

Elementary particle-based models for dark matter on galaxy scales

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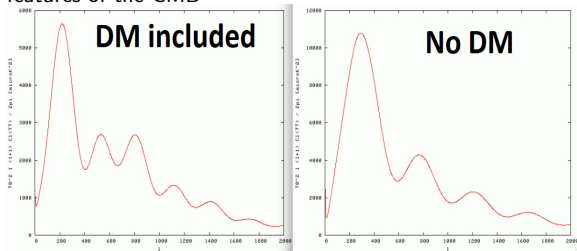
Advanced Workshop on Accelerating the Search for DM with Machine Learning
8 - 12 April, 2019, ICTP-Trieste

R. Ruffini, J. Rueda, A. Krut, R. Yunis (ICRANet), R., N. Mavromatos (King's College), L. G. Gómez (UIS), A. Molinè (IFT)

- 1 The Λ CDM paradigm: success & challenges
- 2 Alternatives to CDM: Ultra light DM, WDM
 - Bosons: Fuzzy Dark Matter
 - Fermions: WDM \rightarrow the RAR model for relaxed halos
- 3 Conclusions

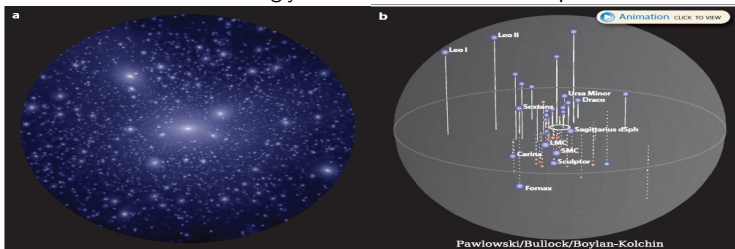
Success of Λ CDM Cosmology

- Astrophysical observations (CMB, BAO, Ly- α forest, local distribution and evolution of galaxies, etc) ranging from horizon scale (~ 15000 Mpc) to the typical scale between galaxies (1 Mpc) are *all* consistent with a Universe that was seeded by a scale invariant primordial spectrum, and that is dominated by dark energy $\sim 70\%$ followed by $\sim 25\%$ of Cold Dark Matter (CDM) and only $\sim 5\%$ of baryons plus radiation [Planck Collaboration et al., 2016]; [Vogelsberger et al., 2014]; [Kitaura, Angulo, et al., 2012]
- Being the most compelling evidence for the existence of CDM the precise features of the CMB

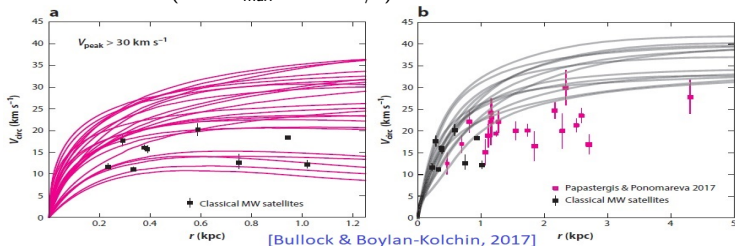


Small-scale challenges to Λ CDM

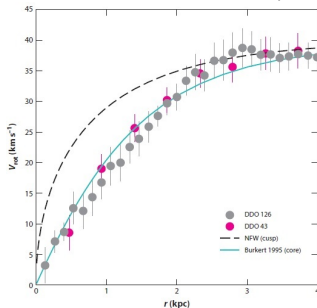
- **LOST SATELLITE PROBLEM:** possible solution is to argue that galaxy formation becomes increasingly inefficient as DM mass drops



- **TOO BIG TO FAIL:** large population of predicted (Aquarius-simulations) massive subhalos (with $V_{max} > 30$ km/s) which don't fit the data

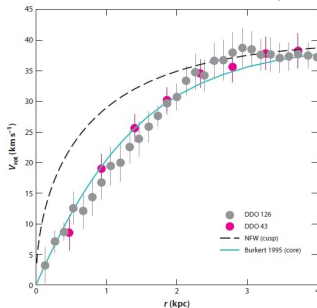


- CORE-CUSP PROBLEM: Central regions of DM-dominated galaxies (as inferred from rotation curves) tend to be less cuspy than in LCDM halos



[Oh et al. AJ 2015]

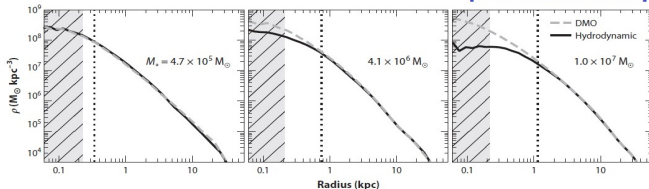
- CORE-CUSP PROBLEM: Central regions of DM-dominated galaxies (as inferred from rotation curves) tend to be less cuspy than in LCDM halos



[Oh et al. AJ 2015]

- BARYONIC FEEDBACK becomes inefficient for low (stellar mass M_*) galaxies

[Fitts et al. MNRAS 2017]



Alternatives to Λ CDM

- Different outcomes respect to standard cosmological simulations when dropping one or more assumptions

Cold

\rightarrow Warm

Collisionless

\rightarrow Self-Interacting DM

DM only

\rightarrow DM plus neutrinos

Gaussian IC

\rightarrow Primordial NG

GR

\rightarrow Modified gravity
simulations

Classical
particles

\rightarrow axion/wave/fussy DM

Bosons: Fuzzy Dark Matter

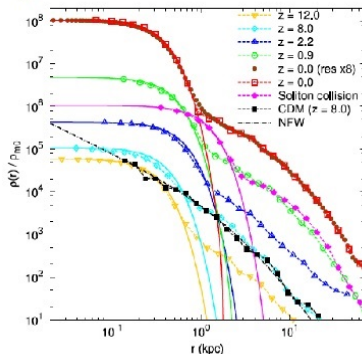
- From VLASOV-POISSON to SCHRÖDINGER-POISSON

$$\frac{i\hbar}{a^{3/2}} \frac{\partial}{\partial t} (a^{3/2} \Psi) = \left[-\frac{\hbar^2}{2a^2 m_\Phi} \nabla^2 + m_\Phi \phi_N \right] \Psi$$

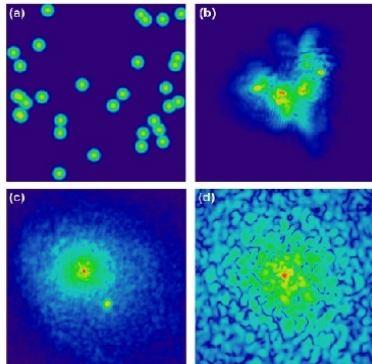
$$m_\Phi \sim 10^{-22} \text{eV}$$

$$\frac{\nabla^2}{a^2} \phi_N = 4\pi G \rho_N \leftarrow \rho_N = \frac{2}{c\lambda_c^2} |\Psi|^2$$

$$\lambda_{db} \sim 10^{16} \text{km} \left(\frac{10^{-4} c}{v} \right) \left(\frac{10^{-22} \text{eV}}{cm_\Phi} \right)$$



[Schive et al. (2014)]



Theory: The RAR model

- Relaxed systems of Fermions under self-gravity DO ADMIT a perfect fluid approximation (differently to the case of bosons) [Ruffini et al. *Phys.Rev* 1969]
- Challenge: Solve the Einstein equations (i.e. T.O.V) of a semi-degenerate fermion gas in hydrostatic and thermodynamic equilibrium in the background metric $ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$

$$\frac{dM}{dr} = 4\pi r^2 \rho,$$

$$\frac{dP}{dr} = -\frac{1}{2} \frac{d\nu}{dr} (c^2 \rho + P), \quad \frac{d\nu}{dr} = \frac{2G}{c^2} \frac{M + 4\pi r^3 P/c^2}{r^2 [1 - 2GM/(c^2 r)]}$$

$$e^{\nu/2} T = \text{const.}, \quad e^{\nu/2} (\mu + mc^2) = \text{const.}, \quad e^{\nu/2} (\epsilon + mc^2) = \text{constant.}$$

$$\rho = m \frac{2}{h^3} \int f_c(p) \left[1 + \frac{\epsilon(p)}{mc^2} \right] d^3 p,$$

$$P = \frac{1}{3} \frac{2}{h^3} \int f_c(p) \left[1 + \frac{\epsilon(p)}{mc^2} \right]^{-1} \left[1 + \frac{\epsilon(p)}{2mc^2} \right] \epsilon d^3 p,$$

$$f_c(p) = \begin{cases} \frac{1 - e^{(\epsilon - \epsilon_c)/kT}}{e^{(\epsilon - \mu)/kT} + 1}, & \epsilon \leq \epsilon_c \\ 0, & \epsilon > \epsilon_c \end{cases}$$

Theory: Thermodynamics and Statistical physics

- DM halo formation: COLLISIONLESS RELAXATION & ESCAPE OF PARTICLES \rightarrow generalized Fokker-Planck equation for fermions [P.H. Chavanis, *Physica A* \(2004\)](#)

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} - \nabla \Phi(\mathbf{r}, t) \frac{\partial f}{\partial \mathbf{v}} = \frac{\partial}{\partial \mathbf{v}} \mathbf{J}_f \quad \Delta \Phi = 4\pi G \int f d^3\mathbf{v}$$

- Stationary solutions of the form $f = f(\epsilon)$ ($f(\epsilon_c) = 0$, $\epsilon_c = v^2/2 + \Phi$) can be found, satisfying the H-theorem (Entropy Maximization) for an arbitrary functional C

$$S = - \int C(f) d^3\mathbf{r} d^3\mathbf{v}$$

- The generalized Kinetic equation has a thermodynamical structure (corresponding to a canonical description in the case of KRAMERS)
- If $C(f)$ is the Boltzmann Entropy functional

$$f(\epsilon) = A(\exp[-\beta(\epsilon)] - \exp[-\beta(\epsilon_c)]) \quad \text{classical particles}$$

- If $C(f)$ is the Fermi-Dirac Entropy functional

$$f(\epsilon) = \frac{1 - e^{\beta(\epsilon - \epsilon_c)}}{e^{\beta(\epsilon - \mu)} + 1} \quad \text{quantum or classical particles}$$

Theory: The RAR model

- Dimensionless form of the diff. eqtns. \rightarrow Necessary step
 $\hat{r} = r/\chi, \chi \propto m^{-2}$

$$\frac{d\hat{M}}{d\hat{r}} = 4\pi\hat{r}^2\hat{\rho},$$

$$\frac{d\theta}{d\hat{r}} = -\frac{1 - \beta_0(\theta - \theta_0)}{\beta_0} \frac{\hat{M} + 4\pi\hat{P}\hat{r}^3}{\hat{r}^2(1 - 2\hat{M}/\hat{r})},$$

$$\frac{d\nu}{d\hat{r}} = \frac{\hat{M} + 4\pi\hat{P}\hat{r}^3}{\hat{r}^2(1 - 2\hat{M}/\hat{r})},$$

$$\beta(r) = \beta_0 e^{-\frac{\nu(r) + \nu_0}{2}}$$

$$W(r) = W_0 + \theta(r) - \theta_0$$

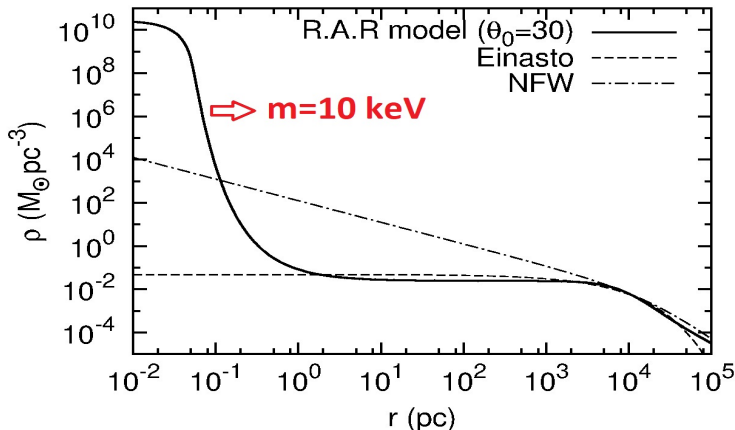
- Free parameters: $m, \beta = kT/mc^2, \theta = \mu/kT$ and $W = \epsilon_c/kT$

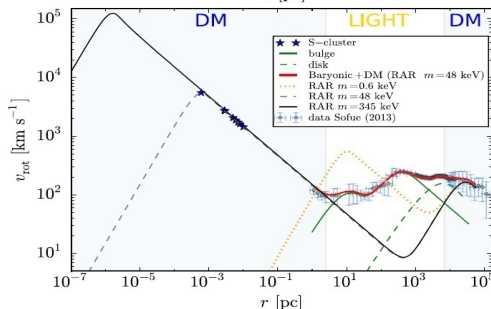
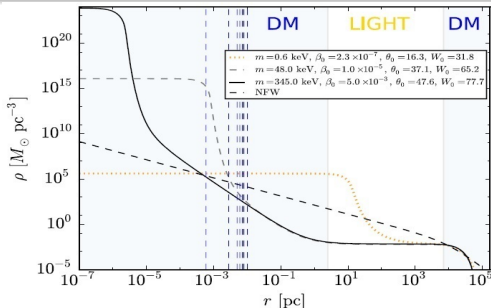
$$M(0) = 0; \quad \nu_0 = 0; \quad \theta(0) = \theta_0 > 0; \quad \beta(0) = \beta_0; \quad W(0) = W_0$$

- BOUNDARY CONDITION problem: Solved systematically for different $(m, \beta_0, \theta_0, W_0)$ such that $M(r = r_h)$ satisfy required observations from DM dominated halos
- MATLAB code: <https://github.com/ankrut/RAR-GetStarted> (Grid coverage using non-linear least squares)

Fermions: WDM & keV particles

- General solutions: Novel CORE-HALO fermionic profiles which depend on the particle mass. Central dense core fulfills 'quantum condition' $\lambda_{db} > 3l_{core}$
 [Ruffini, C.R.A, Rueda *MNRAS* 2015]





Upper bound in m given by critical core mass criterion

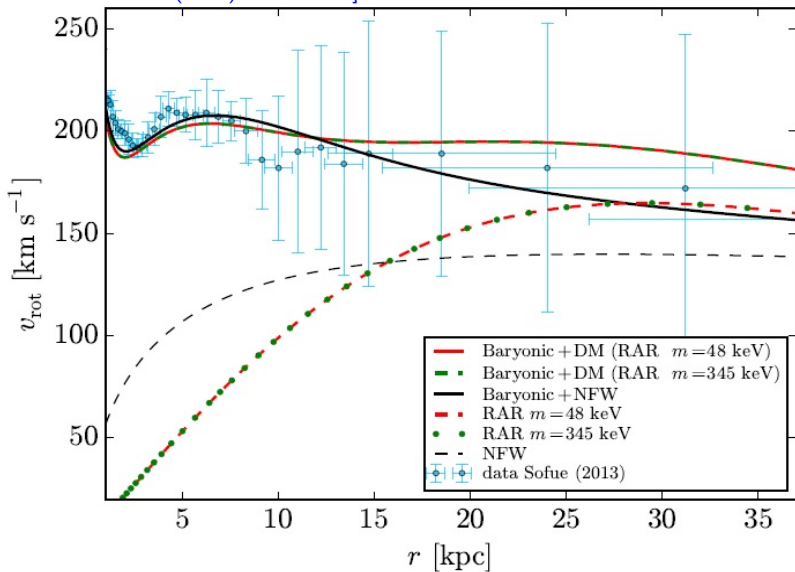
$$M_c^{\text{cr}} \sim M_{\text{pl}}^3 / m^2$$

DM boundary conditions

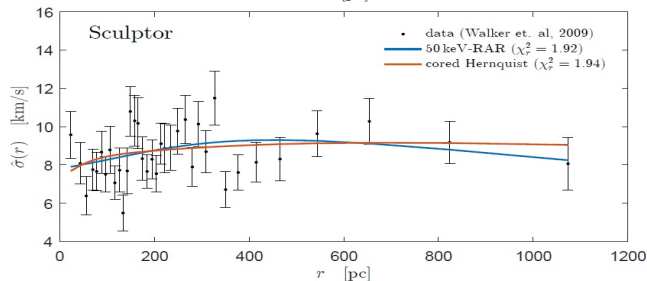
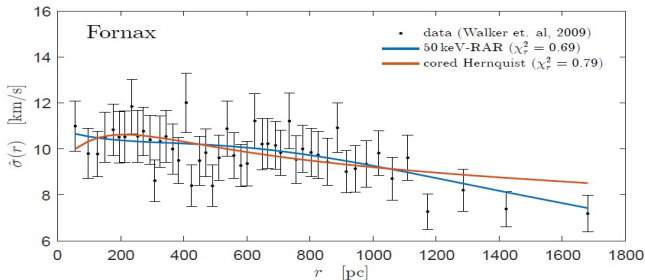
- $M(r=r_c) = 4.2 \times 10^6 M_\odot \quad (r_c < r_{s-2})$
- $M(r=12 \text{ kpc}) = 5 \times 10^{10} M_\odot$
- $M(r=40 \text{ kpc}) = 2 \times 10^{11} M_\odot$

[C.R.A et al. PDU 2018, 1606.07040]

- [C.R.A et al. *PDU* (2018) 1606.07040]



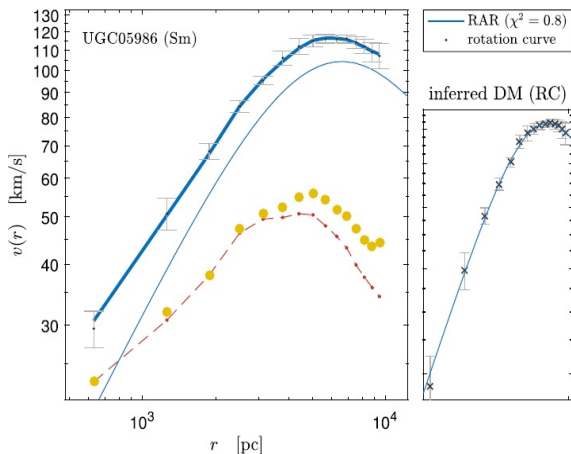
Typical dSphs - Jeans analysis for constant anisotropy



Typical Spiral from SPARC- RC analysis $m = 50$ keV

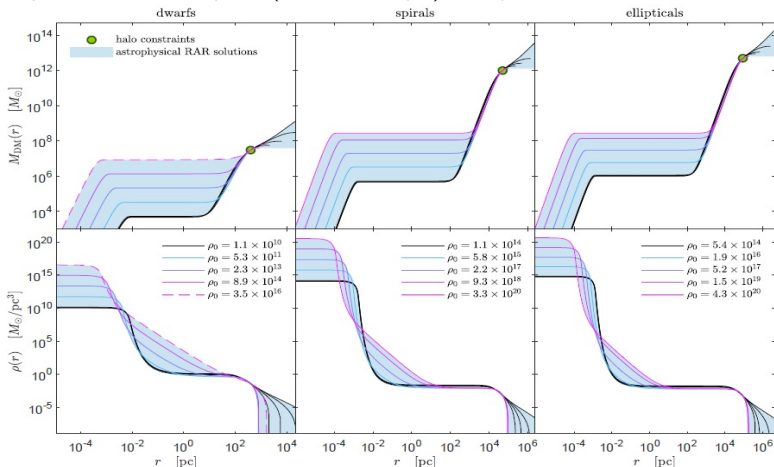
[C.R.A et al. *PDU* (2019) 1810.00405]

$$V_{obs}^2 = (\Upsilon_b V_b^2 + \Upsilon_d V_d^2 + V_g^2) + V_{DM}^2(\theta_0, \beta_0, W_0)$$



From dwarf to ellipticals - THE CASE OF $m = 48$ keV

- Observationally-inferred boundary conditions ($M(r_h), r_h$) from well resolved dSph MW satellites, spirals (THINGS sample) & ellipticals

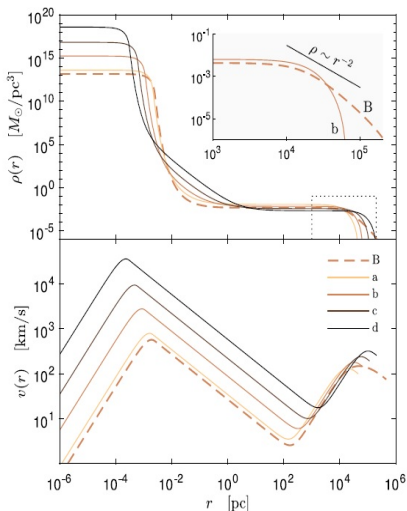
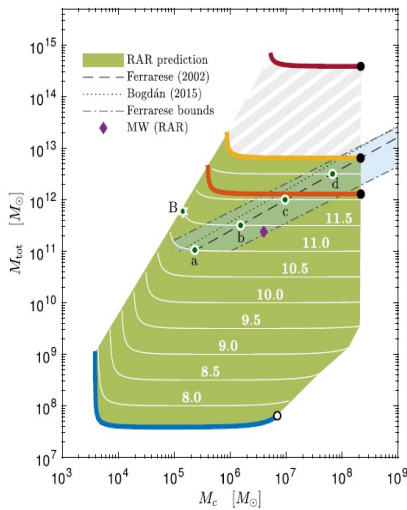


[C.R.A et al. PDU 2019 1810.00405]



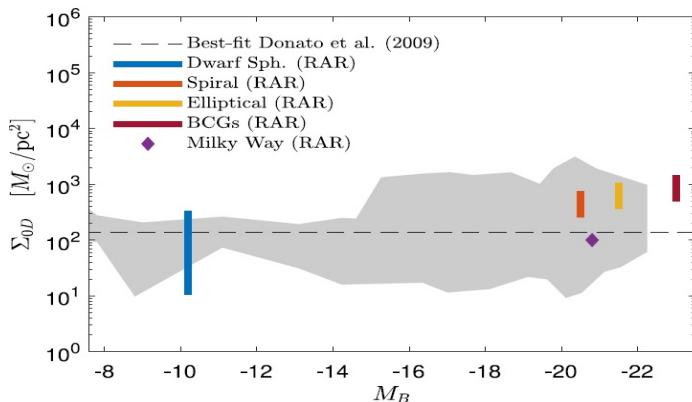
Ferrarese Universal relation - THE CASE OF $m = 48$ keV

[C.R.A et al. *PDU* (2019) 1810.00405]



DM surface density Universal relation - THE CASE OF $m = 48$ keV

$$\Sigma_{0D} = \rho_{0D} r_B = 140^{+80}_{-50} M_{\odot} pc^2 \quad [\text{Donato, Gentile, Salucci, et al. } MNRAS(2009)]$$

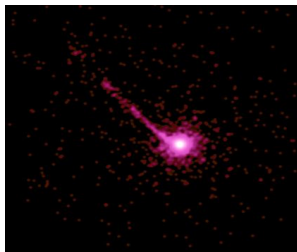


[C.R.A et al. *PDU* (2019) 1810.00405]

- Normal Galaxies \rightarrow NO Active Nuclei
NOR Jets ($M_c \sim 10^{6-7} M_\odot$)



- Active Galaxies \rightarrow YES Active Nuclei AND
Jet emission ($M_c \sim 10^{9-10} M_\odot$)



Conclusions

- Relaxed systems of self-gravitating bosons/fermions arising from *stationary solutions* of Schrödinger-Poisson/ Generalized FP-Poisson provide novel *core-halo* profiles which may offer deep insights to non-linear structure formation
- The fermionic RAR model can be directly confronted with local-Universe galaxy data allowing to put constraints on the particle mass m $10 - 10^2$ keV, without the need of the cosmological history of the particles
- Such bounds are in line with other *independent* cosmological/astrophysical WDM bounds such as phase-space or Ly- α forest constraints
- The RAR is a *predictive* model: (i) novel DM profile morphology with new *degenerate core* and *asymptotic-tail* features which can be Universally contrasted through Universal galaxy scaling relations
- Other predictions are (ii) lensing effects: weak on the halo and strong within the dense core (such as formations of Einstein ring) [Gómez, C.R.A, et al. *PRD* 2016]; (iii) Novel X-ray flux pattern (assuming decaying sterile ν) [Yunis, C.R.A, et al. 1810.05756], AdS/CFT consequences [C.R.A, Grandi *JHEP* 2018]
- It provides a natural for the formation of $\sim 10^8 M_\odot$ SMBH via the gravitational collapse of the DM dense cores

THANK YOU!

"I'm not happy with all the analysis that go with just the classical theory, because nature is not classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy"

Richard P. Feynman

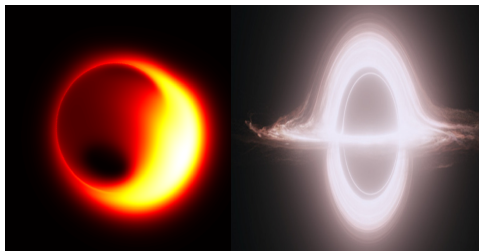
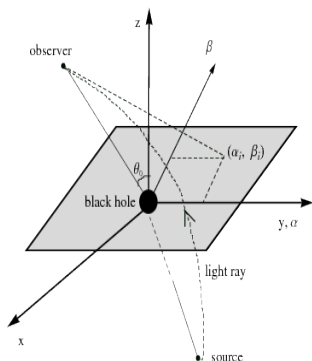
Strong lensing around SgrA*: RAR-core Vs. BH

- Deflection angle $\hat{\alpha}(r_0)$ in the Relativistic Regime

$$\hat{\alpha}(r_0) = 2 \int_{r_0}^{\infty} \frac{e^{\lambda/2} dr}{\sqrt{(r^4/b^2)e^{-\nu} - r^2}} - \pi.$$

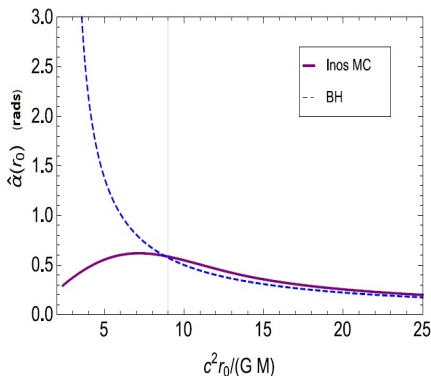
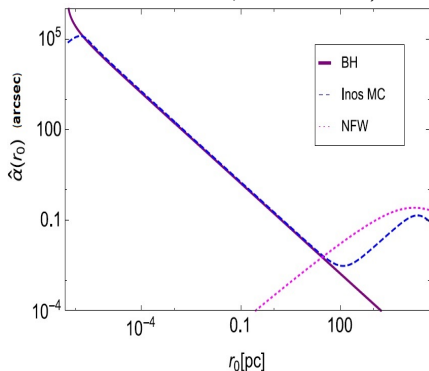
$b = r_0 \exp[-\nu(r_0/2)]$ impact parameter.

- For Schwarzschild BH the deflection angle can reach $\hat{\alpha} > 3/2\pi$ (i.e. relat. images)



Strong lensing around SgrA*: RAR-core Vs. BH

- RAR-core: At $r \sim 10^{-5}$ pc strong lensing effects arises: Einstein ring at $r \sim 4 R_s$
 L. G. Gómez, C. R. Argüelles, V. Perlick, J. A. Rueda, R. Ruffini, PRD (2016)
- DM RAR cores do not show a photon sphere ($\hat{\alpha}(r_0) < 1$), i.e. they do not cast a shadow as the BH does!
- The EHT expect angular resolution of $\sim 30 \mu\text{arcsec}$ (angular diameter θ of BH shadow $\sim 50 \mu\text{arcsec} \sim 6 R_s$)



Bosons: Ultra-light DM or Fuzzy DM

- Main physical motivation for a FDM particle: occurs in QCD-axion

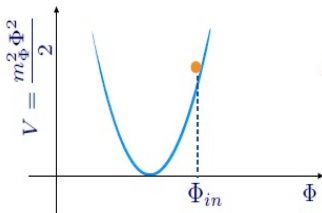
Very light DM with large $n = \rho_{DM}/m_\Phi$

Classical solution to the Klein-Gordon equation $[-\square + m_\Phi^2]\Phi(\vec{x}, t) = 0$

Background: $ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j \longrightarrow \ddot{\Phi} + 3H\dot{\Phi} + m_\Phi^2 \Phi = 0$

Initial conditions set by inflation
 (very homogeneous)

When $H \gg m_\Phi \longrightarrow \Phi \sim \text{constant} \longrightarrow T_\nu^\mu \propto \delta_\nu^\mu$



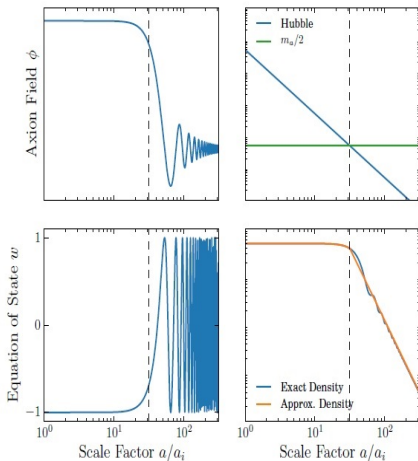
After $H < m_\Phi \longrightarrow \Phi \sim \Phi_0 \cos(m_\Phi t + \Upsilon)$

$$T_{\mu\nu} \begin{cases} \longrightarrow \rho_{DM} = \frac{m_\Phi^2 \Phi_0^2}{2} + \text{oscillations} \\ \longrightarrow p_{DM} = -\rho_{DM} \cos(2m_\Phi t + 2\Upsilon) \end{cases}$$

(oscillations are irrelevant on long time scales)

Bosons: Ultra-light DM or Fuzzy DM

- The temperature T_0 at which $\rho_{FDM} \propto a^{-3}$ is 500 eV ($z \sim 10^6$)
- Evolution of $\rho(r) = m|\Psi|^2$ from radiation era to matter-radiation equality (at ~ 1 eV) implies $m \sim 10^{-22}$ eV



[Marsh, Phys. Rep 2016]

- SELF-GRAVITY: A large collection of bosons in the same state: described by a classical field Ψ - SCHRÖEDINGER-POISSON ($\rho(r) = m|\Psi|^2$) -

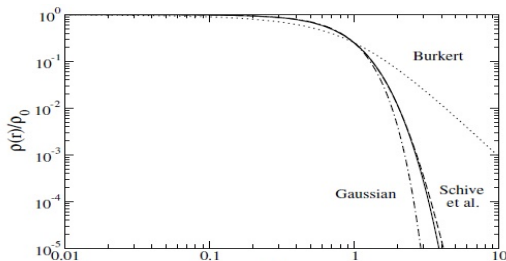
$$-\hbar^2/2m\nabla^2\Psi + Vm\Psi = mE\Psi \quad \nabla^2 V = 4\pi Gm|\Psi|^2 \quad (1)$$

- SELF-GRAVITY: A large collection of bosons in the same state: described by a classical field Ψ - SCHRÖDINGER-POISSON ($\rho(r) = m|\Psi|^2$) -

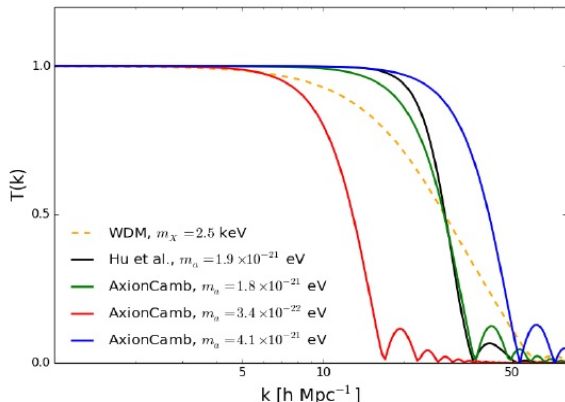
$$-\hbar/2m\nabla^2\Psi + Vm\Psi = mE\Psi \quad \nabla^2 V = 4\pi Gm|\Psi|^2 \quad (1)$$

- The lowest eigenstate solution ($E_n; n = 0$) is stable. The nth excited states are unstable and **decay to the ground state** \rightarrow Gravitational cooling! [Seidel, *PRL* 1994]; [Schwabe, *PRD* 2016]
- Central density eigenstates: $\rho_c \propto m^3 M^4 \rho_n$. For ($n = 0$)
 $M \sim 10^9 M_\odot$; $\rho_c \sim 5 M_\odot / \text{pc}^3 \Rightarrow m \sim 10^{-22} \text{ eV}$ [Hui, et al. *PRD* 2017]

$$\frac{\lambda_{db}}{2\pi} = \frac{\hbar}{mv} = 1.9 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m} \right) \left(\frac{10 \text{ km s}^{-1}}{v} \right) \quad \text{de - Broglie wavelength} \quad (2)$$

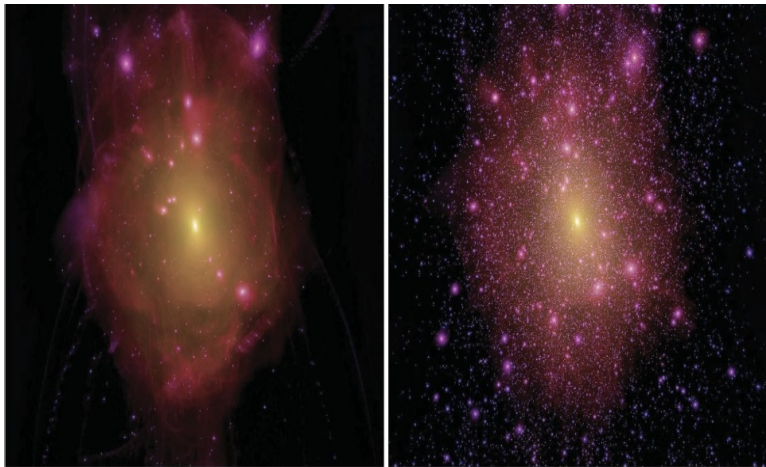


- FDM is unstable ONLY for masses larger than Jeans mass
 $M_J \sim 10^7 M_\odot (10^{-22} \text{ eV/m})^{3/2}$
- The number of CDM sub-halos RISES for M_h below $10^8 M_\odot$, while for FDM halts
- The power spectrum in FDM is SUPPRESSED relative to CDM at small scales
- Ly- α** forest offers additional prove of power spectrum: $m_{\text{WDM}} > 3.3 \text{ keV}$ translates in $m_{\text{FDM}} > 2 \times 10^{-21} \text{ eV} \Rightarrow$ Tension!! [Hui, et al. *PRD* 2017]



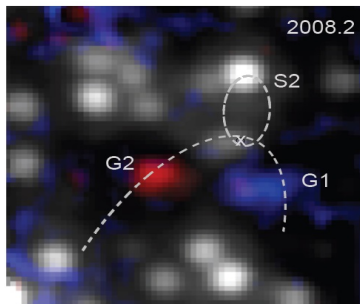
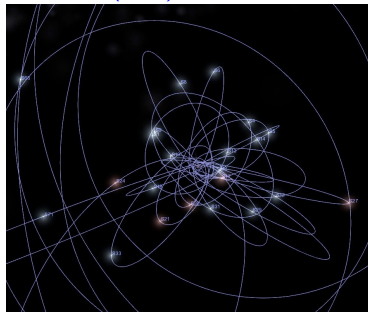
WDM & FDM: Suppression of structure below Mpc

- Matter power spectrum which are cut-off below given scale produce less amount of substructure and less concentrated [Lovell, et al. *MNRAS* 2012]
- THIS PICTURE: High-res. N-body simulations: $z = 0$; box of 1.5 Mpc side



The S-stellar cluster & central gas

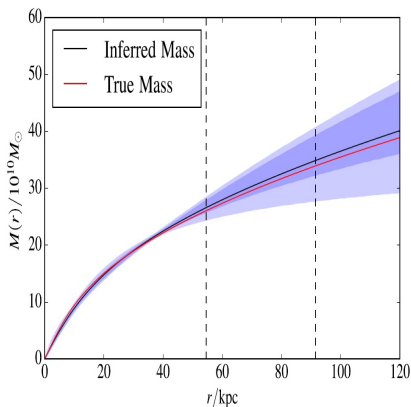
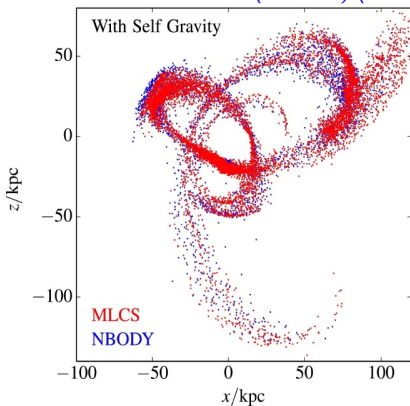
- The central $10^{-3} \text{ pc} \lesssim r \lesssim 2 \text{ pc}$ consist in young S-stars and molecular gas obeying a Keplerian law ($v \propto r^{-1/2}$)
- The observational near-IR technics were developed in [S. Gillessen et al. \(Apj\) \(2009\)](#) and in [S. Gillessen et al. \(Apj\) \(2015\)](#) for S-stars and gas cloud G2



Observations implies $M_c \approx 4.2 \times 10^6 M_\odot$ within the smallest pericenter
 $r_p(S2) \approx 6 \times 10^{-4} \text{ pc}$

The outermost DM halo constraints: Sgr-dwarf

- The outermost satellite galaxies of the MW are excellent total DM tracers
- The Sgr-dwarf satellite with its stream motion of tidally disrupted stars was well observed and well reproduced numerically *Belokurov et al. (MNRAS) (2014)*, *S. Gibbons et al. (MNRAS) (2014)*



Theory: Thermodynamics and Statistical physics

- COLLISIONLESS RELAXATION: described by the VLASOV-POISSON equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} - \nabla \Phi(\mathbf{r}, t) \frac{\partial f}{\partial \mathbf{v}} = 0 \quad \Delta \Phi = 4\pi G \int f d^3 \mathbf{v}$$

- Main collisionless mechanisms: phase mixing & violent relaxation. Defined over macroscopic (averaged) states: $f \rightarrow \bar{f}$
- VIOLENT RELAXATION [Lynden Bell, *MNRAS* 1967]: the total energy of the bodies is NOT conserved

$$\frac{dE}{dt} = \frac{\partial \Phi}{\partial t} |_{r(t)}$$

- COLLISIONLESS RELAXATION TIME proper of violently changing Φ is the Dynamical time $t_D \ll t_R \rightarrow$ Relaxation in galaxies can be approached without the need for collisions
- (Macroscopic) Maximization entropy principle at fixed total mass and energy (η is a phase-space patch or macrocell)

$$S = \int \rho(\mathbf{r}, \mathbf{v}, \eta) \ln \rho(\mathbf{r}, \mathbf{v}, \eta) d\eta d^3 \mathbf{r} d^3 \mathbf{v} \quad \bar{f}(\mathbf{r}, \mathbf{v}) = \int \rho(\mathbf{r}, \mathbf{v}, \eta) \eta d\eta$$

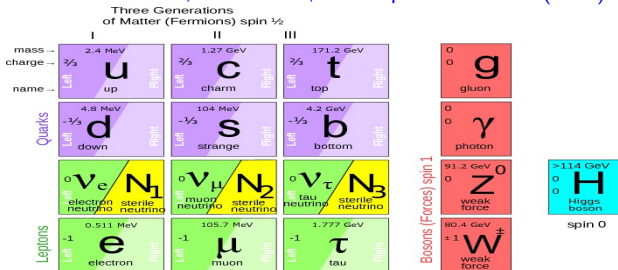
$$\delta S = 0 \Rightarrow \bar{f} = \frac{1}{e^{\beta[\epsilon(\mathbf{p}) - \alpha]} + 1}$$

CONSTRAINTS ON PARTICLE PHYSICS BEYOND SM

- Coupling with Higgs provides (through SSB mechanism) the Quark, Lepton (e, μ, τ) and gauge boson - mass generation

$$\mathcal{L}_\psi \propto -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \quad \text{BUT in SM } \nexists \nu_R \quad (3)$$

- Minimal extension of SM (ν MSM) adding 3 right-handed STERILE ($Q_{SM} = 0$) neutrinos [T. Asaka, S. Blanchet, M. Shaposhnikov *PLB* \(2005\) 0503065](#)



- Group-invariance in ν MSM model: $SU(3) \times SU(2) \times U(1)$ remains unchanged!

$$\mathcal{L} = \mathcal{L}_{SM} + i\nu_R \partial_\mu \gamma^\mu \nu_R - g \bar{L} \nu_R \phi - M/2 \bar{\nu}_R^c \nu_R \quad (4)$$

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Three Generations of Matter (Fermions) spin $\frac{1}{2}$

	I	II	III		
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0	0
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
name →	u up	c charm	t top	g gluon	
	Left Right	Left Right	Left Right		
Quarks				0	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	γ photon	
	Left Right	Left Right	Left Right		
	d down	s strange	b bottom	91.2 GeV	0
	Left Right	Left Right	Left Right	Z^0 weak force	
	0	0	0		0
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	H Higgs boson	
	Left Right	Left Right	Left Right		
	0	0	0		
	N_1 sterile neutrino	N_2 sterile neutrino	N_3 sterile neutrino		
	Left Right	Left Right	Left Right		
Leptons	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV	± 1
	e electron	μ muon	τ tau	W weak force	
	Left Right	Left Right	Left Right		
	+1	-1	-1		
	Left Right	Left Right	Left Right		

Bosons (Forces) spin 1

spin 0

- Group-invariance in ν MSM model: $SU(3) \times SU(2) \times U(1)$ remains unchanged!

$$\mathcal{L} = \mathcal{L}_{SM} + i\nu_R \partial_\mu \gamma^\mu \nu_R - g \bar{L}_R \phi - M/2 \bar{\nu}_R^c \nu_R \quad (4)$$

- A Lagrangian extension including for self-interactions \mathcal{L}_I under self-gravity was analyzed [C. Argüelles, N. Mavromatos, et al. JCAP \(2016\) 1502.00136](#)

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{\nu_R} + \mathcal{L}_V - g_V V_\mu J^\mu$$

Indirect ν_s detection from GC

- Sterile neutrino Decay channel: $N_1 \rightarrow \nu_\alpha + \gamma$

$$f = \frac{\Gamma_\gamma}{4\pi M_{N_1}} \int d\Omega \int dx \rho_{DM}(x) = \frac{\Gamma_\gamma}{4\pi M_{N_1}} S_{DM}$$

- DM density profile assumption (i.e. RAR model) $\rightarrow \rho_{DM} \equiv \rho_{DM}(m_s)$

Then...

$$F_{max}^{obs} \geq F = \frac{\Gamma}{4\pi M_1} S$$

$$\Gamma_{N_1 \rightarrow \gamma \nu}(\theta, M_1) = \frac{9\alpha G_F^2 M_1^5}{256\pi^4} \theta^2$$

Indirect ν_s detection from GC

- DM halos in terms of self-gravitating neutral fermions can put constraints on particular DM models such as ν MSM [R. Yunis, C.R.A. et al., (2018) 1810.05756]

