Two general classes of DM models:
- Light, cold, bosonic field $\longrightarrow$ example: axion
- Heavy (MeV+), can be fermionic/bosonic, thermal/non-thermal $\rightarrow$ example: WIMP

Production by thermal freezeout

Suppose DM has a number-changing <u>annihilation</u> reaction:

$$DM + DM \longrightarrow X \text{ (in SM)}$$

In absence of annihilation, DM # density dilutes w/ redshift,

$$\frac{d}{dt}(na^3) = 0 \implies a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0 \implies \frac{dn}{dt} + 3Hn = 0$$

With annihilation, add depletion term, + DM injection term for $X \to DM + DM$

$$\frac{dn}{dt} + 3Hn = \underbrace{-\frac{n^2}{2}\langle\sigma v\rangle \times 2}_{\substack{\text{annihilation rate,} \\ \text{2 particles removed} \\ \text{per annihilation}}} + \text{production term (independent of } n\text{)}$$

call this $\langle\sigma v\rangle x$

$$= \langle\sigma v\rangle (x - n^2)$$
$$= \langle\sigma v\rangle (n_{eq}^2 - n^2)$$

As $\langle\sigma v\rangle \to \infty$, ~~will~~ will drive $n^2 \to x$, & DM to equilibrium with SM
$\implies x$ must equal n_{eq}^2

n_{eq} set by Boltzmann distribution,

$$n_{eq} \sim \begin{cases} (m_{DM} T)^{3/2} e^{-m_{DM}/T} , & m_{DM} \gg T \quad \text{non-relativistic} \\ T^3 , & m_{DM} \ll T \quad \text{relativistic} \end{cases}$$

$\langle \sigma v \rangle \ll 3Hn \implies n \propto \frac{1}{a^3}$, $n^2 \langle \sigma v \rangle \gg 3Hn \implies n \to n_{eq}$

...ossover at $n \langle \sigma v \rangle \sim H$ – freezeout/decoupling.

At early times, $n \sim n_{eq}$, at freezeout n stops tracing n_{eq} & $n a^3 \sim$ constant

i.e $n \sim \frac{n_f a_f^3}{a^3} \sim \frac{n_{eq,f} a_f^3}{a^3} \sim n_{eq,f} \left(\frac{T}{T_f} \right)^3$ (Note: this ignores changes in temperature due to entropy injection – OK for order-of-magnitude estimates.

Hot relic: relativistic at freezeout, $n_{eq,f} \sim T_f^3$

$\implies n \sim T^3$ after freezeout – abundance similar to photon! Overcloses universe

unless $m_{DM} \lesssim 1 \, eV$

Cold relic: non-relativistic at freezeout, abundance exponentially suppressed

once $T \ll m_{DM}$, rapidly drives $n \langle \sigma v \rangle$ below $H \implies T_f \sim m_{DM}$ (really

$T_f \sim m_{DM}/20$ for classic WIMPs).

To get correct relic density, require $m_{DM} n \sim T^4$ at matter-radiation equality

matter density ↗   ↗ energy density

(assume freezeout occurs during radiation domination)

Denote temperature of MRE by T_{eq}.

$\implies m_{DM} n_{eq,f} \left(\frac{T_{eq}}{T_f} \right)^3 \sim T_{eq}^4 \implies n_{eq,f} \sim T_{eq} m_{DM}^2$

But $\langle \sigma v \rangle n_{eq,f} \sim H_f$ by definition

During radiation domination,

$H^2 \sim \frac{8\pi G}{3} \rho \sim \frac{T^4}{m_{PL}^2} \implies H_f \sim \frac{T_f^2}{m_{PL}} \sim \frac{m_{DM}^2}{m_{PL}}$

$\implies \frac{H_f}{\langle \sigma v \rangle} \sim \frac{1}{\langle \sigma v \rangle} \frac{m_{DM}^2}{m_{PL}} \sim n_{eq,f} \sim T_{eq} m_{DM}^2$

$\implies \langle \sigma v \rangle \sim 1/(m_{PL} T_{eq})$

Now $T_{eq} \sim 1$ eV (prove it!), $m_{Pl} \sim 10^{19}$ GeV $\sim 10^{28}$ eV

$$\Rightarrow \langle \sigma v \rangle \sim \frac{1}{(10^{14} \text{eV})^2} \sim \frac{1}{(100 \text{ TeV})^2}$$ — roughly independent of DM mass

But if $\langle \sigma v \rangle \sim \frac{\alpha^2}{m_{DM}^2}$, works for $\alpha \sim 10^{-2}$, $m_{DM} \sim$ TeV

"WIMP miracle"

Works for mass scales up to ~ 100 TeV (then required xsec runs into unitarity bounds)

Predictive annihilation signal.

Classic example: lightest supersymmetric particles.

Axions

SM Lagrangian should in principle have a term of the form:

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} G_{\mu\nu} \widetilde{G}^{\mu\nu}$$

$\hookrightarrow$ gluon field strength

No particular reason to have $\theta = 0$, CP violation is generated in quark sector of SM.

If present, this terms induces a neutron electric dipole moment

$$d_n = 5.2 \times 10^{-16} \, e \, cm$$

Experimentally, $d_n < 3 \times 10^{-26} \, e \, cm \Rightarrow \theta \lesssim 10^{-10}$

Why so small?

Axion solution: replace θ by a dynamical field $\frac{\widetilde{a}}{f_a}$, $\widetilde{a}$ = axion field, $\frac{1}{f_a}$ = coupling to SM. $\widetilde{a}$ can dynamically evolve to a small value.

There is an effective potential for $\tilde{a}$ (for derivation, see e.g Dine's TASI lecture notes, hep-ph/0011376), given by

$$V(\tilde{a}) = -m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \cos(\tilde{a}/f_a) ,$$

$f_\pi \approx 93 \text{ MeV} - \text{pion decay constant}$

$m_\pi \approx 135 \text{ MeV} - \text{pion mass}$

Examine minimum at $n=0$. Coefficient of $\tilde{a}^2$ term in $V(\tilde{a})$ gives axion mass:

$$V(\tilde{a}) \approx -m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} + \frac{1}{2} \tilde{a}^2 \left(\frac{f_\pi}{f_a}\right)^2 m_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d}$$

$$\Rightarrow m_a = \frac{f_\pi m_\pi}{f_a} \left(\frac{m_u m_d}{(m_u + m_d)^2}\right)^{1/4} \approx 0.6 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a}\right)$$

$\frac{1}{f_a}$ controls coupling of $\tilde{a}$ to SM fields (coupling coefficients are model-dependent but all contain $\frac{1}{f_a}$), thus higher mass $\Rightarrow$ smaller f_a $\Rightarrow$ larger coupling. Very light axions are very weakly coupled - good trait for DM! (to avoid bounds on warm DM, light degrees of freedom). Are axions a plausible DM candidate? Questions to answer:

(1) Are they stable? $\tilde{a}$ can decay to $\gamma\gamma$. Lifetime $\sim 10^{24} \text{s} \left(\frac{m_A}{eV}\right)^{-5}$. Age of universe $\sim 10^{10}$ yr $\sim 3 \times 10^{17}$ s. Thus $\tilde{a}$ lives at least as long as the universe for $m_{\tilde{a}} \lesssim 20$ eV. Above this mass, $\tilde{a}$ is unstable (cosmologically).

(2) Were they ever in thermal equilibrium with SM? Answer is yes for $m_a \gtrsim 10^{-3} - 10^{-2}$ eV - in this case $\tilde{a}$ is hot dark matter. $\Omega_{axions} \sim O\left(\frac{m_a}{100 \text{ eV}}\right)$ (via thermal processes), $m_a \lesssim 1$ eV has less than $\sim 1\%$ abundance, OK for HDM. But cannot be 100% of DM. For $m_a \lesssim 10^{-3}$ eV, the axion is non-thermal - can be cold dark matter (& 100%) but we need an alternate production mechanism.

Consider the energy stored in the vacuum expectation value of the axion field, which we will denote $\Theta(t) = \langle a(t) \rangle$ and treat as a classical scalar field. The equation of motion for Θ is

$$\frac{d^2\Theta}{dt^2} + 3H\frac{d\Theta}{dt} + m_a^2\,\Theta = 0$$

$\qquad\qquad\qquad\qquad\qquad\hookrightarrow$ set by shape of potential near minimum

For $m_a \ll H$, $\frac{d\Theta}{dt} \approx 0$ is a solution — the field is "frozen" at a constant value Θ_0

For $m_a > H$, the field begins to oscillate in the potential, with approximate solution

$$\Theta(t) = \Theta_0 \cos(m_a t)\, f(t)$$

$\quad\downarrow$ initial condition $\qquad\hookrightarrow$ slowly varying function, scales as $a^{-3/2}$

The total energy density in the field is given by

$$\rho = PE + KE = \frac{1}{2}m_a^2\,\Theta^2 + \frac{1}{2}\dot{\Theta}^2$$

$\qquad\qquad\qquad\quad\underset{\text{from potential}}{\downarrow}\qquad\quad\underset{\text{kinetic term}}{\downarrow}$

But $\dot\Theta \approx -m_a \sin(m_a t)\,\Theta_0 f(t)$ (dropping subdominant terms from $f'(t)$)

$$= \frac{1}{2}\left(m_a^2\,\Theta_0^2 f^2 \cos^2(m_a t) + m_a^2\,\Theta_0^2 f^2 \sin^2(m_a t)\right)$$

$$= \frac{1}{2}\Theta_0^2 m_a^2 f(t)^2$$

$$\propto a^{-3}$$

i.e. the energy density obeys the equation of state for pressureless matter, and can behave as cold dark matter.

The initial energy density stored in the field is $\rho = \frac{1}{2}m_a^2\,\Theta_0^2$ (at frozen stage, so KE term $=0$).

$\Theta_0 =$ initial value of $\langle a \rangle$ — can vary between $\pm f_a \pi$. Write $\Theta = \frac{\Theta_0}{f_a} =$ "misalignment angle".

Then energy density $= \frac{1}{2}m_a^2 f_a^2 \Theta^2$ initially, redshifts as $\frac{1}{a^3}$ once oscillations begin.

Final result controlled by initial condition Θ, + when oscillations start (set by m_a).

Note for QCD axion ma is actually also a function of T (turns on during QCD phase transition) — makes calculation more subtle. Careful calculation assuming universal value of θ gives

$$\frac{\Omega_{axions}}{\Omega_{DM}} \approx \theta^2 \left(\frac{6\,\mu eV}{m_a}\right)^{1.165} \quad \text{for QCD axion (PDG review on axions, 2018)}$$

θ is an angle — can be $\ll 1$, but at most π. Thus if a very small angle θ is acceptable, axion can be arbitrarily light (small mass gives high abundance naturally, suppressed by θ^2), but cannot be much heavier than $\sim 50\,\mu eV$, if it is to be 100% of DM. Implies $f_a \gtrsim 10^{11}$ GeV — high-scale physics.

Why would θ be so small? Maybe anthropically selected? (in our case sq. patch)

θ will only have uniform value throughout universe if its value is fixed^prior to inflation — region with a given value of θ will inflate to cover observable universe (& much more).

Alternative: θ is fixed after inflation, different patches of cosmos have different misalignment angles. Effectively take average over values of θ^2 — plus the varying values of θ lead to formation of axion string network, can decay producing more axions. Recent simulations find in this case

$$\frac{\Omega_{axions}}{\Omega_{DM}} \approx 0.4 \left(\frac{f_a}{10^{11}\,GeV}\right)^{1.187}$$

Buschmann et al 1906.00967
Klaer & Moore '17
(although see also Gorghetto et al 1806.04677)

Fixes preferred mass to $\sim 25\,\mu eV$.