

# Noisy Quantum Measurements: a nuisance or fundamental physics?

Wolfgang Belzig  
Universität Konstanz

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# Content

- Quantum measurement: projection and weak measurements
- Quantum dynamics: Keldysh contour
- Facets of weak quantum measurements
  1. Time-resolved counting statistics and quantum transport
  2. Keldysh-ordered expectations are quasiprobabilities
  3. Time-reversal symmetry breaking
  4. General non-markovian weak measurement

Quantum measurement and correlations?      Order of operators matters!

$$\langle a(t)b(s) \rangle \xrightarrow{?} \begin{cases} i\langle [\hat{B}(s), \hat{A}(t)] \rangle / 2 \\ \langle \hat{B}(s)\hat{A}(t) \rangle \\ \langle \hat{A}(t)\hat{B}(s) \rangle \\ \langle \{\hat{B}(s), \hat{A}(t)\} \rangle / 2 \end{cases}$$

Textbook (LL Vol. V):

The operators  $\hat{x}(t)$  and  $\hat{x}(t')$  relating to different instants do not in general commute, and the correlation function must now be defined as

$$\phi(t' - t) = \frac{1}{2}[\overline{\hat{x}(t)\hat{x}(t')} + \overline{\hat{x}(t')\hat{x}(t)}], \quad (121.9)$$

Quantum optics:

photodetector measures ,normal ordered' expectations (one click)  
homodyning and heterodyning are highly specific

## Von Neumann measurement: from strong to weak

Idea: couple system ( $\hat{A}$ ) to a pointer wavefunction  $\sqrt{P(x)}$

$$|\psi_i\rangle \otimes \sqrt{P(x)}$$

$$|\psi_i\rangle = \alpha_1 |A_1\rangle + \alpha_2 |A_2\rangle$$

$$\hat{U}_{int} = e^{ig\hat{p}\hat{A}}$$

$$\rightarrow \alpha_1 |A_1\rangle \sqrt{P(x + gA_1)} + \alpha_2 |A_2\rangle \sqrt{P(x + gA_2)}$$

Strong measurement (large g): projective measurement on well separated pointer positions implies projection of system state

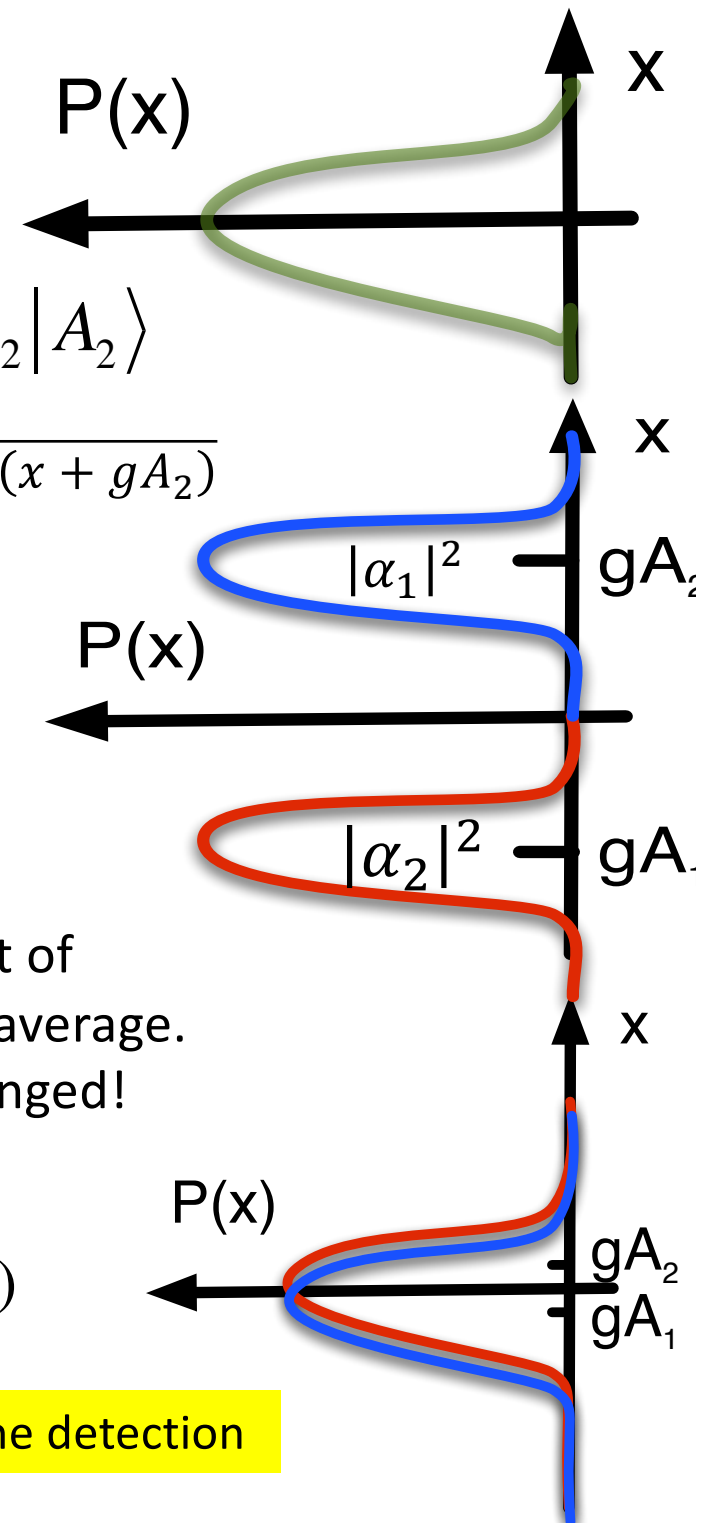
$$|\psi_f\rangle = |A_1\rangle \quad \text{or} \quad |\psi_f\rangle = |A_2\rangle$$

Weak measurement (small g): projective measurement of pointer state gives almost no information, but correct average. The system state in one measurement is almost unchanged!

After reading the pointer

$$|\psi_f\rangle \approx \alpha_1 |A_1\rangle + \alpha_2 |A_2\rangle + O(g^2)$$

Price to pay for non-invasiveness: large uncertainty of the detection



## Quantum dynamics: time evolution of a quantum system

$i\hbar \frac{\partial}{\partial t}  \Psi(t)\rangle = \hat{H}  \Psi(t)\rangle$	$\longrightarrow$	$ \Psi(t)\rangle = U_{\rightarrow}(t)  \Psi(0)\rangle$ <p><b>Forward</b> time-evolution</p>
$-i\hbar \frac{\partial}{\partial t} \langle\Psi(t)  = \langle\Psi(t) \hat{H}$	$\longrightarrow$	$\langle\Psi(t)  = \langle\Psi(0) U_{\leftarrow}(t)$ <p><b>Backward</b> time-evolution</p>

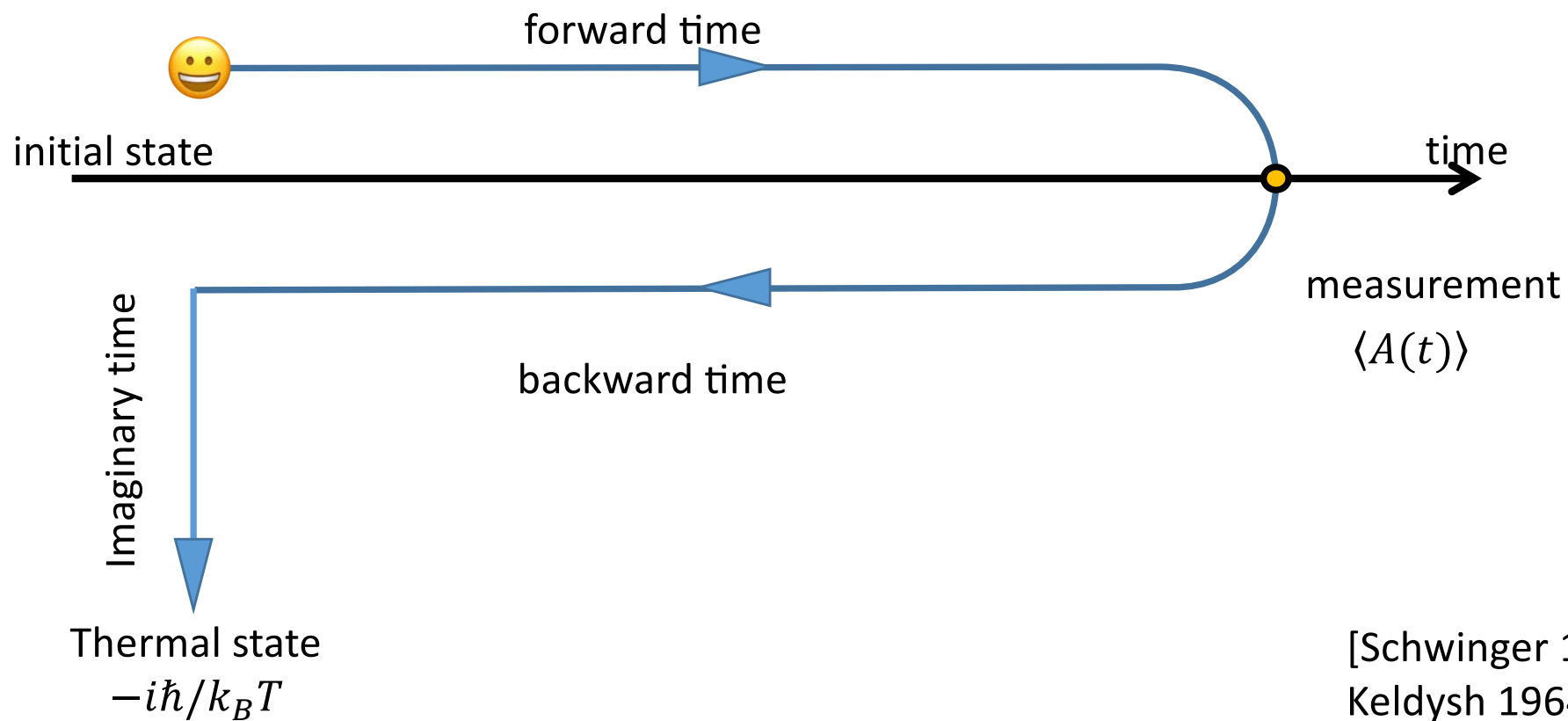
Physical expectations

$$\langle A(t) \rangle = \langle \Psi(t) | \hat{A} | \Psi(t) \rangle = \langle \Psi(0) | U_{\leftarrow}(t) \hat{A} U_{\rightarrow}(t) | \Psi(0) \rangle$$

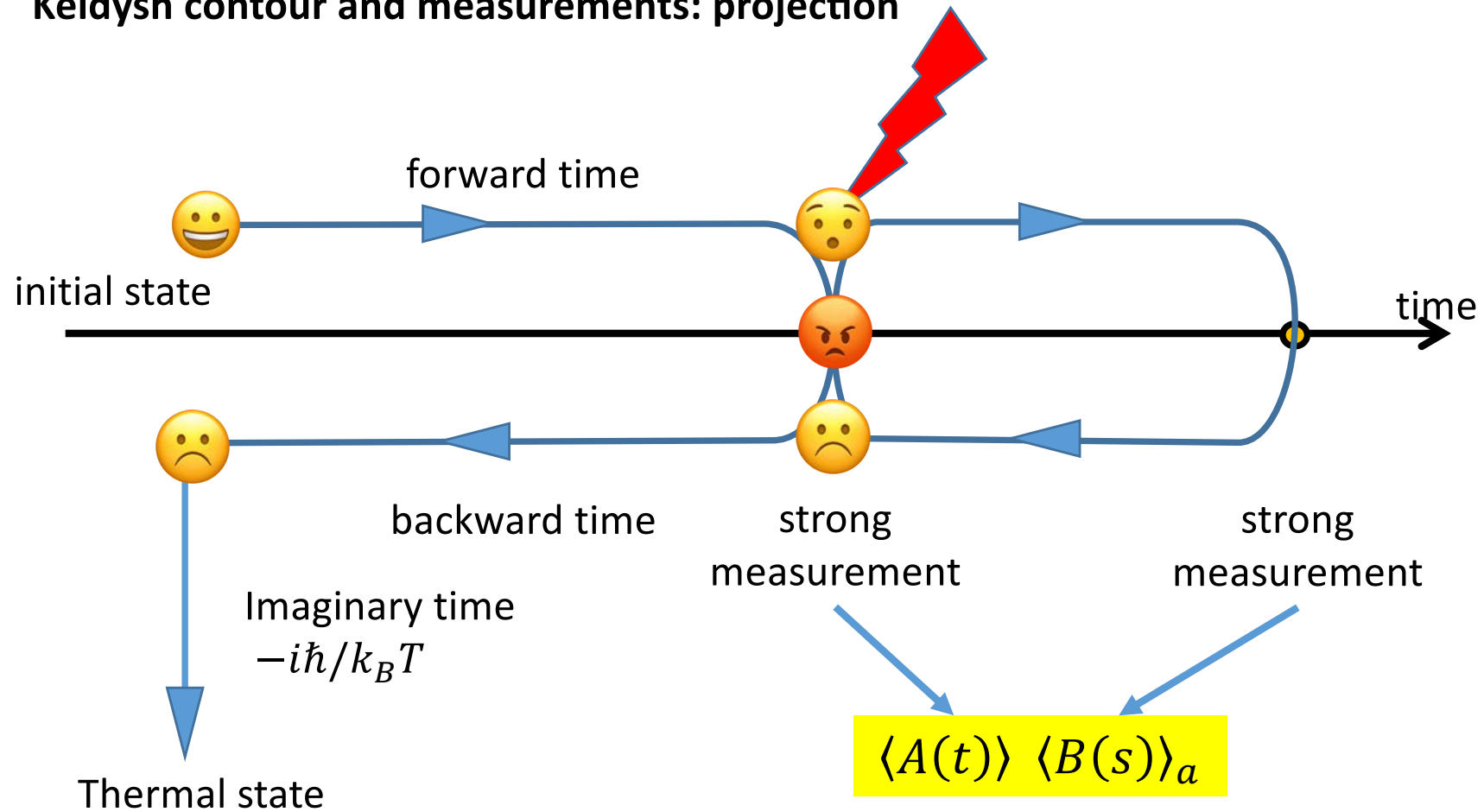
**Backward** and **Forward**  
time-evolution

Quantum dynamics requires **Forward** and **Backward** time-evolution  $\rightarrow$  **Keldysh contour**

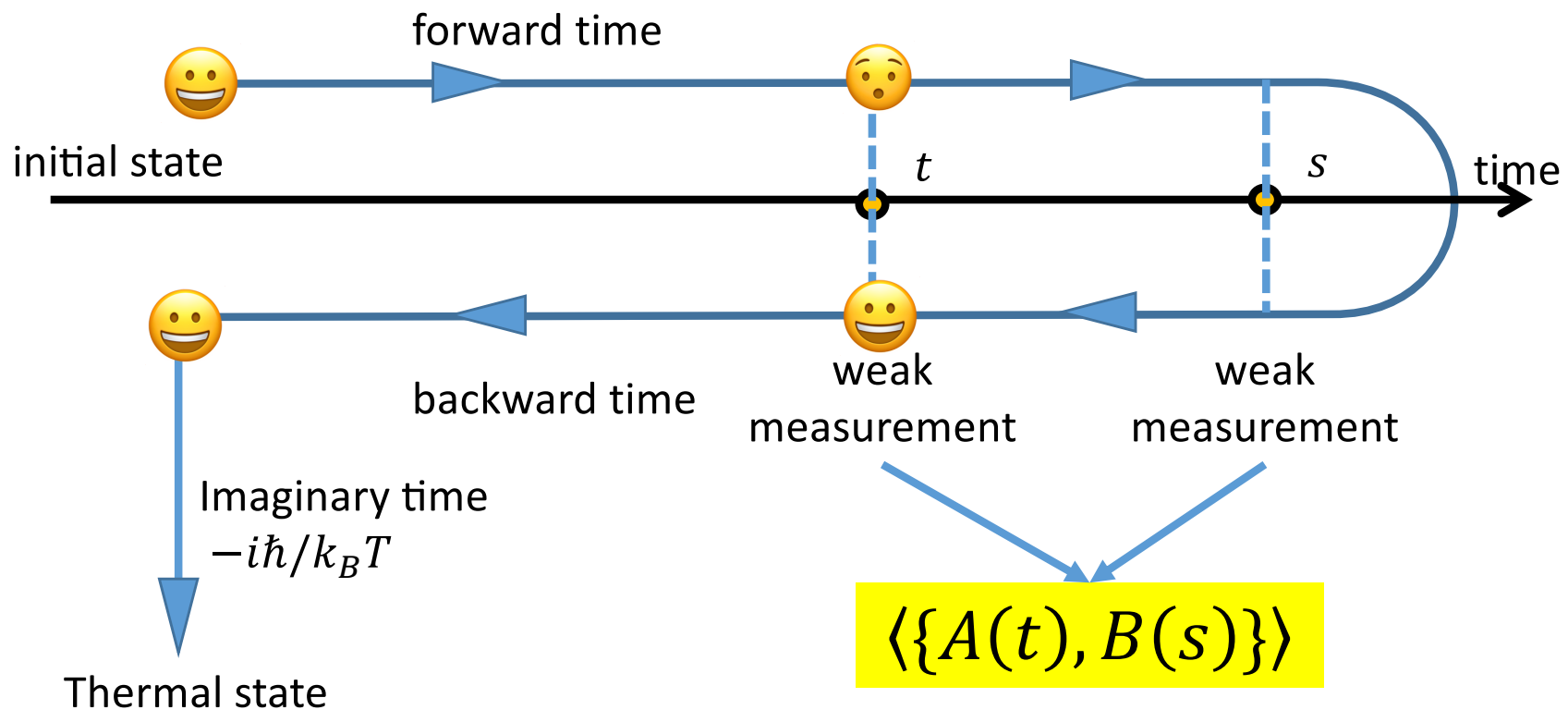
## The Keldysh contour: expanding the time dimension



## Keldysh contour and measurements: projection

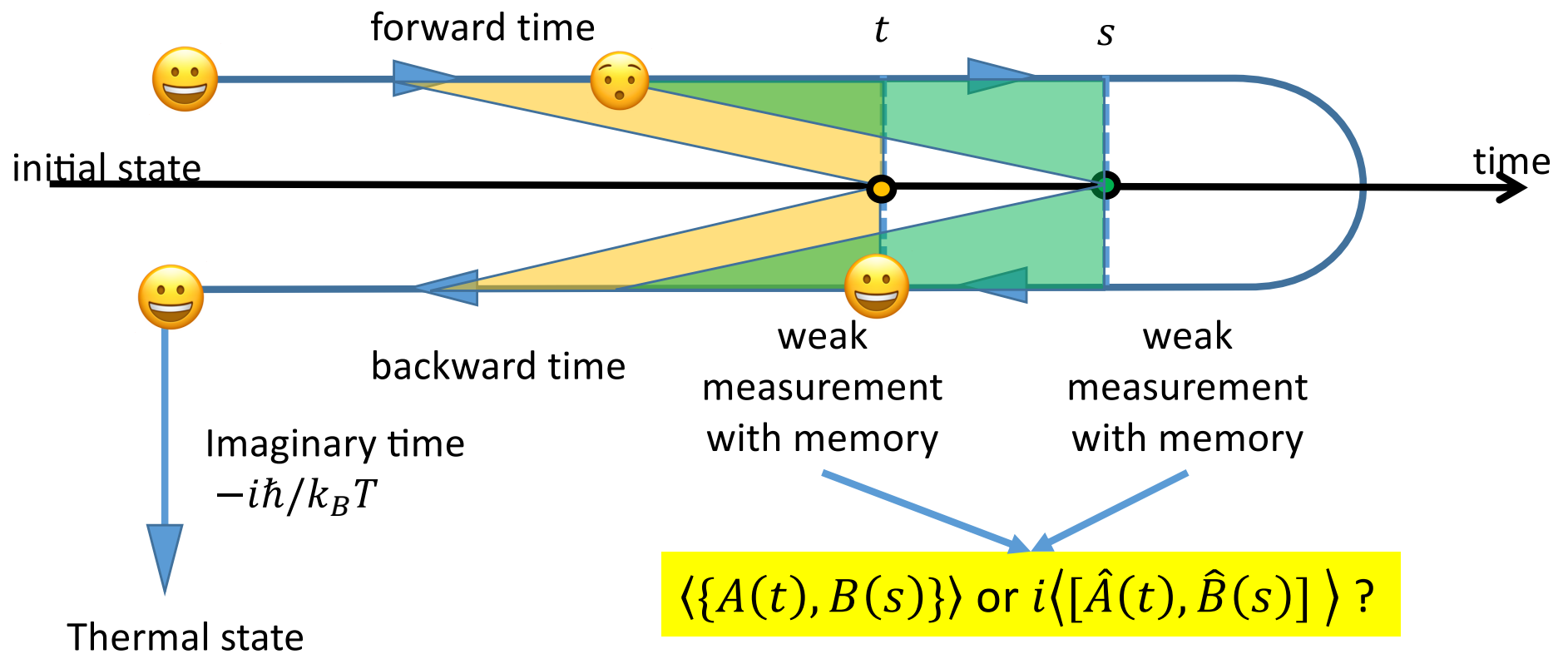


## Keldysh contour and measurements: weak and markovian (instantaneous)





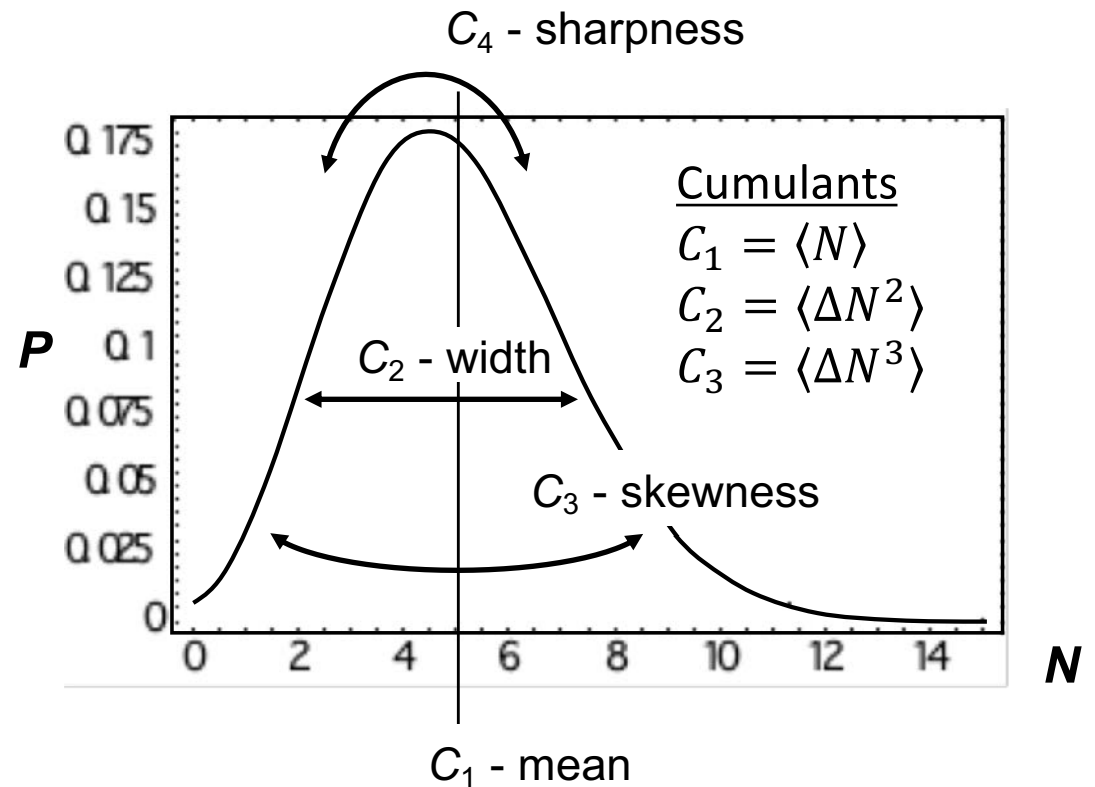
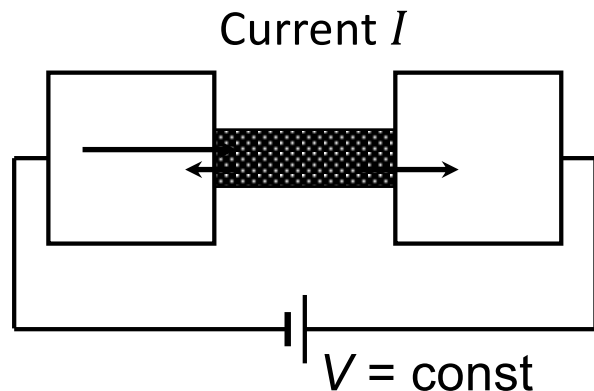
## Keldysh contour and measurements: weak and continuous



# 1) Full Counting Statistics:

$$P_{t_0}(N)$$

Probability that a total charge  $Ne$  is transferred in given time  $t_0$



(Quantum) definition  $N = \int dt I(t)$

Definition through Cumulant Generating function (CGF):

$$P(N) = \int d\chi e^{iN\chi} e^{-S(\chi)}$$

$$e^{-S(\chi)} = \langle e^{i\hat{N}\chi} \rangle = \langle e^{i\chi \int dt \hat{I}(t)} \rangle$$

Is this correct in the quantum case?

## How to calculate the CGF quantum mechanically?

Quantum mechanical current detection has to account for non-commuting current operators!

$$[\hat{I}(t), \hat{I}(t')] \neq 0$$

$$e^{S(\chi)} = \text{Tr} \left[ \hat{\rho} \tilde{T} e^{i \frac{\chi}{2} \int_0^{t_0} \hat{I}(t) dt} \mathcal{T} e^{i \frac{\chi}{2} \int_0^{t_0} \hat{I}(t) dt} \right]$$

Belzig, Nazarov, PRL 2001

Microscopic justification: time evolution of **ideal current detector** and **projective measurement** [Levitov et al. 1997; Kindermann, Nazarov 2003].  
(Projection can be problematic for superconductors, due to charge-phase uncertainty [Belzig, Nazarov, PRL 2001])

### Important difference

to classical definition

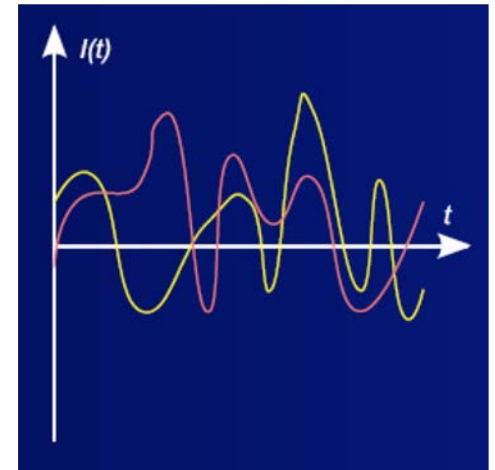
(see also Levitov, Lesovik 93)

$$e^{S_{cl}(\chi)} = \left\langle e^{i\chi \int_0^{t_0} \hat{I}(t) dt} \right\rangle$$

## Generalization of FCS to Time-Dependent Counting:

“Probability” density functional for given current profile  $I(t)$ :

$$\varrho[I] = \int \mathcal{D}\chi \, e^{S[\chi] - \frac{i}{e} \int dt \chi(t) I(t)}$$



Inverse transformation

$$e^{S[\chi]} = \int \mathcal{D}I \, \varrho[I] e^{\frac{i}{e} \int dt \chi(t) I(t)} \equiv \left\langle e^{\frac{i}{e} \int dt \chi(t) I(t)} \right\rangle_{\varrho}$$



classical average

[c.f. stochastic path integral Sukhorukov, Jordan, et al. 2003]

Keldysh ordered!

Quantum definition of CGF for time-independent FCS

$$e^{S(\chi)} = \text{Tr} \left[ \hat{\rho} \tilde{\mathcal{T}} e^{i \frac{\chi}{2} \int_0^{t_0} \hat{I}(t) dt} \mathcal{T} e^{i \frac{\chi}{2} \int_0^{t_0} \hat{I}(t) dt} \right]$$

Generalization of standard Keldysh functional **to time dependent counting**

$$e^{S[\chi]} = \text{Tr} \left[ \hat{\rho} \tilde{\mathcal{T}} e^{\frac{i}{2e} \int dt \chi(t) \hat{I}(t)} \mathcal{T} e^{\frac{i}{2e} \int dt \chi(t) \hat{I}(t)} \right]$$

[see also e.g. Nazarov and Kindermann, PRL 2004]

Can we interpret this as probability density generating functional?

**No!** Analogous to Wigner function we can have **negative probabilities**

Problem: current operators at different times do **not commute**

The current cannot be measured at all times, but only up to some uncertainty

## Handling non-projective (weak) measurements:

Orthogonal measurements

$$\{\hat{P}_A = |A\rangle\langle A|\} \sum_A \hat{P}_A = \hat{1}$$

$$\hat{P}_A \hat{P}_B = \hat{P}_A \delta_{A,B}$$

Probability to find A

$$p_A = \text{Tr} \hat{\rho} \hat{P}_A$$

State after measurement

$$\hat{\rho}_A = \hat{P}_A \hat{\rho} \hat{P}_A / p_A$$

Non-projective measurements:

$$\text{Kraus operators } \{\hat{K}_A\} \quad \hat{F}_A = \hat{K}_A^\dagger \hat{K}_A$$

$$\sum_A \hat{K}_A \hat{K}_A^\dagger = \hat{1}$$

$$\hat{F}_A \hat{F}_B \neq \hat{F}_A \delta_{A,B}$$

**P**ositive  
**O**perator  
**V**alued  
**M**easure

$$p_A = \text{Tr} \hat{\rho} \hat{K}_A^\dagger \hat{K}_A$$

$$\hat{\rho}_A = \hat{K}_A \hat{\rho} \hat{K}_A^\dagger / p_A$$

**Neumarks Theorem:** Every POVM corresponds to a projective measurement in some extended Hilbert space

See e.g. Milburn & Wiseman, Quantum Measurement and Control (Cambridge, 2009)

**Proposed solution:** weak Markovian measurement a la **POVM**

Kraus operator (instead of projection operator) for Markovian measurement

$$\hat{K}[I] = \int \mathcal{D}\varphi \mathcal{T} e^{\int dt \frac{i}{\epsilon} \varphi(t) (\hat{I}(t) - I(t)) - \frac{\varphi^2(t)}{\tau}}$$

Causality

Noise of the detector  
+ uncertainty

Positive operator valued probability measure (=projection in extended space)

Neumarks theorem

**Positive definite probability distribution:**

$$\rho[I] = \text{Tr} \left[ \hat{\rho} \hat{K}^\dagger[I] \hat{K}[I] \right]$$

## Final result for current generating functional

Generalized Keldysh functional

$$\varphi(\varphi^\dagger) = \phi \pm \chi/2$$

$$e^{S[\chi, \phi]} = \text{Tr} \left[ \hat{\rho} \tilde{\mathcal{T}} e^{\frac{i}{e} \int dt (\chi(t)/2 + \phi(t)) \hat{I}(t)} \mathcal{T} e^{\frac{i}{e} \int dt (\chi(t)/2 - \phi(t)) \hat{I}(t)} \right]$$

Current generating functional with additional backaction and noise due to detector

$$e^{S[\chi]} = \int \mathcal{D}\phi e^{S[\chi, \phi]} e^{-\int dt (2\phi^2(t) + \chi^2(t)/2)/\tau}$$

Gaussian noise of the detector

Backaction of the detector (partial projection)

Limiting cases:

$\tau \rightarrow \infty$  full projection  $\longrightarrow$  Strong backaction

$\tau \rightarrow 0$  large detector noise  $\longrightarrow$  Weak measurement

$$\varrho[I] = \int \mathcal{D}\chi e^{-i \int dt \chi(t) I(t)} \Phi[\chi] \quad \text{with} \quad \Phi[\chi] = e^{S[\chi, 0] + \int dt \chi^2(t)/2\tau} \quad \tau \rightarrow 0: \text{Large Gaussian noise subtracted!}$$



## Generalized Wigner functional

WB and Y. V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001)

Phys. Rev. Lett. **87**, 067006 (2001)

A. Bednorz and WB, PRL (2008,2010)

The generating function of a **markovian** quantum measurement is Keldysh-ordered:

$$\Phi[\chi] = e^{S[\chi,0]-S_{det}} = \langle \tilde{\mathcal{T}}[e^{i\int dt \chi(t)\hat{A}(t)}] \mathcal{T}[e^{i\int dt \chi(t)\hat{A}(t)}] \rangle$$

Quasiprobability density generating functional!

Analogously to Wigner function we can have **negative probabilities**

The generating function of a **non-markovian** quantum measurement is ...

... (even) more complicated

The **answer** to the question of operator order:  $C \sim \delta^2 \Phi[\chi] / \delta \chi(t) \delta \chi(s)$

Markovian:

$$\langle a(t)a(s) \rangle \rightarrow \langle \{\hat{A}(s), \hat{A}(t)\} \rangle / 2$$

Higher order Markovian:

$$\langle abc \rangle \rightarrow \frac{1}{4} \langle \{ \hat{A}, \{ \hat{B}, \hat{C} \} \} \rangle$$

Non-Markovian:

$$\langle a(t)a(s) \rangle \rightarrow g \otimes \langle \{\hat{A}(t), \hat{A}(s)\} \rangle + f \otimes \langle [\hat{A}(t), \hat{A}(s)] \rangle$$

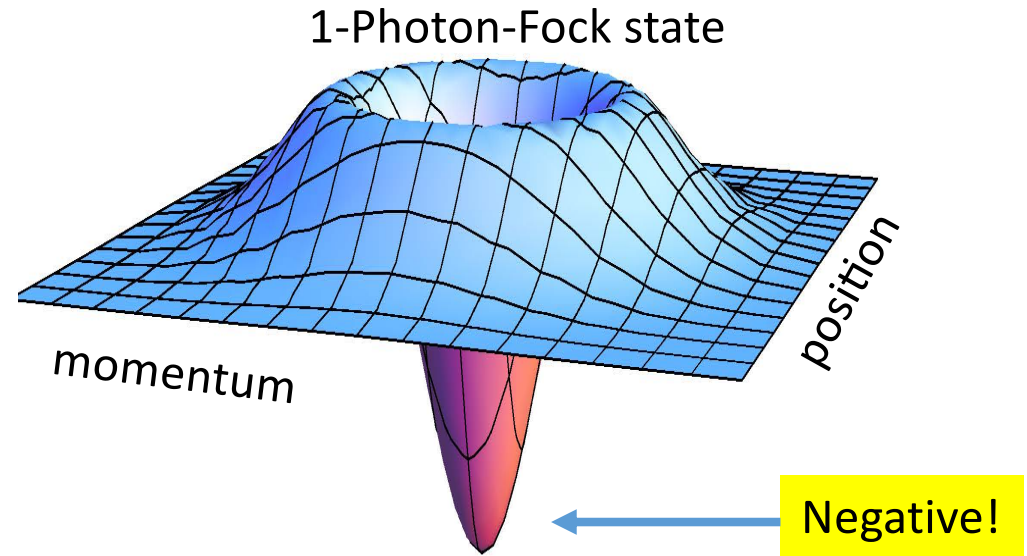
$g, f$  depend on the detector, but arbitrary ordering possible ( $\rightarrow$  engineering)

## 2) Keldysh-ordered expectations are quasiprobabilities

Quasiprobability?

Example:

Wigner-function  $W(x, p)$   
= “Probability” for  $x$  **and**  $p$



Cannot be measured directly, but through a **noisy and weak measurement**

Signatures of negativity (=non-classicality)?

**Violation** of classical inequalities, e.g. Bell, CHSH, Leggett-Garg, weak values....

# Weak positivity of the Wigner-Keldysh quasiprobability

Weak markovian measurement scheme:

[Bednorz & Belzig, PRB 2011]

$$C_{ij} = \langle A_i A_j \rangle = \frac{1}{2} \langle \{ \hat{A}_i, \hat{A}_j \} \rangle = \text{positive definite correlation matrix}$$

C can be simulated by classical probability distribution, e.g.

$$p(A_1, A_2, \dots) \sim e^{-\sum_{ij} A_i C_{ij}^{-1} A_j / 2} \geq 0$$

With symmetrized **second order** correlation functions a violation of classical inequalities is impossible  $\rightarrow$  the corresponding quasiprobability is **weakly positive**

Note: does not assume dichotomy, corresponding e.g. to  $\langle (A^2 - 1)^2 \rangle = 0$

Possible inequality → Cauchy-Bunyakowski-Schwarz (CBS) inequality

$$\langle X^2 \rangle \langle Y^2 \rangle \geq \langle XY \rangle^2$$

→ Fullfilled for all **positive** probabilities  $P(X, Y)$

# Test of CBS with Wigner functional for current fluctuations

Current operator in frequency space:  $\hat{I}_\omega = \int dt e^{i\omega t} \hat{I}(t)$

We choose:  $\hat{X} = \int_{\omega_X - \Delta_X/2}^{\omega_X + \Delta_X/2} d\omega \delta \hat{I}_\omega \delta \hat{I}_{-\omega}$  and  $\hat{Y} = \dots$

→ measurement bandwidth  $\Delta_{X/Y}$  centered at  $\omega_{X/Y}$

$$\langle XY \rangle = \Delta_X \Delta_Y \langle \delta \hat{I}_{\omega_X} \delta \hat{I}_{-\omega_X} \delta \hat{I}_{\omega_Y} \delta \hat{I}_{-\omega_Y} \rangle$$

$$\langle X^2 \rangle = \Delta_X^2 \left\langle (\delta \hat{I}_{\omega_X} \delta \hat{I}_{-\omega_X})^2 \right\rangle + \Delta_X \langle \delta \hat{I}_{\omega_X} \delta \hat{I}_{-\omega_X} \rangle^2$$

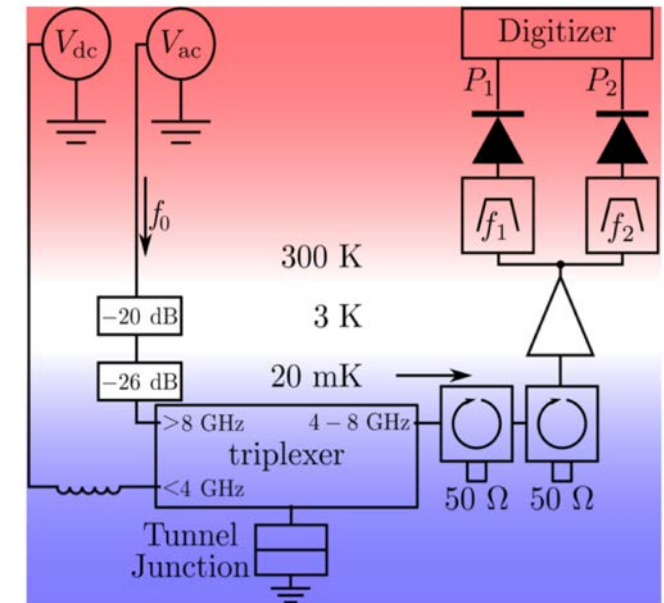
2<sup>nd</sup> and 4<sup>th</sup>-order correlators from tunnel Hamiltonian

$$H_T = \sum_{kq} t_{kq} c_{kL}^+ c_{qR} + h.c.$$

Violation of CBS would be a proof of negativity of Wigner functional!

$$? \quad \langle X^2 \rangle \langle Y^2 \rangle \geq \langle XY \rangle^2 \quad ?$$

Typical experimental setup



Forgues, Lupien, Reulet, PRL (2014)

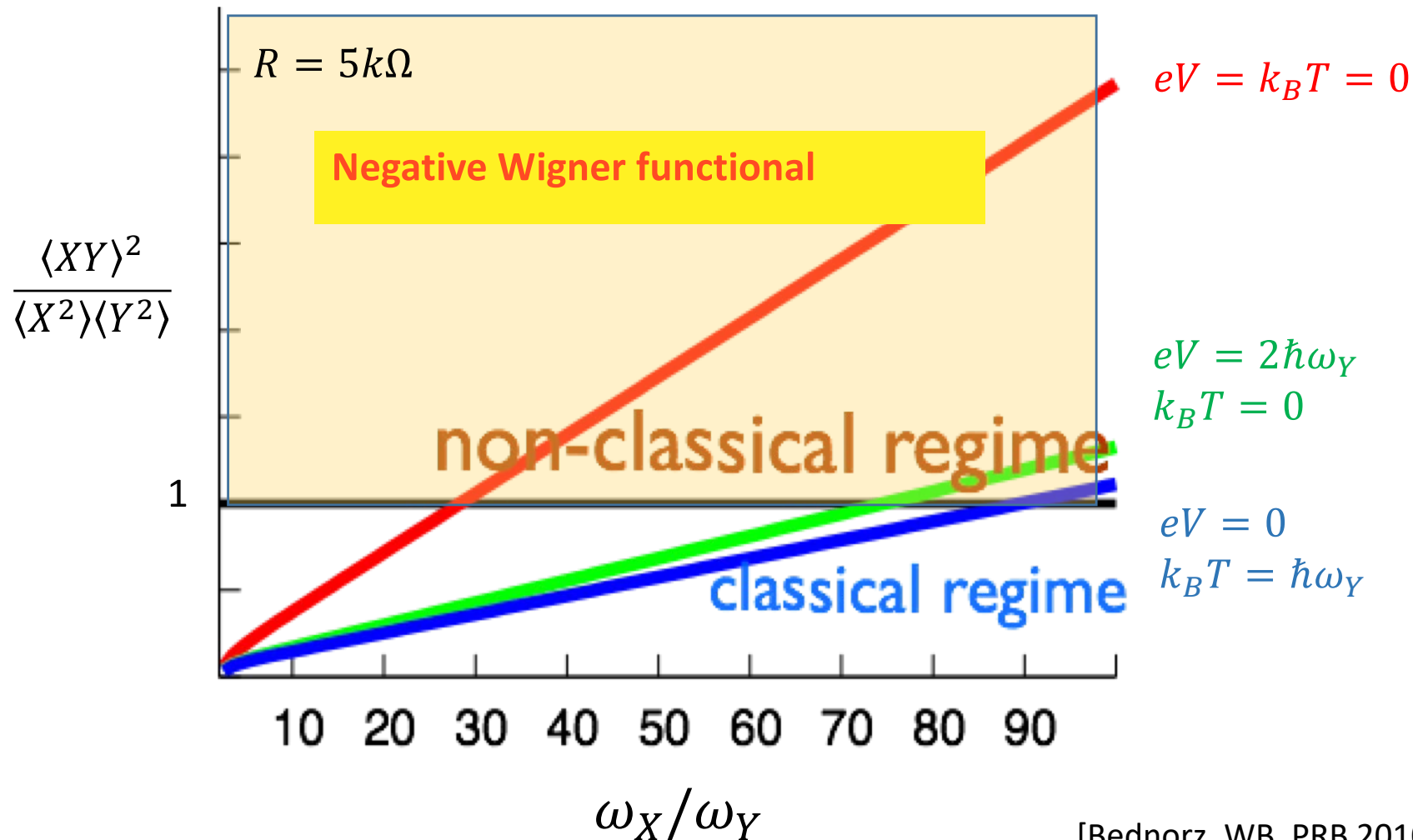
See also

Zakka-Bajjani et al. PRL (2010)

Bednorz and WB,  
Phys. Rev. Lett. **105**, (2010)  
Phys. Rev. B **81**, 125112 (2010)

## Violation of CBS for a tunnel junction

Maximally extended non-overlapping frequency intervals  $\omega_X \approx 2\Delta_Y + \Delta_X$ ,  $\omega_Y \approx \Delta_Y$



[Bednorz, WB, PRB 2010, PRL 2010]

Violation: Quantum many-body entanglement of electrons in different dynamical modes

E.g. nonequilibrium many-body wave function, Vanevic, Gabelli, Belzig, Reulet, PRB 2016

### 3) Time-reversal symmetry breaking

Does the observation of a system in thermal equilibrium show time-reversal symmetry ( $T$ )?

Measurement	Classical	Quantum
strong (invasive)	$T$ is broken (order of disturbances influences the dynamics)	$T$ is broken (order of projections influences the state)
weak (non invasive)	$T$ is observed (measurement is completely independent of the dynamics)	?

## Time-resolved weak measurements

Quantum prediction for three measurements?

$$A \rightarrow B \rightarrow C \longrightarrow \langle \{A, \{B, C\}\} \rangle$$

$\neq$

Opposite order:

$$C \rightarrow B \rightarrow A \longrightarrow \langle \{C, \{B, A\}\} \rangle$$

Three point correlator for  $t', t > 0$  (e.g. thermal equilibrium)

$$\langle \{A, \{A(t), A(t + t')\}\} \rangle \neq \langle \{A, \{A(t'), A(t + t')\}\} \rangle$$

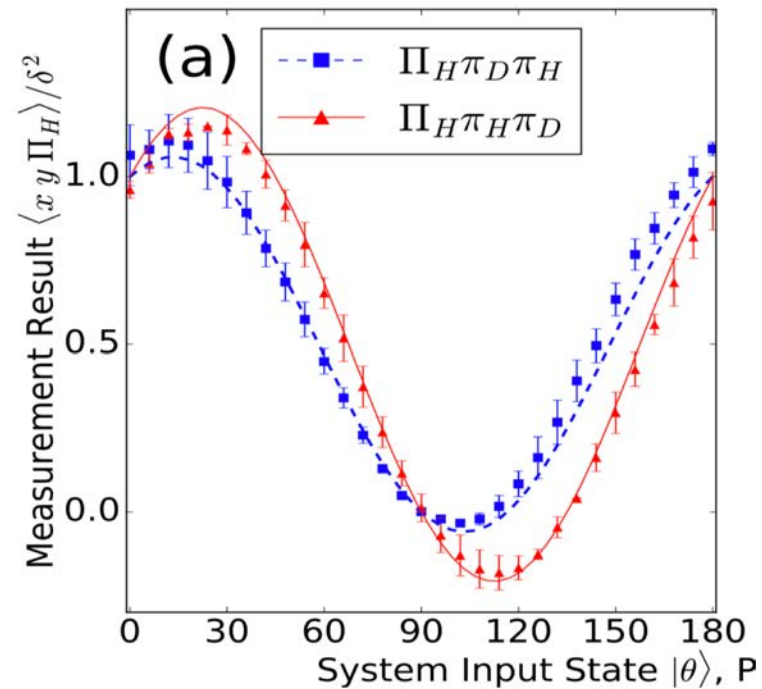
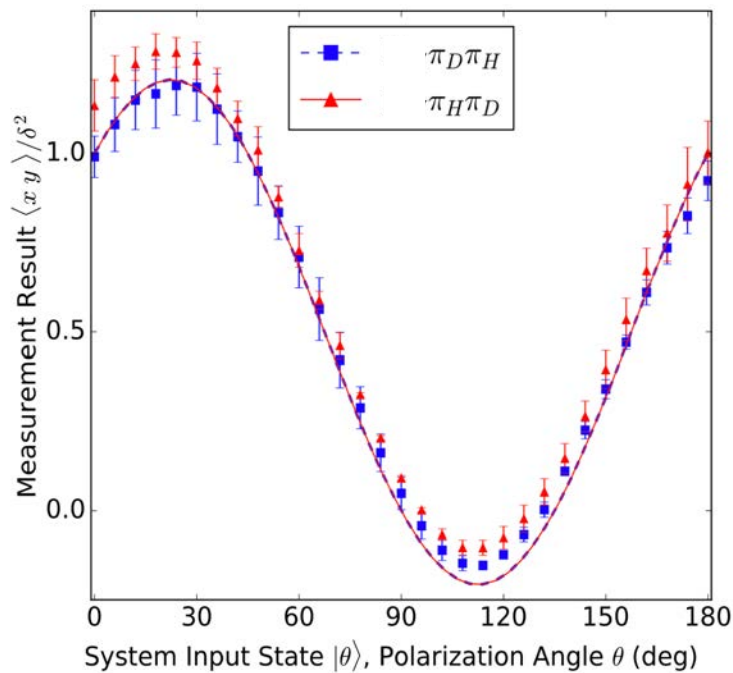
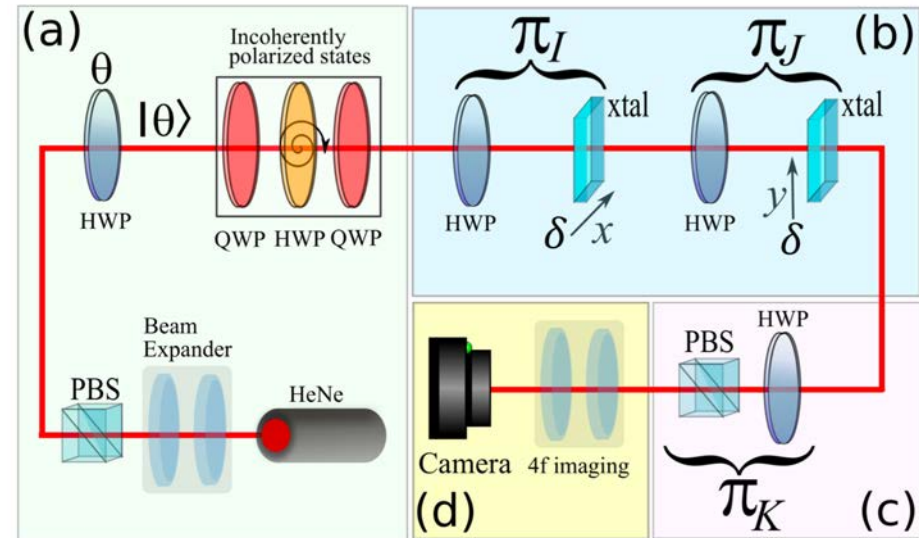
time-reversal (and shift by  $t + t'$ )

Classical expectation is not matched:  
A quantum system observed weakly in equilibrium  
**seemingly breaks time-reversal symmetry**



# Experimental confirmation that time-ordering matters in third order weak measurements

Curic, Richardson, Thekkadath, Flórez, Giner, Lundeen, Phys. Rev. A (2018)



$$\langle \{A, B\} \rangle = \langle \{B, A\} \rangle \xrightarrow{\text{+ third measurement}} \langle \{A, \{B, C\}\} \rangle \neq \langle \{B, \{A, C\}\} \rangle$$

## 4) General non-markovian weak measurement

The measured observable depends on the history!

A single measurement (of A):

$$\langle a(t) \rangle = \int_{-\infty}^t dt' g(t-t') \langle \hat{A}(t') \rangle$$

Two measurements (first A, then B)

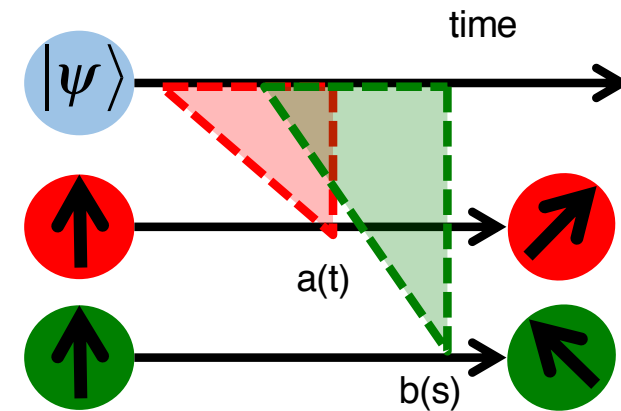
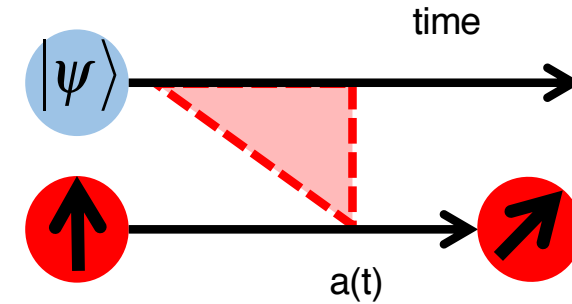
Derived using time-non-local Kraus operators

$$\langle a(t)b(s) \rangle = g \otimes \langle \{\hat{B}, \hat{A}\} \rangle(t, s) + f \otimes \langle [\hat{B}, \hat{A}] \rangle(t, s)$$

$\otimes$  = time convolution

memory functions

Standard Markovian



Result: Introducing memory function allows measurement of the commutator  
 → non-Markovian scheme

## Microscopic picture of non-Markovian weak measurements

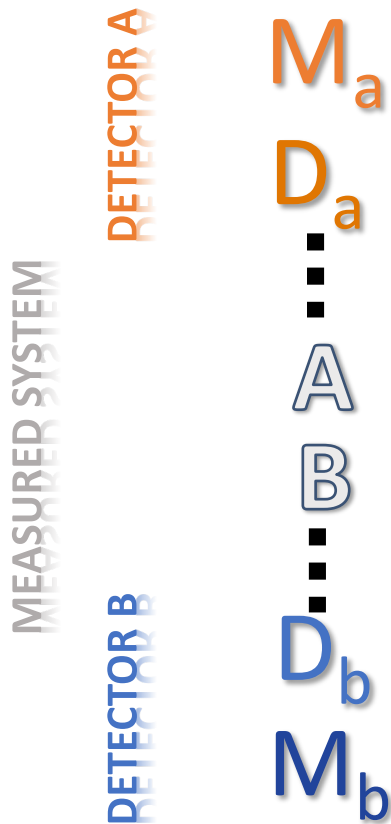
- One system, two detectors weakly coupled:  $\hat{H} = \hat{H}_{sys} + \hat{H}_a + \hat{H}_b + \hat{H}_{int}$
- Initial product state of the density matrices
- Unitary time evolution, interrupted by readout of the detectors (Kraus operators  $\rightarrow$  taken as weak measurements)
- Expansion of the time evolution to 2<sup>nd</sup> order in the coupling constant
- Final density matrix provides probability for the correlation function

Non-Markovian:  $\langle a(t)b(s) \rangle \rightarrow g \otimes \langle \{\hat{A}, \hat{B}\} \rangle(t, s) + f \otimes \langle [\hat{A}, \hat{B}] \rangle(t, s)$

Result: Separation into three processes  $C = \langle a(t)b(s) \rangle = C^{sym} + C_a^{det} + C_b^{det}$

J. Bülte, A. Bednorz, C. Bruder, and WB, Phys. Rev. Lett. **120**, 140407 (2018).

## Interaction Hamiltonian



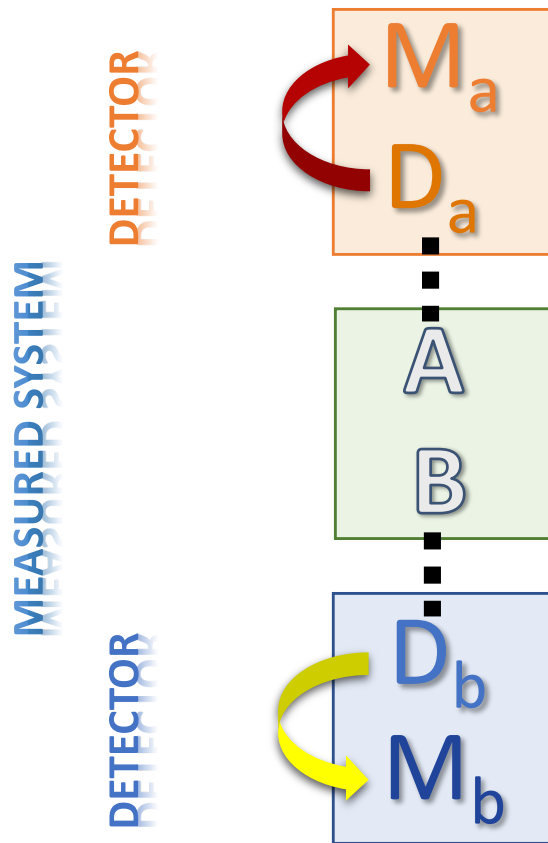
Interaction:

$$\hat{H}_{int} = \lambda_a \hat{D}_a \hat{A} + \lambda_b \hat{D}_b \hat{B}$$

The meter variables are  $\hat{M}_a(\hat{M}_b)$ :

$$C = \langle a(t)b(s) \rangle = \frac{1}{\lambda_a \lambda_b} \langle \{ \hat{M}_a(t), \hat{M}_b(s) \} \rangle$$

## Decomposition into elementary processes

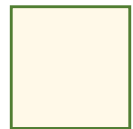


$$C = C^{\text{sym}} + C_a^{\text{det}} + C_b^{\text{det}}$$

All contributions are expressed by ( $\alpha = a, b, \text{sys}$ )

- Symmetrized noise

$$S_{XY}^{\alpha}(t, t') = \frac{1}{2} \langle \{ \hat{X}_{\alpha}(t), \hat{Y}_{\alpha}(t') \} \rangle$$

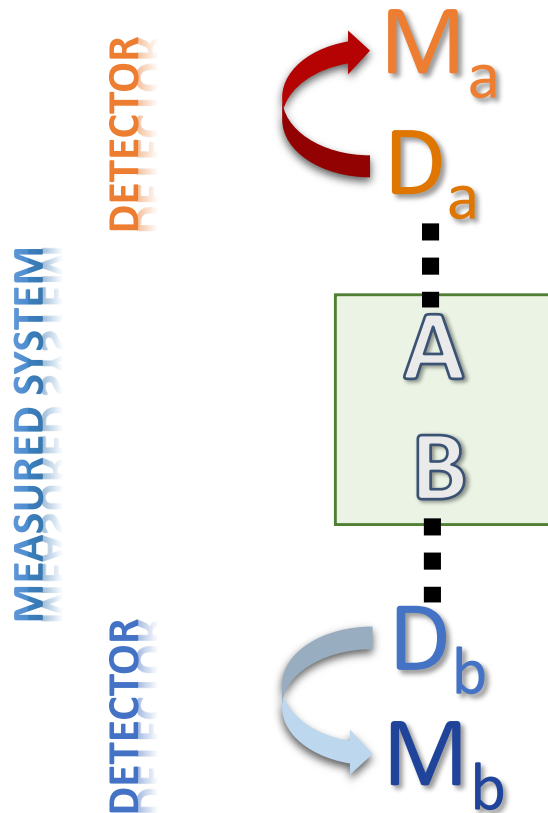


- Response function

$$\chi_{XY}^{\alpha}(t, t') = -\frac{i}{\hbar} \theta(t - t') \langle [\hat{X}_{\alpha}(t), \hat{Y}_{\alpha}(t')] \rangle$$



## The markovian (symmetrized) contribution

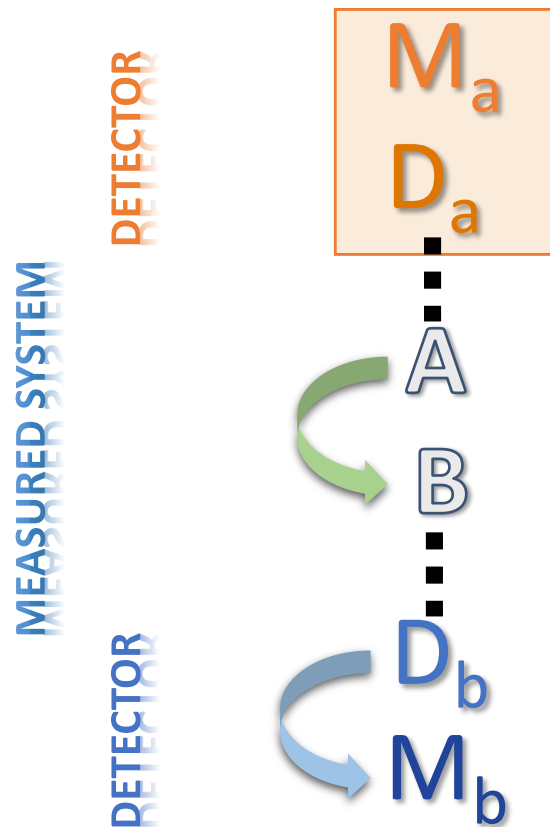


$$C^{\text{sym}} = \int dt dt' \chi_{MD}^a(t_a, t) \chi_{MD}^b(t_b, t') S_{AB}^0(t, t')$$

$$"g \otimes \langle \{\hat{A}, \hat{B}\} \rangle(t, s)"$$

→ Corresponds to classical frequency filter!

## The non-markovian (non-symmetrized) contribution



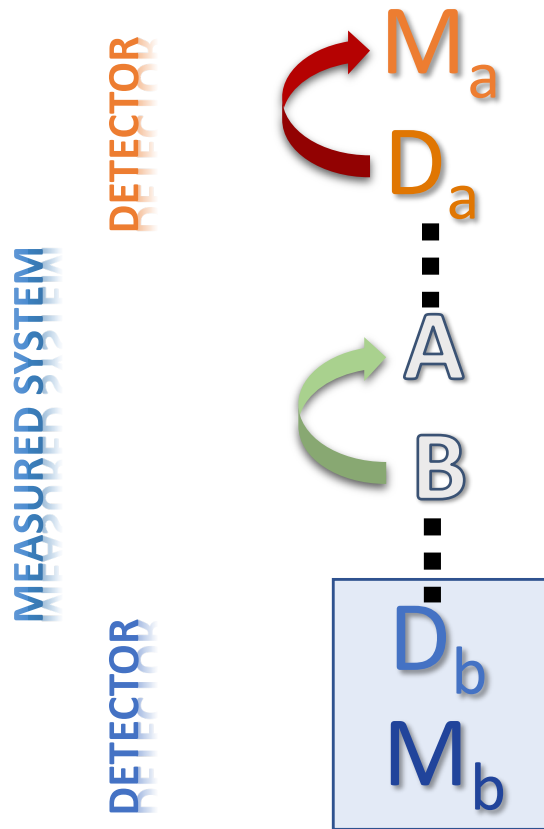
$$C_a^{\text{det}} = \int dt dt' S_{MD}^a(t_a, t) \chi_{MD}^b(t_b, t') \chi_{BA}^0(t', t)$$

$$\langle a(t)b(s) \rangle \sim f \otimes \langle [\hat{A}, \hat{B}] \rangle(t, s)$$

### System-mediated detector-detector interaction:

The noise of detector a measured by the response of the system seen by detector b.

## The non-markovian (non-symmetrized) contribution (part II)



$$C_b^{\text{det}} = \int dt dt' \chi_{MD}^a(t_a, t) S_{MD}^b(t_b, t') \chi_{AB}^0(t, t')$$

$$\langle a(t)b(s) \rangle \sim f \otimes \langle [\hat{A}, \hat{B}] \rangle(t, s)$$

The other way round.....

**System-mediated detector-detector interaction:**  
The noise of detector b measured by the response of  
of the system seen by detector a



## Result of microscopic treatment

Expressed by **noises** and **responses** of the **system** and the **detectors**:

Symmetrized noise

$$S_{sys} = \langle \{\hat{A}(s), \hat{B}(t)\} \rangle_0 / 2 \quad \text{or} \quad S_a = \langle \{\hat{M}_A, \hat{D}_A\} \rangle_A / 2$$

Response function

$$\chi_{sys} = i \langle [\hat{A}(s), \hat{B}(t)] \rangle_0 \quad \text{or} \quad \chi_a = i \langle [\hat{M}_A, \hat{D}_A] \rangle_A$$

$$\begin{aligned} C = \langle a(t)b(s) \rangle &= C^{sym} + C_a^{det} + C_b^{det} \\ &= \chi_a \chi_b \otimes S_{sys} + \chi_a \chi_{sys} \otimes S_b + \chi_b \chi_{sys} \otimes S_a \end{aligned}$$

Frequency-filtered  
markovian response

System-mediated  
detector-detector interaction

$$\langle a(t)b(s) \rangle$$

Detector engineering

$$\langle \{\hat{B}(s), \hat{A}(t)\} \rangle / 2$$

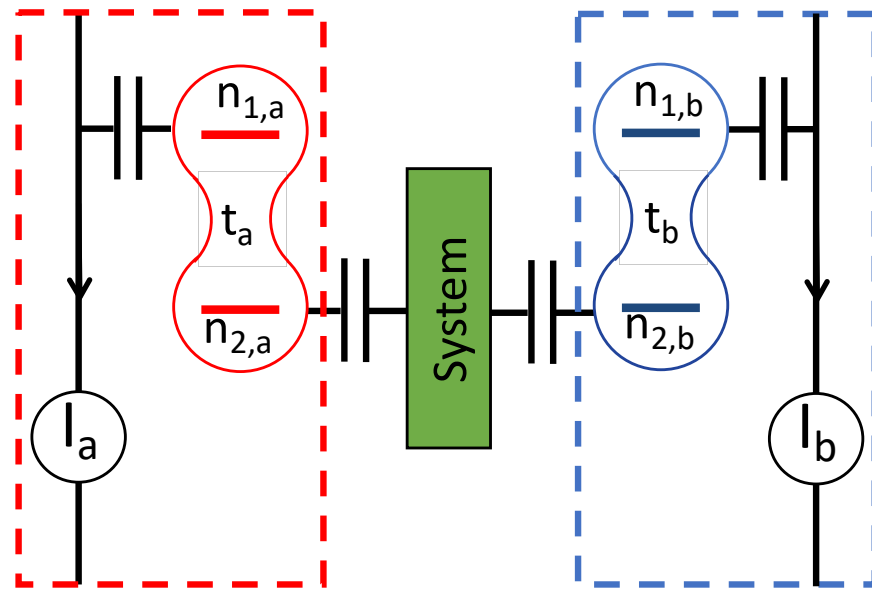
$$\xi_s \langle \{\hat{B}(s), \hat{A}(t)\} \rangle + i \xi_a \langle [\hat{B}(s), \hat{A}(t)] \rangle$$

$$i \langle [\hat{B}(s), \hat{A}(t)] \rangle / 2$$

Corresponds to a family of quasiprobabilities (Wigner, Q, P,...)

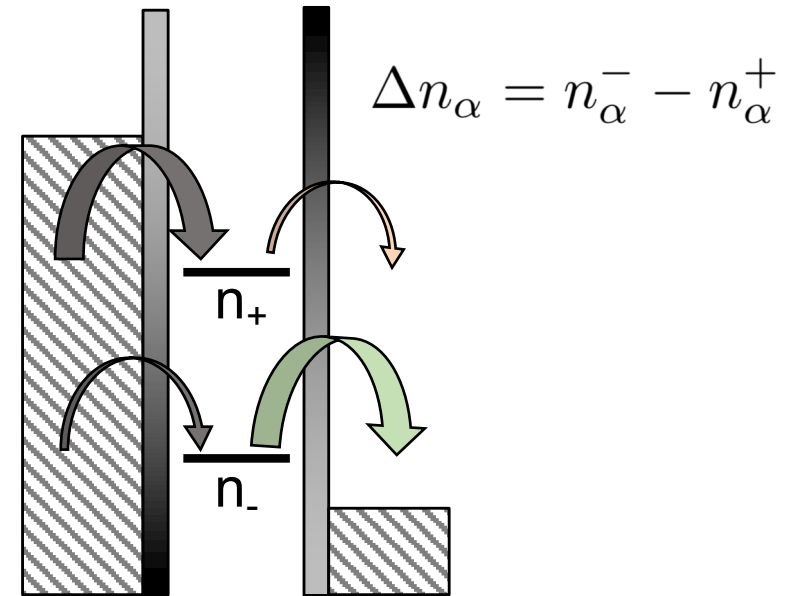
# Proposed implementation: Two double-dot detectors measuring a single quantum system

Occupation recorded by a bypassing current



$$\hat{H}_\alpha = \epsilon_\alpha \hat{\sigma}_z^\alpha + t_\alpha \hat{\sigma}_x^\alpha \quad \sigma_z^\alpha = n_{1,\alpha} - n_{2,\alpha}$$

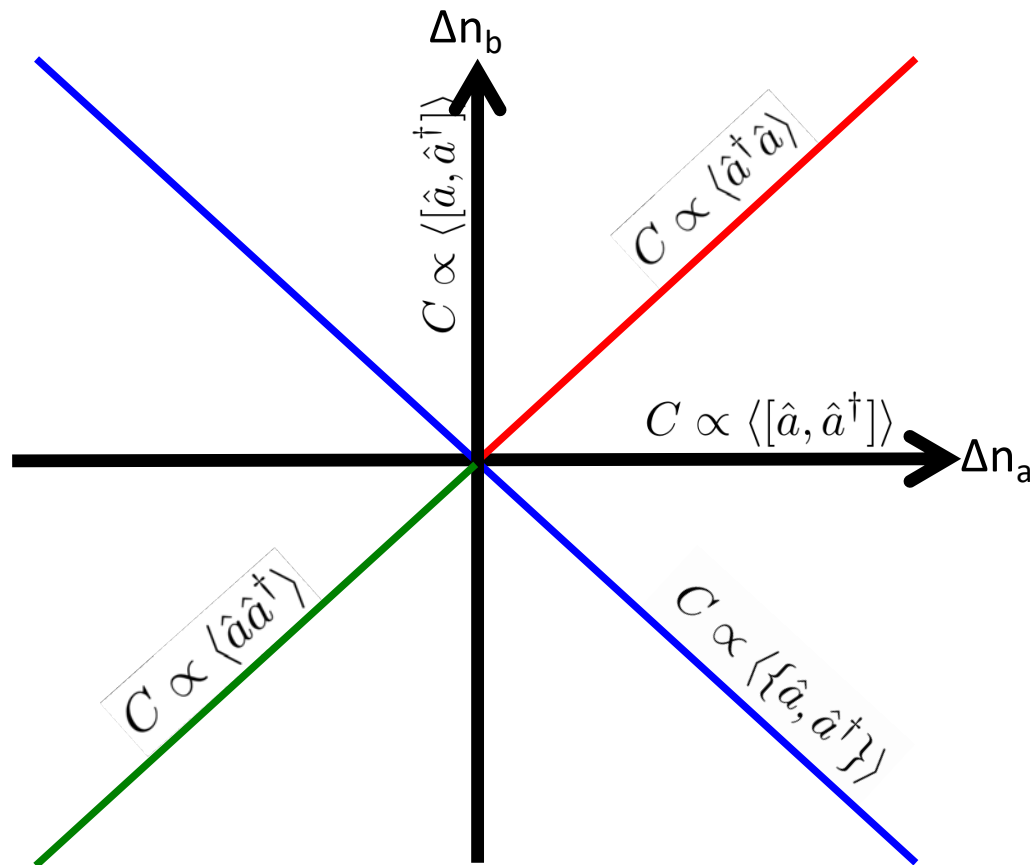
$$\hat{H}_{\text{int}} = \eta_a \hat{\sigma}_z^a \hat{A} + \eta_b \hat{\sigma}_z^b \hat{B}$$



- Double dot characterized by occupation difference of the energy eigen levels
- Tuning  $\Delta n_\alpha$  from positive to negative switches the detector from absorption to emission mode

c.f. double dot detectors Aguado, Kouwenhoven

Measurement of a bosonic system:  $a = \frac{1}{\sqrt{2}}(x + ip)$



$$C = C^{\text{sym}} + C_a^{\text{det}} + C_b^{\text{det}}$$

By tuning  $\Delta n_a$  and  $\Delta n_b$  different system operator orders are obtained

- **Wigner**
- **normal**
- **antinormal**
- **Kubo**

J. Bülte, A. Bednorz, C. Bruder, and W. Belzig,  
Phys. Rev. Lett. **120**, 140407 (2018).

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## The Quantum Transport Group with guests

**A. Bednorz**  
(Warsaw)

**C. Bruder**  
(Basel)

**A. Nitzan**  
(Tel Aviv/Phil.)

**B. Reulet**  
(Sherbrooke)

**Yu. Nazarov** (Delft)



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Summary of *Noisy Quantum Measurements: a nuisance or fundamental physics?*

- Quantum measurement: projection vs. weak measurements
  - (Noisy) non-invasive measurements offer another (new) perspective on the quantum measurement problem
- Quantum dynamics: Keldysh contour
- Generalized Keldysh-ordered functional
- Keldysh-ordered expectations are quasiprobabilities
  - Weakly measured non-commuting variables violate classicality (in the forth order)
- Keldysh-ordered third cumulant
  - Time-reversal symmetry
  - Violation of conservation laws
- General non-markovian weak measurement
  - System mediated detector-detector interaction
  - Detector engineering allows tailored operator order
  - Unusual third-order correlators

**THE END**

