Probabilistic Programming and Inference in Particle Physics

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International Centre for Theoretical Physics Trieste, Italy, 9 April 2019



About me

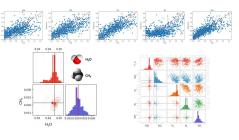
http://www.robots.ox.ac.uk/~gunes/

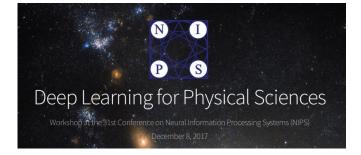
I work in probabilistic programming and machine learning for science

- High-energy physics
- Space sciences, NASA Frontier Development Lab, ESA Gaia collaboration
- Workshop in Deep Learning for Physical Sciences at NeurIPS conference Other interests: automatic differentiation, hyperparameter optimization, evolutionary

algorithms, computational physics







NASA FDL frontierdevelopmentlab.org Exoplanetary atmospheres <u>https://arxiv.org/abs/1811.03390</u>

https://dl4physicalsciences.github.io/

About me

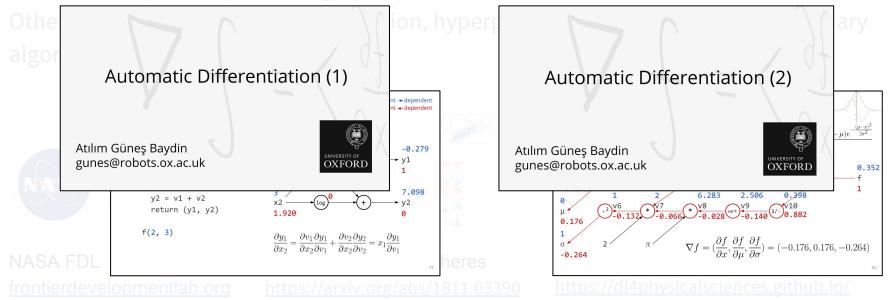
Automatic differentiation / differentiable programming

http://www.robots.ox.ac.uk/~gunes

Baydin, A.G., Pearlmutter, B.A., Radul, A.A. and Siskind, J.M., 2018. Automatic differentiation in machine learning: a survey. *Journal of Machine Learning Research*, 18, pp.1-43. <u>https://arxiv.org/abs/1502.05767</u> High-energy physics

Space sciences, NASA Frontier Development Lab, ESA Gaia collaboration

https://docs.google.com/presentation/d/1aBX-wgGmO8Gfl2bdZQBWd AlQjP_nj8_TLLceAbC-pKA/edit?usp=sharing https://docs.google.com/presentation/d/1NTodzA0vp6zLljl0v4vXpbz9z Pe8mWaNDtD5QdK3v4/edit?usp=sharing



Probabilistic models define a set of random variables and their relationships

- Observed variables
- Unobserved (hidden, latent) variables

Probabilistic models define a set of random variables and their relationships

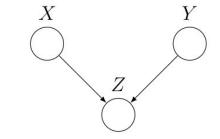
- Observed variables
- Unobserved (hidden, latent) variables HEP: Monte Carlo truth

Probabilistic models define a set of random variables and their relationships

- Observed variables
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Probabilistic graphical models use graphs to express conditional dependence

- Bayesian networks
- Markov random fields (undirected)



p(x, y, z) = p(x)p(y)p(z|x, y)

Probabilistic models define a set of random variables and their relationships

- Observed variables
- Unobserved (hidden, latent) variables HEP: Monte Carlo truth

Probabilistic programming extends this to *"ordinary programming with two added constructs"* (Gordon et al. 2014):

- Sampling from distributions
- **Conditioning** random variables by specifying observed values

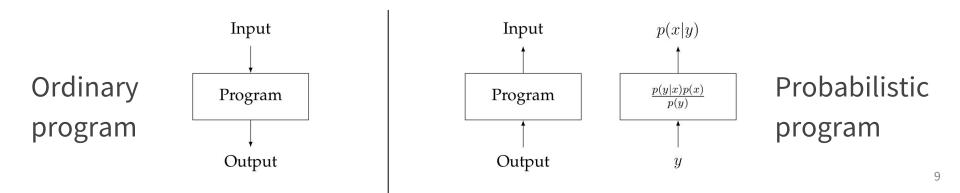
```
1: bool c1, c2;
2: c1 = Bernoulli(0.5);
3: c2 = Bernoulli(0.5);
4: return(c1, c2);
```

```
1: bool c1, c2;
2: c1 = Bernoulli(0.5);
3: c2 = Bernoulli(0.5);
4: observe(c1 || c2);
5: return(c1, c2);
```

Inference

With a probabilistic program, we define a joint distribution of **unobserved** and **observed** variables p(x,y)

Inference engines give us distributions over **unobserved** variables, given **observed** variables (data) $p(x|y) = \frac{p(y|x)p(x)}{p(y)}$



Inference engines

Model writing is decoupled from running inference

After writing the program, we execute it using an **inference engine**

- Exact (limited applicability)
 - Belief propagation
 - Junction tree algorithm
- Approximate (very common)
 - Deterministic
 - Variational methods
 - Stochastic (sampling-based)
 - Monte Carlo methods
 - Markov chain Monte Carlo (MCMC)
 - Sequential Monte Carlo (SMC)

Probabilistic programming languages (PPLs)

- Anglican (Clojure)
- Church (Scheme)
- Edward, TensorFlow Probability (Python, TensorFlow)
- Pyro (Python, PyTorch)
- Figaro (Scala)
- LibBi (C++ template library)
- PyMC3 (Python)
- Stan (C++)
- WebPPL (JavaScript)

For more, see http://probabilistic-programming.org

Large-scale simulators as probabilistic programs

Interpreting simulators as probprog

A stochastic simulator implicitly defines a probability distribution by **sampling** (pseudo-)random numbers → already satisfying one requirement for probprog



Idea:

- Interpret all RNG calls as **sampling** from a prior distribution
- Introduce **conditioning** functionality to the simulator
- Execute under the control of general-purpose inference engines
- Get posterior distributions over all simulator latents conditioned on observations

Interpreting simulators as probprog

A stochastic simulator implicitly defines a probability distribution by **sampling** (pseudo-)random numbers → already satisfying one requirement for probprog



Advantages:

- Vast body of existing scientific simulators (accurate generative models) with years of development: MadGraph, Sherpa, Geant4
- Enable model-based (Bayesian) machine learning in these
- Explainable predictions directly reaching into the simulator (simulator is not used as a black box)
- Results are still from the simulator and meaningful

Coupling probprog and simulators

Several things are needed:

• A PPL with with simulator control incorporated into design

• A language-agnostic interface for connecting PPLs to simulators

• Front ends in languages commonly used for coding simulators

Coupling probprog and simulators

Several things are needed:

- A PPL with with simulator control incorporated into design pyprob
- A language-agnostic interface for connecting PPLs to simulators **PPX - the Probabilistic Programming eXecution protocol**
- Front ends in languages commonly used for coding simulators
 pyprob_cpp

pyprob

https://github.com/probprog/pyprob

A PyTorch-based PPL



Inference engines:

- Markov chain Monte Carlo
 - Lightweight Metropolis Hastings (LMH)
 - Random-walk Metropolis Hastings (RMH)
- Importance Sampling
 - Regular (proposals from prior)
 - Inference compilation (IC)

pyprob

https://github.com/probprog/pyprob

A PyTorch-based PPL



Inference engines:

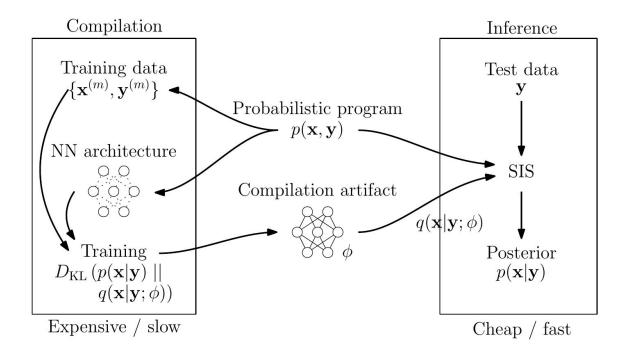
- Markov chain Monte Carlo
 - Lightweight Metropolis Hastings (LMH)
 - Random-walk Metropolis Hastings (RMH)
- Importance Sampling
 - Regular (proposals from prior)

• Inference compilation (IC)

Le, Baydin and Wood. Inference Compilation and Universal Probabilistic Programming. AISTATS 2017 *arXiv:1610.09900*.

Inference compilation

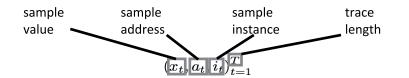
Transform a generative model implemented as a probabilistic program into a trained neural network artifact for performing inference



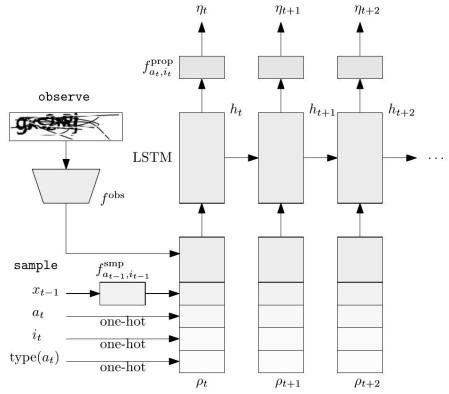
Inference compilation

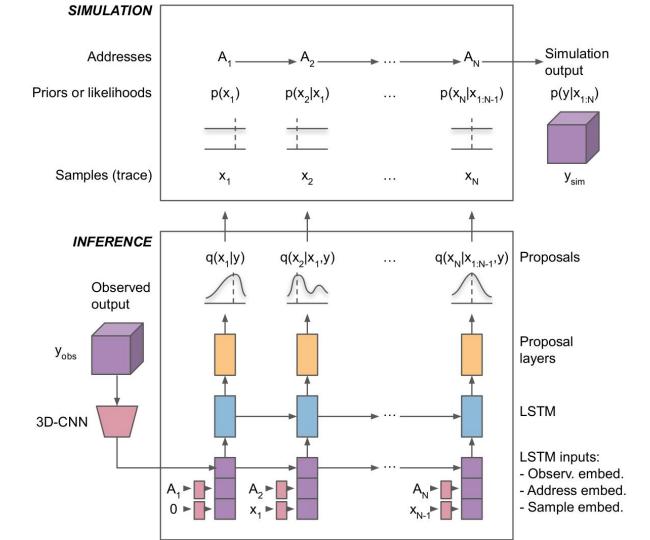
- A stacked LSTM core
- Observation embeddings, sample embeddings, and proposal layers specified by the probabilistic program

$$\begin{aligned} \mathcal{L}(\phi) &= \mathbb{E}_{p(\mathbf{y})} \left[\mathrm{KL}(p(\mathbf{x}|\mathbf{y})||q(\mathbf{x}|\mathbf{y};\phi)) \right] \\ &= \int_{\mathbf{y}} p(\mathbf{y}) \int_{\mathbf{x}} p(\mathbf{x}|\mathbf{y}) \log \frac{p(\mathbf{x}|\mathbf{y})}{q(\mathbf{x}|\mathbf{y};\phi)} \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{y} \\ &= -\mathbb{E}_{p(\mathbf{x},\mathbf{y})} \left[\log q(\mathbf{x}|\mathbf{y};\phi) \right] + \mathrm{const.} \end{aligned}$$



Proposal distribution parameters







https://github.com/probprog/ppx

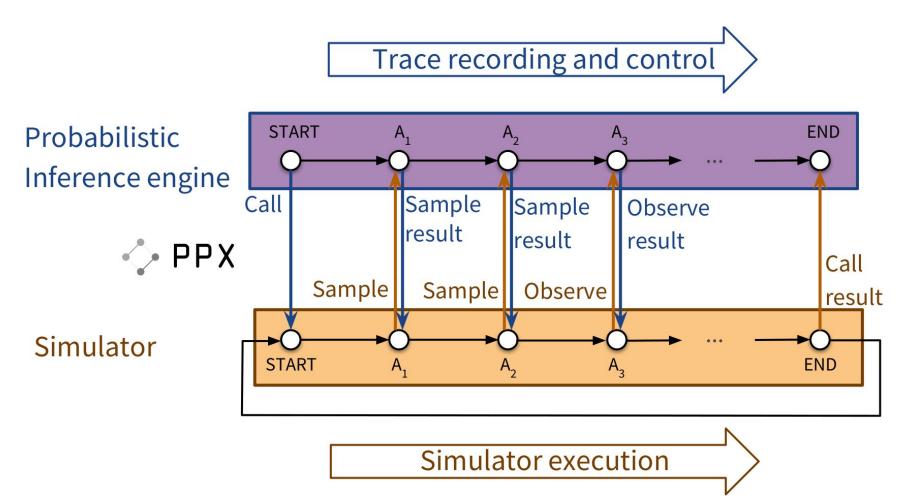


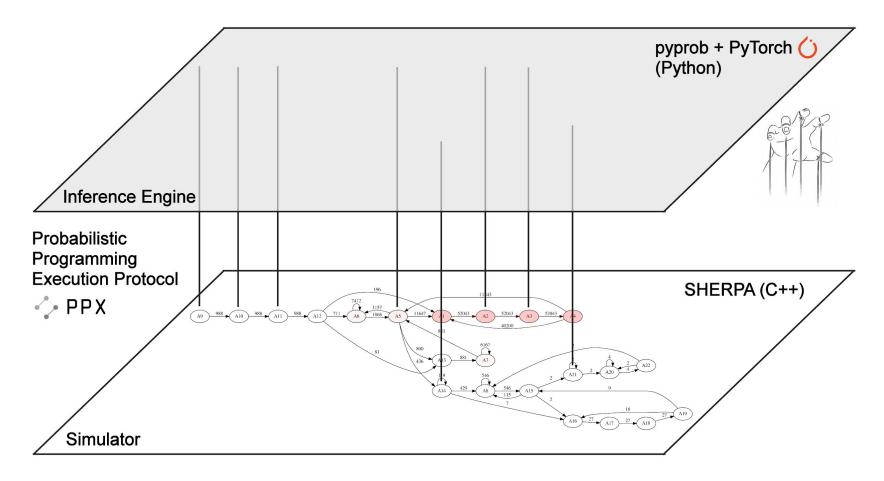
Probabilistic Programming eXecution protocol

- Cross-platform, via flatbuffers: <u>http://google.github.io/flatbuffers/</u>
- Supported languages: C++, C#, Go, Java, JavaScript, PHP, Python, TypeScript, Rust, Lua
- Similar to Open Neural Network Exchange (ONNX) for deep learning

Enables inference engines and simulators to be

- implemented in different programming languages
- executed in separate processes, separate machines across networks





pyprob_cpp

https://github.com/probprog/pyprob_cpp

A lightweight C++ front end for PPX

#include <pyprob_cpp.h>

```
// Gaussian with unkown mean
// http://www.robots.ox.ac.uk/~fwood/assets/pdf/Wood-AISTATS-2014.pdf
xt::xarray<double> forward(xt::xarray<double> observation)
  auto prior mean = 1;
  auto prior_stddev = std::sqrt(5);
  auto likelihood_stddev = std::sqrt(2);
  auto prior = pyprob_cpp::distributions::Normal(prior_mean, prior_stddev);
  auto mu = pyprob_cpp::sample(prior);
  auto likelihood = pyprob cpp::distributions::Normal(mu, likelihood stddev);
  for (auto & o : observation)
    pyprob_cpp::observe(likelihood, o);
  }
  return mu;
```

Probprog and high-energy physics "etalumis"

etalumis simulate









Atılım Güneş Baydin Bradley Gram-Hansen Lukas Heinrich

Kyle Cranmer

Frank Wood Andreas Munk Saeid Naderiparizi



Wahid Bhimji Jialin Liu Prabhat



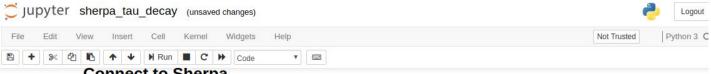
(intel)

Gilles Louppe

Lei Shao Larry Meadows Victor Lee

```
#include <pyprob cpp.h>
 1
 2
                                                                                                       pyprob_cpp and
Sherpa
    xt::xarray<double> forward()
 3
 4
 5
      int channel index;
       std::vector<double> mother momentum;
 6
       std::vector<std::vector<double>> final state particles;
 7
       std::tie(channel_index, mother_momentum, final_state_particles) = sherpa.Generate();
 8
      pyprob cpp::tag(xt::xarray<double>({(double)(channel_index)}), "channel_index");
 9
      pyprob cpp::tag(xt::xarray<double>(mother momentum[0]), "mother momentum x");
10
      pyprob cpp::tag(xt::xarray<double>(mother momentum[1]), "mother momentum y");
11
      pyprob_cpp::tag(xt::xarray<double>(mother_momentum[2]), "mother_momentum_z");
12
13
      pyprob_cpp::tag(xt::adapt(flatten(final_state_particles), std::vector<std::size_t> { 30, 8 }), "final_state_particles");
14
      auto calo histo = calorimeter.calo simulation(final state particles);
15
16
17
      xt::xarray<double> mean_n_deposits = calo_histo / caloutils::minEnergyDeposit;
      //flatten
18
      mean_n_deposits.reshape({uint(caloutils::NBINX*caloutils::NBINY*caloutils::NBINZ)});
19
      auto likelihood = pyprob cpp::distributions::Poisson(mean n deposits + 1E-19L);
20
21
      pyprob cpp::observe(likelihood, "calorimeter n deposits");
22
       return xt::xarray<double>({(double)(channel index)});
23
24
25
26
    int main(int argc, char *argv[])
27
28
    {
       auto serverAddress = (argc > 1) ? argv[1] : "ipc://@sherpa_tau_decay";
29
      pyprob cpp::Model model = pyprob cpp::Model(forward, "SHERPA tau lepton decay");
30
31
      model.startServer(serverAddress);
32
      return 0;
33 }
```

28



pyprob and Sherpa

Connect to Sherpa

If you started this notebook using the following, SHERPA is already running:

docker run --rm -it --net=host etalumis/sherpa tmuxp load ./sherpa tau decay.yaml

If you are doing something else, and SHERPA is not already running, start it as follows:

docker run -it --rm --net=host etalumis/sherpa ./sherpa tau decay

Note: by default sherpa tau decay uses the Unix domain sockets (IPC) address ipc://@sherpa tau decay. You can run it with other addresses, e.g., TCP, using: ./sherpa tau decay tcp://*:5555

In [3]: model = ModelRemote('ipc://@sherpa tau decay') M

```
ppx (Python): zmg.REQ socket connecting to server ipc://@sherpa tau decay
                                : pyprob 0.10.0
ppx (Python): This system
ppx (Python): Connected to system: pyprob cpp 0.1.3 (master:8110b71)
ppx (Python): Model name
                               : SHERPA tau lepton decay
```

Inspect the prior

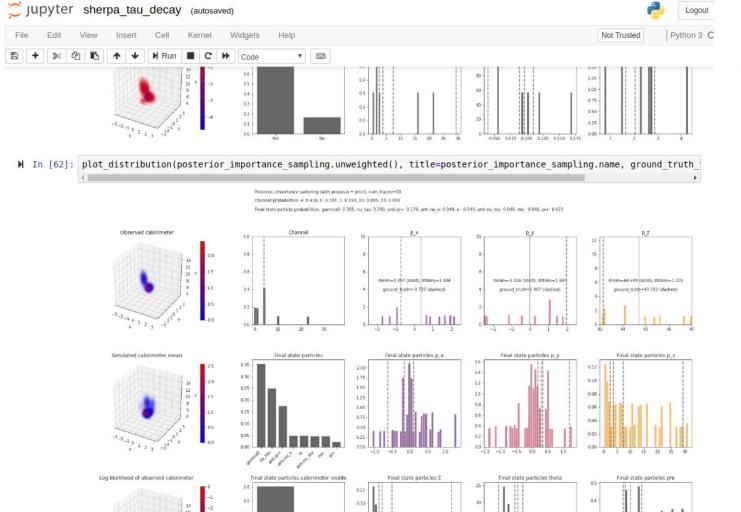
We construct an empirical distribution of the prior, using a given number of traces. Beware, large number of traces can take a long time. traces=500 gives a reasonable-looking plot.

[4]: prior dist = model.prior distribution(num traces=5) H In

Time spent	Time remain.	Progress	Trace	Traces/sec
0d:00:00:01	0d:00:00:00	#######################################	5/5	2.60

Separate (marginalize) the distribution into distributions over momenta and channel, by mapping a function,

```
M
  In [5]: prior dist px = prior dist.map(lambda x: float(x[0]))
           prior dist py = prior dist.map(lambda x: float(x[1]))
           prior dist pz = prior dist.map(lambda x: float(x[2]))
            prior dist channel = prior dist map(lambda x: int(x[3]))
```



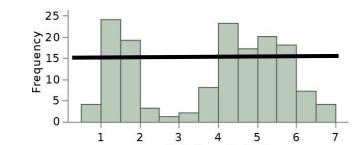
0.2

pyprob and Sherpa

Main challenges

Working with large-scale HEP simulators requires several innovations

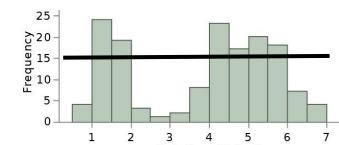
- Wide range of prior probabilities, some events highly unlikely and not learned by IC neural network
- Solution: "prior inflation"
 - Training: modify prior distributions to be uninformative
 - Inference: use the unmodified (real) prior for weighting proposals



Main challenges

Working with large-scale HEP simulators requires several innovations

- Wide range of prior probabilities, some events highly unlikely and not learned by IC neural network
- Solution: "prior inflation"
 - Training: modify prior distributions to be uninformative
 HEP: sample according to phase space
 - Inference: use the unmodified (real) prior for weighting proposals
 *HEP: differential cross-section = phase space * matrix element*

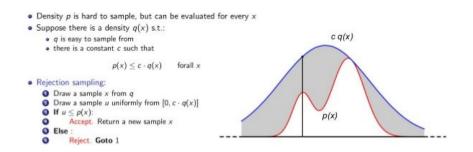


Main challenges

Working with large-scale HEP simulators requires several innovations

- Potentially very long execution traces due to rejection sampling loops
- Solution: "replace" (or "rejection-sampling") mode
 - Training: only consider the last (accepted) values within loops
 - Inference: use the same proposal distribution for these samples

Rejection sampling



Experiments

Tau lepton decay

Tau decay in Sherpa, 38 decay channels, coupled with an approximate calorimeter simulation in C++

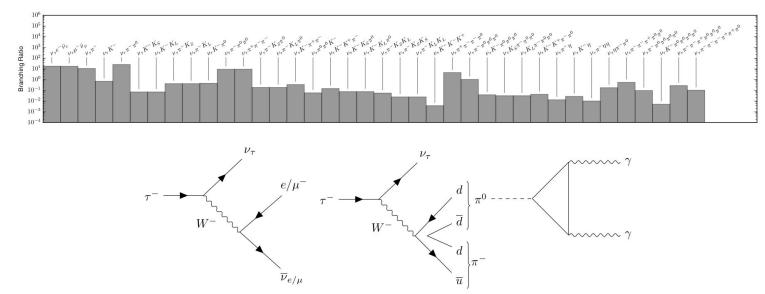


Figure 2: *Top:* branching ratios of the τ lepton, effectively the prior distribution of the decay channels in SHERPA. Note that the scale is logarithmic. *Bottom:* Feynman diagrams for τ decays illustrating that these can produce multiple detected particles.

Tau lepton decay

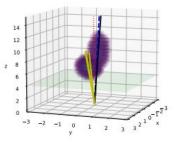
Tau decay in Sherpa, 38 decay channels, coupled with an approximate calorimeter simulation in C++

Observation: 3D calorimeter depositions (Poisson)

- Particle showers modeled as Gaussian blobs, deposited energy parameterizes a multivariate Poisson
- Shower shape variables and sampling fraction based on final state particle

Monte Carlo truth (latent variables) of interest:

- Decay channel (Categorical)
- px, py, pz momenta of tau particle (Continuous uniform)
- Final state momenta and IDs



Probabilistic addresses in Sherpa

Approximately 25,000 addresses encountered

Address ID Full address

- A1 [forward(xt:: xarray_container<xt:: uvector<double, std:: allocator<double> >, (xt:: layout_type)1, xt:: svector<unsigned long, 4ul, std:: allocator<unsigned long>, true>, xt:: xtensor_expression_tag>)+0x5f; SherpaGenerator:: Generate()+0x36; SHERPA:: Sherpa:: GenerateOneEvent(bool)+0x2fa; SHERPA:: Event_Handler:: GenerateEvent(SHERPA:: eventtype:: code)+0x44d; SHERPA:: Event_Handler:: GenerateHadronDecayEvent(SHERPA:: eventtype:: code&)+0x45f; ATOOLS:: Random:: Get(bool, bool)+0x1d5; probprog_RNG:: Get(bool, bool)+0xf9]_Uniform_1
- A6 [forward(xt:: xarray_container<xt:: uvector<double, std:: allocator<double> >, (xt:: layout_type)1, xt:: svector<unsigned long, 4ul, std:: allocator<unsigned long>, true>, xt:: xtensor_expression_tag>)+0x5f; SherpaGenerator:: Generate()+0x36; SHERPA:: Sherpa:: GenerateOneEvent(bool)+0x2fa; SHERPA:: Event_Handler:: GenerateEvent(SHERPA:: eventtype:: code)+0x44d; SHERPA:: Event_Handler:: GenerateHadronDecayEvent(SHERPA:: eventtype:: code&)+0x982; SHERPA:: Event_Handler:: IterateEventPhases(SHERPA:: eventtype:: code&, double&)+0x1d2; SHERPA:: Hadron_Decays:: Treat(ATOOLS:: Blob_List*, double&)+0x975; SHERPA:: Decay_Handler_Base:: TreatInitialBlob(ATOOLS:: Blob*, METOOLS:: Amplitude2_Tensor*, std:: vector<ATOOLS:: Particle*, std:: allocator<ATOOLS:: Particle*> > const&)+0x1ab1; SHERPA:: Hadron_Decay_Handler:: CreateDecayBlob(ATOOLS:: Particle*)+0x4cd; PHASIC:: Decay_Table:: Select() const+0x9d7; ATOOLS:: Random:: GetCategorical(std:: vector<double, std:: allocator<double> > const&, bool, bool)+0x1a5; probprog_RNG:: GetCategorical(std:: vector<double, std:: allocator<double> > const&, bool, bool)+0x1a5; probprog_RNG:: GetCategorical(length_categories:38)_1

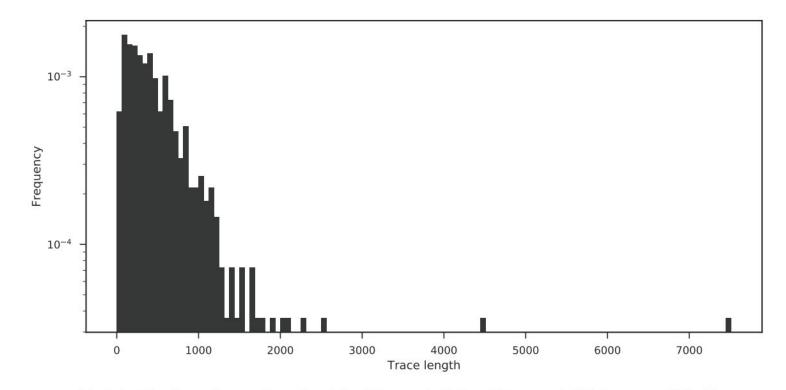
Approximately 450 trace types encountered

Trace type: unique sequencing of addresses (with different sampled values)

Freq.	Length	Addresses (showing controlled only)
0.106	72	A1, A2, A3, A5, A6, A32, A33, A31
0.105	41	A1, A2, A3, A5, A6, A499, A31
0.078	1,780	A1, A2, A3, A5, A6, A7, A8, A9, A10, A31
0.053	188	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A26, A31
0.053	100	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A99, A100, A101, A102, A31
0.039	56	A1, A2, A3, A5, A6, A499, A17, A18, A26, A31
0.039	592	A1, A2, A3, A5, A6, A499, A17, A18, A99, A100, A101, A102, A31
0.038	162	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A500, A99, A100, A101, A102, A31
0.030	240	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A20, A21, A41, A42, A26, A99, A100, A101, A102, A31
0.029	836	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A20, A21, A41, A42, A99, A100, A101, A102, A26, A31
0.027	643	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A507, A99, A100, A101, A102, A31
0.023	135	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A20, A21, A41, A42, A44, A45, A26, A99, A100, A101, A102, A31
0.023	485	A1, A2, A3, A5, A6, A7, A8, A9, A10, A17, A18, A20, A21, A41, A42, A44, A45, A99, A100, A101, A102, A26, A31

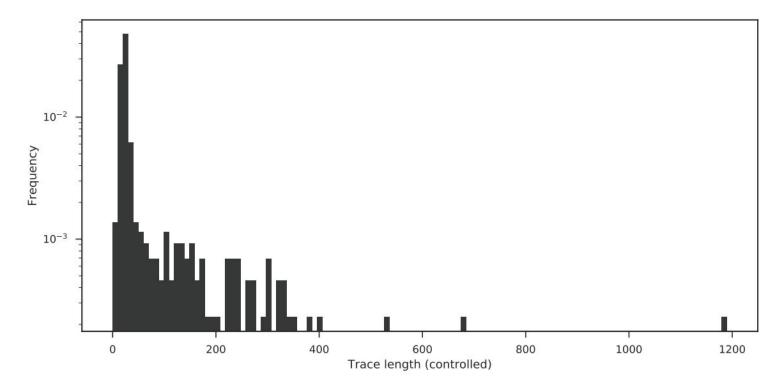
...

Approximately 450 trace types encountered



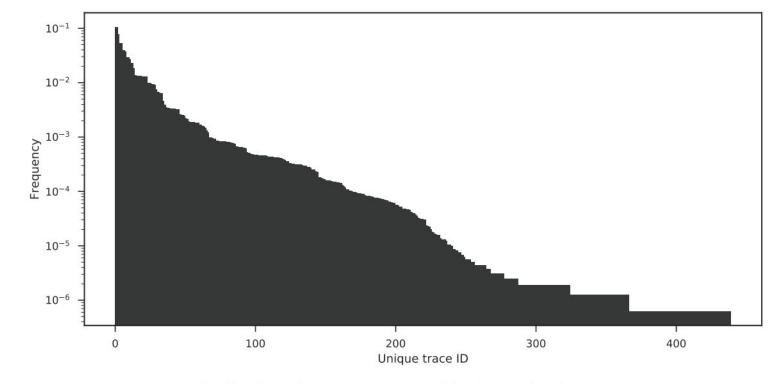
(a) Distribution of trace lengths (all addresses). Min: 13, max: 7,514, mean: 383.58.

Approximately 450 trace types encountered



(b) Distribution of trace lengths (controlled addresses only). Min: 6, max: 1,190, mean: 13.61.

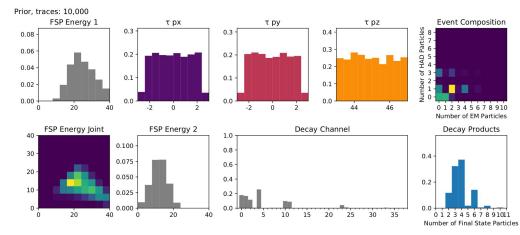
Approximately 450 trace types encountered



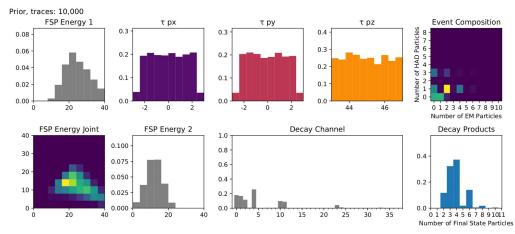
(c) Distribution of trace types, sorted in decreasing frequency.

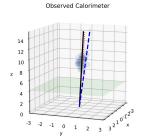
Inference results with MCMC engine

Prior



Inference results with MCMC engine

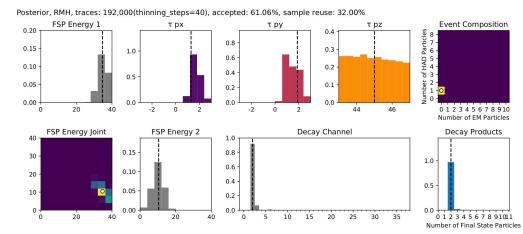




MCMC Posterior conditioned on calorimeter

7,700,000 samples Slow and has to run single node

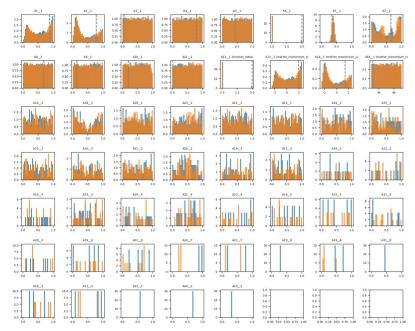
Prior



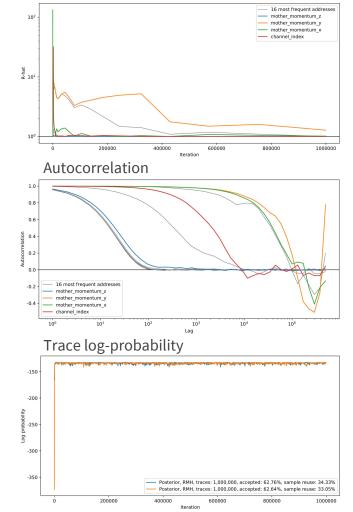
Convergence to true posterior

We establish that two independent RMH MCMC chains converge to the same posterior for all addresses in Sherpa

- Chain initialized with random trace from prior
- Chain initialized with known ground-truth trace



Gelman-Rubin convergence diagnostic

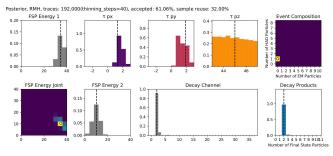


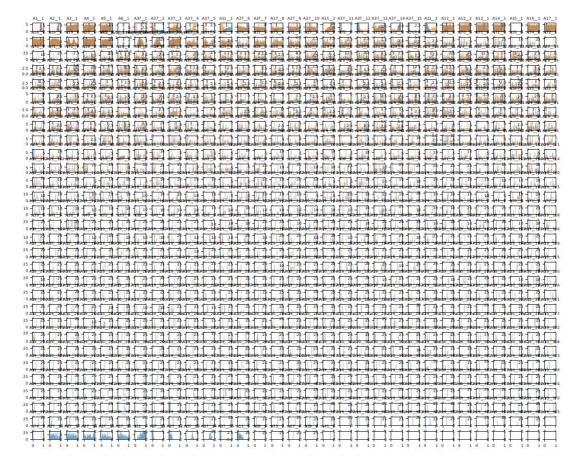
Convergence to true posterior

Important:

- We get posteriors over the whole Sherpa address space, 1000s of addresses
- Trace complexity varies depending on observed event

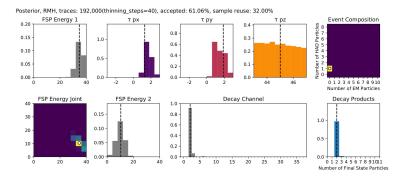
This is just a selected subset:





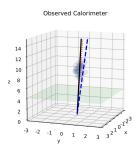
Inference results with IC engine

MCMC true posterior (7.7M single node)



Inference results with IC engine

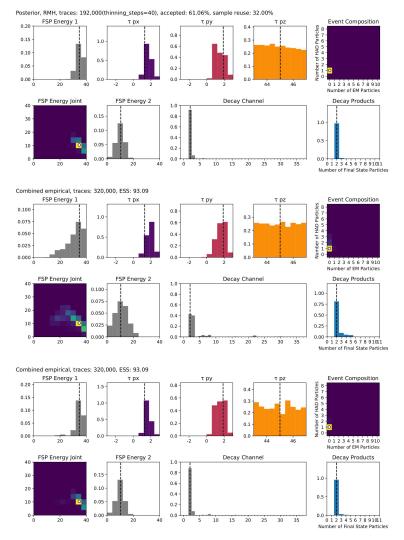
MCMC true posterior (7.7M single node)



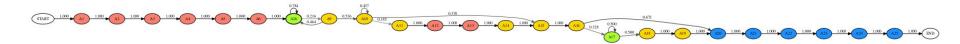
IC proposal

from trained NN

IC posterior after importance 320,000 samples Fast "embarrassingly" parallel multi-node



Latent probabilistic structure of 10 most frequent trace types

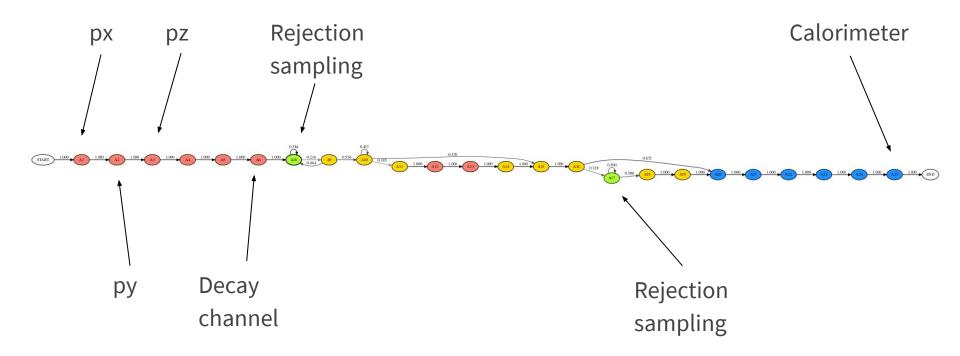


Latent probabilistic structure of 10 most frequent trace types

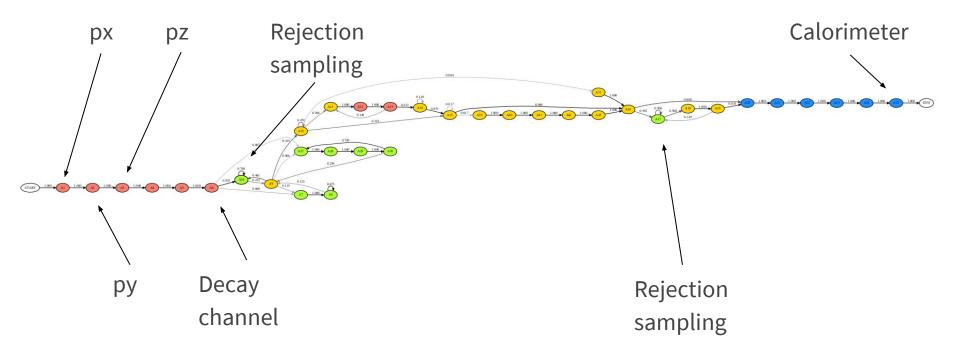
[forward(xt:: xarray_container<xt:: uvector<double, std:: allocator<double> >, (xt:: layout_type)1, xt:: svector<unsigned long, 4ul, std:: allocator<unsigned long>, true>, xt:: xtensor_expression_tag>)+0x5f; SherpaGenerator:: Generate()+0x36; SHERPA:: Sherpa:: GenerateOneEvent(bool)+0x2fa; SHERPA:: Event_Handler:: GenerateEvent(SHERPA:: eventtype:: code)+0x44d; SHERPA:: Event_Handler:: GenerateHadronDecayEvent(SHERPA:: eventtype:: code&)+0x45f; ATOOLS:: Random:: Get(bool, bool)+0x1d5; probprog_RNG:: Get(bool, bool)+0xf9]_Uniform_1

> [forward(xt:: xarray_container<xt:: uvector<double, std:: allocator<double> >, (xt:: layout_type)1, xt:: svector<unsigned long, 4ul, std:: allocator<unsigned long>, true>, xt:: xtensor_expression_tag>)+0x5f; SherpaGenerator:: Generate()+0x36; SHERPA:: Sherpa:: GenerateOneEvent(bool)+0x2fa; SHERPA:: Event_Handler:: GenerateEvent(SHERPA:: eventtype:: code)+0x44d; SHERPA:: Event_Handler:: GenerateEvent(SHERPA:: eventtype:: code&)+0x982; SHERPA:: Event_Handler:: IterateEventPhases(SHERPA:: eventype:: code&, double&)+0x1d2; SHERPA:: Hadron_Decays:: Treat(ATOOLS:: Blob_List*, double&)+0x975; SHERPA:: Decay_Handler_Base:: TreatInitialBlob(ATOOLS:: Blob_, METOOLS:: Amplitude2_Tensor*, std:: vector<ATOOLS:: Particle*, std:: allocator<ATOOLS:: Particle*> > const&)+0x1ab1; SHERPA:: Hadron_Decay_Handler:: CreateDecayBlob(ATOOLS:: Particle*)+0x4cd; PHASIC:: Decay_Table:: Select() const+0x9d7; ATOOLS:: Random:: GetCategorical(std:: vector<double, std:: allocator<double> > const&, bool, bool)+0x1a5; probprog_RNG:: GetCategorical(std:: vector<double, std:: allocator<double> > const&, bool, bool)+0x111]_Categorical(length_categories:38)_1

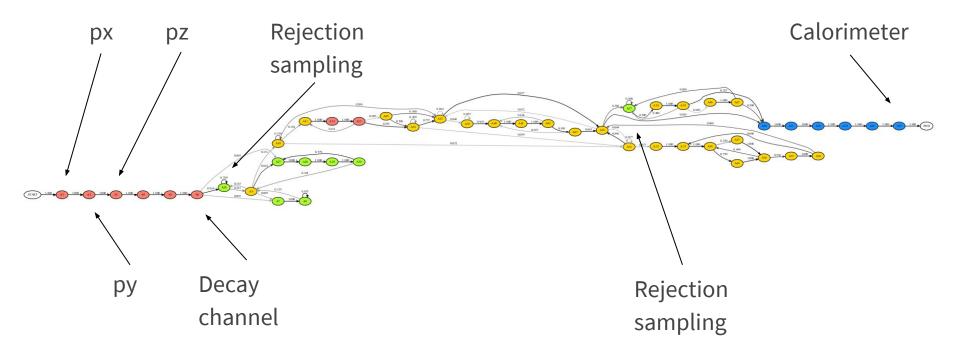
Latent probabilistic structure of 10 most frequent trace types



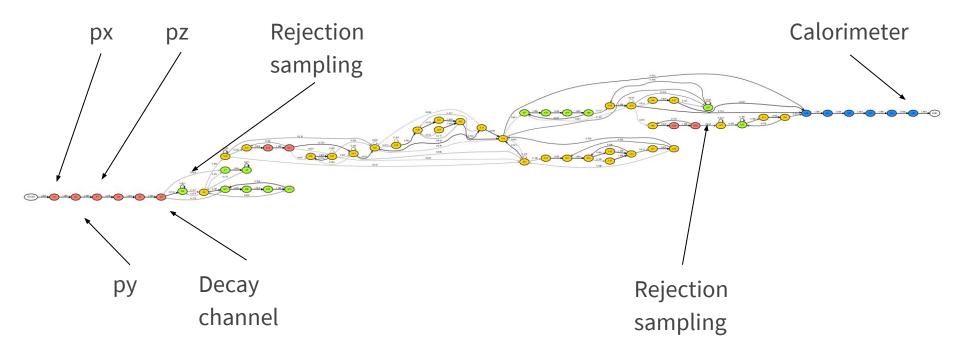
Latent probabilistic structure of 25 most frequent trace types

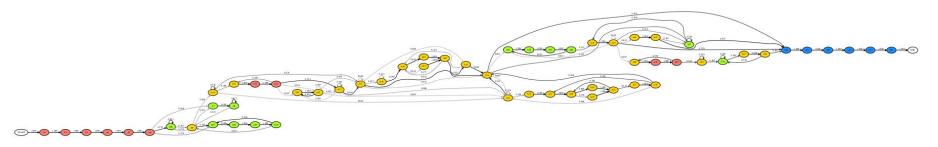


Latent probabilistic structure of 100 most frequent trace types

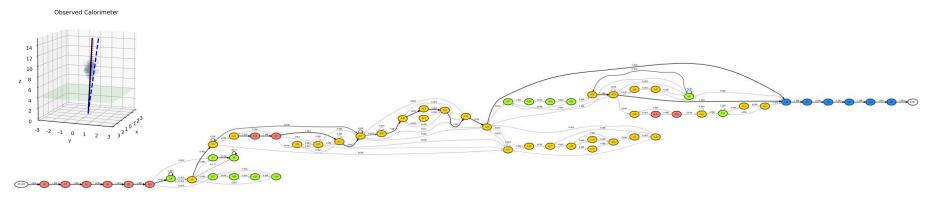


Latent probabilistic structure of 250 most frequent trace types





(a) Prior execution $p(\mathbf{x})$.



(b) Posterior execution $p(\mathbf{x}|\mathbf{y})$ conditioned on a given calorimeter observation \mathbf{y} .

What's next?

Current and upcoming work

- Science
 - Statistically measure distance between RMH and IC results
 - Uniform(0,1)-only control
 - Rare event simulation for compilation ("prior inflation")
 - Control / not control
- Engineering
 - Batching of open-ended traces for NN training
 - Distributed training of dynamic networks (thanks to PyTorch)
 - Balancing distributed data generation and training nodes
 - User-friendly features: posterior code highlighting, etc.
 - Other simulators

Thank you for listening

International Centre for Theoretical Physics Trieste, Italy, 9 April 2019



References

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Extra slides

Calorimeter

For each particle in the final state coming from Sherpa:

- Determine whether it interacts with the calorimeter at all (muons and neutrinos don't)
- 2. Calculate the total mean number and spatial distribution of energy depositions from the calorimeter shower (simulating combined effect of secondary particles)
- 3. Draw a number of actual depositions from the total mean and then draw that number of energy depositions according to the spatial distribution

Training objective and data for IC

• Minimize

$$\begin{aligned} \mathcal{L}(\phi) &= \mathbb{E}_{p(\mathbf{y})} \left[\mathrm{KL}(p(\mathbf{x}|\mathbf{y})||q(\mathbf{x}|\mathbf{y};\phi)) \right] \\ &= \int_{\mathbf{y}} p(\mathbf{y}) \int_{\mathbf{x}} p(\mathbf{x}|\mathbf{y}) \log \frac{p(\mathbf{x}|\mathbf{y})}{q(\mathbf{x}|\mathbf{y};\phi)} \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{y} \end{aligned}$$

- Using stochastic gradient descent with Adam
- Infinite stream of minibatches

sampled from the model
$$p(\mathbf{x}, \mathbf{y})$$

$$p(\mathbf{x}, \mathbf{y})$$

Gelman-Rubin and autocorrelation formulae

Gelman-Rubin diagnostic (\hat{R})

- Compute *m* independent Markov chains
- Compares variance of each chain to pooled variance
- If initial states (θ_{1j}) are overdispersed, then \hat{R} approaches unity from above
- Provides estimate of how much variance could be reduced by running chains longer
- It is an estimate!

$$W = \frac{1}{m} \sum_{j=1}^{m} s_j^2 \qquad \qquad \bar{\bar{\theta}} = \frac{1}{m} \sum_{j=1}^{m} \bar{\theta}_j$$
$$B = \frac{n}{m-1} \sum_{j=1}^{m} (\bar{\theta}_j - \bar{\bar{\theta}})^2 \qquad \qquad s_j^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\theta_{ij} - \bar{\theta}_j)^2$$
$$\hat{Var}(\theta) = (1 - \frac{1}{n})W + \frac{1}{n}B \qquad \qquad \hat{R} = \sqrt{\frac{\hat{Var}(\theta)}{W}}$$

From Eric B. Ford (Penn State): Bayesian Computing for Astronomical Data Analysis http://astrostatistics.psu.edu/RLectures/diagnosticsMCMC.pdf

Gelman-Rubin and autocorrelation formulae

Check Autocorrelation of Markov chain

• Autocorrelation as a function of lag

$$\rho_{lag} = \frac{\sum_{i}^{N-lag} (\theta_i - \bar{\theta})(\theta_{i+lag} - \bar{\theta})}{\sum_{i}^{N} (\theta_i - \bar{\theta})^2}$$

- What is smallest lag to give an $\rho_{lag} \approx 0$?
- One of several methods for estimating how many iterations of Markov chain are needed for *effectively* independent samples

From Eric B. Ford (Penn State): Bayesian Computing for Astronomical Data Analysis http://astrostatistics.psu.edu/RLectures/diagnosticsMCMC.pdf