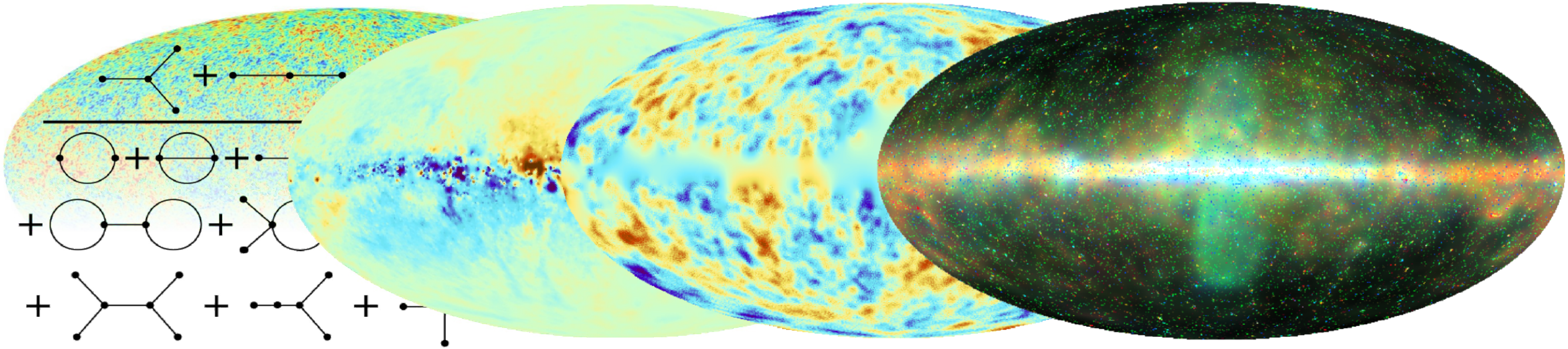


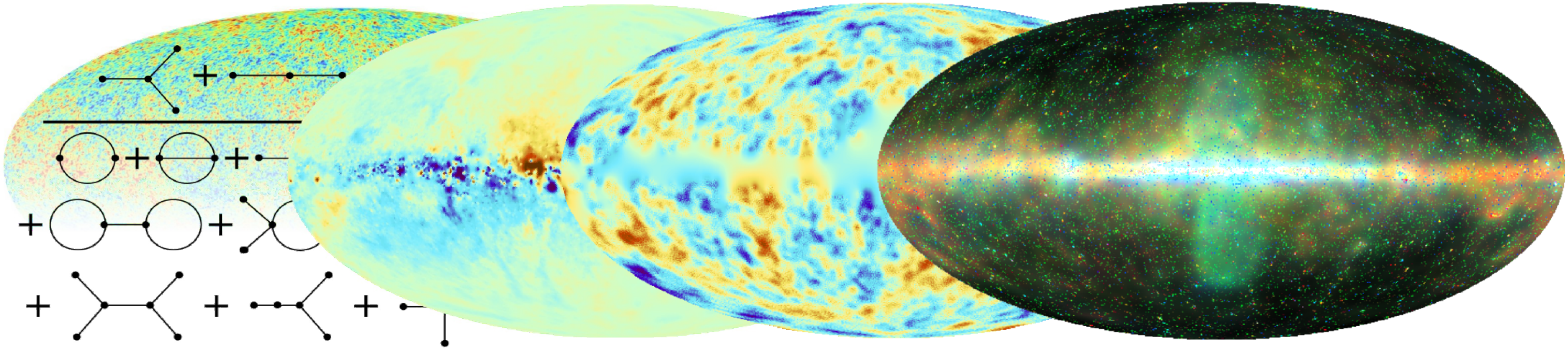
Information field theory

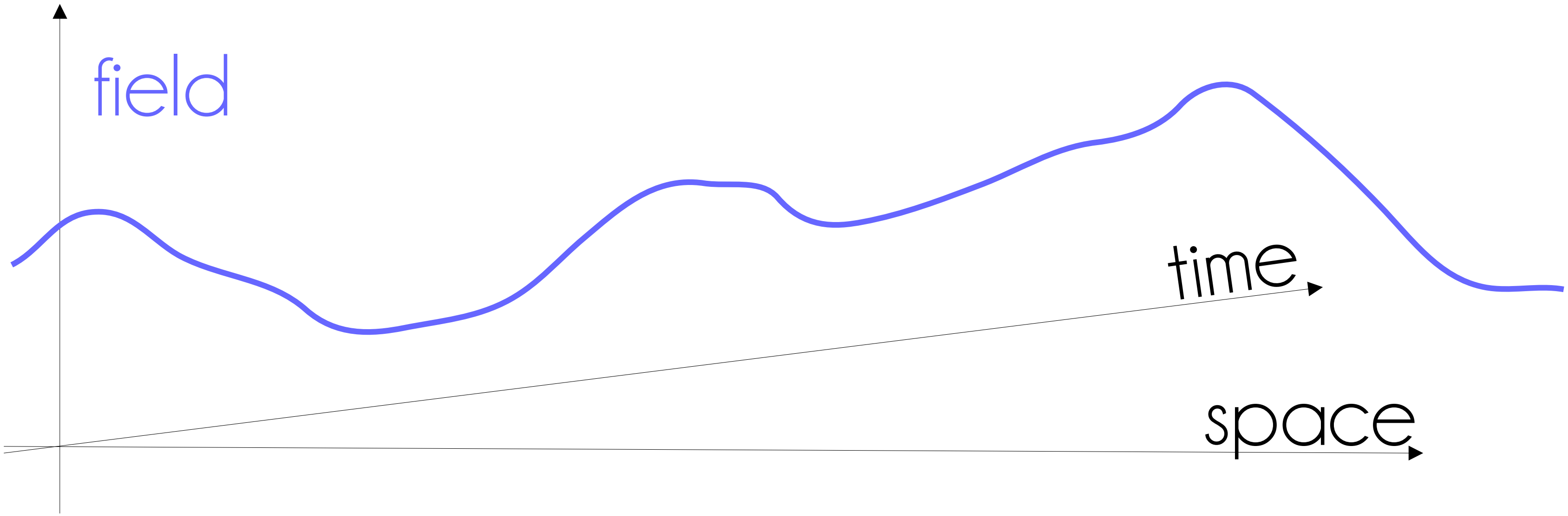
Torsten Enßlin

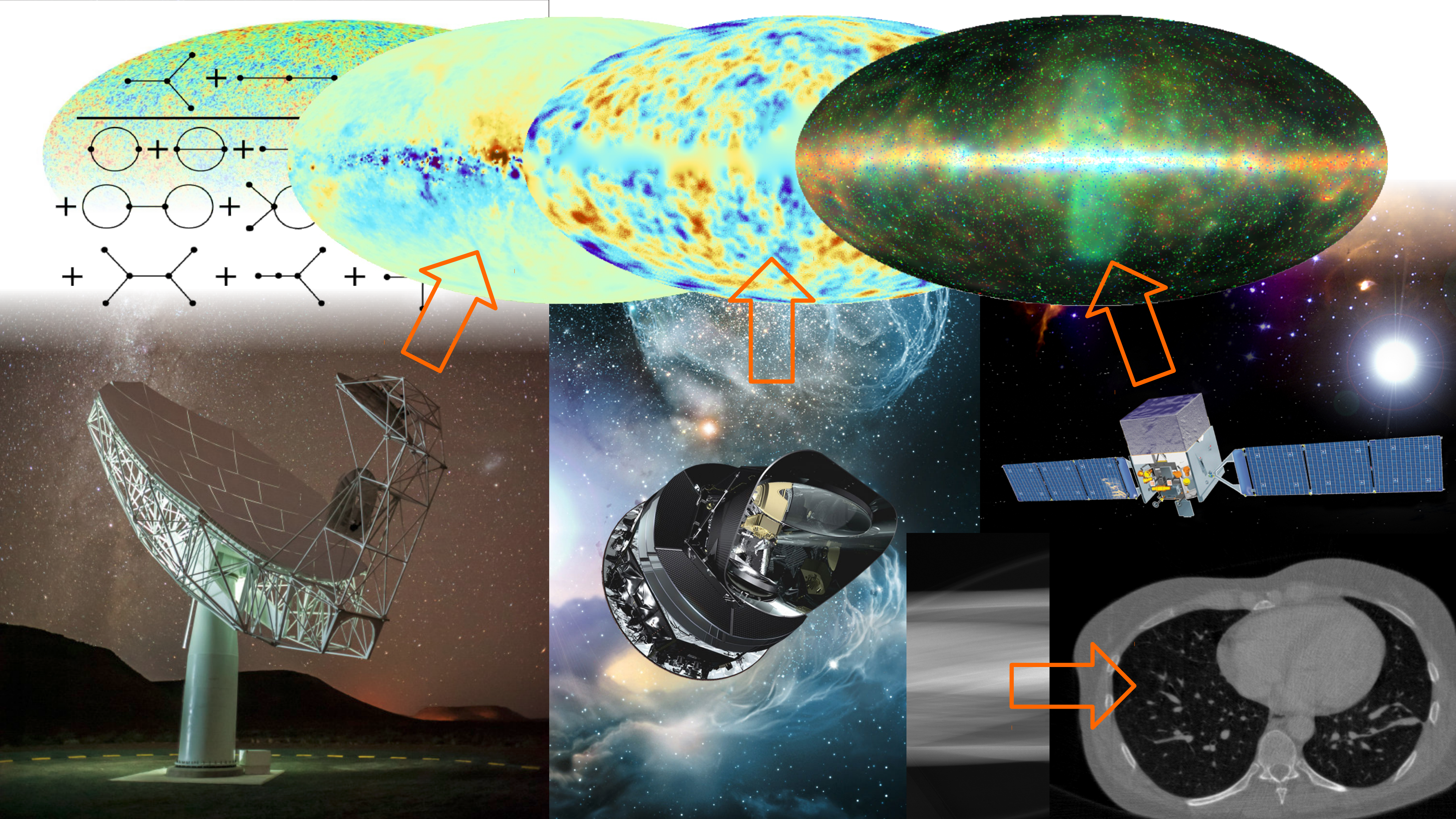
MPI for Astrophysics

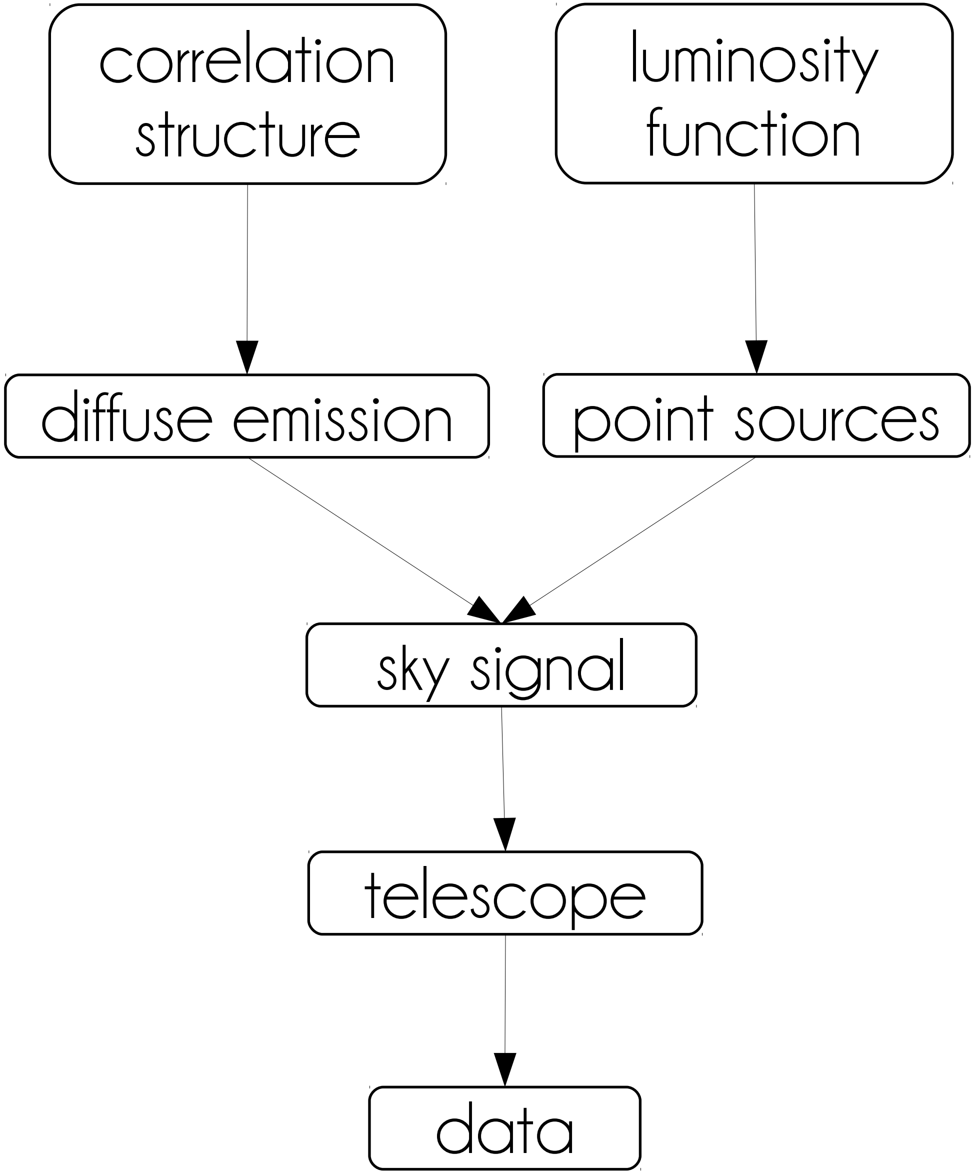
Ludwig Maximilian University Munich

IFT Team: Philipp Arras, Michael Bell, Vanessa Böhm, Sebastian Dorn, Martin Dupont, Mona Frommert, Philipp Frank, Mahsa Ghaempanah, Maksim Greiner, Sebastian Hutschenreuter, Henrik Junklewitz, Francisco-Shu Kitaura, Jakob Knollmüller, Christoph Lienhard, Reimar Leike, Marco Selig, Theo Steininger, Johannes Oberpriller, Niels Oppermann, Natalia Porquerese, Daniel Pumpe, Tiago Ramalho, Martin Reinecke, Julia Stadler, Valentina Vacca, Cornelius Weig, Margret Westerkamp, & many more



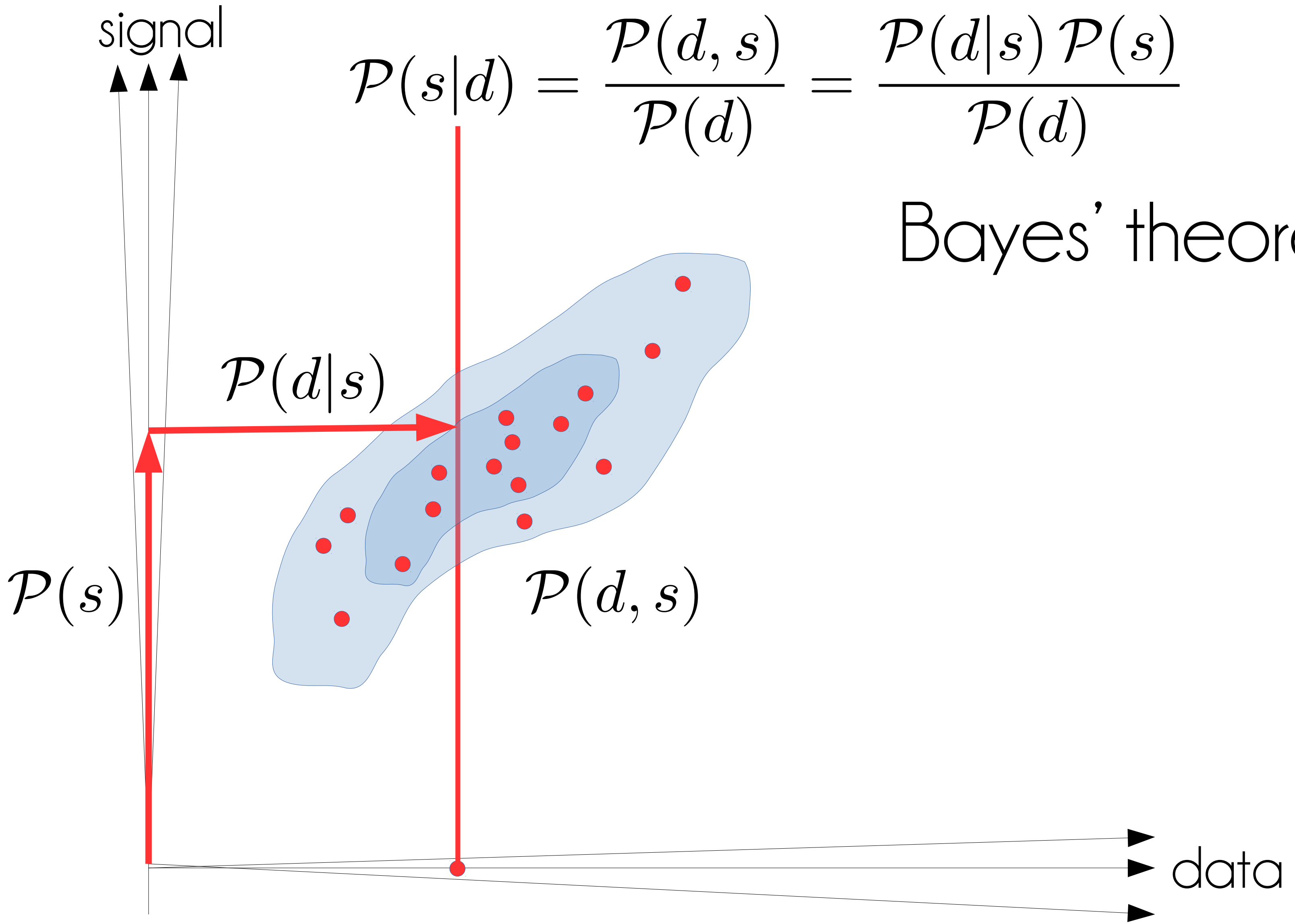

field
time
space

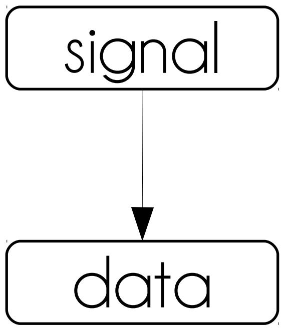



correlation structure
luminosity function
diffuse emission
point sources
sky signal
telescope
data

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d,s)}{\mathcal{P}(d)} = \frac{\mathcal{P}(d|s)\,\mathcal{P}(s)}{\mathcal{P}(d)}$$

Bayes' theorem





Information theory

$$
\begin{aligned}
\mathcal{P}(s|d) &= \frac{\mathcal{P}(d,s)}{\mathcal{P}(d)} = \frac{e^{-\mathcal{H}(d,s)}}{\mathcal{Z}(d)} \\
\mathcal{H}(d,s) &= -\log \mathcal{P}(d,s) \\
\mathcal{Z}(d) &= \mathcal{P}(d) \\
&= \int \mathcal{D}s \, \mathcal{P}(d,s) \\
\mathcal{P}(d,s) &= \mathcal{P}(d|s)\,\mathcal{P}(s) \\
\mathcal{H}(d,s) &= \mathcal{H}(d|s) + \mathcal{H}(s)
\end{aligned}
$$

Information

is additive

Probability & Information

$$\mathcal{P}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{s^2}{2\sigma^2}}$$

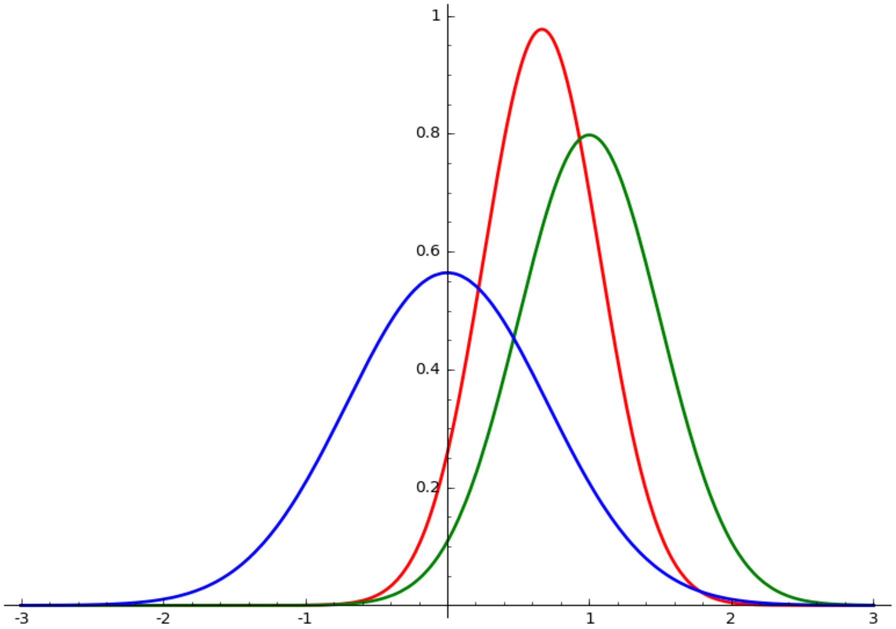
$$\mathcal{P}(d|s) \propto e^{-\frac{(s-d)^2}{2\sigma'^2}}$$

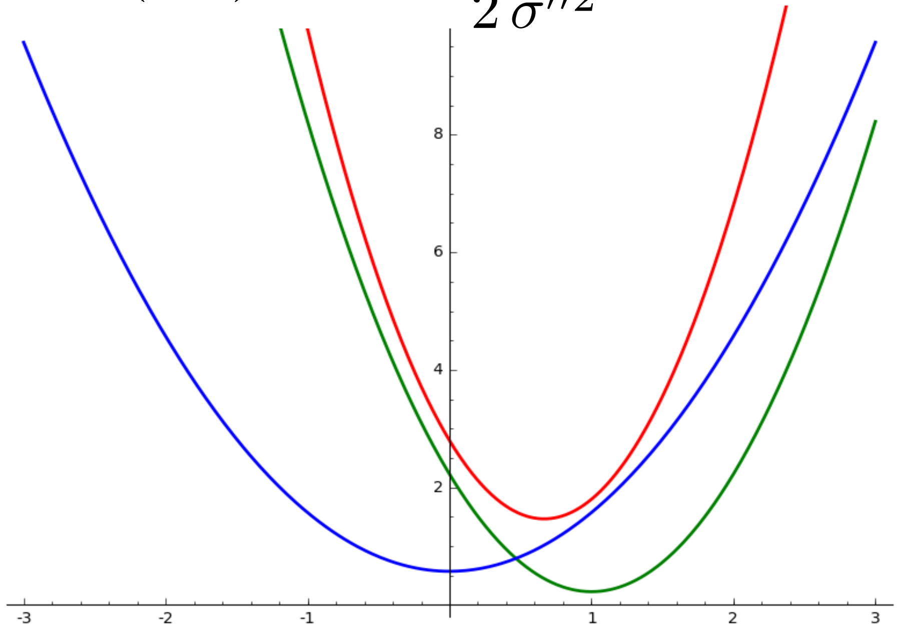
$$\mathcal{P}(s|d) \propto e^{-\frac{(s-m)^2}{2\sigma''^2}}$$

$$\mathcal{H}(s) \widehat{=} \frac{s^2}{2\,\sigma^2}$$

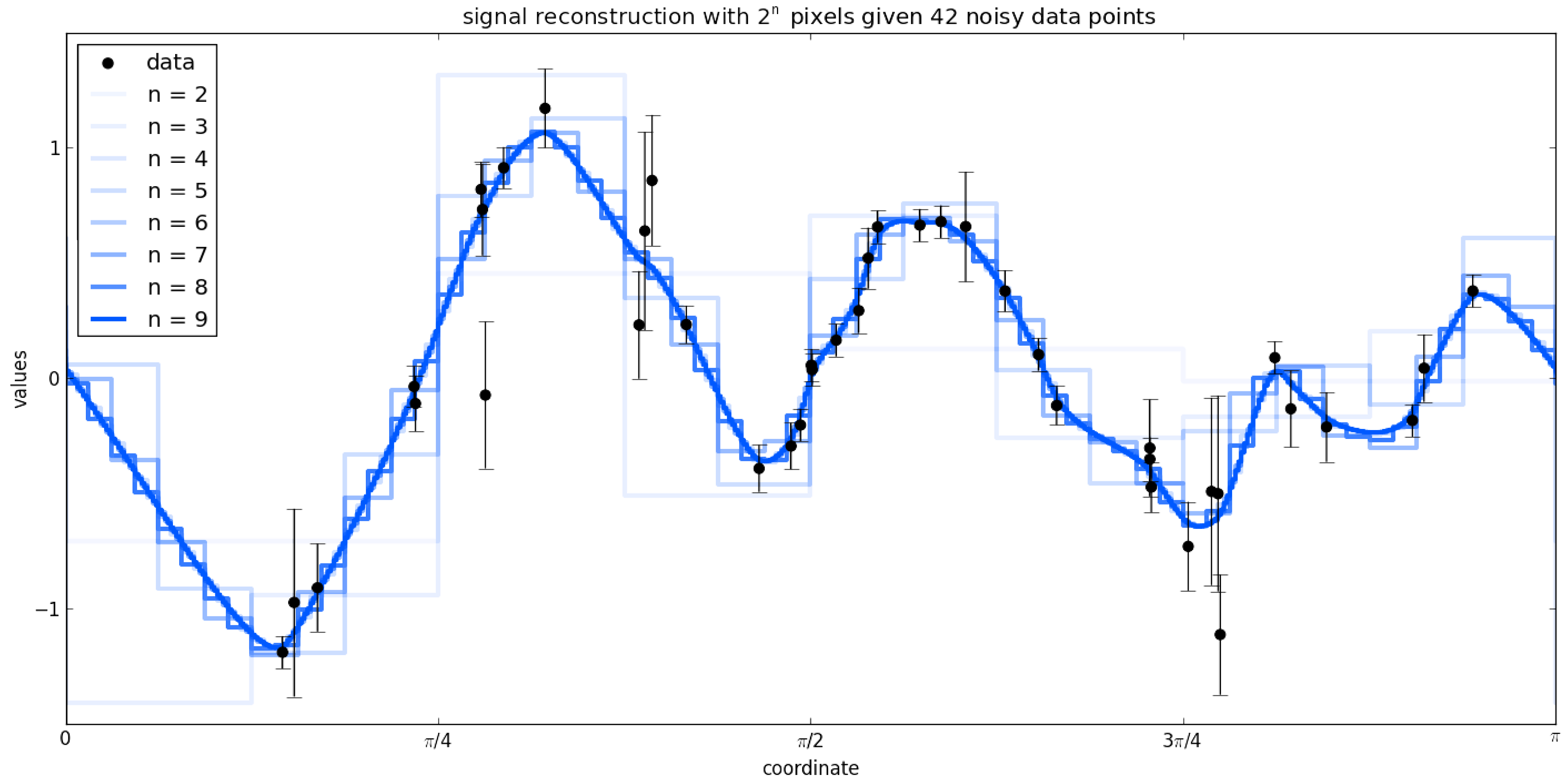
$$\mathcal{H}(d|s) \widehat{=} \frac{(s-d)^2}{2\,\sigma'^2}$$

$$\mathcal{H}(d,s) \widehat{=} \frac{(s-m)^2}{2\,\sigma''^2}$$






Correlations



signal reconstruction with 2^n pixels given 42 noisy data points
data
n = 2
n = 3
n = 4
n = 5
n = 6
n = 7
n = 8
n = 9
values
1
0
−1
coordinate
0
π/4
π/2
3π/4
π

NIFTy – Numerical Information Field Theory

NIFTy [1], [2], "**Numerical Information Field Theory**" is a versatile library designed to enable the development of signal inference algorithms that are independent of the underlying grids (spatial, spectral, temporal, …) and their resolutions. Its object-oriented framework is written in Python, although it accesses libraries written in C++ and C for efficiency.

NIFTy offers a toolkit that abstracts discretized representations of continuous spaces, fields in these spaces, and operators acting on these fields into classes. This allows for an abstract formulation and programming of inference algorithms, including those derived within information field theory. NIFTy's interface is designed to resemble IFT formulae in the sense that the user implements algorithms in NIFTy independent of the topology of the underlying spaces and the discretization scheme. Thus, the user can develop algorithms on subsets of problems and on spaces where the detailed performance of the algorithm can be properly evaluated and then easily generalize them to other, more complex spaces and the full problem, respectively.

The set of spaces on which NIFTy operates comprises point sets, n-dimensional regular grids, spherical spaces, their harmonic counterparts, and product spaces constructed as combinations of those. NIFTy takes care of numerical subtleties like the normalization of operations on fields and the numerical representation of model components, allowing the user to focus on formulating the abstract inference procedures and process-specific model properties.

Probabilistic programming with auto-differentiation

References

[1] Selig et al., "NIFTY - Numerical Information Field Theory. A versatile PYTHON library for signal inference ", 2013, Astronmy and Astrophysics 554, 26; [DOI], [arXiv:1301.4499]

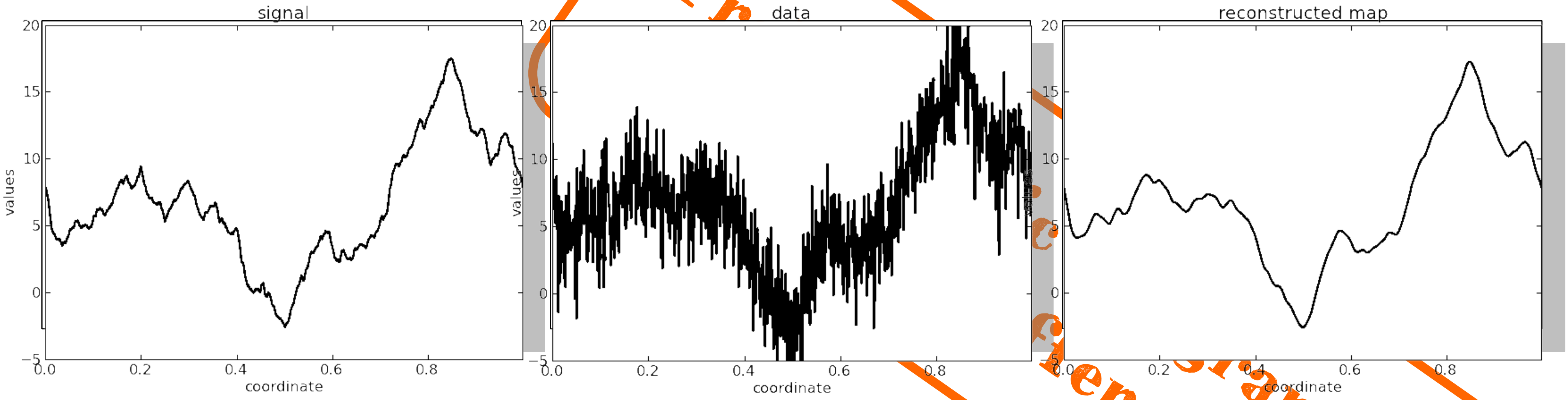
[2] Steininger et al., "NIFTy 3 - Numerical Information Field Theory - A Python framework for multicomponent signal inference on HPC clusters", 2017, accepted by Annalen der Physik; [arXiv:1708.01073]

Contents

NIFTy5

NIFTy – Numerical Information Field Theory

NIFTy [1], [2], "Numerical Information Field Theory" is a versatile library designed to enable the development of signal inference algorithms that are independent of the underlying grids (spatial, spectral, temporal, …) and their resolutions. Its object-oriented framework is written in Pytho

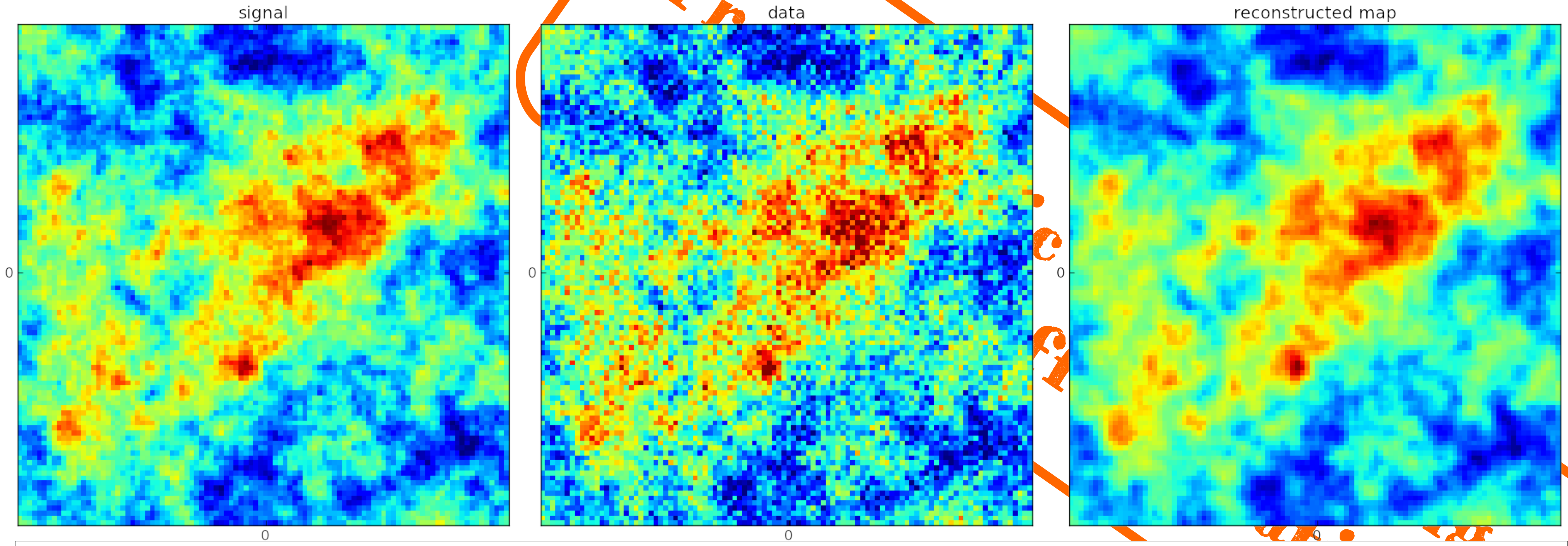



```
import nifty5 as ift
s_space = ift.RGSpace([N])
```

NIFTy5

NIFTy – Numerical Information Field Theory

NIFTy [1], [2], “Numerical Information Field Theory” is a versatile library designed to enable the development of signal inference algorithms that are independent of the underlying grids (spatial, spectral, temporal, …) and their resolutions. Its object-oriented framework is written in Pytho

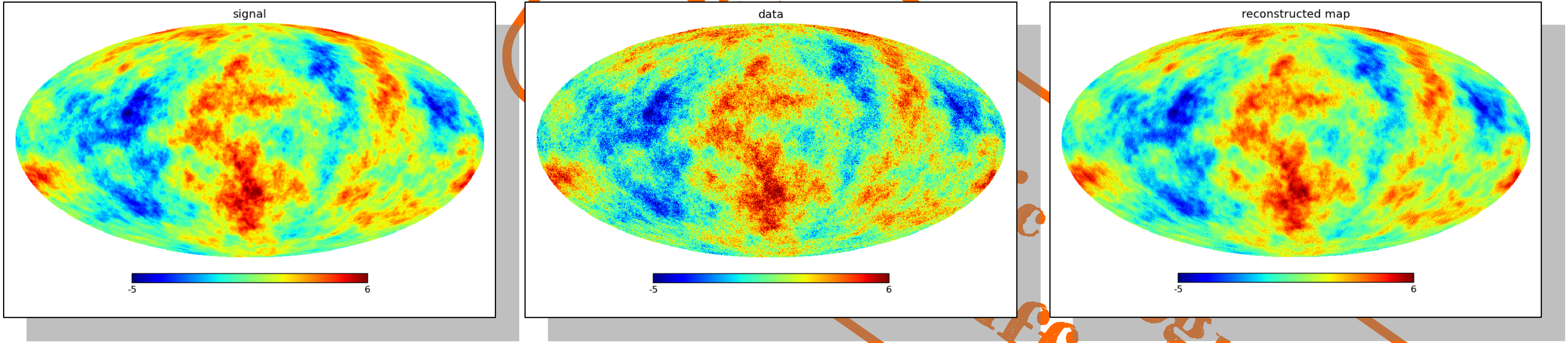



```
import nifty5 as ift
s_space = ift.RGSpace([N,N])
```

NIFTy5

NIFTy – Numerical Information Field Theory

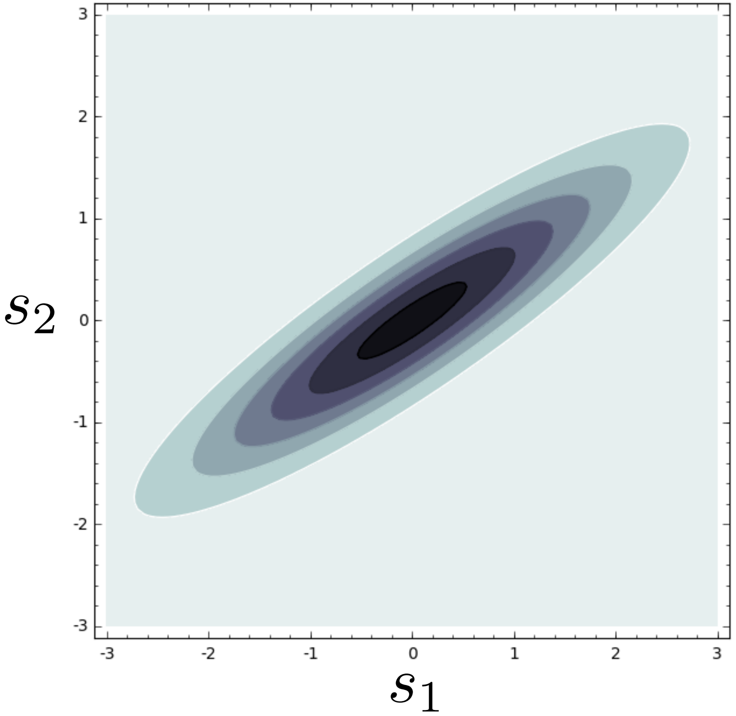
NIFTy [1], [2], "Numerical Information Field Theory" is a versatile library designed to enable the development of signal inference algorithms that are independent of the underlying grids (spatial, spectral, temporal, …) and their resolutions. Its object-oriented framework is written in Python




```
import nifty5 as ift
s_space = ift.HPSpace(NSide)
```

Correlations

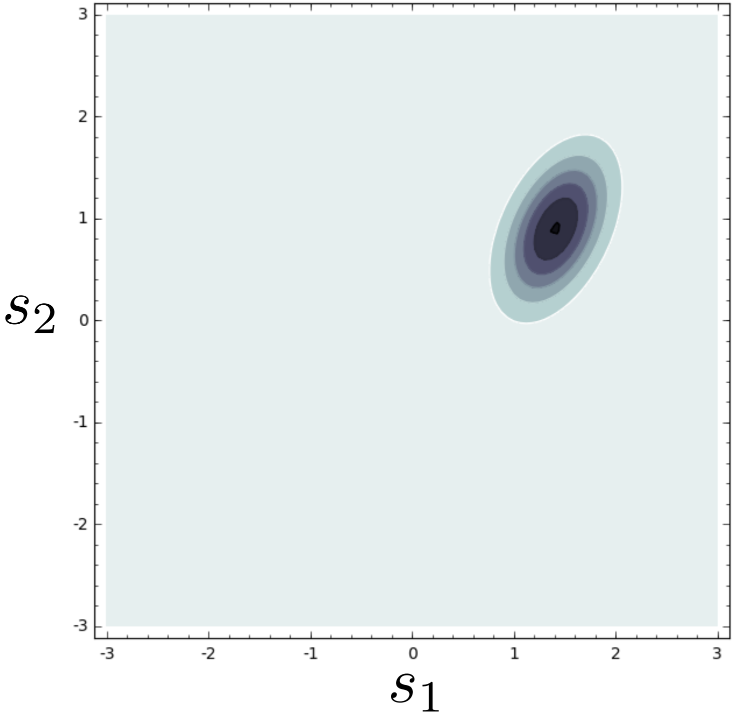
$$\mathcal{P}(s) \qquad s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

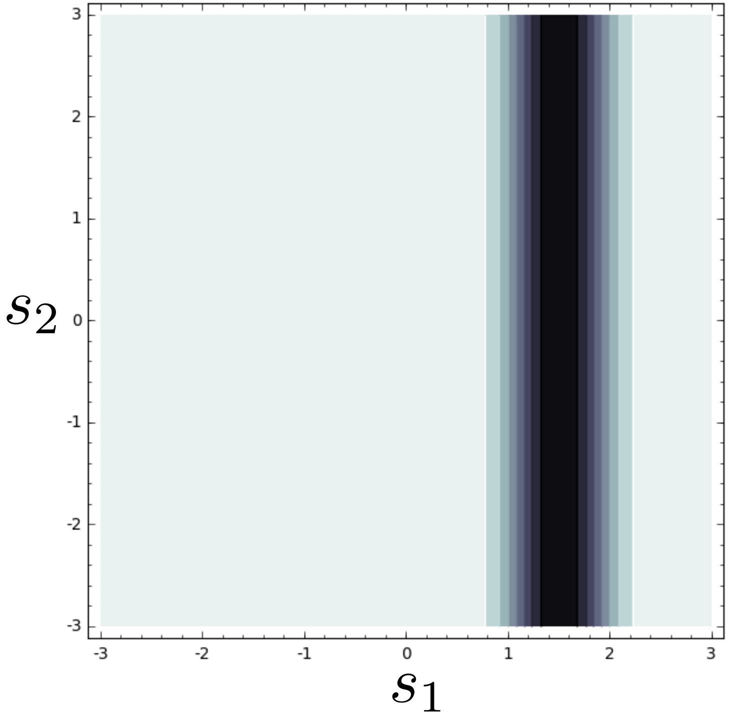


Correlations

$$\mathcal{P}(s|d) \qquad s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \qquad \mathcal{P}(d|s)$$

$$d = s_1 + n$$

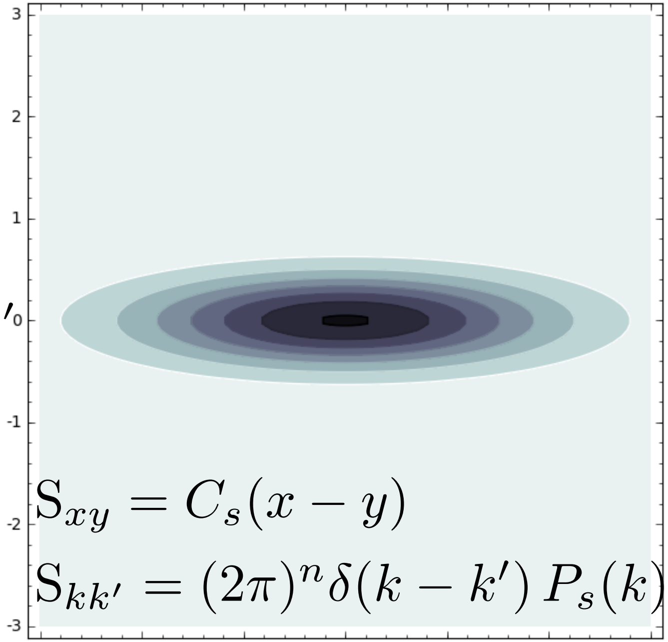




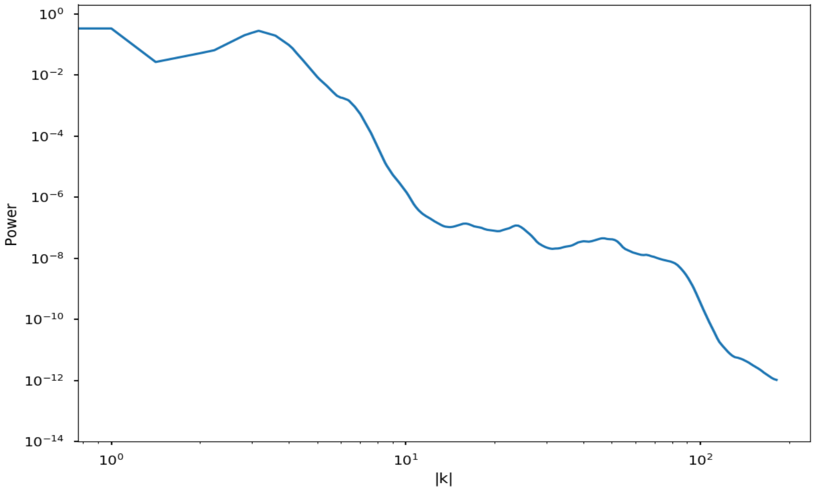
Correlations

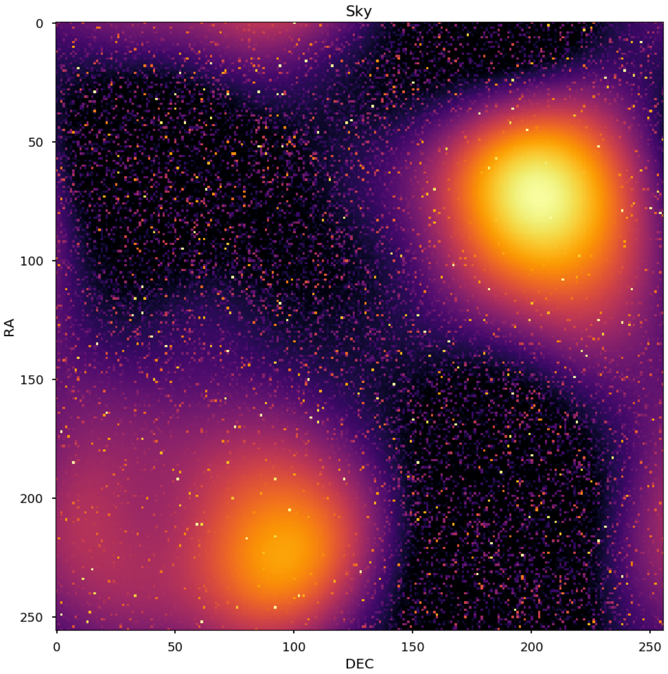
$$\mathcal{P}(s)$$

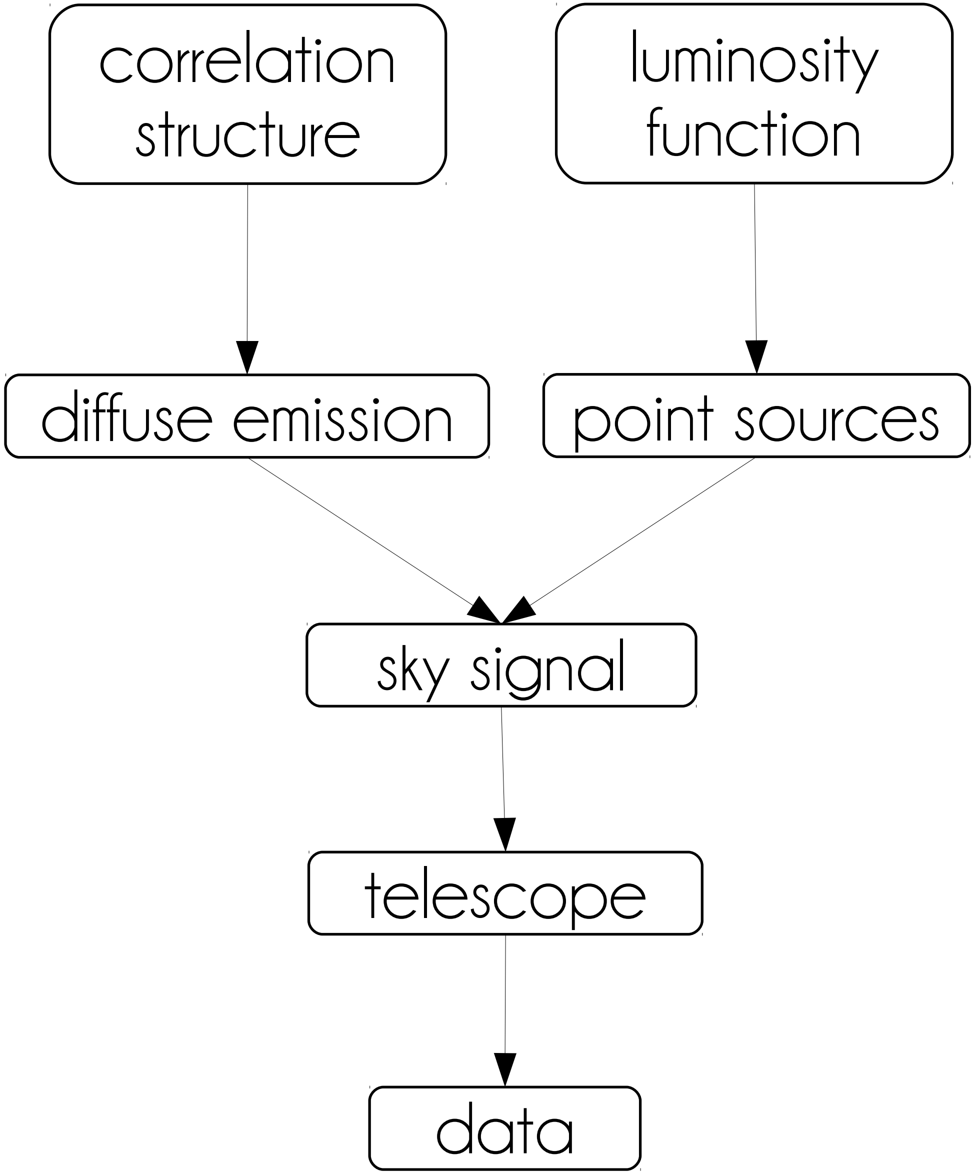
$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$



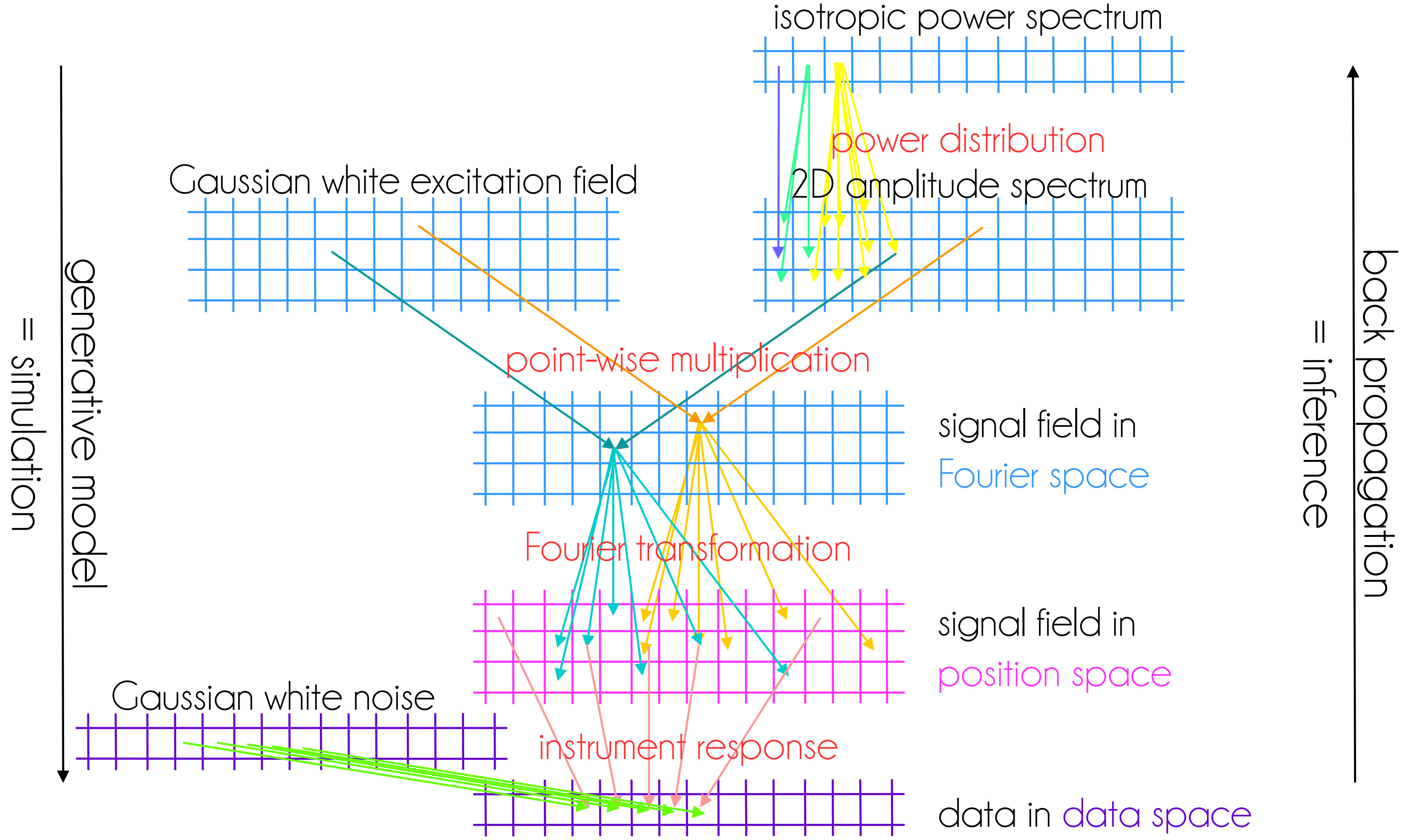
$$\begin{aligned}
\mathcal{P}(s) &= \mathcal{G}(s, S) \\
&= \frac{1}{\sqrt{|2\pi S|}} \exp\left(-\frac{1}{2} s^\dagger S^{-1} s\right) \\
S &= \begin{pmatrix} \langle s_1 s_1 \rangle & \langle s_1 s_2 \rangle \\ \langle s_2 s_1 \rangle & \langle s_2 s_2 \rangle \end{pmatrix} \quad \text{2-dim.} \\
S_{ij} &= \langle s_i s_j \rangle \qquad n\text{-dim.} \\
S_{xy} &= \langle s_x s_y \rangle,\ x \in \mathbb{R}^n \qquad \infty\text{-dim.}
\end{aligned}$$


Power
|k|
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{0}
10^{1}
10^{2}


Sky
RA
DEC
0
50
100
150
200
250


correlation structure
luminosity function
diffuse emission
point sources
sky signal
telescope
data

IFT as neural network



Data model

known $\rightarrow$ $d = R\,e^{s} + n$

known response

unknown

$\lambda = R\,e^{s}$

$\mathcal{P}(s) = \mathcal{G}(s,\, S)$ unknown

$$\mathcal{P}(d|\lambda) = \prod_i \frac{\lambda_i^{d_i}}{d_i!}\, e^{-\lambda_i}$$

Information

$$
\begin{aligned}
\mathcal{H}(\boldsymbol{d}, \boldsymbol{s}, \boldsymbol{\tau}) &= -\log \mathcal{P}(\boldsymbol{d}, \boldsymbol{s}, \boldsymbol{\tau}) \\
&= 1^\dagger \left[\log(d!) + \boldsymbol{R}\left(\mathrm{e}^{\boldsymbol{s}} + \mathrm{e}^{\boldsymbol{u}}\right)\right] - \boldsymbol{d}^\dagger \log\left[\boldsymbol{R}\left(\mathrm{e}^{\boldsymbol{s}} + \mathrm{e}^{\boldsymbol{u}}\right)\right] \\
&\quad + \frac{1}{2}\boldsymbol{s}^\dagger \boldsymbol{S}^{-1}\boldsymbol{s} + \frac{1}{2}\log\left(\det\left[\boldsymbol{S}\right]\right) \\
&\quad + (\boldsymbol{\alpha} - \mathbf{1})^\dagger \\
&\quad + (\boldsymbol{\beta} - \mathbf{1})^\dagger
\end{aligned}
$$

$$
\boldsymbol{S} = \sum_k \mathrm{e}^{\tau_k}\boldsymbol{S}_k
$$

- Convert into **generative model**
- Compress information into Gaussian via **Metric Gaussian Variational Inference**

Variational Bayes

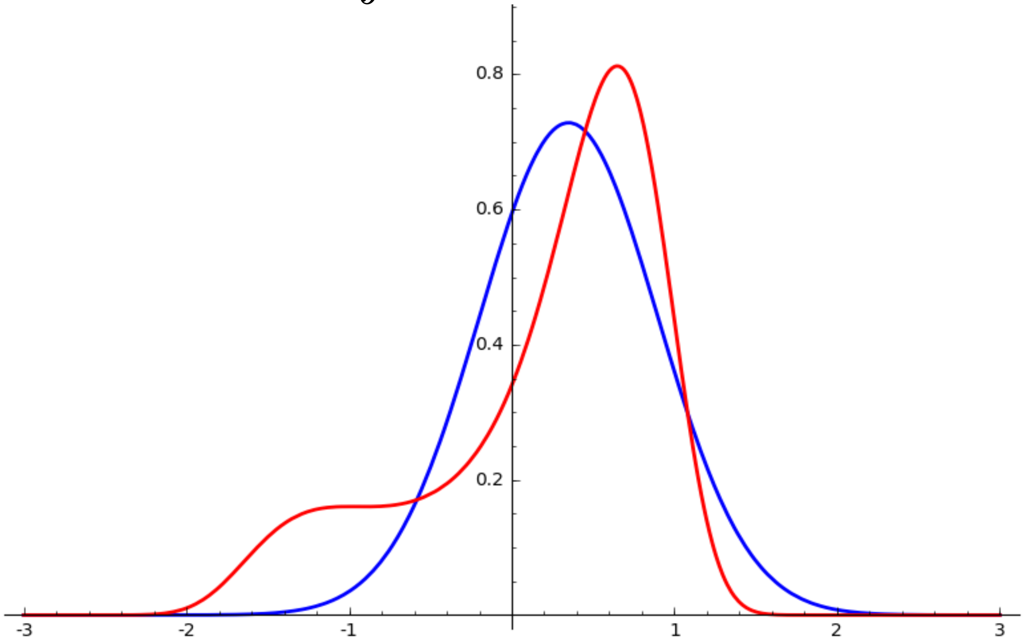
$\mathcal{P}(s|d)$

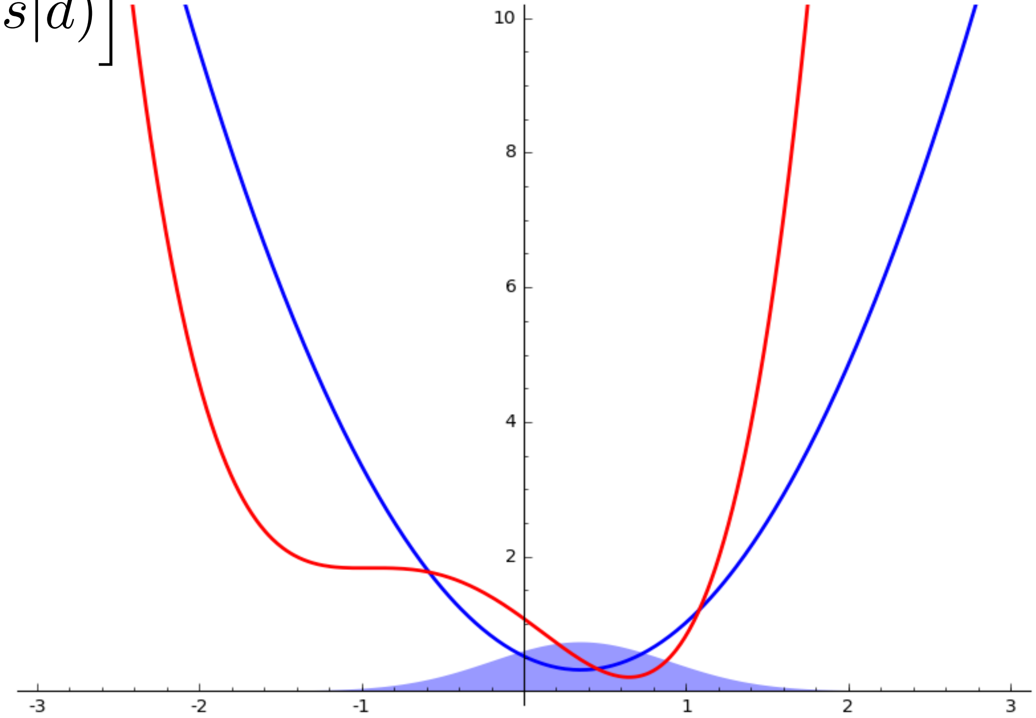
$\widetilde{\mathcal{P}}(s|d) = \mathcal{G}(s-m, D)$

$\mathcal{H}(s|d)$

$\widetilde{\mathcal{H}}(s|d) \widehat{=} \frac{1}{2}(s-m)^\dagger D^{-1}(s-m)$

$$\mathrm{KL}(\widetilde{\mathcal{P}}, \mathcal{P}) = \int \mathcal{D}s\, \widetilde{\mathcal{P}}(s|d) \left[\mathcal{H}(s|d) - \widetilde{\mathcal{H}}(s|d)\right]$$





Metric Gaussian Variational Bayes

$\mathcal{P}(s|d)$

$\widetilde{\mathcal{P}}(s|d) = \mathcal{G}(s-m, D)$

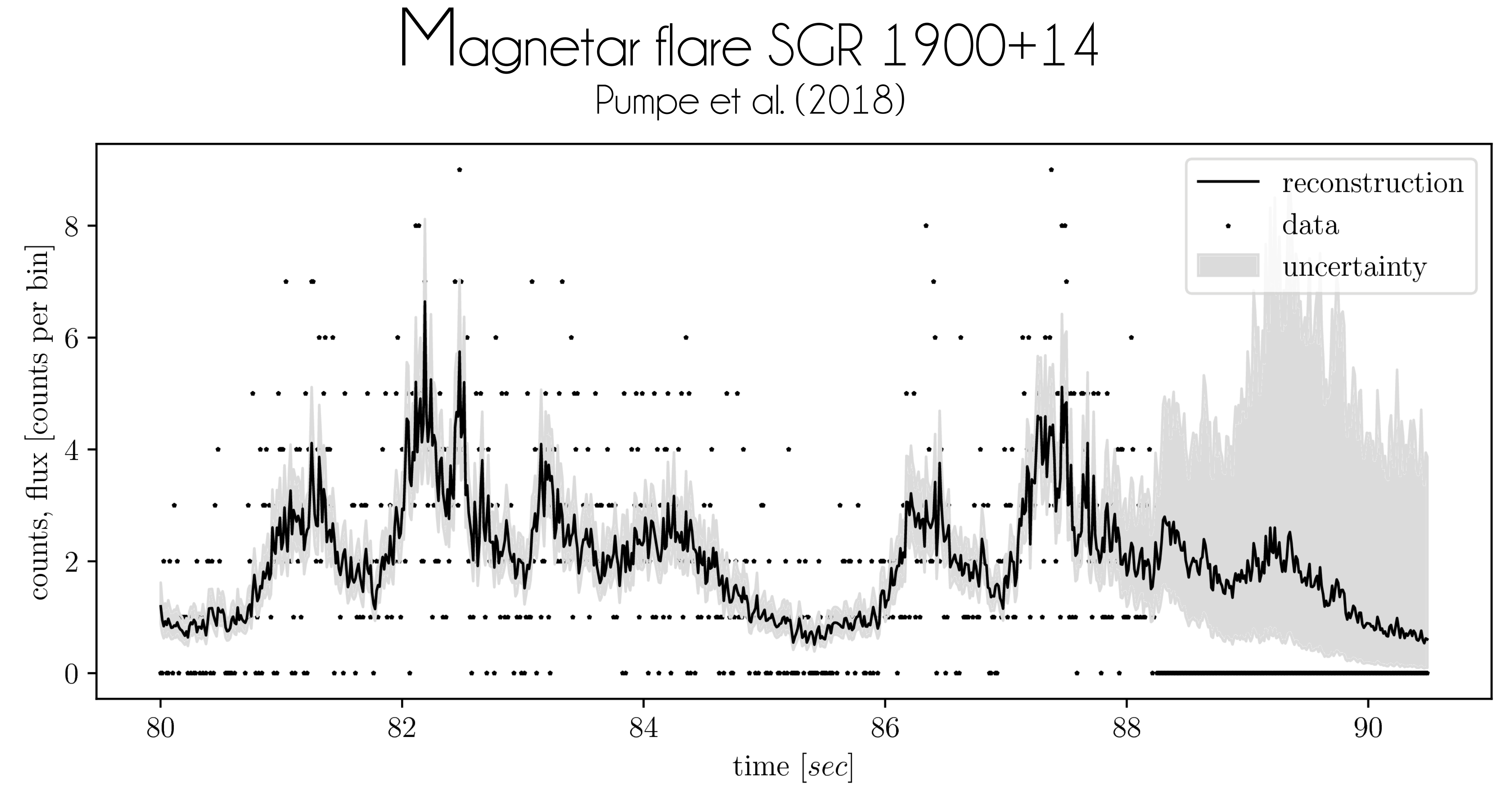
$\mathcal{H}(s|d)$

Knollmüller & Enßlin (2019)

$\widetilde{\mathcal{H}}(s|d) \widehat{=} \frac{1}{2}(s-m)^\dagger D^{-1}(s-m)$

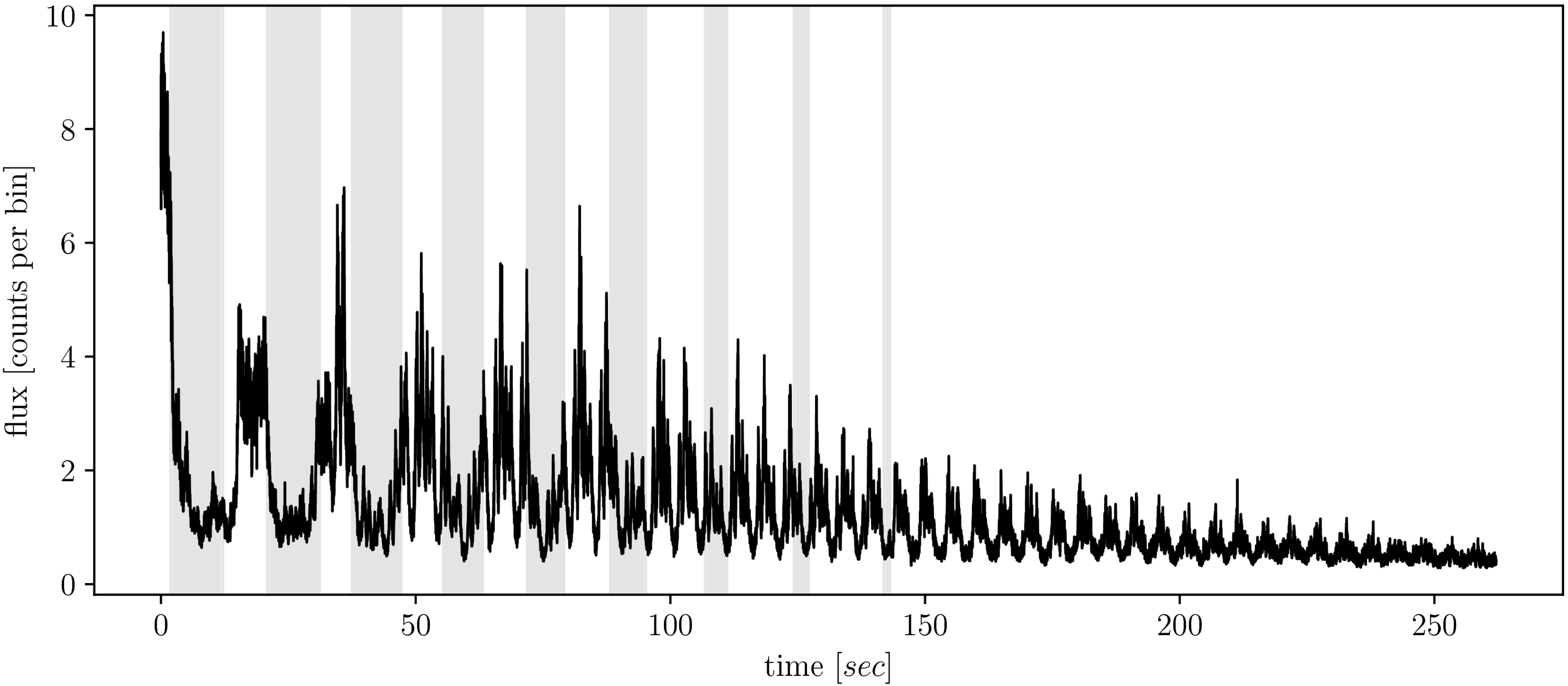
$$\mathrm{KL}(\widetilde{\mathcal{P}}, \mathcal{P}) = \int \mathcal{D}s\, \widetilde{\mathcal{P}}(s|d) \left[\mathcal{H}(s|d) - \widetilde{\mathcal{H}}(s|d)\right]$$

$$D \approx \left\langle \frac{\partial \mathcal{H}(d,s)}{\partial s} \frac{\partial \mathcal{H}(d,s)}{\partial s}^\dagger \right\rangle^{-1}_{(d|s=m)}$$


Magnetar flare SGR 1900+14
Pumpe et al. (2018)
reconstruction
data
uncertainty
counts, flux [counts per bin]
time [sec]
0
2
4
6
8
80
82
84
86
88
90

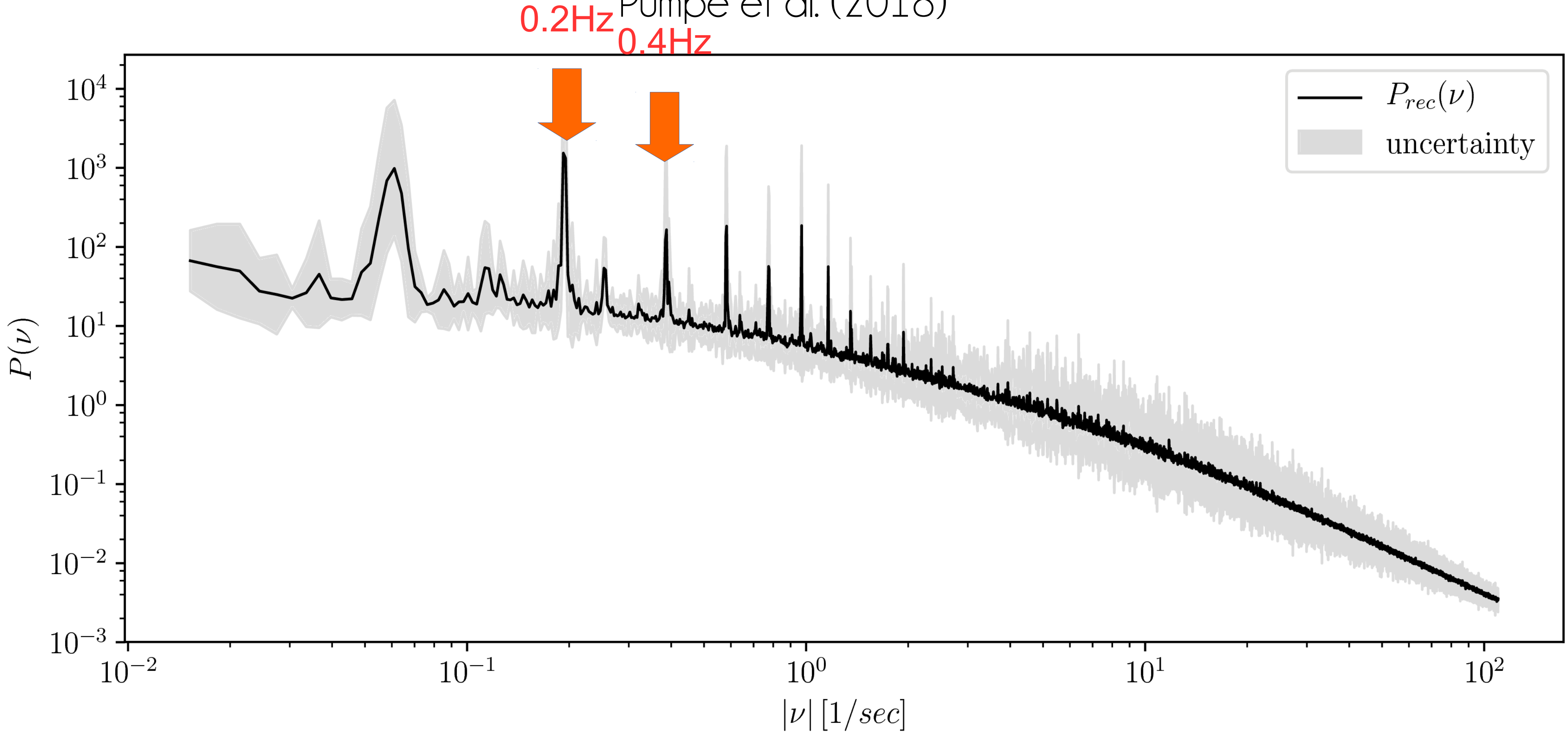
Magnetar flare SGR 1900+14

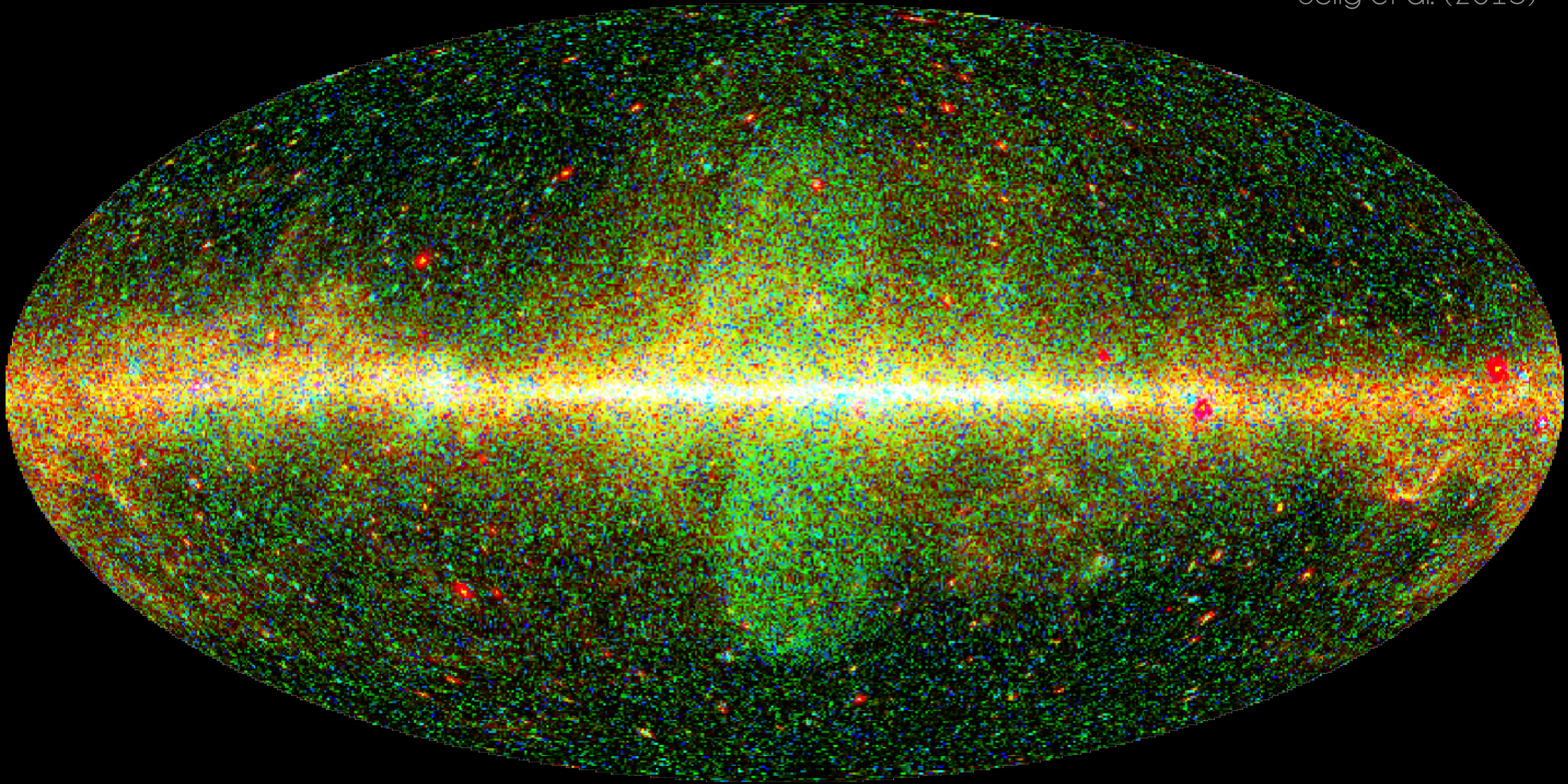
Pumpe et al. (2018)

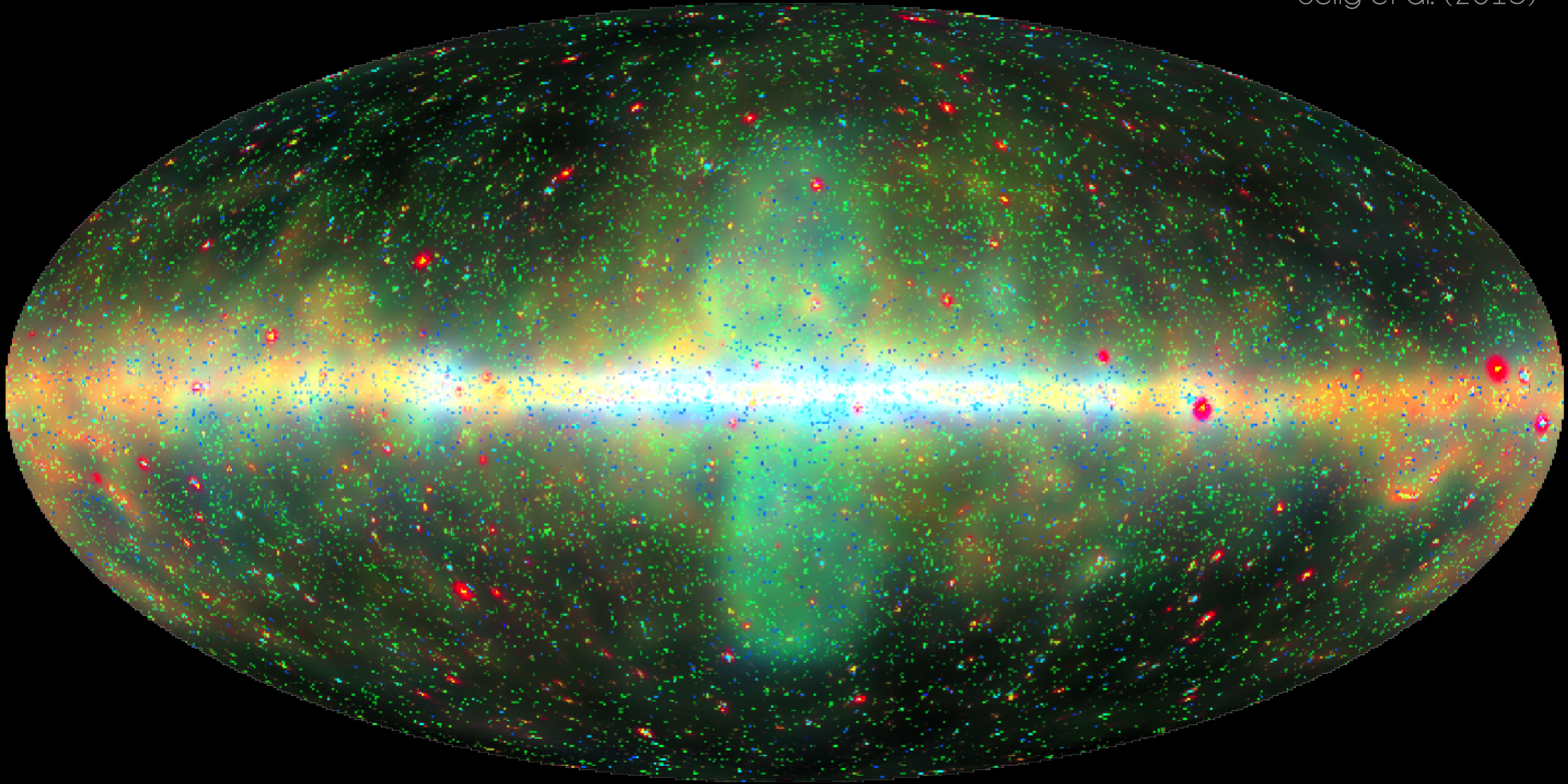


Magnetar flare SGR 1900+14

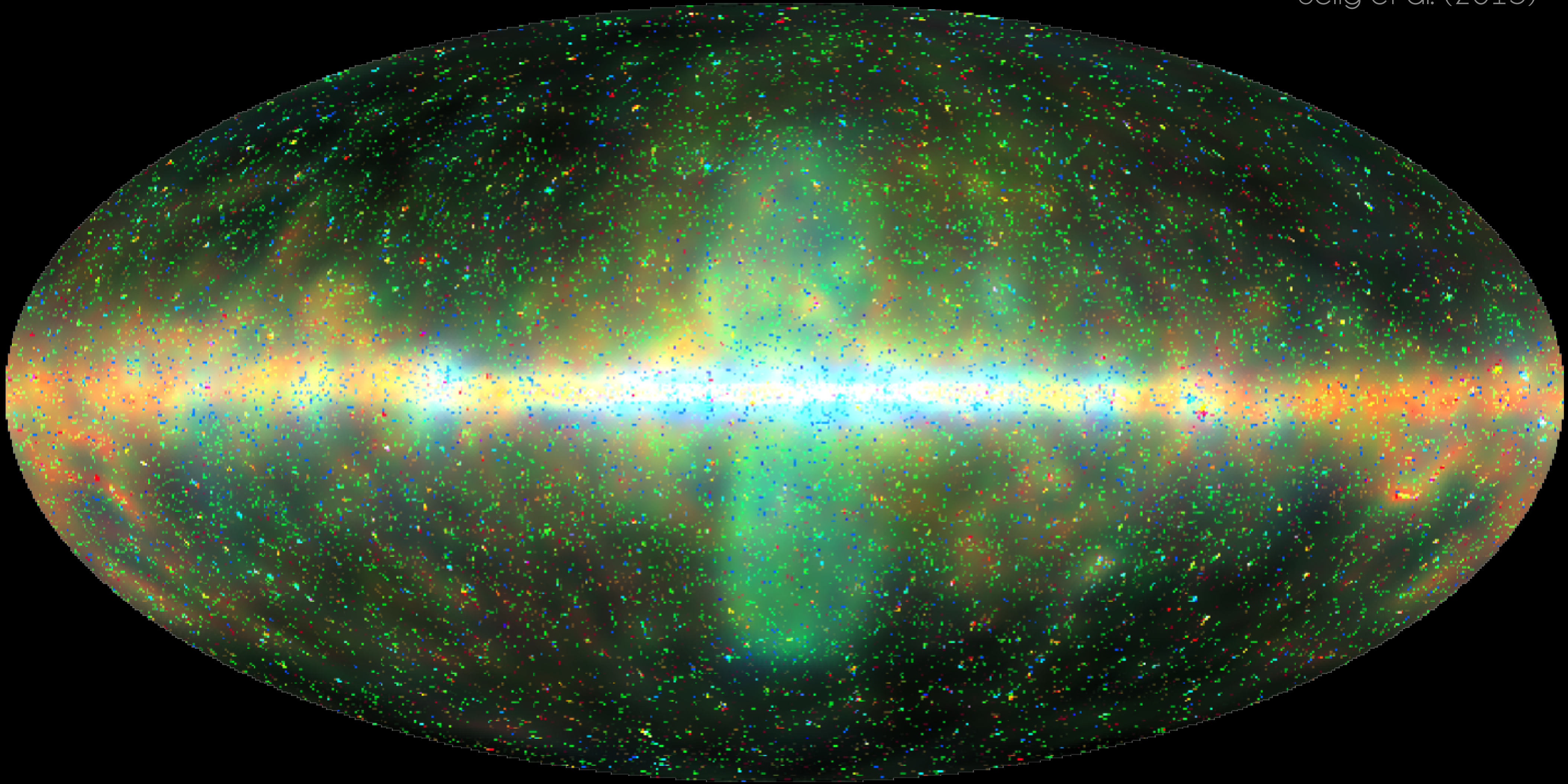
Pumpe et al. (2018)



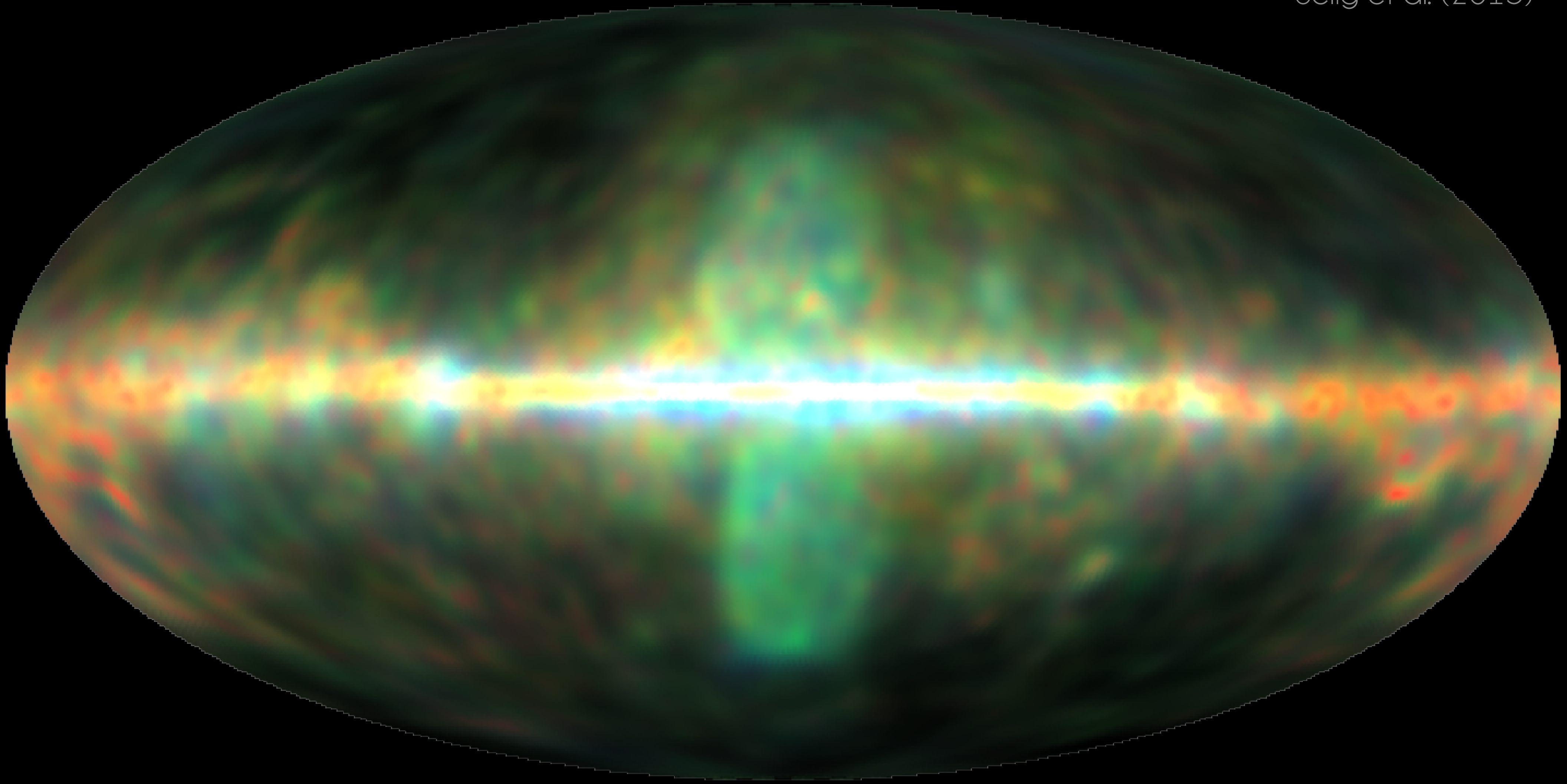

Selig et al. (2015)



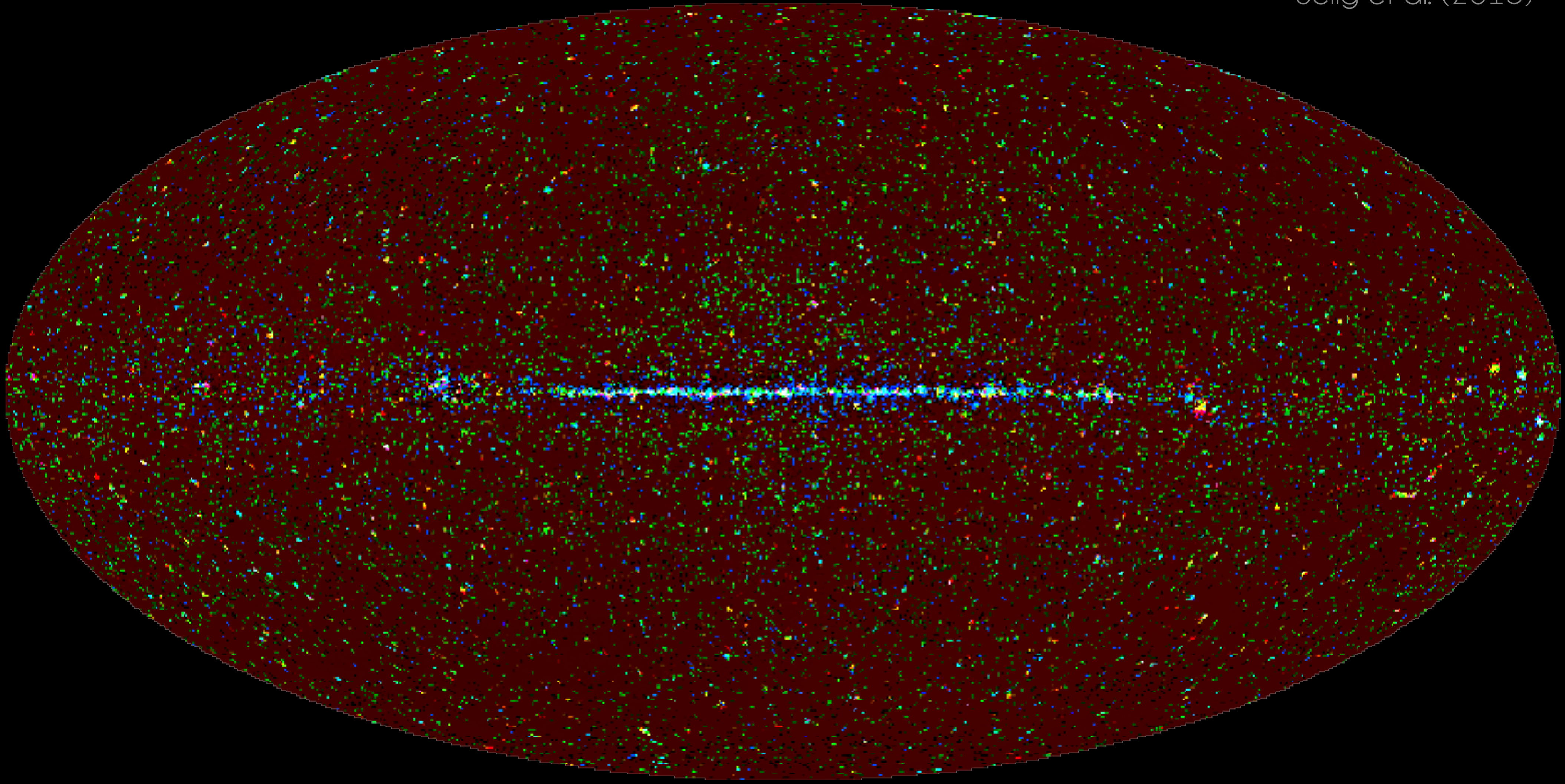
Selig et al. (2015)



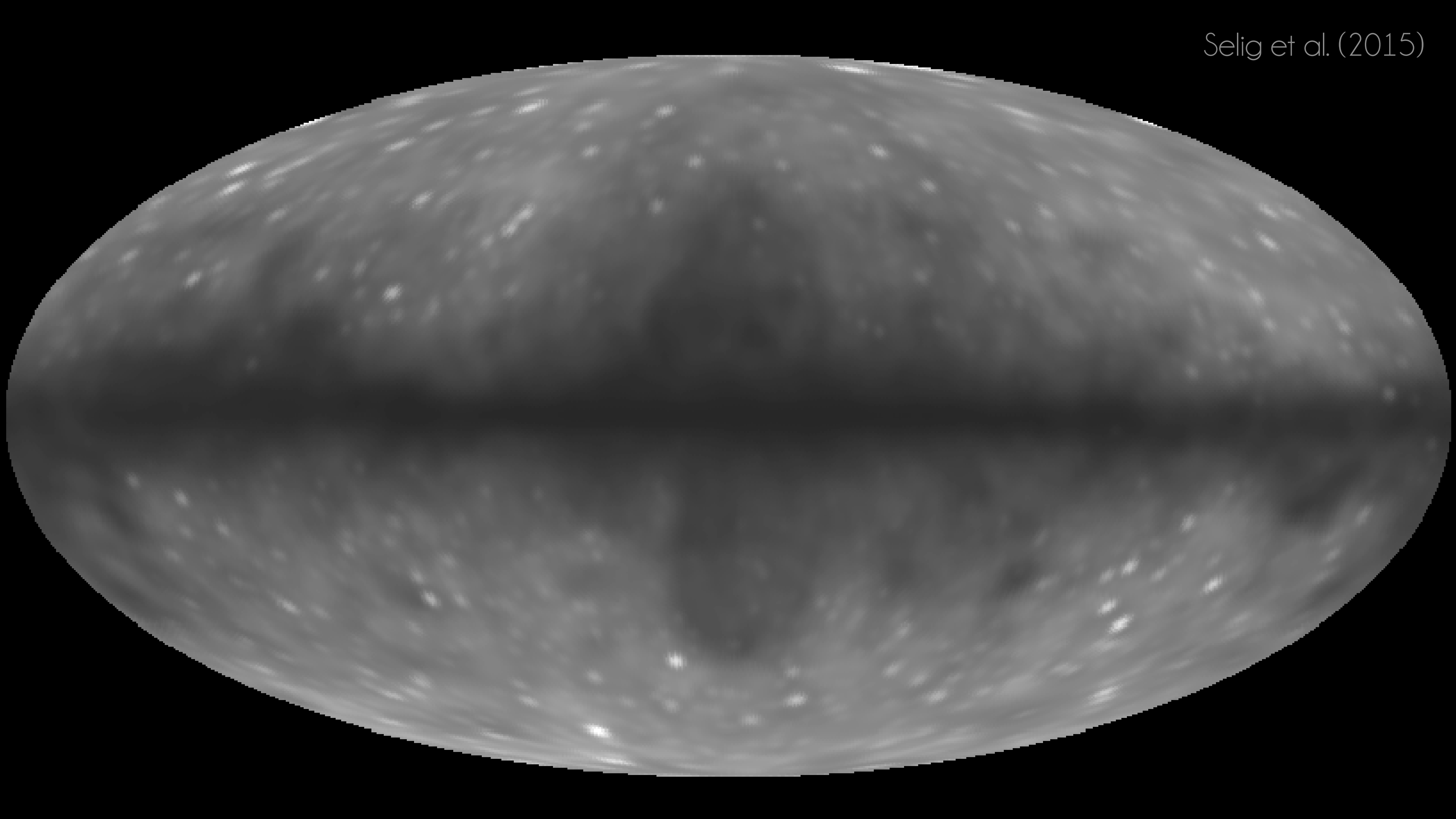
Selig et al. (2015)

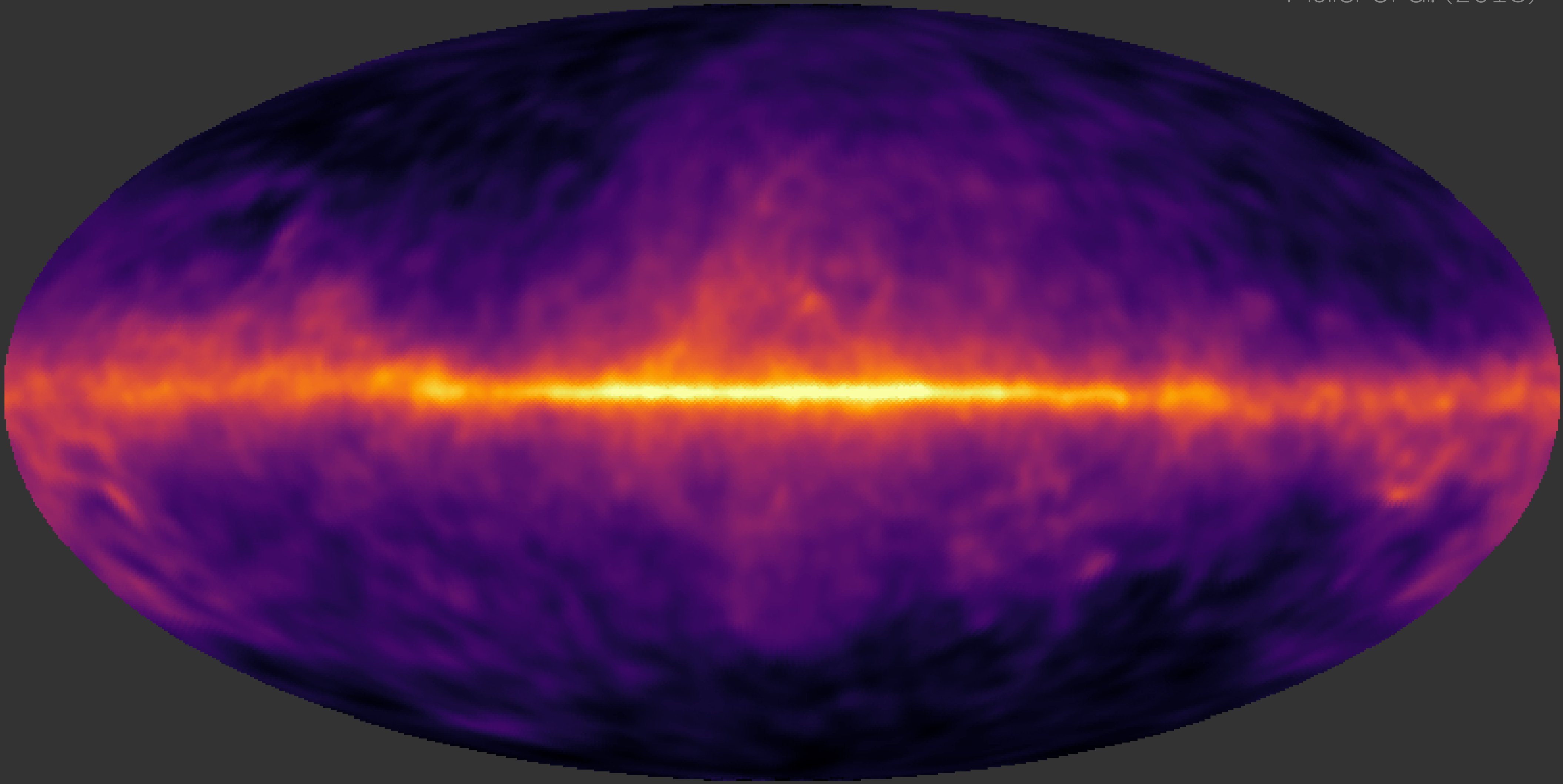


Selig et al. (2015)

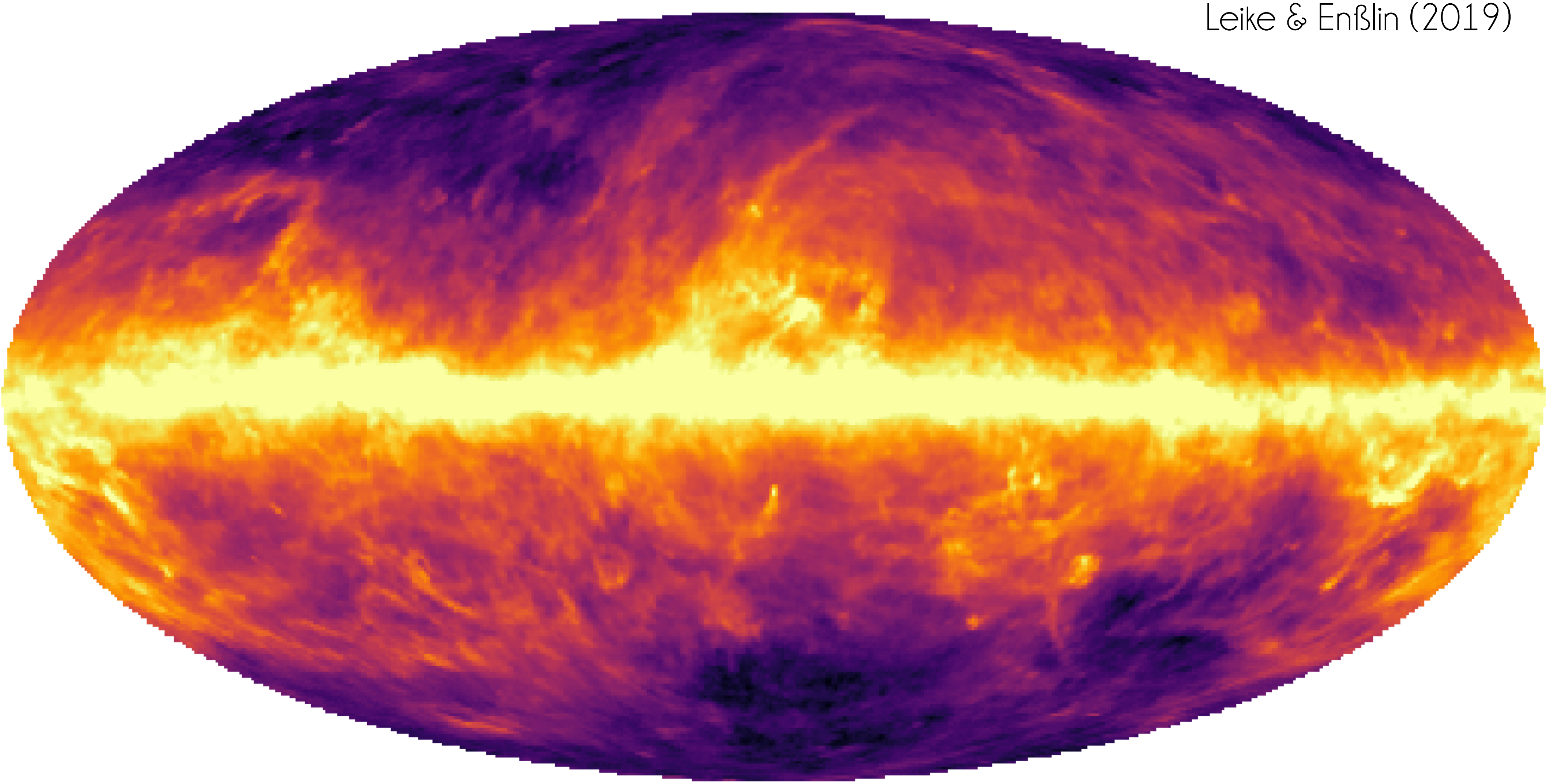


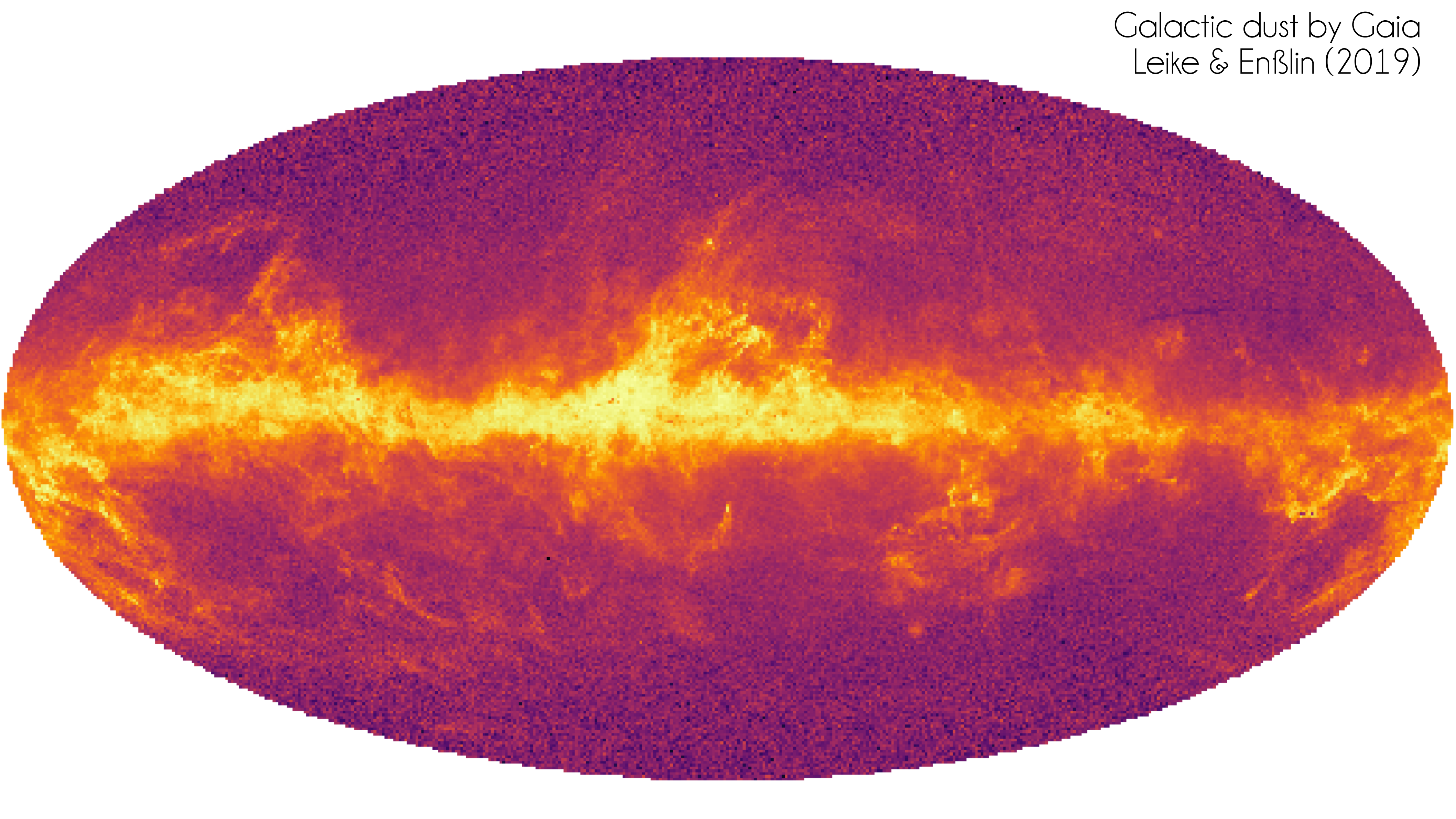
Selig et al. (2015)


Selig et al. (2015)

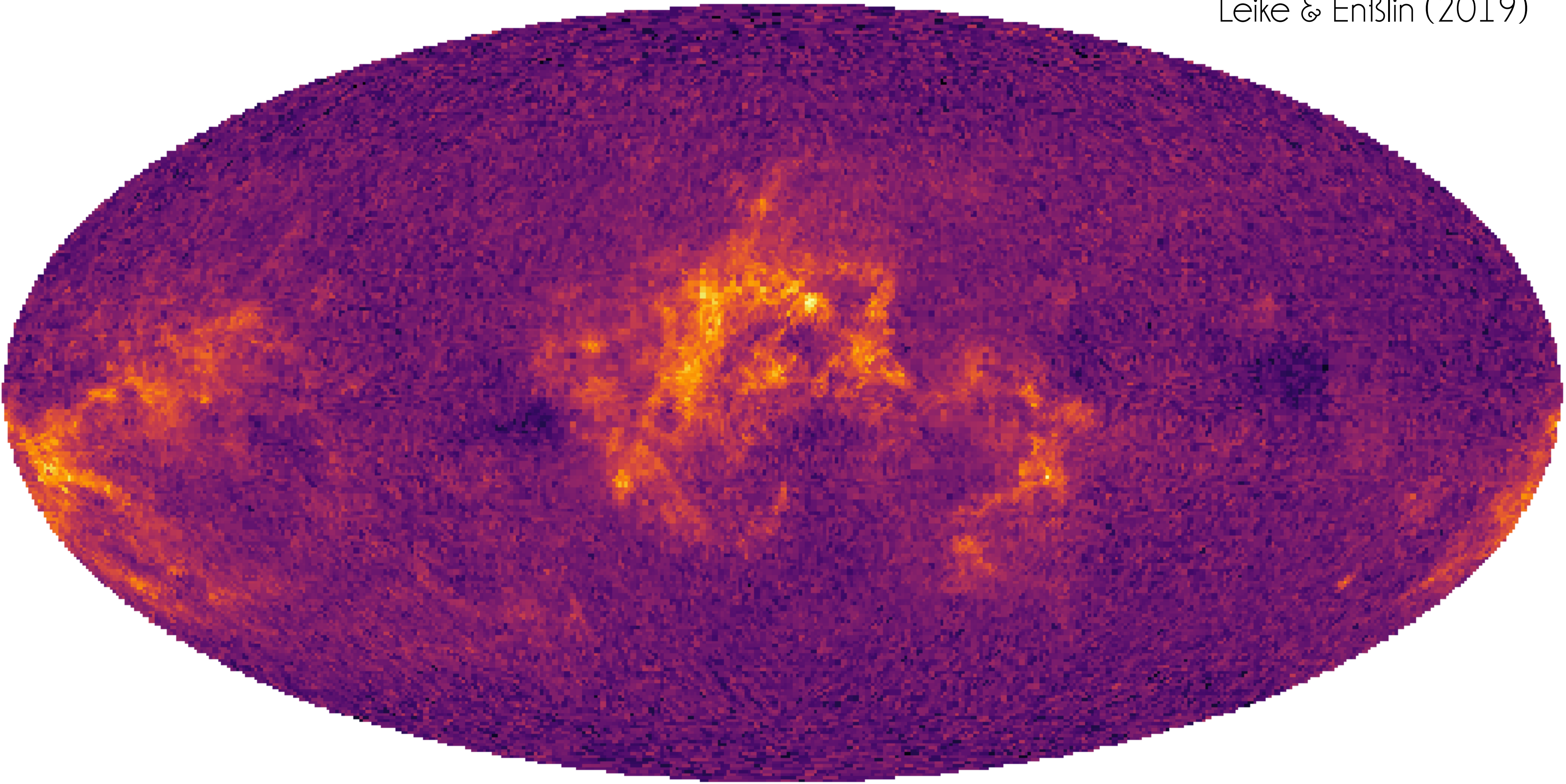


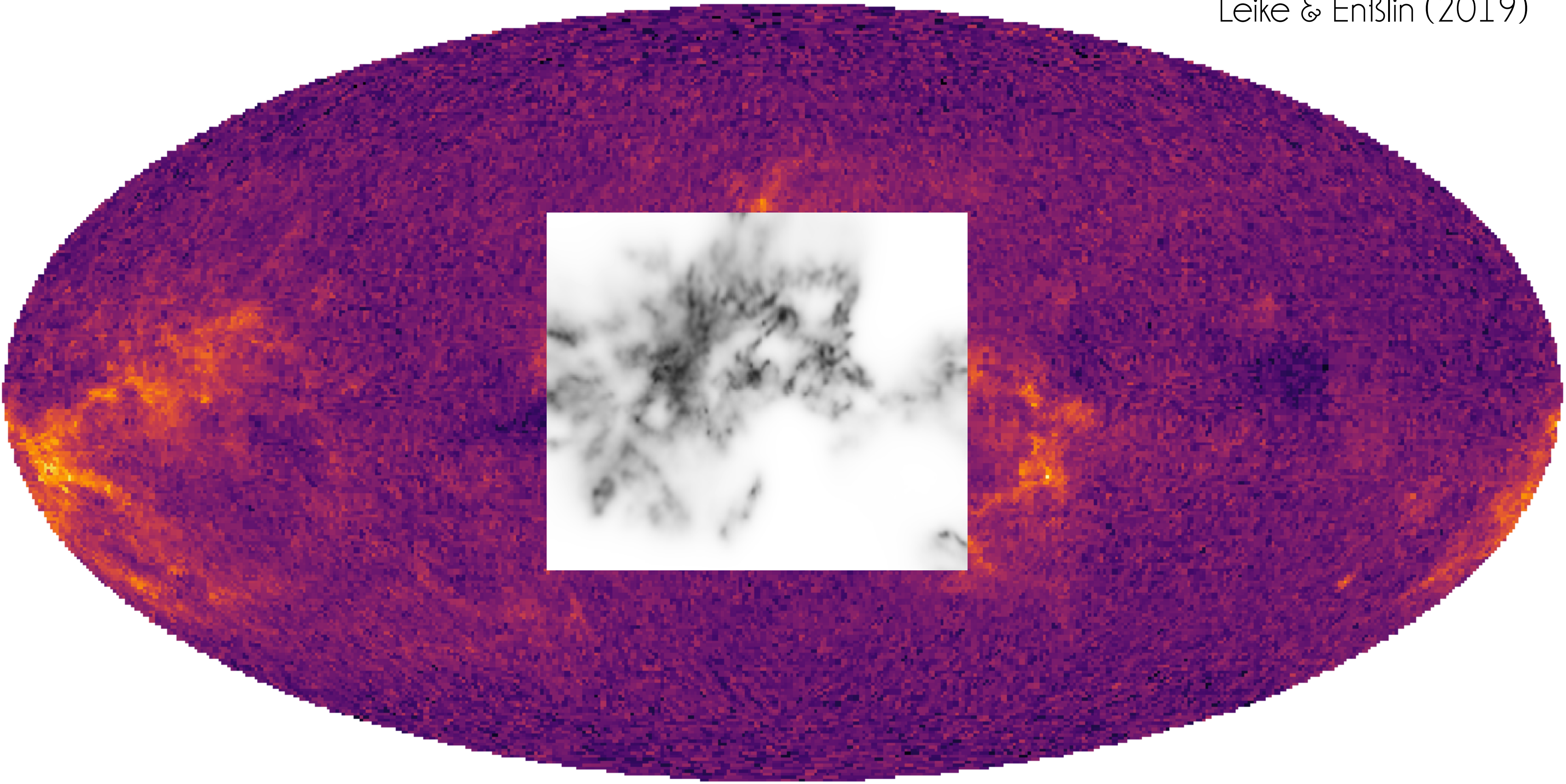
Müller et al. (2018)


Galactic dust by Planck
Leike & Enßlin (2019)


Galactic dust by Gaia
Leike & Enßlin (2019)

Galactic dust by Gaia
Leike & Enßlin (2019)

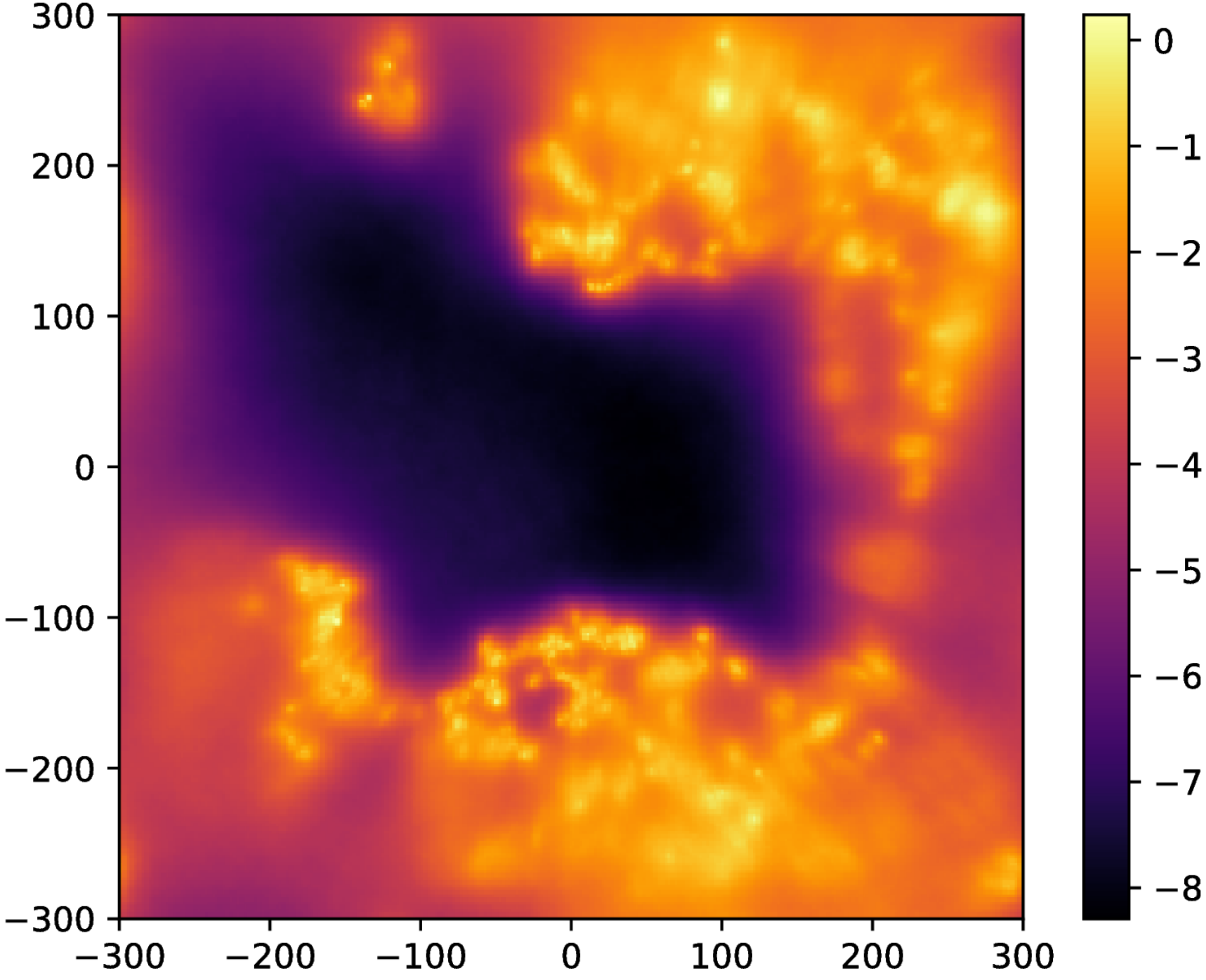




Galactic dust by Gaia
Leike & Enßlin (2019)

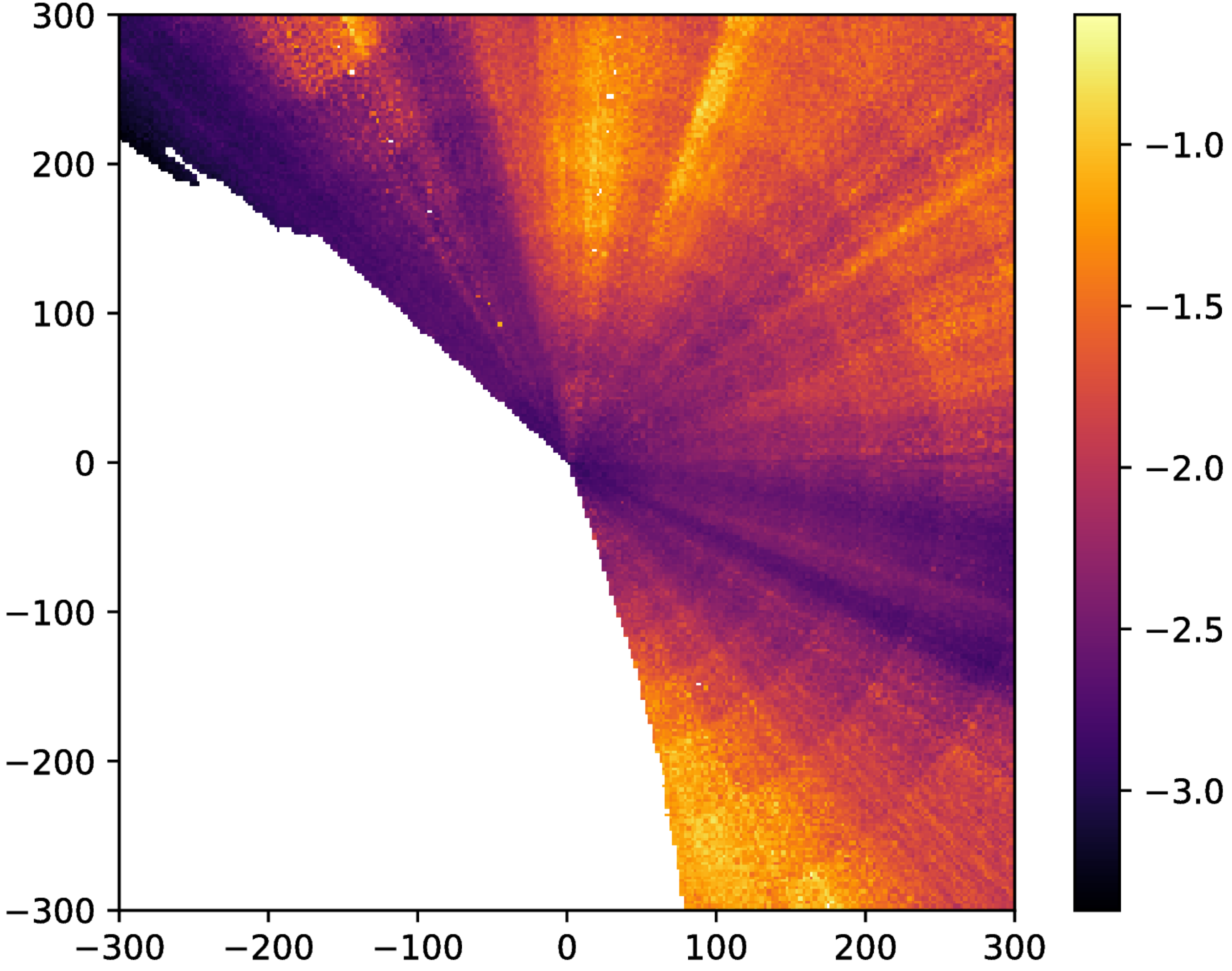
log dust density

Reimar Leike et al. (2019)



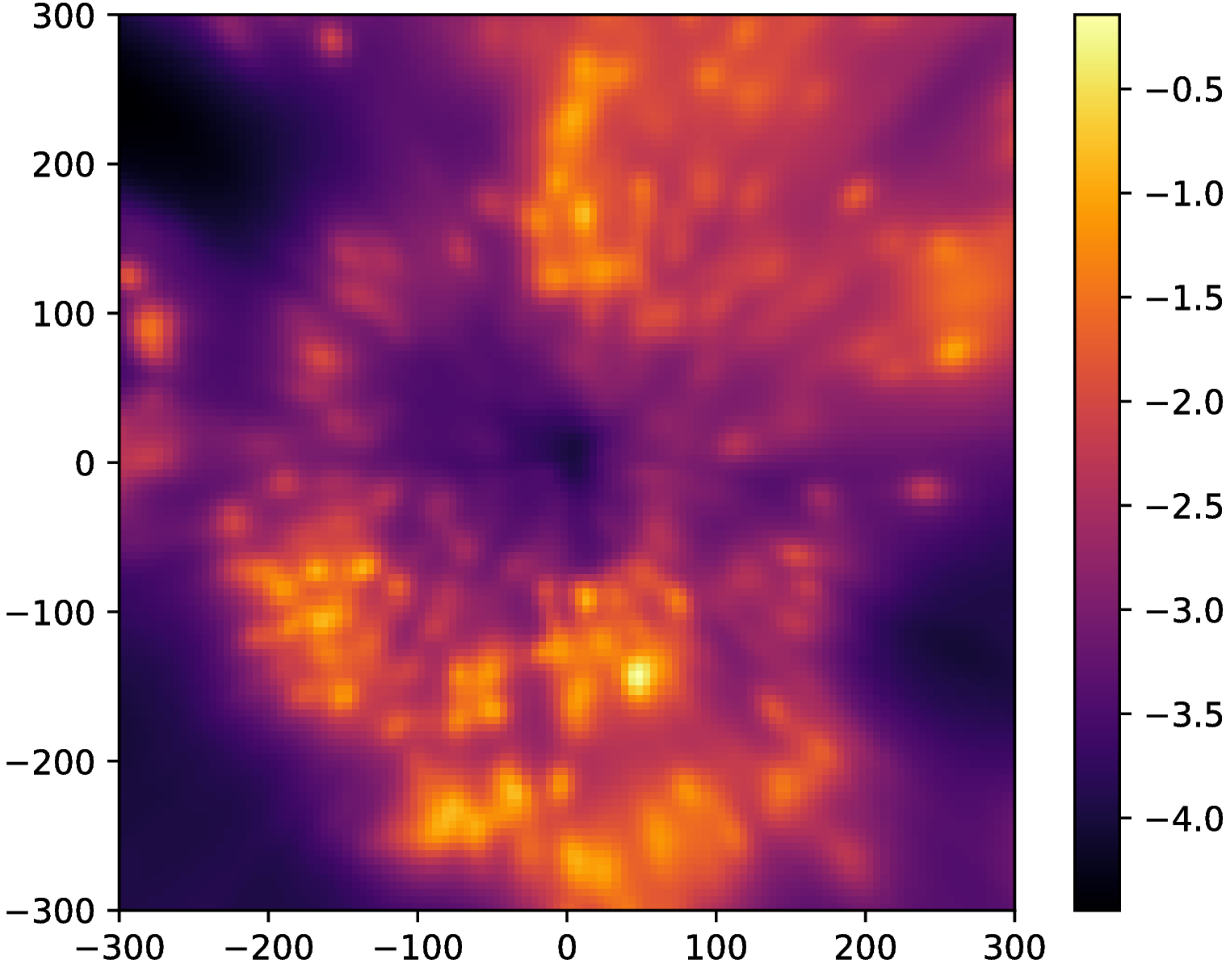
log dust density

Green et al. (2018.)



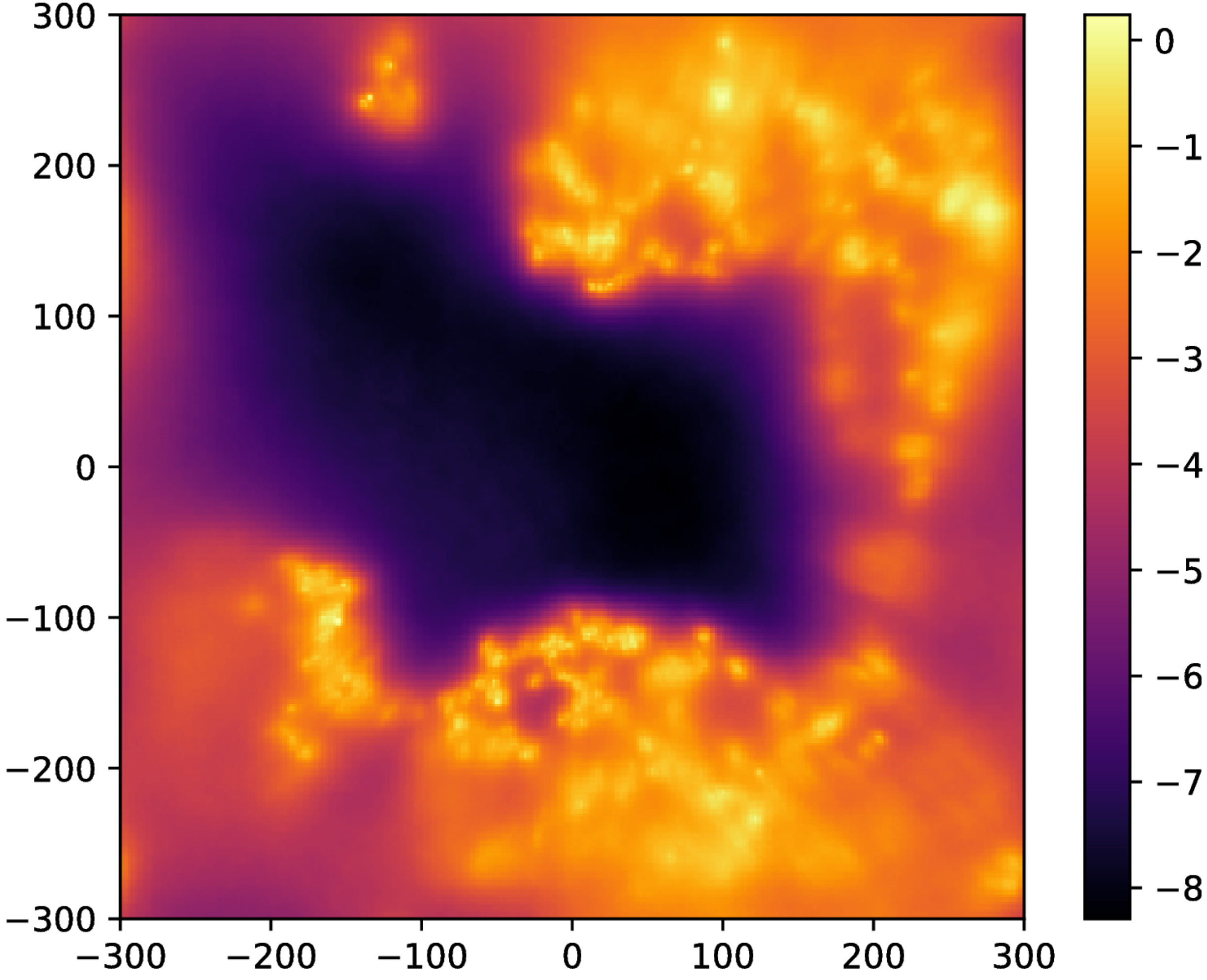
log dust density

Lallement et al. (2018)

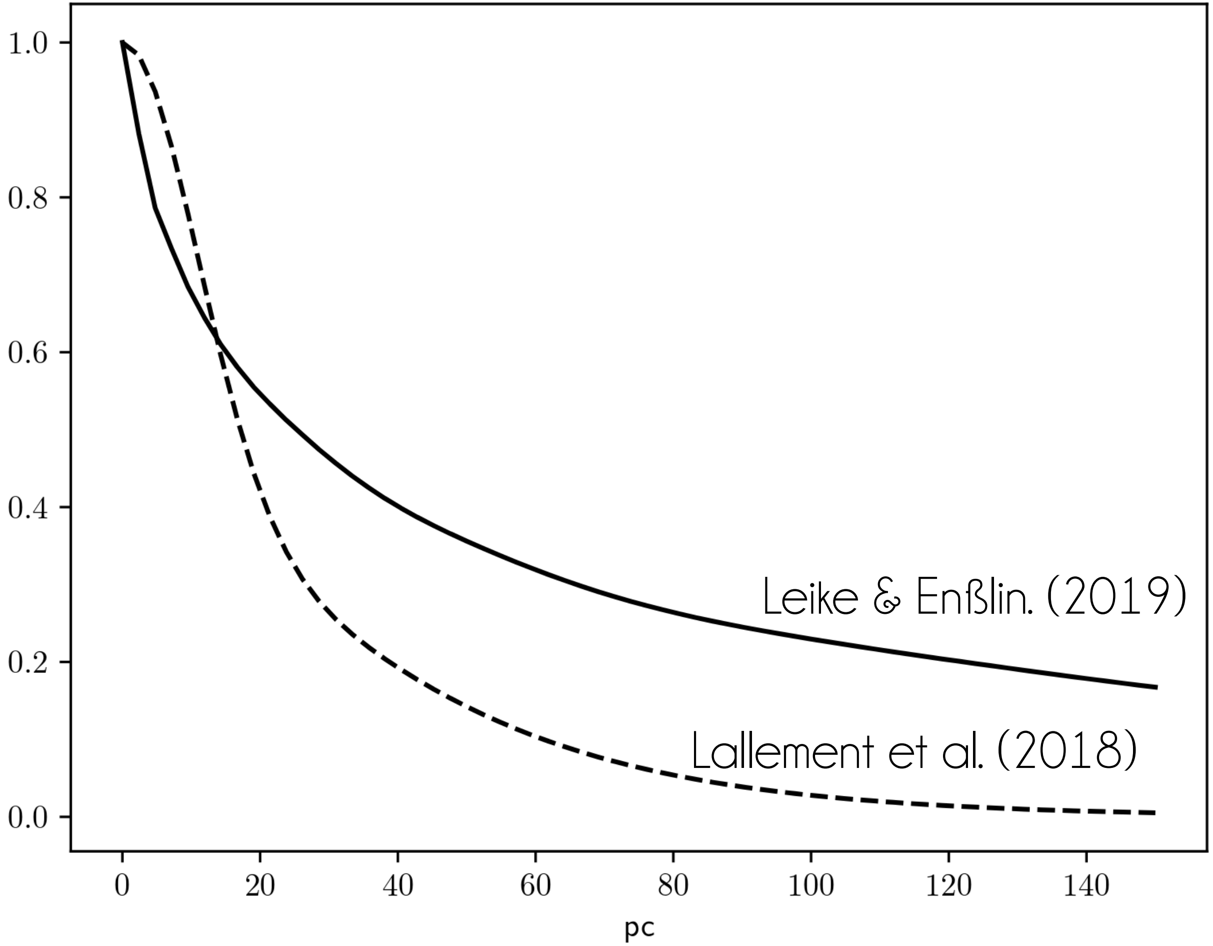


log dust density

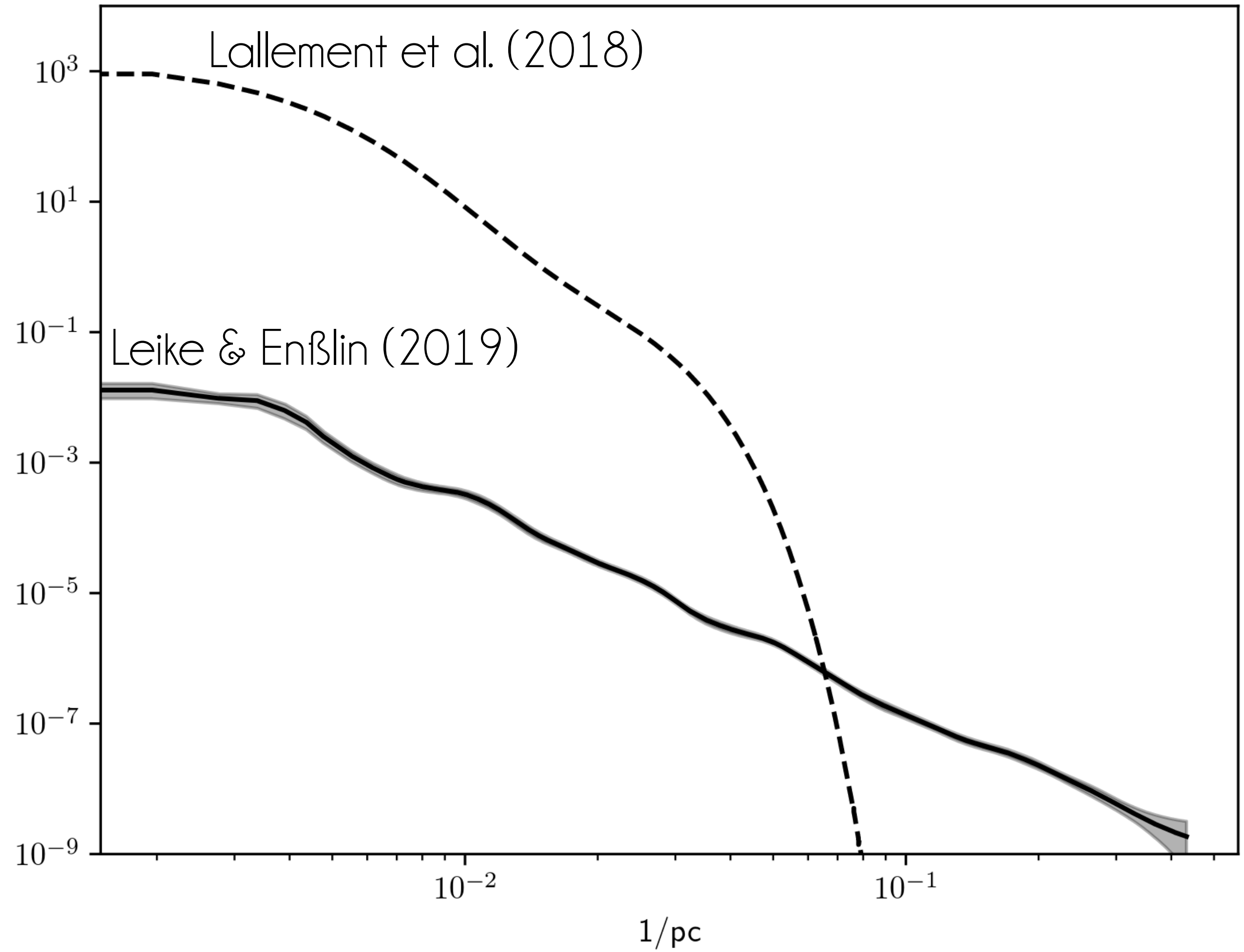
Reimar Leike et al. (2019)



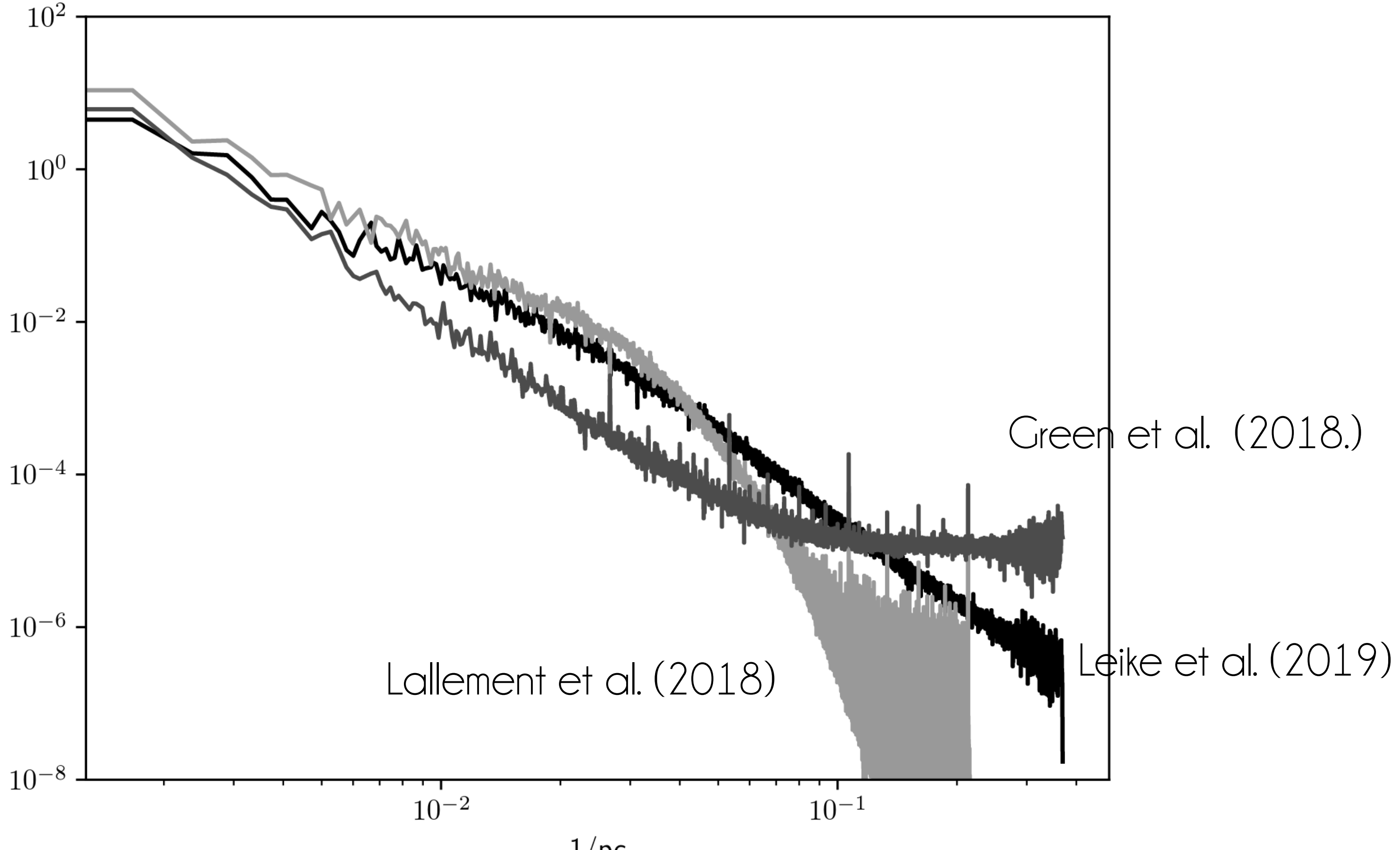
assumed 2-point correlation

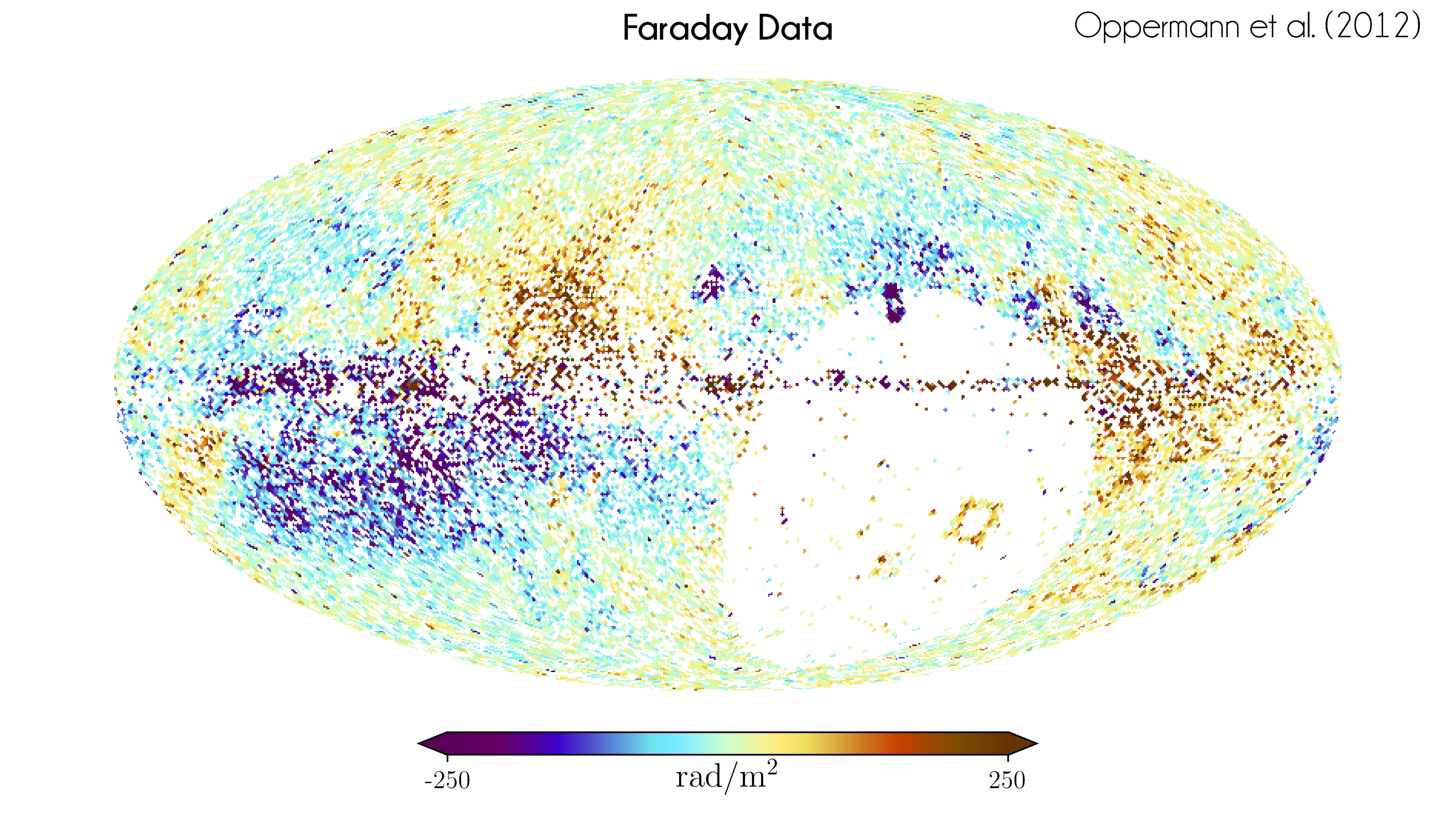


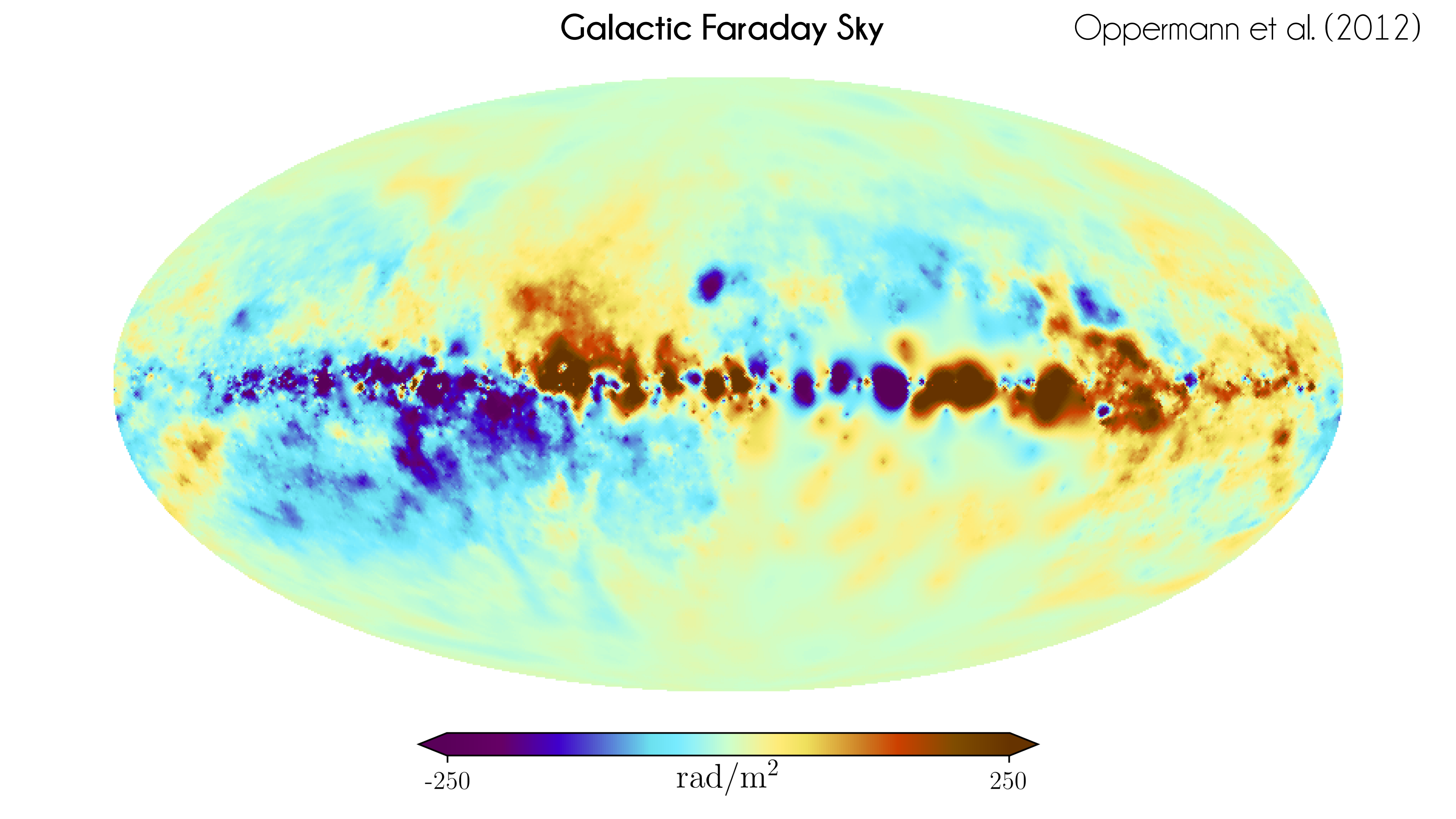
assumed power spectra

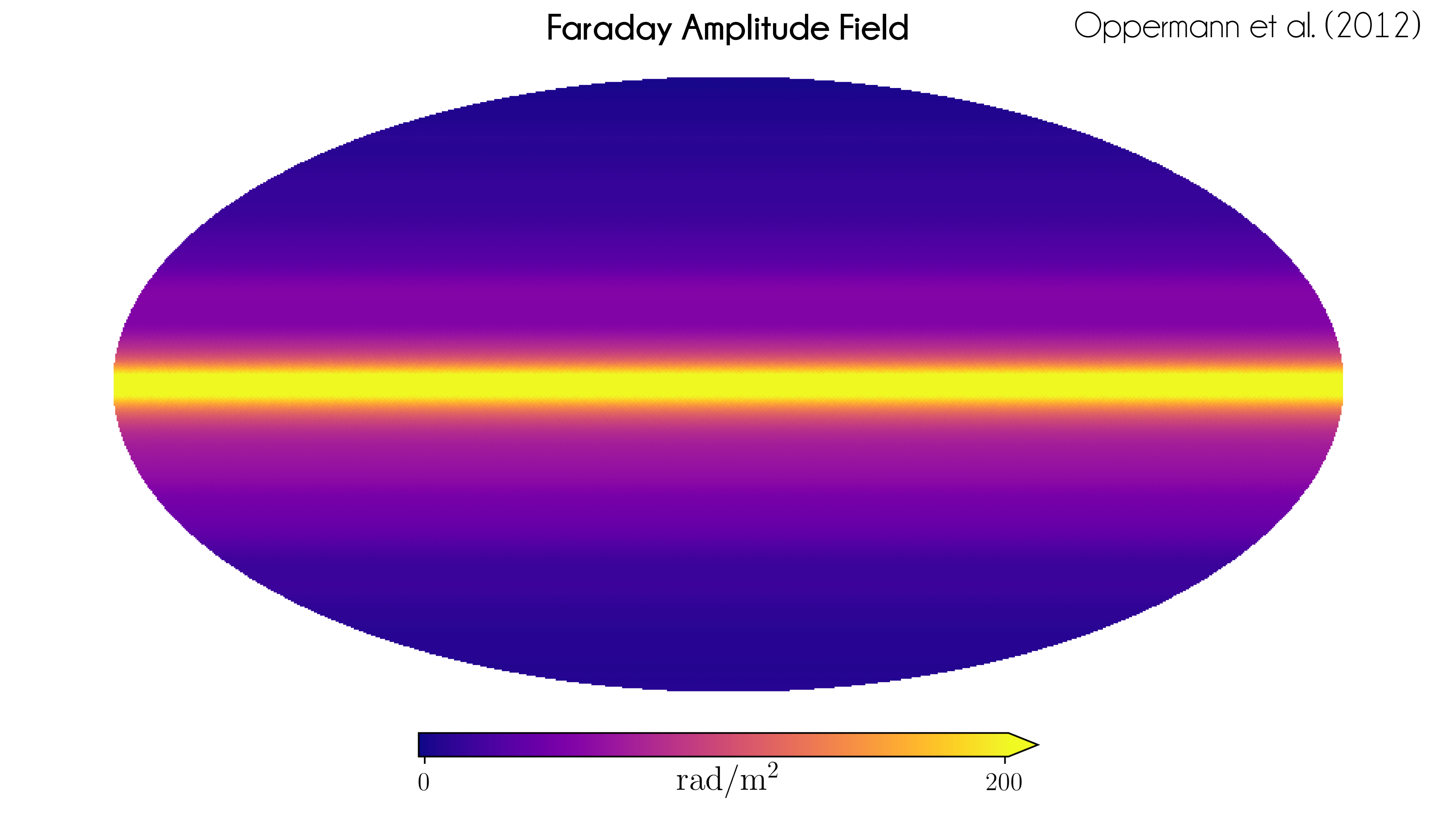


observed power spectra




Faraday Data
Oppermann et al. (2012)
-250
rad/m^2
250

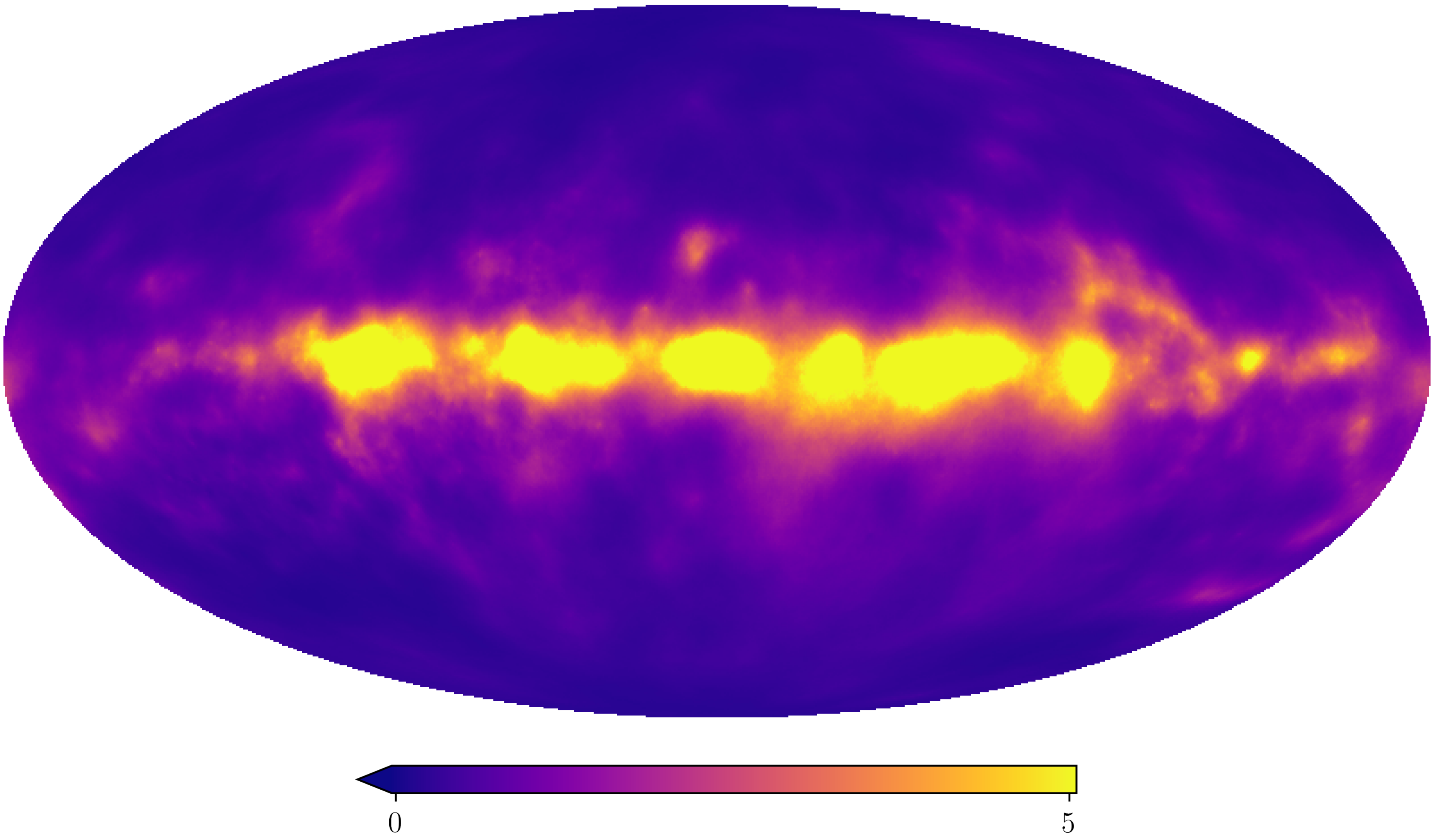

Galactic Faraday Sky
Oppermann et al. (2012)
-250
rad/m^2
250

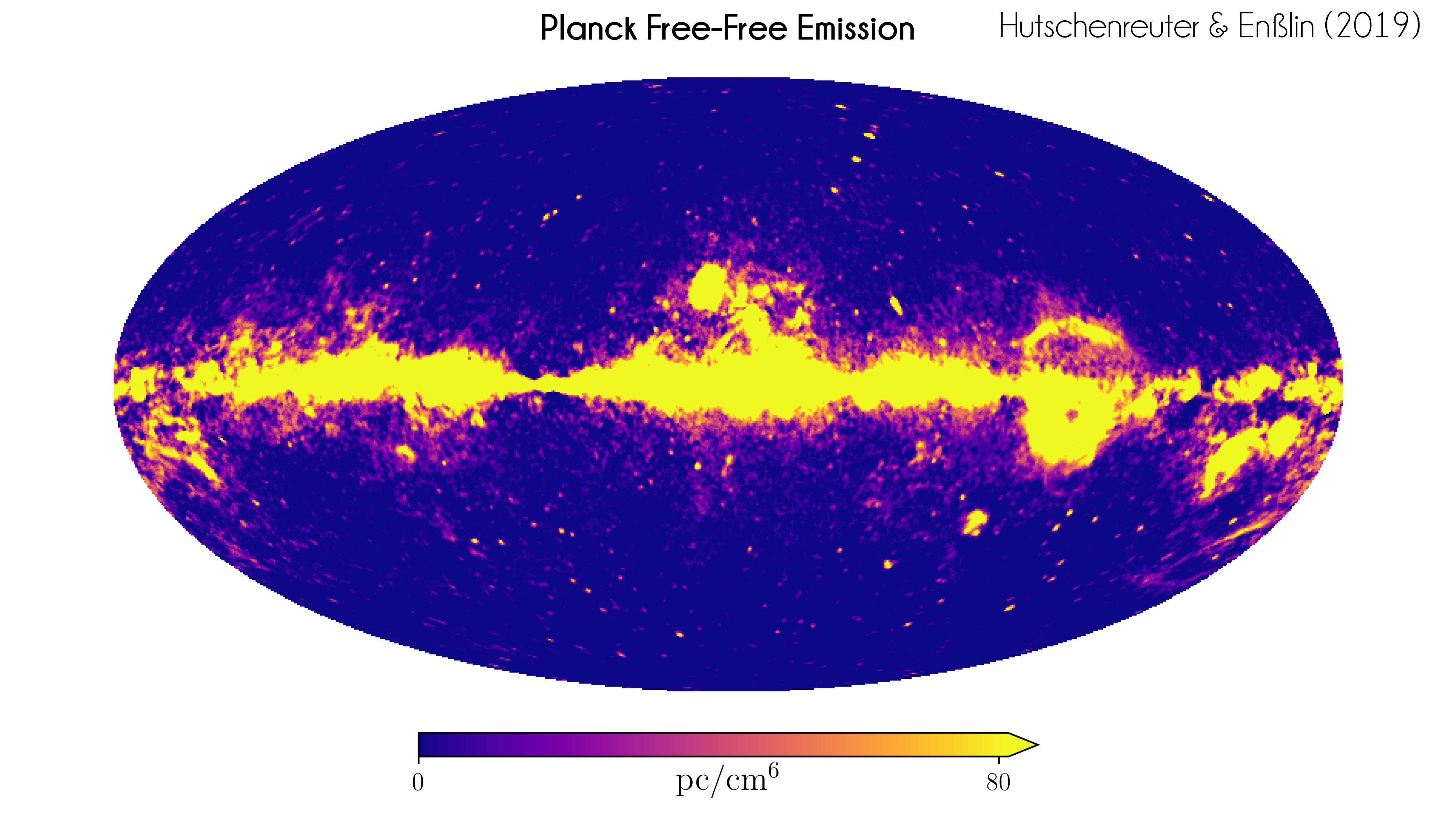


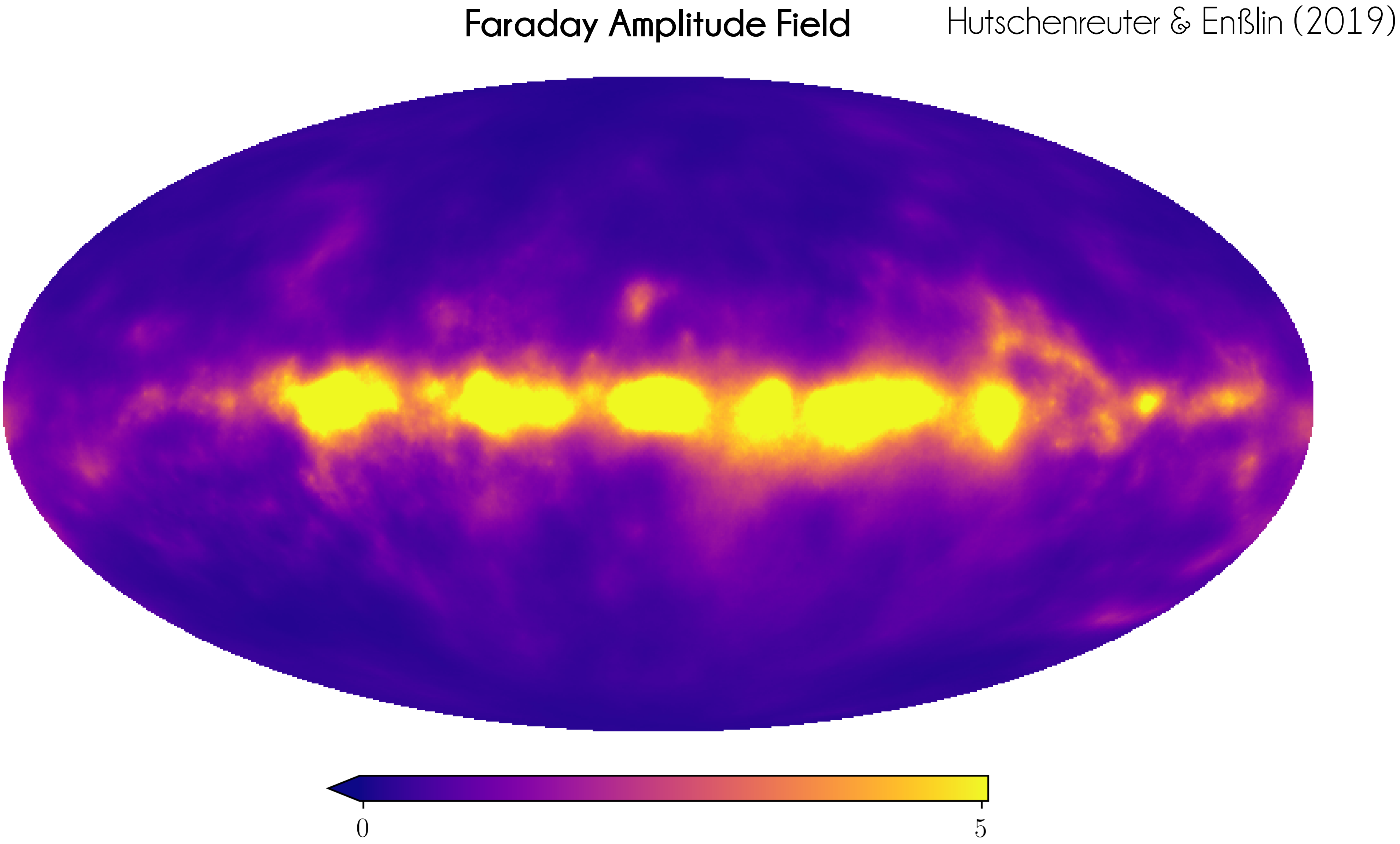
Faraday Amplitude Field
Oppermann et al. (2012)
0
rad/m^2
200

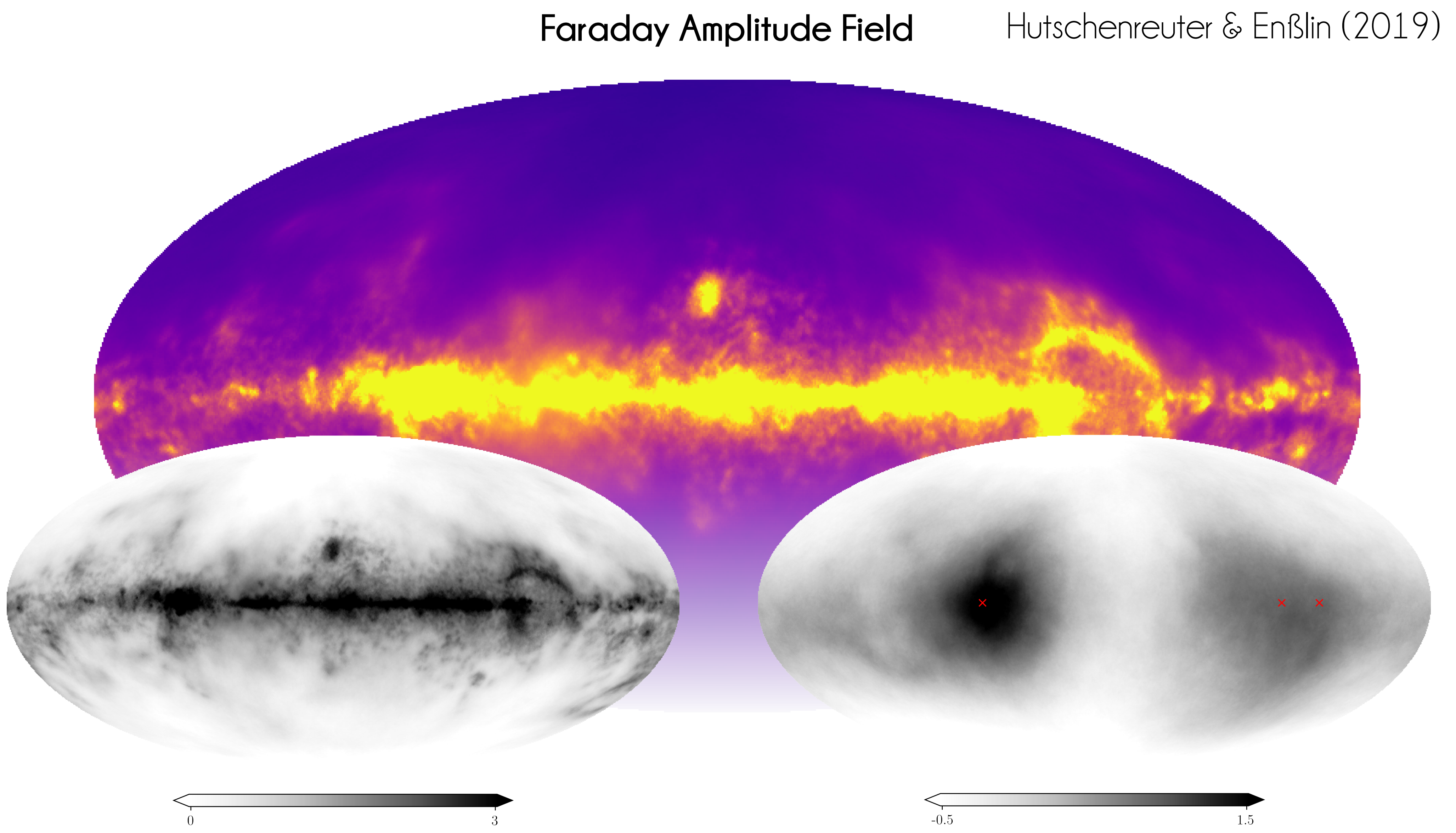
Faraday Amplitude Field

Hutschenreuter & Enßlin (2019)



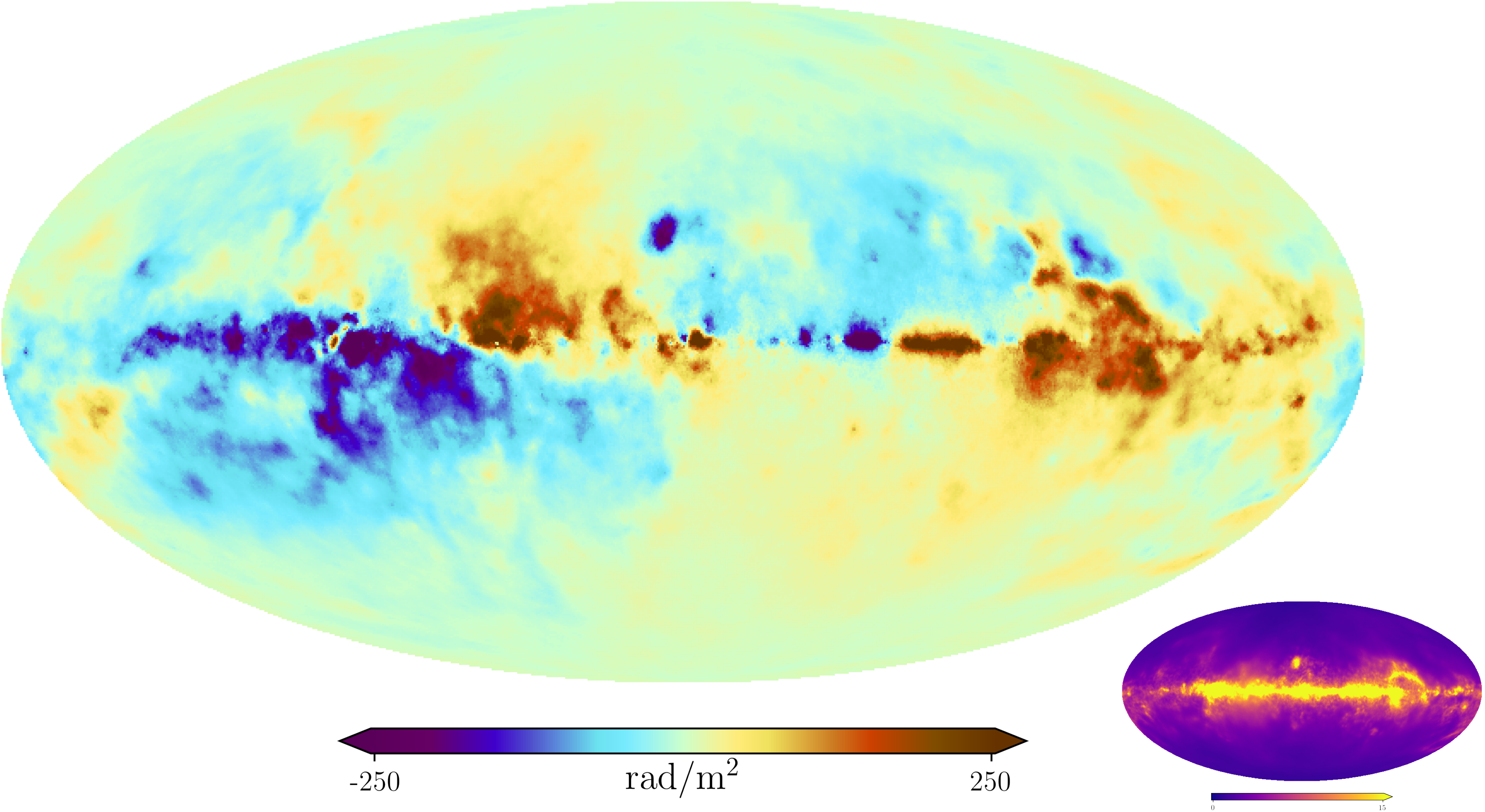

Planck Free-Free Emission
Hutschenreuter & Enßlin (2019)
0
pc/cm^6
80


Faraday Amplitude Field
Hutschenreuter & Enßlin (2019)
0
5


Faraday Amplitude Field
Hutschenreuter & Enßlin (2019)
0
3
-0.5
1.5

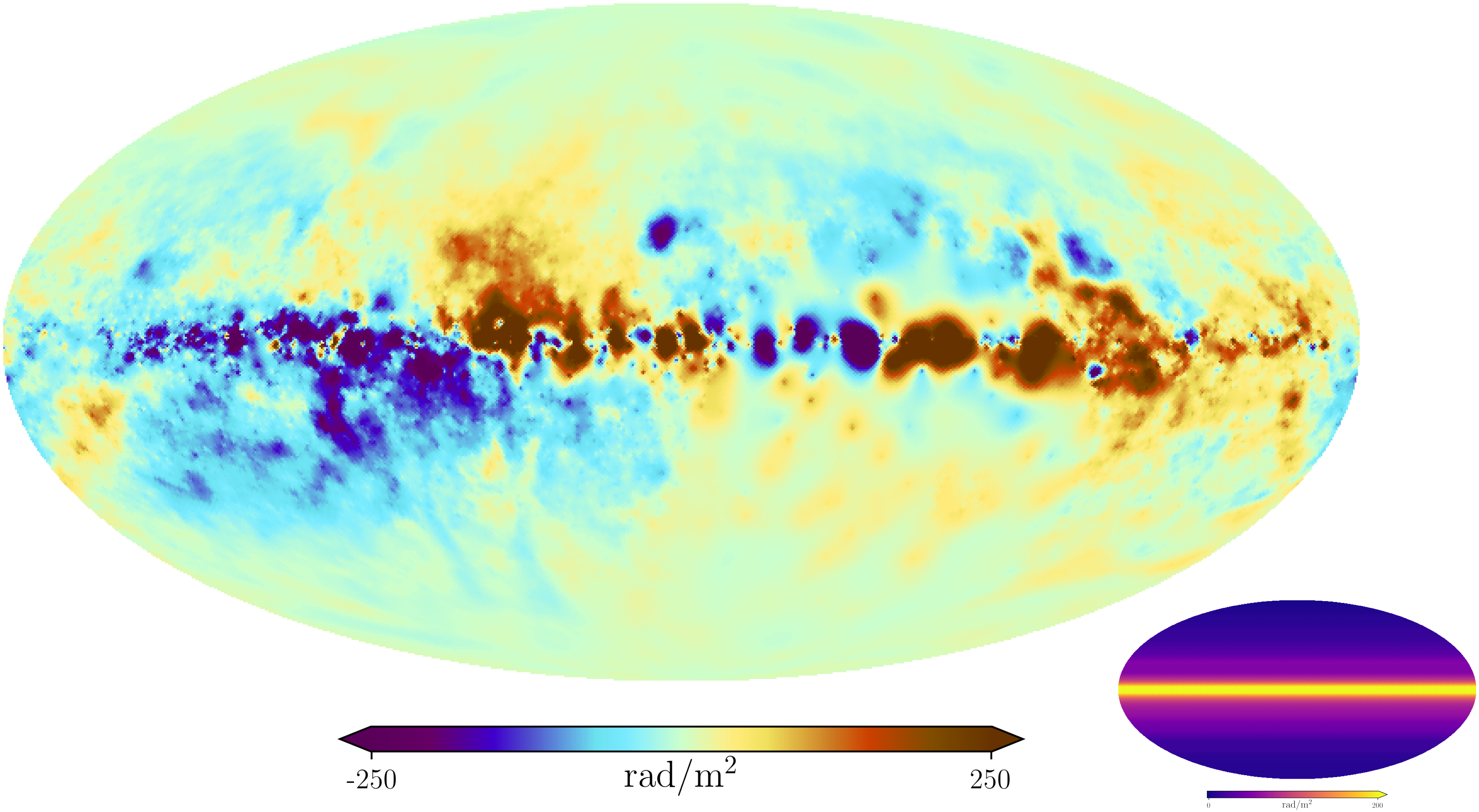
Galactic Faraday Sky

Hutschenreuter & Enßlin (2019)

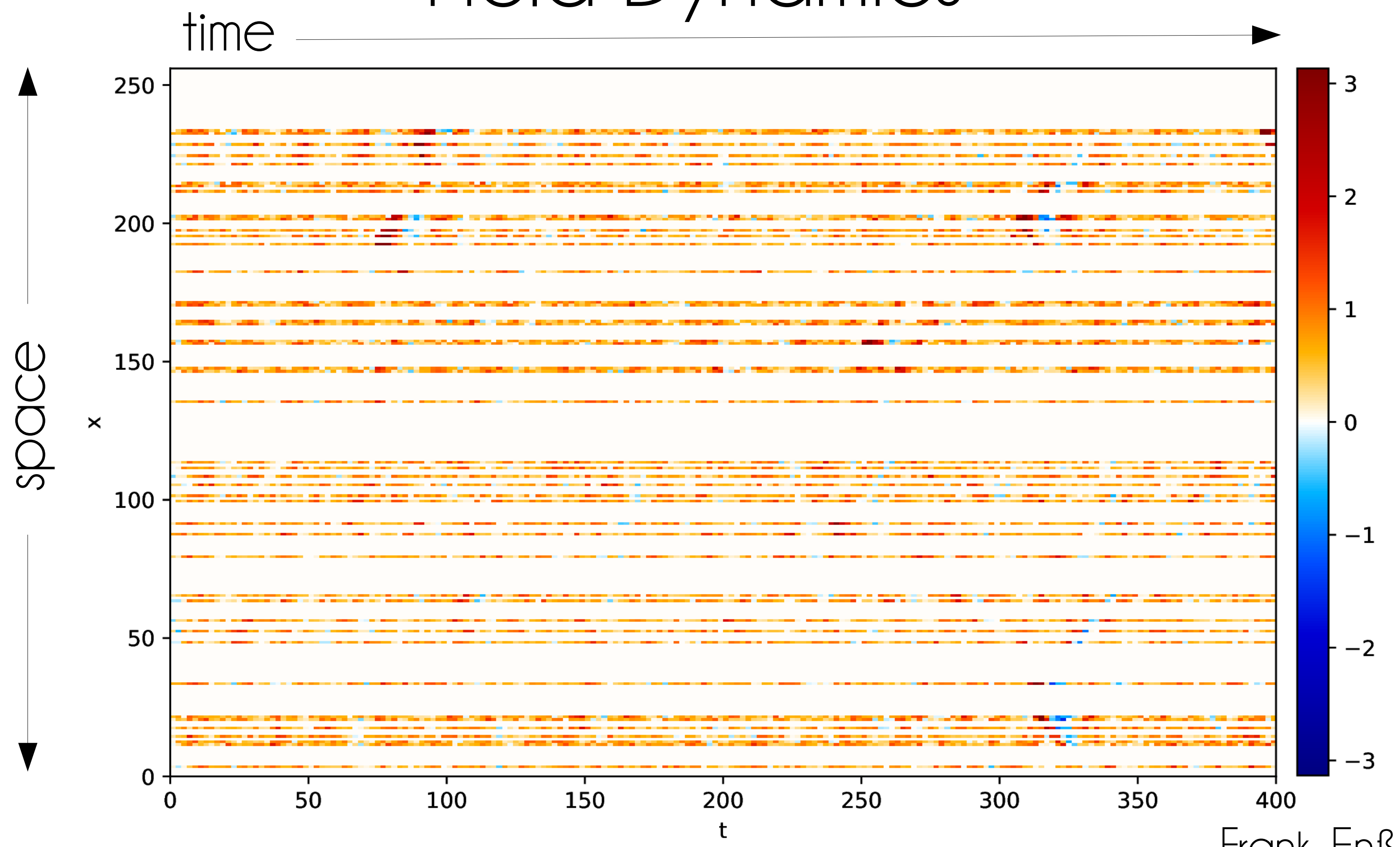


Galactic Faraday Sky

Oppermann et al. (2012)

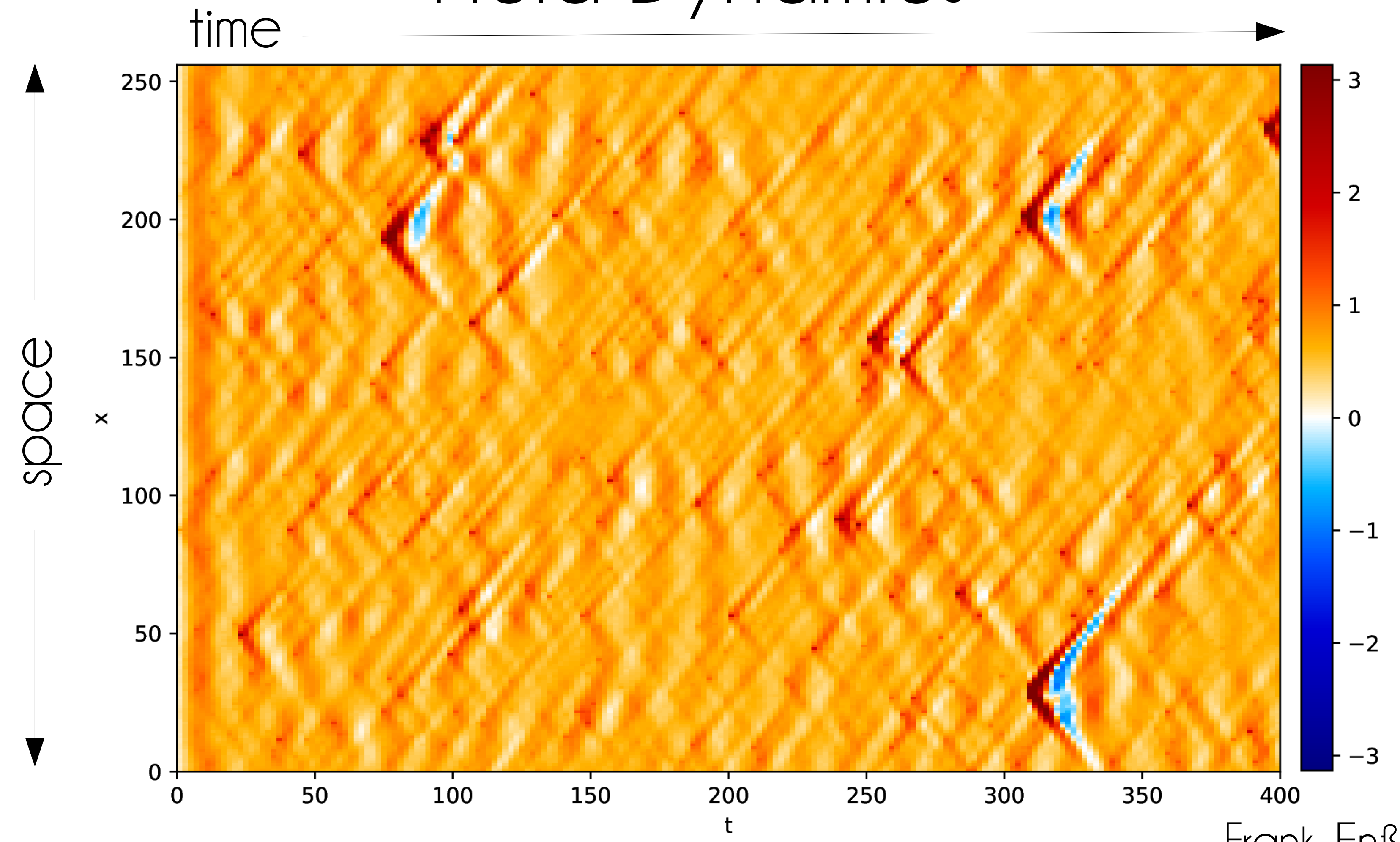


Field Dynamics



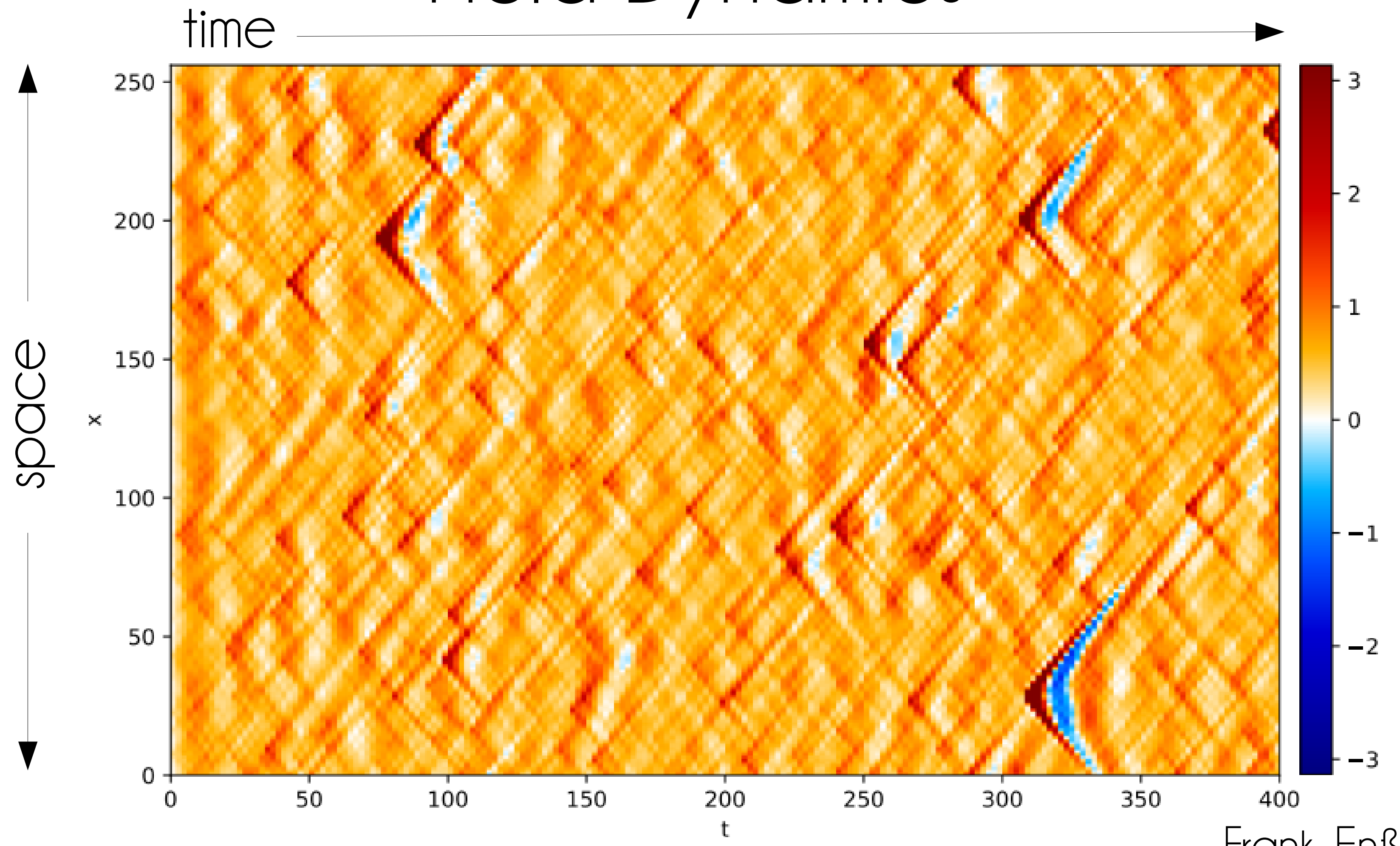
Frank, Enßlin (2019)

Field Dynamics



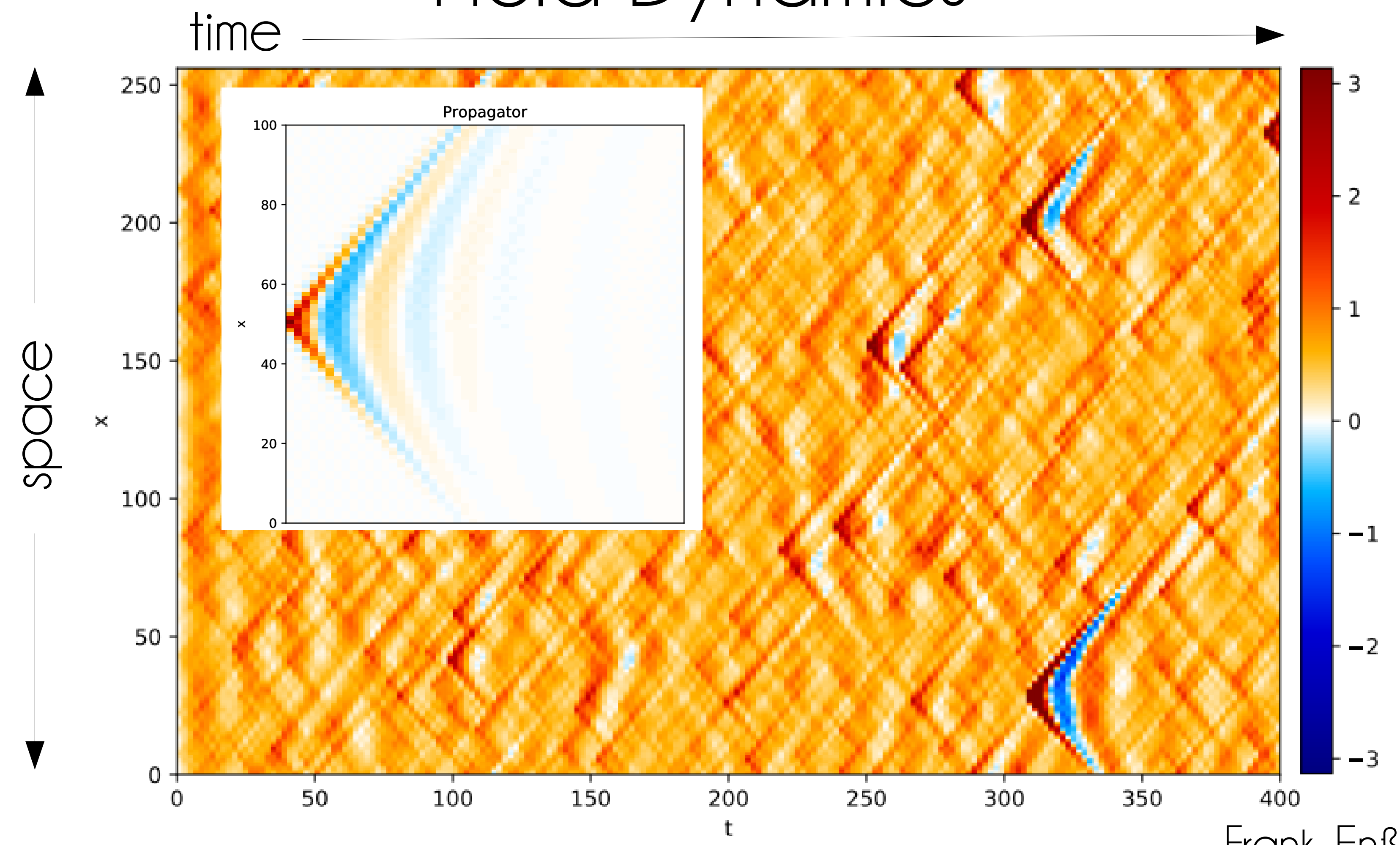
Frank, Enßlin (2019)

Field Dynamics



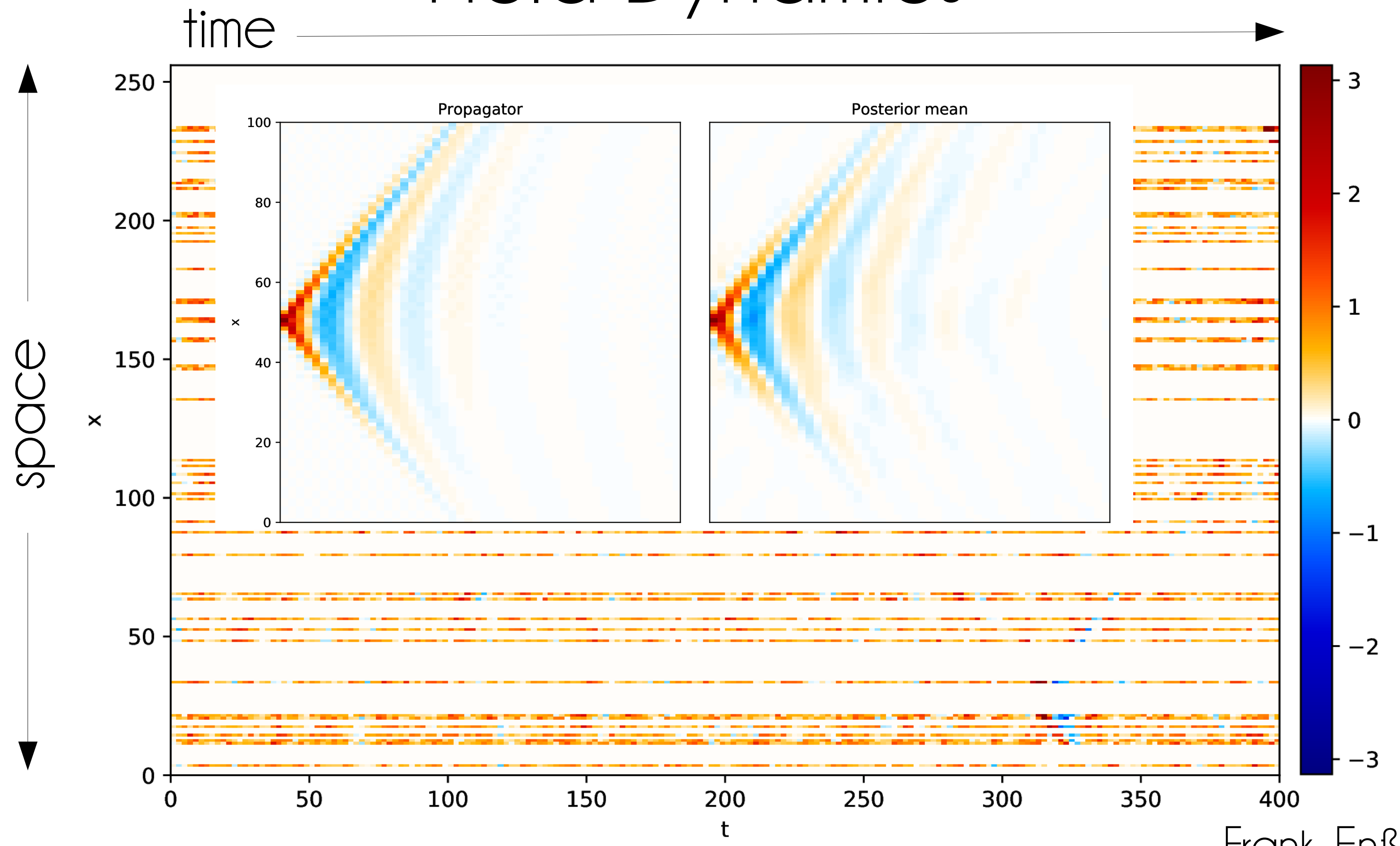
Frank, Enßlin (2019)

Field Dynamics

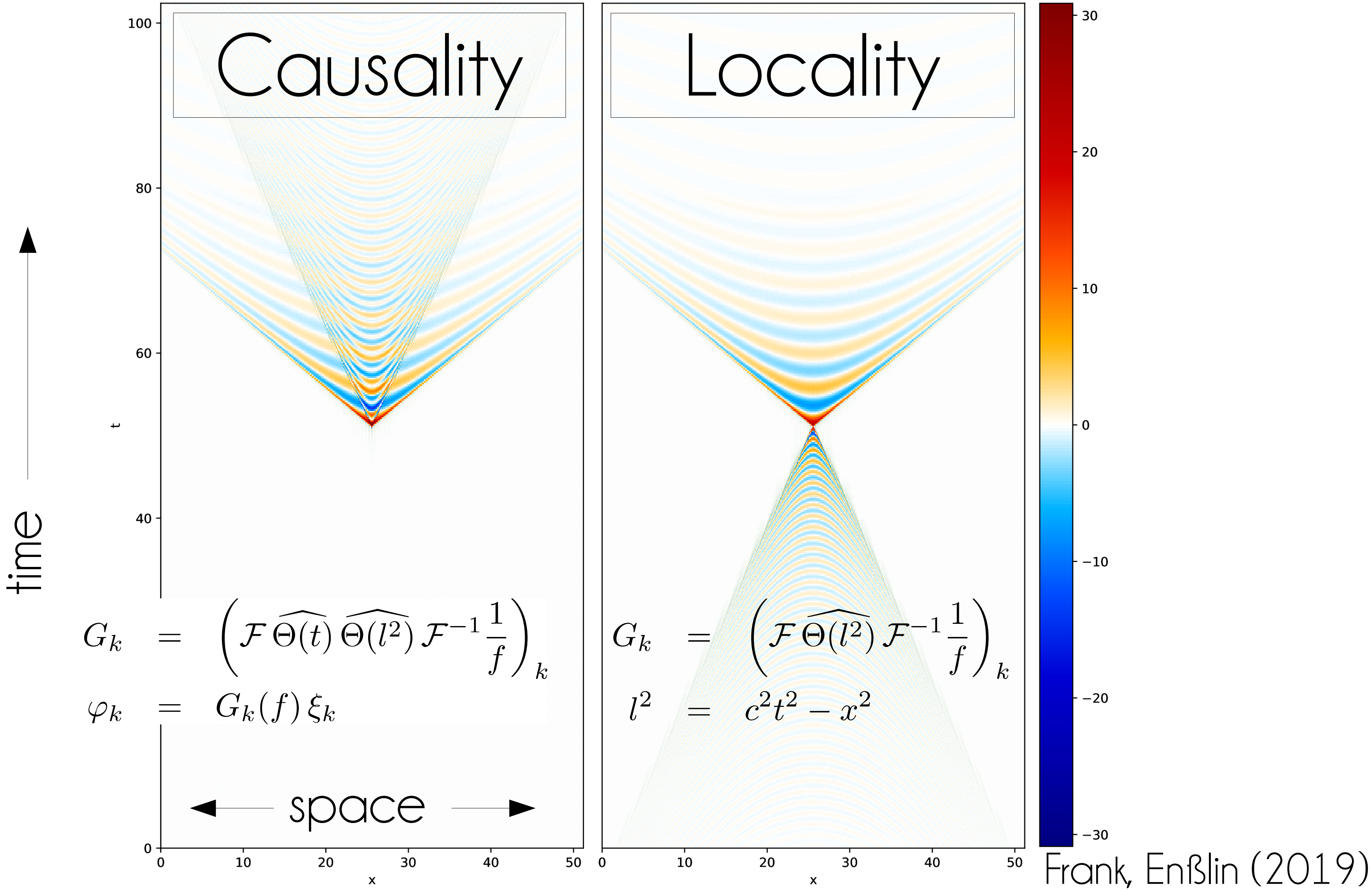


Frank, Enßlin (2019)

Field Dynamics



Frank, Enßlin (2019)

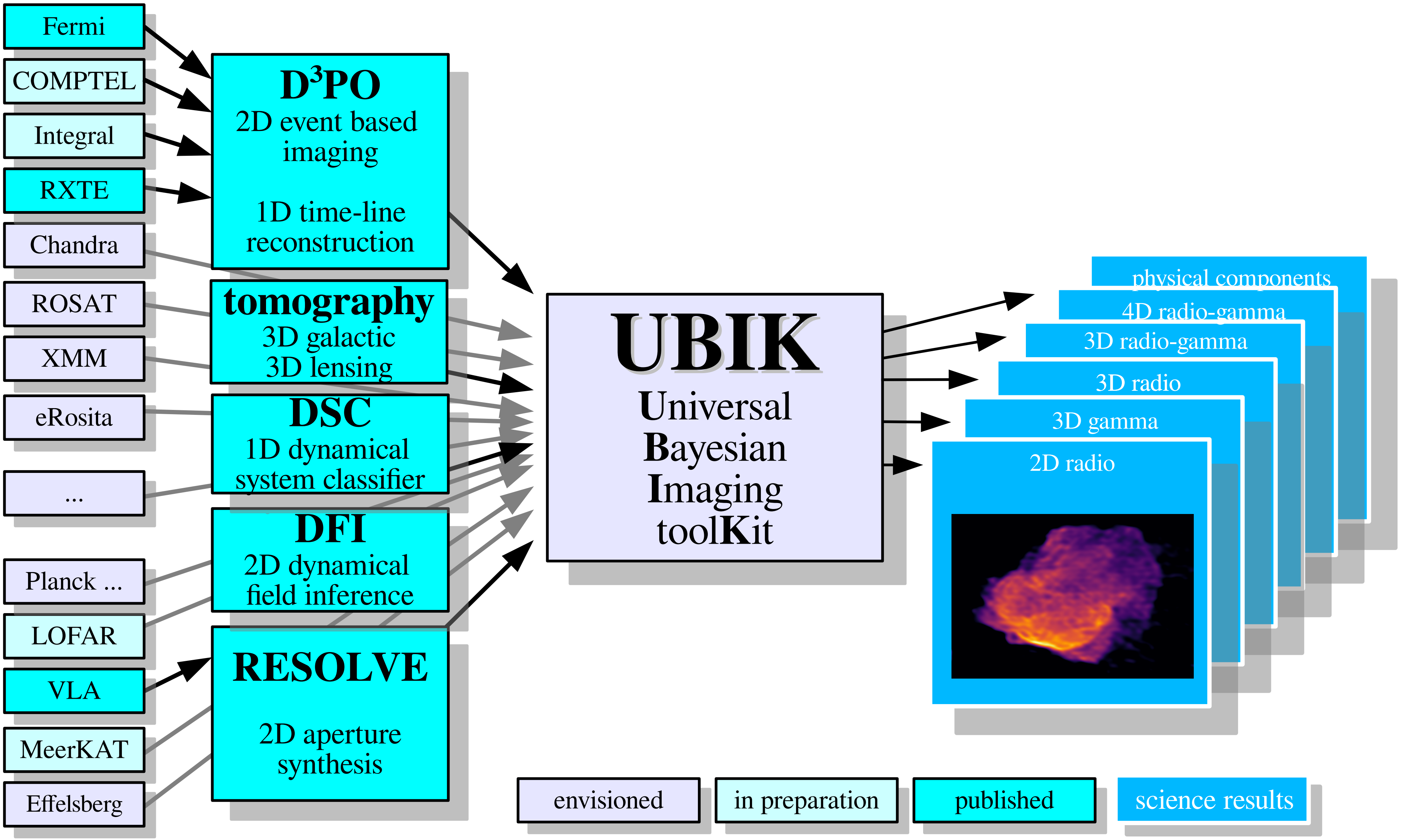


$$G_k = \left(\mathcal{F}\,\widehat{\Theta(t)}\,\widehat{\Theta(l^2)}\,\mathcal{F}^{-1}\frac{1}{f}\right)_k$$

$$\varphi_k = G_k(f)\,\xi_k$$

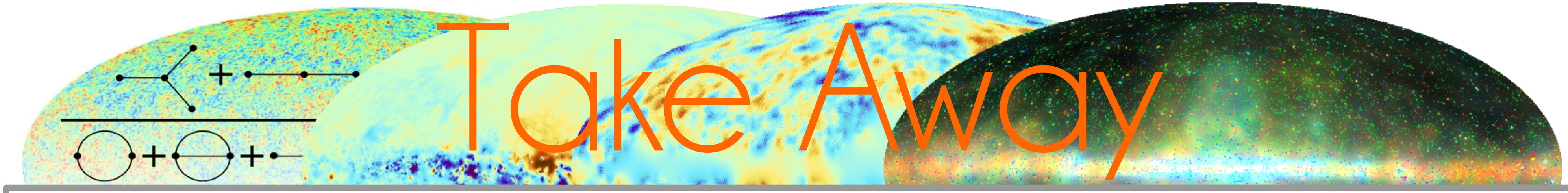
$$G_k = \left(\mathcal{F}\,\widehat{\Theta(l^2)}\,\mathcal{F}^{-1}\frac{1}{f}\right)_k$$

$$l^2 = c^2t^2 - x^2$$



Fermi
COMPTEL
Integral
RXTE
Chandra
ROSAT
XMM
eRosita
...
Planck ...
LOFAR
VLA
MeerKAT
Effelsberg
D3PO
2D event based imaging
1D time-line reconstruction
tomography
3D galactic
3D lensing
DSC
1D dynamical system classifier
DFI
2D dynamical field inference
RESOLVE
2D aperture synthesis
UBIK
Universal Bayesian Imaging toolKit
physical components
4D radio-gamma
3D radio-gamma
3D radio
3D gamma
2D radio
envisioned
in preparation
published
science results

Take Away

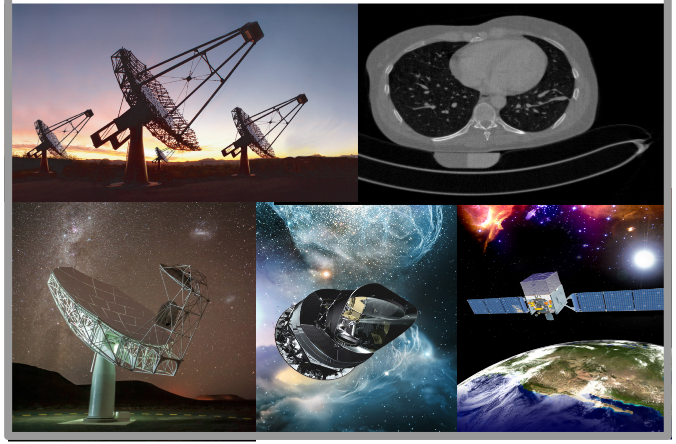


IFT = Information field theory

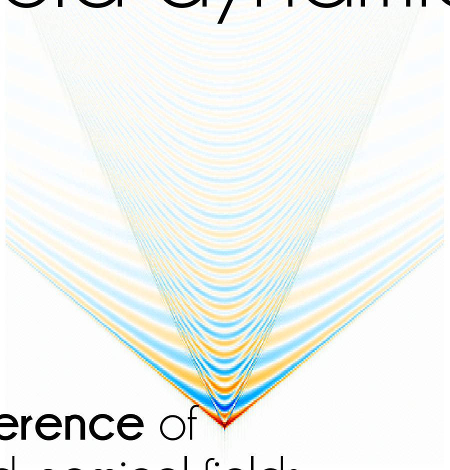
NIFTy = Numerical Information Field Theoy

Unified imaging

UBIK



Field dynamics

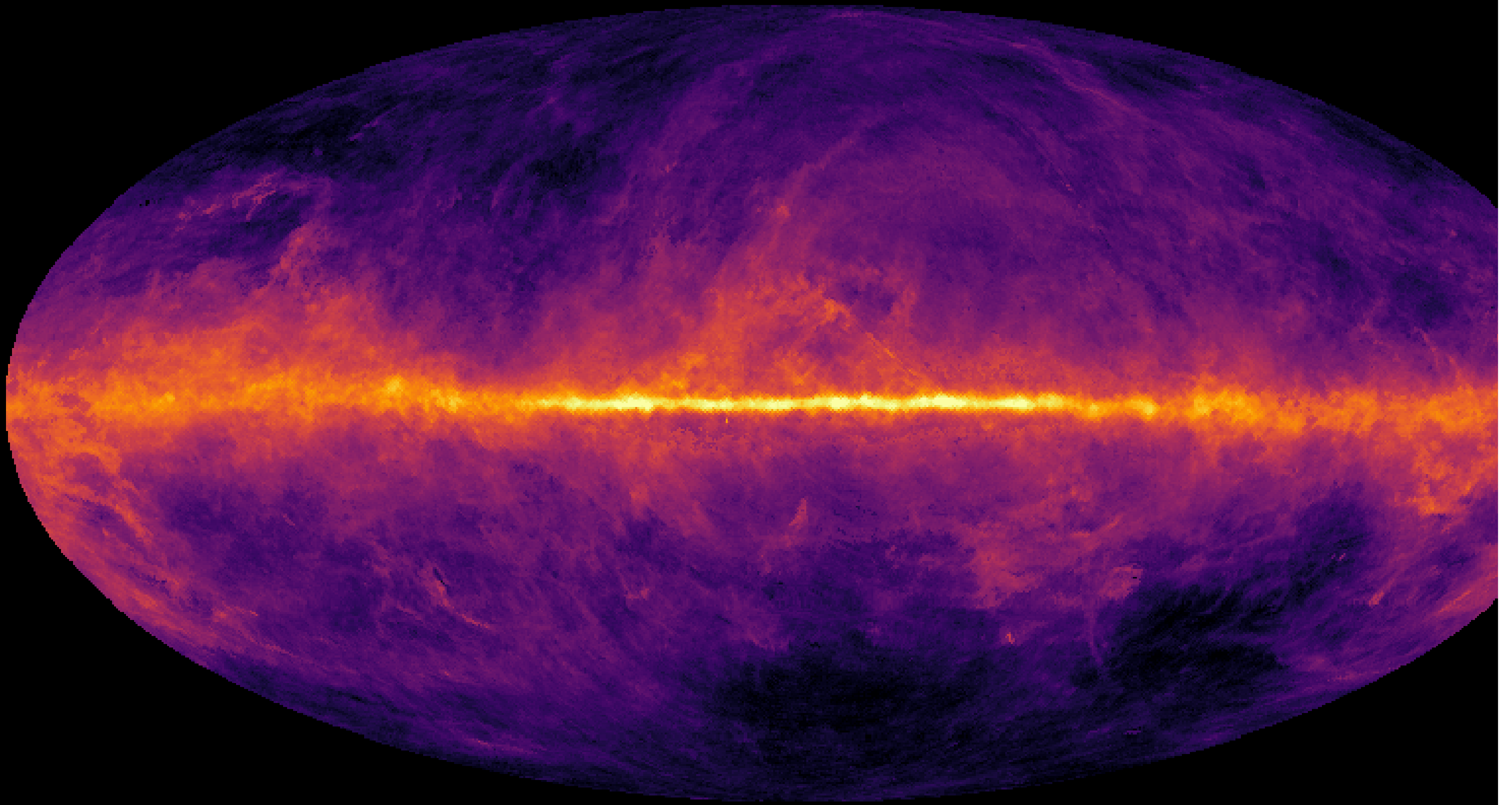


Inference of
- dynamical fields
- & their dynamics

Use it!

IFT:
- Theory papers
- Application papers

NIFTy:
- Python
- freely available
- documented
- tutorials
- demos
- codes



Thank you!

www.mpa-garching.mpg.de/ift