

Random search challenges and new directions

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Joining the dots



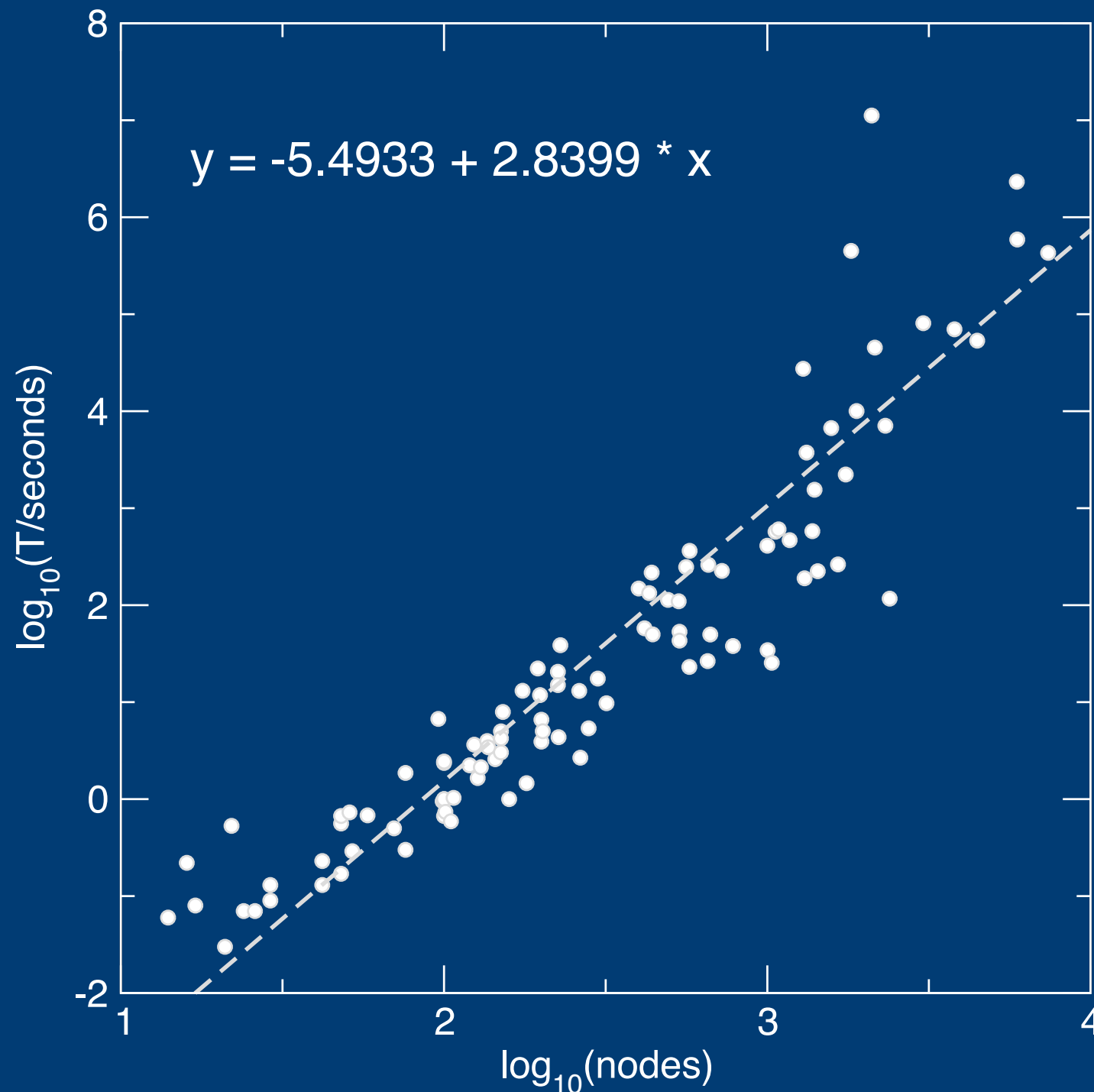
$(n-1)! = 50! \sim 3 \times 10^{64}$
age of universe $\sim 4 \times 10^{17}$ seconds

Joining the dots



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age of universe $\sim 4 \times 10^{17}$ seconds

Joining the dots



**Reasonable random
guess**



**Go “downhill” as far
as you can**



**Do something
- or nothing**

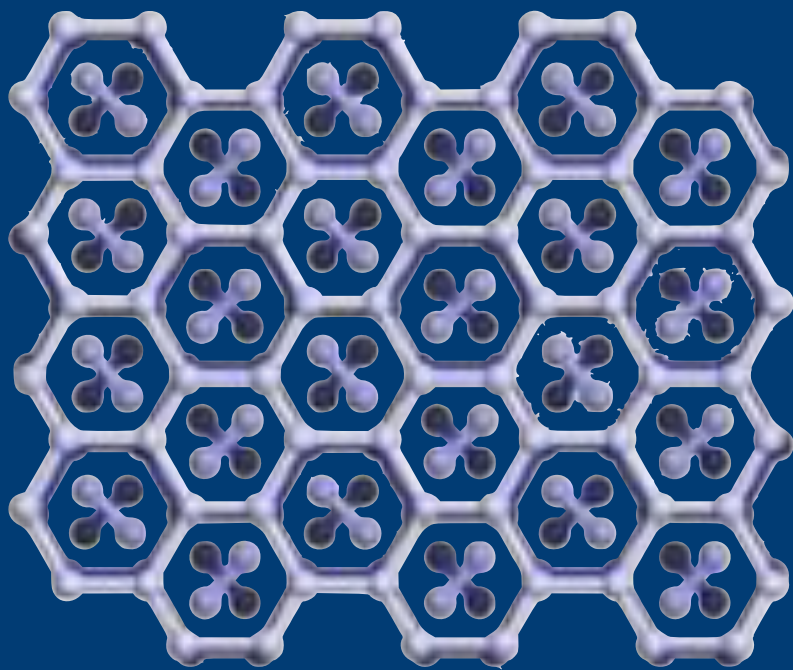


Analyse and Repeat

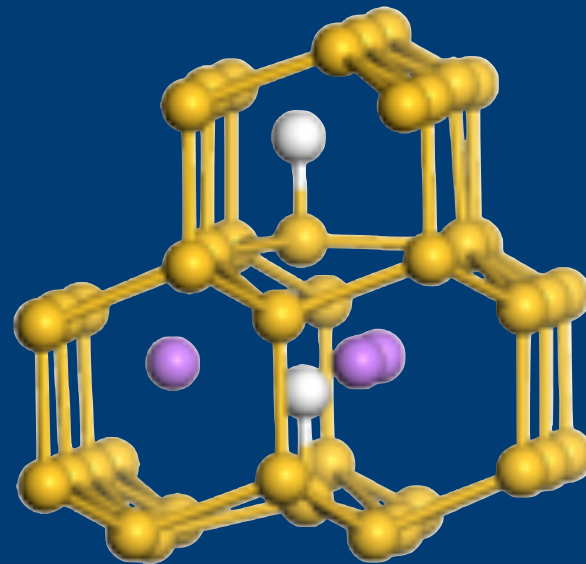
Data from William Cook:
<http://www.math.uwaterloo.ca/tsp/concorde/benchmarks/bench99.html>

Structure Prediction

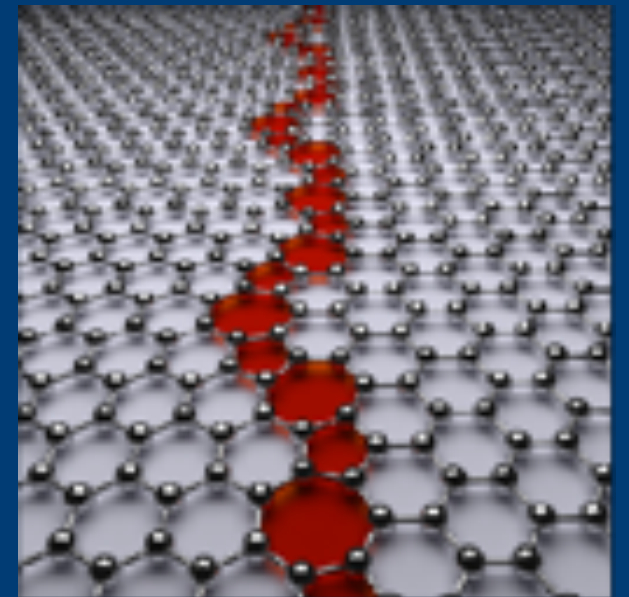
What structure will a collection of atoms adopt?



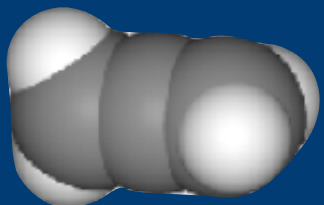
Crystal



Point Defect



Interfaces



Allene

"The Arrangement of Atoms in Space"
Jacobus Henricus van 't Hoff, 1898



Energy

Interpolative

Empirical or data driven

Extrapolative and predictive

First principles or theory driven

Periodic Table

Reference point: **WIEN2k**13.1 with basis LAPW/APW+lo and potential all-electron

Code: **CASTEP**9.0 with basis plane waves and potential OTFG CASTEP 9.0

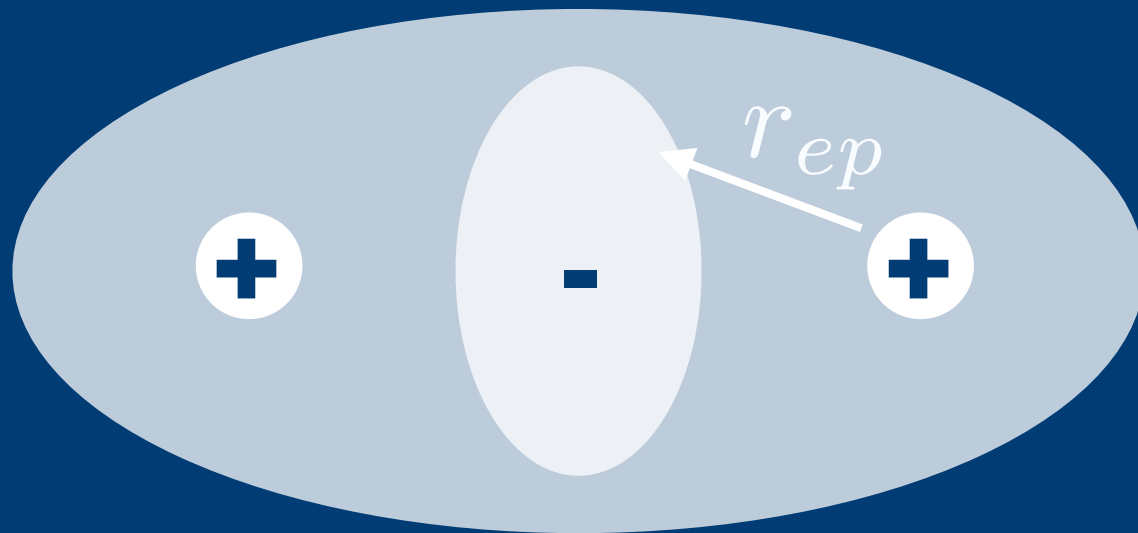
Maximum at Mn, Minimum at V

All values are in meV.

H 0.05																		He 0.03
Li 0.08	Be 0.26											B 0.81	C 0.37	N 0.66	O 1.17	F 0.52	Ne 0.16	
Na 0.44	Mg 0.27											Al 0.11	Si 0.24	P 0.64	S 0.28	Cl 0.74	Ar 0.12	
K 0.12	Ca 0.07	Sc 0.82	Ti 0.35	V 0.03	Cr 1.19	Mn 2.42	Fe 0.62	Co 0.44	Ni 0.38	Cu 0.51	Zn 0.40	Ga 0.06	Ge 0.18	As 0.35	Se 0.58	Br 0.11	Kr 0.04	
Rb 0.21	Sr 1.23	Y 0.53	Zr 0.33	Nb 0.30	Mo 0.46	Tc 0.56	Ru 0.86	Rh 0.37	Pd 0.52	Ag 0.46	Cd 1.35	In 0.32	Sn 0.28	Sb 0.30	Te 0.08	I 0.72	Xe 0.11	
Cs 0.13	Ba 0.11		Hf 0.36	Ta 0.85	W 1.05	Re 0.75	Os 0.25	Ir 0.23	Pt 0.24	Au 1.15	Hg 1.02	Tl 0.19	Pb 0.67	Bi 1.09	Po 1.03	At 0.72	Rn 0.13	
Fr	Ra																	
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lr 0.54		
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Kurt Lejaeghere, et al., Reproducibility in density functional theory calculations of solids, Science 351 (6280), aad3000 (2016).

Smooth interactions



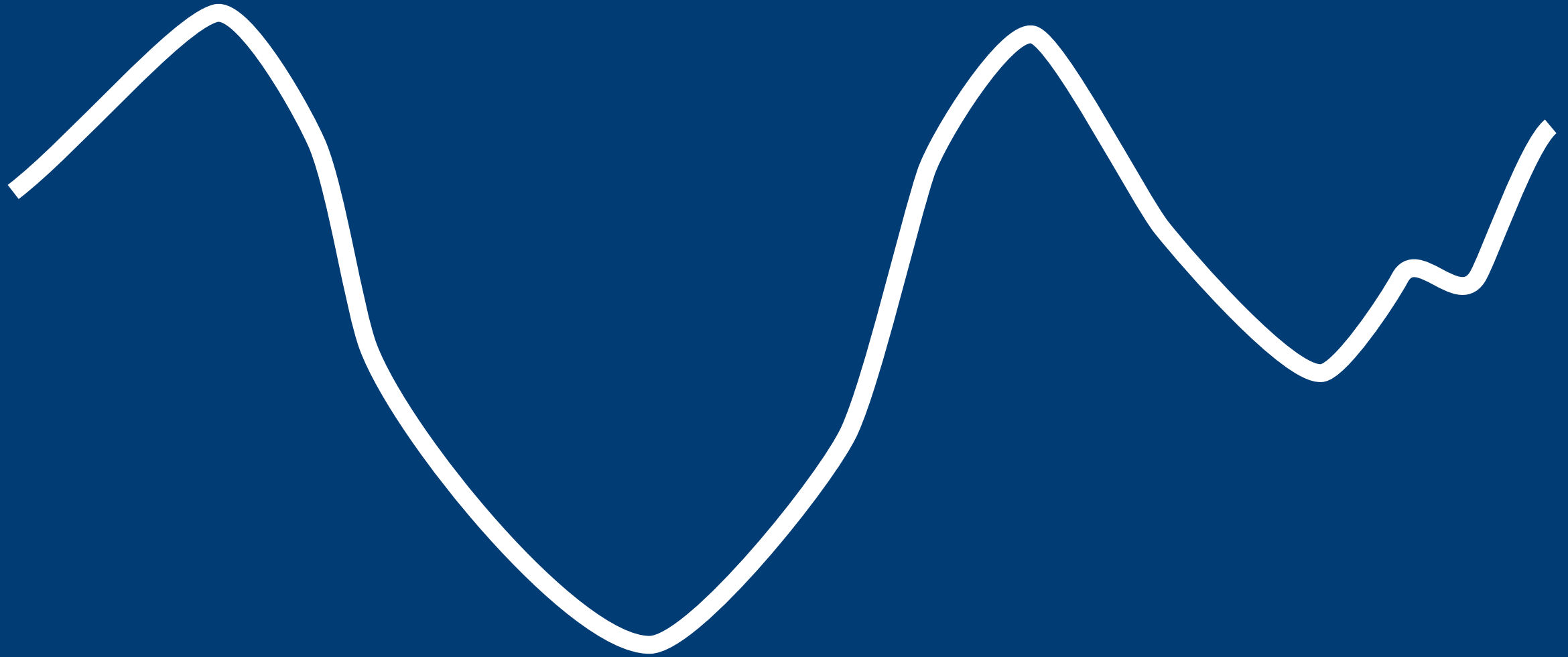
$$E_{ep} = -\frac{1}{r_{ep}}$$

Coulomb

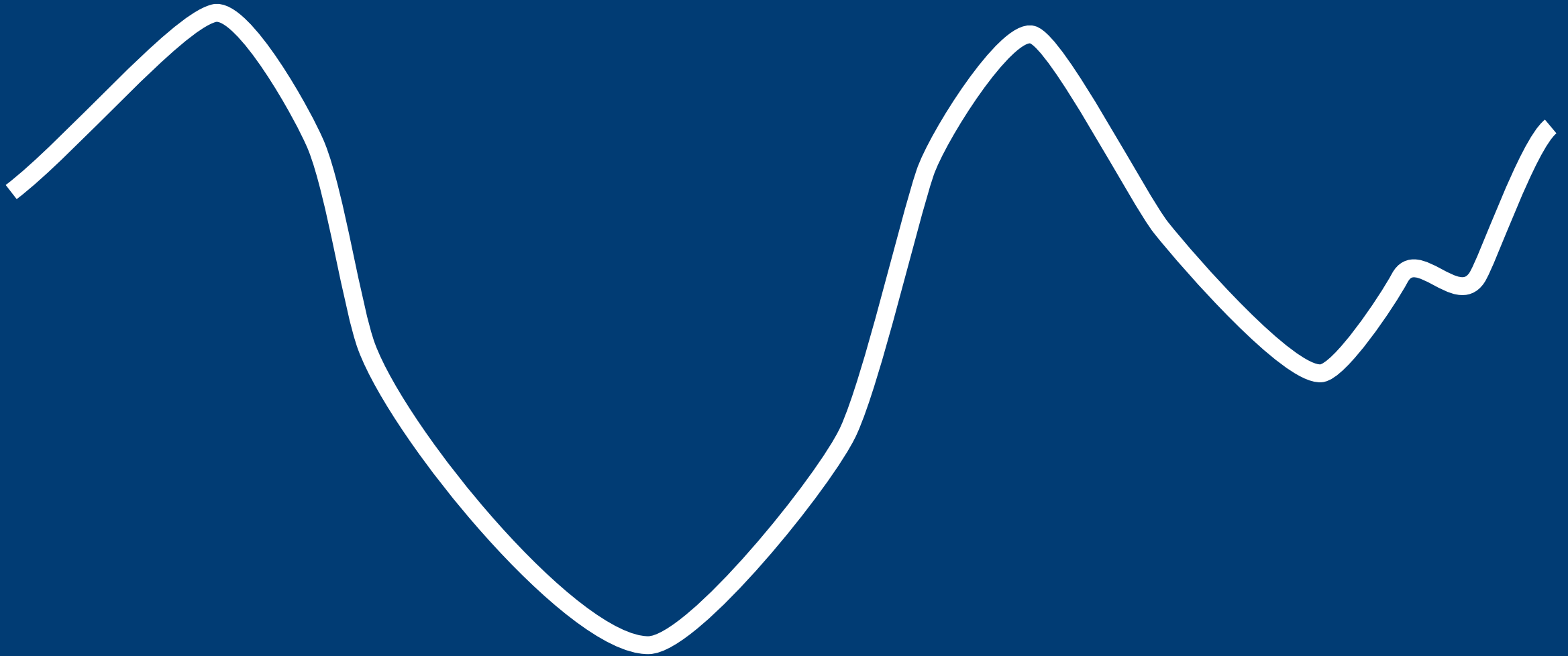
$$H\Psi = -\frac{1}{2}\nabla^2\Psi + V\Psi = E\Psi$$

Schroedinger

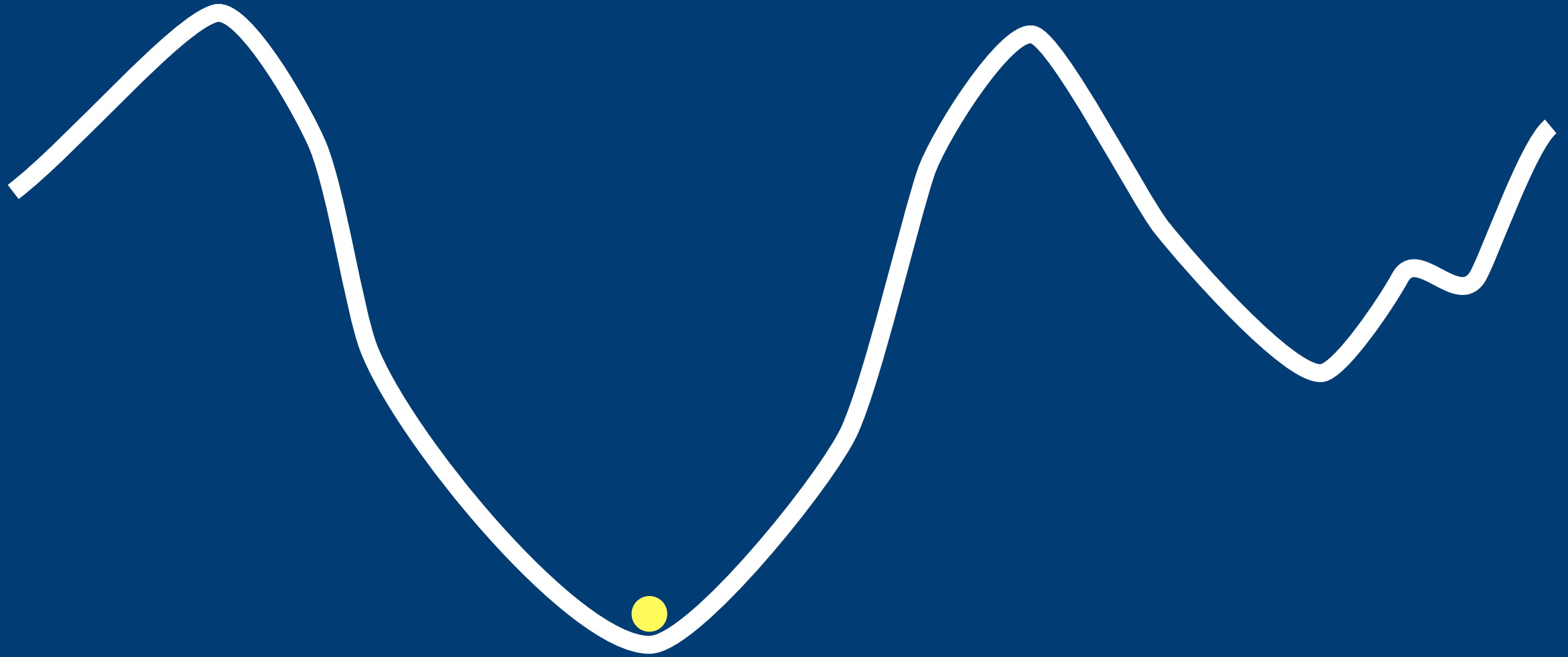
Smooth Landscapes



Smooth Landscapes

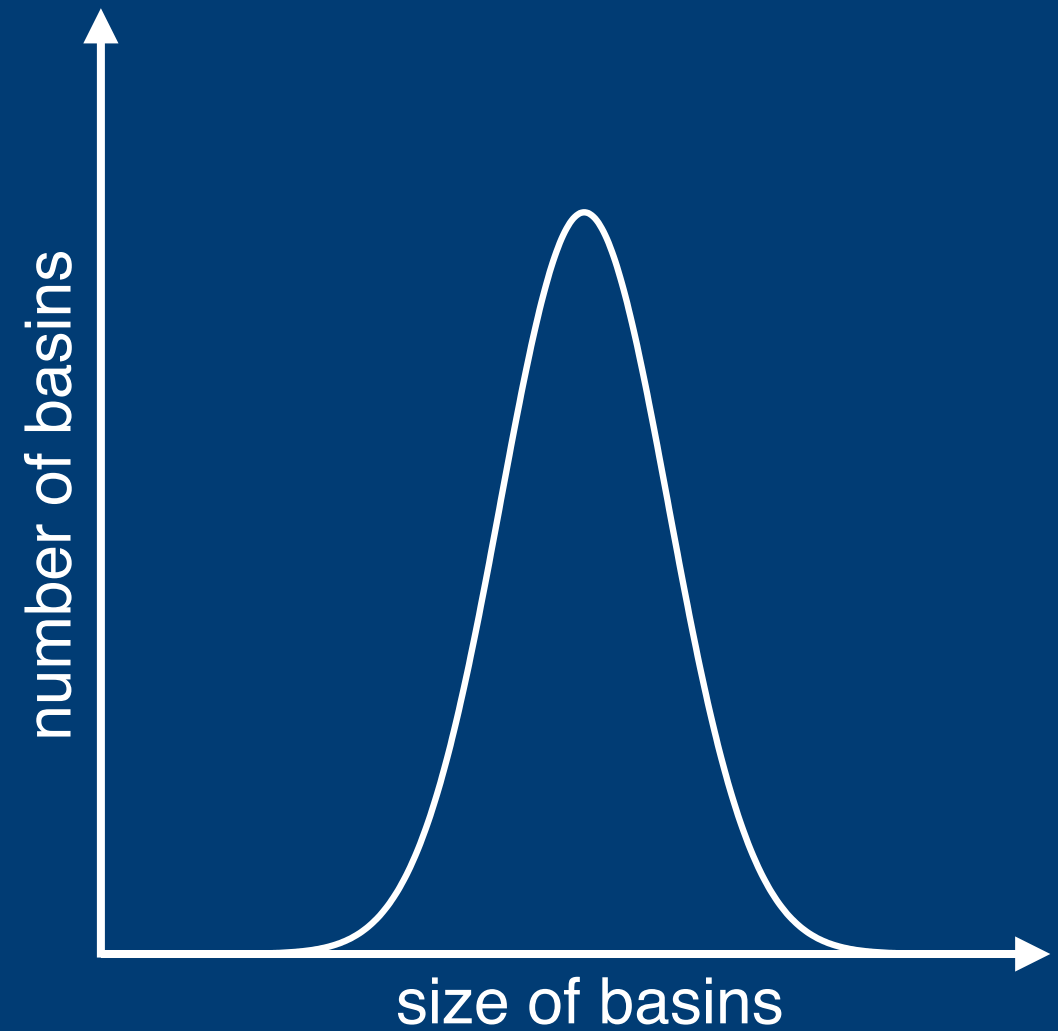
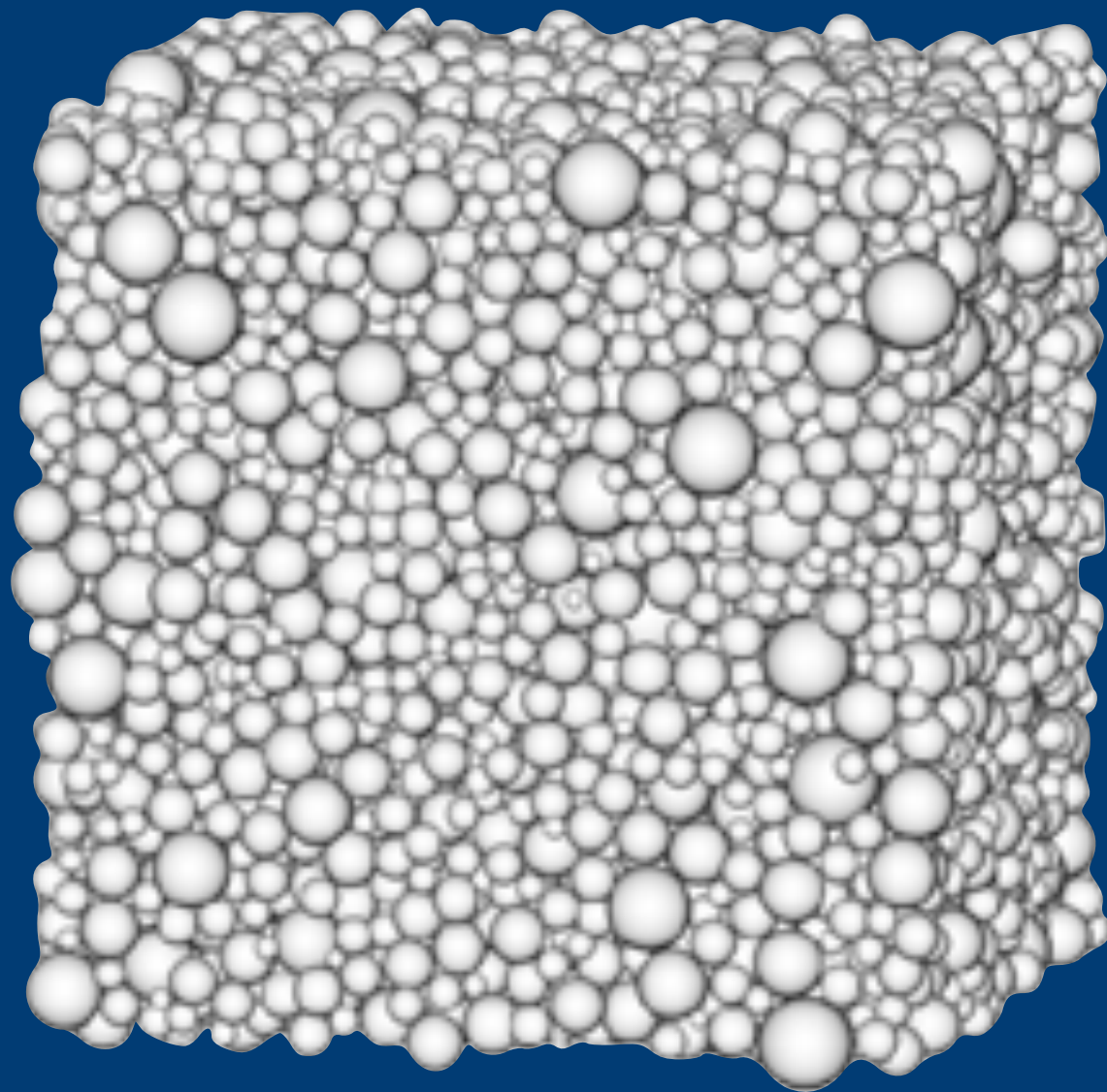


Smooth Landscapes

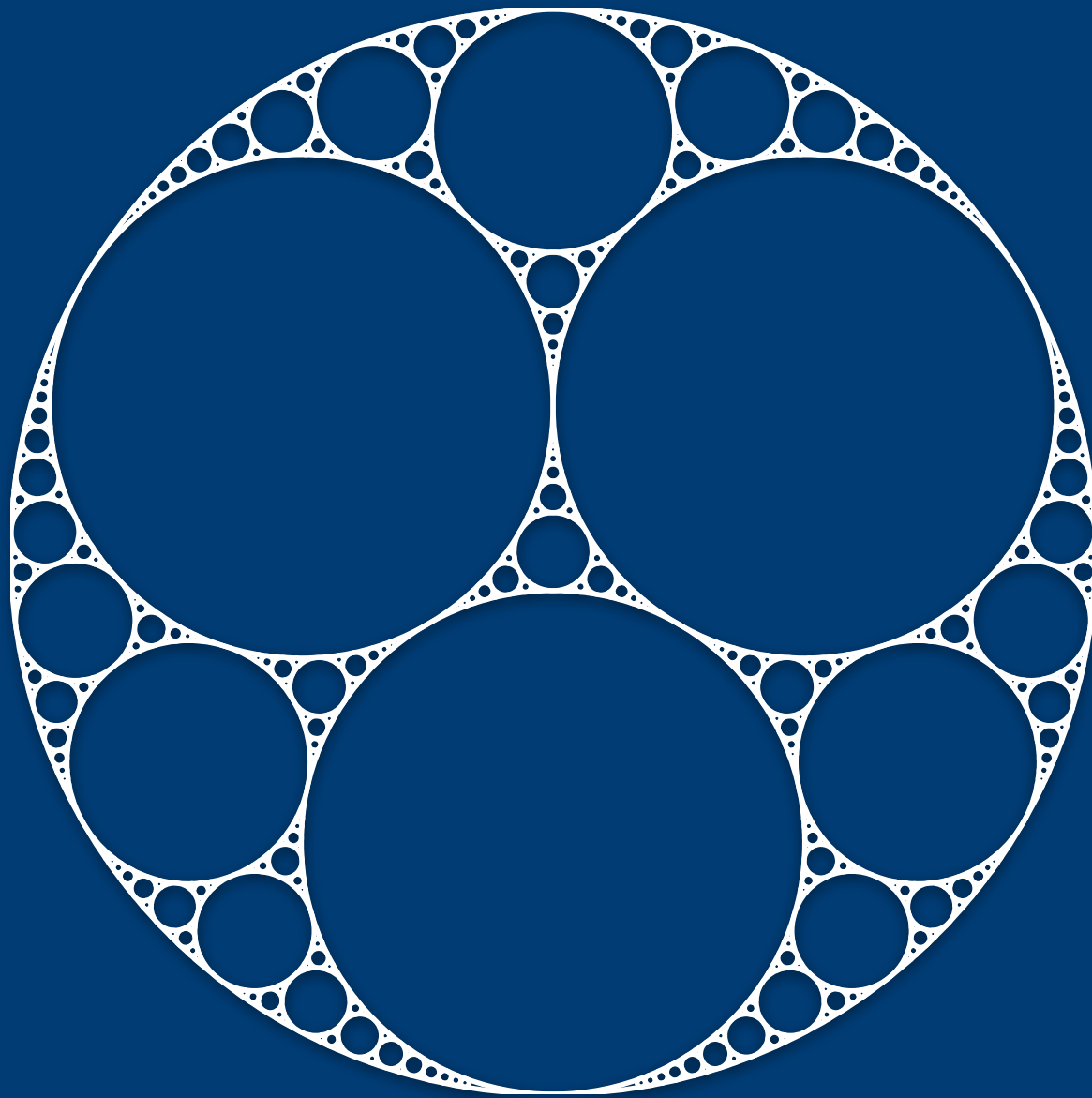


Deep basins are large

Basin Volumes

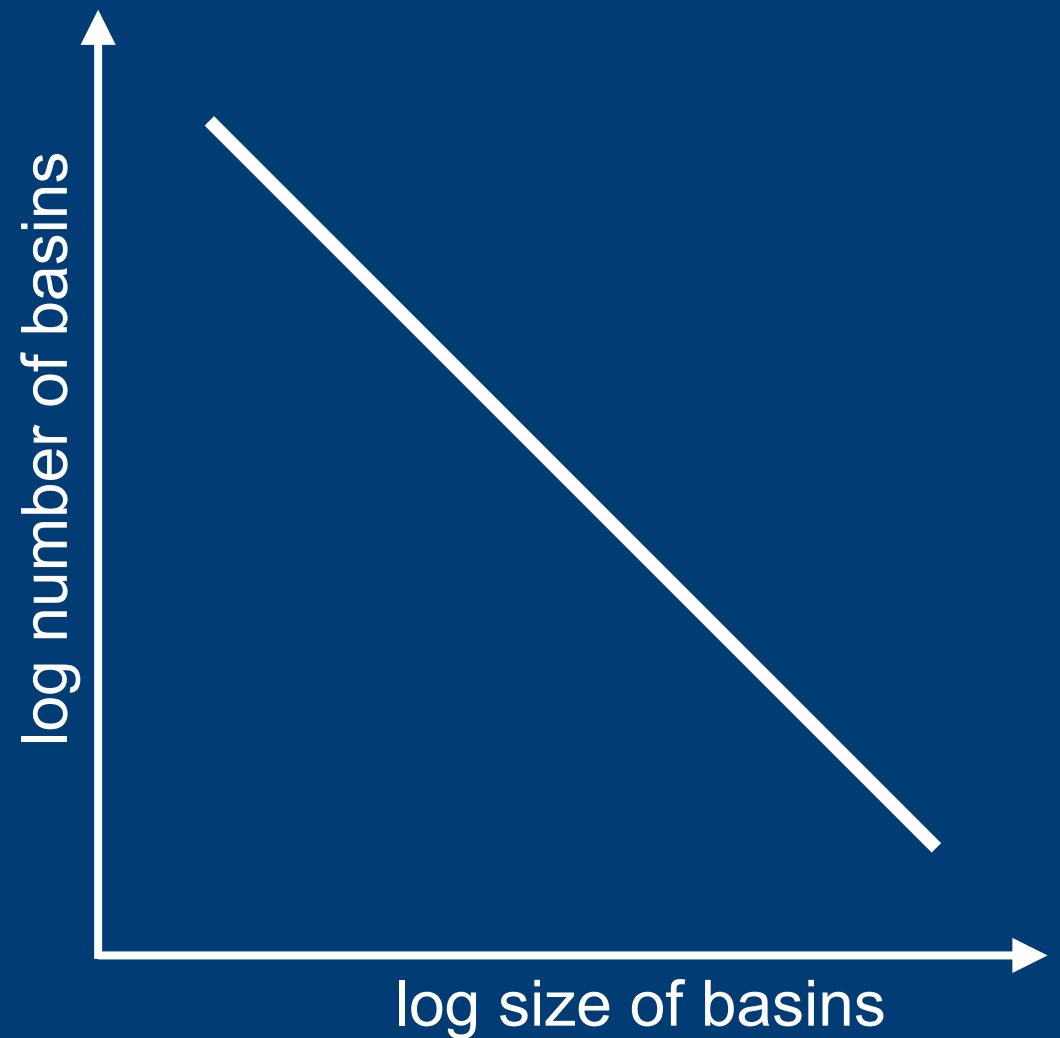


Basin Volumes



Apollonian Gasket

a Fractal



Doye and Massen, J. Chem. Phys. 122, 084105 (2005)

Massen and Doye, PRE, 75, 037101 (2007)

Random Sampling

Uniform distribution
(*exploration not exploitation*)

Intrinsically parallel

Uncorrelated

Clear when (not) to stop

Robust and communicable

Being Sensible

When you don't know anything:
rough volume, avoid overlap

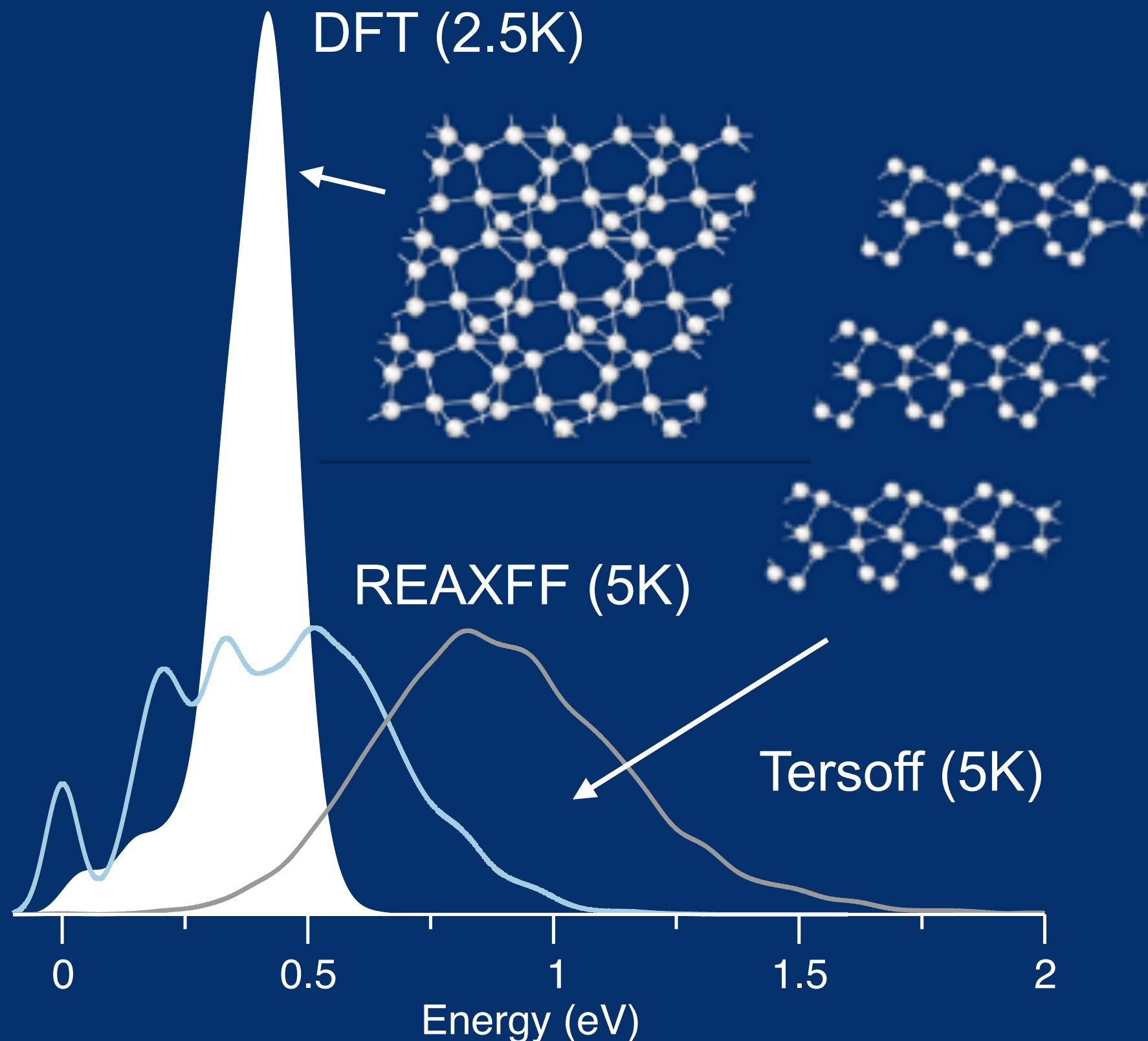
Impose chemical ideas
molecules, fragments, distances, connectivity

Impose symmetry
space, wallpaper, point group

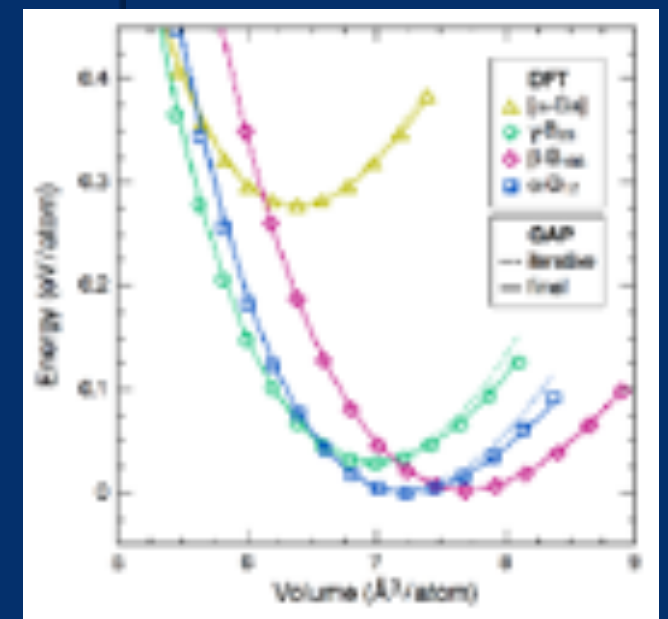
Use experimental data
lattice parameters

Empirical or First Principles?

Eight atoms of Silicon



Data-Driven Learning
of Total and Local
Energies in Elemental
Boron



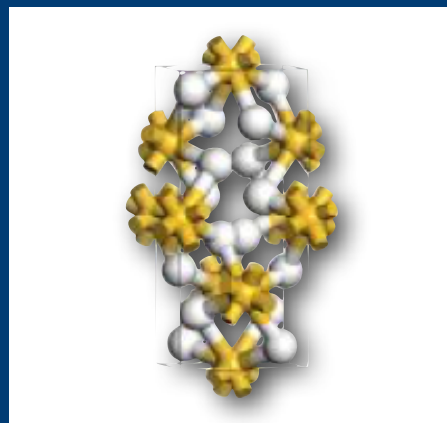
Deringer, Pickard,
Csanyi, PRL, 2018

**Learn entire energy
landscape**

Ab Initio Random Structure Searching

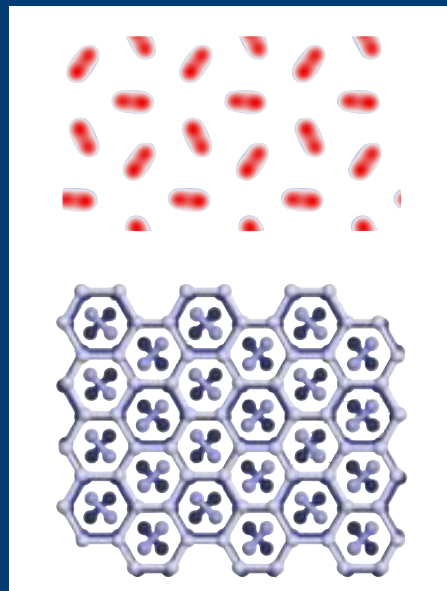
Superconducting
hydrides

Physical Review Letters,
2006



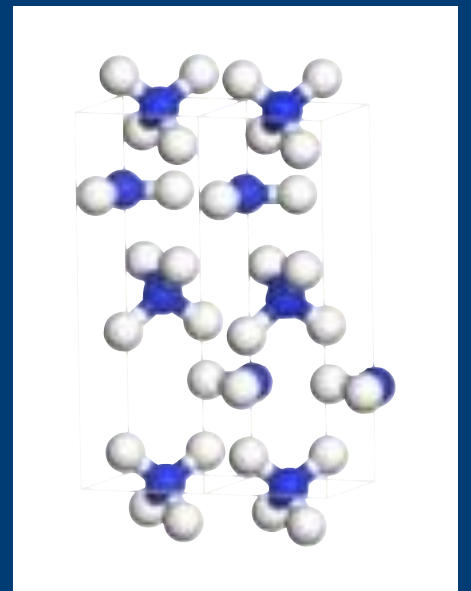
Hydrogen is
polar and
“graphene”

Nature Physics,
2007



Ammonia is ionic

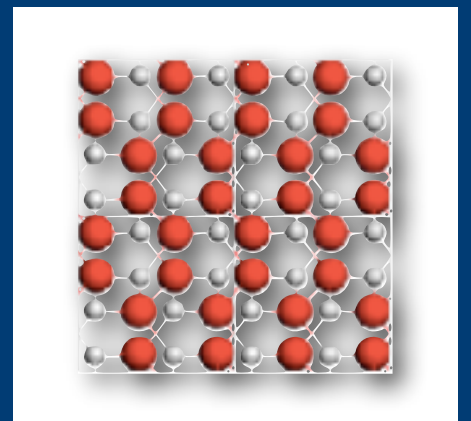
Nature Materials,
2008



The end of water

Physical Review Letters,
2013

+ Martinez-Canales



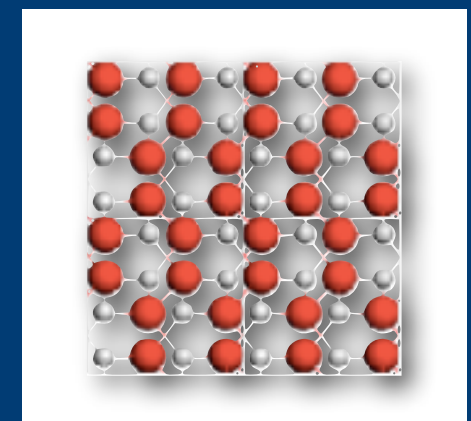
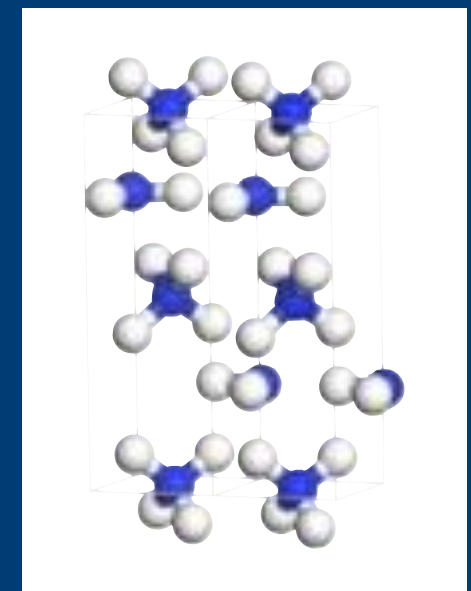
Pickard & Needs, *PRL* 2006 and *JPCM* 2011

see also: basin/minima hopping, GA/EAs, particle swarms

Ab Initio Random

Searching

... is ionic



PRL 119, 107001 (2017)

PHYSICAL REVIEW LETTERS

week ending
8 SEPTEMBER 2017

Hydrogen Clathrate Structures in Rare Earth Hydrides at High Pressures: Possible Route to Room-Temperature Superconductivity

Feng Peng,^{1,2,3} Ying Sun,² Chris J. Pickard,⁴ Richard J. Needs,⁵ Qiang Wu,⁶ and Yanning Ma^{1,2,3}

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³State Key Laboratory of Electromagnetic Transformation and Detection, Linying 475022, China

⁴State Key Laboratory of Superhard Materials, College of Physics, Jilin University, Changchun 130012, China

⁵Department of Materials Science and Metallurgy, University of Cambridge, Cambridge CB3 0PS, United Kingdom

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⁷Department of Condensed Matter Physics, Graduate School of Science, Tohoku University, 2-1-1 Katahira, Aoba, Sendai 980-8577, Japan

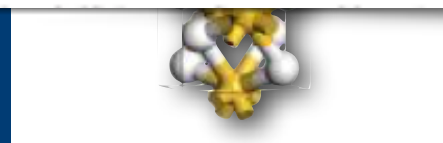
⁸Department of Condensed Matter Physics, Graduate School of Science, Tohoku University, 2-1-1 Katahira, Aoba, Sendai 980-8577, Japan

⁹National Key Laboratory of Shock Wave and Detonation Physics, Institute of Fluid Physics, CAFEP, Mianyang 621000, China

¹⁰International Center of Future Science, Abo University, Guangzhou 510012, China

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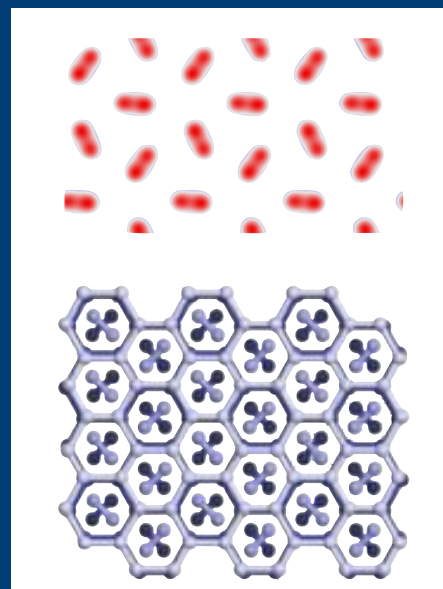
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Searching

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and Advanced

Theory of Condensed

Matter Key Laboratory

Superconductivity at 215 K in lanthanum hydride at high pressures

A. P. Drozdov, V. S. Minkov, S. P. Besedin, P. P. Kozh, M. A. Kuzovnikov, D. A. Knyazev,
M. I. Erements

Max-Planck-Institut für Chemie, Hahn-Meitner Weg 1, 55128 Mainz, Germany

Physical Review
2017

Hydrogen is
polar and
“graphene”

Nature Physics,
2007

Evidence for superconductivity above 260 K in lanthanum superhydride at megabar pressures

Madhury Somayajulu^{1,2}, Muktar Abart¹, Ajay K. Mishra², Zachary M. Geballe², Maria Radtke²,
Yue Meng², Viktor V. Struzhkin³, and Russell J. Hemley^{1,2}

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³IPCAT, X-ray Science Division, Argonne National Laboratory, Argonne IL 60439, USA

Pickard & Needs, *PRL* 2006 and *JPCM* 2011

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Theory of Condensed

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Superconductivity at 250 K in lanthanum hydride under high pressures

A. P. Drozdov¹, P. P. Kong¹, V. S. Minkov¹, S. P. Besedin¹, M. A. Kuzovnikov^{1,6}, S. Mozaffari², L. Balicas², F. Balakirev³, D. Graf², V. B. Prakapenka⁴, E. Greenberg⁴, D. A. Knyazev¹, M. Tkacz⁵, and M. I. Erements¹

Physical Review Letters
2017

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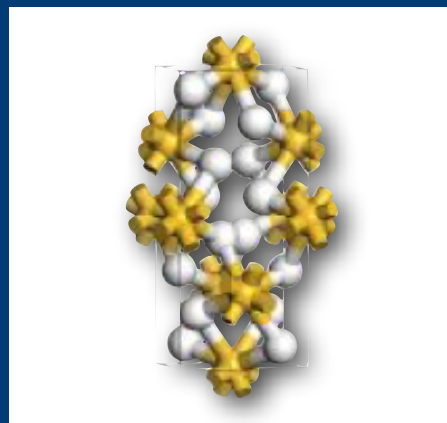
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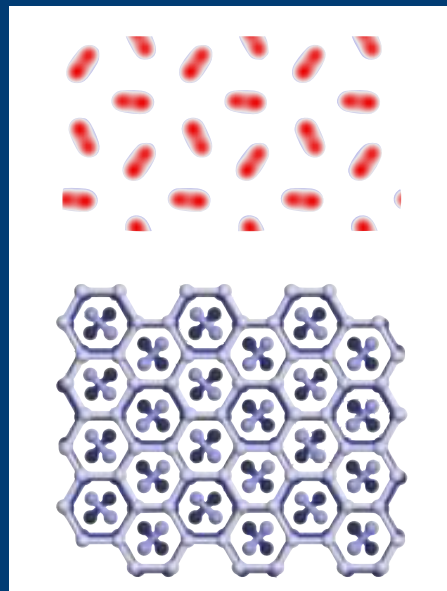
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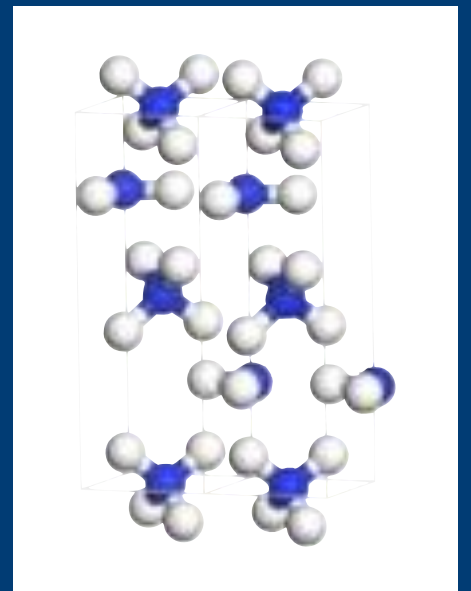
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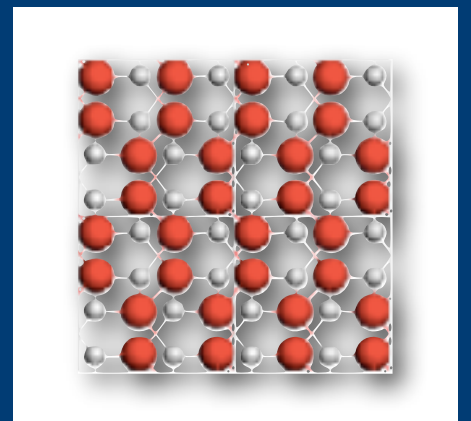
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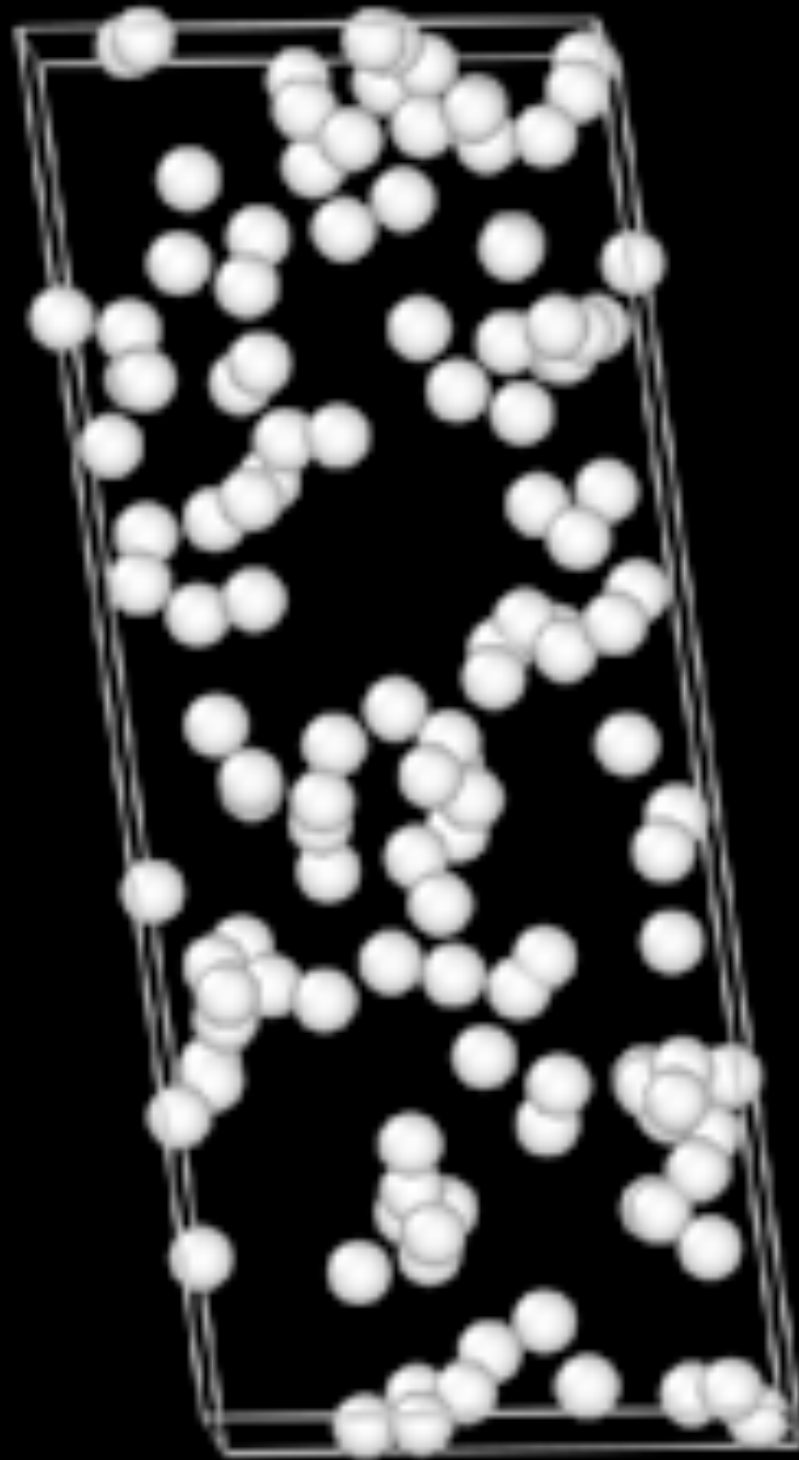
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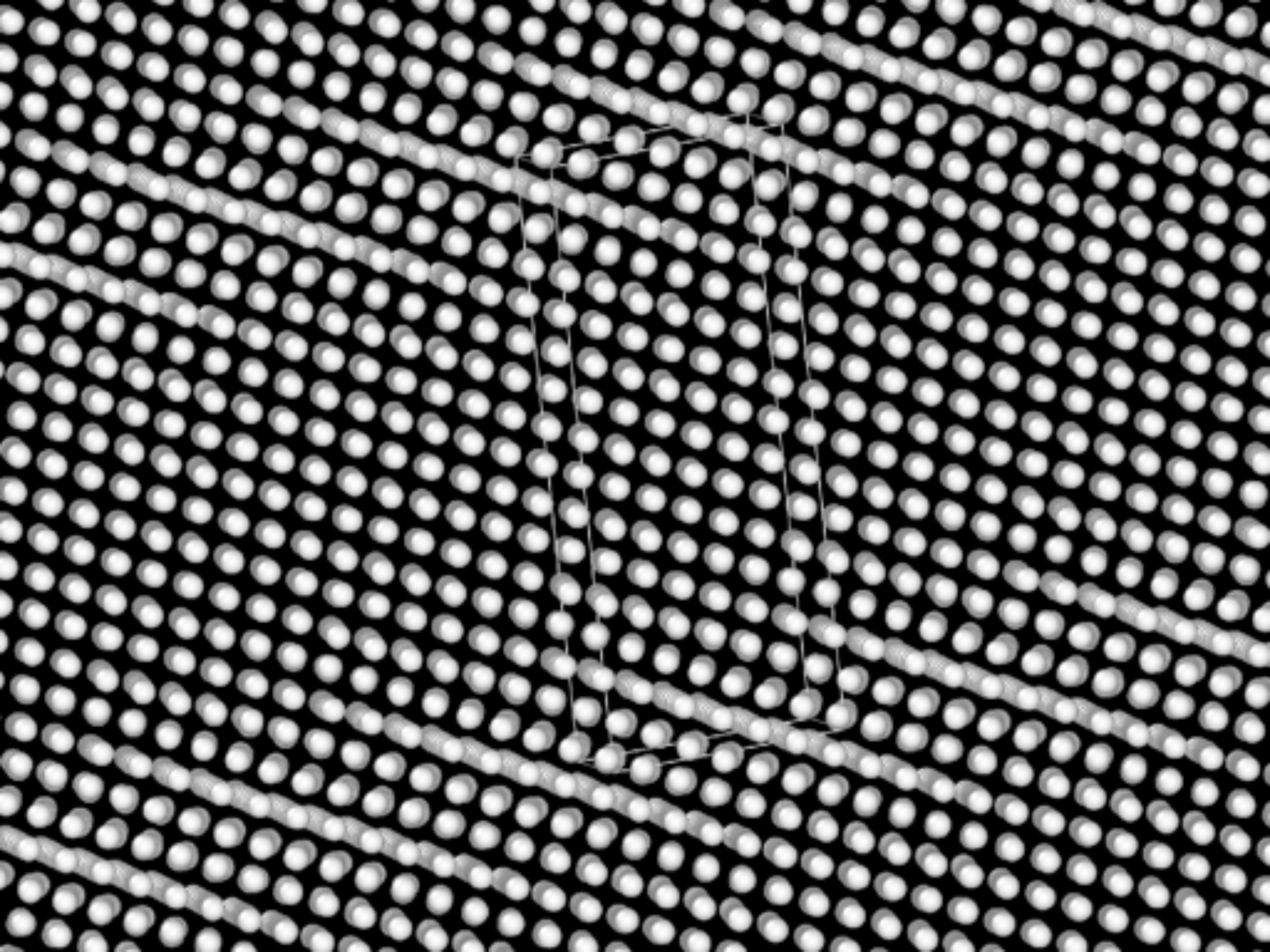
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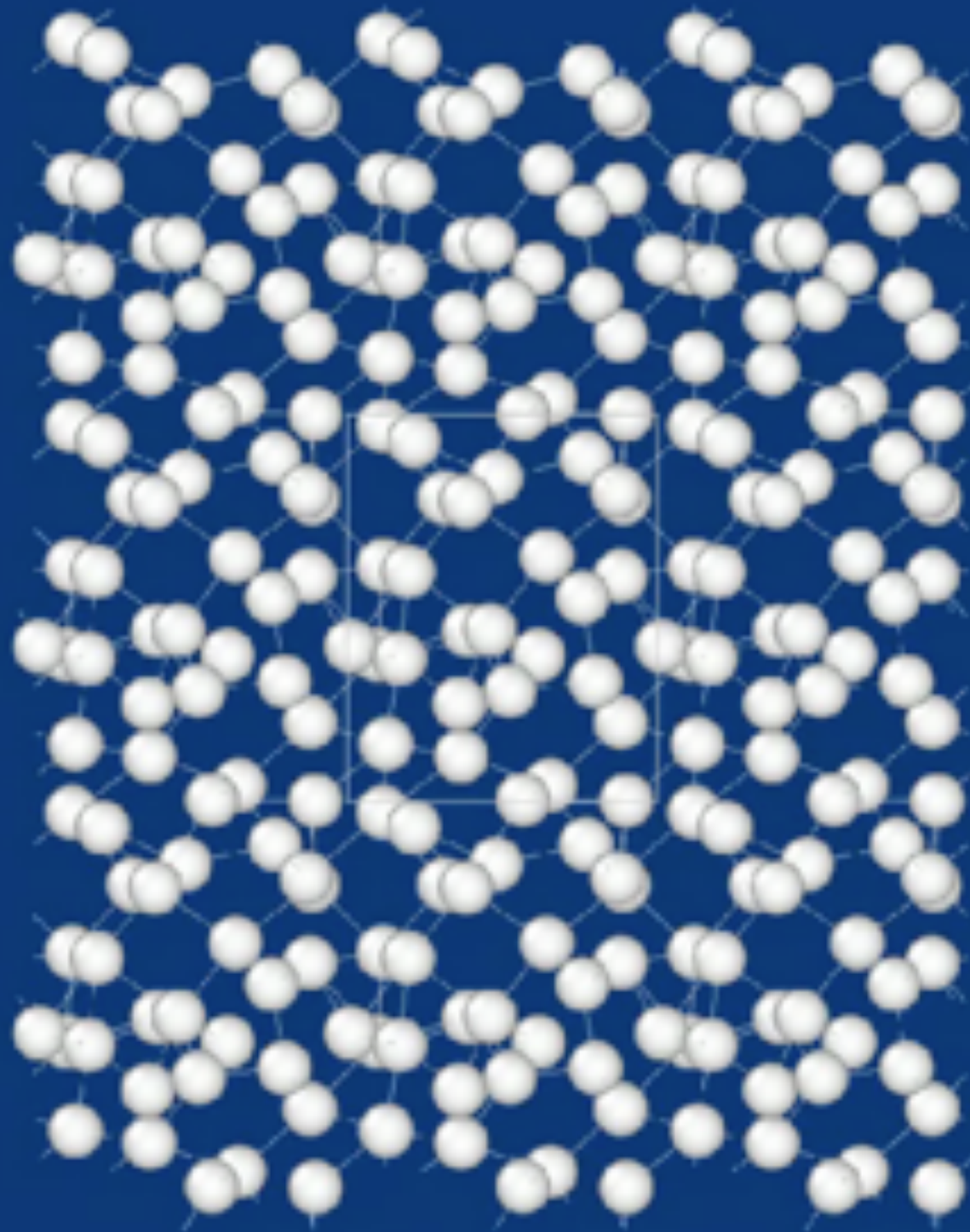
Thomas-Fermi-Dirac

simplified 128 atoms ($V \sim L^{384}$)

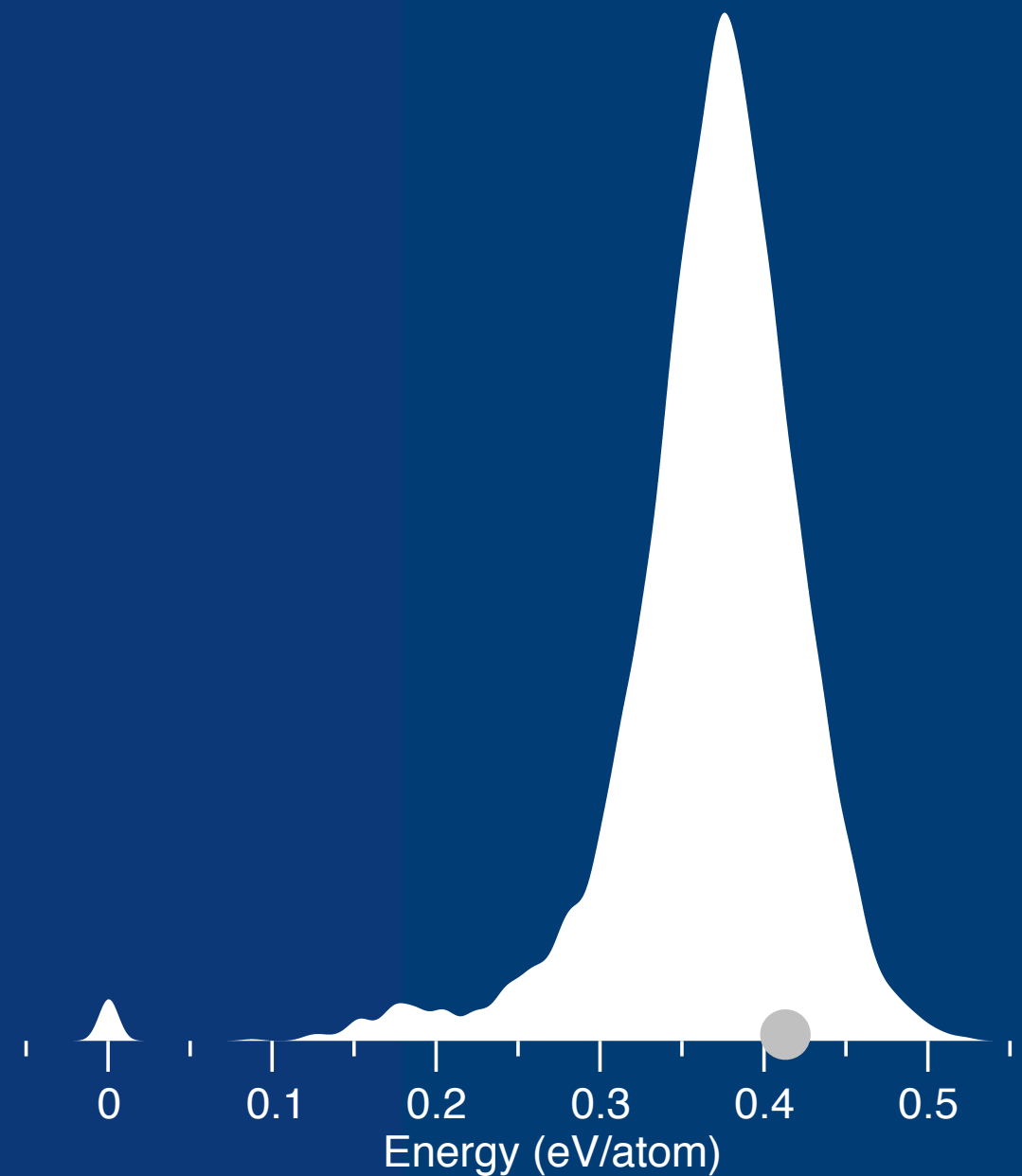
C.J. Pickard, *Physical Review Materials*, **2**, 013806 (2018)



Gamma Boron

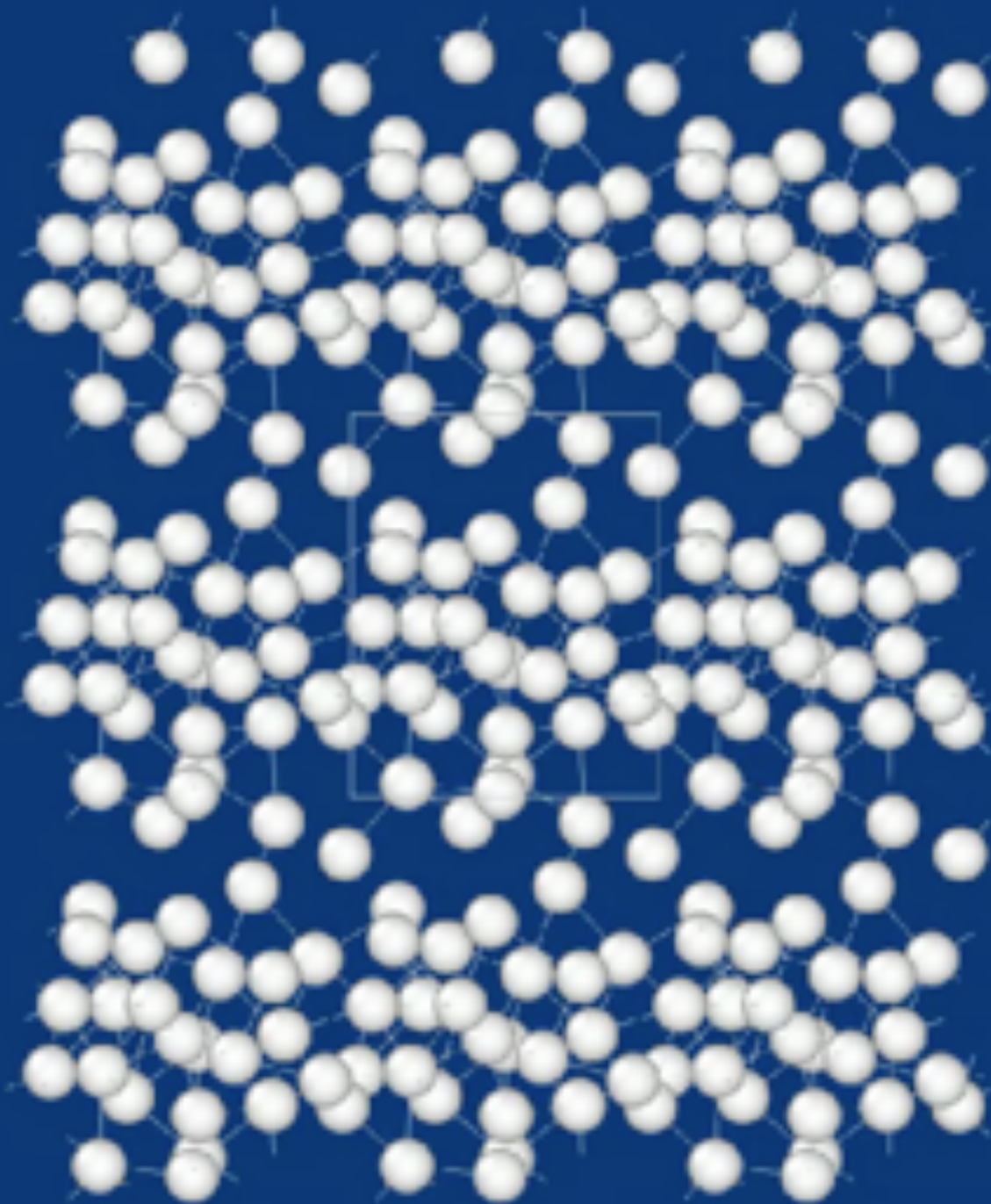


Fixed Cell
AIRSS Example 2.3

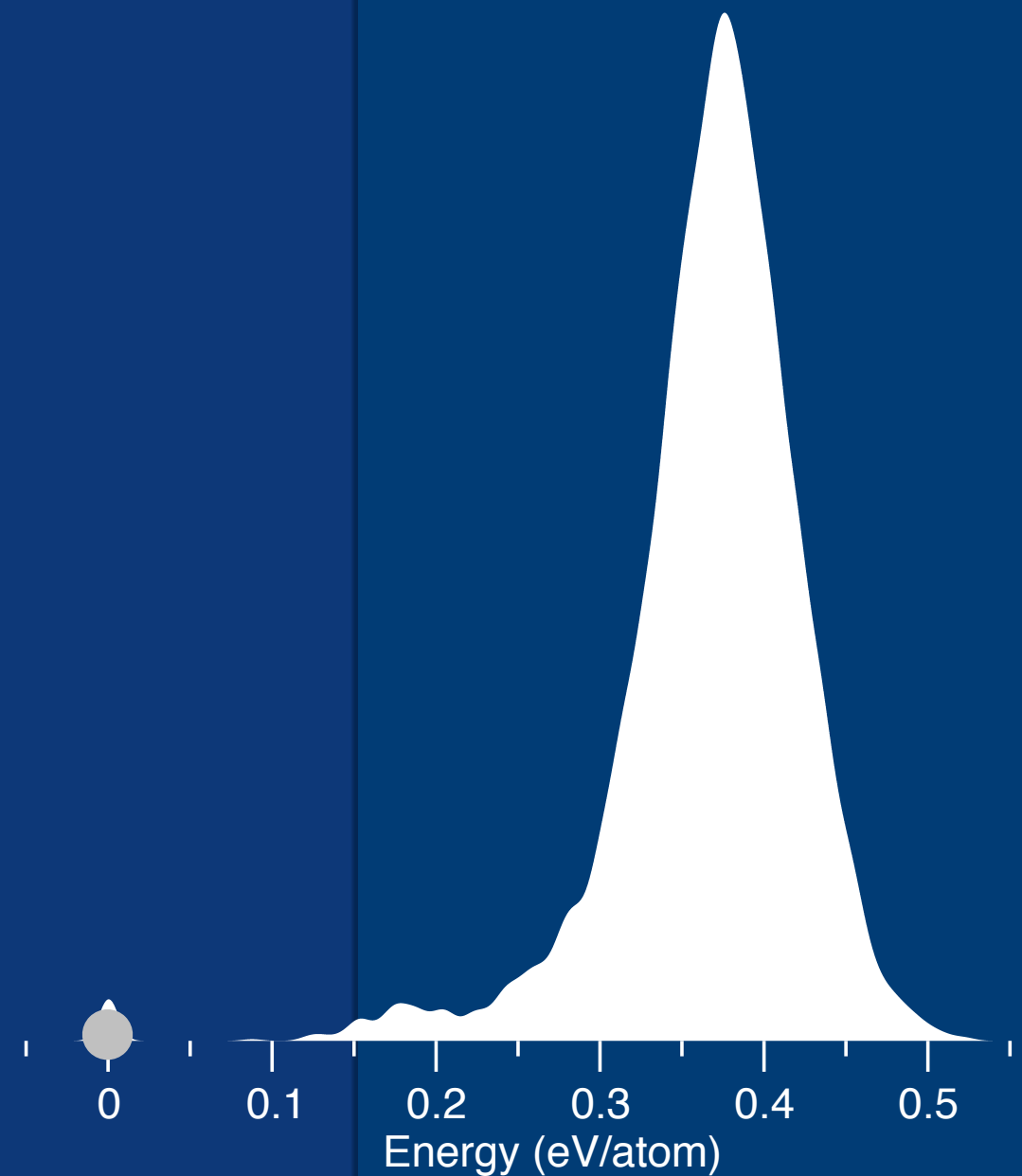


Oganov et al *Nature* 2009

Gamma Boron



Fixed Cell
AIRSS Example 2.3



Oganov et al *Nature* 2009

Random “sensible” structures

#VARVOL=11.8

#SPECIES=B%NUM=1,N%NUM=1

#SYMMOPS=1

#NFORM=128

#MINSEP=1.0 B-B=2.6 N-N=2.6 B-N=1.6

KPOINTS_MP_SPACING 0.07

SYMMETRY_GENERATE

SNAP_TO_SYMMETRY

%BLOCK SPECIES_POT

%ENDBLOCK SPECIES_POT

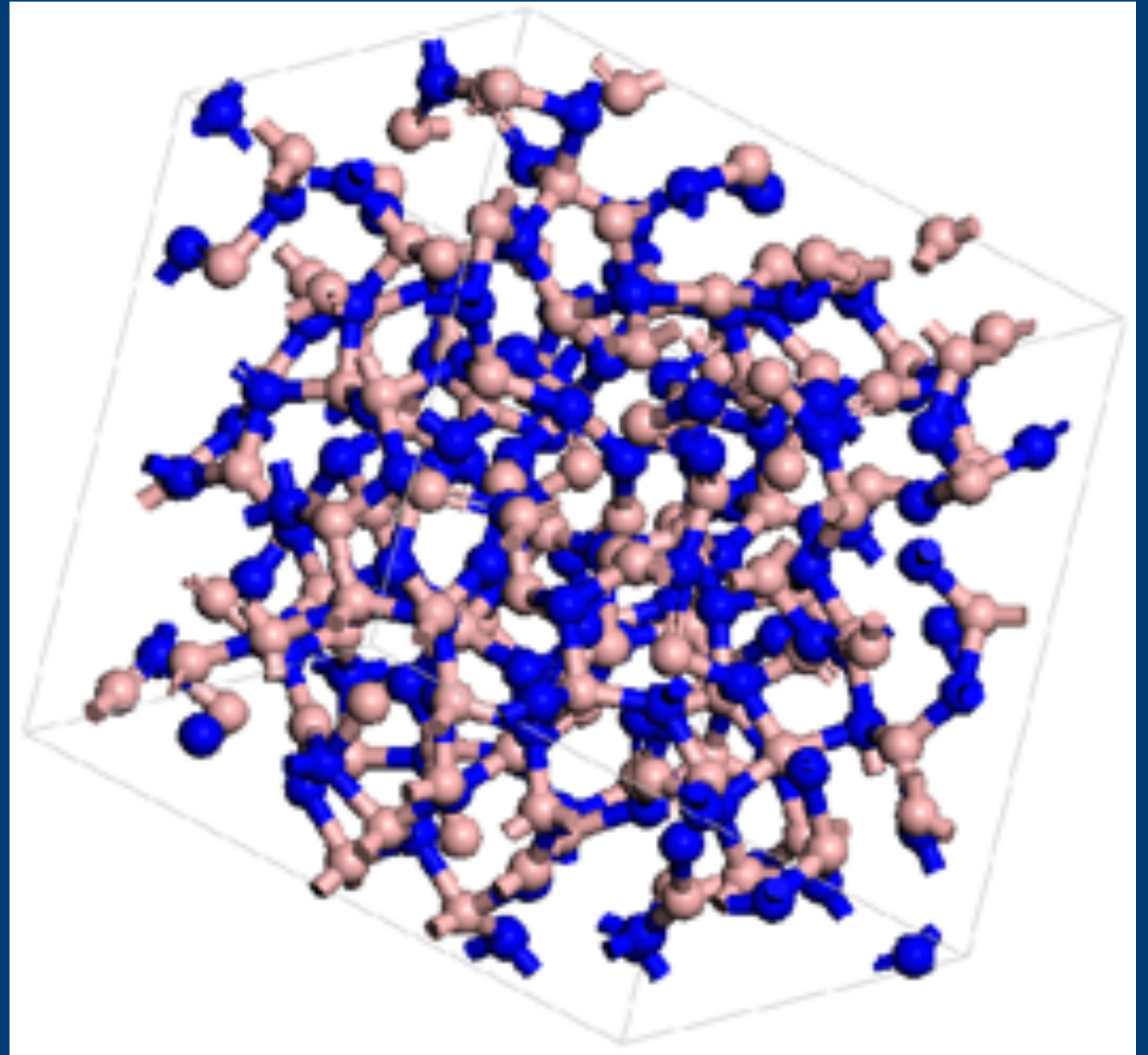
%BLOCK EXTERNAL_PRESSURE

0 0 0

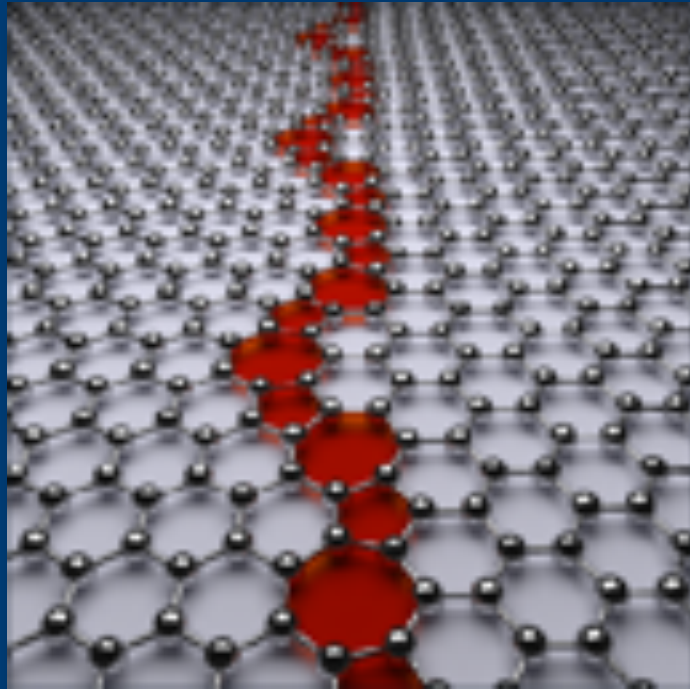
0 0

0

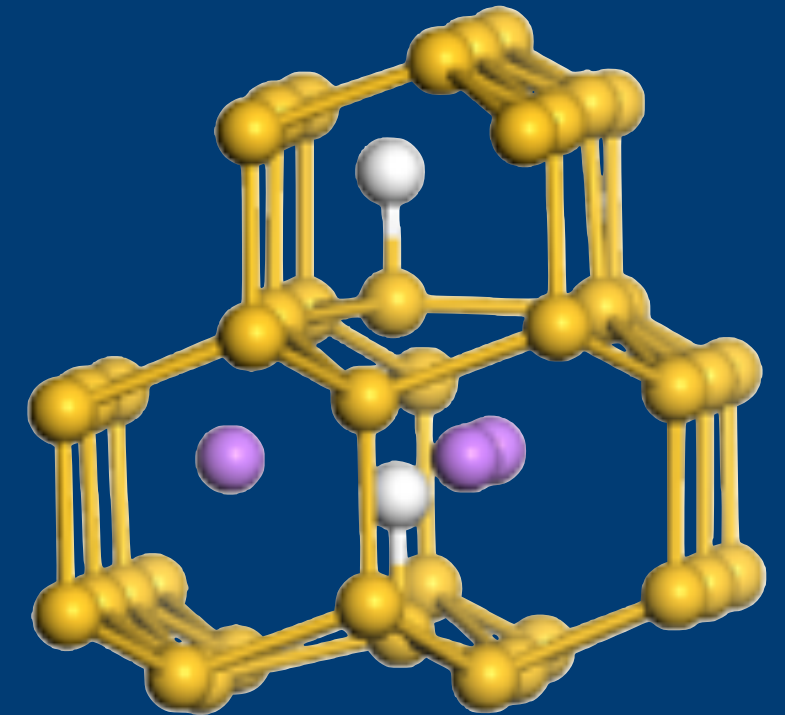
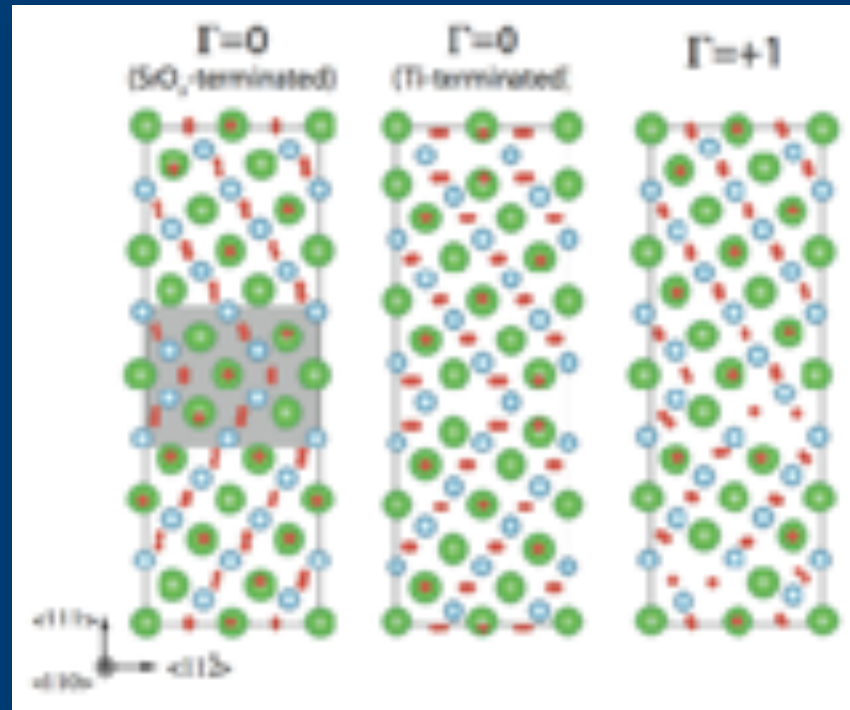
%ENDBLOCK EXTERNAL_PRESSURE



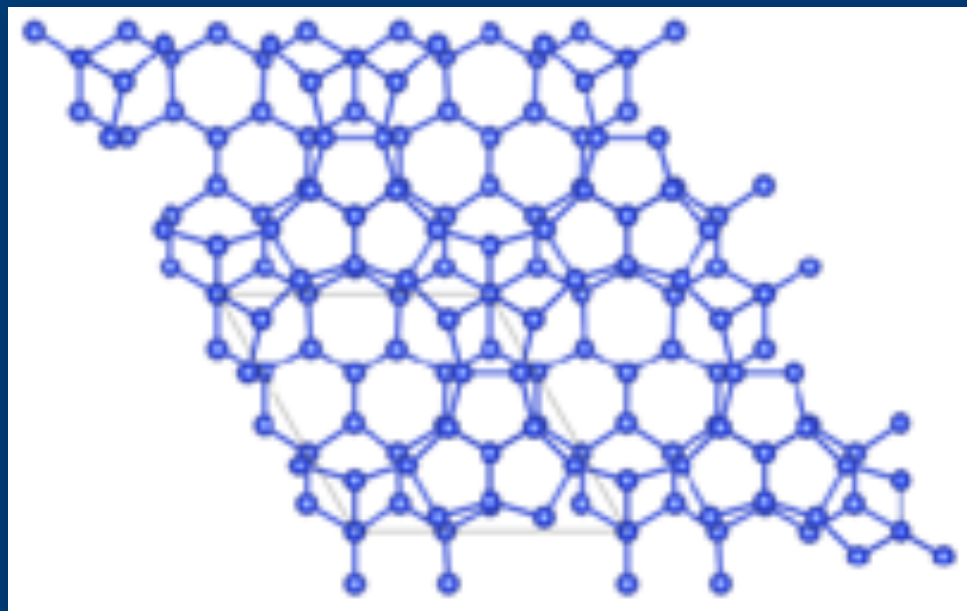
Beyond Crystals



Schusteritsch & Pickard, "Predicting interface structures: From SrTiO3 to graphene", Physical Review B (2014)

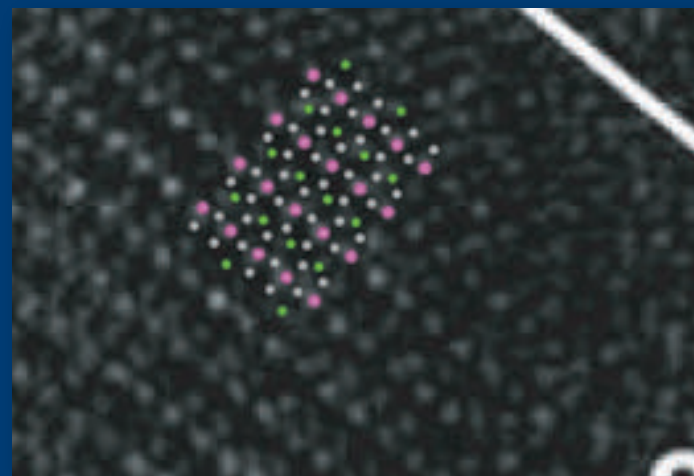


{2H,3Li} complex in silicon
Morris, Grey, Needs & Pickard, 2014

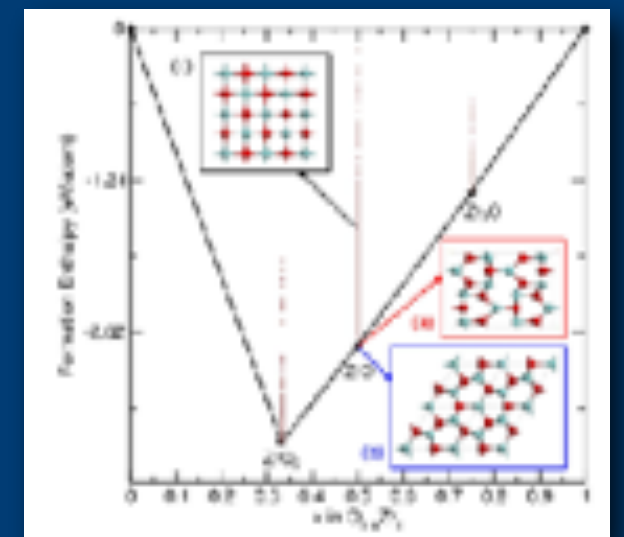


Silicon(111) 3x3 reconstruction

Interfacial Materials

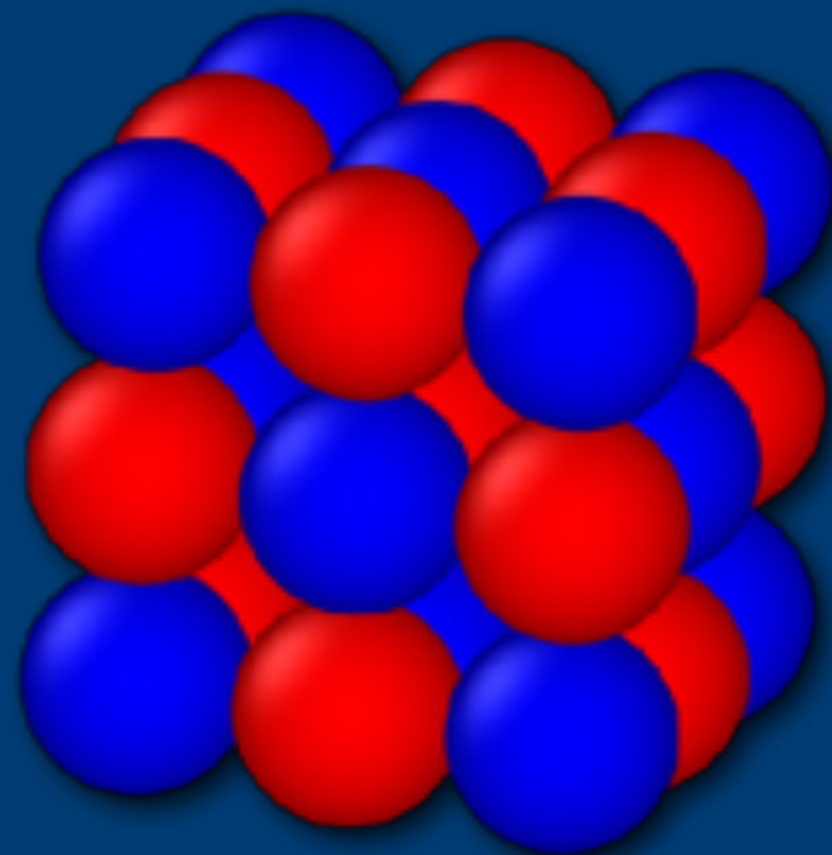
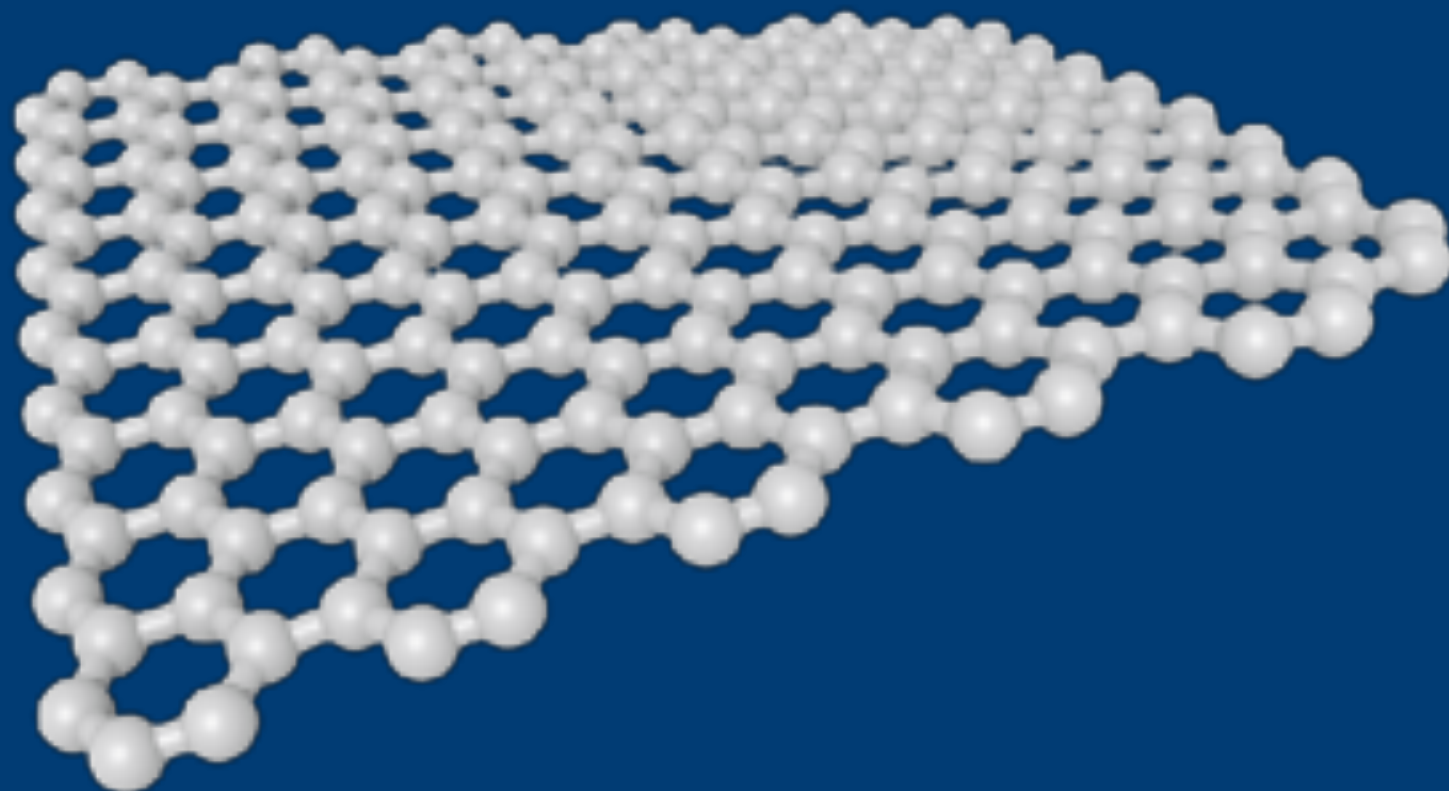


Ni₃InAs
Schusteritsch et al, 2015



Zirconium suboxide
Nicholls et al, 2015

Challenges



New directions

Allow the particles making up a structure to explore a higher dimensional physical space.

$$d = d_0 + d_+$$

Hyperspace

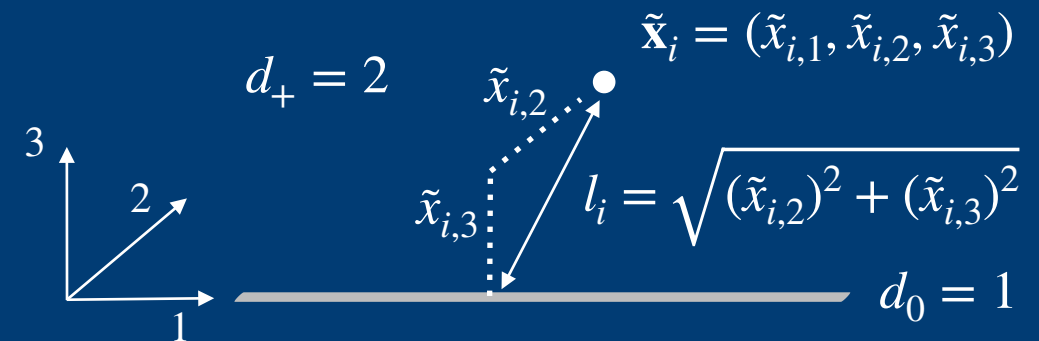
Normal space

Extra dimensions

$$\bar{E}(\{\tilde{\mathbf{x}}_i\}) = \tilde{E}(\{\tilde{\mathbf{x}}_i\}) + \frac{1}{2}\mu \sum_i l_i^2$$

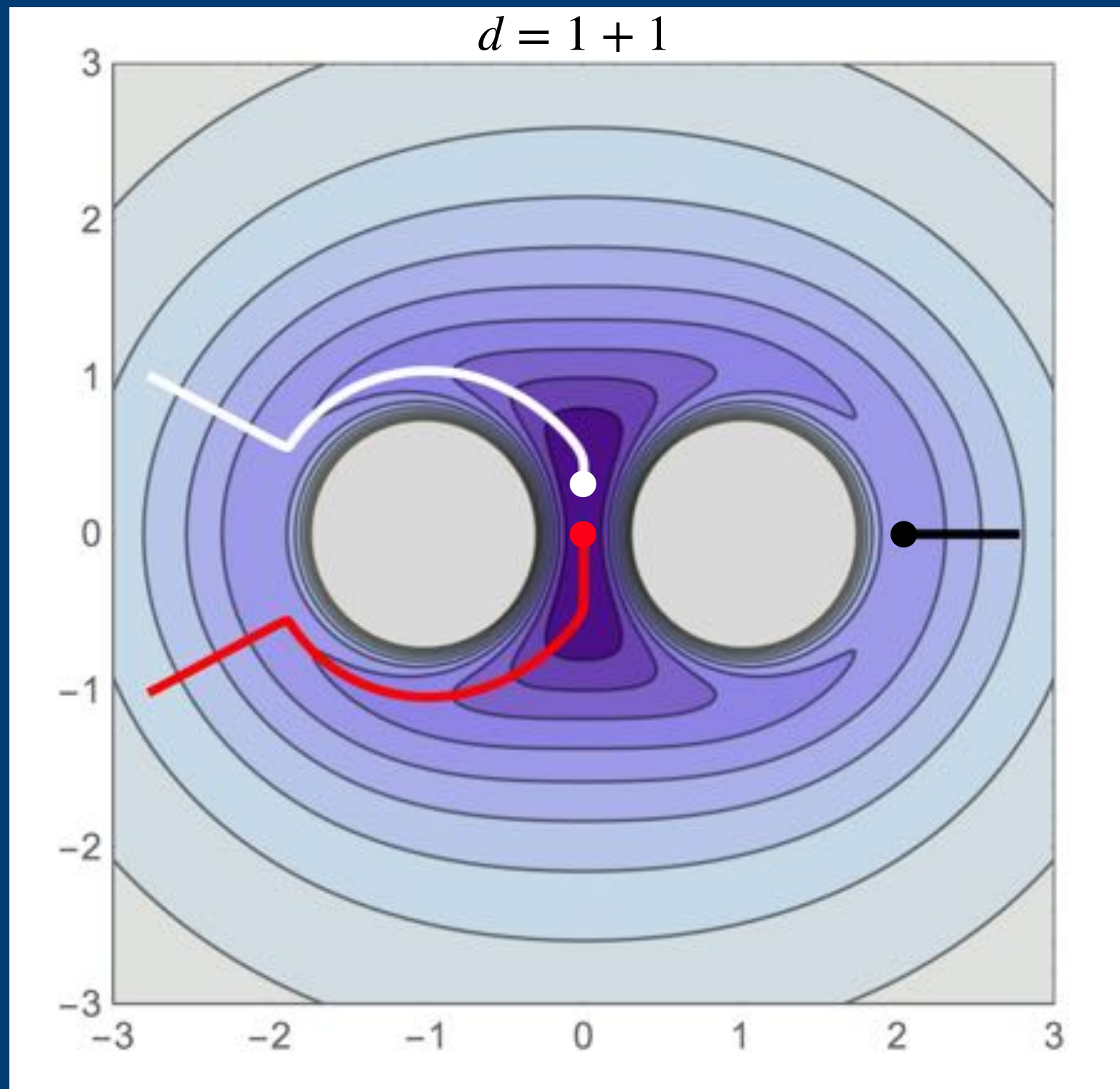
Energy
extended to
hyperspace

Increasing
penalty

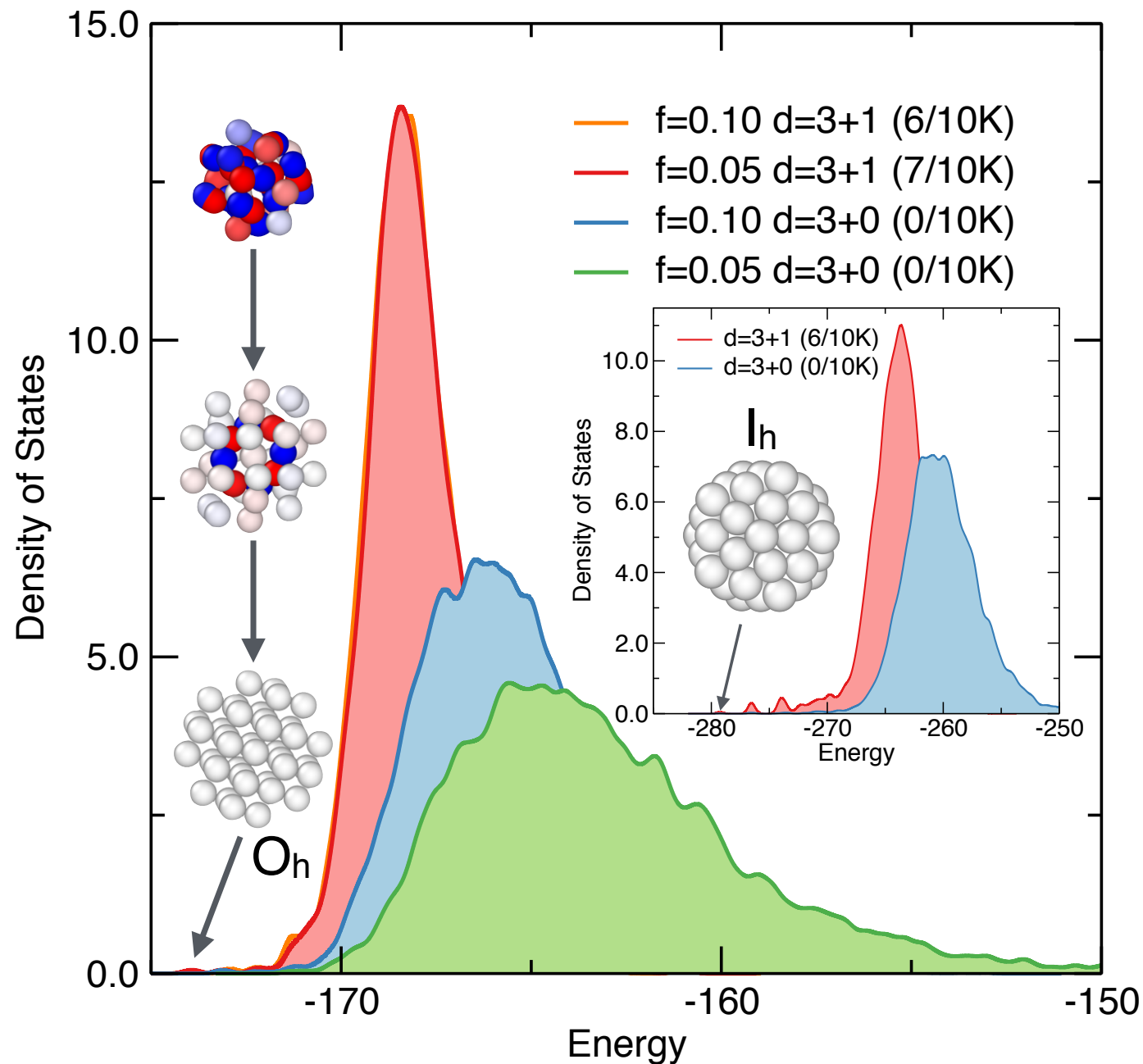


$$\mu = \mu_0 \beta^{n-1} \quad \mu_0 = 10, \beta = 1.001$$

Toy example



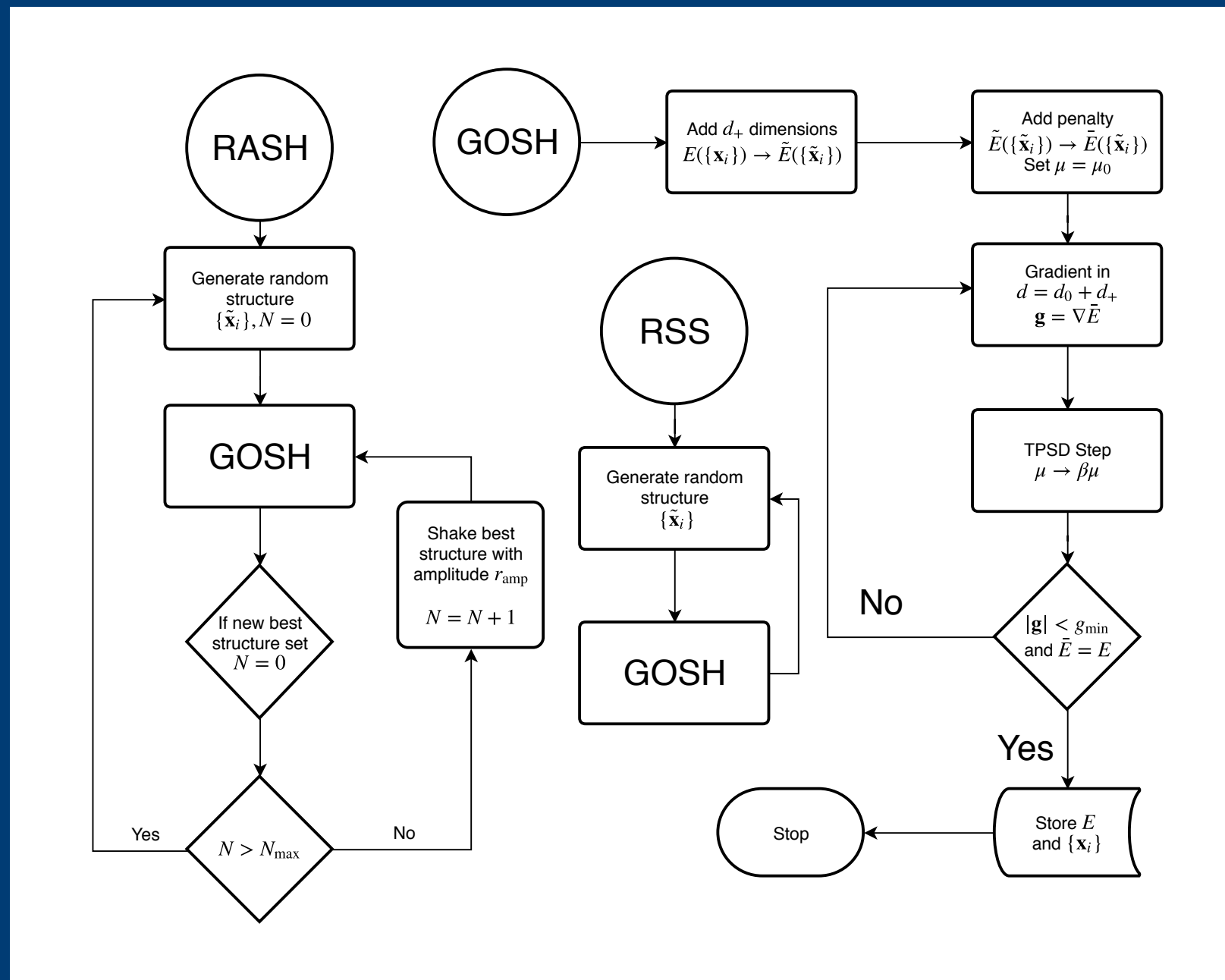
Lennard-Jones Clusters



d	f	Num. of Lennard-Jones atoms (PG of GS)					
		37 (C_1)	38 (O_h)	39 (C_{5v})	47 (C_1)	55 (I_h)	69 (C_{5v})
3+0	0.05	4.5	6.0	4.4	4.6	5.6	7.0 [†]
	0.10	3.8	5.0	3.8	4.1	4.8	6.5 [†]
	0.20	3.2	3.9	3.1	3.5	3.6	5.9 [†]
3+1	0.05	3.3	3.4	3.1	3.6	3.3	5.9 [†]
	0.10	3.2	3.3	3.2	3.6	3.2	5.9 [†]
	0.20	3.2	3.2	3.2	3.6	3.2	5.9 [†]
3+2	0.10	3.5	3.8	3.2	3.7	3.3	5.8 [†]

1. When $d_+ > 0$ the dependence on system and packing (f) is much reduced
2. Performance for LJ38 comparable to MH and EA ($-\log_{10}(p_e) = 3.1$)^[1]
3. Adjusting parameters for LJ38 leads to $-\log_{10}(p_e) = 2.8$ (at cost of more GO steps)
4. Combining with Relax and Shake (RASH), for LJ55 $-\log_{10}(p_e) = 1.9$
5. **Reality check:** $-\log_{10}(p_e) = 5.8$ for LJ75 (RASH) while it is 4.4 for MH

Acronyms

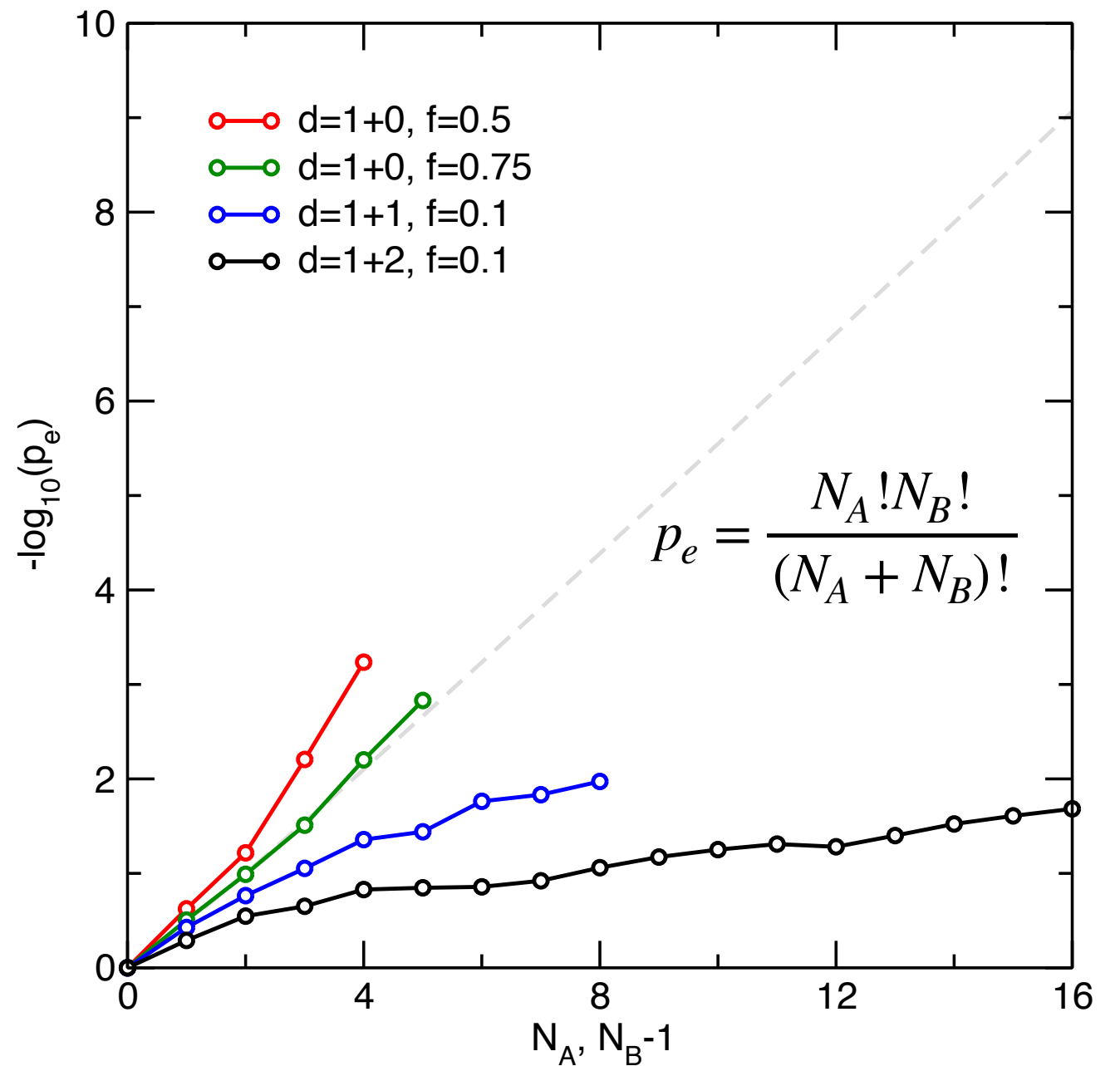
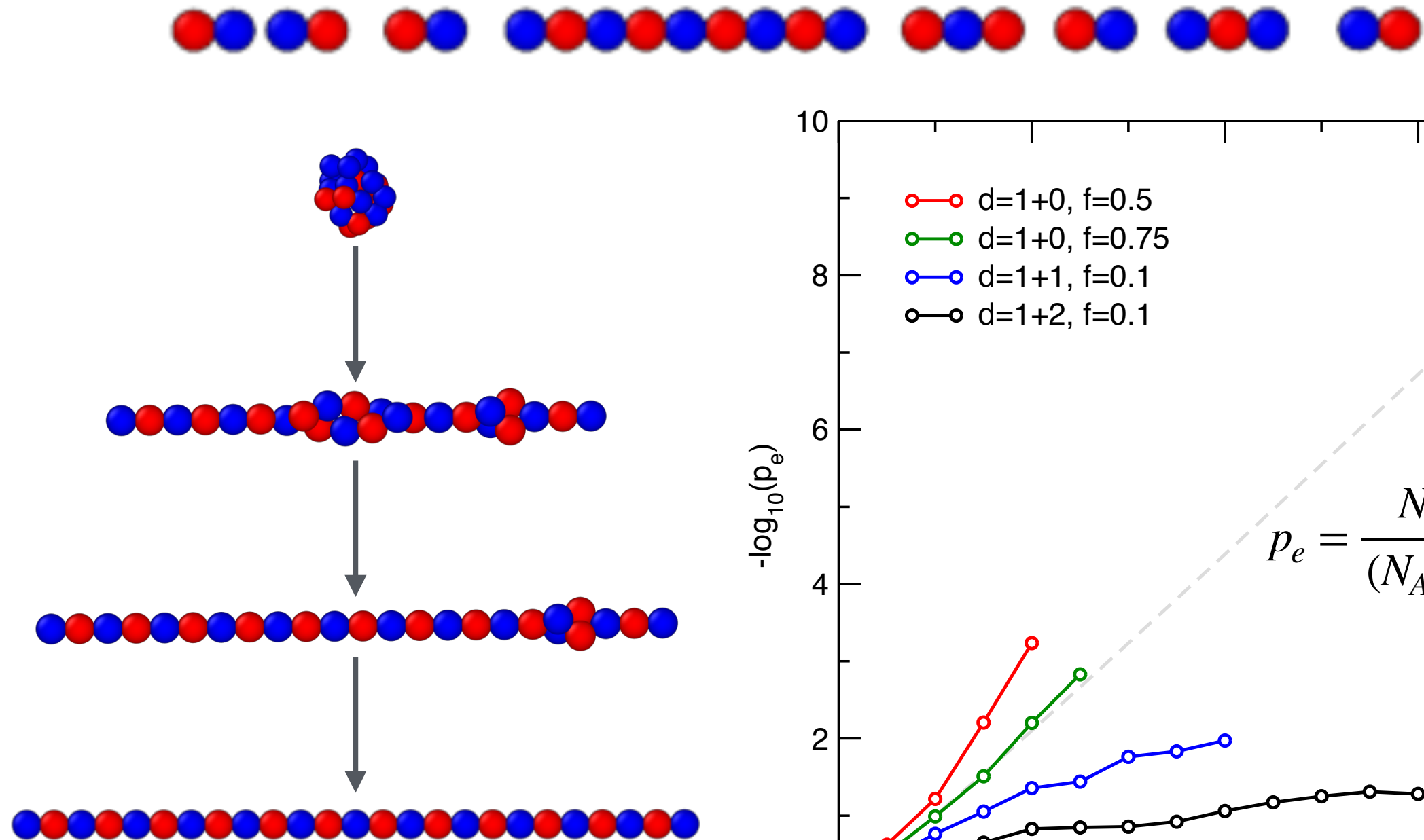


Random structure search - RSS

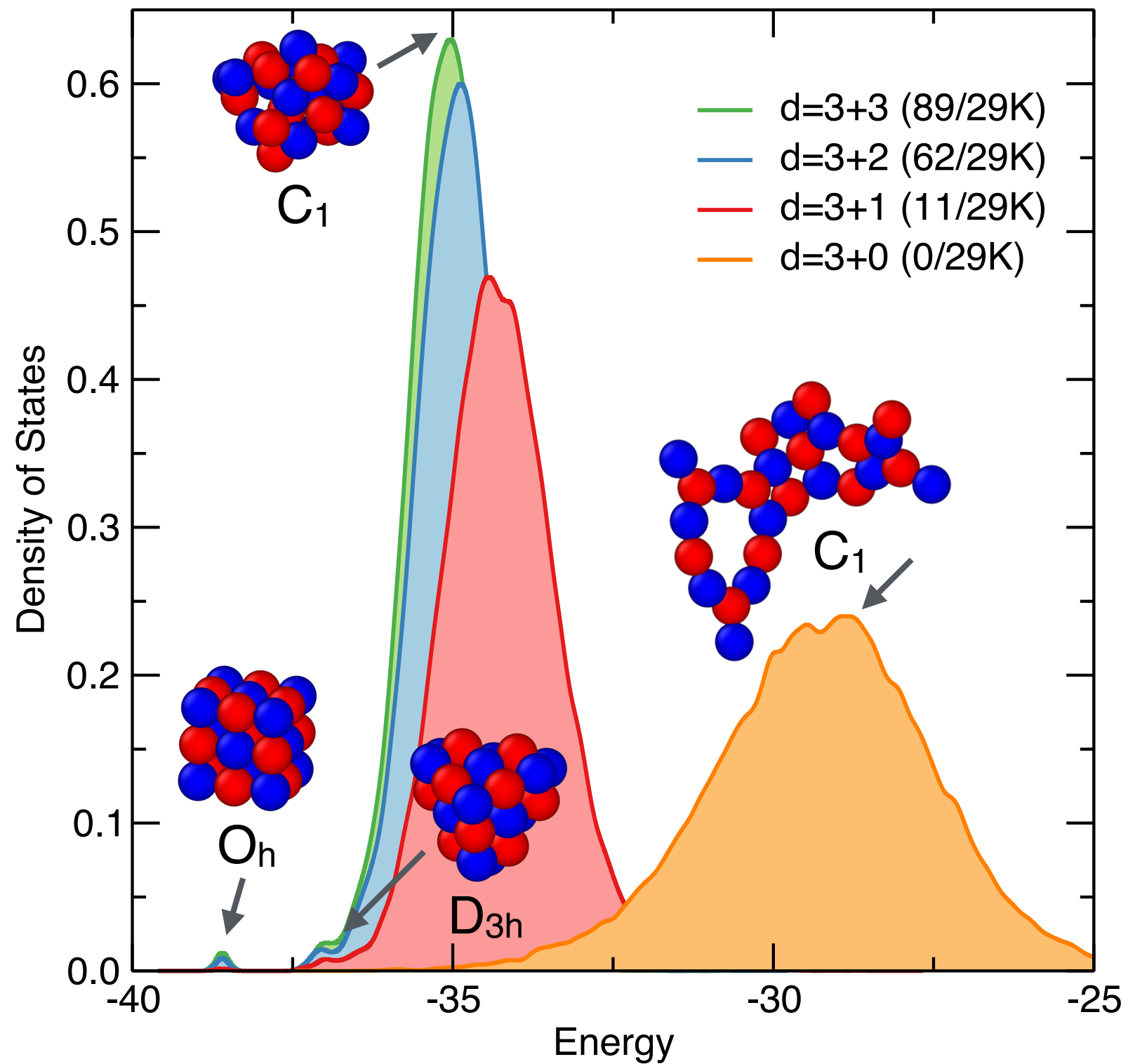
Geometry optimisation of structures from hyperspace - GOSH

Relax and shake - RASH

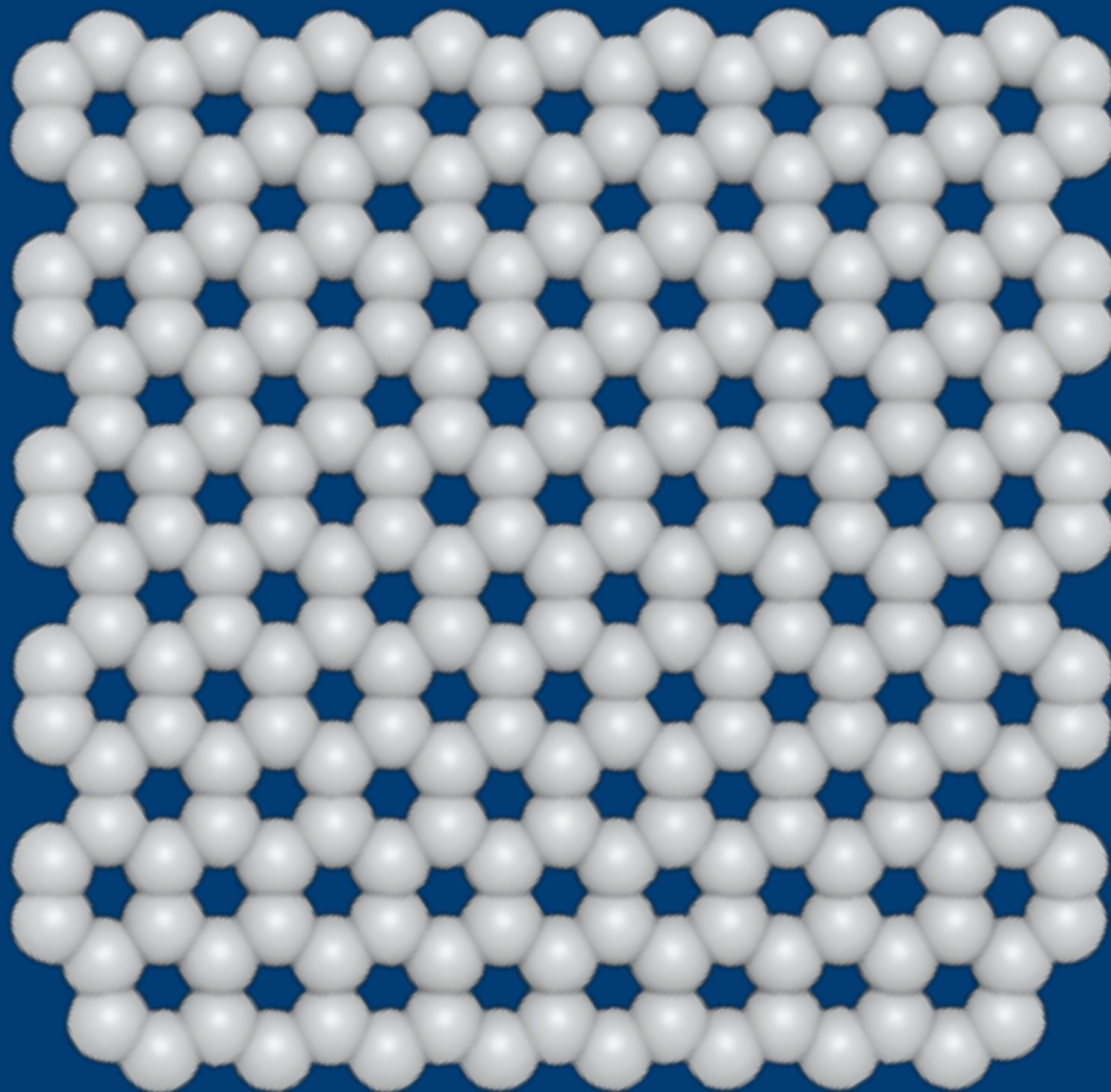
Binary chain



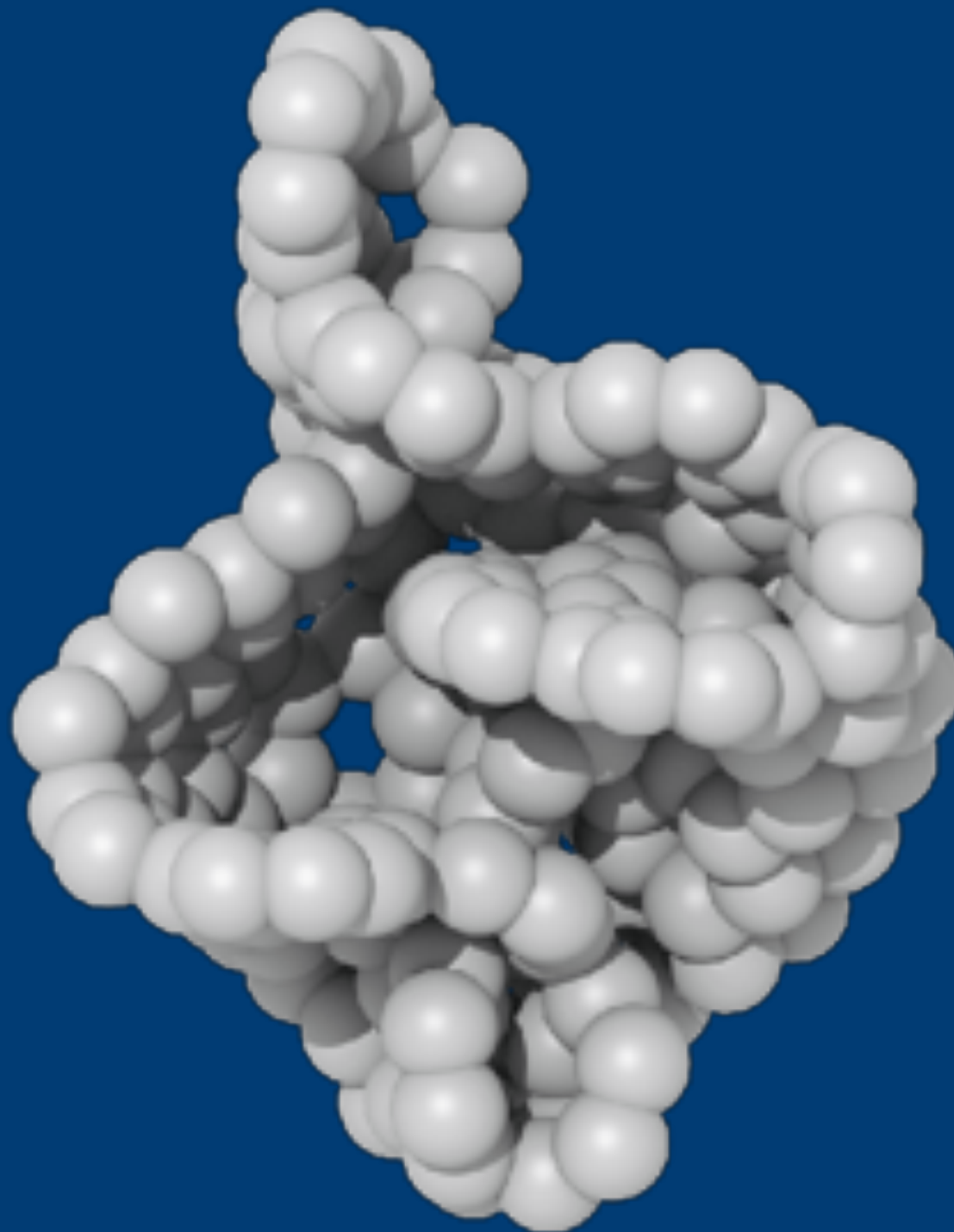
Binary cubes



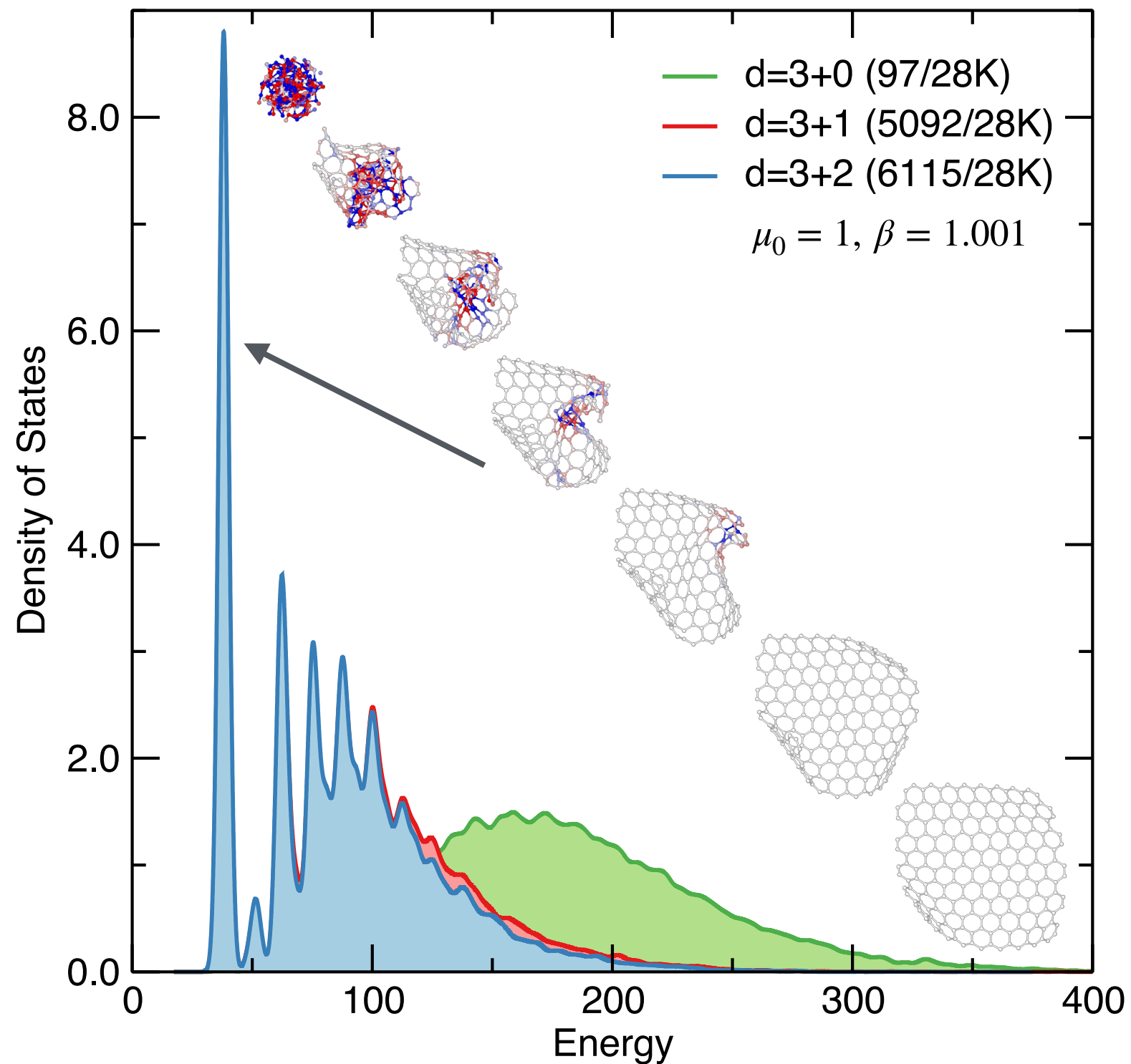
Covalent bonds

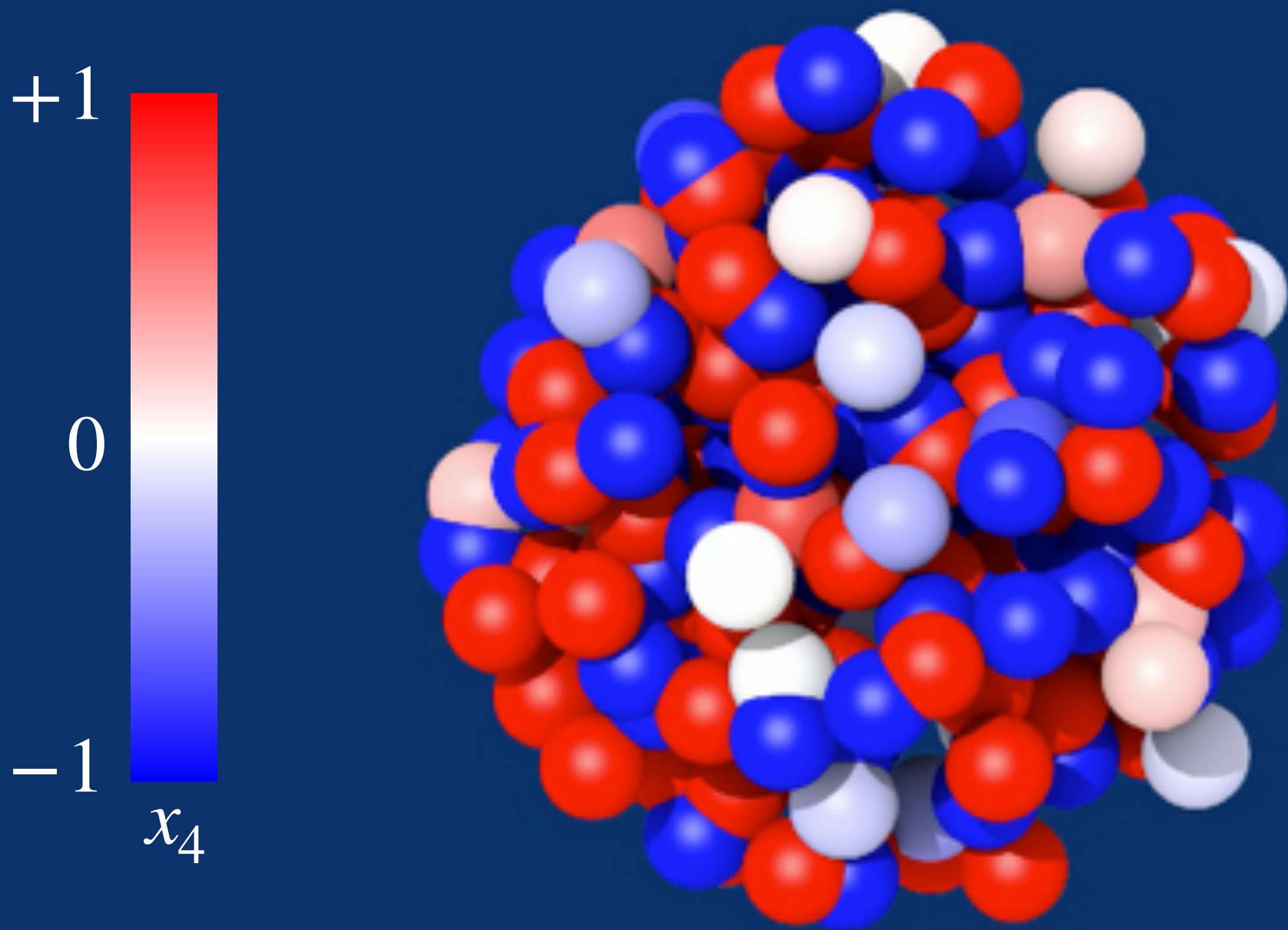


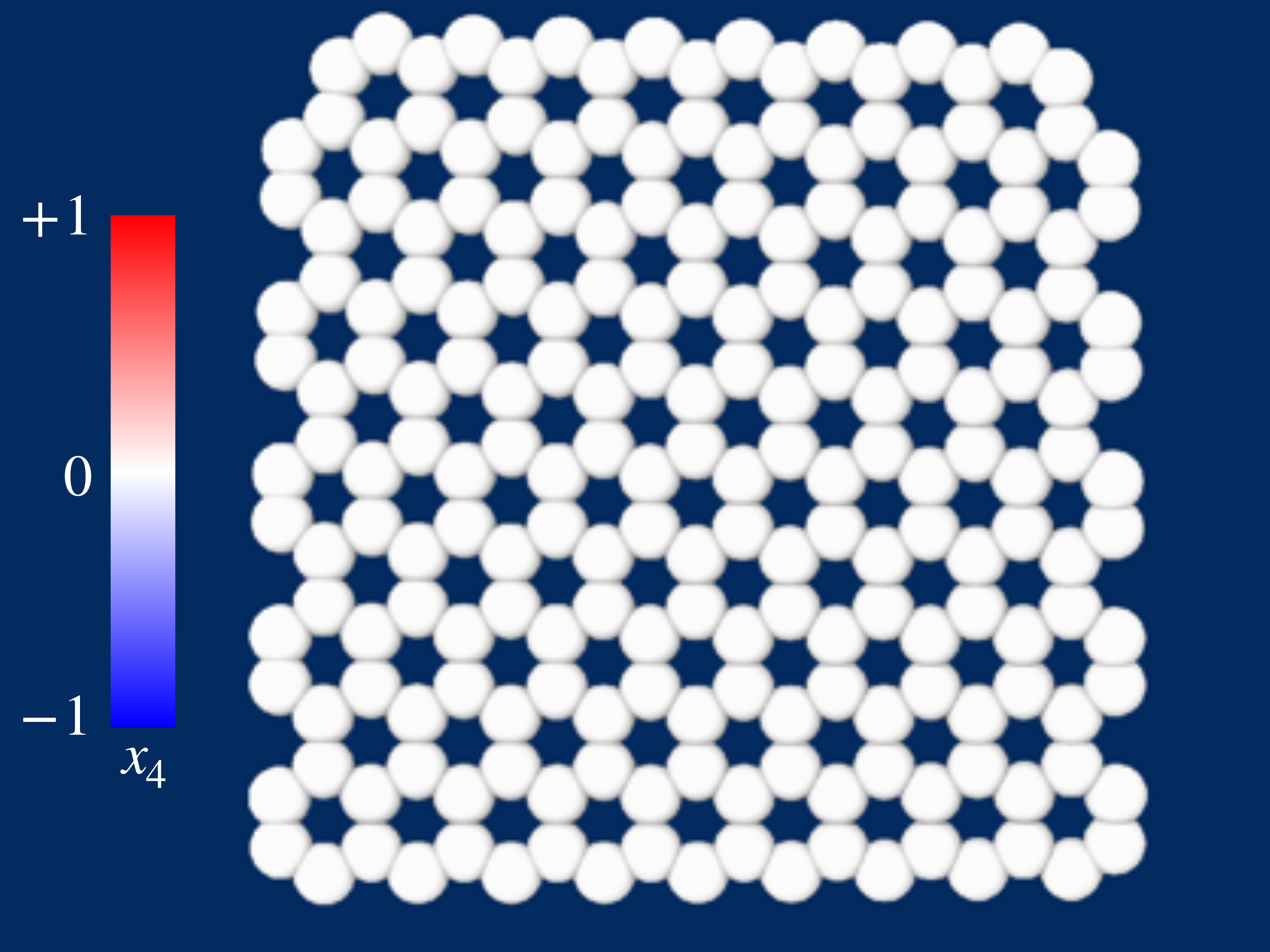
Covalent bonds



Covalent bonds







Conclusion

Random search is *better* than you would think

Stochastic search and first principles approaches can *discover*

Hyperspatial optimisation provides promising new directions



AIRSS package available from:

<http://www.mtg.msm.cam.ac.uk/Codes/AIRSS>
under the GPL 2.0 licence

Please direct queries to airss@msm.cam.ac.uk