# **Energy harvesting and energy transfer in molecular quantum systems**

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## Overview

Part I: Light-harvesting & quantum coherence in biomolecular systems

- 1. Introduction: Excitons, dynamics, environment
- 2. Open quantum systems: System-Bath-Model
- 3. Path integral approach: Feynman-Vernon influence functional
- 4. Numerically exact: Quasiadiabatic Propagator Path Integral (QUAPI)
- 5. Application: Fenna-Matthews-Olson (FMO) complex
- 6. Experiment: 2D nonlinear spectroscopy
- 7. Is exciton transfer quantum coherent?







Hy Bath, temperatu

eaction centre

## Overview

Part II: Role of vibrational modes & vibronic coupling

- 1. Small dye molecule: strong coupling to vibrations
- 2. Electronic vs vibrational vs vibronic coherence
- 3. Role of coherence on exciton transfer efficiency



- 4. Can long-live vibrational coherence enhance exciton coherence?
- 5. Can strong exciton coherence enhance vibrations?
- 6. Can strong vibronic coupling speed-up exciton transfer?





## Overview

Part III: Förster transfer in molecules with orthogonal dipole moments

- 1. Dye molecules with orthogonal dipole moments: no dipole coupling
- 2. Role of angle fluctuations: induce fast transfer
- 3. Presence of vibrations of molecular backbone

Part IV: Exciton transfer & conical intersections

- Vibrational coherence at a conical intersection: overdamped / underdamped
- 2. Holstein & Peierls phonon viewed at conical intersection





### Literature

#### Textbooks:

- ☆ U. Weiss: Quantum Dissipative Systems (World Scientific)
- \* A. O. Caldeira: Macroscopic Quantum Phenomena and Quantum Dissipation (Cambridge)
- ☆ H.-P. Breuer, F. Petruccione, The Theory of Open Quantum Systems (Oxford)
- ☆ T. Dittrich, P. Hänggi, G.-L. Ingold, B. Kramer, G. Schön, and W. Zwerger, Quantum Transport and Dissipation (Wiley-VCH)
- ☆ V. May, O. Kühn, Charge and Energy Transfer Dynamics in Molecular Systems (Wiley-VCH)

#### Original works (incomplete):

- ☆ A.O. Caldeira and A.J. Leggett, Influence of Dissipation on Quantum Tunneling in a Macroscopic System, Phys. Rev. Lett. 46, 211 (1981)
- ☆ H. Grabert, P. Schramm und G.-L. Ingold, Quantum Brownian Motion: The Functional Integral Approach, Phys. Rep. 168, 115 (1988)
- ☆ R.P. Feynman and F.L. Vernon, The theory of a general quantum system interacting with a linear dissipative system, Ann. Phys. 24, 118 (1963)
- ☆ N. Makri, Numerical path integral techniques for long time dynamics of quantum dissipative systems, J. Math. Phys. 36, 2430 (1995)
- ☆ M. Thorwart, P. Reimann, P. Hänggi, Iterative algorithm versus analytic solutions of the parametrically driven dissipative quantum harmonic oscillator, Phys. Rev. E 62, 5808 (2000)

#### Literature

- ☆ P. Nalbach, D. Braun, M. Thorwart, Exciton transfer dynamics and quantumness of energy transfer in the Fenna-Matthews-Olson complex, Phys. Rev. E 84, 041926 (2011)
- ☆ C. Mujica-Martinez, P. Nalbach, M. Thorwart, Quantification of non-Markovian effects in the Fenna-Matthews-Olson complex, Phys. Rev. E 88, 062719 (2013)
- ☆ H.-G. Duan, V. Prokhorenko, R. Cogdell, K. Ashraf, A. Stevens, M. Thorwart, R.J.D. Miller, Nature does not rely on long-lived electronic quantum coherence for photosynthetic energy transfer, Proc. Natl. Acad.Sci. 114, 8493 (2017)
- ☆ H.-G. Duan, P. Nalbach, V. Prokhorenko, S. Mukamel, M. Thorwart, On the nature of oscillations in twodimensional spectra of excitonically-coupled molecular systems, New J. Phys. 17, 072002 (2015)
- ☆ H.-G. Duan, A. Stevens, P. Nalbach, M. Thorwart, V. Prokhorenko, R.J.D. Miller, Two-dimensional electronic spectroscopy of Light Harvesting Complex II at ambient temperature: a joint experimental and theoretical study, J. Phys. Chem. B 119, 12017 (2015)
- ☆ H.-G. Duan, V. Prokhorenko, E. Wientjes, R. Croce, M. Thorwart, R.J.D. Miller, Primary charge separation in the reaction center revealed by two-dimensional electronic spectroscopy, Sci. Rep. 7, 12347 (2017)
- ☆ P. Nalbach, C. Mujica-Martinez, M. Thorwart, Vibronic speed-up of the excitation energy transfer in the Fenna-Matthews-Olson complex, Phys. Rev. E 91, 022706 (2015)
- ☆ P. Nalbach, I. Pugliesi, H. Langhals, M. Thorwart, Noise-induced Förster resonant energy transfer between orthogonal dipoles in photoexcited molecules, Phys. Rev. Lett. 108, 218302 (2012)
- ☆ H.-G. Duan, M. Thorwart, Quantum Mechanical Wavepacket Dynamics at a Conical Intersection with Strong Vibrational Dissipation, J. Chem. Phys. Lett. 7, 382 (2016)
- ☆ H.-G. Duan, R.J.D. Miller, M. Thorwart, Impact of Vibrational Coherence on the Quantum Yield at a Conical Intersection, J. Chem. Phys. Lett. 7, 3491 (2016)

#### **Photosynthesis**

Energy capture per year: 100 TW

Human energy consumption per year: 15 TW

6CO2<br/>Carbon dioxide+6H2O<br/>WaterLight<br/>SugarC6H12O6<br/>Sugar+6O2<br/>Oxygen

#### **Photosynthesis**

Biomass production: 105 billion tons per year

or 12 Mio tons per hour



## **Light-harvesting in plants**



Blankenship, Molecular Mechanisms of Photosynthesis

## **Light-harvesting in plants**

A structural phylogenetic map for chloroplast photosynthesis

John F. Allen, Wilson B. M. de Paula, Sujith Puthiyaveetil, Jon Nield School of Biological and Chemical Sciences, Queen Mary University of London

TRENDS in Plant Science



See online version for legend

Trends in Plant Science, December 2011, Vol. 16 (No. 12)

## **Energy transfer in biomolecules**



LH-2 antenna complex of Rps. Acidophila K. Schulten, UIUC

- Molecular structure often ring-shaped
- Photon => Exciton
- Transfer of excitation energy
- Different "hopping" time scales:
  - 100 fs to few ps



Blankenship, Molecular Mechanisms of Photosynthesis

## **Energy transfer in biomolecules**

A physicist's view:

- ☆ formation of an exciton (quasiparticle, bound e-h pair)
- radiationless transfer of excitation energy on a network of molecular sites via dipole coupling
- $\Rightarrow$  final step: reaction center, energy sink  $\Rightarrow$  electron transfer
- $\Rightarrow$  important: polar solvent or vibrational effects  $\Rightarrow$  fluctuations





FMO complex

#### Förster mechanism (1946)

- Exciton wave function unperturbed by coupling
- **Born-Oppenheimer** approximation:
  - nuclei provide static potential energy surfaces for electrons
- ☆ Classical occupation probability
- ransfer rate in lowest order in coupling:



### **Energy transfer in biomolecules**

Are there nontrivial quantum effects in biomolecular energy transfer?

classical "hopping", or,quantum coherent wave-like energy transfer?

New field of quantum biology?
 ☆ fundamental: Role of quantum coherence in energy transfer?





Aharonov, Rohrlich, Quantum Paradoxes, Wiley-VCH

Quietschgrüne Quantenwesen

FRANKFURTER ALLGEMEINE SONNTAGSZEITUNG, 30. JULI 2017, NR. 30

**58 WISSENSCHAFT SPEZIAL** 

### **Trivial vs. nontrivial**

Are there nontrivial quantum effects in biomolecular energy transfer?

Septem Artes Liberales ("curriculum proper to a free man")

Tri-vium (three-fold way): Grammar, Rhetoric, Dialectic (Logic)
 Quadri-vium (four-fold way): Arithmetic, Geometry, Music, Astronomy



Trivial = Part of the tri-vium = Basic



## **Biomolecular quantum coherence: Experiments**



Reaction centre

#### **Biomolecular quantum coherence: experiments**

G. Engel et al. (Berkeley), Nature 446, 782 (2007)

☆ long-lived excitonic quantum coherence
☆ up to 660 fs (system: 100 fs)
☆ temperature 77 K









dicted<sup>12,13</sup> and indirectly observed<sup>14</sup>. Here we extend previous twodimensional electronic spectroscopy investigations of the FMO bacteriochlorophyll complex, and obtain direct evidence for remarkably long-lived electronic quantum coherence playing an important part in energy transfer processes within this system. The quantum coherence manifests itself in characteristic, directly observable quantum beating signals among the excitons within the *Chlorobium tepidum* FMO complex at 77 K. This wavelike characteristic of the energy transfer within the photosynthetic complex can explain its extreme efficiency, in that it allows the complexes to sample vast areas of phase space to find the most efficient path.

### **Biomolecular quantum coherence: experiments**

#### ★ FMO @ room temperature:

#### G. Panitchayangkoon et al. (G. Engel) PNAS **107**, 12766 (2010)



### Quantum transfer of excitation energy: exciton model

 $|g_i\rangle \quad |e_i\rangle$ 

Dimer formed by two monomers:

(neglect doubly excited state, high energy!)

 $\Rightarrow$  A single chromophore at site i:

$$H_i = \frac{E_i}{2} \begin{pmatrix} 1 & 0\\ 0 & -1 \end{pmatrix}$$



#### $\Rightarrow$ Pair of chromophores with dipole coupling:

$$H = \frac{E_1}{2}\tau_z^1 + \frac{E_2}{2}\tau_z^2 + \Delta(\tau_x^1\tau_x^2 + \tau_y^1\tau_y^2)$$

 $\Delta = \frac{\kappa \mu_1 \mu_2}{R^3}$ 



Two quantum states:





## Quantum transfer of excitation energy: exciton model

#### Dimer in an environment:



- Protein-solvent environment:
  - Polar solvent = dielectric continuum
  - Protein host = discrete vibrational modes
- System-bath model

## **System-bath model**

$$H_{\rm tot} = H_S(t) + H_B + H_{SB}$$

System: (for example)  $\mathbf{H}_{\mathrm{S}}(t) = \frac{\mathbf{p}^2}{2\mathcal{M}} + V(\mathbf{q}, t)$ 

System Hs HB Bake, seuperature T

Bath + interaction:

$$\mathbf{H}_{\mathrm{B}} = \sum_{j=1}^{\mathcal{N}} \mathbf{H}_{j}(\mathbf{q}) = \sum_{j=1}^{\mathcal{N}} \frac{1}{2} \left[ \frac{\mathbf{p}_{j}^{2}}{m_{j}} + m_{j} \omega_{j}^{2} \left( \mathbf{x}_{j} - \frac{c_{j}}{m_{j} \omega_{j}^{2}} \mathbf{q} \right)^{2} \right]$$

- set of uncoupled harmonic oscillators
- bilinear coupling to the system
- effect of bath can be strong since (infinitely) many oscillators couple to system

#### Initial condition:

$$\mathbf{W}(t_0) = \rho_{\rm S}(t_0) \otimes \rho_{\rm B}^0$$

- required for dynamics
- uncoupled at t=0
- instantaneous switching on at t=0+

$$\rho_{\rm B}^0 = Z_{\rm B}^{-1} \exp\left(-\beta \mathbf{H}_{\rm B}^0\right)$$

### **Quantum Langevin equation**

Heisenberg equations of motion:

System: 
$$\mathcal{M}\ddot{\mathbf{q}}(t) + V'(\mathbf{q}, t) + \mathbf{q}(t) \sum_{j=1}^{\mathcal{N}} \frac{c_j^2}{m_j \omega_j^2} = \sum_{j=1}^{\mathcal{N}} c_j \mathbf{x}_j(t) \quad \underbrace{\text{insert here}}_{j=1}$$
  
Bath:  $m_j \ddot{\mathbf{x}}_j(t) + m_j \omega_j^2 \mathbf{x}_j(t) = c_j \mathbf{q}(t), \quad j = 1, ..., \mathcal{N}$ 

harmonic oscillator driven by the system displacement, solution:

$$\begin{aligned} \mathbf{x}_{j}(t) &= \mathbf{x}_{j}(t_{0}) \cos \omega_{j}(t-t_{0}) + \frac{\mathbf{p}_{j}(t_{0})}{m_{j}\omega_{j}} \sin \omega_{j}(t-t_{0}) + \frac{c_{j}}{m_{j}\omega_{j}} \int_{t_{0}}^{t} ds \sin \left[\omega_{j}(t-s)\right] \mathbf{q}(s) \\ &= \left[ \mathbf{x}_{j}(t_{0}) - \frac{c_{j}}{m_{j}\omega_{j}^{2}} \mathbf{q}(t_{0}) \right] \cos \omega_{j}(t-t_{0}) + \frac{\mathbf{p}_{j}(t_{0})}{m_{j}\omega_{j}} \sin \omega_{j}(t-t_{0}) \\ &- \frac{c_{j}}{m_{j}\omega_{j}^{2}} \int_{t_{0}}^{t} ds \cos \left[\omega_{j}(t-s)\right] \dot{\mathbf{q}}(s) + \frac{c_{j}}{m_{j}\omega_{j}^{2}} \mathbf{q}(t) \end{aligned}$$

Result:

$$\mathcal{M}\ddot{\mathbf{q}}(t) + \int_{t_0}^t ds \begin{bmatrix} \mathcal{M} & \frac{c_j^2}{m_j \omega_j^2} \cos \omega_j (t-s) \\ j=1 \end{bmatrix} \dot{\mathbf{q}}(s) + V'(\mathbf{q},t) = \begin{bmatrix} \mathcal{N} & \sum_{j=1}^{\mathcal{N}} c_j \left[ \mathbf{x}_j(t_0) \cos \omega_j (t-t_0) + \frac{\mathbf{p}_j(t_0)}{m_j \omega_j} \sin \omega_j (t-t_0) \right] \\ = \Gamma(t) \end{bmatrix}$$

### **Quantum Langevin equation**

Heisenberg equations of motion:

System: 
$$\mathcal{M}\ddot{\mathbf{q}}(t) + V'(\mathbf{q}, t) + \mathbf{q}(t) \sum_{j=1}^{\mathcal{N}} \frac{c_j^2}{m_j \omega_j^2} = \sum_{j=1}^{\mathcal{N}} c_j \mathbf{x}_j(t) \quad \underbrace{\text{insert here}}_{j=1}$$
  
Bath:  $m_j \ddot{\mathbf{x}}_j(t) + m_j \omega_j^2 \mathbf{x}_j(t) = c_j \mathbf{q}(t), \quad j = 1, ..., \mathcal{N}$ 

harmonic oscillator driven by the system displacement, solution:

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Result: Quantum Langevin equation

$$\mathcal{M}\ddot{\mathbf{q}}(t) + \mathcal{M}\int_{t_0}^t ds\,\gamma(t-s)\dot{\mathbf{q}}(s) + V'(\mathbf{q},t) = \mathbf{\Gamma}(t) - \mathcal{M}\gamma(t-t_0)\mathbf{q}(t_0)$$

#### **Quantum Langevin equation**

$$\mathcal{M}\ddot{\mathbf{q}}(t) + \mathcal{M}\int_{t_0}^t ds\,\gamma(t-s)\dot{\mathbf{q}}(s) + V'(\mathbf{q},t) = \mathbf{\Gamma}(t) - \mathcal{M}\gamma(t-t_0)\mathbf{q}(t_0)$$

Operator-valued stochastic force, since drawn from thermal ensemble:  $\mathbf{\Gamma}(t) = \sum_{j=1}^{\mathcal{N}} c_j \left[ \mathbf{x}_j(t_0) \cos \omega_j(t-t_0) + \frac{\mathbf{p}_j(t_0)}{m_j \omega_j} \sin \omega_j(t-t_0) \right]$ <u>**1. Mean:**</u>  $\langle \mathbf{\Gamma}(t) \rangle_{\mathrm{B}} = 0$ 

Gaussian statistics:

2. Variance:

Bath autocorrelation function:

$$m_{j}\omega_{j}\langle \mathbf{x}_{j}(t_{0})\mathbf{x}_{k}(t_{0})\rangle_{\mathrm{B}} = \frac{1}{m_{j}\omega_{j}}\langle \mathbf{p}_{j}(t_{0})\mathbf{p}_{k}(t_{0})\rangle_{\mathrm{B}} = \delta_{jk}\frac{\hbar}{2}\operatorname{coth}\frac{\hbar\omega_{j}\beta}{2}$$
$$\langle \mathbf{x}_{j}(t_{0})\mathbf{p}_{k}(t_{0})\rangle_{\mathrm{B}} = -\langle \mathbf{p}_{j}(t_{0})\mathbf{x}_{k}(t_{0})\rangle_{\mathrm{B}} = i\,\delta_{jk}\frac{\hbar}{2}$$

$$\langle \mathbf{\Gamma}(t-s+t_0)\mathbf{\Gamma}(t_0)\rangle_{\mathrm{B}} = \hbar \sum_{j=1}^{\mathcal{N}} \frac{c_j^2}{2m_j\omega_j} \left[ \coth \frac{\hbar\omega_j\beta}{2} \cos \omega_j(t-s) - i\sin \omega_j(t-s) \right]$$

## **Bath spectral density**

All bath parameters come in a specific combination: Bath spectral density:

$$J(\omega) = \frac{\pi}{2} \sum_{j=1}^{N} \frac{c_j^2}{m_j \omega_j} \delta(\omega - \omega_j)$$
Continuum limit:  $\langle \mathbf{\Gamma}(t) \mathbf{\Gamma}(s) \rangle_{\mathrm{B}} = \hbar L(t-s) = \frac{1}{\pi} \int_0^{\infty} d\omega J(\omega) \begin{bmatrix} \coth \frac{\hbar \omega \beta}{2} \cos \omega t - i \sin \omega t \end{bmatrix}$ 
weight function
Damping kernel:
$$\gamma(t) = \frac{2}{\pi \mathcal{M}} \int_0^{\infty} d\omega \frac{J(\omega)}{\omega} \cos \omega t$$
Damping becomes Markovian:
$$\gamma(t-s) = 2\gamma \delta(t-s)$$

Dielectric environment:

$$J(\omega) \propto (\Delta \mu)^2 \operatorname{Im} \frac{2(\epsilon(\omega) - \epsilon_p)}{2\epsilon(\omega) + \epsilon_p}$$

 $J(\omega) \propto \delta(\omega - \Omega)$ 

Quantum Langevin equation is impractical (Heisenberg picture)! Alternative: Schrödinger picture: Feynman-Vernon influence functional

#### Path integral approach to open quantum systems

Full density operator of system+bath at time t in coordinate space:

$$\langle q_{\mathbf{f}} \boldsymbol{x}_{\mathbf{f}} | \mathbf{W}(t) | q'_{\mathbf{f}} \boldsymbol{x}'_{\mathbf{f}} \rangle = \langle q_{\mathbf{f}} \boldsymbol{x}_{\mathbf{f}} | \mathbf{U}(t, t_0) \mathbf{W}(t_0) \mathbf{U}^{-1}(t, t_0) | q'_{\mathbf{f}} \boldsymbol{x}'_{\mathbf{f}} \rangle$$

$$= \int dq_{\mathbf{i}} dq'_{\mathbf{i}} \prod_{j=1}^{\mathcal{N}} dx_{j,\mathbf{i}} \prod_{j=1}^{\mathcal{N}} dx'_{j,\mathbf{i}} U(q_{\mathbf{f}}, \boldsymbol{x}_{\mathbf{f}}, t; q_{\mathbf{i}}, \boldsymbol{x}_{\mathbf{i}}, t_0) \langle q_{\mathbf{i}} \boldsymbol{x}_{\mathbf{i}} | \mathbf{W}(t_0) | q'_{\mathbf{i}} \boldsymbol{x}'_{\mathbf{i}} \rangle U^*(q'_{\mathbf{f}}, \boldsymbol{x}'_{\mathbf{f}}, t; q'_{\mathbf{i}} \boldsymbol{x}'_{\mathbf{i}}, t_0)$$

with propagator as path integral:

$$U(q_{\rm f}, \boldsymbol{x}_{\rm f}, t; q_{\rm i}, \boldsymbol{x}_{\rm i}, t_{\rm 0}) = \langle q_{\rm f} \boldsymbol{x}_{\rm f} | \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_{t_0}^t ds \, \mathbf{H}(s) \right] | q_{\rm i} \boldsymbol{x}_{\rm i} \rangle$$
$$= \int_{q(t_0)=q_{\rm i}}^{q(t)=q_{\rm f}} \mathcal{D}q \prod_{j=1}^{\mathcal{N}} \int_{x_j(t_0)=x_{j,\rm i}}^{x_j(t)=x_{j,\rm f}} \mathcal{D}x_j \exp \left\{ \frac{i}{\hbar} \left[ S_{\rm S}[q] + \sum_{j=1}^{\mathcal{N}} S_{\rm B}[q, x_j] \right] \right\}$$

with the classical actions:

$$S_{\rm S}[q] = \int_{t_0}^t ds \left\{ \frac{\mathcal{M}}{2} \dot{q}^2(s) - V(q(s), t) \right\}$$
$$S_{\rm B}[q, x_j] = \int_{t_0}^t ds \left\{ \frac{m_j}{2} \dot{x}_j^2(s) - \frac{m_j}{2} \omega_j^2 [x_j(s) - \frac{c_j}{m_j \omega_j^2} q(s)]^2 \right\}$$

#### Path integral approach to open quantum systems

Bath d.o.f. not under control & not of interest: average over them:

Reduced density operator in system Hilbert space:

 $\rho(t) \equiv \mathrm{tr}_{\mathrm{B}} \mathbf{W}(t)$ 

possible, since action quadratic: Gaussian integral

$$\begin{aligned} (q_{\rm f}, q_{\rm f}', t) &= \prod_{j=1}^{\mathcal{N}} \int dx_{j,\rm f} \langle q_{\rm f} \boldsymbol{x}_{\rm f} | \mathbf{W}(t) | q_{\rm f}' \boldsymbol{x}_{\rm f} \rangle & \text{ac} \\ &= \int dq_{\rm i} \int dq_{\rm i}' \, \mathcal{G}(q_{\rm f}, q_{\rm f}', t; q_{\rm i}, q_{\rm i}', t_0) \rho_{\rm S}(q_{\rm i}, q_{\rm i}', t_0) \end{aligned}$$

$$\mathcal{G}(q_{\rm f}, q_{\rm f}', t; q_{\rm i}, q_{\rm i}', t_0) = \int_{q(t_0)=q_{\rm i}}^{q(t)=q_{\rm f}} \mathcal{D}q \int_{q'(t_0)=q_{\rm i}'}^{q'(t)=q_{\rm f}'} \mathcal{D}q' \exp\left\{\frac{i}{\hbar} \left(S_{\rm S}[q] - S_{\rm S}[q']\right)\right\} \mathcal{F}_{\rm FV}[q, q']$$

Feynman-Vernon influence functional:

ρ

$$\mathcal{F}_{\rm FV}[q,q'] = e^{-\frac{1}{\hbar}\phi_{\rm FV}[q,q']}$$

### **Feynman-Vernon influence functional**

$$\mathcal{F}_{\rm FV}[q,q'] = e^{-\frac{1}{\hbar}\phi_{\rm FV}[q,q']}$$

with

$$\phi_{\rm FV}[q,q'] = \int_{t_0}^t dt' \int_{t_0}^{t'} dt'' \{q(t') - q'(t')\} \{\eta(t'-t'')q(t'') - \eta^*(t'-t'')q'(t'')\}$$

#### with integral kernel



Consequences of bath fluctuations:

- time correlations between paths of system dynamics
- complex correlation function

N. Makri, J. Math. Phys. 36, 2430 (1995)



<u>1. Step:</u> Symmetric Trotter splitting of short-time propagator:

$$\mathbf{U}(t_{k+1}, t_k) \approx \exp(-i\mathbf{H}_{\mathrm{B}}\Delta t/2\hbar)\mathbf{U}_{\mathrm{S}}(t_{k+1}, t_k) \exp(-i\mathbf{H}_{\mathrm{B}}\Delta t/2\hbar)$$
  
General time-ordered system propagator  
$$\mathbf{U}_{\mathrm{S}}(t_{k+1}, t_k) = \mathcal{T} \exp\left\{-\frac{i}{\hbar}\int_{t_k}^{t_{k+1}} dt' \,\mathbf{H}_{\mathrm{S}}(t')\right\}$$

Trotter error:

$$\sim \mathcal{O}([\mathbf{H}_{\mathrm{B}}, [\mathbf{H}_{\mathrm{S}}, \mathbf{H}_{\mathrm{B}}]]\Delta t^{3})$$

N. Makri, J. Math. Phys. 36, 2430 (1995)







<u>1. Step:</u> Symmetric Trotter splitting of short-time propagator:

Short-time propagator factorizes:

 $\langle q \Pi_j x_j | \mathbf{U}(t_{k+1}, t_k) | q' \Pi_j x'_j \rangle \approx \langle q | \mathbf{U}_{\mathrm{S}}(t_{k+1}, t_k) | q' \rangle \prod_{j=1}^{\mathcal{N}} \langle x_j | e^{-i\mathbf{H}_j(q)} \Delta t/2\hbar e^{-i\mathbf{H}_j(q')} \Delta t/2\hbar | x'_j \rangle$ 



System coordinate treated quasi-adiabatically -> QUAPI

N. Makri, J. Math. Phys. 36, 2430 (1995)

Combine to full propagator from initial to final time (use completeness...)

& carry out integration over bath d.o.f. (partial trace):

Reduced density operator at time t:

$$\rho(q_{\rm f},q_{\rm f}';t) = \int dq_0 \dots \int dq_N \int dq_0' \dots \int dq_N' \,\delta(q_{\rm f}'-q_N') \,\delta(q_{\rm f}-q_N)$$

$$\times \langle q_N | \mathbf{U}_{\rm S}(t,t-\Delta t) | q_{N-1} \rangle \dots \langle q_1 | \mathbf{U}_{\rm S}(t_0+\Delta t,t_0) | q_0 \rangle$$

$$\times \langle q_0 | \rho_{\rm S}(t_0) | q_0' \rangle \langle q_0' | \mathbf{U}_{\rm S}^{-1}(t_0+\Delta t,t_0) | q_1' \rangle \dots \langle q_{N-1}' | \mathbf{U}_{\rm S}^{-1}(t,t-\Delta t) | q_N' \rangle$$

$$\times \mathcal{F}_{FV}^{(N)}(q_0,q_0',\dots,q_N,q_N')$$

discrete Feynman-Vernon influence functional:

$$\mathcal{F}_{FV}^{(N)}(q_0, ..., q'_N) = \exp\left\{-\frac{1}{\hbar} \sum_{k=0}^N \sum_{k'=0}^k [q_k - q'_k] [\eta_{kk'} q_{k'} - \eta^*_{kk'} q'_{k'}]\right\}$$

$$\eta_{kk'} = \eta(t_k - t'_k)$$
  
 $\eta(t) = L(t) + i\delta(t)\frac{2}{\pi}\int_0^\infty d\omega \frac{J(\omega)}{\omega}$ 

N. Makri, J. Math. Phys. 36, 2430 (1995)

2. Step: Cut memory when it is negligible:

For any finite temperature: memory decays exponentially, i.e., there exists a memory time scale!



N. Makri, J. Math. Phys. 36, 2430 (1995)

2. Step: Cut memory when it is negligible:  $\tau_{\rm mem} = K\Delta t$  $N \min\{N,K\}$  $\mathcal{F}_{FV}^{(N)}(q_0, ..., q'_N) \approx \prod_{k=0}^{N} \prod_{k=0}^{N} \exp\left\{-\frac{1}{\hbar}[q_k - q'_k][\eta_{kk'}q_{k'} - \eta^*_{kk'}q'_{k'}]\right\}$ k=0k'=00.4 0.3 in practice: increase K until — Re,  $\hbar\omega_c/k_BT 
ightarrow \infty$ 0.2 numerical convergence is — Re,  $\hbar\omega_c/k_BT = 10$ established 0.1 –- Im L(t) 0.0 -0.1 2 8 10 6 0 4  $\omega_{c} t$ **Remember:**  $\mathcal{F}_{FV}^{(N)}(q_0, ..., q'_N) = \exp\left\{-\frac{1}{\hbar} \sum_{k=0}^N \sum_{k'=0}^k [q_k - q'_k] [\eta_{kk'} q_{k'} - \eta^*_{kk'} q'_{k'}]\right\}.$ 

N. Makri, J. Math. Phys. 36, 2430 (1995)

Iterative tensor multiplication scheme:

$$\rho(q,q',t) \equiv \rho_k(q_k,q'_k)$$

reduced density matrix (local in time)

$$A_k(q_k, ..., q'_{k+K-1})$$

propagate numerically: iteration:

$$A_{k+1}(q_{k+1},...,q'_{k+K}) = \int dq_k \int dq_{k'} \Lambda_k(q_k,...,q'_{k+K}) A_k(q_k,...,q'_{k+K-1})$$

#### with propagating tensor:

$$\Lambda_k(q_k, \dots, q'_{k+K}) = \langle q_{k+1} | \mathbf{U}_{\mathrm{S}}(t_{k+1}, t_k) | q_k \rangle \langle q'_k | \mathbf{U}_{\mathrm{S}}^{-1}(t_{k+1}, t_k) | q'_{k+1} \rangle \prod_{k'=0}^K \exp\left\{-\frac{1}{\hbar} [q_k - q'_k] [\eta_{kk'} q_{k'} - \eta^*_{kk'} q'_{k'}]\right\}$$

At final time: 
$$\rho(q_f, q'_f, t) = A_N(q_f, q'_f, \hat{q}, ..., \hat{q}) \exp\left\{-\frac{1}{\hbar}[q_f - q'_f][\eta_{NN}q_f - \eta^*_{NN}q'_f]\right\}$$

N. Makri, J. Math. Phys. 36, 2430 (1995)

3. Step: Discrete variable representation

 $dq_k \int dq_{k'} \dots$ 

continuous integration in coordinate space dimension of system Hilbert space  $\sum_{m_k=1}^{M} \sum_{m'_k=1}^{M} \cdots$ discrete summation in eigenspace of system-bath coupling operator

Careful check for convergence:

- Trotter increment as small as possible
- memory time as large as possible
- optimum in between

M. Thorwart, P. Reimann, P. Hänggi, Rev. E 62, 5808 (2000)



#### **QUAPI: Verification & tests**

Parametrically driven quantum dissipative harmonic oscillator

$$\mathbf{H}_{\mathrm{S}}(t) = \frac{\mathbf{p}^2}{2\mathcal{M}} + \frac{\mathcal{M}}{2} [\omega_0^2 + \epsilon \cos \Omega t] \mathbf{q}^2$$



$$\sigma_{qq}(t) \equiv \langle \mathbf{q}^2(t) \rangle - \langle \mathbf{q}(t) \rangle^2$$
  
$$\sigma_{qp}(t) \equiv \frac{1}{2} \langle \mathbf{q}(t) \mathbf{p}(t) + \mathbf{p}(t) \mathbf{q}(t) \rangle - \langle \mathbf{q}(t) \rangle \langle \mathbf{p}(t) \rangle$$
  
$$\sigma_{pp}(t) \equiv \langle \mathbf{p}^2(t) \rangle - \langle \mathbf{p}(t) \rangle^2$$

M. Thorwart, P. Reimann, P. Hänggi, Rev. E 62, 5808 (2000)
#### **QUAPI: Verification & tests**



$$\mathbf{H}_{\mathrm{S}}(t) = \frac{\mathbf{p}^2}{2\mathcal{M}} + \frac{\mathcal{M}}{2} [\omega_0^2 + \epsilon \cos \Omega t] \mathbf{q}^2$$

 $ω_0$ =1.0, ε =0.5, T=0.1, γ=1.0,  $ω_c$ =50.0 Ohmic bath





M. Thorwart, P. Reimann, P. Hänggi, Rev. E 62, 5808 (2000)

#### **QUAPI vs. Hierarchy equation of motion**

P. Nalbach, A. Ishizaki, G.R. Fleming, M. Thorwart, New J. Phys. (2011)

☆ Dimer in an Ohmic bath:



$$\varepsilon = 2.5\Delta, \omega_c = 1.3\Delta, T = 5.2\Delta, \Delta = 40 \text{cm}^{-1} \qquad J(\omega) = \frac{\lambda}{\omega_c} \omega e^{-\omega/\omega_c}$$
$$\omega_c = 53.1 \text{cm}^{-1}, J_{12} = 100 \text{cm}^{-1} = \Delta/2 \qquad \lambda = 35 \text{cm}^{-1}$$

## **Other techniques (personal selection)**

- Quantum Monte Carlo: stochastic sampling of path integral Egger, Mak, ... (+: very general, numerically exact, -: sign problem in real time)
- Stochastic Schrödinger Equation & HOPS (+: very general, very efficient, numerically exact) Djosi, Strunz, Eisfeld
- Hierarchy Equation of Motion Tanimura (+: very general, numerically exact)
- Renormalization Groups: DMRG, NRG, fRG, ... Plenio, Burghardt, ...
- Flow Equation Kehrein (+: numerical exact, -: limited to smaller systems)
- Time-Nonlocal Quantum Master Equations Meir, Tannor, ... (+: very efficient, -: approximative, weak coupling)
- Redfield Equation (Born-Markov master equation) (+: very efficient, -: approximative, weak coupling) Redfield, Bloch, ...
- Analytical: NIBA, generalized master equation (+: analytical, -: approximative/perturbative) Legget

Leggett, Weiss, Grabert, Grifoni, Hänggi,...

#### **FMO protein complex: Calculations**

Nalbach, Braun, Thorwart, Phys. Rev. E (2011)

☆ Quantum dissipative exciton dynamics
 ☆ Monomer: 7 chromophore / sites
 ☆ Numerically exact path-integrals QUAPI
 ☆ Hamiltonian:



$$H = \begin{pmatrix} 240 & -87.7 & 5.5 & -5.9 & 6.7 & -13.7 & -9.9 \\ 315 & 30.8 & 8.2 & 0.7 & 11.8 & 4.3 \\ 0 & -53.5 & -2.2 & -9.6 & 6.0 \\ 130 & -70.7 & -17.0 & -63.3 \\ 285 & 81.1 & -1.3 \\ 435 & 39.7 \\ 245 \end{pmatrix} \text{ cm}^{-1}$$

Wendling et al. J. Phys. Chem. B (2000), Adolphs, Renger, Biophys. J. (2006), Renger, Marcus, J. Chem. Phys. (2002)

## **FMO protein complex: Calculations**

Nalbach, Braun, Thorwart, Phys. Rev. E (2011)

#### ☆ Bath spectral density (from expts. & calcs):

$$J(\omega) = 0.5 \left( c_1 \frac{\omega^5}{\omega_1^4} e^{-\sqrt{\omega/\omega_1}} + c_2 \frac{\omega^5}{\omega_2^4} e^{-\sqrt{\omega/\omega_2}} \right) + 0.22 \omega^2 \delta \left( \omega - 180 \mathrm{cm}^{-1} \right)$$

$$\overset{30}{=} \frac{-\mathrm{Cho \ et \ al.}}{-\mathrm{Cho \ et \ al.}} + c_2 \frac{\omega^5}{\omega_2^4} e^{-\sqrt{\omega/\omega_2}} \right) + 0.22 \omega^2 \delta \left( \omega - 180 \mathrm{cm}^{-1} \right)$$

$$\overset{30}{=} -\mathrm{Cho \ et \ al.}} + c_2 \frac{\omega^5}{\omega_2^4} e^{-\sqrt{\omega/\omega_2}} \right) + 0.22 \omega^2 \delta \left( \omega - 180 \mathrm{cm}^{-1} \right)$$

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Wendling et al. J. Phys. Chem. B (2000), Adolphs, Renger, Biophys. J. (2006), Renger, Marcus, J. Chem. Phys. (2002)

## **Dynamics of FMO site populations**

Nalbach, Braun, Thorwart, Phys. Rev. E (2011)

#### Phenomenological Ohmic bath spectrum (Ishizaki & Fleming)



300 K: 350 fs

77 K: 700 fs

Ishizaki, Fleming, PNAS 106, 17255 (2009)

# **FMO protein complex: Calculations**

Nalbach, Braun, Thorwart, Phys. Rev. E (2011)

#### Measured / parametrized bath spectrum (Adolphs & Renger)



No exceptionally long-lived coherence
 Electronic coherence times as expected!

#### How "quantum" is the FMO exciton?

 $\Rightarrow$  Energy current: Define qm energy current operator  $\hat{j}$  via continuity equation

$$\frac{\partial}{\partial t}H(\mathbf{x},t) + \operatorname{div}\,\hat{j}(\mathbf{x},t) = 0$$

 $\bigstar$  Decompose Hamiltonian  $H = \sum_{i=1}^{N} h_i$   $h_i = \frac{1}{2} \sum_{k=1}^{N} h_{ik} |i\rangle \langle k| + \text{H.c.}$ 

😪 Current at site i

$$\frac{\partial}{\partial t}h_i = \frac{1}{2}\sum_{k\neq i} (s_{k\to i} - s_{i\to k}) \qquad \qquad s_{i\to k} = \frac{i}{2\hbar}\sum_l (h_{ik}h_{kl}|i\rangle\langle l| + \text{H.c.})$$

 $\bigstar$  eigenvectors of  $\hat{j}$  are pointer states, classical

☆ Define Quantumness Q[ρ] = minimal Hilbert-Schmidt distance of a state ρ to the pointer state
 ☆ Q[ρ] = 0 for a classical state

#### How "quantum" is the FMO exciton?

Nalbach, Braun, Thorwart, Phys. Rev. E (2011)



## Is the FMO exciton dynamics non-Markovian?

Mujica-Martinez, Nalbach, Thorwart, Phys. Rev. E (2013)

- Non-Ohmic spectral density: Exciton dynamics could be non-Markovian
- Quantify this by non-Markovianity measure
   Breuer et al., PRL 2009
- based on trace distance between two mixed states:

$$D(\rho_1, \rho_2) = \frac{1}{2} \operatorname{tr} |\rho_1 - \rho_2|$$
 with  $|O| = \sqrt{O^{\dagger}O}$ 

- measures distinguishability of quantum states
- $0 \le D \le 1$
- Markovian dynamics: any two different initial states become less distinguishable with time and approach to the steady state,
   i.e., information flows from system to environment
- non-Markovian dynamics: states may also evolve away from each other,
   i.e., information backflow from bath to system, i.e, D increases with time

#### Is the FMO exciton dynamics non-Markovian?

• non-Markovianity measure:

Mujica-Martinez, Nalbach, Thorwart, Phys. Rev. E (2013)

$$\mathcal{N}(\Phi) = \sum_{i} \left[ D\left(\rho_1(b_i), \rho_2(b_i)\right) - D\left(\rho_1(a_i), \rho_2(a_i)\right) \right]$$

with sum over those time intervals  $(a_i, b_i)$ , over which D increases

• Result: non-Markovianity is zero at low and room temperature:



 same when vibrational mode is artificially set in resonance with exciton transitions

## Summary up to this point

- Theoretical calculations:
  - no exceptionally long coherence (certainly not at room temperature)
  - Excitons delocalized over a few sites at low and room temperatures
  - Exciton dynamics fully Markovian
- BUT: experiment predicts long-lived quantum coherence
- So: Let us repeat the experiment!





FMO complex

☆ Experiment repeated at room temperature
 ☆ 2D ultrafast optical spectroscopy
 ☆ compared with theoretical model
 ☆ 2D: phase-matching approach
 Gelin, Egorova, Domcke, J. Chem. Phys (2005)



FMO complex



Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

#### Laser excitation strength low enough





#### **FMO: Theory: Spectral density**



#### Time evolution of coherence signal: electronic dephasing time ~ 60 fs



Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Self-consistence check:

Anti-diagonal width = electronic dephasing time ~ 60 fs



Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Cross peak at spectral position of Ref. [1]:



[1] G. Panitchayangkoon et al. (G. Engel), PNAS 107, 12766 (2010)

Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Comparison with independent previous exact calculation of Ref. [2]



No signatures of long-lived coherence

[2] Nalbach, Braun, Thorwart, Phys. Rev. E (2011)

Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Calculate low temperature T=77 K and compare to exp. of Ref. [3]



[3] Zigmantas et al., J. Chem. Phys. Lett. (2016)

Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Calculate low temperature T=77 K and compare to exp. of Ref. [3]



Agreement: No signatures of long-lived coherence even at 77 K

[3] Zigmantas et al., J. Chem. Phys. Lett. (2016)

Duan, Prokhorenko, Cogdell, Ashraf, Stevens, Thorwart, Miller, PNAS 114, 8493 (2017)

Simulate long-lived electronic coherence: choose weak damping by hand



Unrealistically small anti-diagonal width, sharp ridge in 2D spectrum (clear difference to experiment)

# 2) Role of vibrations & Vibronic coherence







Duan, Nalbach, Prokhorenko, Mukamel, Thorwart, NJP 17, 072002 (2015)





Other molecular complexes:

- ☆ Cyanine dye:
  - much smaller (dimer), strong electron-nuclear coupling
  - ☆ no protein environment
  - ☆ no conformational statistics
  - ☆ no long-lived electronic coherence
  - ☆ vibronic coherence lasts longer

Duan, Nalbach, Prokhorenko, Mukamel, Thorwart, NJP 17, 072002 (2015)



$$H_{\text{mono}} = H_g + H_e = |g\rangle h_g \langle g| + |e\rangle (h_e + E) \langle e|$$
$$h_g = \Omega \left( b^+ b + 1/2 \right) \qquad h_e = \Omega \left( b^+ b + 1/2 \right) + g(b^+ + b)$$

$$H_{SB} = |e\rangle\langle e|\xi_{\rm el}(t) + (b^+ + b)\xi_{\rm vib}(t)$$

Duan, Nalbach, Prokhorenko, Mukamel, Thorwart, NJP 17, 072002 (2015)



$$\begin{aligned} H_{\rm dim} &= \sum_{\alpha,\beta=g,e} |\alpha\beta\rangle (h_{\alpha}^{A} + h_{\beta}^{B}) \langle\alpha\beta| + |ge\rangle U \langle eg| + |eg\rangle U \langle ge| \\ h_{g}^{A/B} &= \Omega \left( b_{A/B}^{+} b_{A/B} + 1/2 \right) \\ h_{e}^{A/B} &= E + \Omega \left( b_{A/B}^{+} b_{A/B} + 1/2 \right) + g \left( b_{A/B}^{+} + b_{A/B} \right) \end{aligned}$$



Duan, Nalbach, Prokhorenko, Mukamel, Thorwart, NJP 17, 072002 (2015)

Duan, Nalbach, Prokhorenko, Mukamel, Thorwart, NJP 17, 072002 (2015)

Generic picture (= orthodox picture):

- Electronic coherence
  - short-lived (<100 fs)</li>
  - large amplitudes
  - rapid decay

- Vibrational coherence
  - longer lived (few ps)
  - small amplitudes
  - slow decay



## Are these results generic?

Other biomolecular complexes: LHII complex under ambient conditions:
no long lived quantum coherence
Duen Stevens Nelbert Therwort Drekberenke

Duan, Stevens, Nalbach, Thorwart, Prokhorenko, Miller, J. Phys. Chem. B **119**, 12017 (2015)



PSII Reaction Center under ambient conditions:

☆ no long lived quantum coherence

Duan, Prokhorenko, Wientjes, Croce, Thorwart, Miller, Sci. Rep. **7**, 12347 (2017)

Consistent with very recent experiment with FMO: ☆ purely vibrational coherence, 240 fs @77 K ☆ no long lived electronic coherence

Thyrhaug et al. (Zigmantas), Nature Chem. (2018)





FMO complex

# Are these results generic?

| sample                    | Lifetime (fs) | Reference                                                                    |
|---------------------------|---------------|------------------------------------------------------------------------------|
| LHCII                     | ~65 fs        | JPCB <b>119</b> , 12017 (2015).                                              |
| Reaction center           | ~56 fs        | Sci. Rep. <b>7</b> , 12347 (2017).                                           |
| Indocarbocyanine<br>dimer | ~50 fs        | New J. Phys. <b>17</b> , 072002 (2015).<br>Nat. Chem. <b>6</b> , 196 (2014). |
| FMO                       | ~60 fs        | PNAS <b>114</b> , 8493 (2017).                                               |
| Perovskite<br>solar cell  | ~45 fs        | ACS Photonics <b>5</b> , 852 (2018)                                          |

# Can long-lived vibrational coherence enhance electronic coherence?

Nalbach, Thorwart, J.Phys. B: At. Mol. Opt. Phys. 45, 154009 (2012)

Can we use long-lived vibrations to induce long-lived coherence in excitons? ("vibrational laser" drive?)

• Again FMO complex, QUAPI plus nontrivial environment (MD simulations)



Olbrich, Kleinekathoefer, Schulten et al., J. Chem. Phys. Lett. (2011)

each vertically shifted relative to the next by 25 cm<sup>-1</sup>

# Can long-lived vibrational coherence enhance electronic coherence?

Nalbach, Thorwart, J.Phys. B: At. Mol. Opt. Phys. 45, 154009 (2012)

Can we use long-lived vibrations to induce long-lived coherence in excitons? ("vibrational laser" drive?)



# Can long-lived vibrational coherence enhance electronic coherence?

Nalbach, Thorwart, J.Phys. B: At. Mol. Opt. Phys. 45, 154009 (2012)

Can we use long-lived vibrations to induce long-lived coherence in excitons? ("vibrational laser" drive?)



#### All sites:

No coherent oscillations in the populations for full spectral density

# Can electronic coherence enhance vibrational coherence?

Duan, Thorwart, Miller, in review (2018)

#### Electronic resonance with anticorrelated pigment vibrations drives photosynthetic energy transfer outside the adiabatic framework

Vivek Tiwari, William K. Peters, and David M. Jonas<sup>1</sup>

PNAS | January 22, 2013 | vol. 110 | no. 4 | 1203-1208



Pullerits, Zigmantas, Sundström, PNAS 2013



correlated & anti-correlated mode important

 $q_+ = q_A + q_B \quad q_- = q_A - q_B$ 

- vibrational levels on different potential energy surfaces are in resonance
- strong non-adiabatic coupling / mixing
- with strong electronic character of the vibronic, exact non-adiabatic levels

strong coherent electronic admixture enhances amplitude of anti-correlated vibrations

## **Can electronic coherence enhance vibrational coherence?**

Duan, Thorwart, Miller, in review (2018)



#### electronic character of vibronic states

- Idea is in principle correct!
- weight of the electronic component of the vibronic state increases for increasing vibronic coupling

BUT: Fast electronic dephasing destroys coherent coupling in the electronic sector?

Yeh, Hoehn, Allodi, Engel, Kais, PNAS 2018 Weak electronic dephasing
### **Can electronic coherence enhance vibrational coherence?**

Duan, Thorwart, Miller, in review (2018)



## **Can electronic coherence enhance vibrational coherence?**

Duan, Thorwart, Miller, in review (2018)

#### Wave packet tracking on potential energy surfaces

E A в 2000 2000 2000 2000 0.1 0.08 0.15 0.15 0.06 0.1 1500 0.1 1500 1500 1500 0.05 0.04 0.05 0.05 0.02 (s) 1000 (E) 1000 0 1000 0 0 1000 -0.02 -0.05 -0.05 -0.04 -0.05 -0.1 500 -0.1 500 500 500 -0.06 -0.15 -0.15 -0.08 -0.1 -2 2 -2 0 2 -2 0 0 2 2 -2 0 Q Q Q Q off-resonant resonant 0.8 tiny vibrational oscillations 0.8 C G 0.6 30.6 d p (t) 0.4 0.4 0.2 0.20 400 200 400 800 600 1000 0 200 600 800 1000 t (fs) t (fs) Amp (arb. units) 3000 Amp (arb.units) 000 000 000 000 D н 500 1000 1500 2000 200 400 600 800 1000 ω (cm<sup>-1</sup>)  $\omega$  (cm<sup>-1</sup>)

2) Strong (realistic) electronic dephasing from experiment

## **Can electronic coherence enhance vibrational coherence?**

Duan, Thorwart, Miller, in review (2018)

#### Wave packet tracking on potential energy surfaces $6 \frac{10^{4}}{10^{4}}$ Vibrational amplitude weak damping 5 strong damping - - fit 1 --fit 2 weak dephasing 3 strong dephasing 0.2 0.3 0.4 0.5 0.1 0.6 0 **Electron-vibrational coupling**

Conclusion: 1) Picture does not apply under realistic conditions of electronic dephasing 2) same for resonant / off-resonant conditions (not shown here)

Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

- So far:
  - No enhancement of life time of electronic coherence by long-lived vibrational coherence
  - No enhancement of vibrational amplitude due to large amplitude electronic coherence
  - What about effect on efficiency of energ< transfer?
- Non-adiabatic vibration as part of the system (initially not in equilibrium!)
- Cases: Vibrational mode coupled to
  - FMO exit site (= site 3)
  - FMO site 1 (Berkeley experiment)
  - all sites



Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

- Couple an energy sink to exit site 3, no back transfer allowed
- Efficieny of energy transfer given by time constant of its rise kinetics



Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

• Underdamped vibrational mode opens up more transfer channels



Vibrational mode at all sites:

- no enhanced coherence
- speed up of transfer!

Transfer times (300K):

site 1: 2.9 ps (-24%)

site 6: 2.8 ps (-18%)

Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

• Underdamped vibrational mode at entrance: energy is stuck in vibrations!



Vibrational mode at site 1:

- more beatings 1-2
- slow down of transfer!

Transfer times (300K):

site 1: 4.1 ps (+8%)

site 6: 3.6 ps (+6%)

Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

• Underdamped vibrational mode opens up more exit channels



Vibrational mode at site 3:

- no enhanced coherence
- speed up of transfer!

Transfer times (300K):

site 1: 2.7 ps (-29%)

site 6: 2.6 ps (-24%)

• YES!

Nalbach, Mujica-Martinez, Thorwart, PRE 91, 022706 (2015)

#### • More transfer channels open up

| Vibrational<br>mode coupled to | Initial excitation<br>at site | Transfer time<br>(ps) | Change<br>by |
|--------------------------------|-------------------------------|-----------------------|--------------|
| NO vibration                   | 1                             | 3.8                   | <del></del>  |
|                                | 6                             | 3.4                   |              |
| All sites                      | 1                             | 2.9                   | -24 %        |
|                                | 6                             | 2.8                   | -18 %        |
| Site 1                         | 1                             | 4.1                   | +8 %         |
|                                | 6                             | 3.6                   | +6 %         |
| Site 3                         | 1                             | 2.7                   | -29 %        |
|                                | 6                             | 2.6                   | -24 %        |

# 3) FRET in orthogonal dimers



Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)



- But: experimentally measured rather fast exciton transfer
- transfer time: 10 ps

Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)

- Idea: Orientational fluctuations of dipole moments in molecule induce coupling
- strength depends on the variance of orientational fluctuations
- effective phenomenological model: angles of rigid dipole moments fluctuate
- How to obtain spectral density of angular fluctuations?



• dipole coupling:

 $V = J(\sin \phi_2 \cos \alpha_2 - 2 \sin \phi_1 \cos \alpha_1)$ 

- We know energy fluctuations  $\delta E$  due to solvent for general arrangements.
- Onsager theory of solvation:

$$G(\omega) = \lambda \frac{2}{\pi} \frac{\omega \omega_c}{\omega^2 + \omega_c^2}$$

- typical values of reorg. energy:
  - in toluene:  $\lambda = 245 \, \mathrm{cm}^{-1}$
  - in chloroforme:  $\lambda = 133 \, \mathrm{cm}^{-1}$

Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)

- Scaling argument  $\frac{\langle (\delta E)^2 \rangle}{W} = \epsilon_{1,2} \frac{\langle (\delta z)^2 \rangle}{l^2}$  with typical energy/length scales
- Fluctuations of dipole coupling due to angle fluctuations  $\frac{\langle (\delta V)^2 \rangle}{W} = J \frac{\langle (\delta z)^2 \rangle}{12}$
- Corresponding spectral density of angle fluctuations:

$$G_{\phi}(\omega) = \lambda_{\phi} \frac{2}{\pi} \frac{\omega \omega_c}{\omega^2 + \omega_c^2} \qquad \qquad \lambda_{\phi} = \lambda \frac{J}{\epsilon_{1,2}}$$

- typical values of reorg. energy:  $0.5 \,\mathrm{cm}^{-1} \lesssim \lambda_\phi \lesssim 5 \,\mathrm{cm}^{-1}$
- FRET transfer rate by Fermi's Golden Rule:

 $\epsilon = 2448 \,\mathrm{cm}^{-1}$ 

$$\tau^{-1} = \frac{\pi}{2} G_{\phi}(\epsilon) [1 + n_B(\epsilon)]$$

Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)



## **Distance dependence of transfer time**

Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)

Idea: Add spacer molecules between donor & acceptor



• interpolation fits best:  $b_3 R_{DA}^3 + b_6 R_{DA}^6$ 

## **Temperature dependence**

Nalbach, Pugliesi, Langhals, Thorwart, PRL 108, 218302 (2012)

- Standard one-phonon process via Fermi Golden Rule: not optimal fit
- Mimic 3-phonon-process as a guess



#### Noticeable deviations from one-phonon process

Duan, Gerken, Langhals, Miller, Thorwart, unpublished (2019)

• include explicit vibrational effects (same model as before):

$$H_{\rm D} = E_{\rm D} |e_{\rm D}\rangle \langle e_{\rm D}| + \Omega_{\rm D} b_{\rm D}^{\dagger} b_{\rm D} + \lambda |e_{\rm D}\rangle \langle e_{\rm D}| \left( b_{\rm D}^{\dagger} + b_{\rm D} \right)$$
$$H_{\rm A} = E_{\rm A} |e_{\rm A}\rangle \langle e_{\rm A}| + \Omega_{\rm A} b_{\rm A}^{\dagger} b_{\rm A} + \lambda |e_{\rm A}\rangle \langle e_{\rm A}| \left( b_{\rm A}^{\dagger} + b_{\rm A} \right)$$

• fluctuating angles: von Mises-Fisher distribution (sphere!):

$$f(\theta, \phi) = \frac{\kappa}{4\pi \sinh \kappa} e^{\kappa \cos \theta}$$

Duan, Gerken, Langhals, Miller, Thorwart, unpublished (2019)



Duan, Gerken, Langhals, Miller, Thorwart, unpublished (2019)

• Calculate with the extracted parameters the 2D spectra:







#### Duan, Gerken, Langhals, Miller, Thorwart, unpublished (2019)

#### **Prediction**

Duan, Gerken, Langhals, Miller, Thorwart, unpublished (2019)



Strong vibronic coupling (not only fluctuating angles of dipole moments)!

### 4) Exciton transfer & conical intersections



### **Conical intersection: basics**

- Point of degeneracy between two adiabatic potential energy surfaces
- Breakdown of Born-Oppenheimer approximation => Formation of a double cone structure
- explains fast radiationless energy transfer in photoactive molecules
- key: geometric phase of the wavefunction acquired when circulated on a closed loop around the CI (nonadiabatic feature)





F. Temps, Universität Kiel

M.A. Robb, What is a conical intersection, UC London



Duan, Thorwart, J. Chem. Phys. Lett. 7, 382 (2016)

Conical intersection of two electronic potential surfaces

#### Question:

Signatures of CI in presence of strong vibrational dissipation?

$$H_{\rm mol} = H_g + H_e$$

Model: two electronic states plus two harmonic modes:

$$H_g = |g\rangle (h_1 - \epsilon/2) \langle g| \text{ and } H_e = |e\rangle (h_2 + \epsilon/2) \langle e| + (|e\rangle V \langle g| + h.c.)$$
$$h_g = \frac{1}{2} \sum_{i=1,2} \Omega_i (P_i^2 + Q_i^2)$$
plus two Ohmic vibrational baths
$$h_1 = h_g - \kappa Q_2 \quad h_2 = h_g + \kappa Q_2$$



Model: two electronic states plus two harmonic modes:

$$\begin{split} H_g &= |g\rangle (h_1 - \epsilon/2) \langle g| \quad \text{and} \quad H_e = |e\rangle (h_2 + \epsilon/2) \langle e| + (|e\rangle V \langle g| + h.c.) \\ h_g &= \frac{1}{2} \sum_{i=1,2} \Omega_i (P_i^2 + Q_i^2) \\ \text{plus two Ohmic vibrational baths} \\ h_1 &= h_a - \kappa Q_2 \quad h_2 = h_a + \kappa Q_2 \end{split}$$



with two effective baths:

$$J_{1/2}^{\text{eff}}(\omega) = \lambda_{1/2} \frac{\gamma_{1/2} \Omega_{1/2}^2 \omega}{(\Omega_{1/2}^2 - \omega^2)^2 + \gamma_{1/2}^2 \omega^2}$$

overdamped limit easy:  $J_{1/2}^{\text{eff},\infty}(\omega) = \lambda'_{1/2} \frac{\gamma'_{1/2}\omega}{\omega^2 + \gamma'^2_{1/2}}$ 



Duan, Thorwart, J. Chem. Phys. Lett. 7, 382 (2016)

#### unitary equivalent to

#### 1D problem





with two effective baths:

$$J_{1/2}^{\text{eff}}(\omega) = \lambda_{1/2} \frac{\gamma_{1/2} \Omega_{1/2}^2 \omega}{(\Omega_{1/2}^2 - \omega^2)^2 + \gamma_{1/2}^2 \omega^2}$$

overdamped limit easy:  $J_{1/2}^{\text{eff},\infty}(\omega) = \lambda'_{1/2} \frac{\gamma'_{1/2}\omega}{\omega^2 + \gamma'^2_{1/2}}$ 

Duan, Thorwart, J. Chem. Phys. Lett. 7, 382 (2016)

Dissipative quantum two-level system,

#### BUT...

- highly structured bath spectral densities
- non-Ohmic
- => non-Markovian exciton dyanmics
- require advanced numerical tools (QUAPI, hierarchy equation of motion HEOM)
- but they are available!

Duan, Thorwart, J. Chem. Phys. Lett. 7, 382 (2016)

Calculated 2D spectra



#### **Overdamped**

Splitting of the diagonal peak as clear signature of CI in a 2D spectrum!



 $\lambda'_1 = 150\pi \text{ cm}^{-1}, \gamma'_1 = 150 \text{ cm}^{-1}, \lambda'_2 = 150\pi \text{ cm}^{-1}, \gamma'_2 = 20 \text{ cm}^{-1}, \mathcal{T} = 300 \text{ K}$ 

Duan, Thorwart, J. Chem. Phys. Lett. 7, 382 (2016)

Transient absorption spectrum





Two bands with positive & negative amplitude

$$\epsilon = 1000 \text{ cm}^{-1},$$
  
$$\lambda'_1 = 150\pi \text{ cm}^{-1}, \gamma'_1 = 150 \text{ cm}^{-1}, \lambda'_2 = 150\pi \text{ cm}^{-1}, \gamma'_2 = 20 \text{ cm}^{-1}, \mathcal{T} = 300 \text{ K}$$

Duan, Miller, Thorwart, J. Chem. Phys. Lett. 7, 3491 (2016)



#### Quantum yield



Duan, Miller, Thorwart, J. Chem. Phys. Lett. 7, 3491 (2016)

#### Underdamped



Duan, Miller, Thorwart, J. Chem. Phys. Lett. 7, 3491 (2016)

750

T (fs)

1500

 $\omega$  (cm<sup>-1</sup>)

1250

2500

1500

3000

1000

2000



#### Underdamped

Stueckelberg oscillations

#### Quantum yield



Duan, Miller, Thorwart, J. Chem. Phys. Lett. 7, 3491 (2016)

#### Underdamped

Quantum yield

Table 1. Magnitude for Pathways A (1100 cm<sup>-1</sup>) and B (1600 cm<sup>-1</sup>) for Waiting Time T = 5 ps in the Transient Absorption Spectrum with Stable Products . and B<sup>a</sup>

| $\gamma_{c/t}$ (cm <sup>-1</sup> ) | magnitude A | magnitude B | A/(A + B) | $\Gamma_f^{-1}$ (fs) |
|------------------------------------|-------------|-------------|-----------|----------------------|
| 30                                 | -0.1001     | -0.0630     | 0.614     | 285                  |
| 40                                 | -0.1065     | -0.0690     | 0.607     | 268                  |
| 50                                 | -0.1143     | -0.0767     | 0.598     | 240                  |
| 100                                | -0.1601     | -0.1206     | 0.570     | 200                  |
| 150                                | -0.1961     | -0.1672     | 0.540     | 195                  |

"The ratio A/(A + B) shows that the more coherent wave packet produces a higher quantum yield than the less coherent one. In addition, we show the inverse of the isomerization rate  $\Gamma_y$ .

## **Peierls vs. Holstein phonons: conical intersection**

photonicswiki.org

Duan, Nalbach, Miller, Thorwart, submitted (2018)

 $H = \sum_{i} \epsilon_i a_i^{\dagger} a_i + \sum_{i} t_{ij} a_i^{\dagger} a_j$ 

Consider usual tight-binding model of an electron:

Let us include electron-phonon-coupling:

local electron-phonon coupling (intramolecular phonon)

$$\epsilon_i = \varepsilon_{i,0} + \sum_{\alpha} \left( \frac{\partial e_i}{\partial Q_{\alpha}} \right)_0 Q_{\alpha} + \dots$$

Holstein phonon



nonlocal electron-phonon coupling (<u>inter</u>molecular phonon)

$$t_{ij} = t_{ij,0} + \sum_{\alpha} \left( \frac{\partial t_{ij}}{\partial Q_{\alpha}} \right)_{0} Q_{\alpha} + \dots$$

Peierls phonon



## **Peierls vs. Holstein phonons: conical intersection**

Duan, Nalbach, Miller, Thorwart, submitted (2018)

Idea:

- Understand intramolecular (Holstein) mode as tuning mode in a CI picture
- Understand intermolecular (Peierls) mode as coupling mode in a CI picture



Nice, since

Q intra (tuning)

- it explains ultrafast exciton transfer in organic photovoltaics
- experimentally confirmed in singlet fission in a pentacene film (unpublished)

#### **Peierls vs. Holstein phonons: conical intersection**

Duan, Nalbach, Miller, Thorwart, submitted (2018)

Model Hamiltonian

$$H_{\rm mol} = |A\rangle h_A \langle A| + |B\rangle h_B \langle B| + (|A\rangle V \langle B| + h.c.)$$

with

$$h_A = \epsilon_A + h_g - \kappa Q_{\rm H}, \quad h_B = \epsilon_B + h_g + \kappa Q_{\rm H}$$
$$V = V_0 + \Lambda Q_{\rm P}$$
tuning mode  
coupling mode

Peierls & Holstein phonons:

$$h_g = \frac{1}{2} \sum_{i=\mathrm{H,P}} \Omega_i (P_i^2 + Q_i^2)$$

standard Ohmic vibrational baths:

 $J_{\rm P/H}(\omega) = \eta_{\rm P/H}\omega\exp(-\omega/\omega_c)$ 

$$H_{\rm env} = \sum_{i={\rm H,P}} \sum_{\alpha} \left[ \frac{p_{i,\alpha}^2}{2m_{i,\alpha}} + \frac{m_{i,\alpha}\omega_{i,\alpha}^2}{2} \left( x_{i,\alpha} + \frac{c_{i,\alpha}Q_i}{m_{i,\alpha}\omega_{i,\alpha}^2} \right)^2 \right]$$
## **Peierls vs. Holstein phonons: conical intersection**

Duan, Nalbach, Miller, Thorwart, submitted (2018)

Exciton transfer dynamics (with time-nonlocal quantum master equation):



# **Peierls vs. Holstein phonons: conical intersection**

Duan, Nalbach, Miller, Thorwart, submitted (2018)



### Nice, since

- it helps to understand ultrafast exciton transfer in organic photovoltaics
- experimentally confirmed in singlet fission in a pentacene film (unpublished)

## Conclusions

### Quantum transfer of excitation energy / exciton transport



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DAAD





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Thank you for your attention