Realizing effective non-Hermitian time evolution with superconducting circuits

Conference on Quantum Measurement: Fundamentals, Twists, and Applications ICTP 2019

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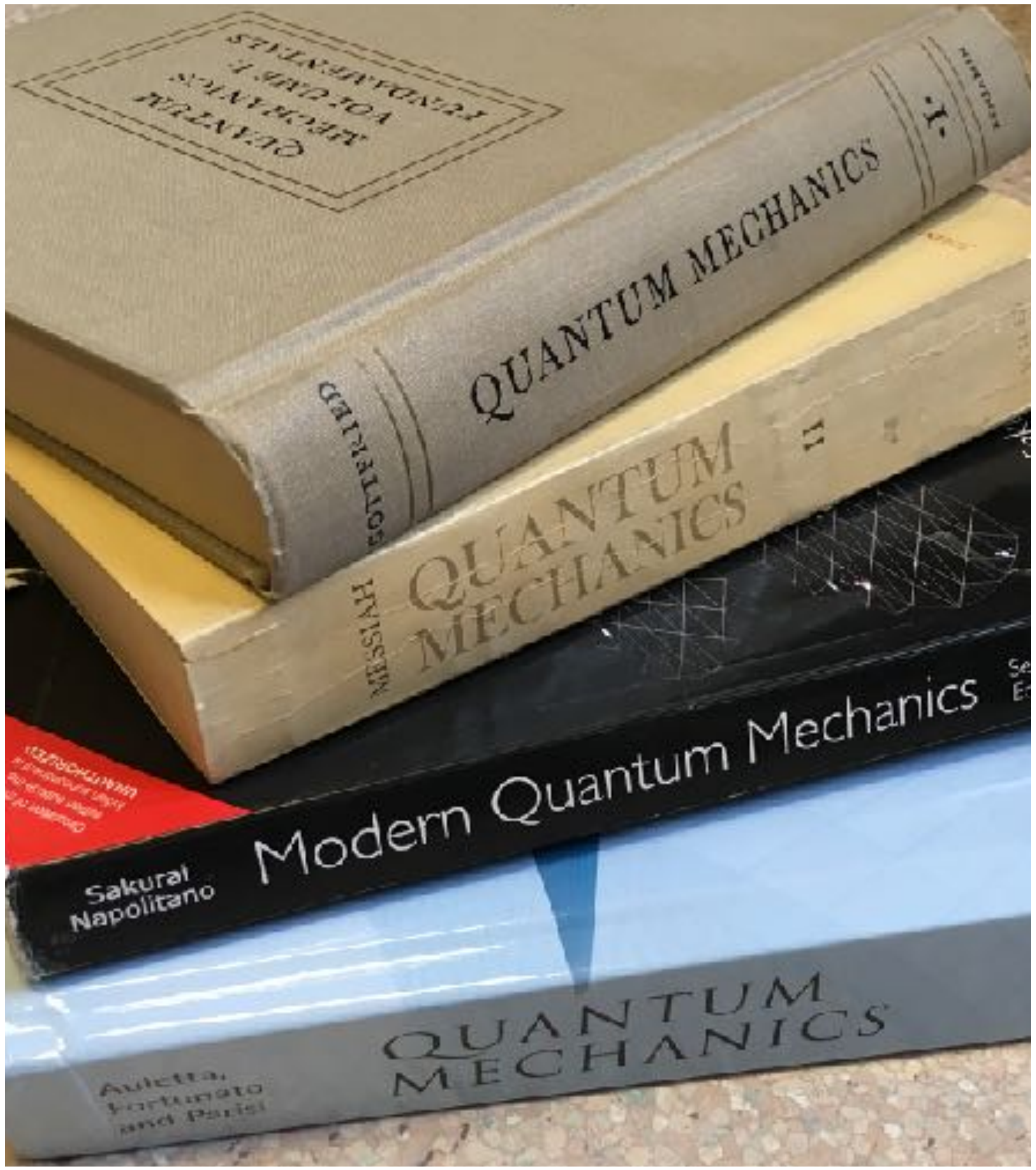
A Hamiltonian must be Hermitian



Hamiltonian described by Hermitian operator

Real eigenvalues
☞ Unitary time evolution

Complete set of eigenvectors
☞ Orthonormal basis

Or must it?

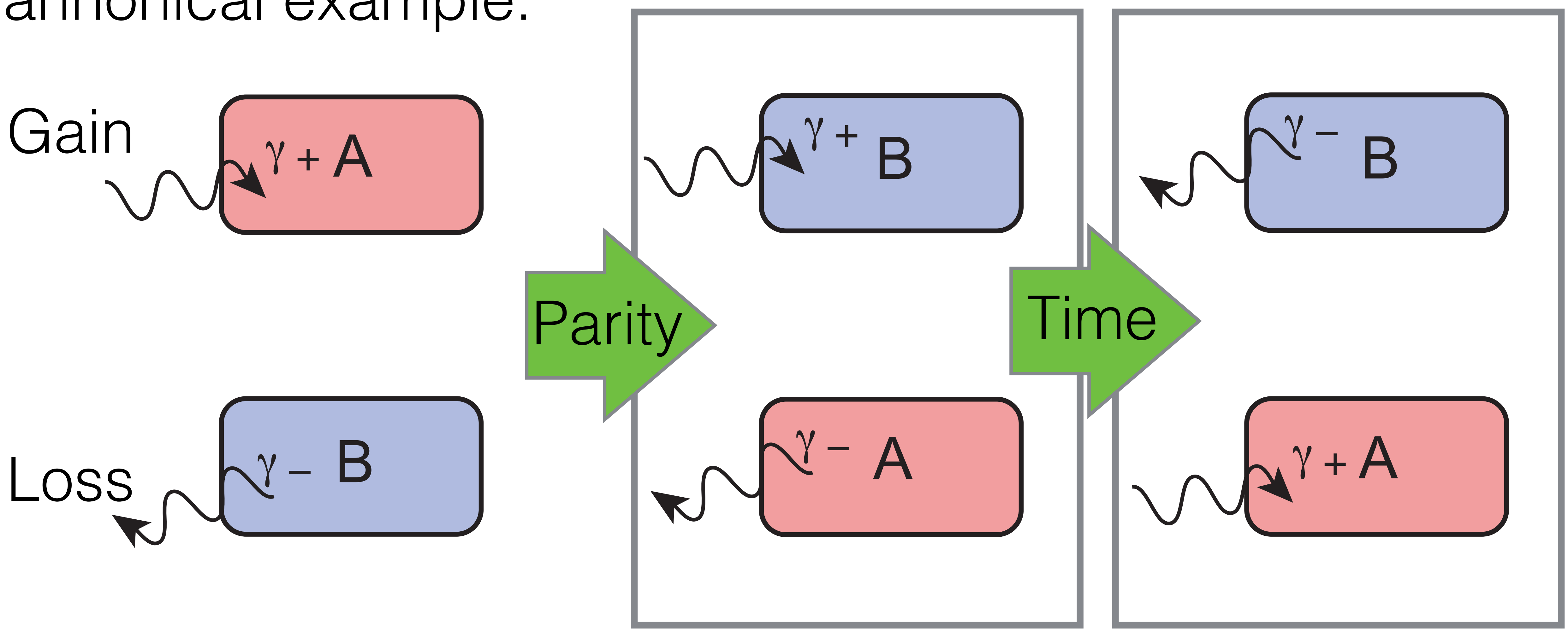
VOLUME 80, NUMBER 24 PHYSICAL REVIEW LETTERS 15 JUNE 1998

Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]

Can have real eigenvalues with non-Hermitian Hamiltonian

Cannonical example:



Non-Hermitian Hamiltonian

VOLUME 80, NUMBER 24 PHYSICAL REVIEW LETTERS 15 JUNE 1998

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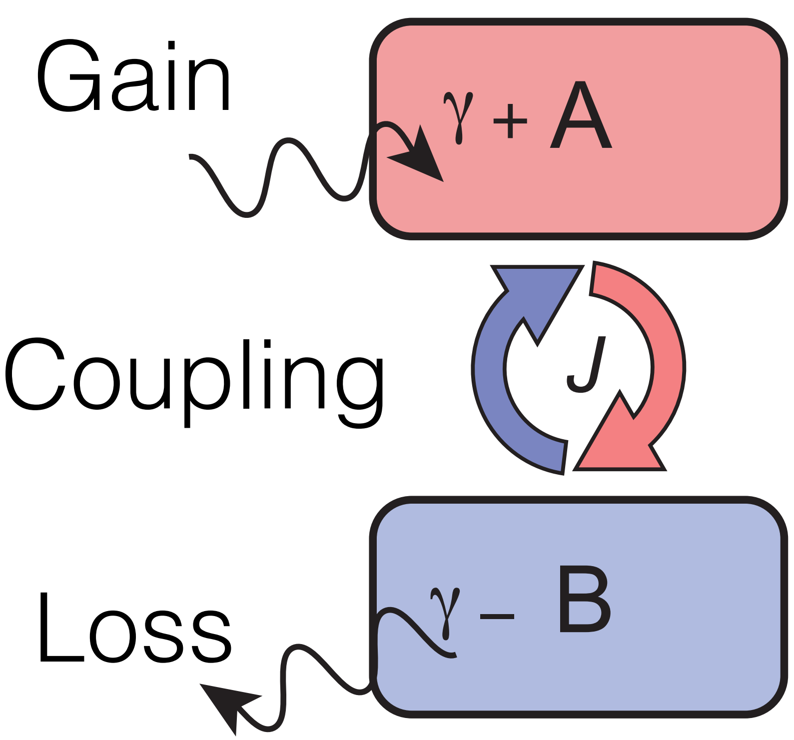


Hamiltonian

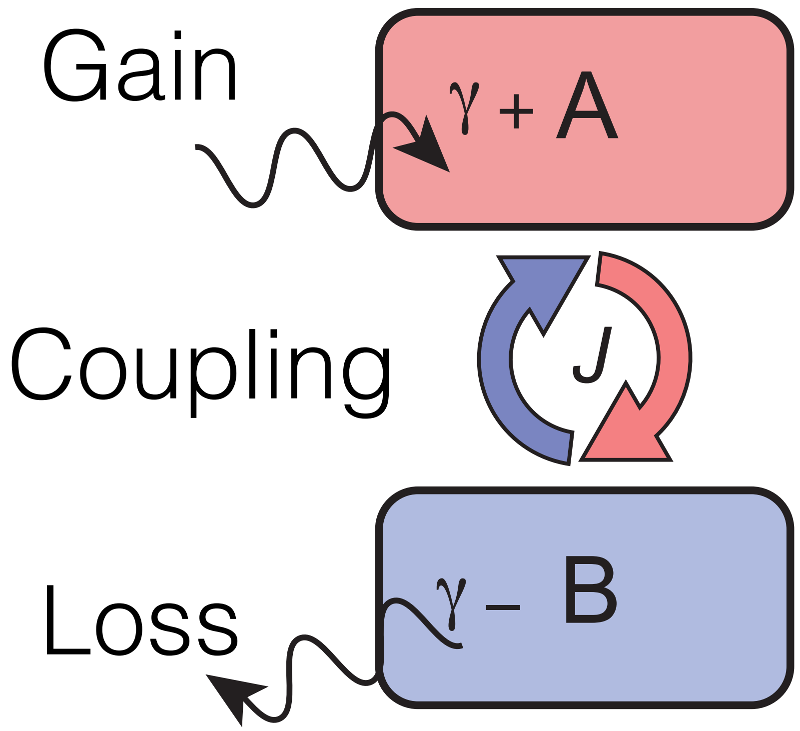
$$i\gamma|A\rangle\langle A| - i\gamma|B\rangle\langle B| + J(|A\rangle\langle B| + |B\rangle\langle A|)$$

Matrix representation

$$\begin{pmatrix} -i\gamma & J \\ J & +i\gamma \end{pmatrix}$$

Eigenvalues: $\lambda = \pm\sqrt{J^2 - \gamma^2}$

But who cares?

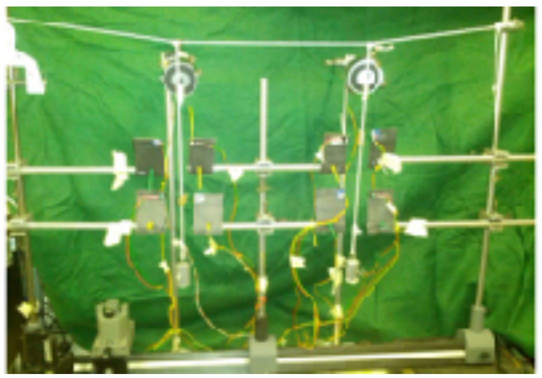


Balanced gain and loss easily achieved in classical systems

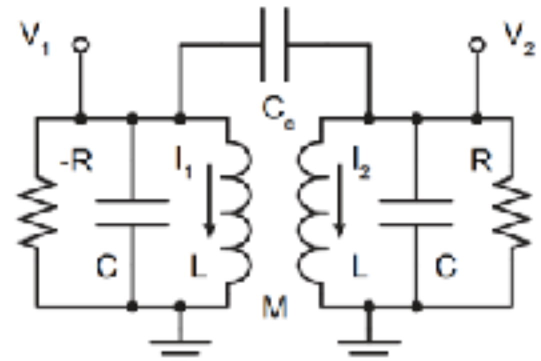
Many experiments…

Non-Hermitian dissipation-experiments and systems

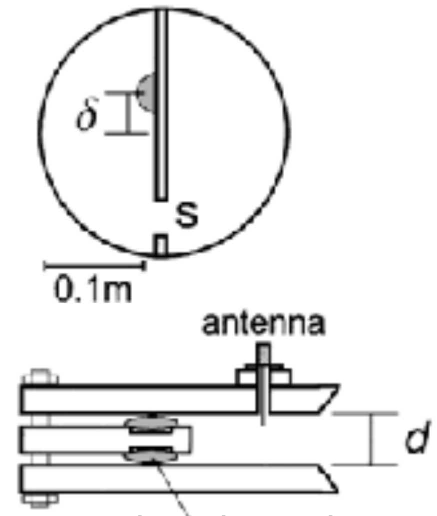
Platforms for non-Hermitian physics: coupled mechanical/electrical oscillators



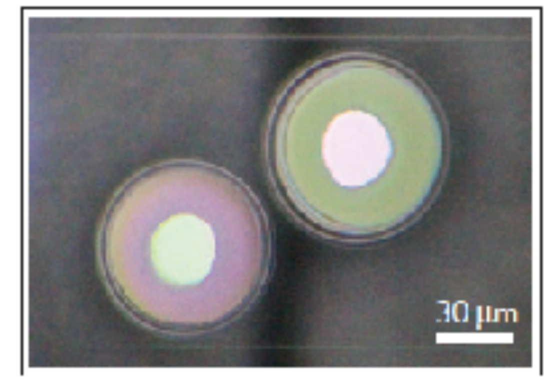
C. M. Bender et al *A. J. Phys.* **81**, 173



J Schindler et al, JPA **45**, 444029

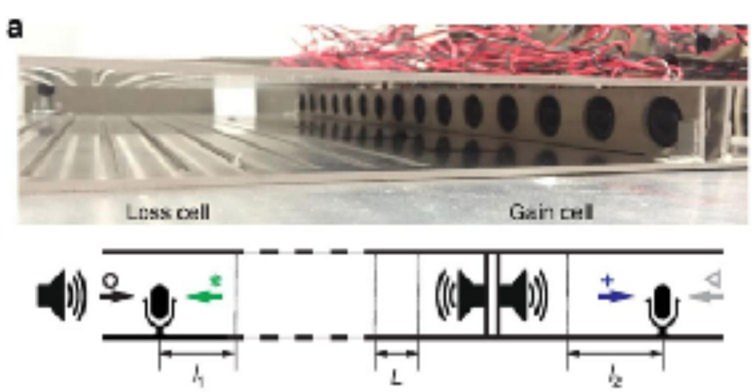


C. Dembowski et al *PRL* **86**, 5 (2001).

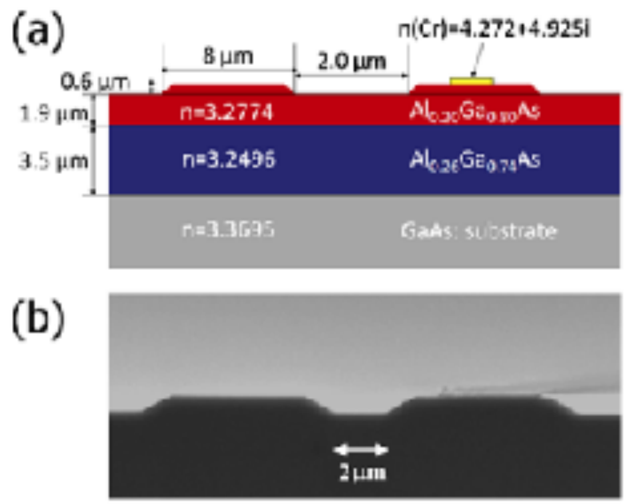


B. Peng et al *Nature Phys.* **10**, 394 (2014).

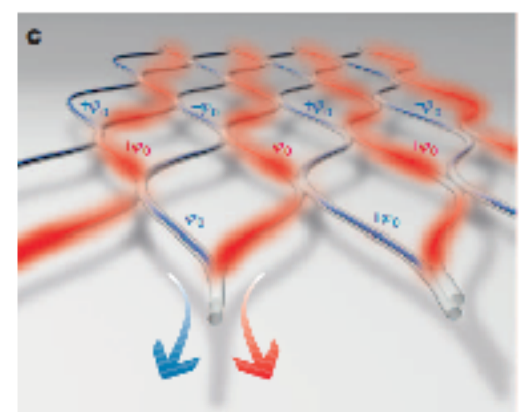
propagating acoustical/optical waves



C. Shi et al *Nat. Commun.* **7**, 11110



A. Guo et al *Phys. Rev. Lett.*



A. Regensburger et al *Nature*

Lasing: B. Peng (14), M. Brandstetter (14), M. Kim (14), Z. Wong (16), B. Peng (16), L. Feng (14), H. Hodaei (14)
Asymmetric mode switching: J. Doppler (16),
Topological energy transfer: Xu (2016)
Enhanced sensing: W. Chen (2017), H. Hodae (17)

PT symmetry: 4 Key phenomena

1

PT breaking transition from Re to Im eigenvalues

$$\lambda = \pm\sqrt{J^2 - \gamma^2}$$

(J>ɣ) PT “un-broken” phase

(J<ɣ) PT “broken” phase

2

“Exceptional” point degeneracy (J=ɣ)

Eigenvectors become degenerate

3

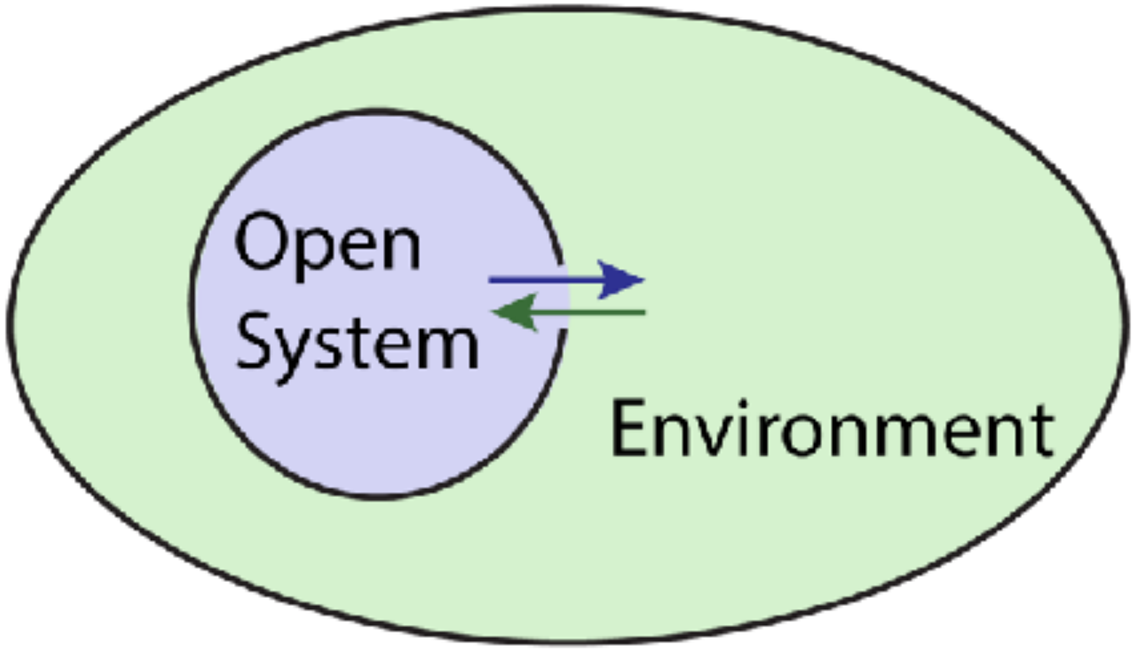
Enhanced sensitivity

4

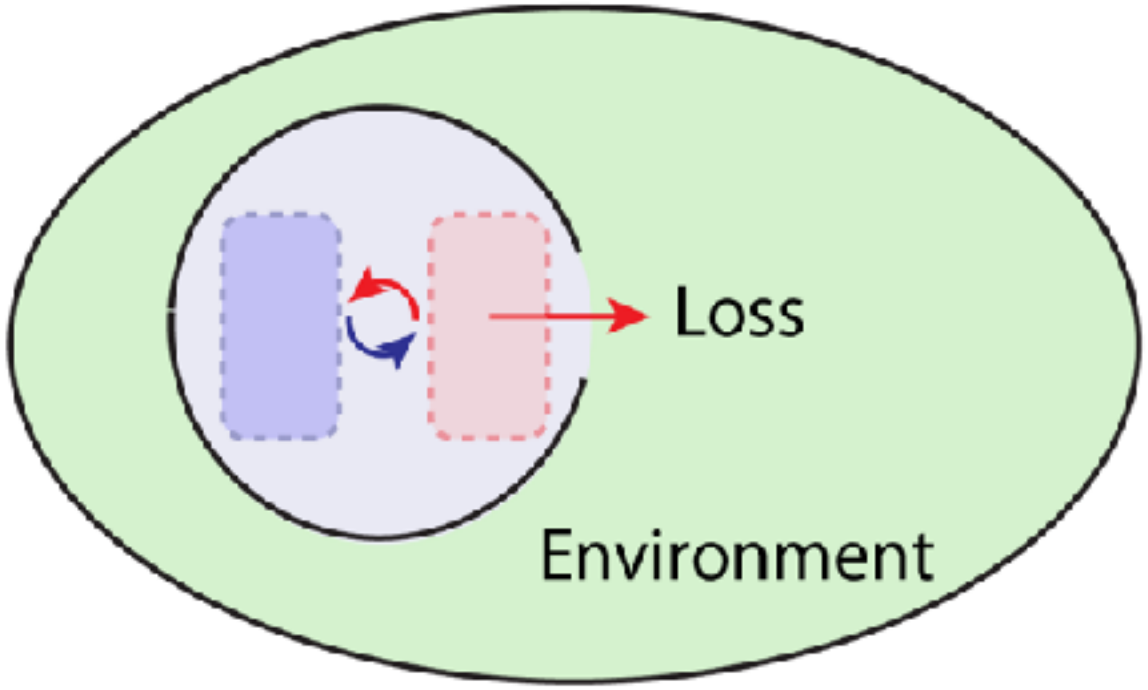
Topological/non-reciprocal

PT symmetric quantum systems?

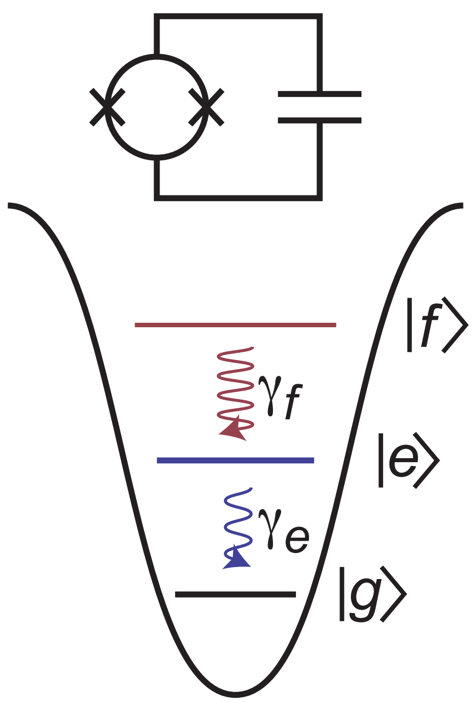
Open systems
Non-unitary dynamics
Master equations
Non-Hermitian Hamiltonian



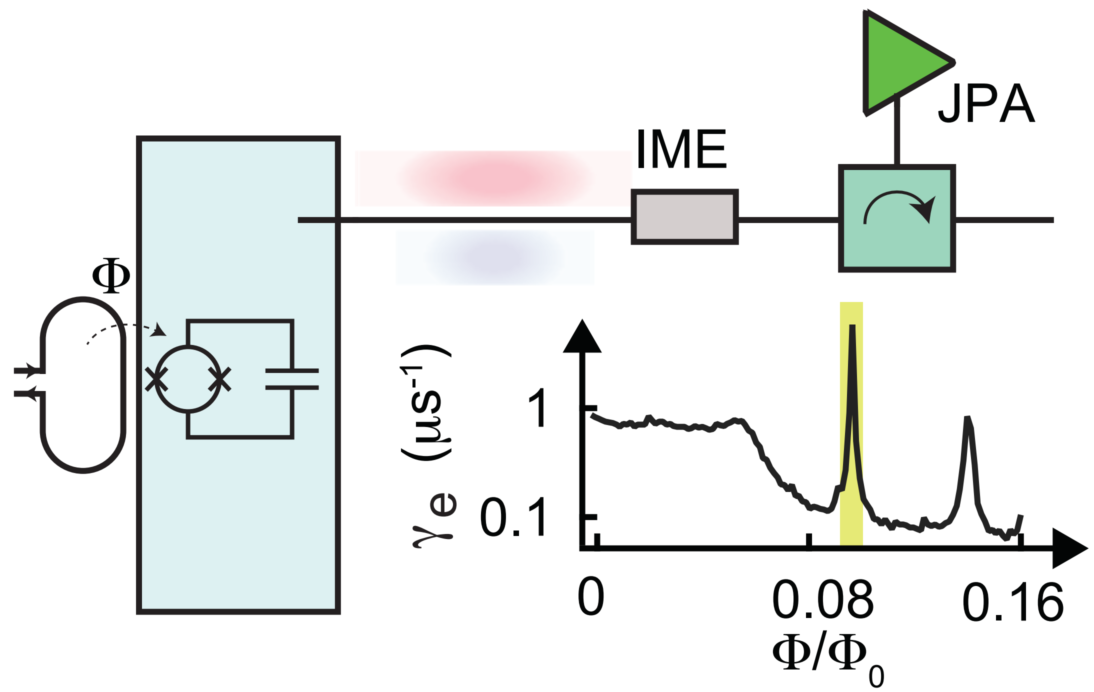
Mode selective loss
(effective PT symmetry)



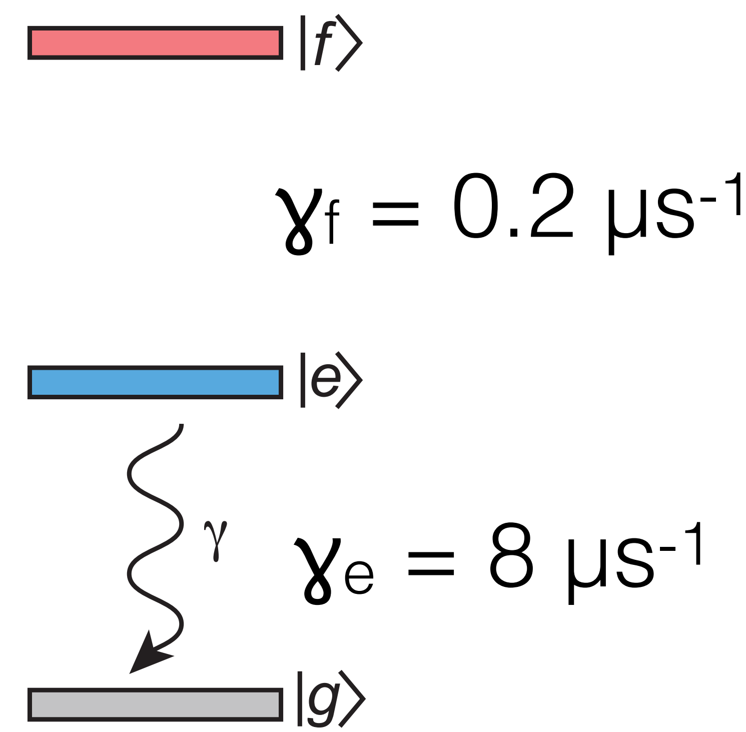
Effective non-Hermitian qubit



Transmon circuit



Engineered decay rates

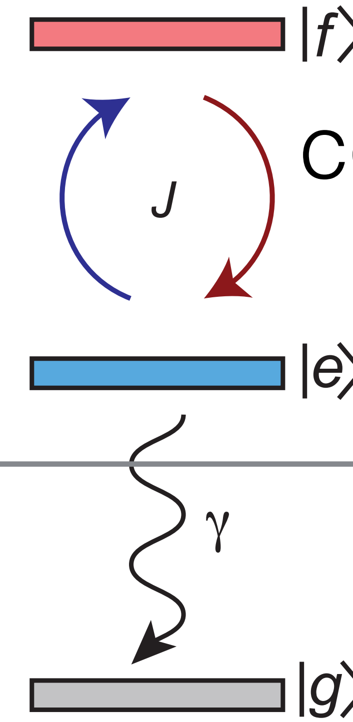


arXiv:1901.07968

Effective non-Hermitian qubit

Sub-manifold evolution given by non-Hermitian Hamiltonian

$$H_{\text{eff}} = J\left(|f\rangle\langle e| + |e\rangle\langle f|\right) - i\gamma_e/2)|e\rangle\langle e|$$



Lindblad Master equation:

$$\dot{\rho} = -i[H_c, \rho] + L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$$

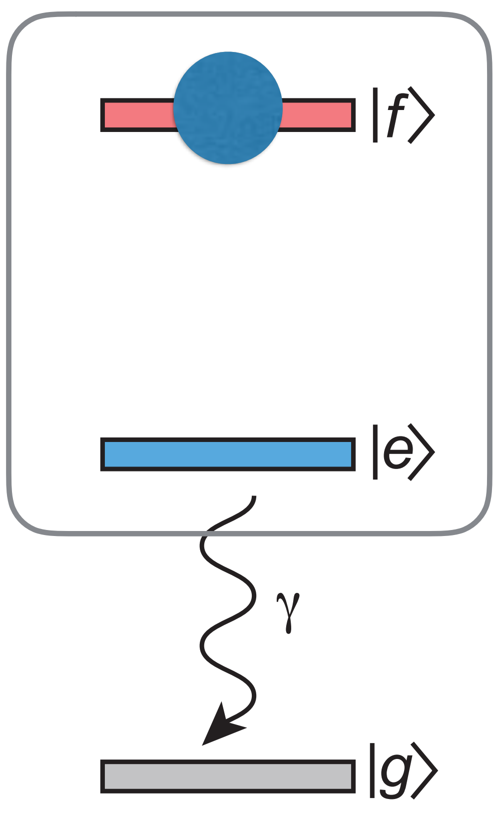
$$H_c = J(|e\rangle\langle f| + |f\rangle\langle e|)$$

$$L = \sqrt{\gamma}|g\rangle\langle e|$$

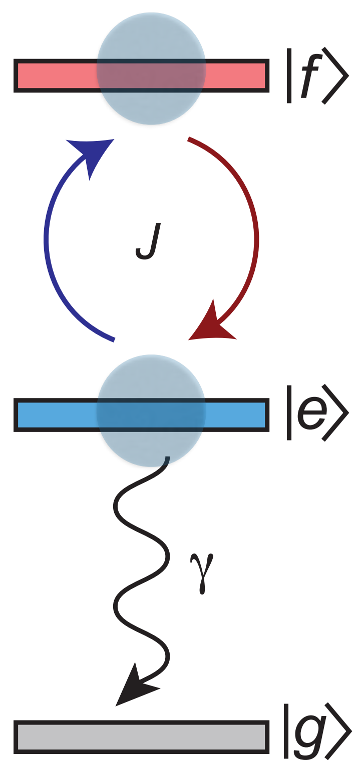
Aharonov et al PRL 96, Weisskopf & Wigner '30

Dynamics of non-Hermitian qubit

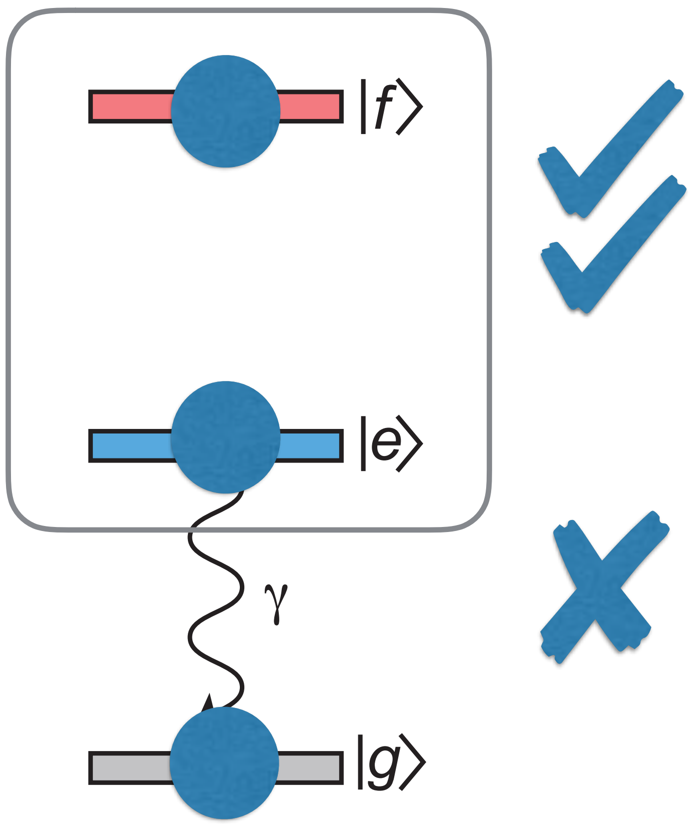
Prepare in qubit e-f manifold



Evolve under H_{eff}



Post-select qubit manifold



Ashida, Furukawa, Ueda Nat. Com. 2017

Effective PT symmetry

In matrix representation:

$$\begin{pmatrix} -i\gamma/2 & J \\ J & 0 \end{pmatrix} = \begin{pmatrix} -i\gamma/4 & J \\ J & +i\gamma/4 \end{pmatrix} - i\frac{\gamma}{2}\mathbb{1}$$

PT symmetric

Overall loss

(EP occurs at J = γ/4)

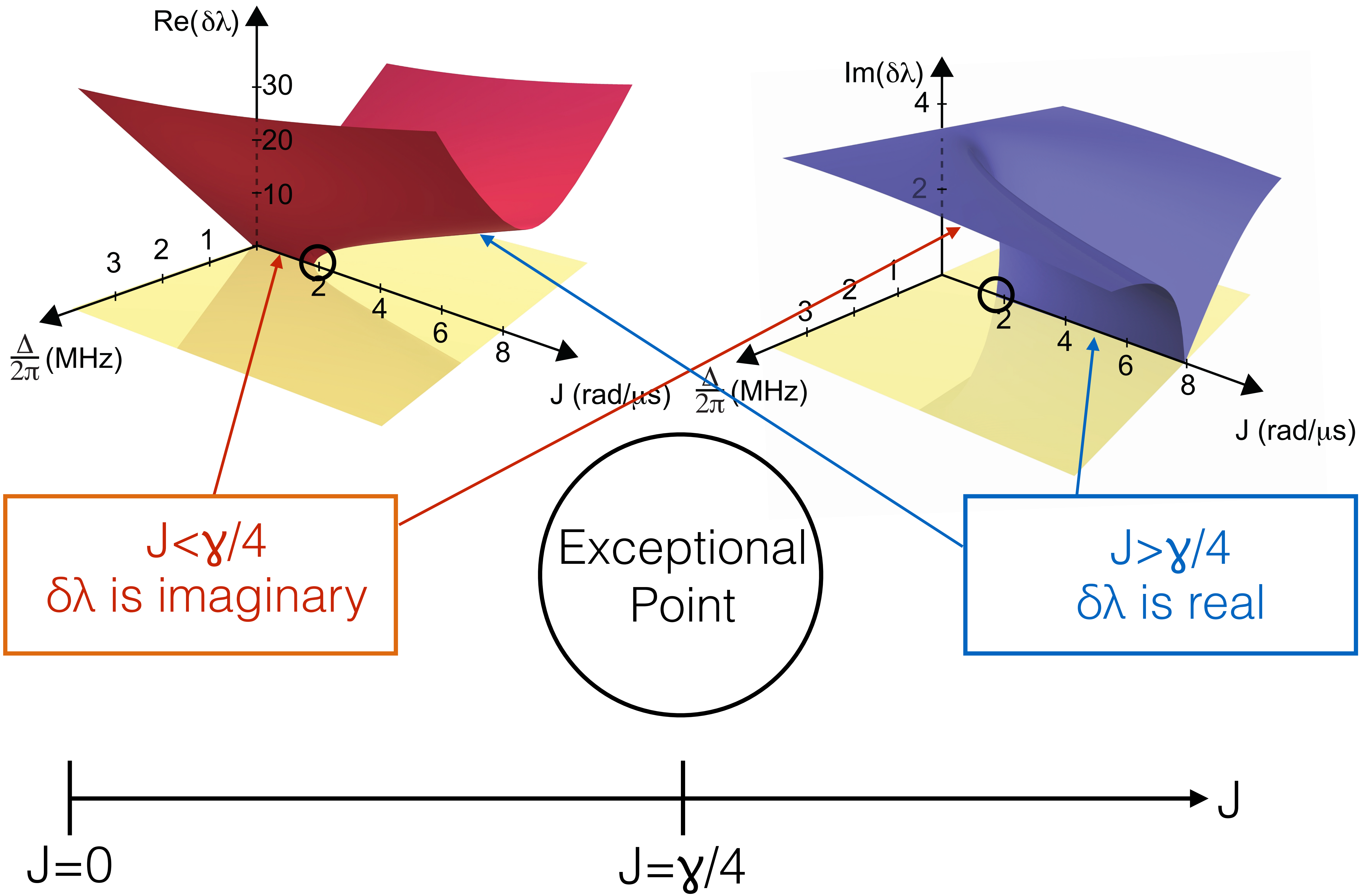
Detuned drive:

$$H_{\text{eff}} = J\left(|f\rangle\langle e| + |e\rangle\langle f|\right) + (\Delta - i\gamma_e/2)|e\rangle\langle e|$$

Time evolution under H_{eff} is governed by the eigenvalue difference:

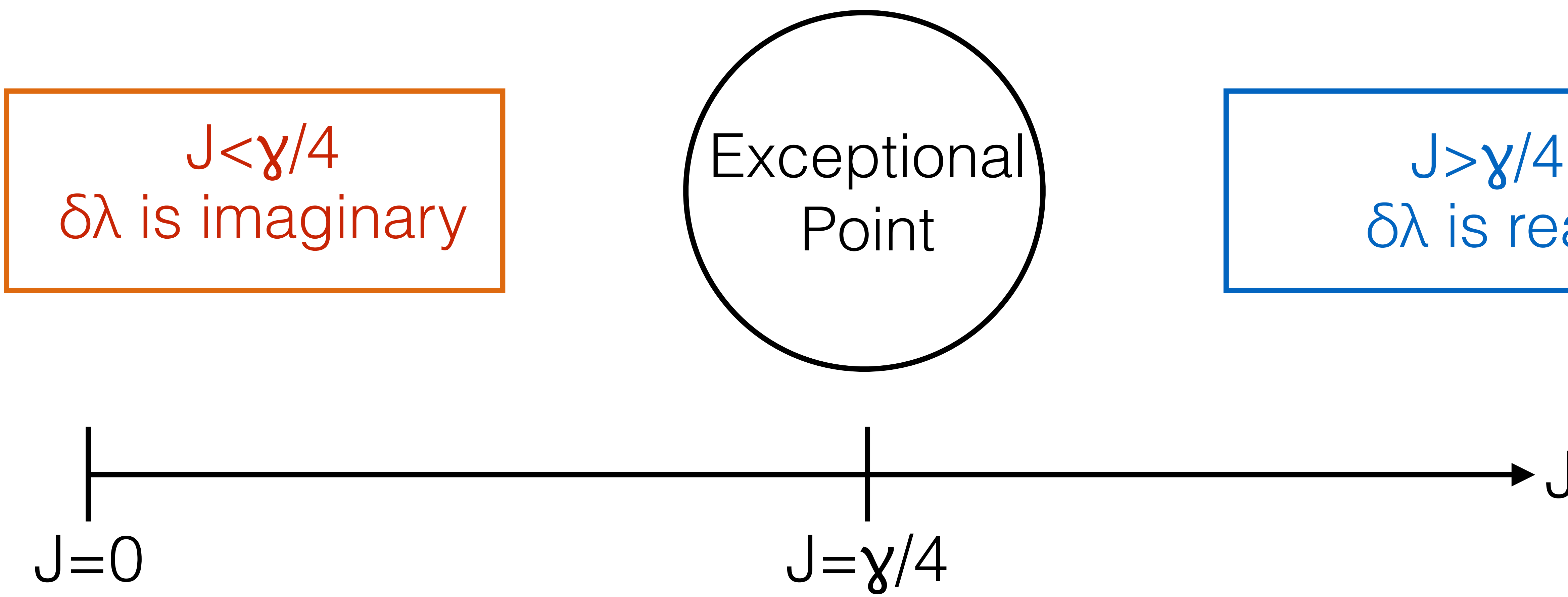
$$\delta\lambda = (\lambda_+ - \lambda_-) = \sqrt{4J^2 - (\Delta - i\gamma_e/2)^2}$$

Effective non-Hermitian qubit overview



Exceptional point

Degeneracy:	**"Diabolic point"**	**"Exceptional Point"**
Hamiltonian:	Hermitian	non-Hermitian
Eigenvalues:	Degenerate	Degenerate
Eigenvectors:	Orthogonal	Degenerate



Key phenomena

1

PT breaking transition from Re to Im eigenvalues

$$\lambda = \pm\sqrt{J^2 - \gamma^2}$$

($J>\gamma$) PT “un-broken” phase

($J<\gamma$) PT “broken” phase

2

“Exceptional” point degeneracy ($J=\gamma$)

Eigenvectors become degenerate

3

Enhanced sensitivity

4

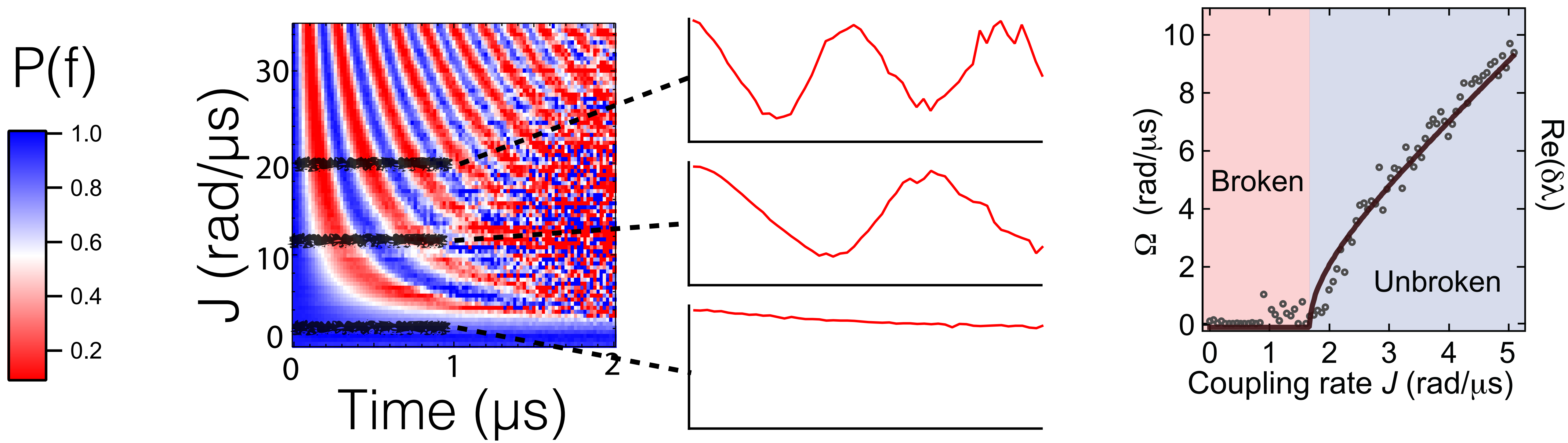
Topological/non-reciprocal

1 Probing the PT breaking transition

Measure time evolution under H_{eff} (Rabi oscillations)

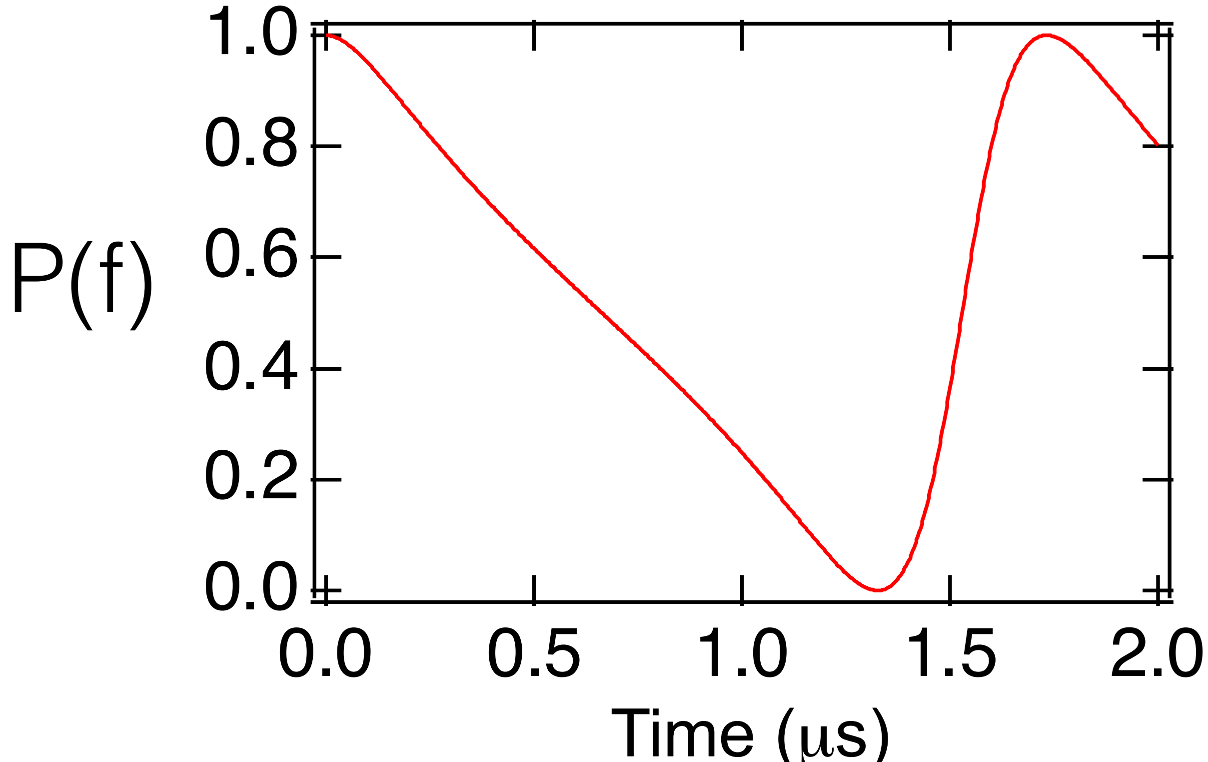
Prepare |f⟩ ········ H_{eff} ········ Readout

← 0-2 μs →

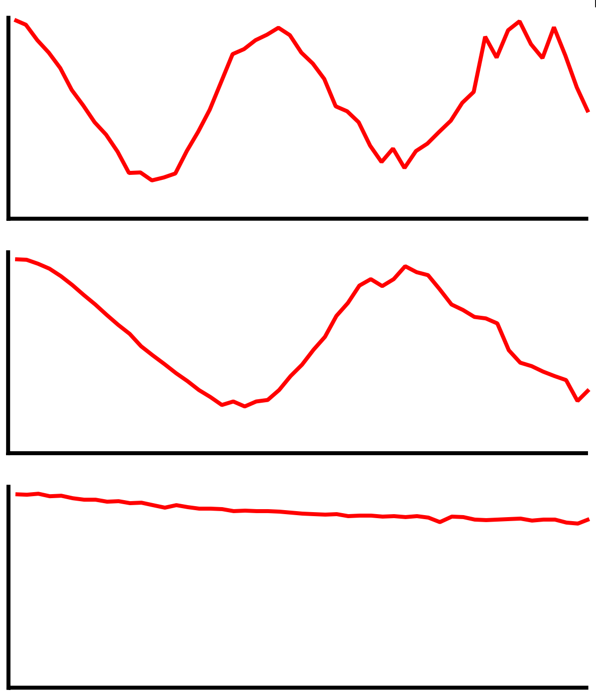


Distorted Rabi oscillations

(calculation from Lindblad Master Equation)



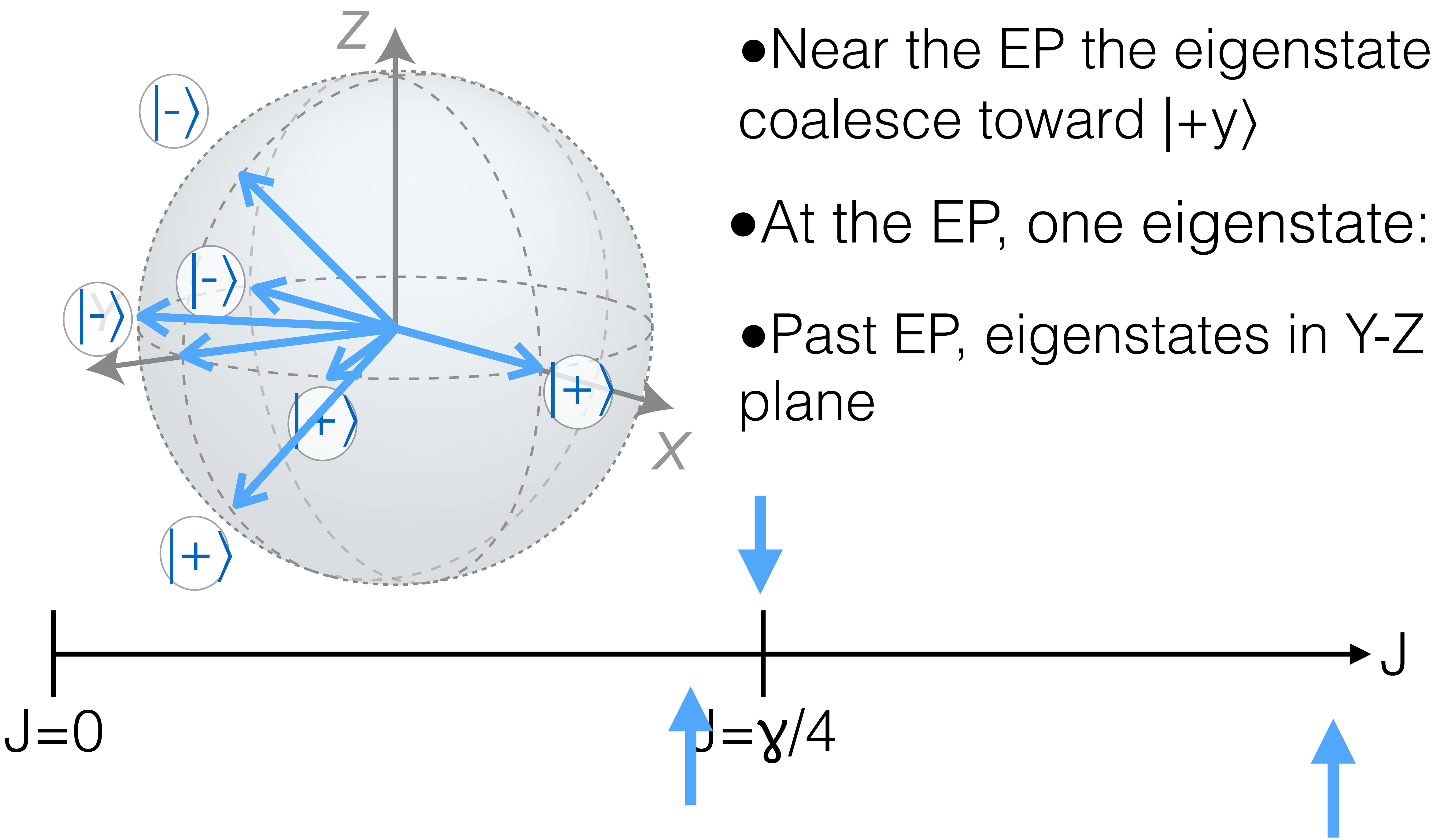
Measurement backaction from post-selection pushes toward |f⟩ state.



Might seem surprising that we observe abrupt transition at J=ɣ/4 even though the qubit never decays in our data set.

2 Degenerate eigenstates at the EP

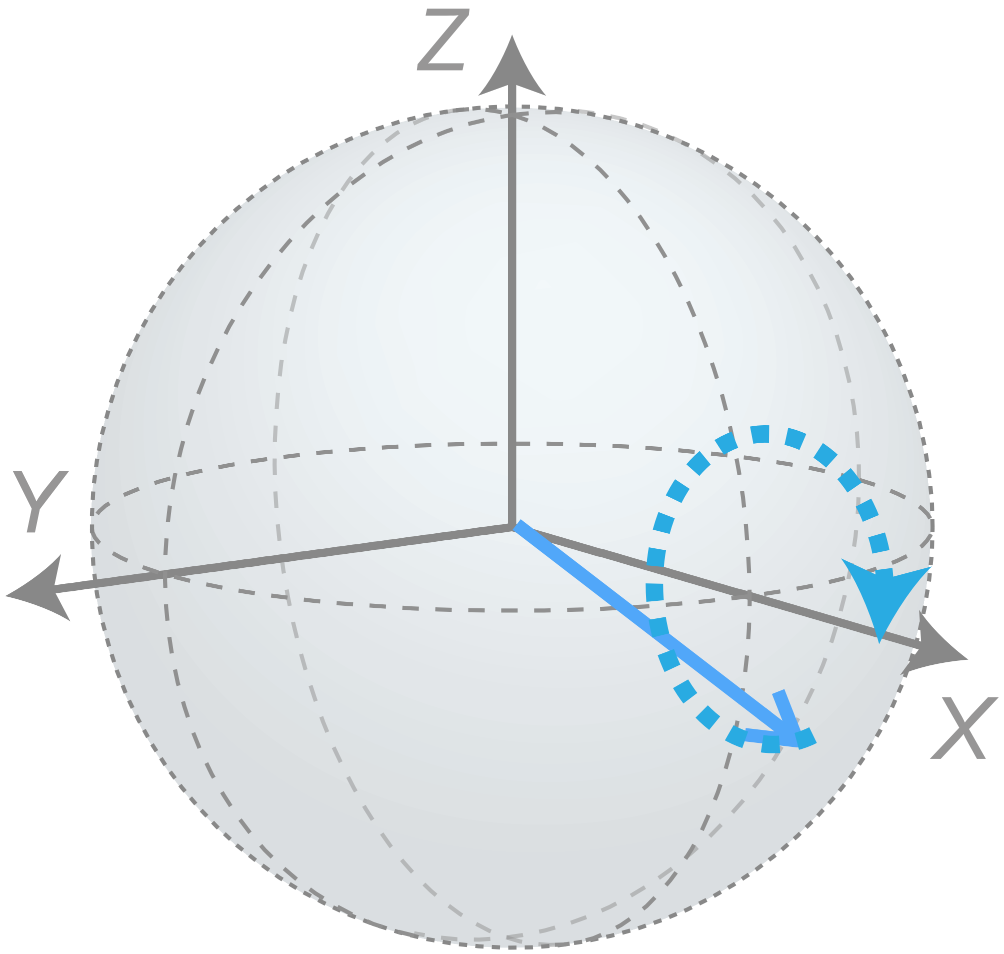
- For J≫ɣ eigenstates are |±x⟩
- Near the EP the eigenstates coalesce toward |+y⟩
- At the EP, one eigenstate: |+y⟩
- Past EP, eigenstates in Y-Z plane

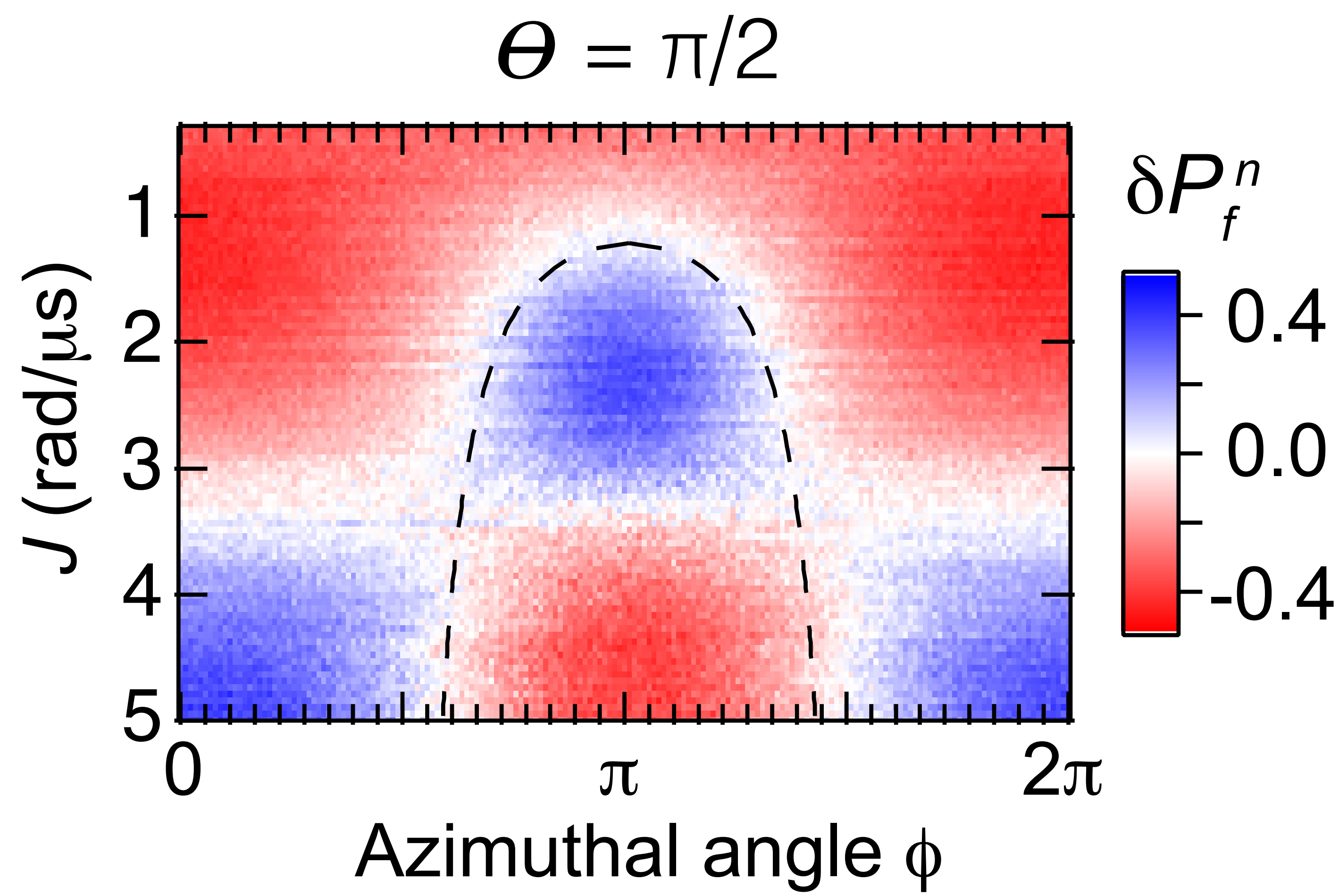


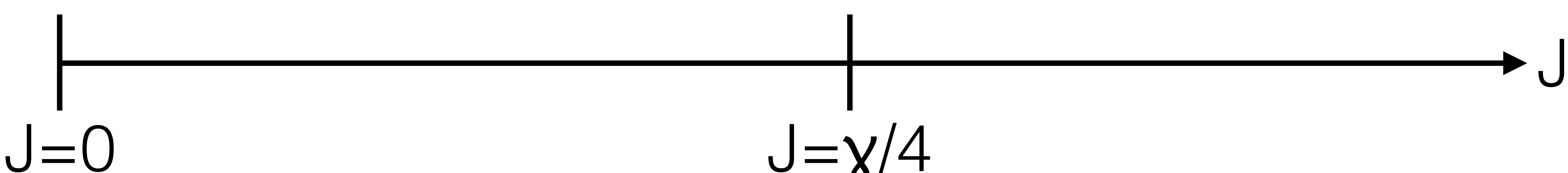
2 Imaging eigenstates (stationary)

Prepare ($\boldsymbol{\theta}$,$\boldsymbol{\phi}$) ········ H_{eff} ········ Readout

←500 ns→







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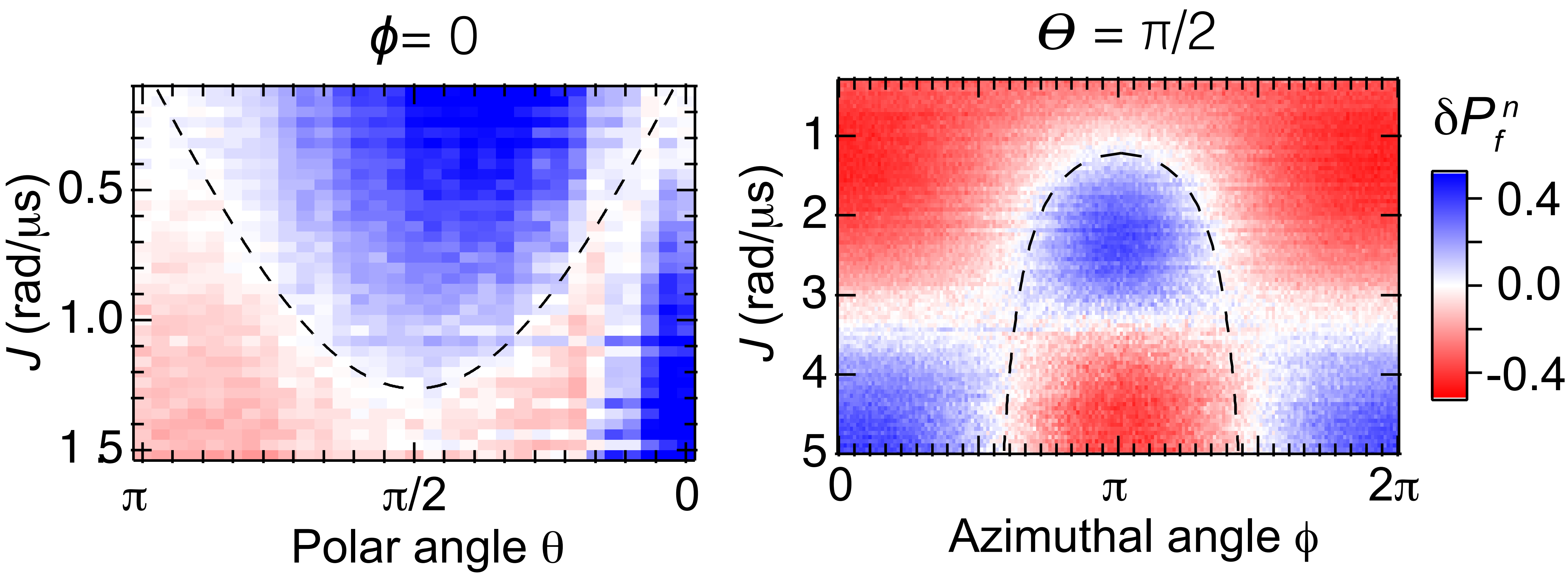
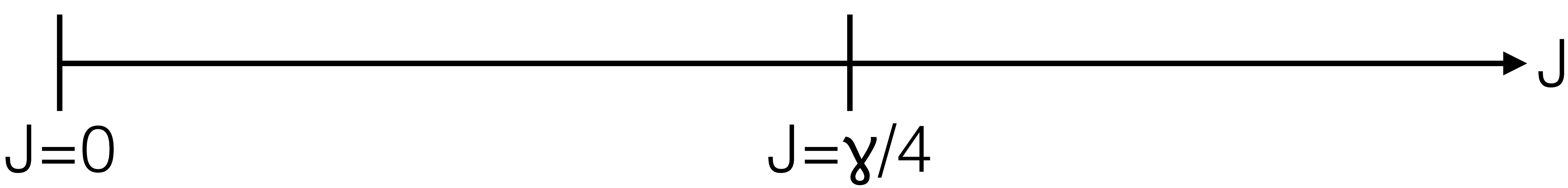
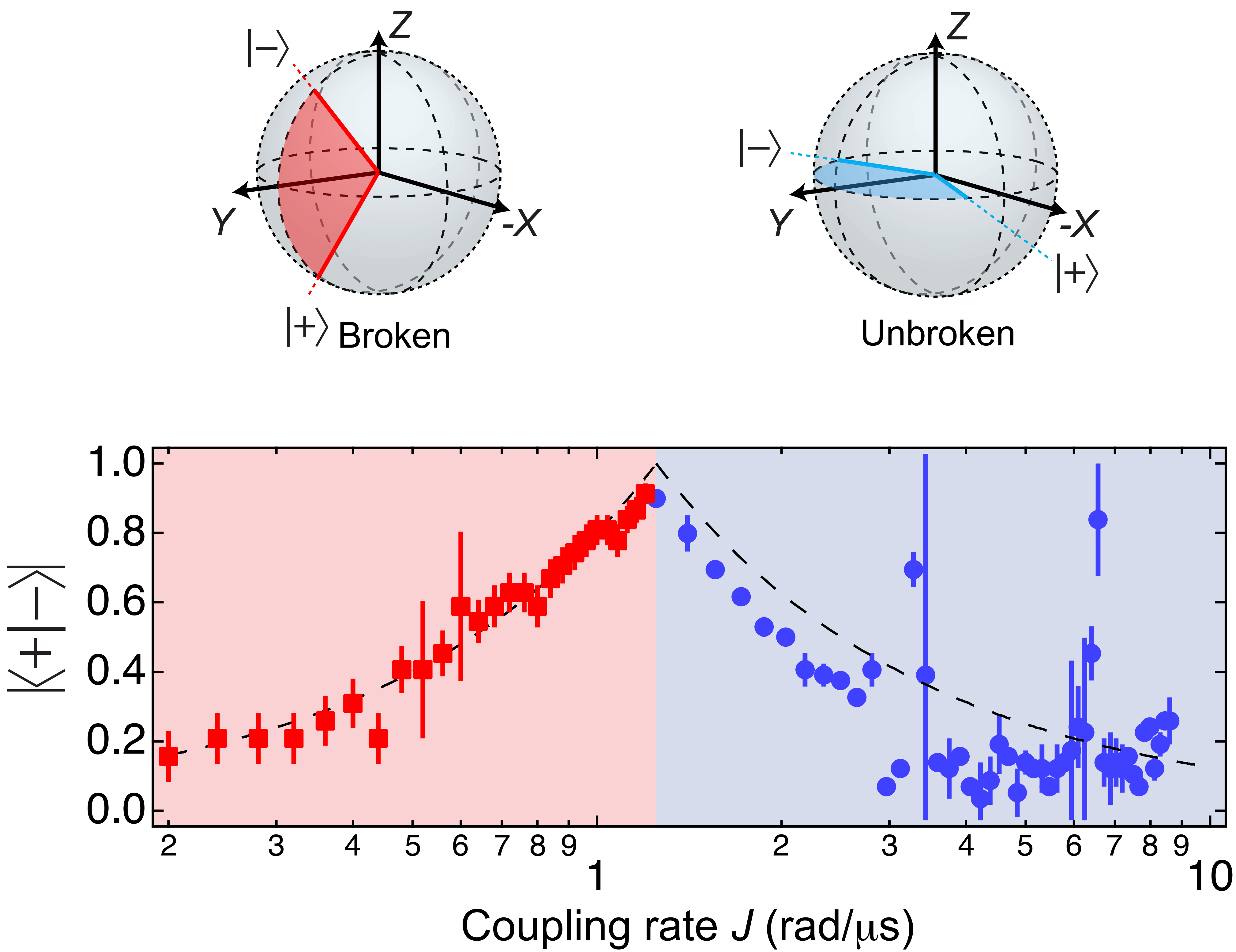


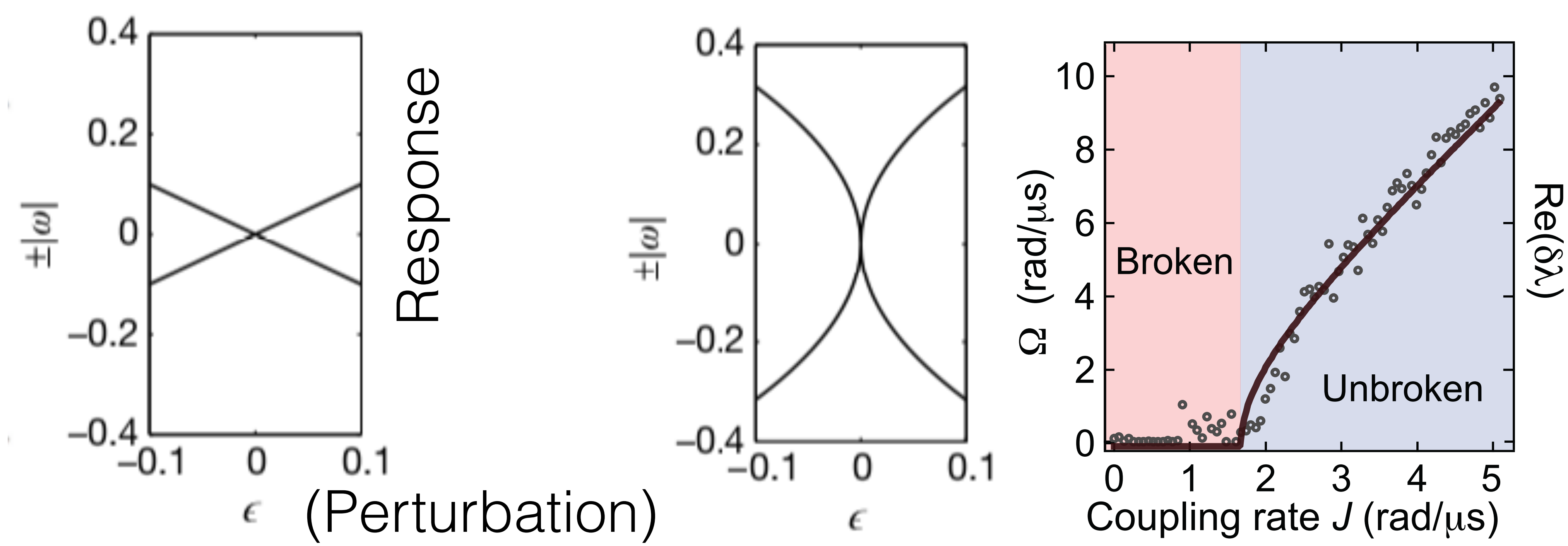


Non-orthogonal eigenstates



3 Sensing advantages with dissipation

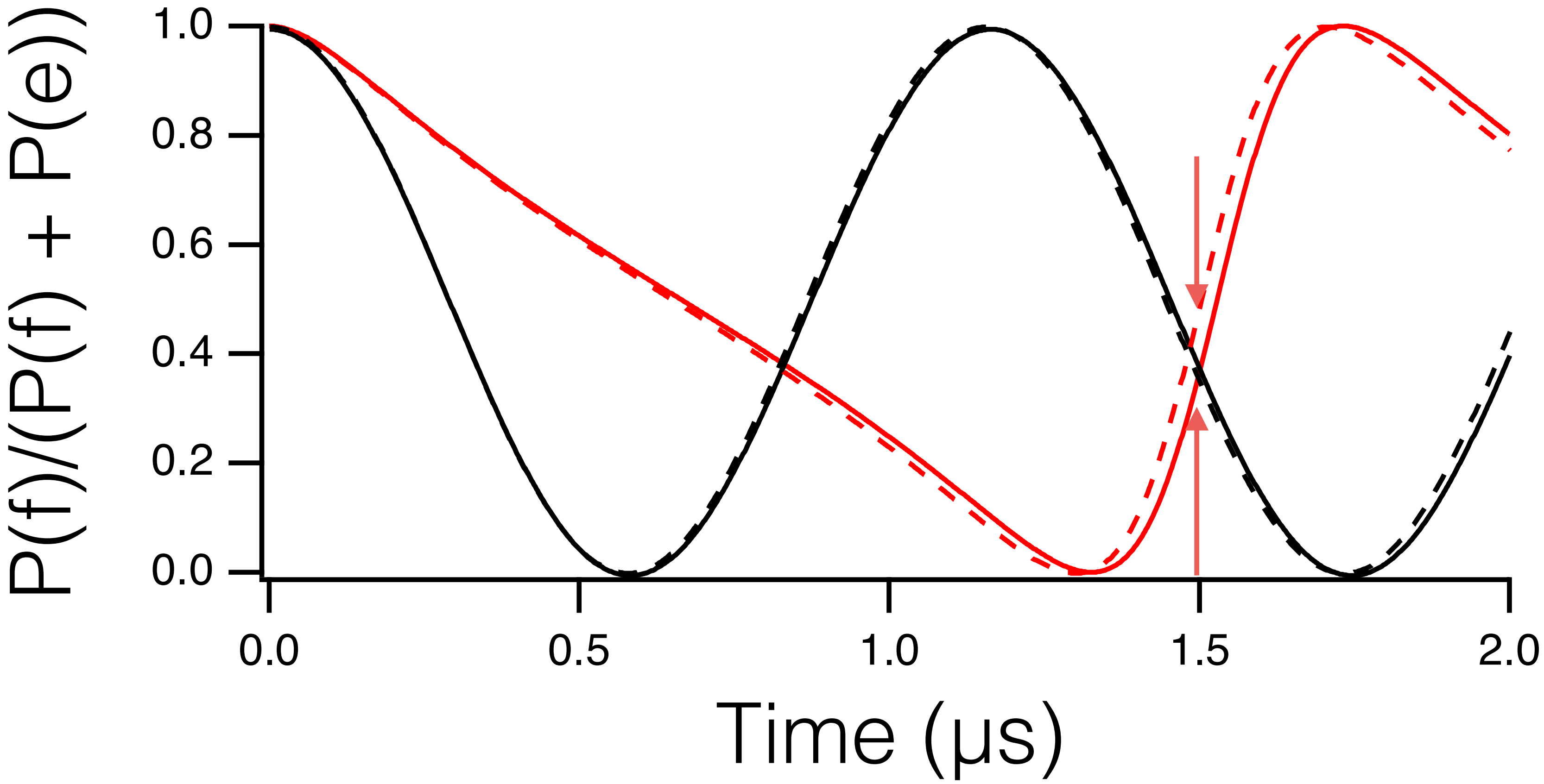
Degeneracy:	**“Diabolic point”**	**“Exceptional Point”**
Hamiltonian:	Hermitian	non-Hermitian
Eigenvalues:	Degenerate	Degenerate
Eigenvectors:	Orthogonal	Degenerate



Wiersig PRL 2014, Chen Nature 2017, Hodaie Nature 2017

3

Time evolution of H_{eff}



- Solve Lindblad Master Equation
- $J/\gamma = 0.33$ (EP at 0.25)

- Distorted trajectories due to H_{eff} (can think of as measurement back-action)

- Response to a small perturbation ($\Delta J/J = 0.7\%$)

3

Cramér-Rao Bound

For large data sets, the Cramer-Rao bound sets a limit on the mean squared deviation of some parameter

$$\langle \delta^2 \hat{J} \rangle \geq \frac{1}{d I_J^{(Q)}}$$

d is the amount of data

$\hat{J}$ is an unbiased estimator of *J*

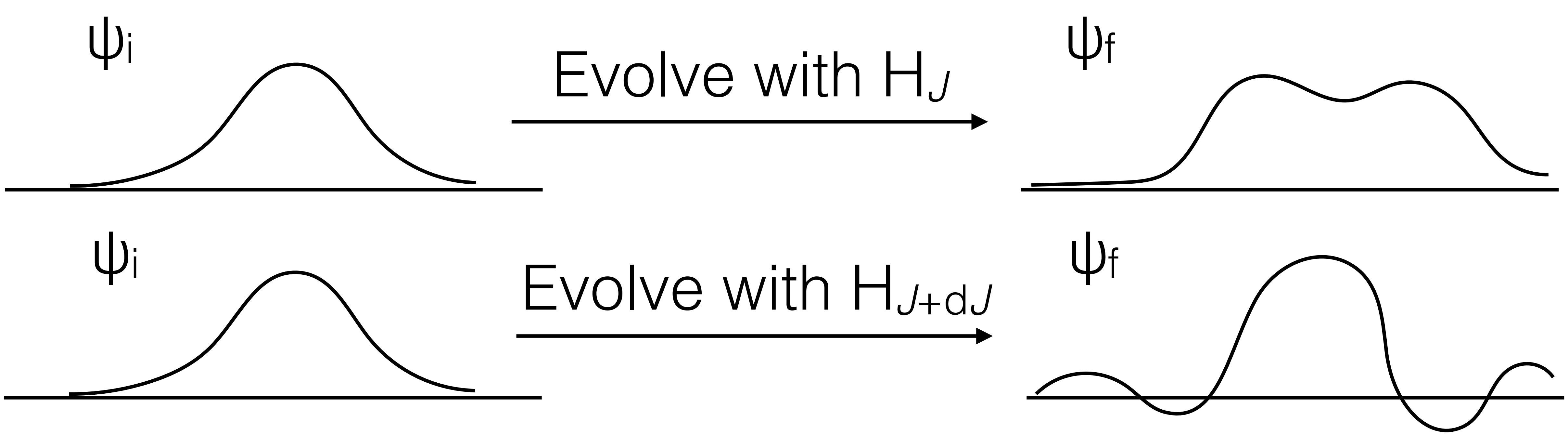
$I_J^{(Q)}$ is the Fisher information

Quantum Fisher Information

$$I_J^{(Q)} = 4 \frac{\mathrm{d}s}{\mathrm{d}J}$$

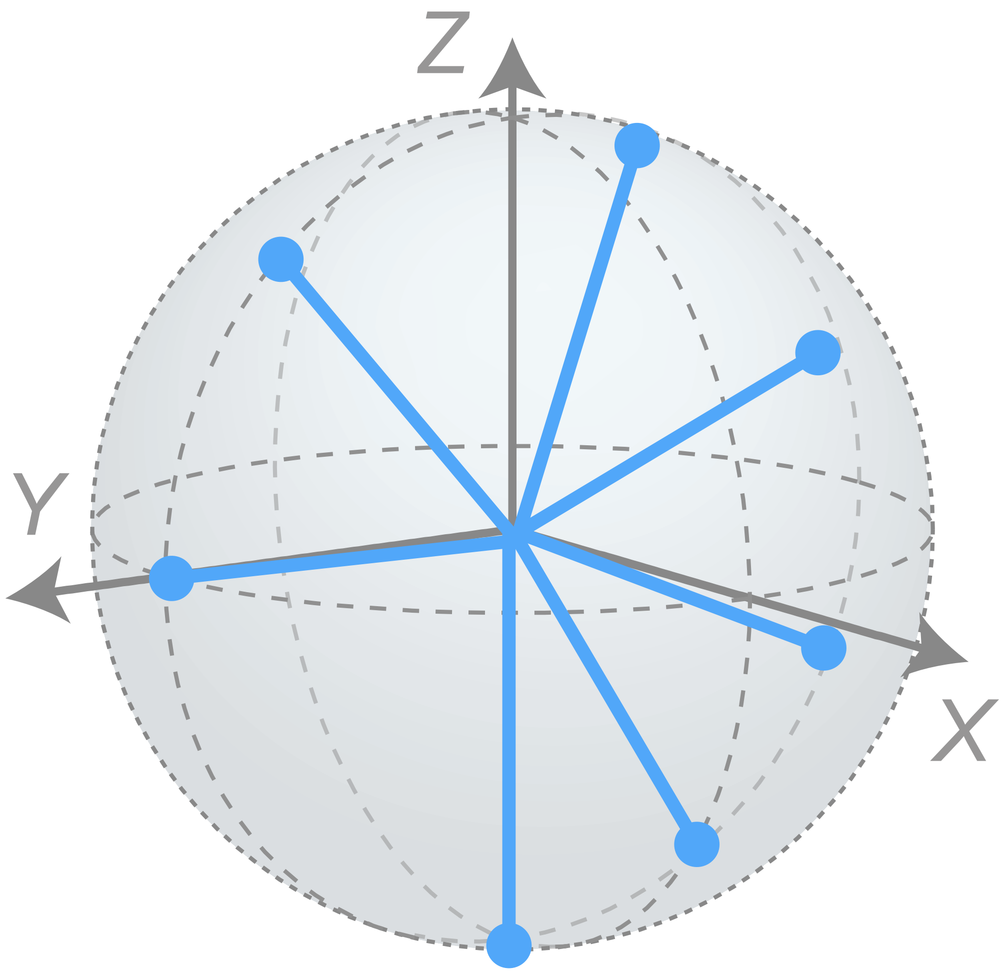
Bures distance:

$$\mathrm{d}s = 2(1 - |\langle \psi_J | \psi_{J+\mathrm{d}J} \rangle|)$$



Cramér 1946

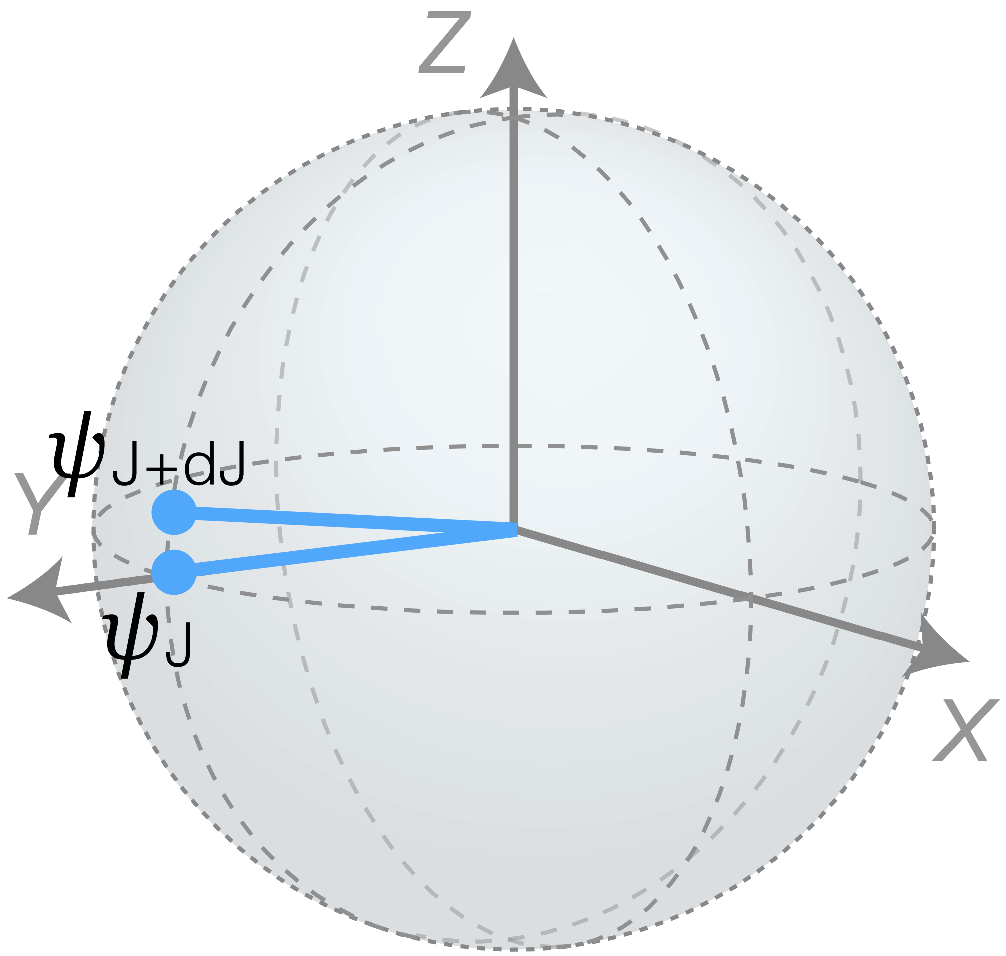
Quantum Fisher Information



Rabi interferometry

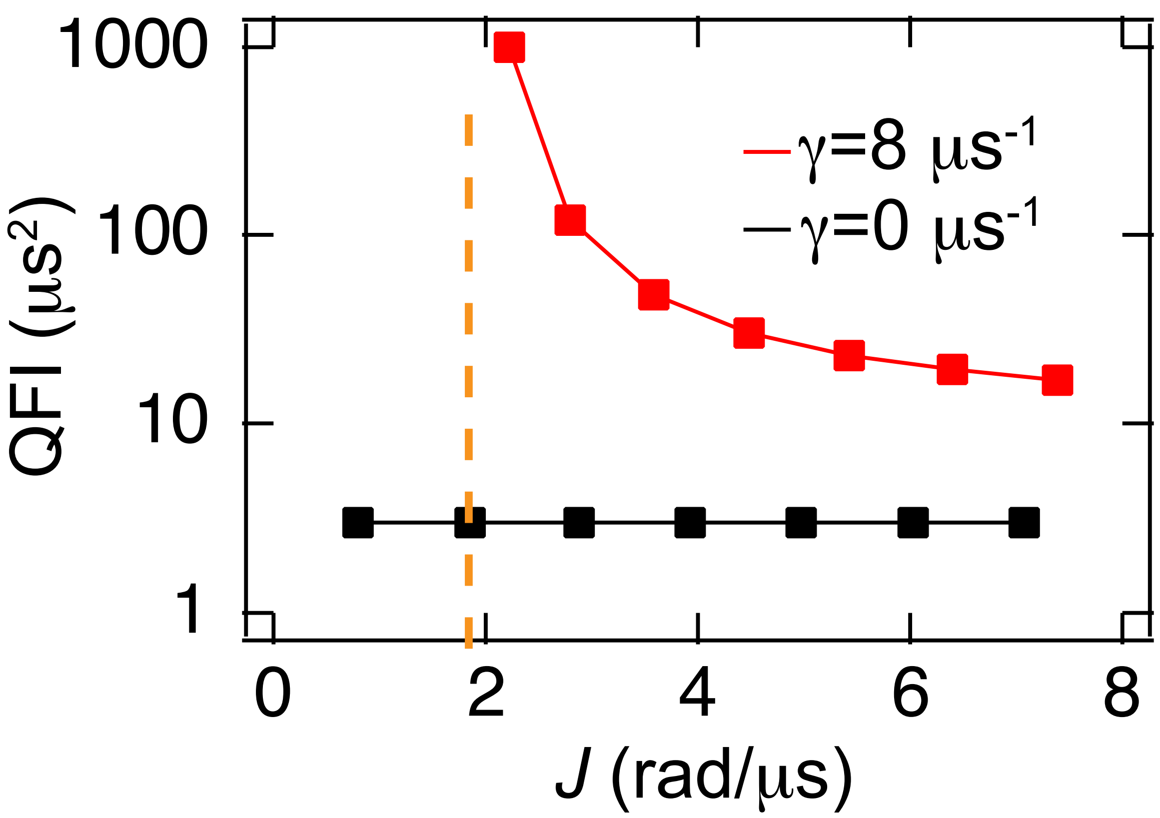
3

Quantum Fisher Information



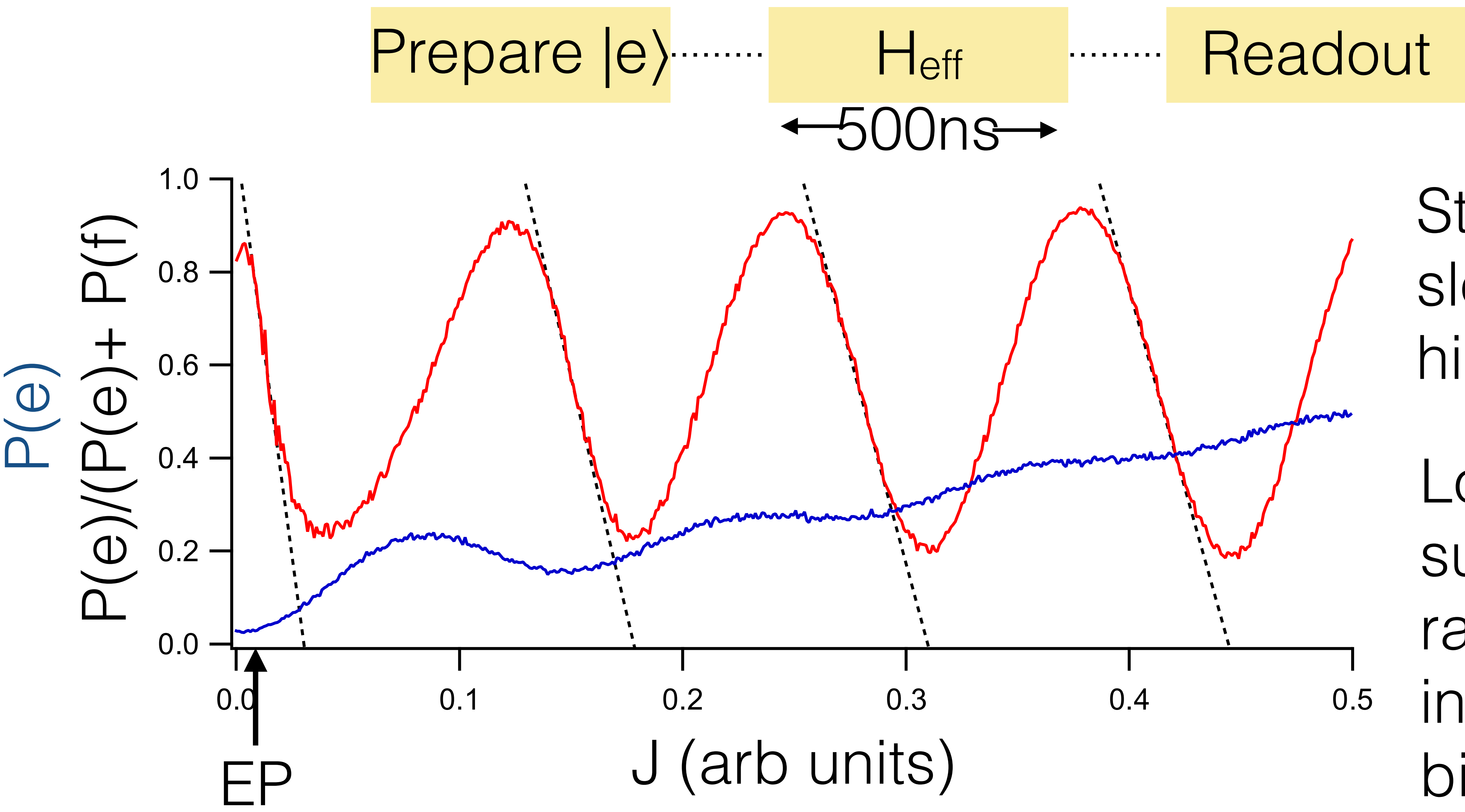
QFI ~ $(dP_f/dJ)^2$

(calculation)



3

Measuring the QFI near the EP



Steeper slope = higher QFI

Low success rate = increased binomial error

Kero Lau, Aash Clerk (Nat. Com. 2018)

3 Non-Hermitian qubit sensing summary

Improvement in the QFI about a perturbation to non-Hermitian Hamiltonian near EP.

(In the post-selected qubit manifold)

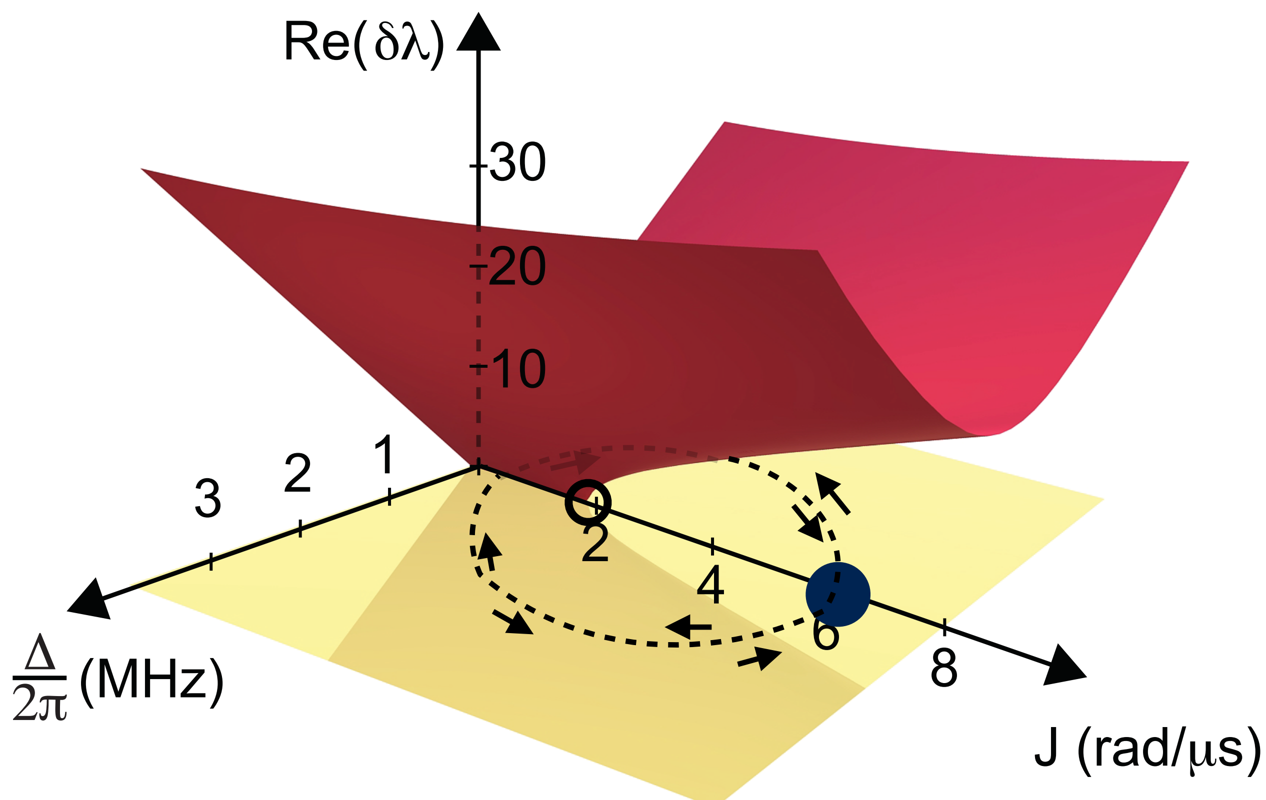
Post-selection introduces a cost due to the dissipation.

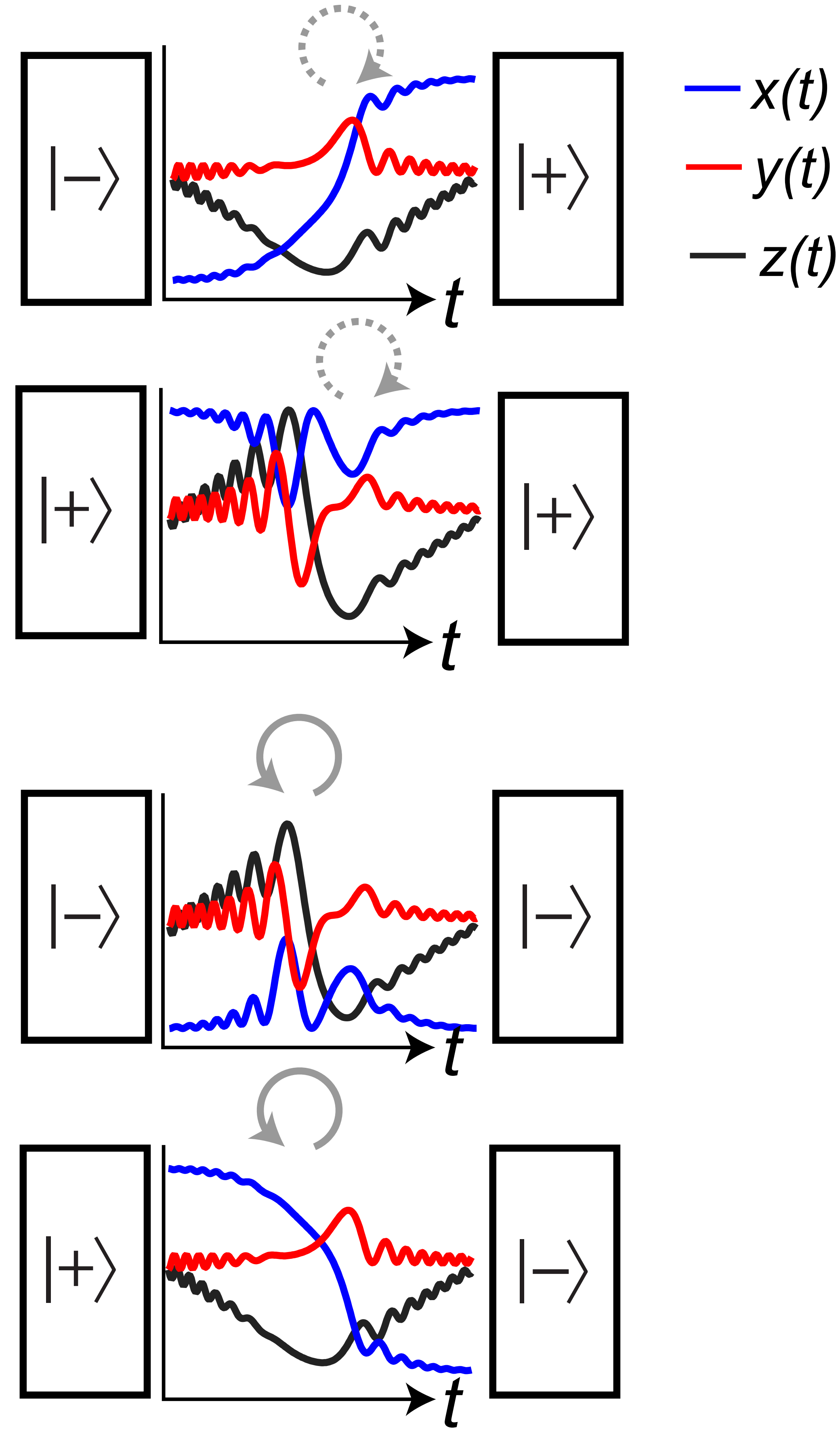
Can be situations (technical noise) where there are still advantages.

Also inspiration to look at non-lossy systems from a new angle.

A. Jordan PRX 2014

Topological features of the EP





Adiabatically tune parameters to encircle the EP

Non-reciprocal behavior

Non-Hermitian qubit

Transition from imaginary to real eigenvalues

($J<\gamma$) PT “broken” phase

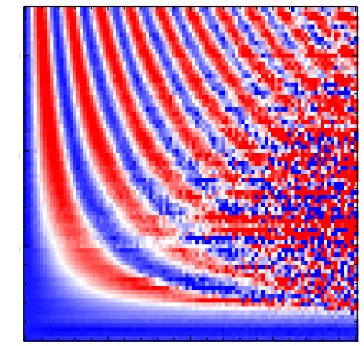
($J>\gamma$) PT “un-broken” phase

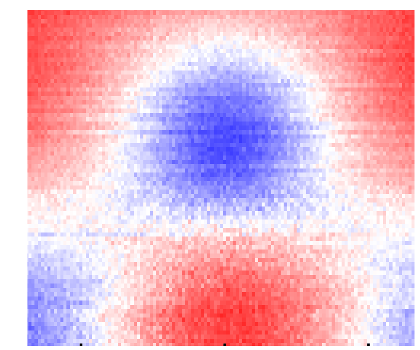
“Exceptional” point degeneracy ($J=\gamma$)

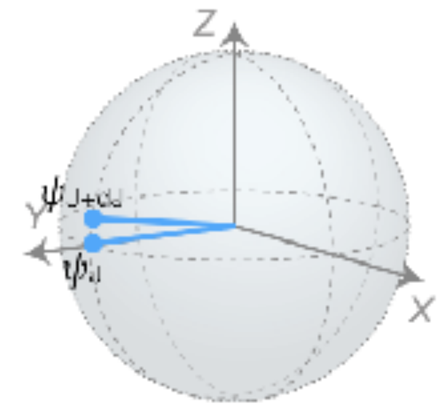
Eigenvectors not orthogonal and become degenerate

Enhanced sensitivity

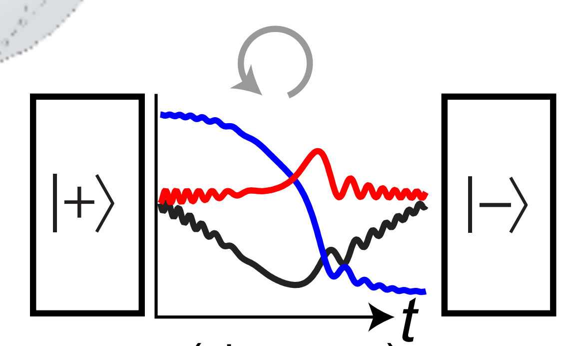
Topological/non-reciprocal features







(prelim)

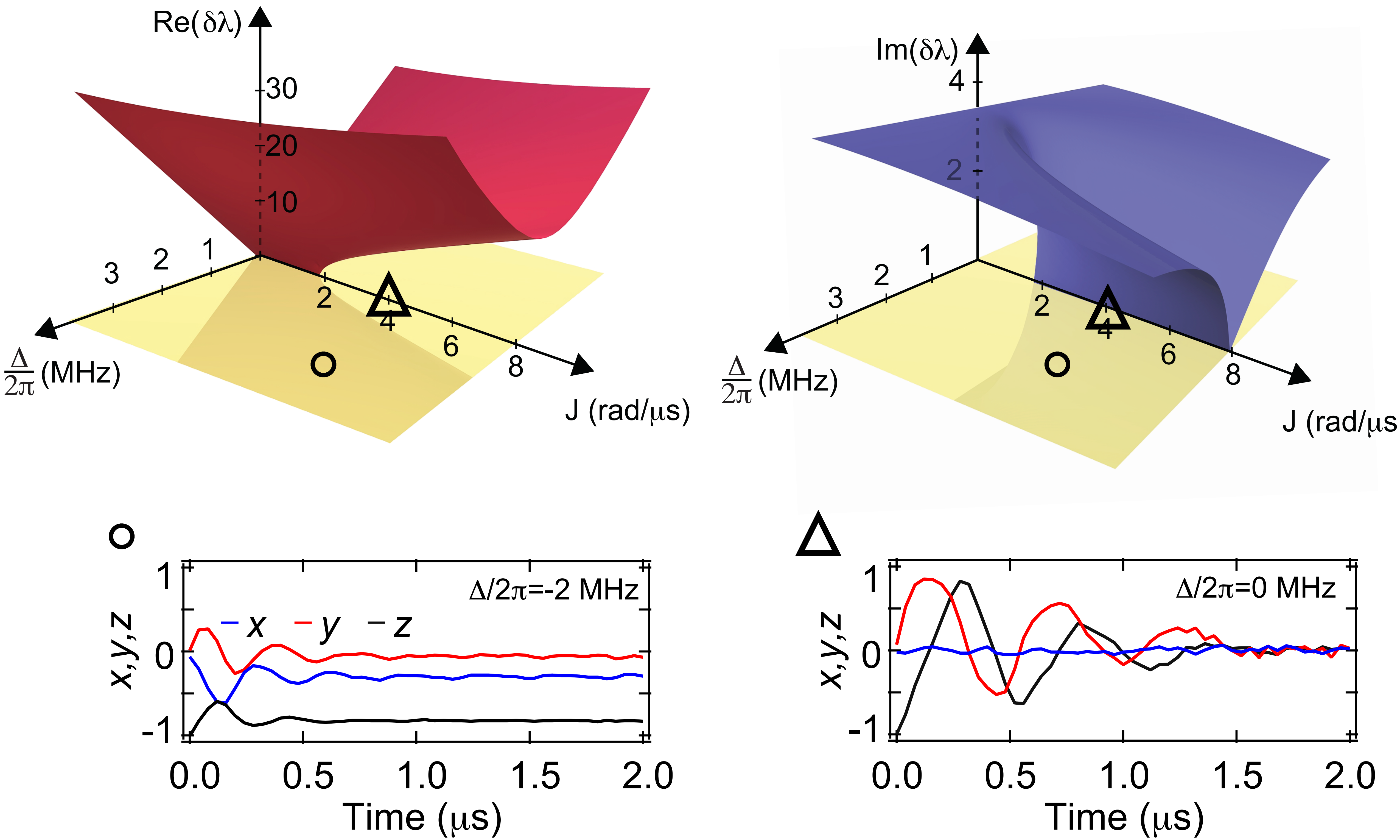


(theory)

Decoherence!

Dissipation: Lindblad vs non-Hermitian

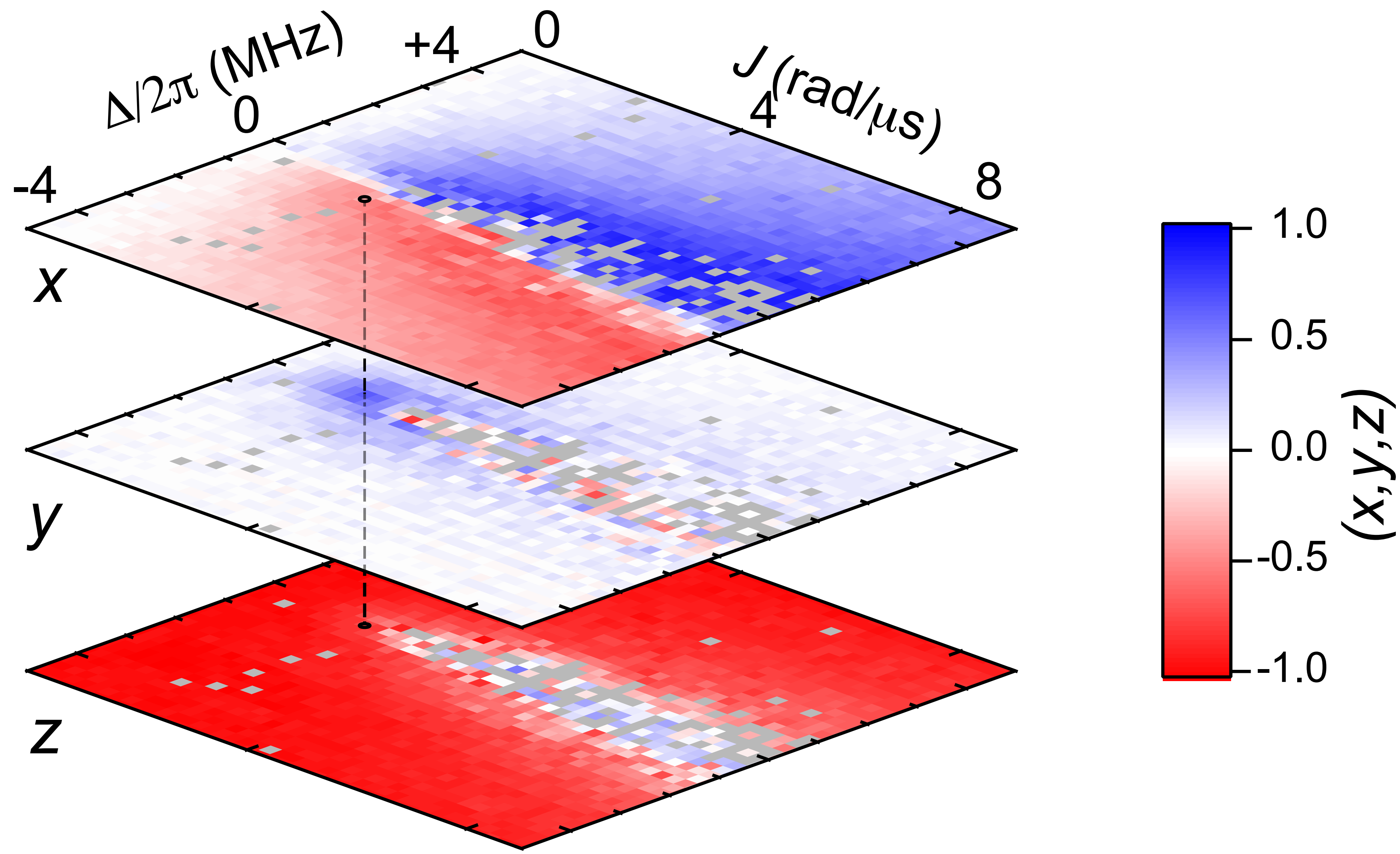
Interplay of two types of dissipation



(additional decoherence)

Steady states from dissipation interplay

Quantum state tomography after 4 μs of evolution

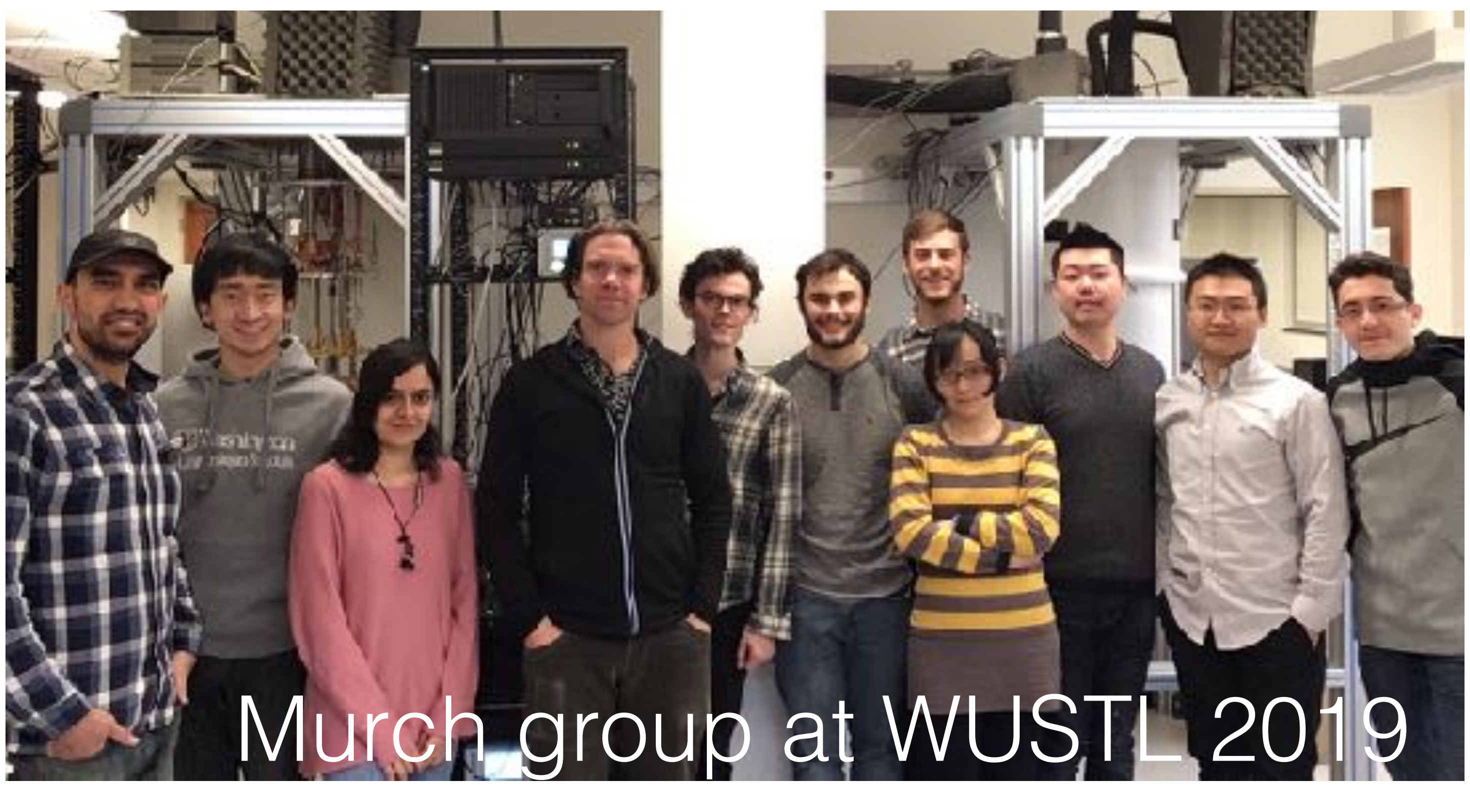


y bump: at the EP dissipation drives system to the single eigenstate.

Summary and thanks

Non-Hermitian qubit: enhanced sensing, non-reciprocal/ topological features, interplay of dissipations.

Naghiloo et al 2019 arXiv:1901.07968



Collaborators

Yogesh Joglekar (IUPUI)

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

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