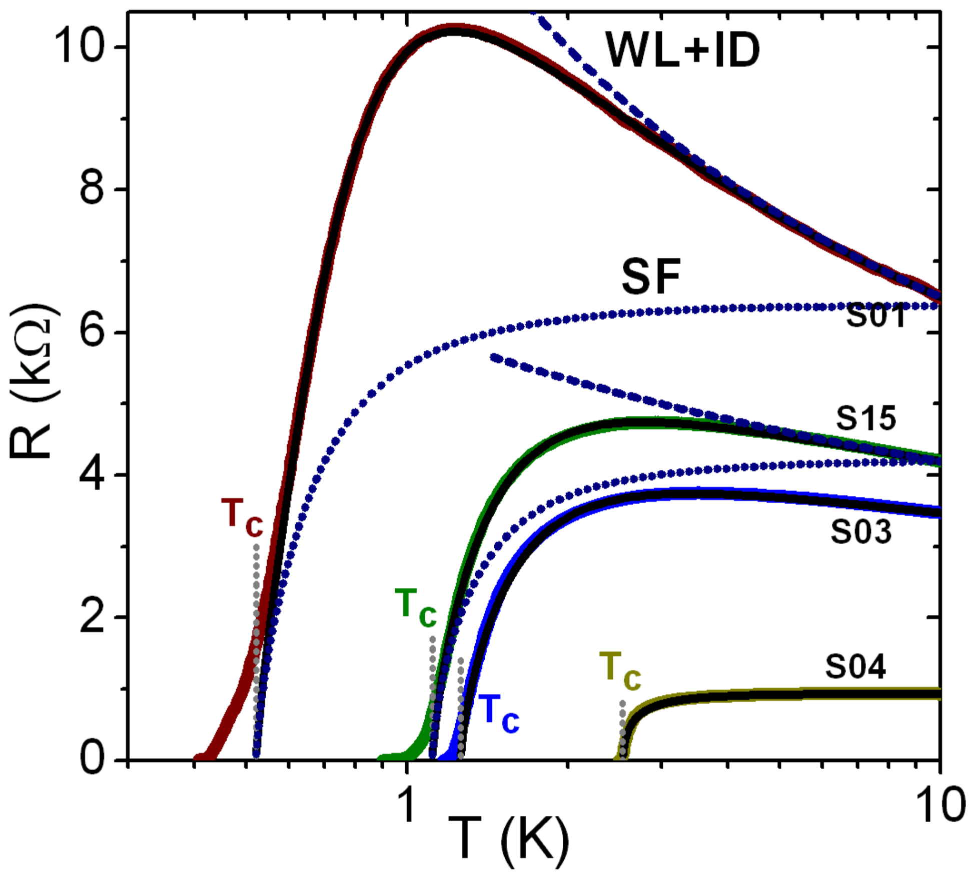
A: Extraction of (microscopic) material parameters by fitting to fluctuation corrections.
Example: T_c from resistance measurements

In the past:

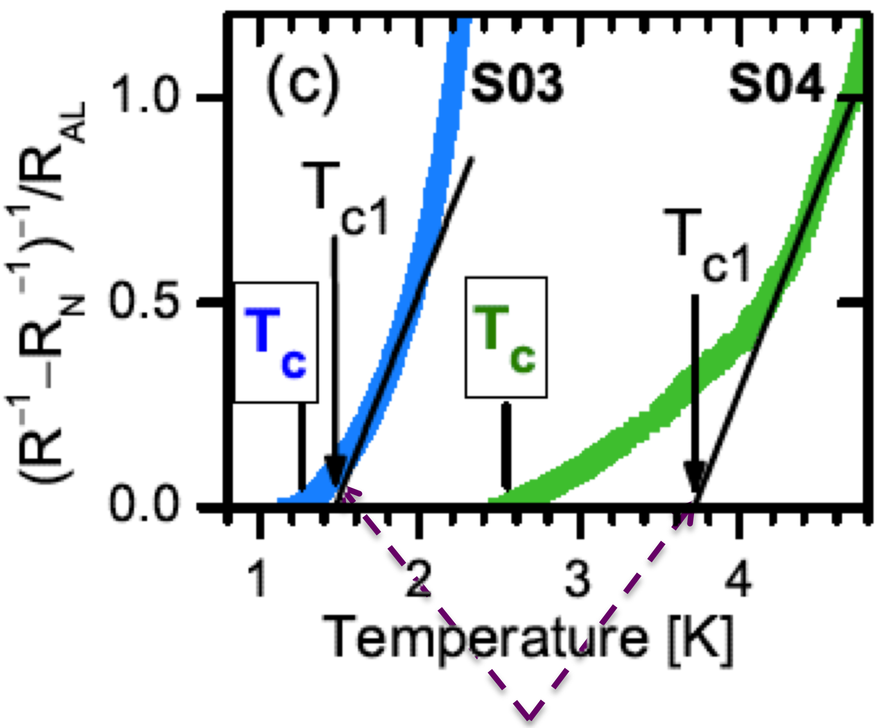


→T_c is assigned, not determined

FLUCTUOSCOPY OF THIN TiN FILMS



S03: T_c=1.260K
S04: T_c=2.538K



Traditional definition of T_c

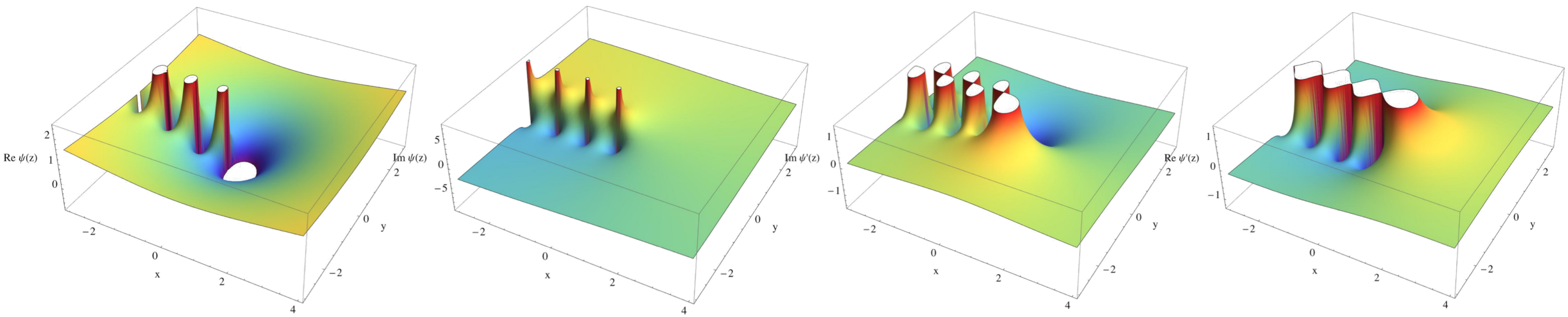
The determination of T_c as the temperature where R(T) drops to 0.9, 0.5 R_N significantly overestimates T_c.

T. I. Baturina, S. V. Postolova, A. Yu. Mironov, A. Glatz, M. R. Baklanov, V. M. Vinokur, EuroPhys. Lett. ***97****, 17012 (2012).*

NUMERICAL FLUCTUOSCOPY

Similar complete expressions were derived for fluctuation corrections to conductivity $\sigma_{xx}(t,h)$, NMR relaxation rate, and tunnel currents.
Its evaluation can only be done numerically:

- Needs precise evaluation of sums and integrals, but essential to analyze experimental results (requires fitting to microscopic parameters)
- Numerically challenging due to slow convergence, large number of Landau levels at low temperatures, and polygamma functions



- Calculation of a single t-h surface plot (10^6 values) takes ~3 months single-core CPU hours (in 2011) ~ 1 month today

→ No numerical cutoff for infinite Matsubara sums & integrals can be used, need to transform ”tails” to inverse parameter integrals

→ Summations need to be done carefully due to oscillations to avoid numerical cancellations

NUMERICAL FLUCTUOSCOPY, EXAMPLE

Transformation of k-sum (MT contribution to NMR relaxation rate):

$$S_m^{(\mathrm{MT})} \equiv \sum_{k=-\infty}^{\infty} \frac{\mathcal{E}_m''(t,h,|k|)}{\mathcal{E}_m(t,h,|k|)}$$

write as

$$S_m^{(\mathrm{MT})} \stackrel{\circ}{=} \left[\sum_{k=0}^{k_{\max}-1}(2-\delta_{0,k}) + 2\int_{k_{\max}}^{\infty} dk\right] \frac{\mathcal{E}_m''(t,h,|k|)}{\mathcal{E}_m(t,h,|k|)}$$
$$\equiv S_m^{(\mathrm{MT})(\mathrm{s})} + S_m^{(\mathrm{MT})(\mathrm{i})}$$

with

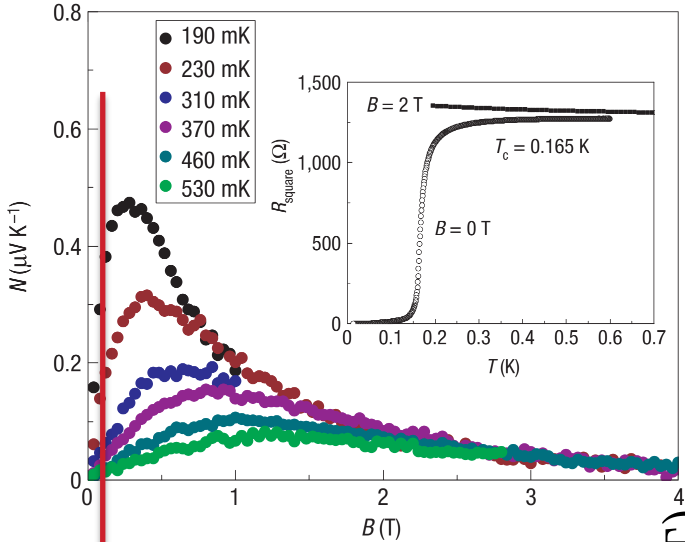
$$k_{\max} = \max\left\{2\Omega - \left\lfloor \frac{4h}{\pi^2\ t}(2m+1)\right\rfloor, 1\right\}.$$

and transform integral to

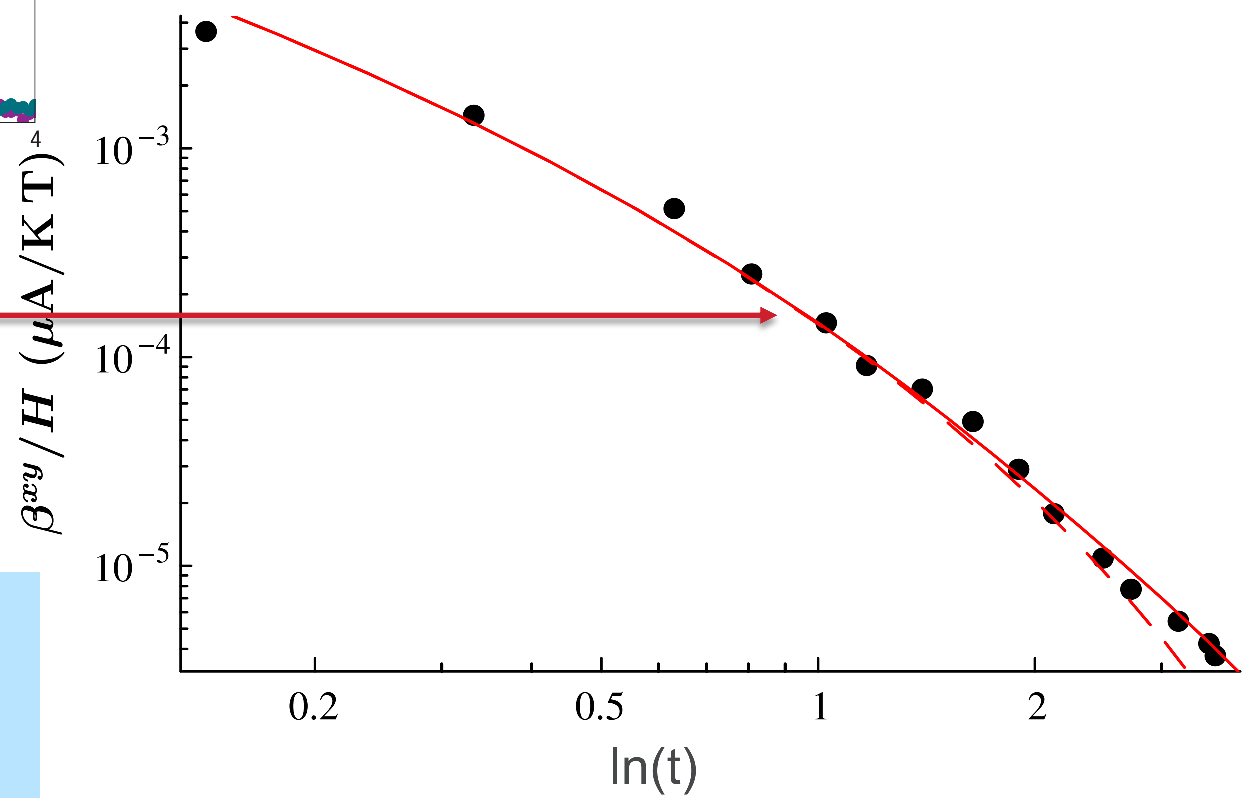
$$S_m^{(\mathrm{MT})(\mathrm{i})} = \frac{\pi^2}{8}\int_{z_{\max}}^{0} \frac{dz}{z^2} \frac{x_m(8z/\pi^2 x_m)^2}{\ln\left(t\pi^2 x_m/4\right) - \psi(1/2) - \ln(2) - \ln(z)}$$
$$= -\frac{8}{\pi^2 x_m}\int_0^{z_{\max}} dz \frac{1}{A_m - \ln z}$$
$$= -\frac{2t}{h(m+1/2)}\int_0^{z_{\max}} dz \frac{1}{A_m - \ln z}$$

$$A_m \equiv \ln\left[h(m+1/2)\right] - \psi(1/2) - \ln(2).$$

COMPARISON EXPERIMENT – THEORY TEMPERATURE DEPENDENCE



Low field fit for 12.5nm thick sample of $Nb_{0.15}Sb_{0.85}$ film

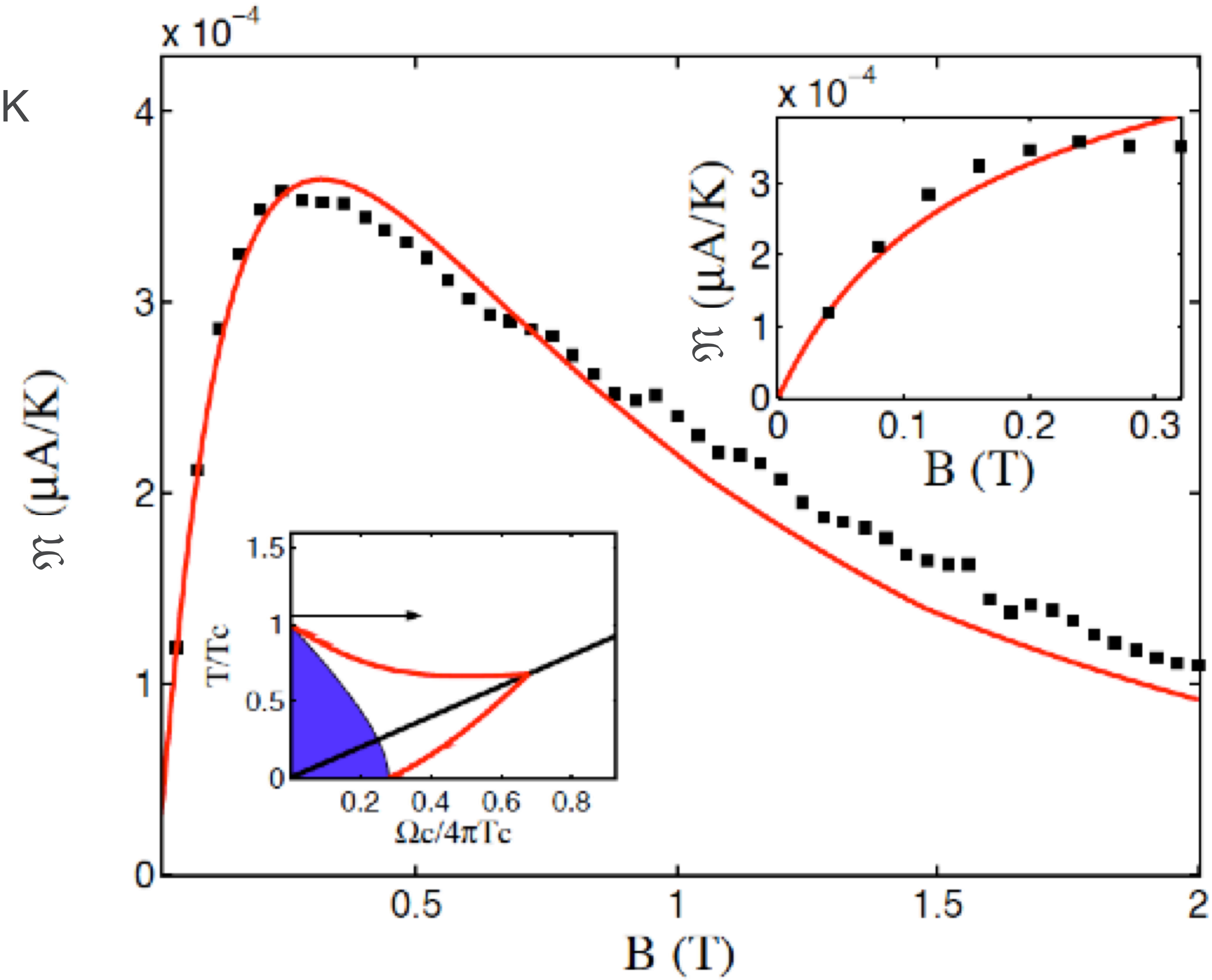


Only one fitting parameter: diffusion coefficient, here 0.087 cm^2/s

COMPARISON EXPERIMENT – THEORY FIELD DEPENDENCE

Field fit at T=410mK

From Michaeli & Finkelstein, 2009



GHOST FIELD, H*

- Characteristic feature of Nernst signal: non-monotonous as function of field
- Size of FCPs determined by $\xi_{GL}(t)$ till $\ell_{\mathrm{H}^*}^{\mathrm{FCP}} = \sqrt{c/(2eH^*)} \sim \xi_{\mathrm{GL}}(\epsilon)$

→ **Nernst signal reaches maximum at field H*(T)** – first called "mirror field", then ghost critical field

Tafti et al. used it to indirectly determine $H_{c2}(0)$ in HTS, which is very high and often inaccessible directly, using

$$H^*(T) = H_{\mathrm{c}2}(0) \ln \frac{T}{T_{\mathrm{c}0}}$$

$H_{c2}(0)$ is used as fitting parameter

However, the complete expression does not allow to extract H*(T) analytically.

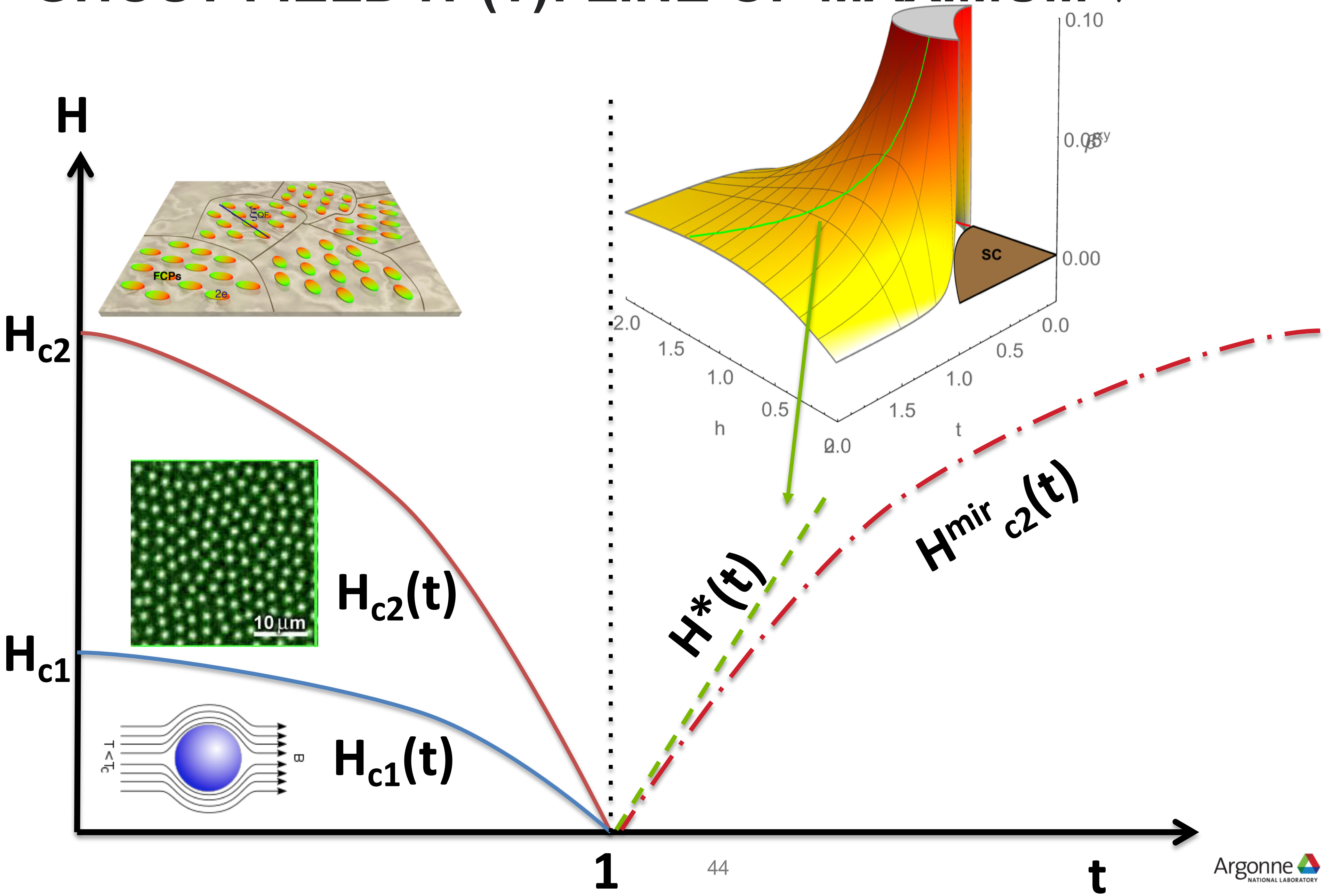
Its analytical structure suggests:

with some function $\varphi(x)$, $\varphi(0)=0$

$$H^*(T) = H_{\mathrm{c}2}(0) \left(\frac{T}{T_{\mathrm{c}0}}\right) \varphi\left(\ln \frac{T}{T_{\mathrm{c}0}}\right)$$

→ **Mainly linear T-dependence**

GHOST FIELD H*(T): LINE OF MAXIMUM $\nu^{(fl)}$



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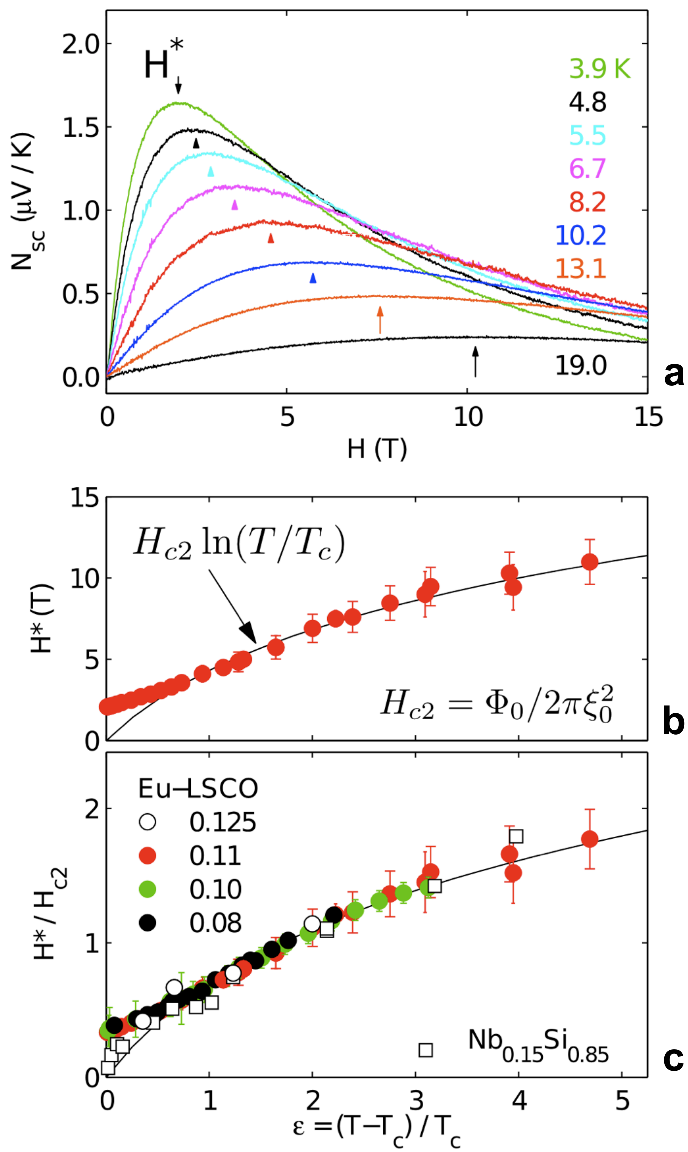
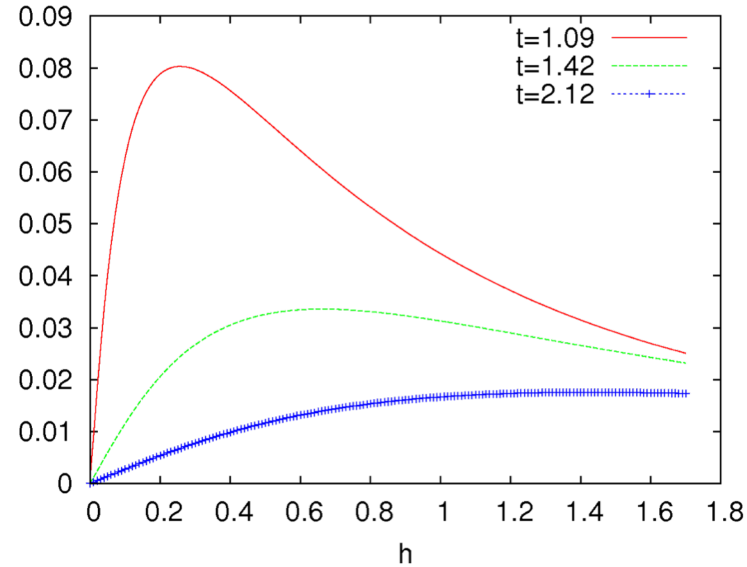
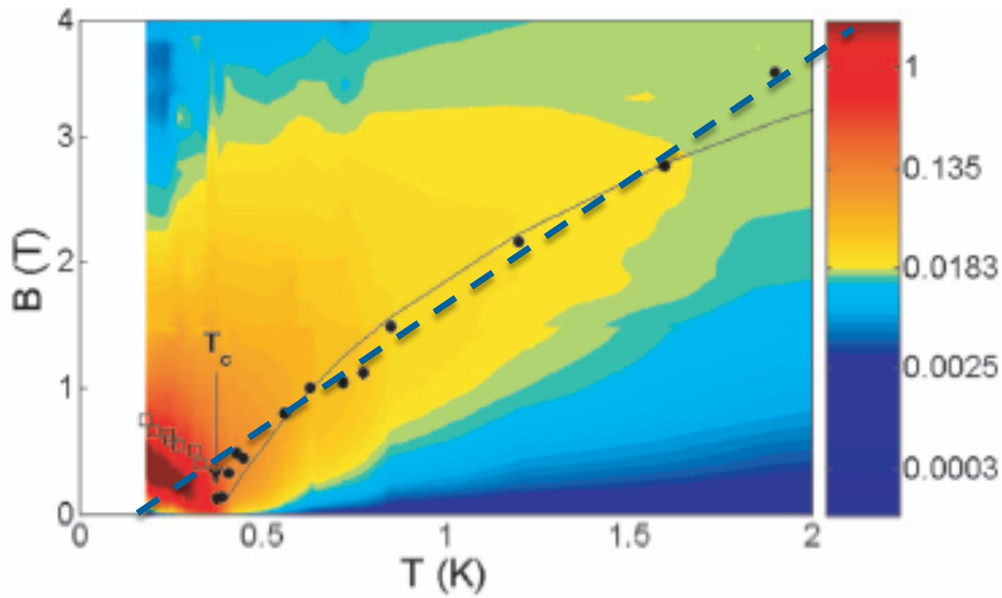
MAX NERNST SIGNAL



J. Chang et al

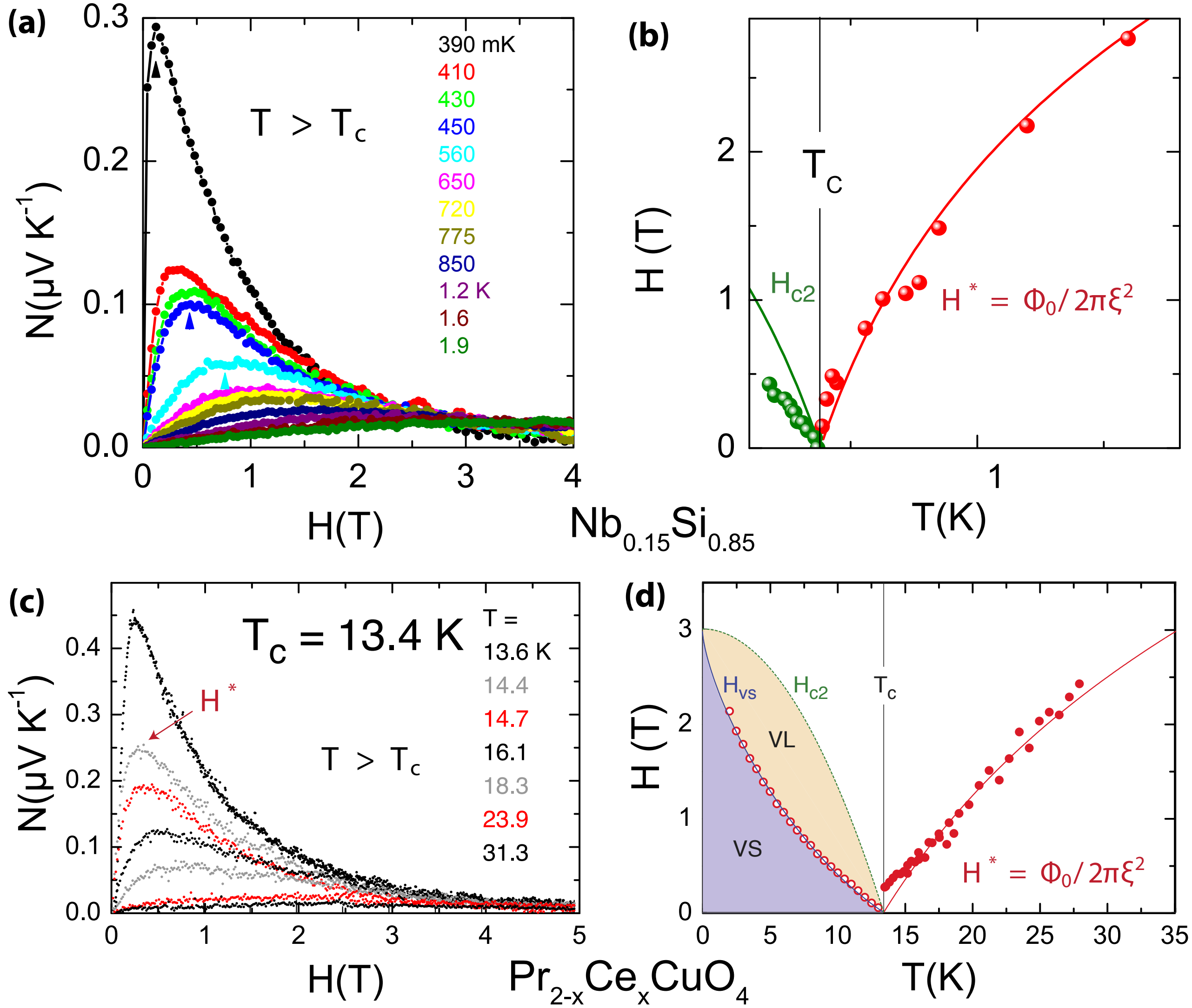




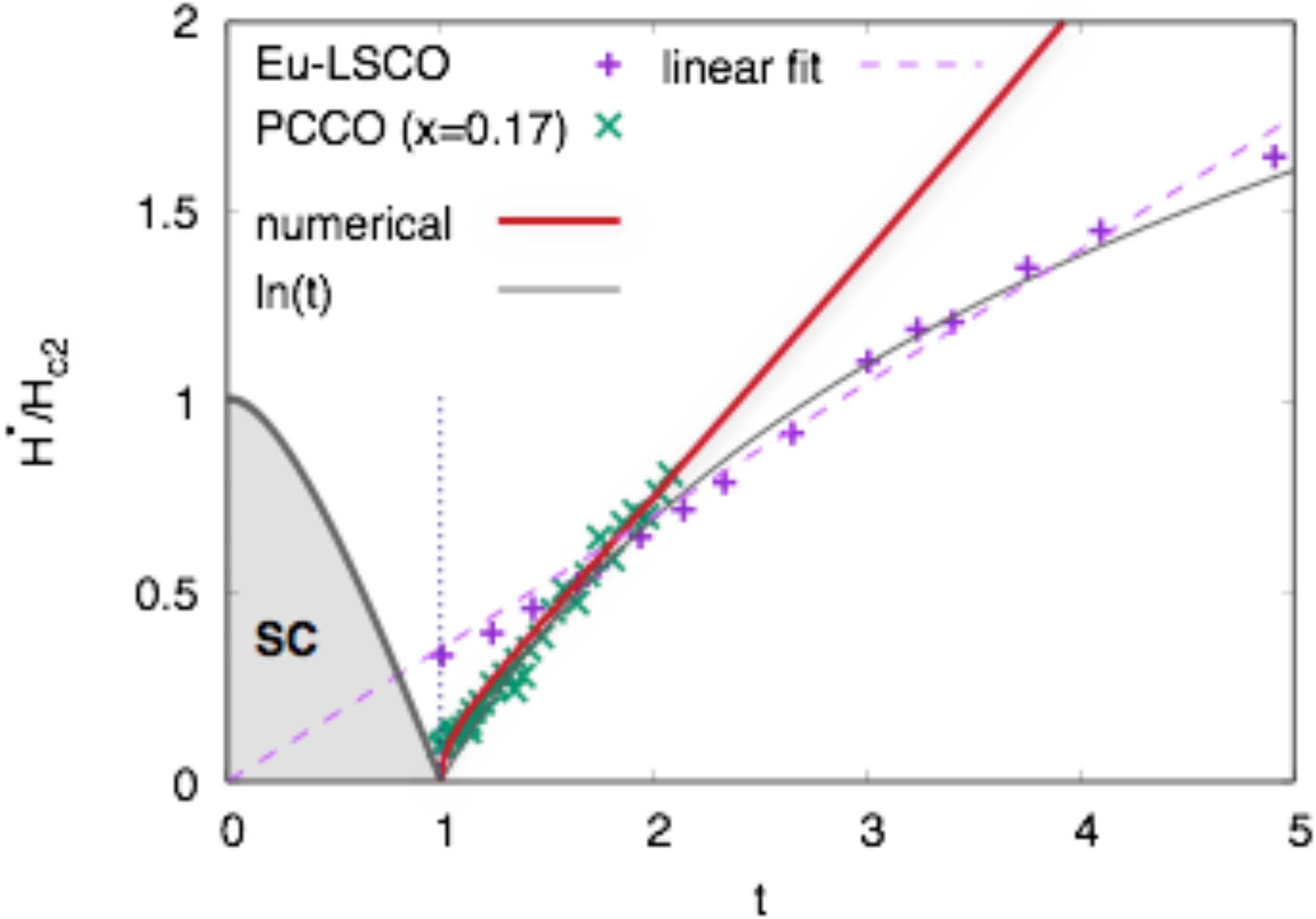
A. Pourret et al.

H* IN CONVENTIONAL AND HTS

$$\mathfrak{N} = \nu H$$



NUMERICAL EXTRACTION OF H* AND FIT



Summary

- Microscopic approach allows to calculate Nernst coefficient in entire h-t domain
- Allows numerical evaluation of the *ghost critical field* H*(T)
- Analytical structure of H*(T) can be obtained

➔ Enables fluctuation spectroscopy to extract microscopic parameters