Ab initio few-mode theory

for quantum potential scattering problems

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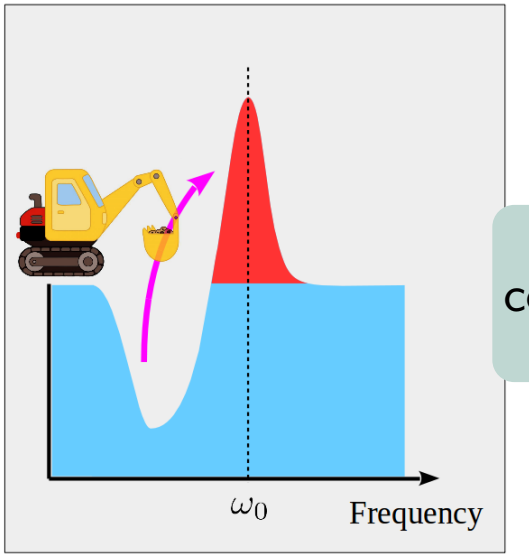
ISOQUANT SFB1225

https://xkcd.com/1070/

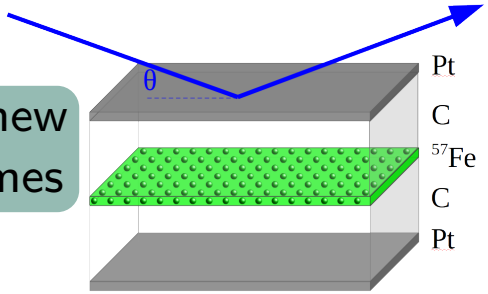
Group of Jörg Evers at MPIK in Heidelberg



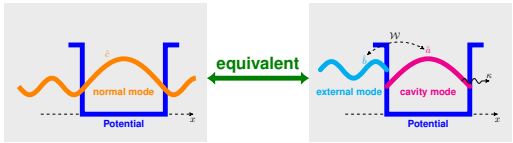
Coherent control of x-rays and nuclei

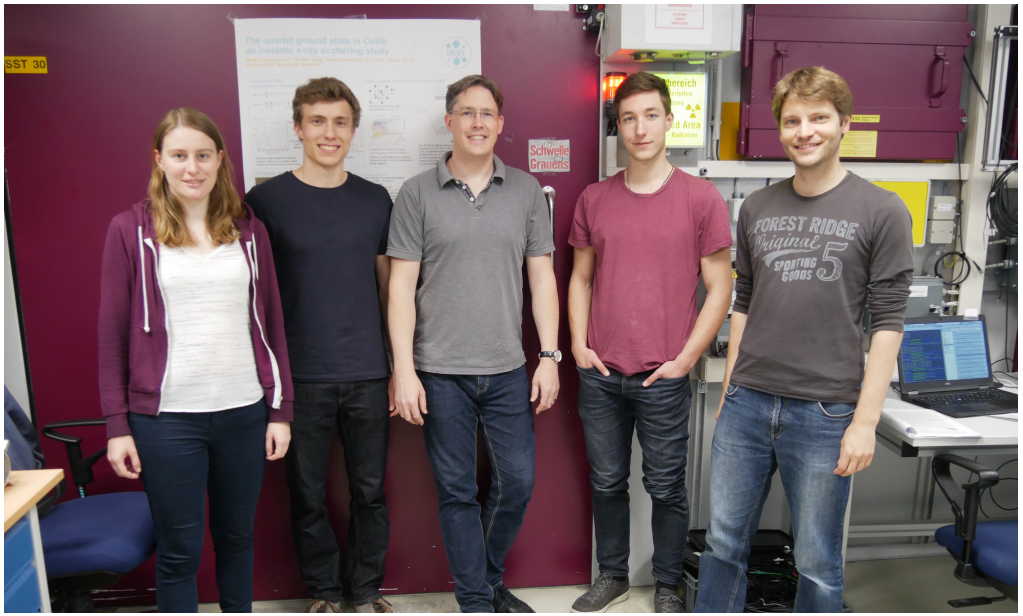
Nuclear quantum optics

X-rays excite new quantum regimes



Understand fundamentals





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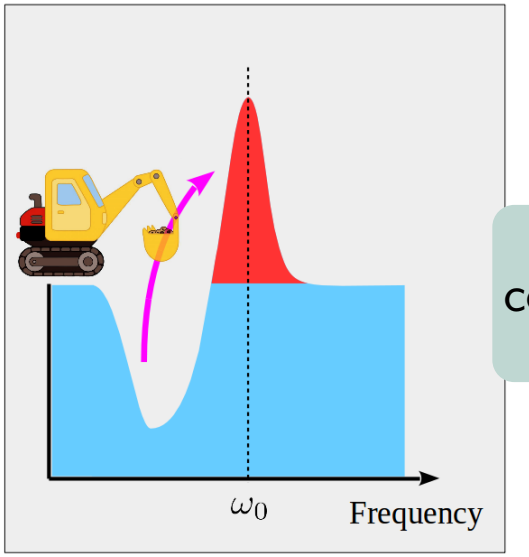

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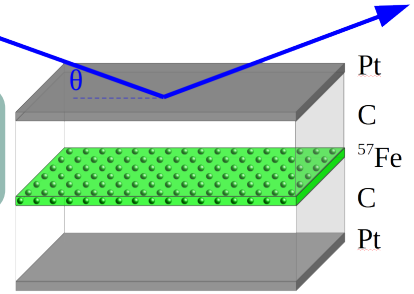
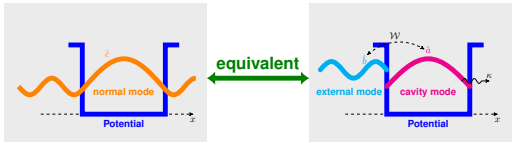
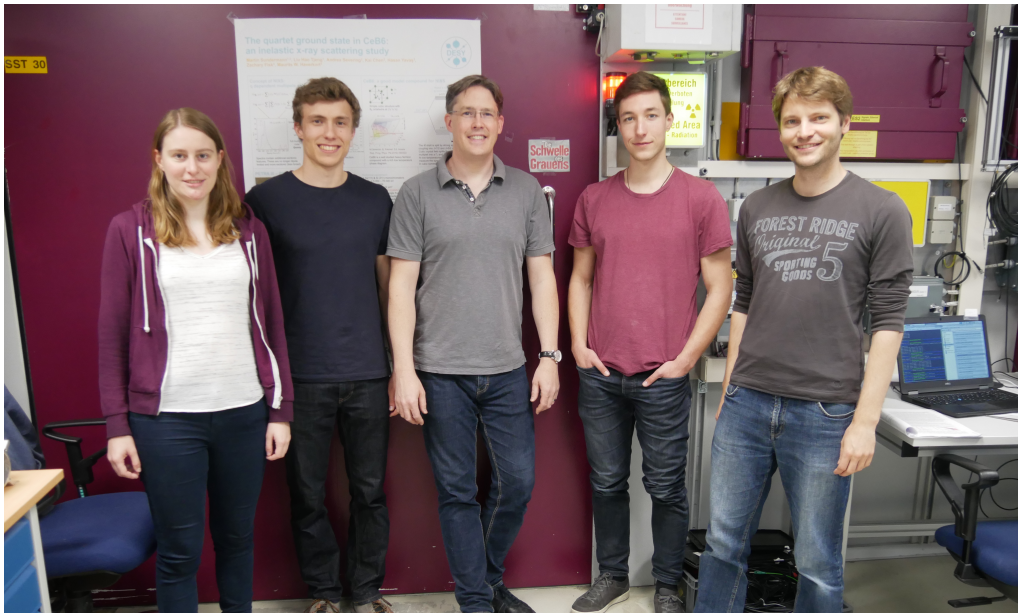


today

Understand fundamentals





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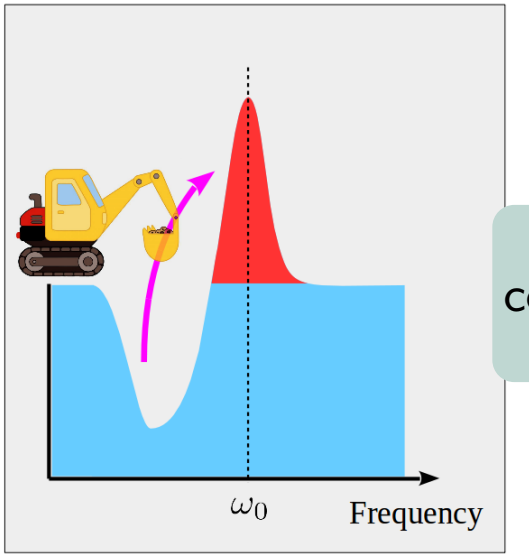

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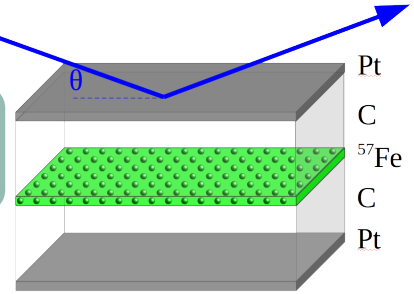

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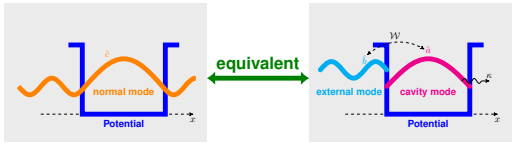
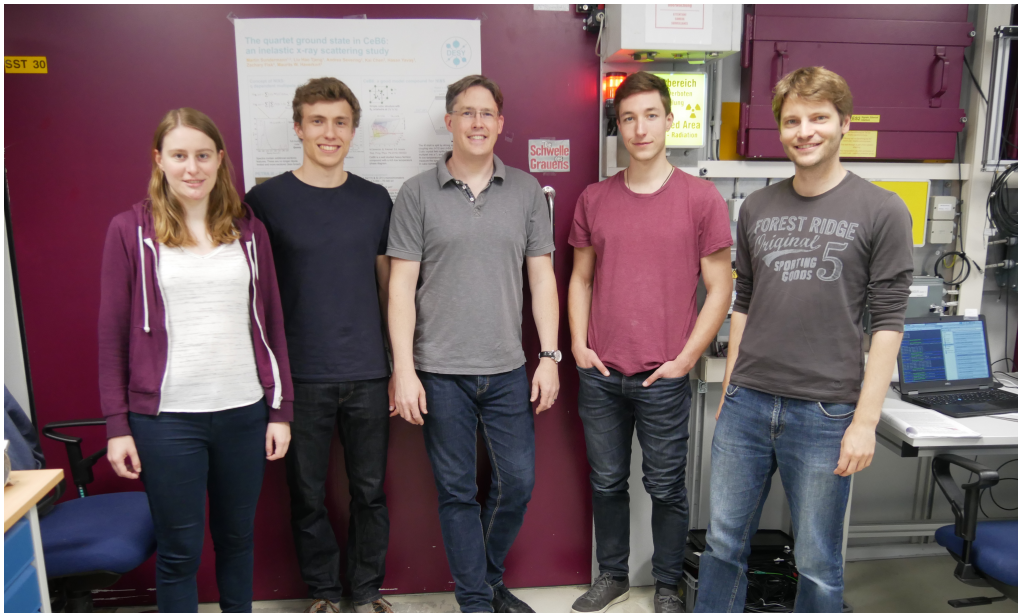


Open quantum systems

Light-matter interactions

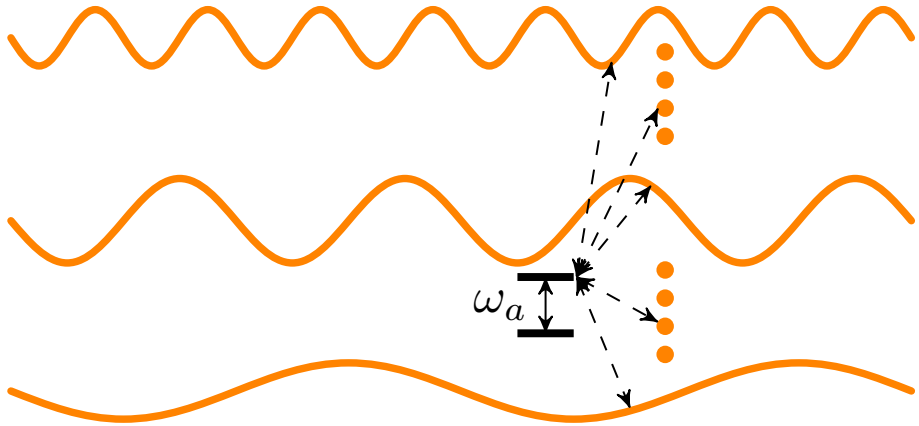
Cavity QED, Circuit QED

...

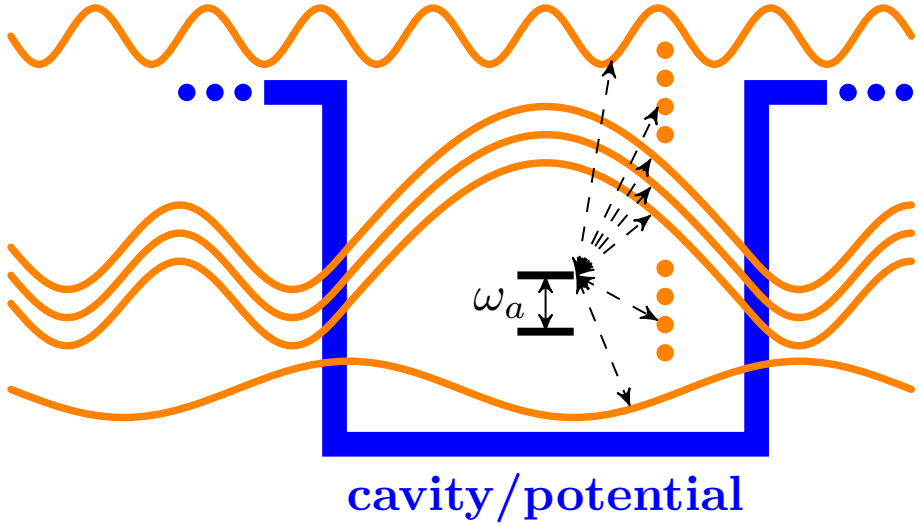


Overview

Matter-continuum coupling



Structured continuum featuring resonances



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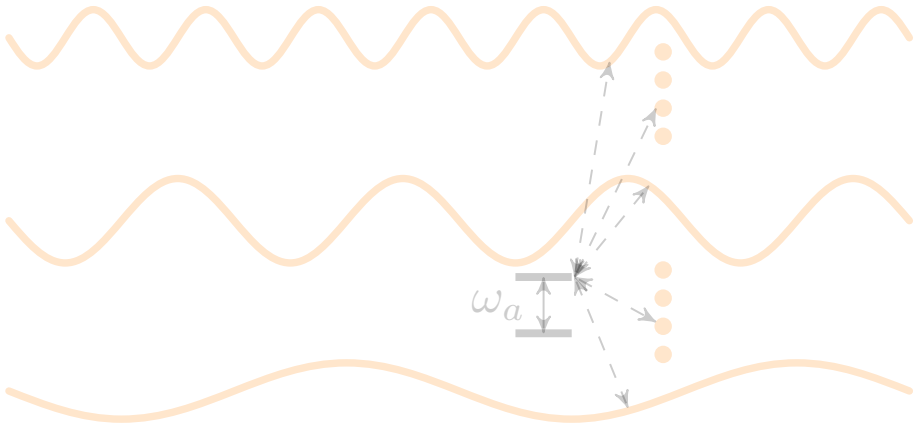
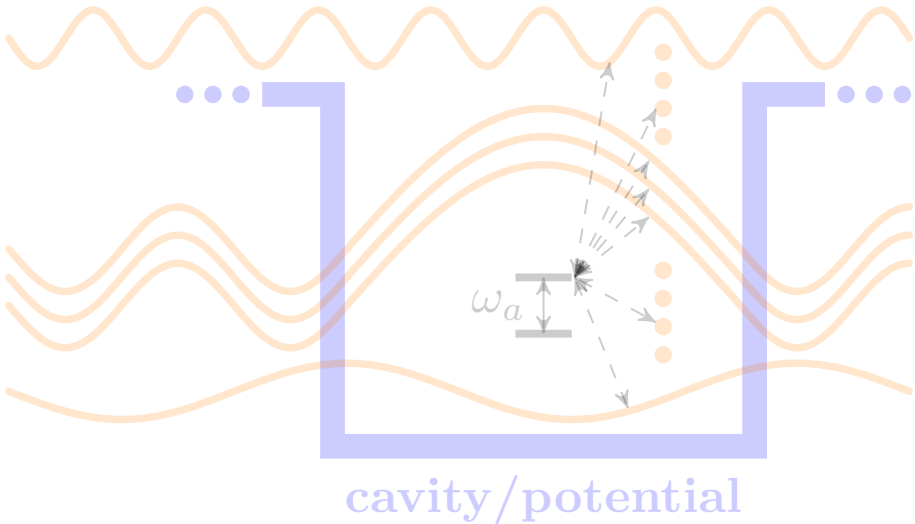
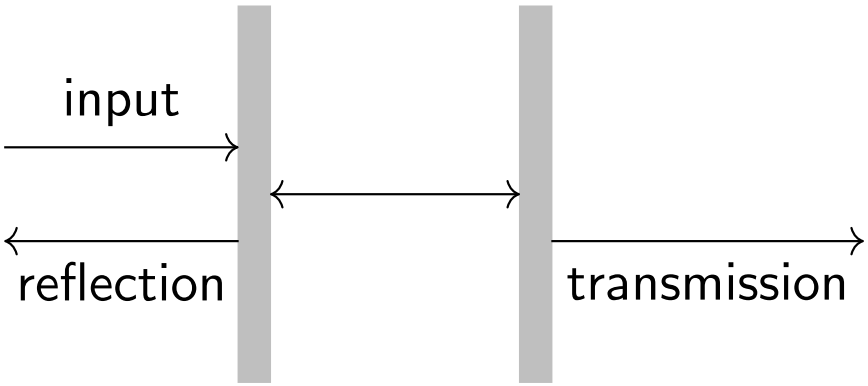


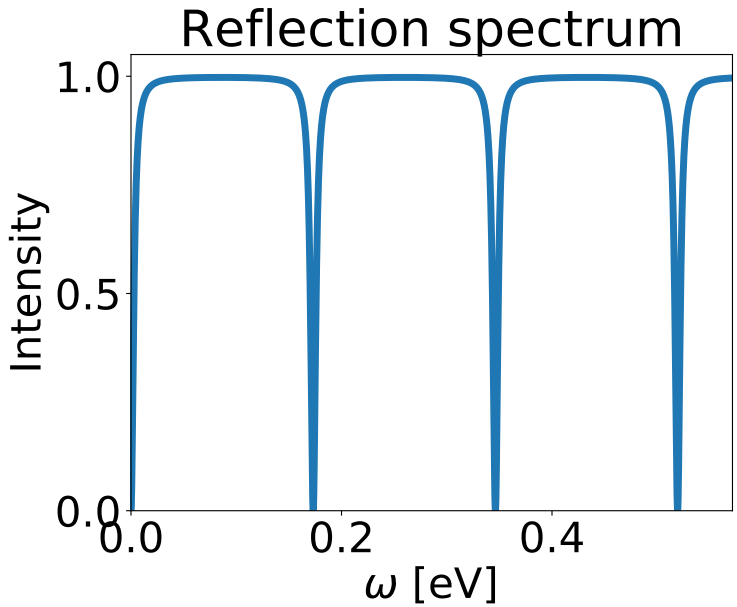
Can we extract **relevant degrees of freedom** from a continuum?

Structured continuum featuring resonances



Example: Fabry-Perot cavity





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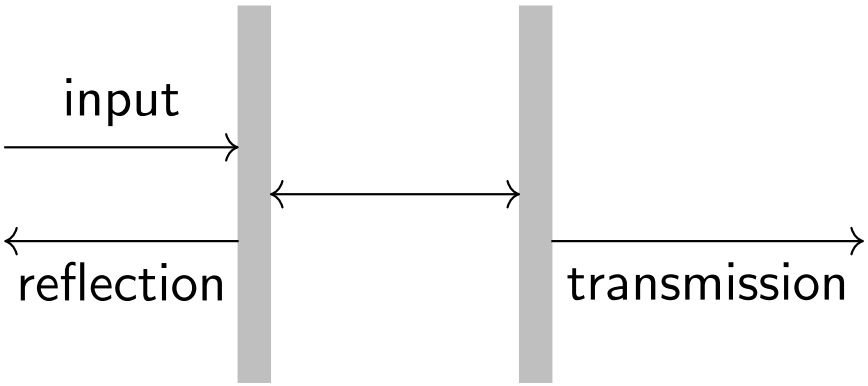

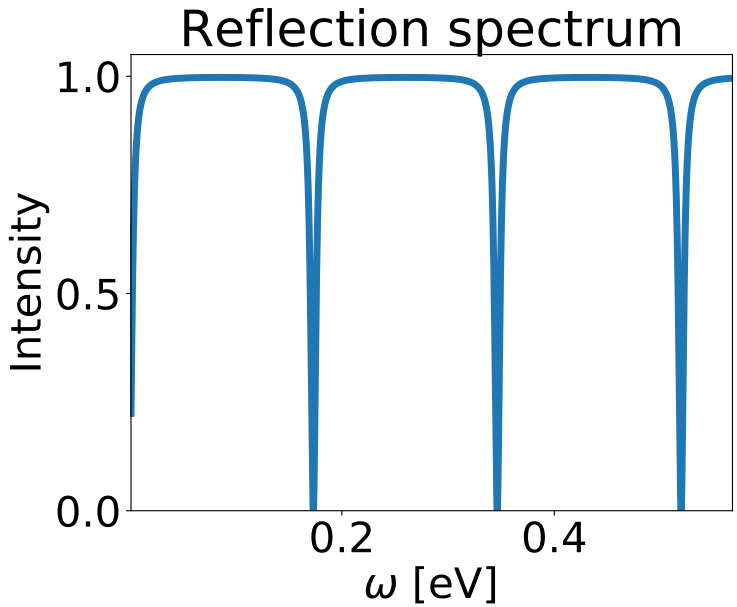

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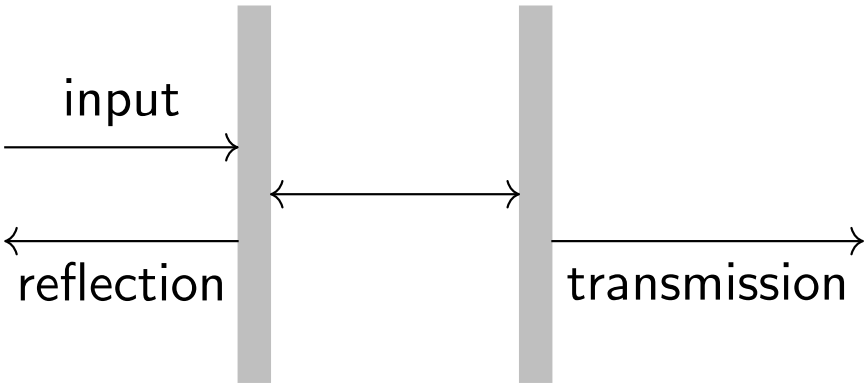

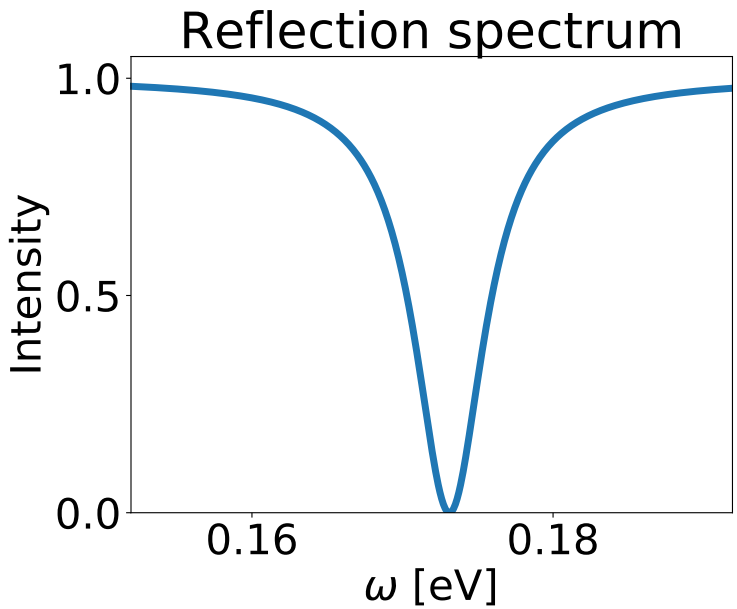

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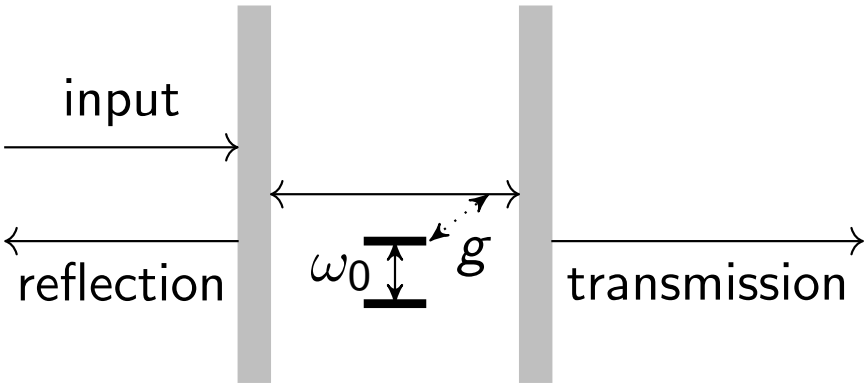

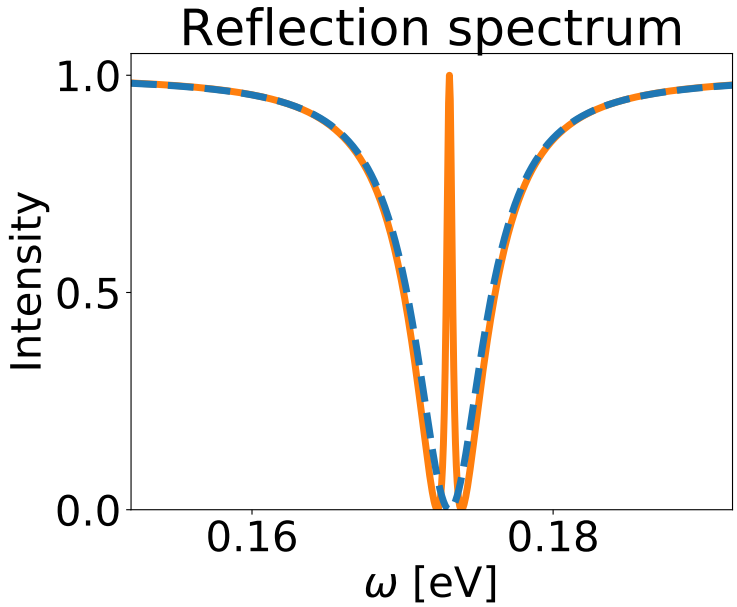

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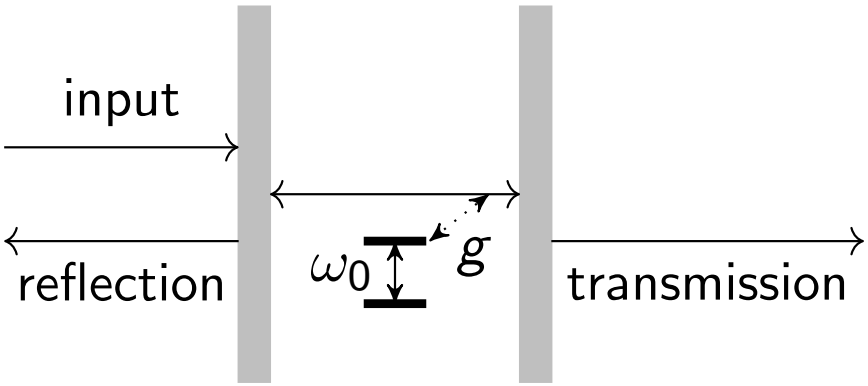
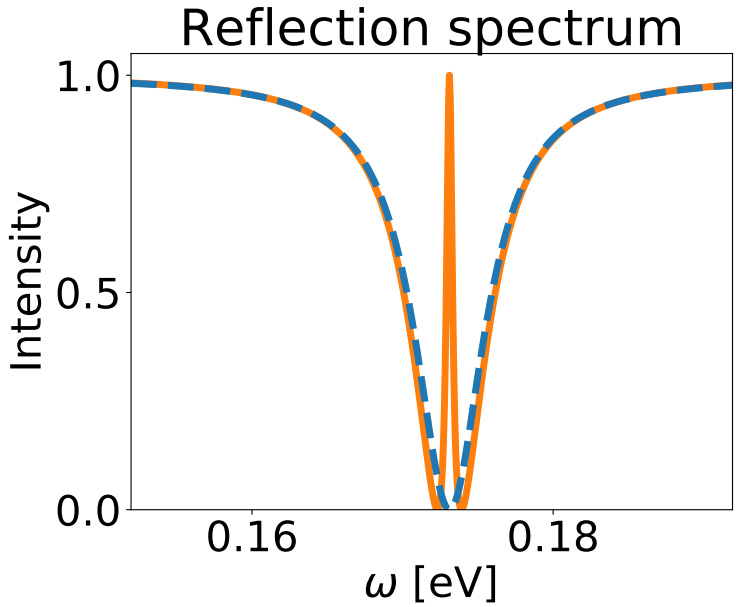
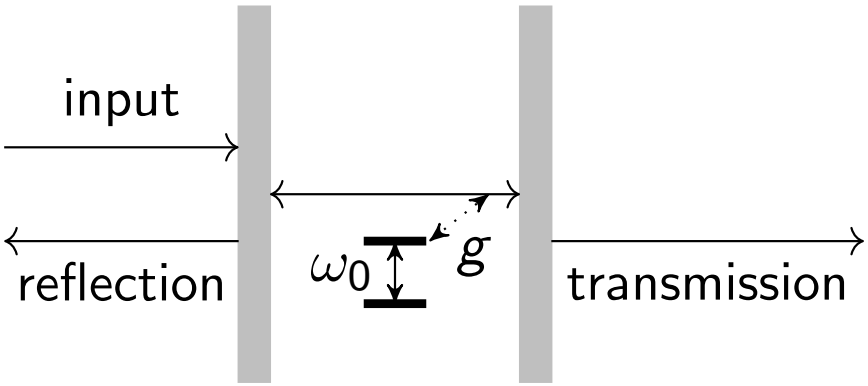
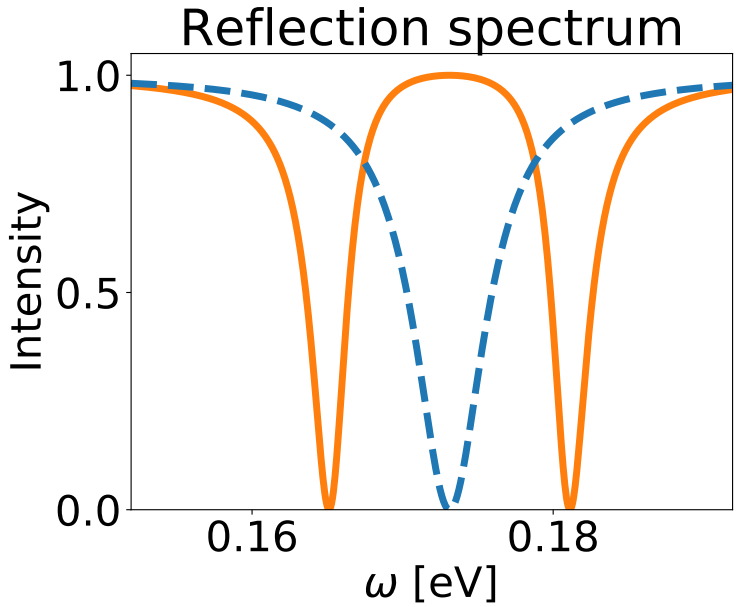




$\rightarrow$ Weak coupling: Purcell effect

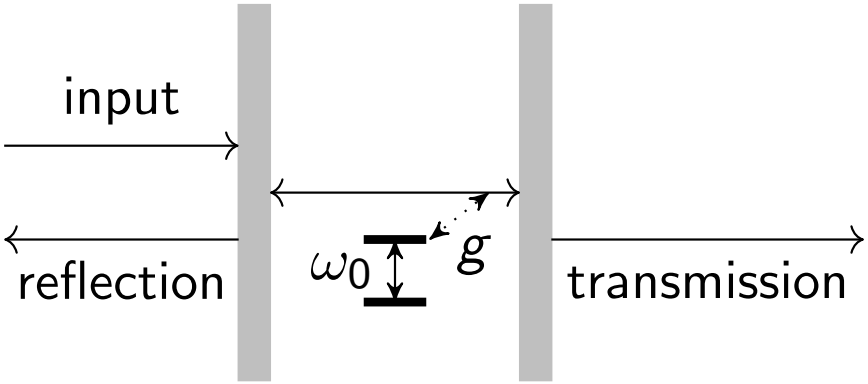
Example: Fabry-Perot cavity

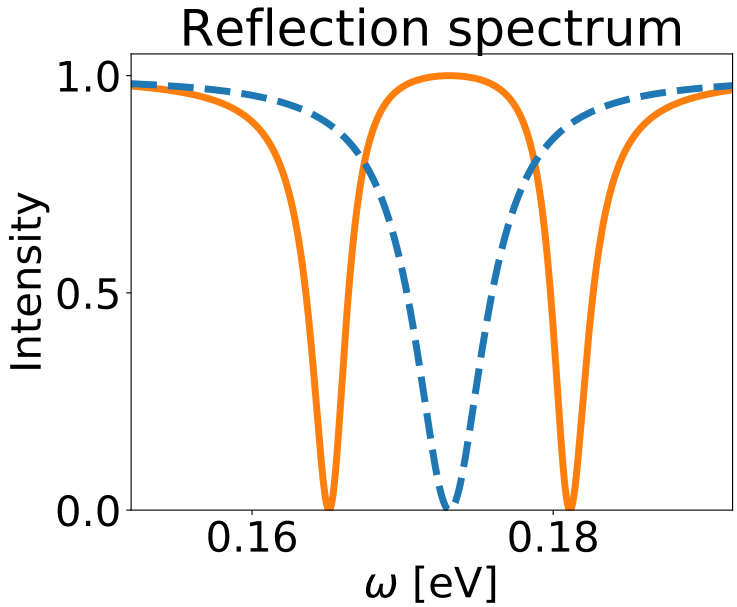




$\rightarrow$ Strong coupling: Vacuum Rabi-splitting

Example: Fabry-Perot cavity

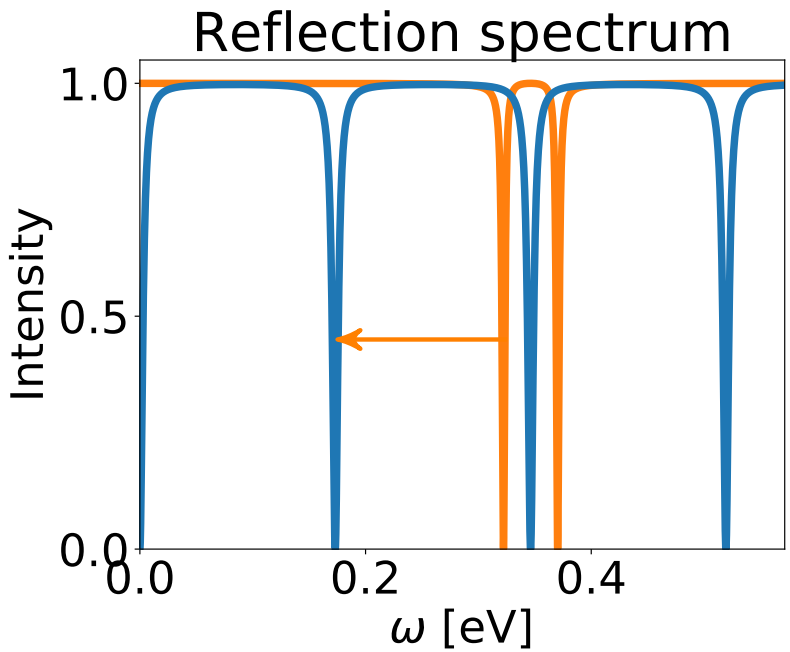




$\Rightarrow$ Enhance and control light-matter interactions!

Extreme regimes

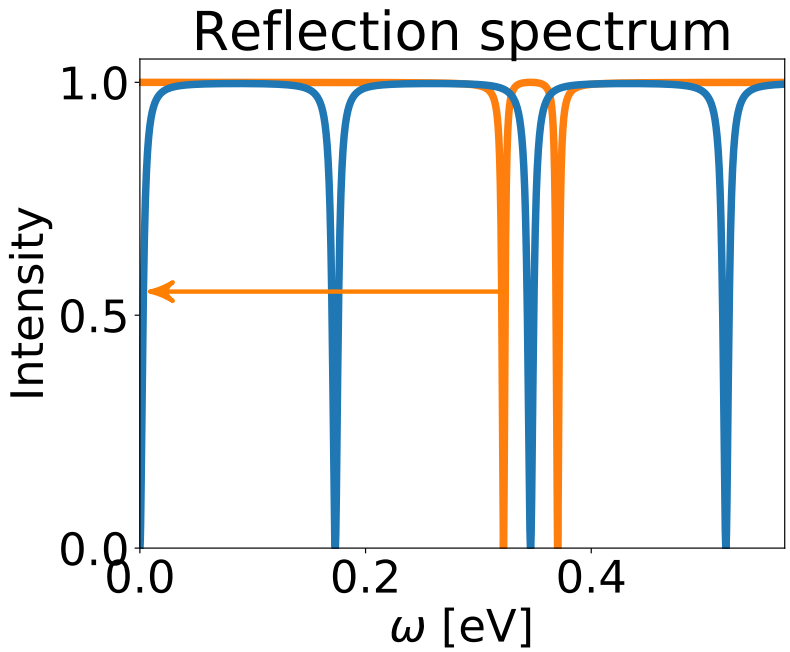
- Multi-mode strong coupling



Türeci et al. *Science* **320**, 643 (2008)
Krimer et al. *Phys. Rev. A* **89**, 033820 (2014)
Sundaresan et al. *Phys. Rev. X* **5**, 021035 (2015)
. . . and many more . . .

Extreme regimes

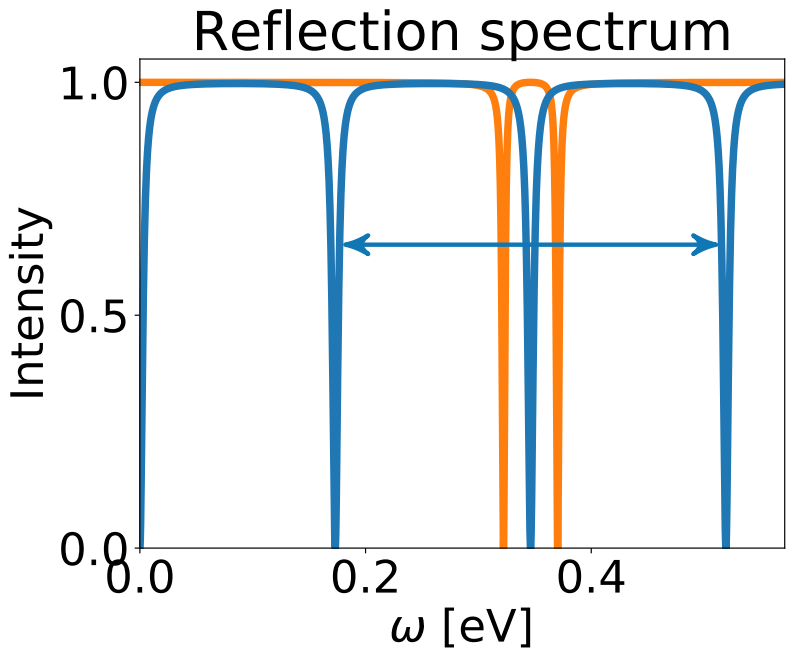
- Multi-mode strong coupling
- Ultra-strong coupling
- Deep-strong coupling



Recent reviews:
Carusotto & Ciuti *Rev. Mod. Phys.* **85**, 299 (2013)
Frisk Kockum et al. *Nat. Rev. Phys.* **1**, 19 (2019)
Forn-Díaz et al. *Rev. Mod. Phys.* **91**, 025005 (2019)
Experimental:
Anappara et al. *Phys. Rev. B* **79**, 201303(R) (2009)
. . . and many more . . .

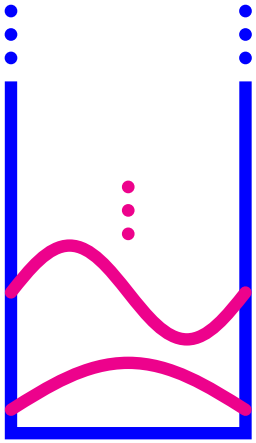
Extreme regimes

- Multi-mode strong coupling
- Ultra-strong coupling
- Deep-strong coupling
- **Overlapping modes**



Petermann *IEEE J. Quantum Electron.* **15**, 566 (1979)
Hackenbroich, Viviescas & Haake *Phys. Rev. Lett.* **89**, 083902 (2002)
I. Rotter *J. Phys. A: Mathematical and Theoretical* **45**, 15 (2009)
Heeg et al. *Phys. Rev. Lett.* **114**, 207401 (2015)
. . . and many more . . .

From closed to open boxes



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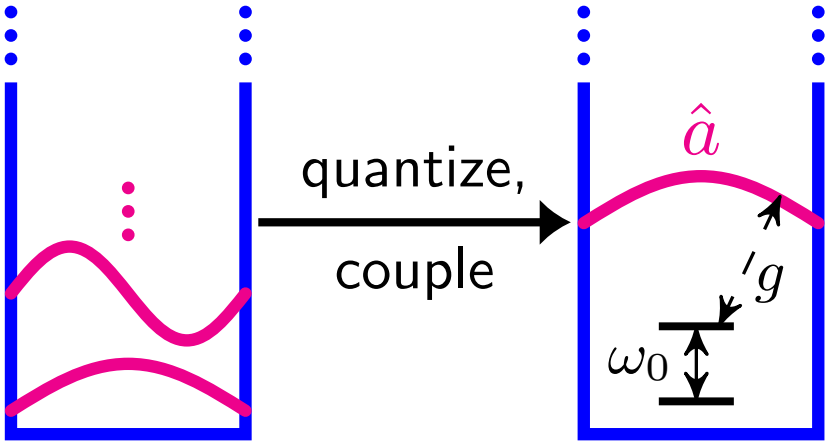

Jaynes-Cummings & friends

$$H = H_{\text{atom}} + H_{\text{cav}} + g\hat{a}\hat{\sigma}^{+} + h.c.$$

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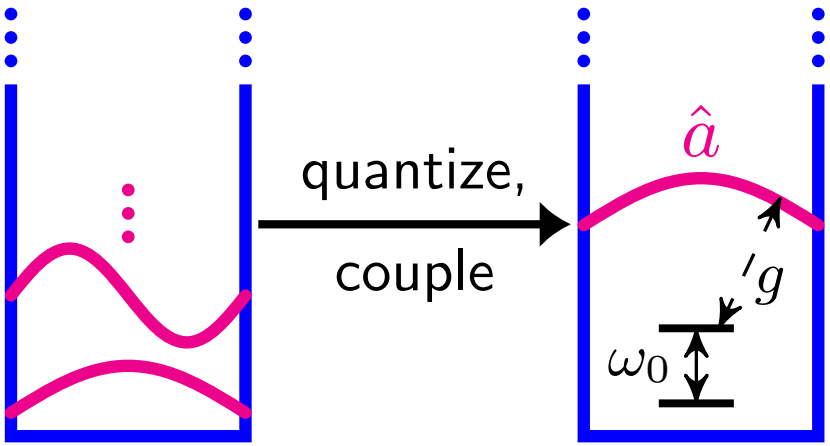

- discrete eigenstates
- closed system
  - × no leakage
  - × no driving
  - × no scattering
  - × no external detection

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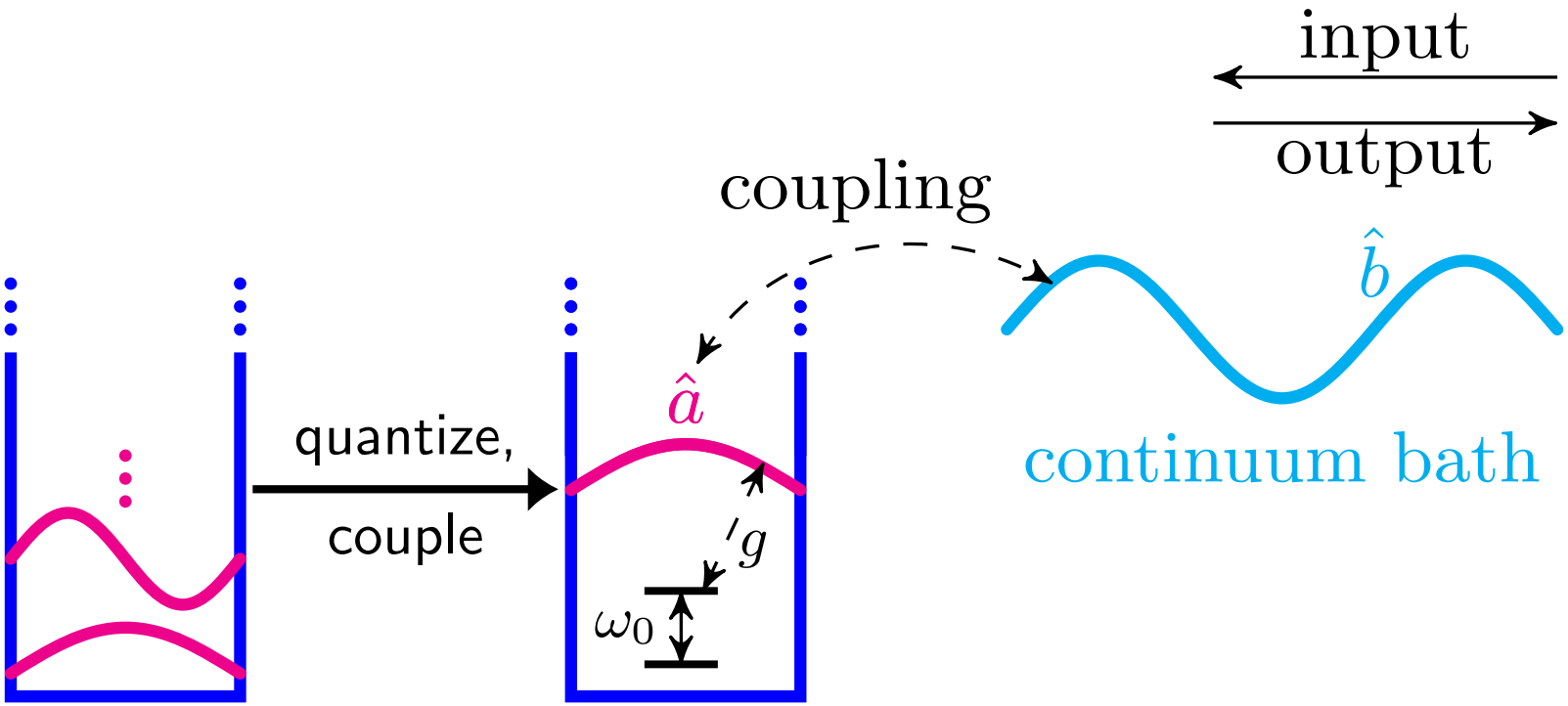

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Input-output theory

$$\hat{b}_{\text{out}} = \hat{b}_{\text{in}} + \kappa\hat{a}$$

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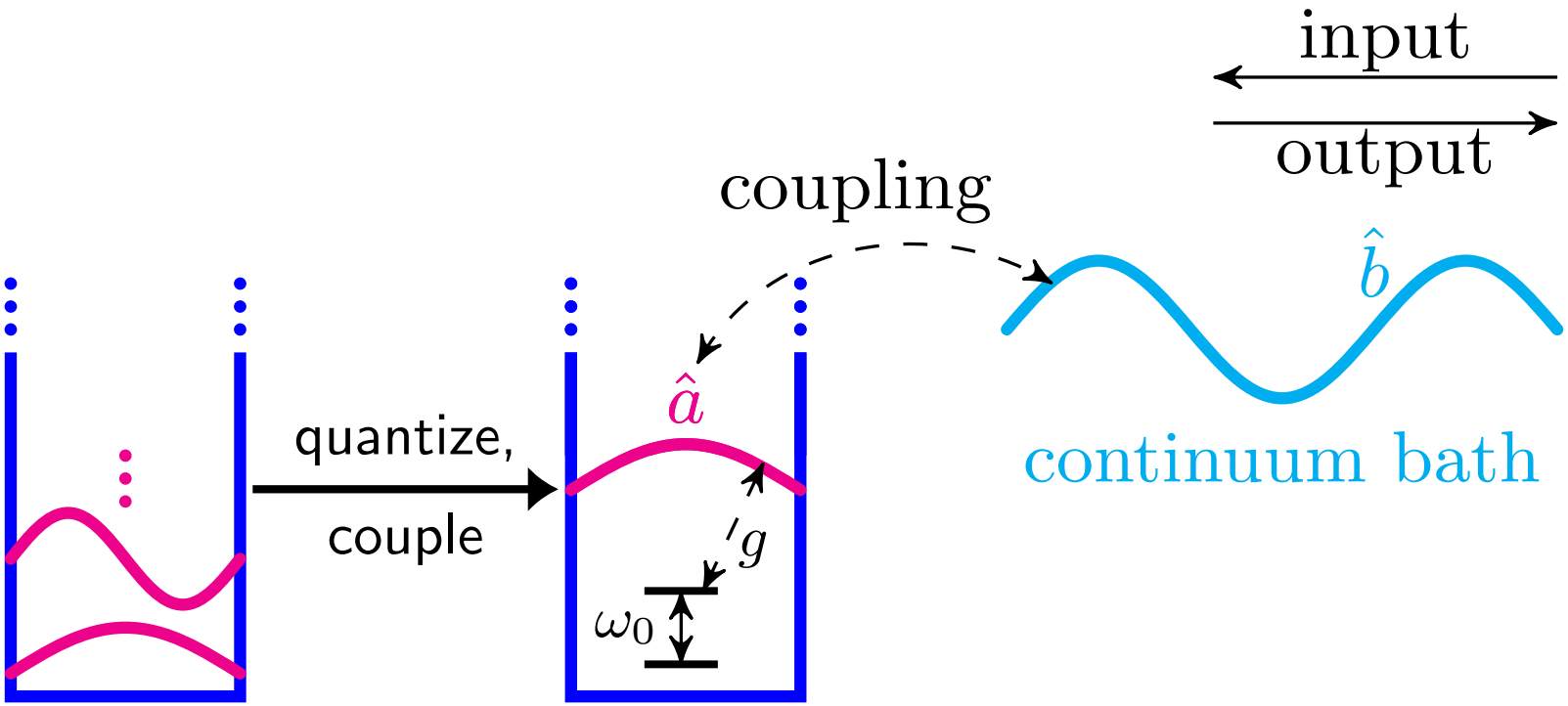

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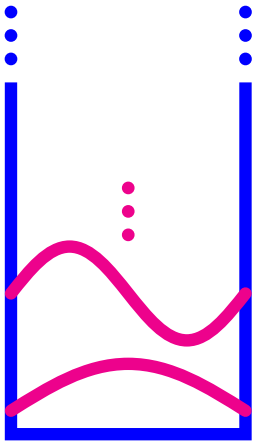
$\rightarrow$ **few-mode concept** $\rightarrow$ **scattering**

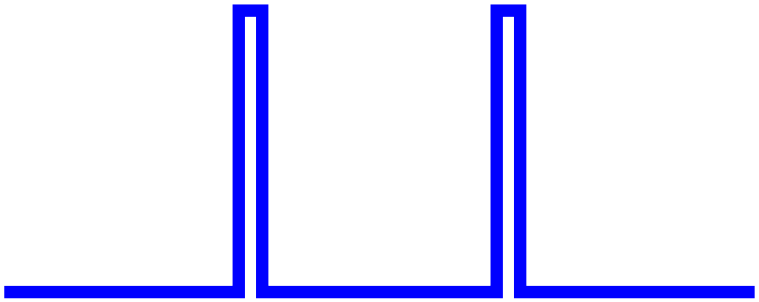
$\Rightarrow$ **Big tool box for quantum optics!**

From closed to open boxes



open system

- ✓ leakage
- ✓ driving
- ✓ scattering



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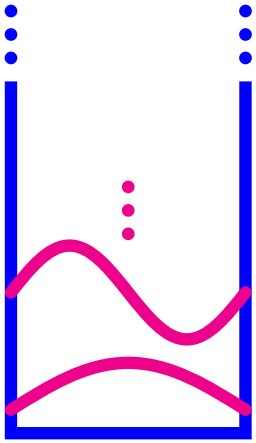

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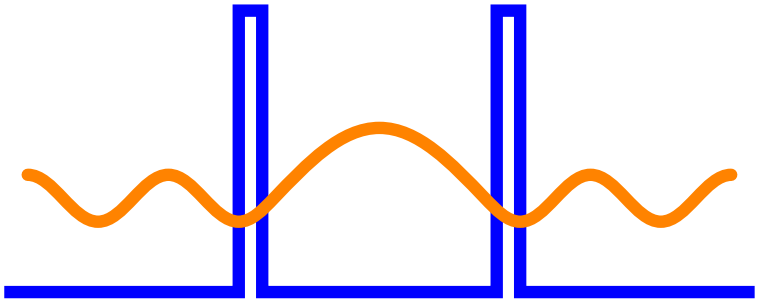

continuum eigenstates

☹ few-mode concept is lost

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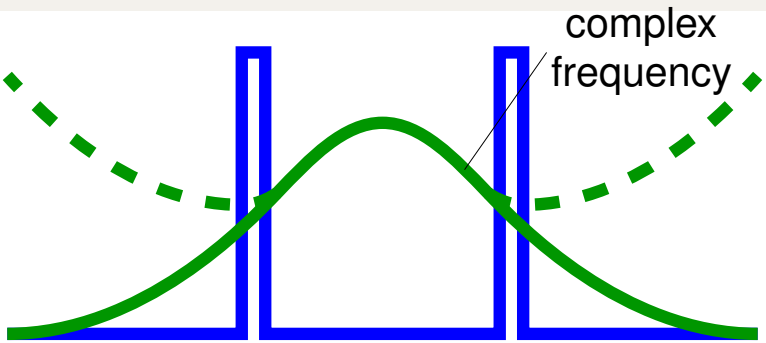
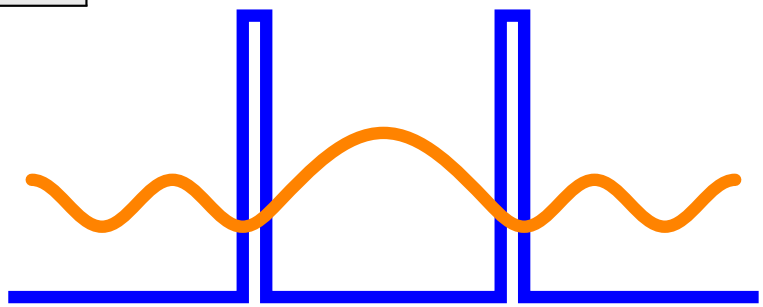


resonant modes



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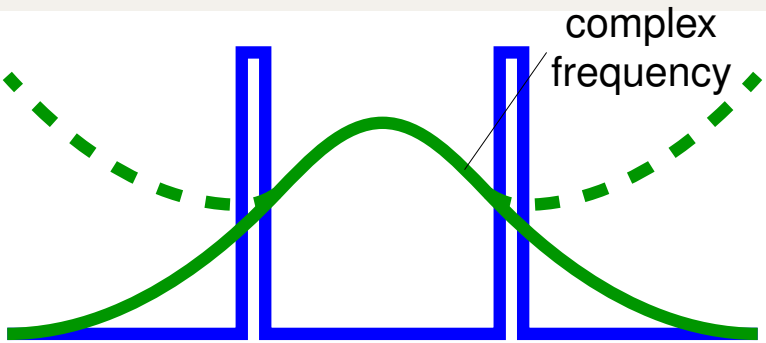

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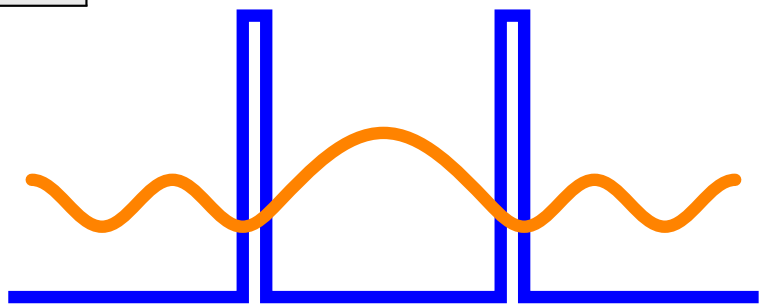

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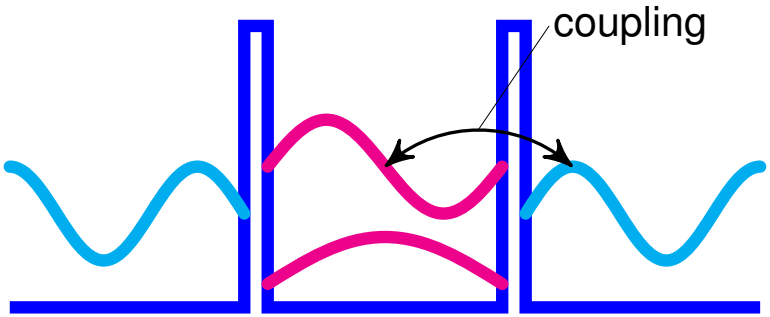
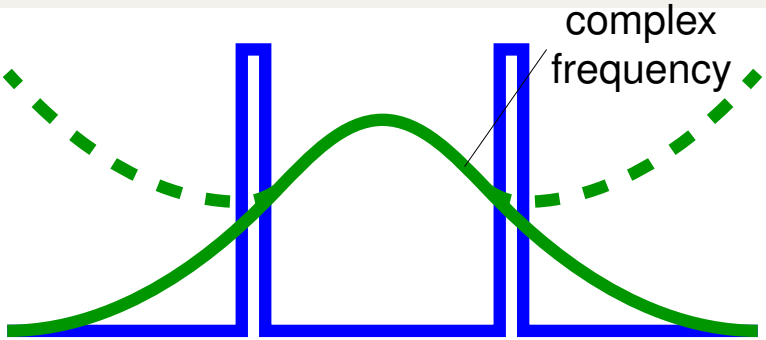


few-mode + bath

From closed to open boxes



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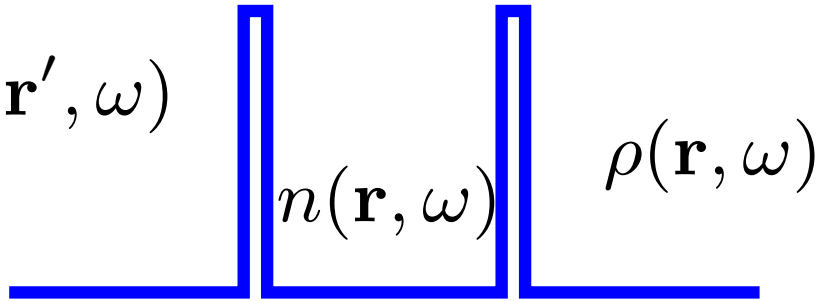
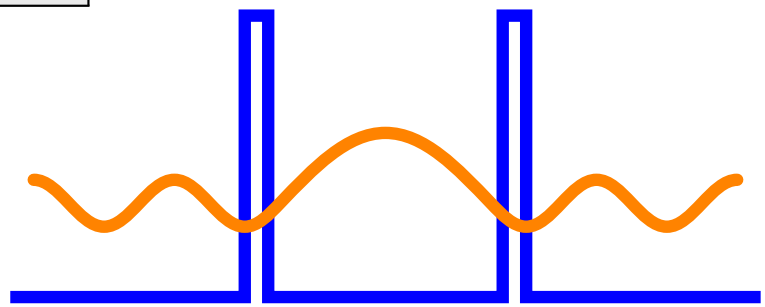


Green fns, LDOS, lin. disp. theory



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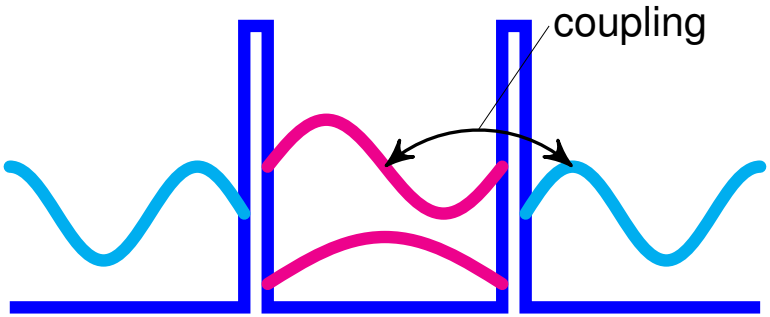
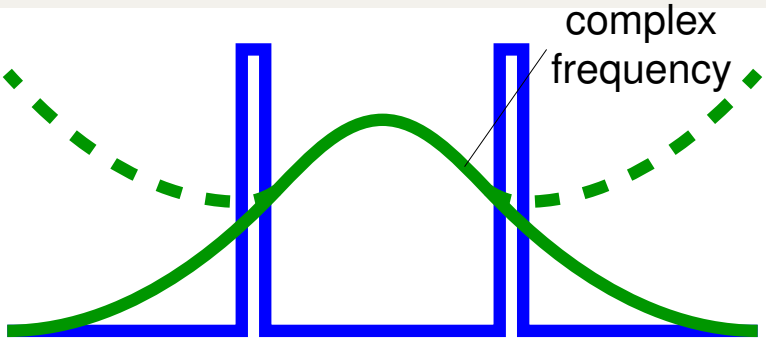


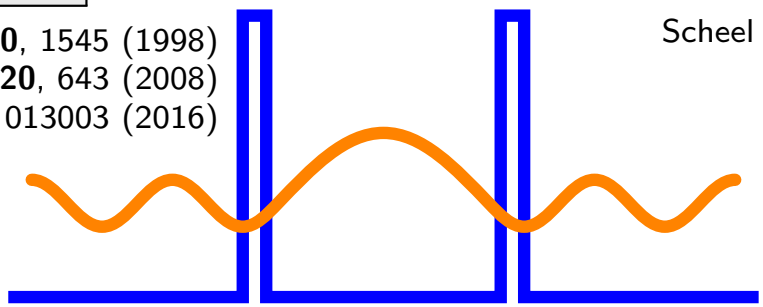
few-mode + bath

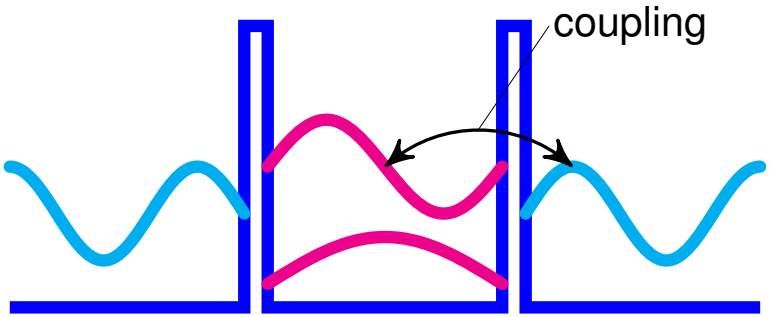
From closed to open boxes



resonant modes

Ching et al. *Rev. Mod. Phys.* **70**, 1545 (1998)
Türeci et al. *Science* **320**, 643 (2008)
Cerjan & Stone *Phys. Scr.* **91** 013003 (2016)





$\underline{\underline{G}}(\mathbf{r}, \mathbf{r}', \omega)$ $n(\mathbf{r}, \omega)$ $\rho(\mathbf{r}, \omega)$

Green fns, LDOS, lin. disp. theory

Scheel & Buhmann *Acta Phys. Slov.* **58**, 675 (2008)
Krimer et al. *Phys. Rev. A* **89**, 033820 (2014)
Zhu et al. *Phys. Rev. Lett.* **64**, 2499 (1990)

continuum eigenstates

☹ few-mode concept is lost

Lang, Scully & Lamb *Phys. Rev. A* **7**, 1788 (1973)
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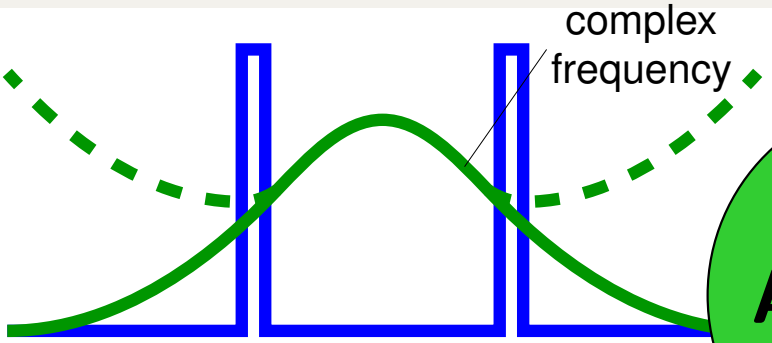

Ab initio

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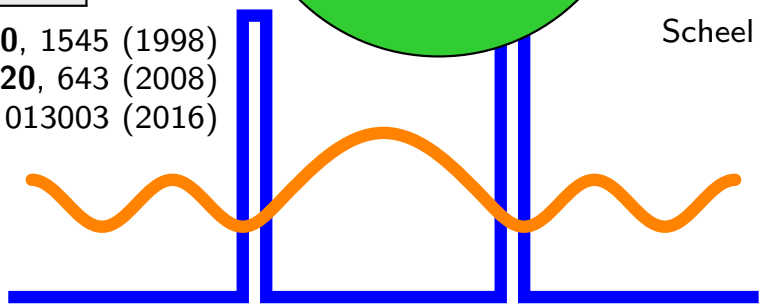

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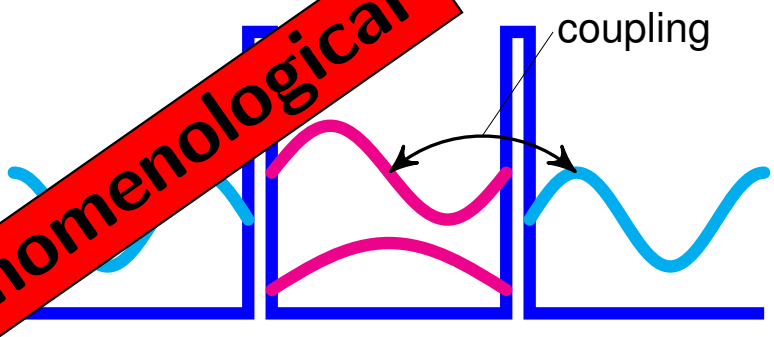

Phenomenological

few-mode + bath

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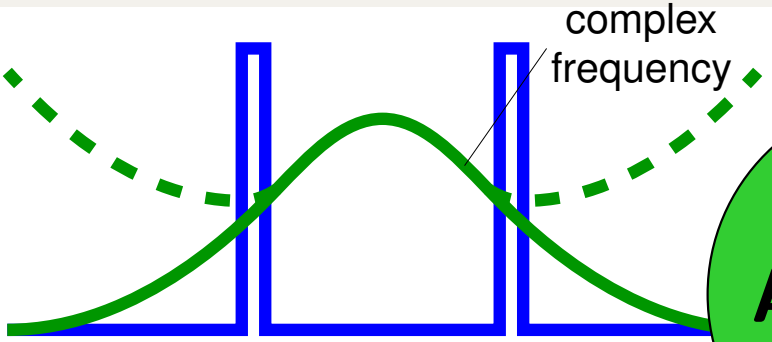
Carmichael *Quantum Optics* (1991)
Gardiner & Zoller *Quantum Noise* (1999)

From closed to open boxes



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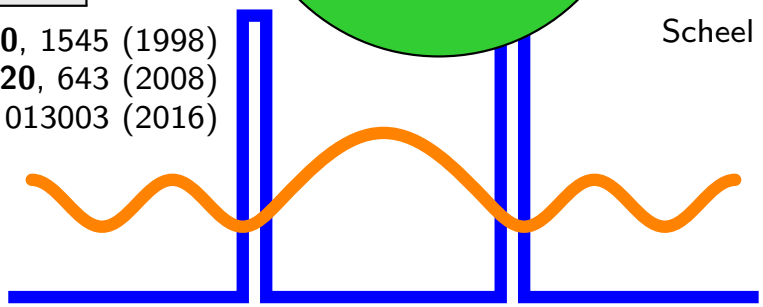

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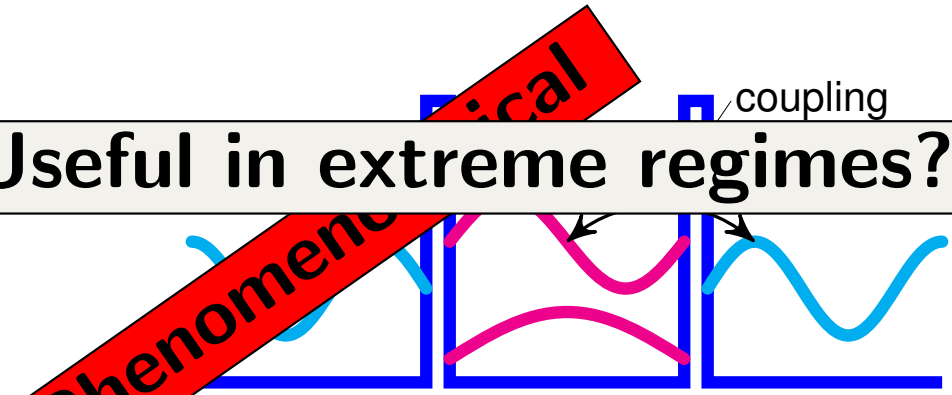
Useful in extreme regimes?

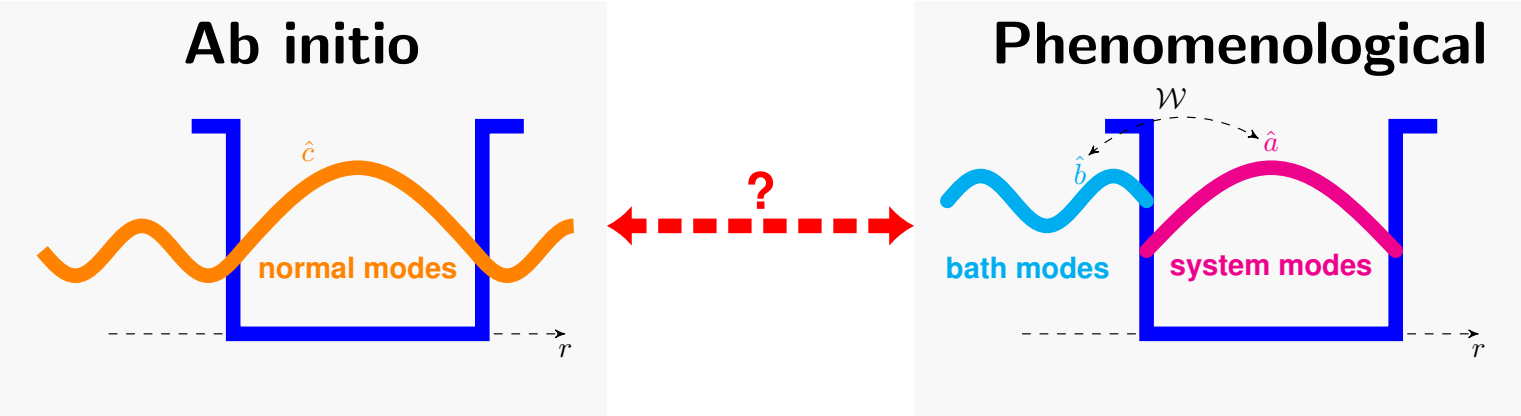


Phenomeno cal

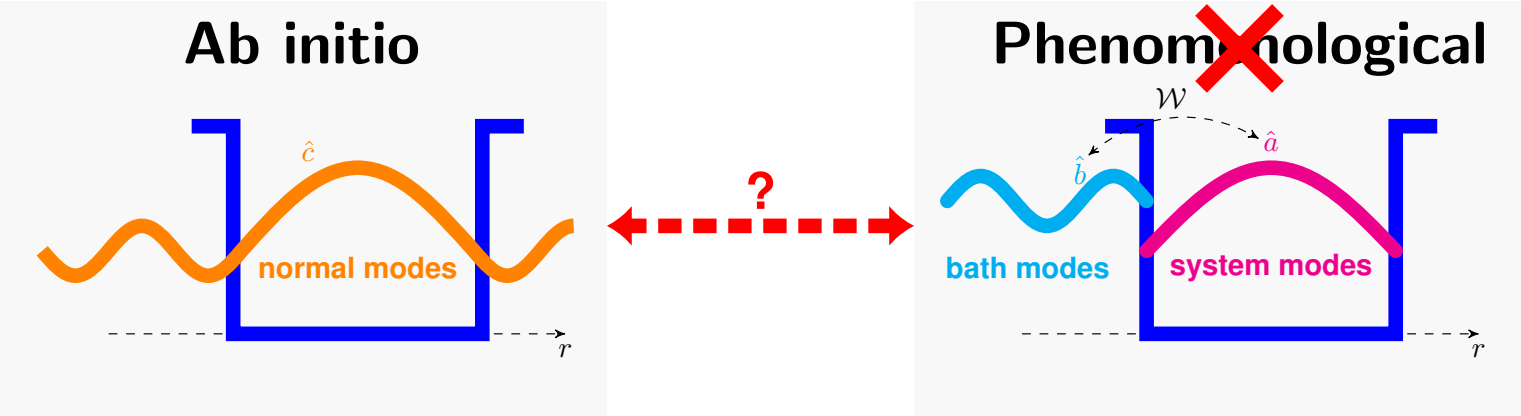
few-mode + bath

Haken *Laser theory* (1984)
Carmichael *Quantum Optics* (1991)
Gardiner & Zoller *Quantum Noise* (1999)

The problem


Ab initio
$\hat{c}$
normal modes
r
?
Phenomenological
$\mathcal{W}$
$\hat{a}$
$\hat{b}$
bath modes
system modes
r

The problem



How to make

- **few-mode**

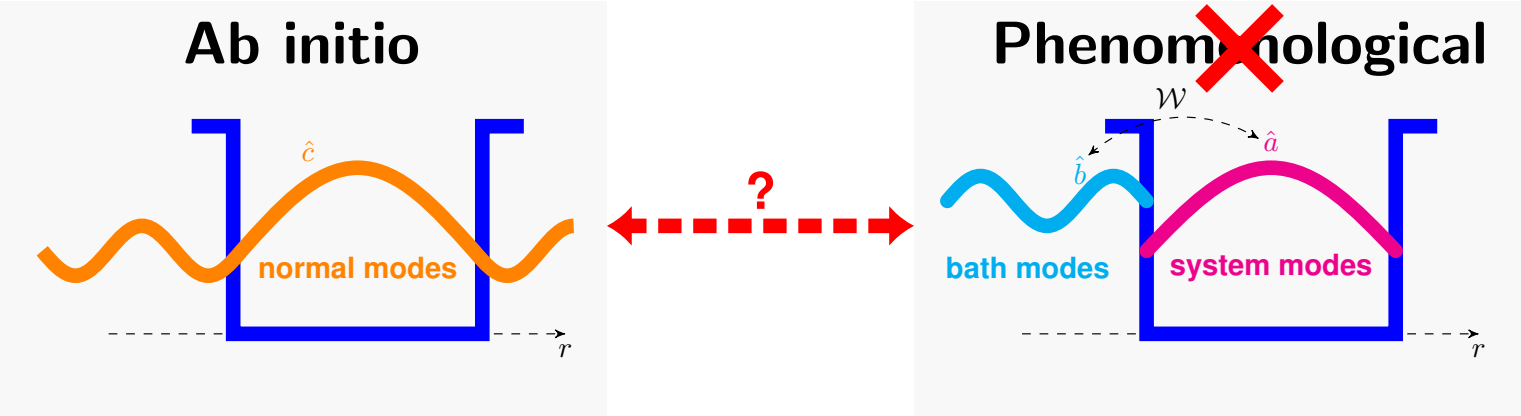
and

- **input-output**

ab initio?

?

The problem



How to make

- **few-mode**

and

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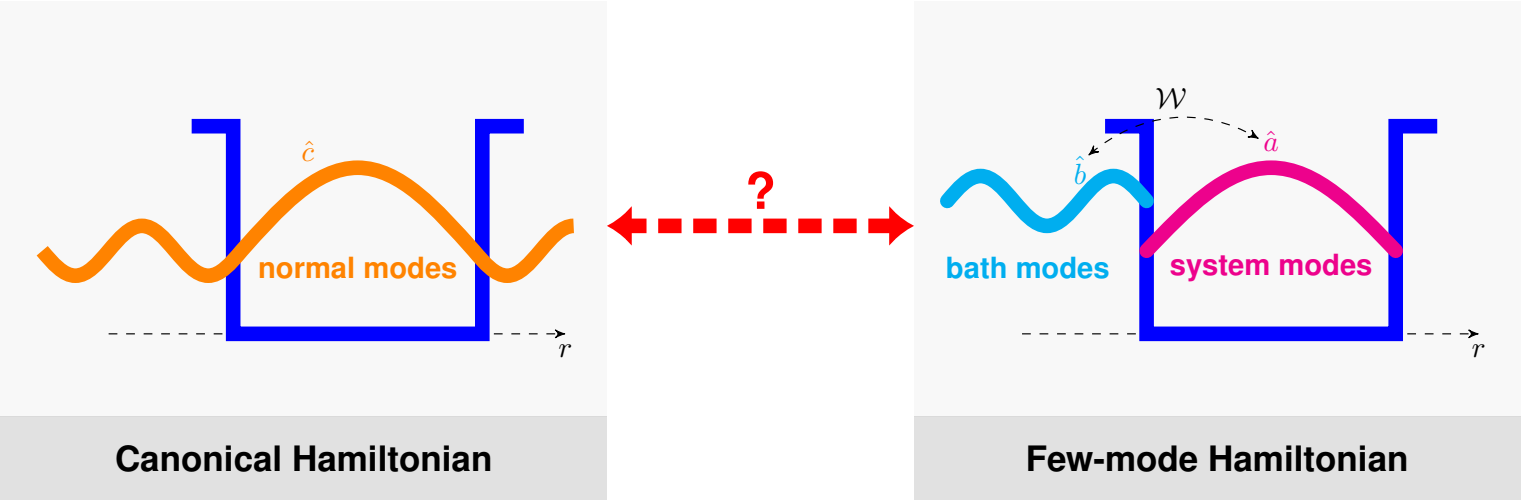
ab initio?

?

⇒ **Ab initio few-mode theory**

Ab initio few-mode Hamiltonians



Glauber & Lewenstein, *Phys. Rev. A* **43**, 467 (1991)
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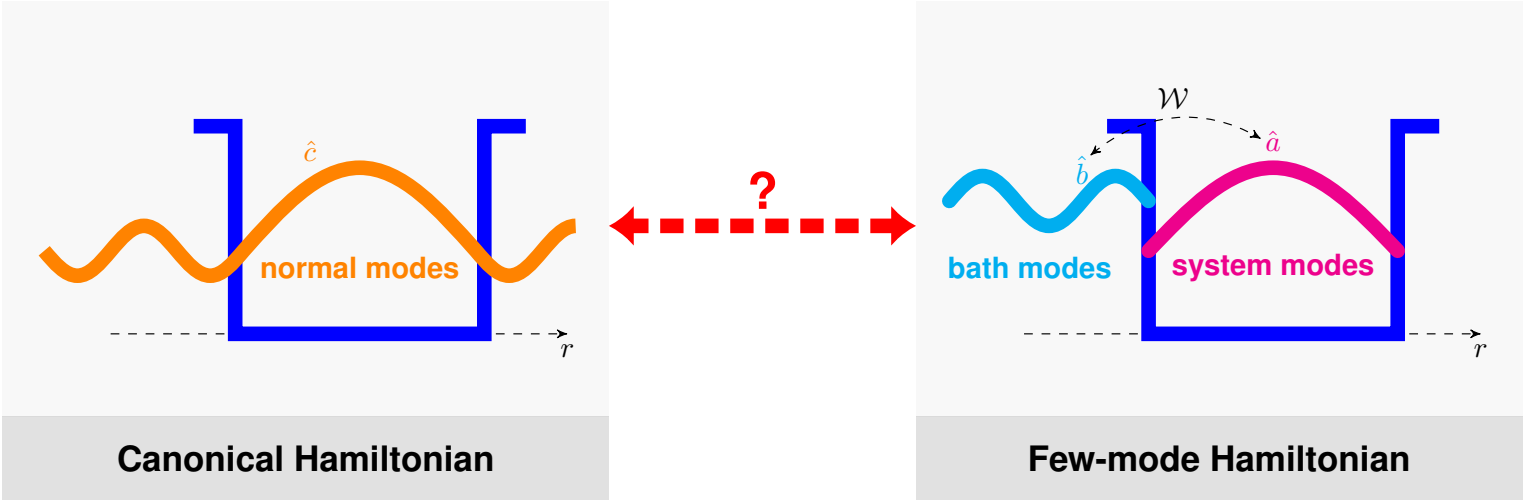
Ab initio few-mode Hamiltonians



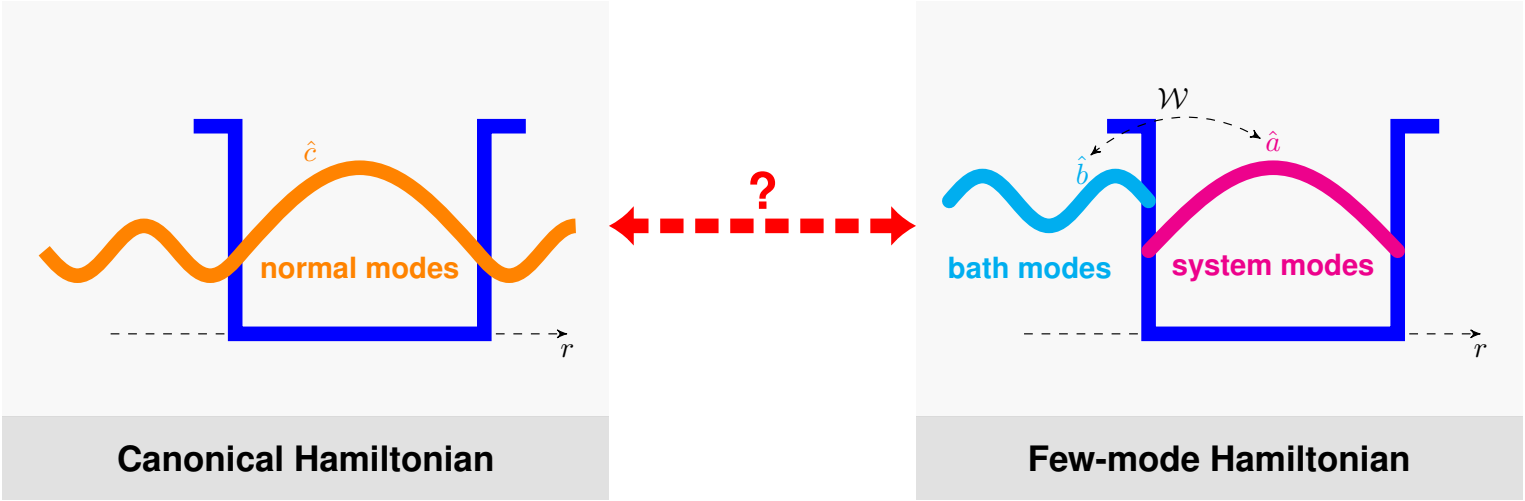
normal modes — discrete modes — external continuum

$$\hat{c}(\omega) = \sum_{\lambda} \alpha_{\lambda}(\omega)\hat{a}_{\lambda} + \int d\omega' \beta(\omega,\omega')\hat{b}(\omega')$$ [1,2]

Glauber & Lewenstein, *Phys. Rev. A* **43**, 467 (1991)
Gardiner & Collett, *Phys. Rev. A* **31**, 3761 (1985)

[1] Viviescas & Hackenbroich, *Phys. Rev. A* **67**, 013805 (2003)
[2] Domcke, *Phys. Rev. A* **28**, 2777 (1982)
[3] DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

Ab initio few-mode Hamiltonians



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$\Rightarrow$ select resonant states as few-mode basis[3]

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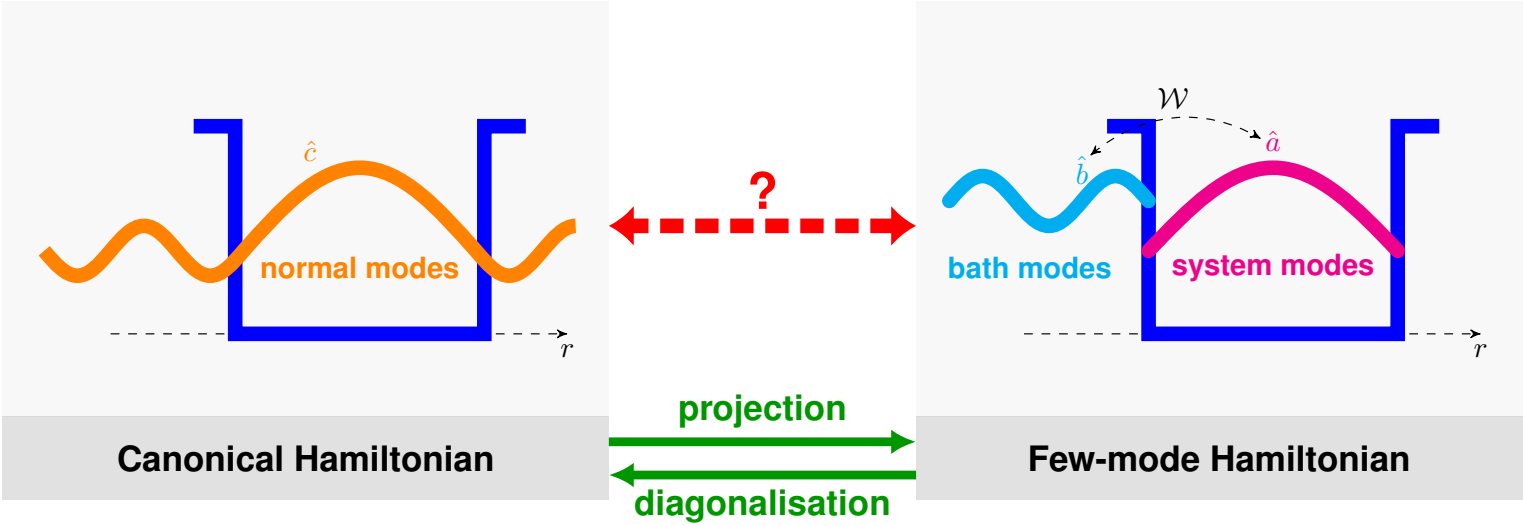

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$\Rightarrow$ **ab initio few-mode Hamiltonians** ☺ [3]

Glauber & Lewenstein, *Phys. Rev. A* **43**, 467 (1991)
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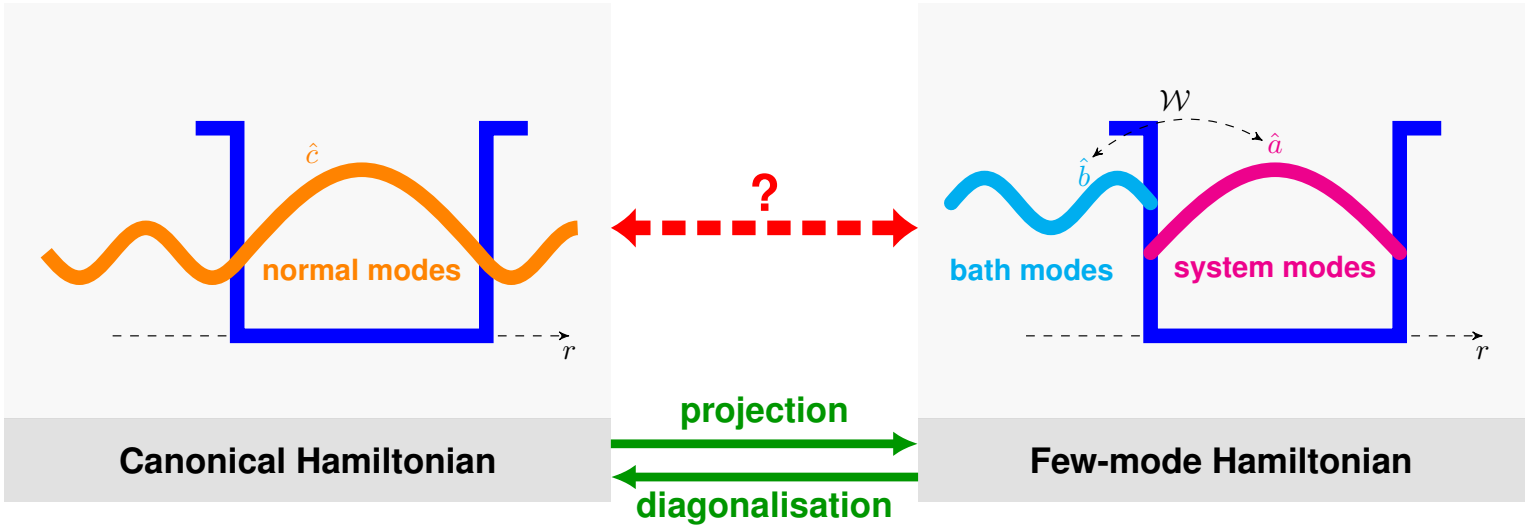
Few-mode scattering



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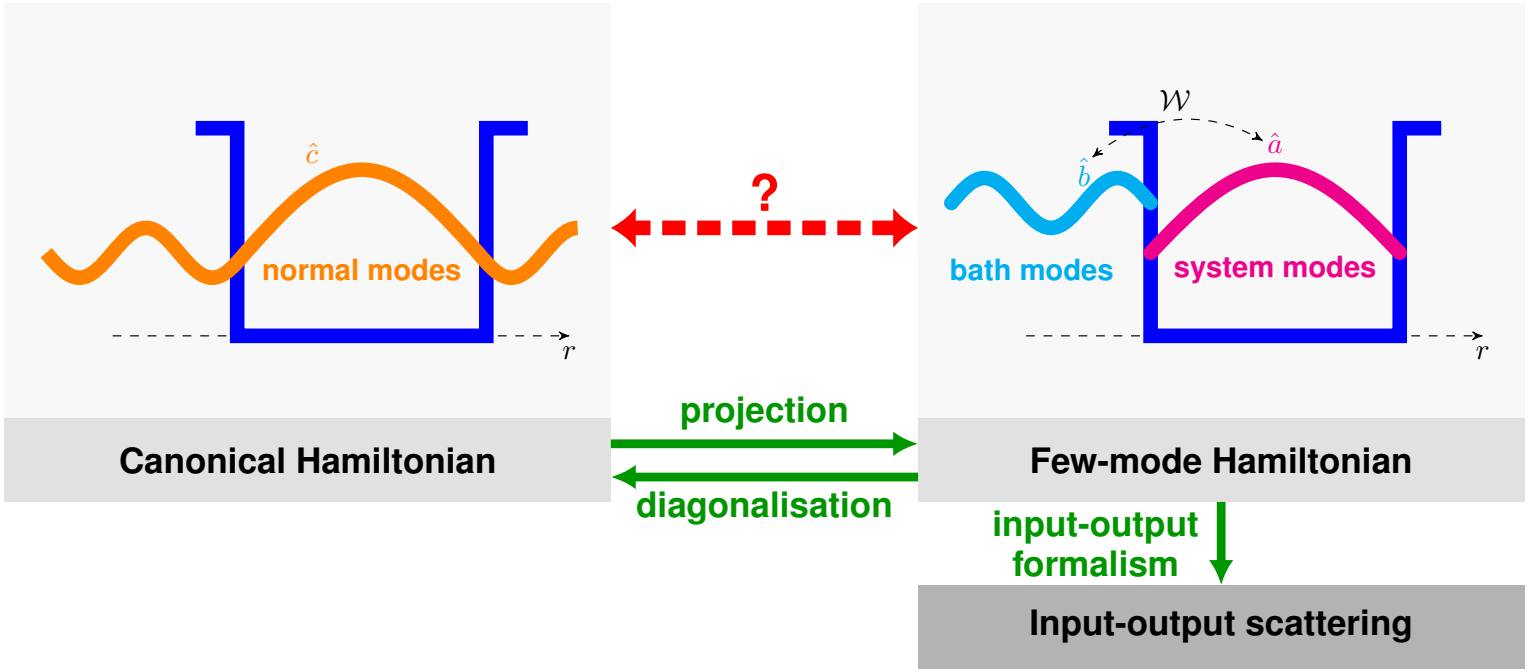

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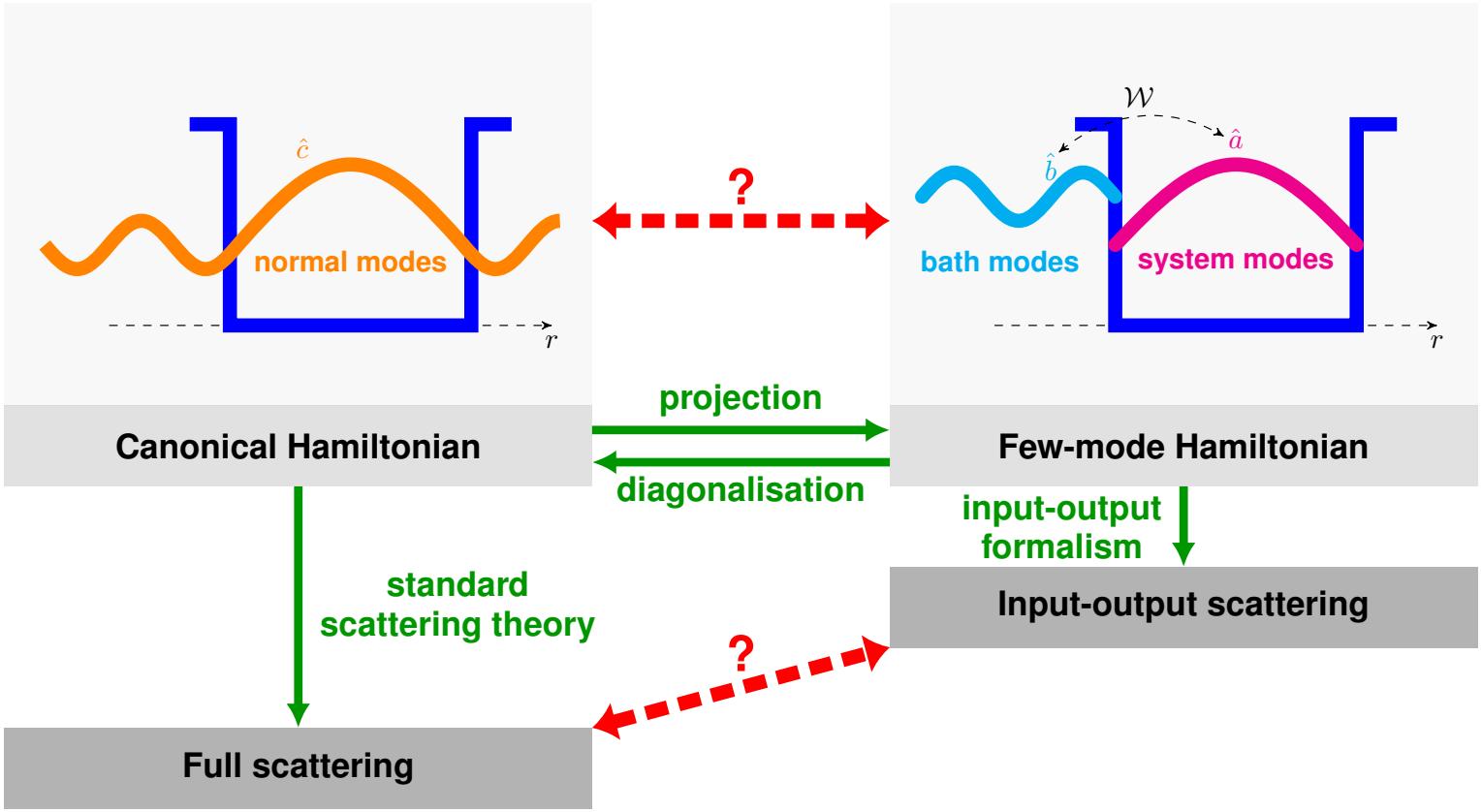

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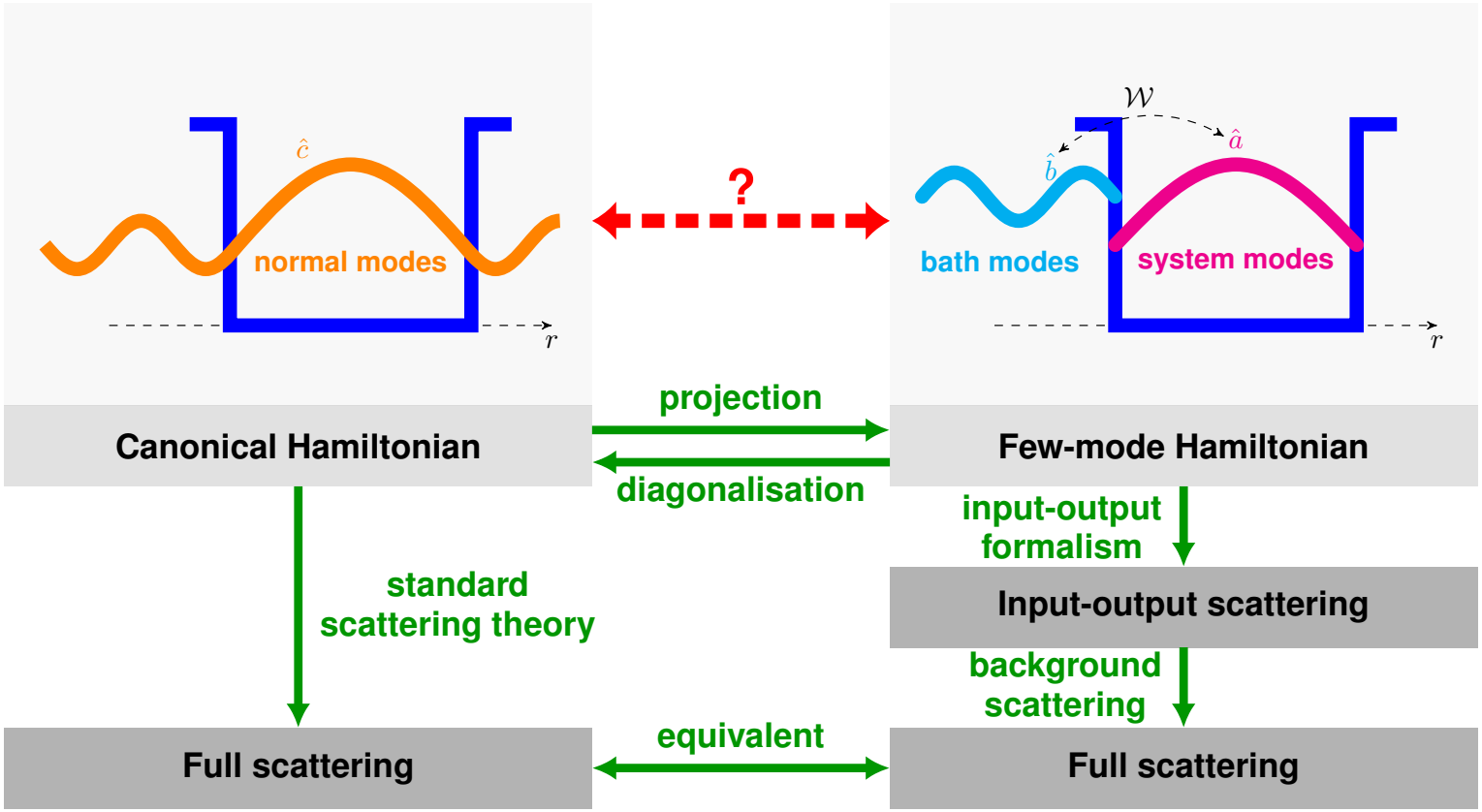

$$S = S_{\mathrm{bg}} S_{\mathrm{io}}$$

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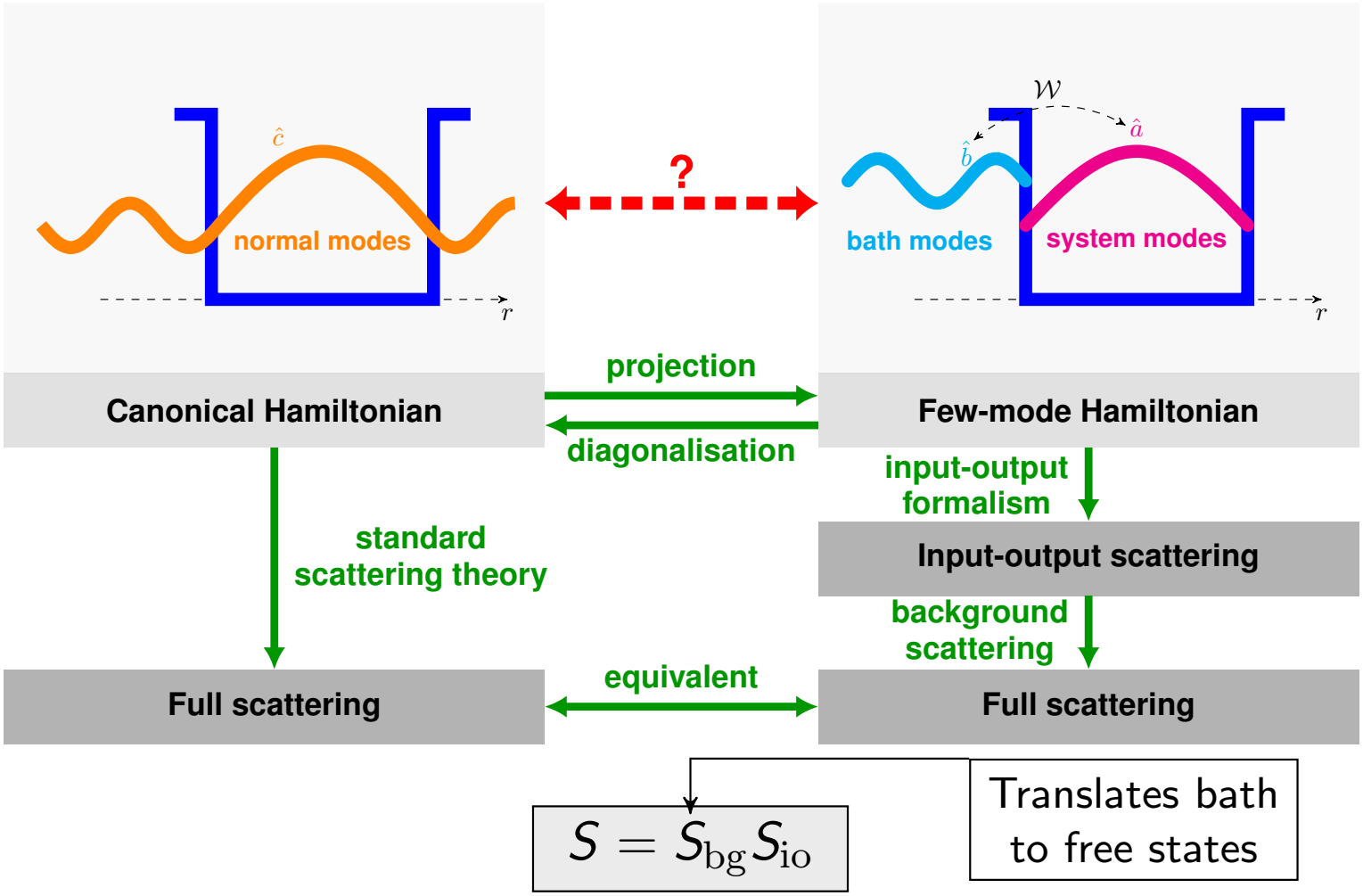
Few-mode scattering



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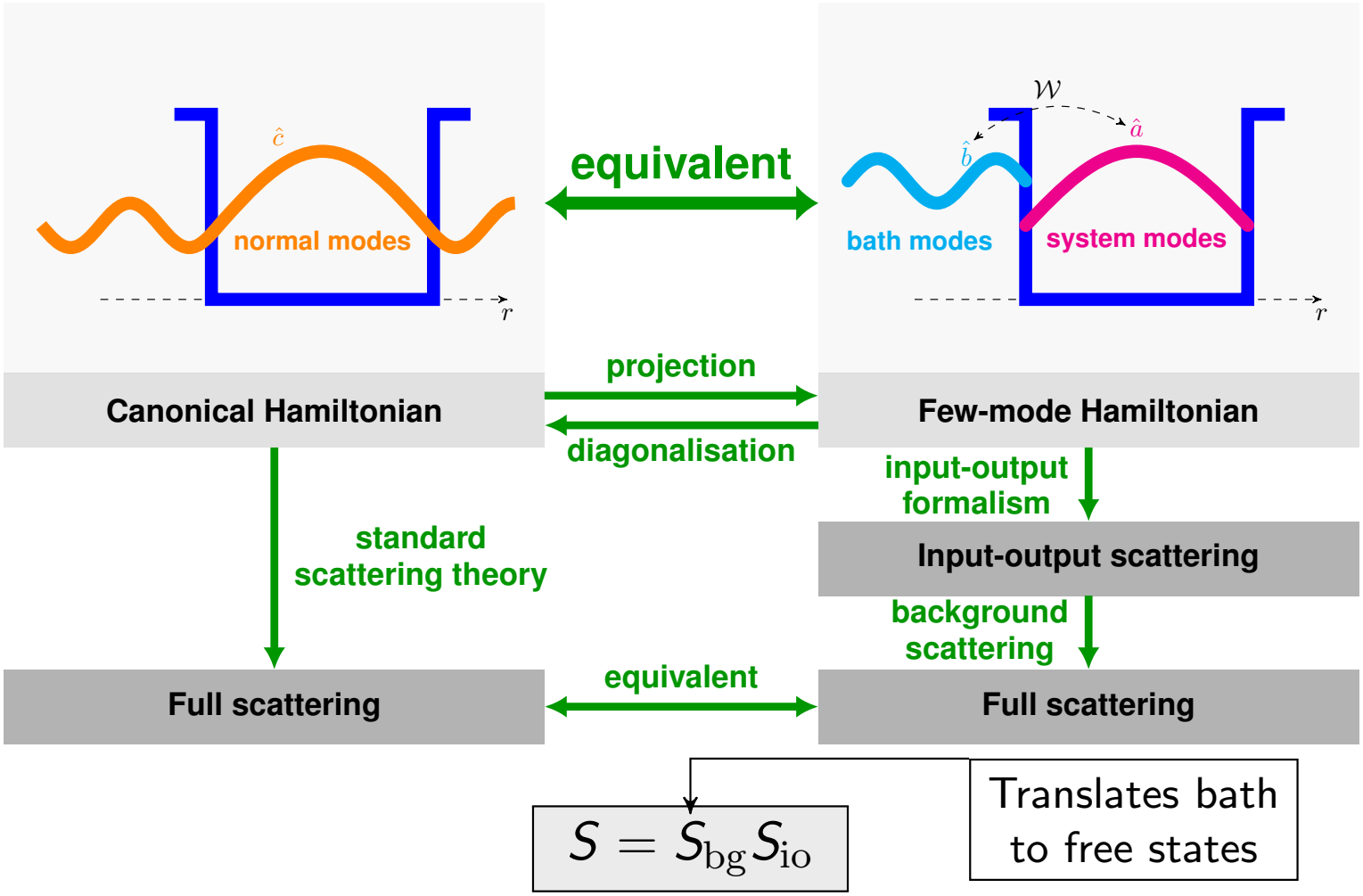

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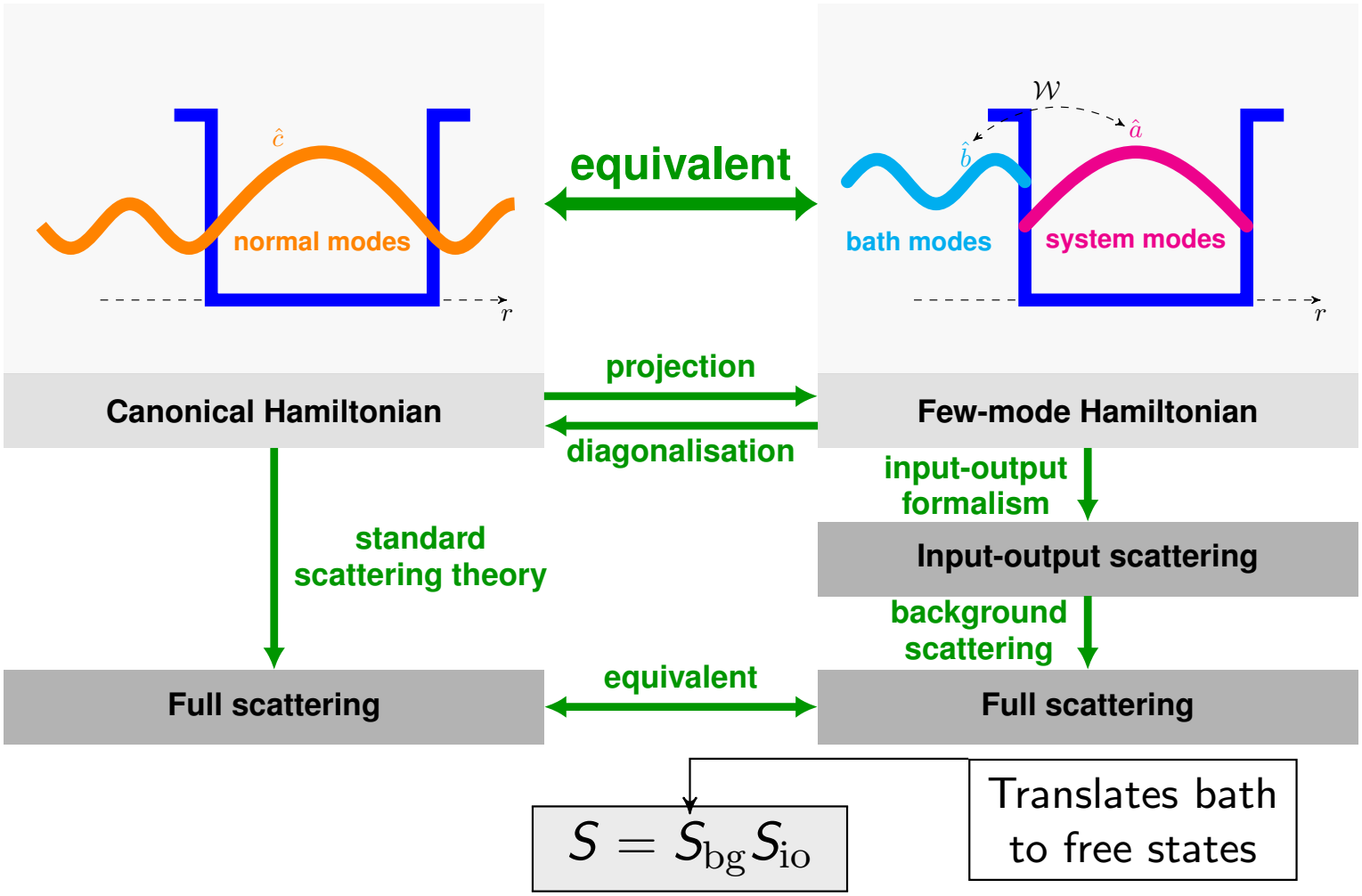
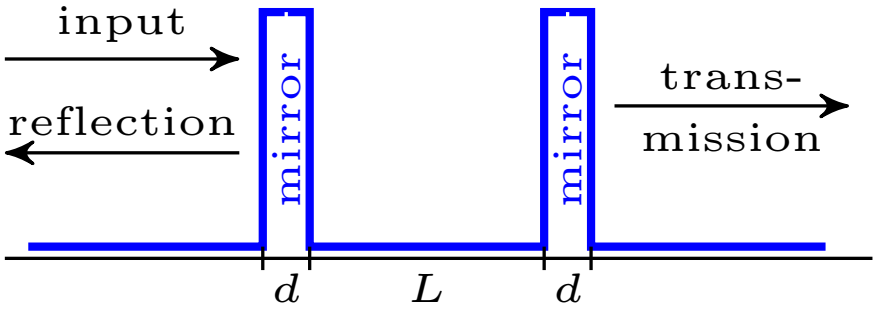


$\Rightarrow$ Few-mode theory is not limited to the good cavity regime!

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

Illustrative example

Example: Two-sided Fabry-Perot cavity



Ley & Loudon *J. Mod. Opt.* **34**, 227-255 (1987)

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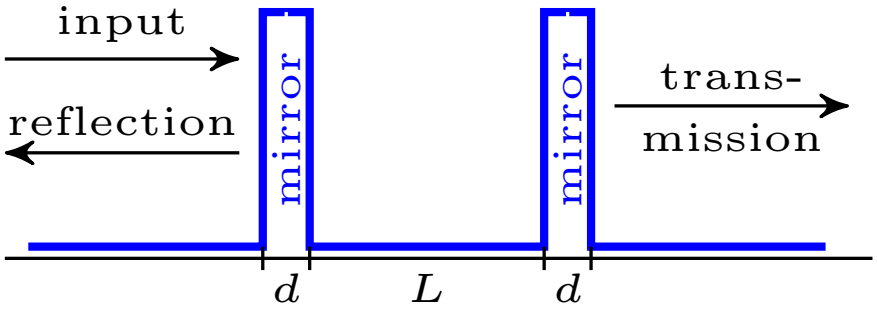

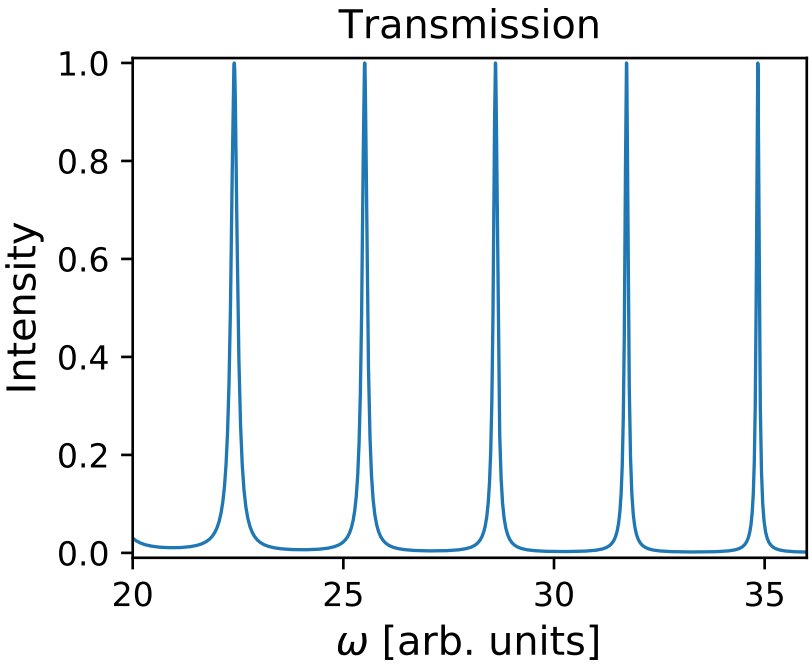

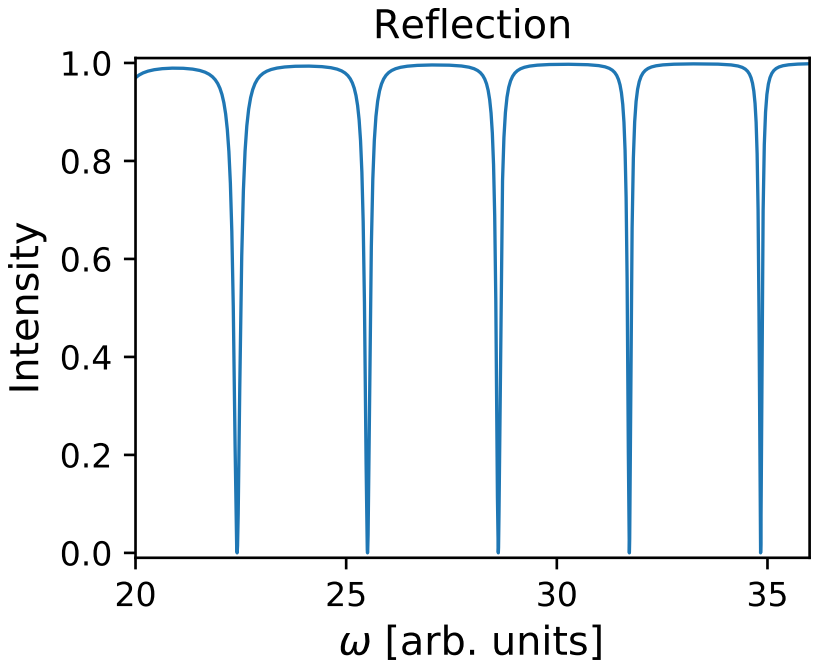

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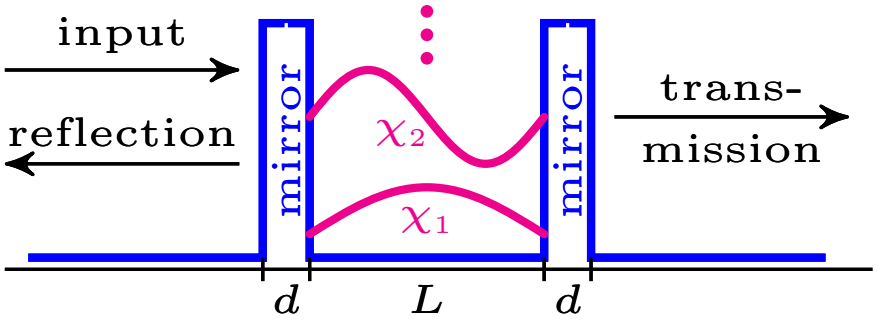

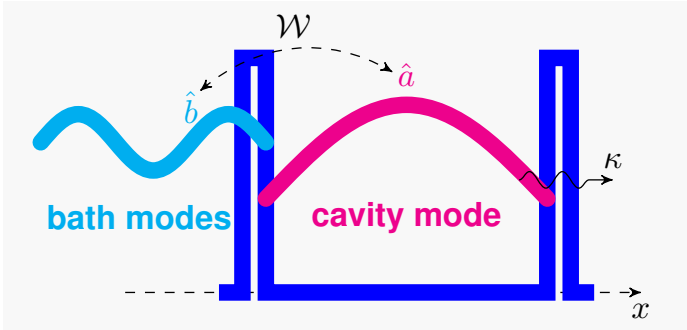

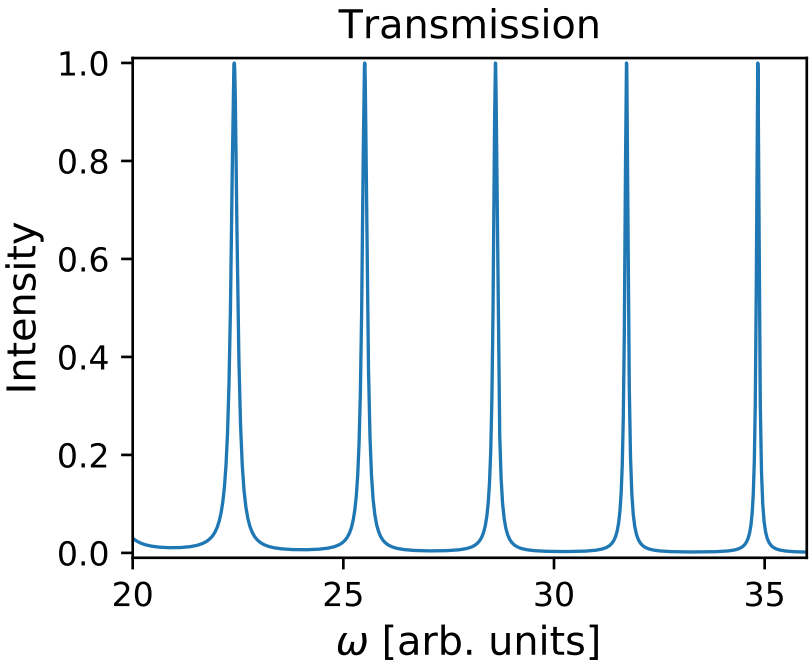

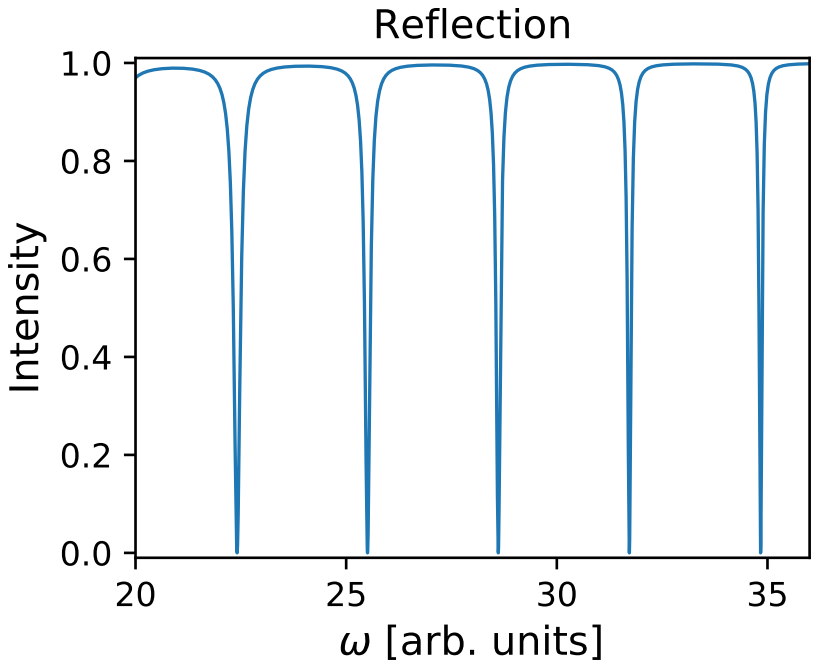

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

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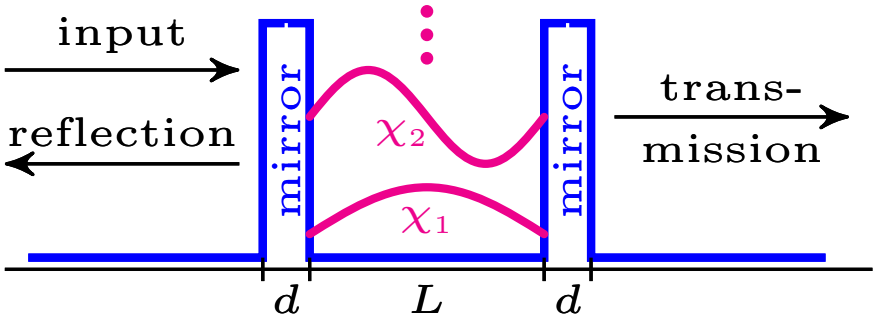
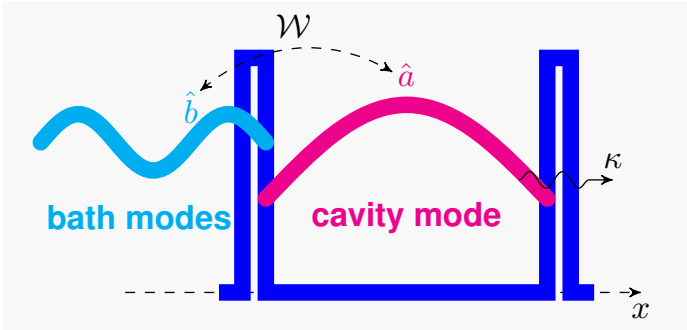
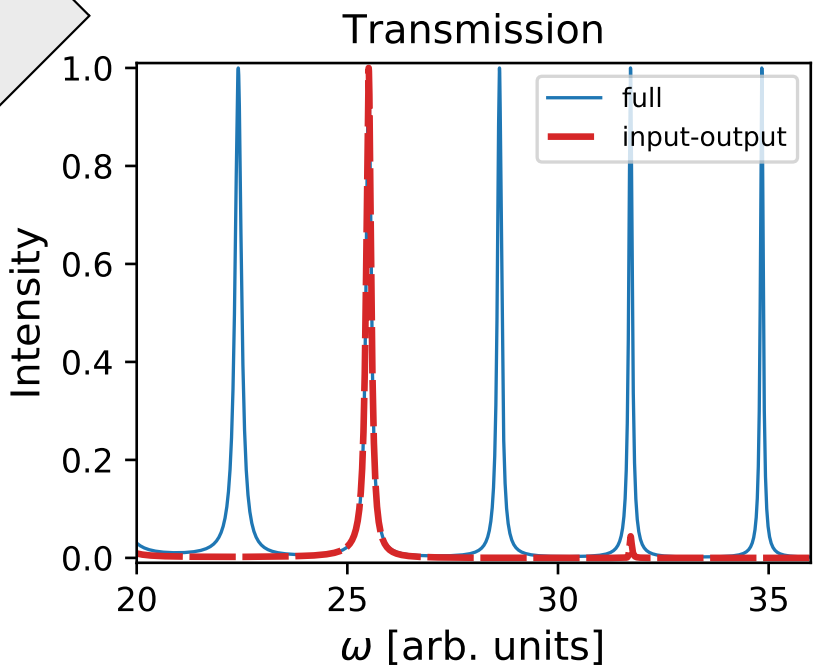
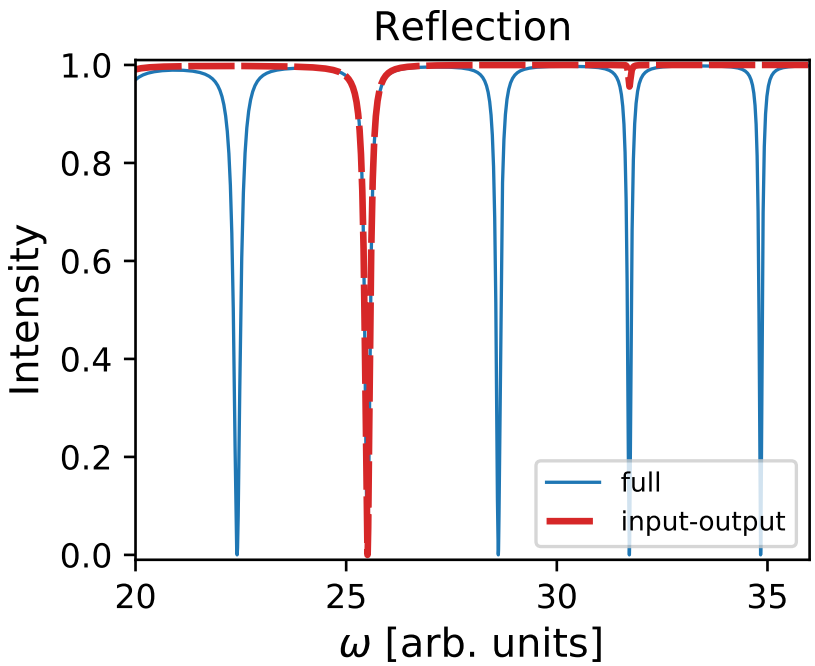




χ_8





$\Rightarrow$ ab initio, not a fit!

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

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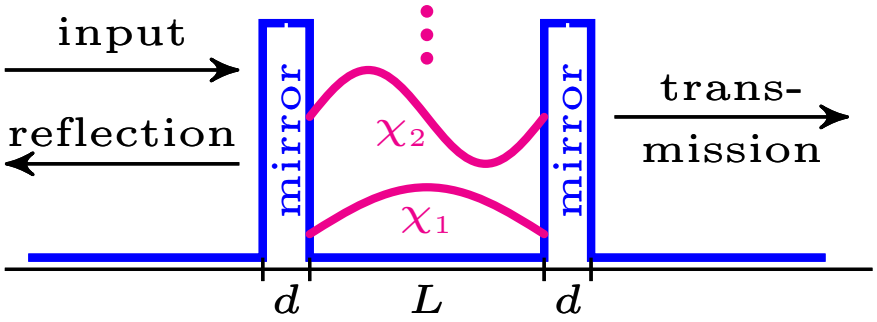
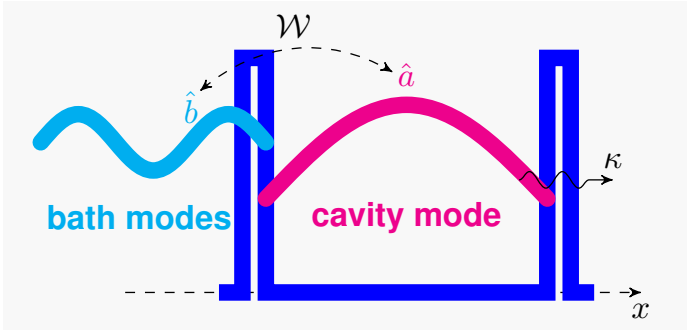
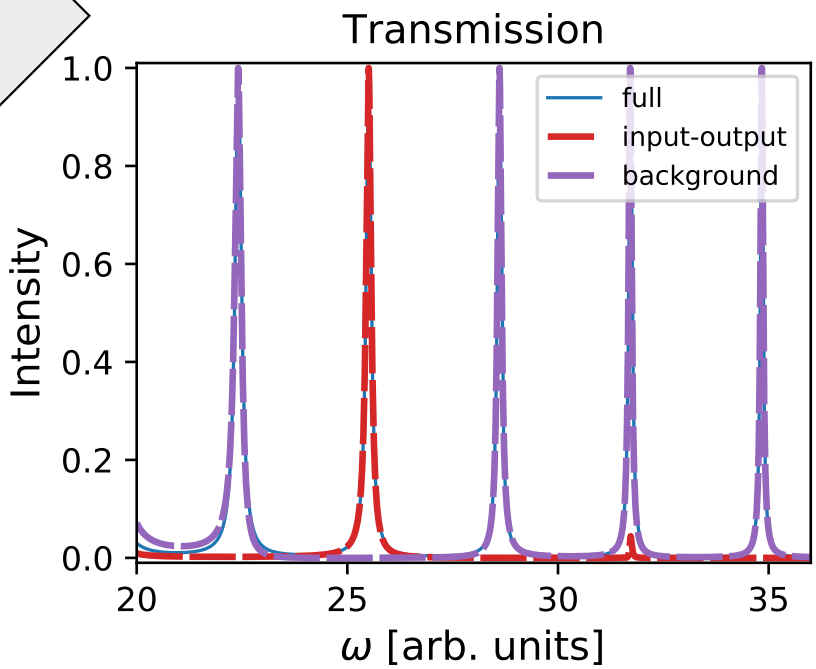
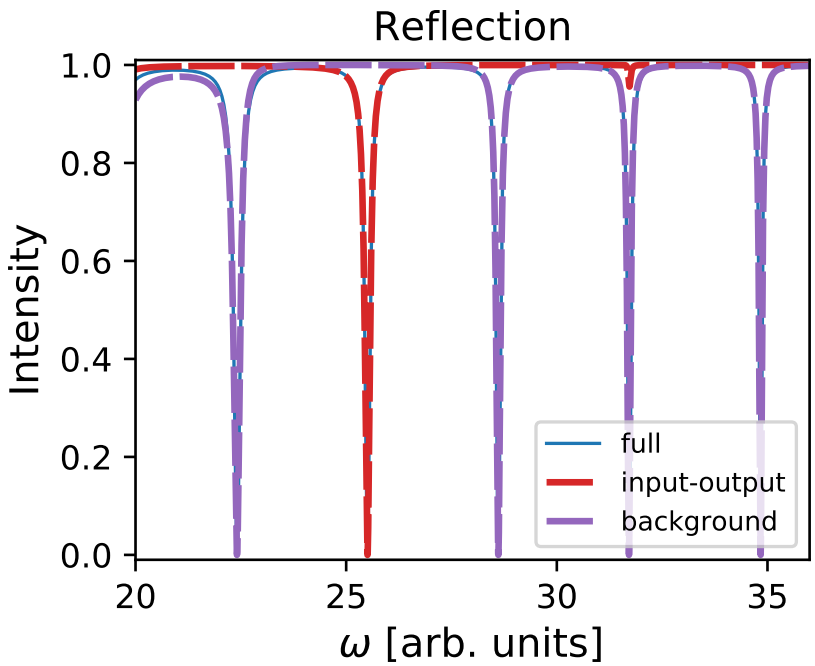




χ_8





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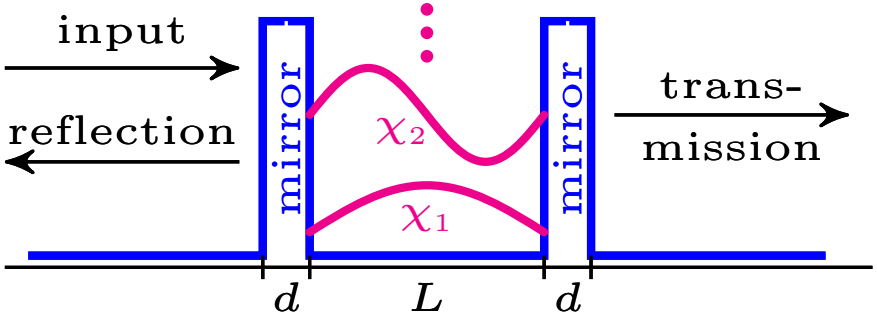
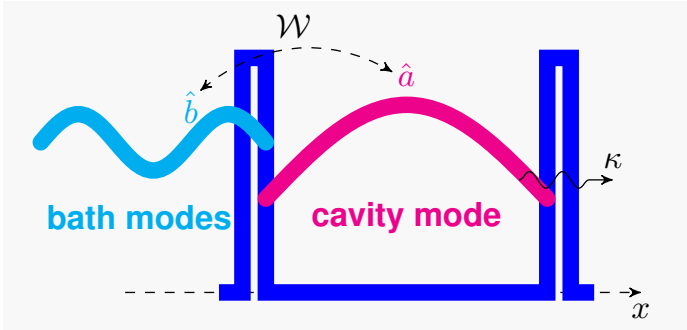
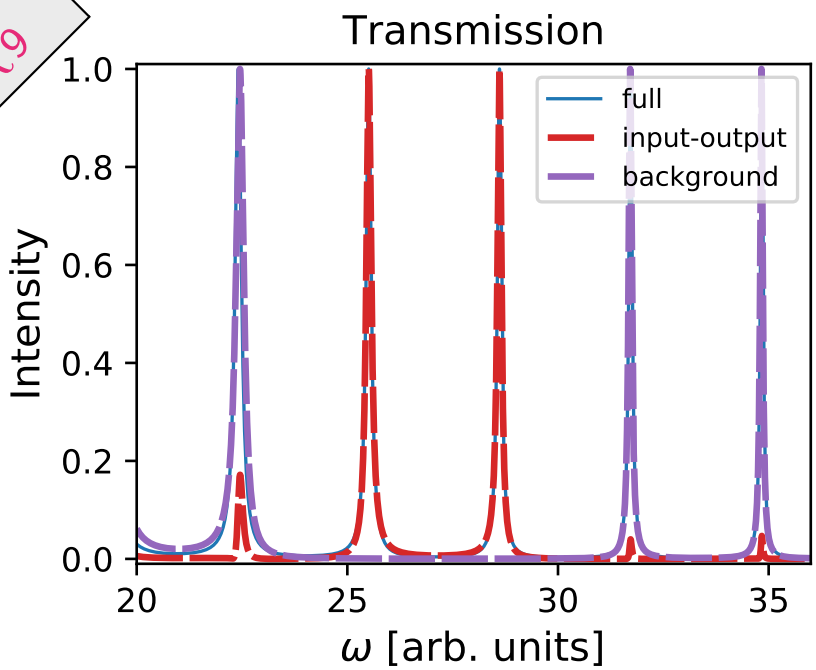
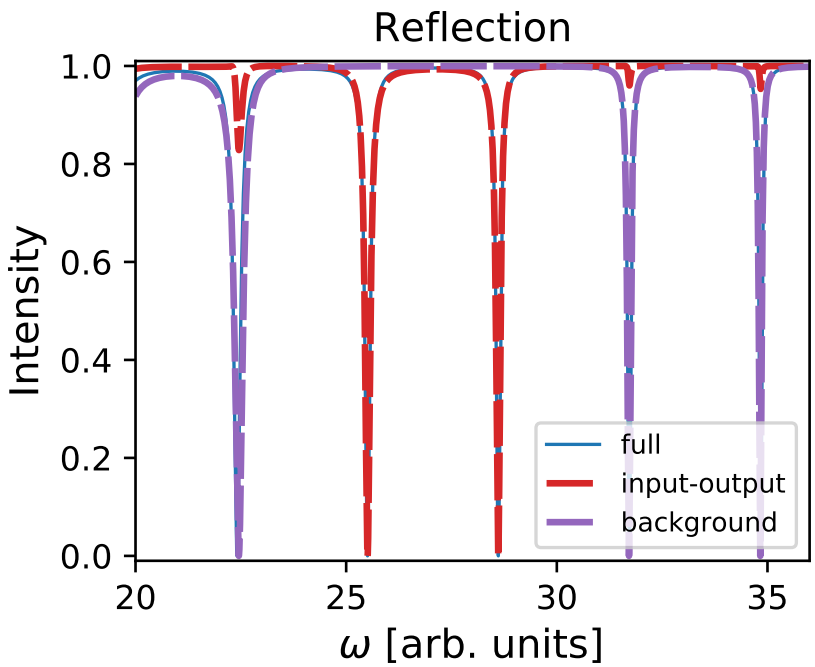




χ_8, χ_9

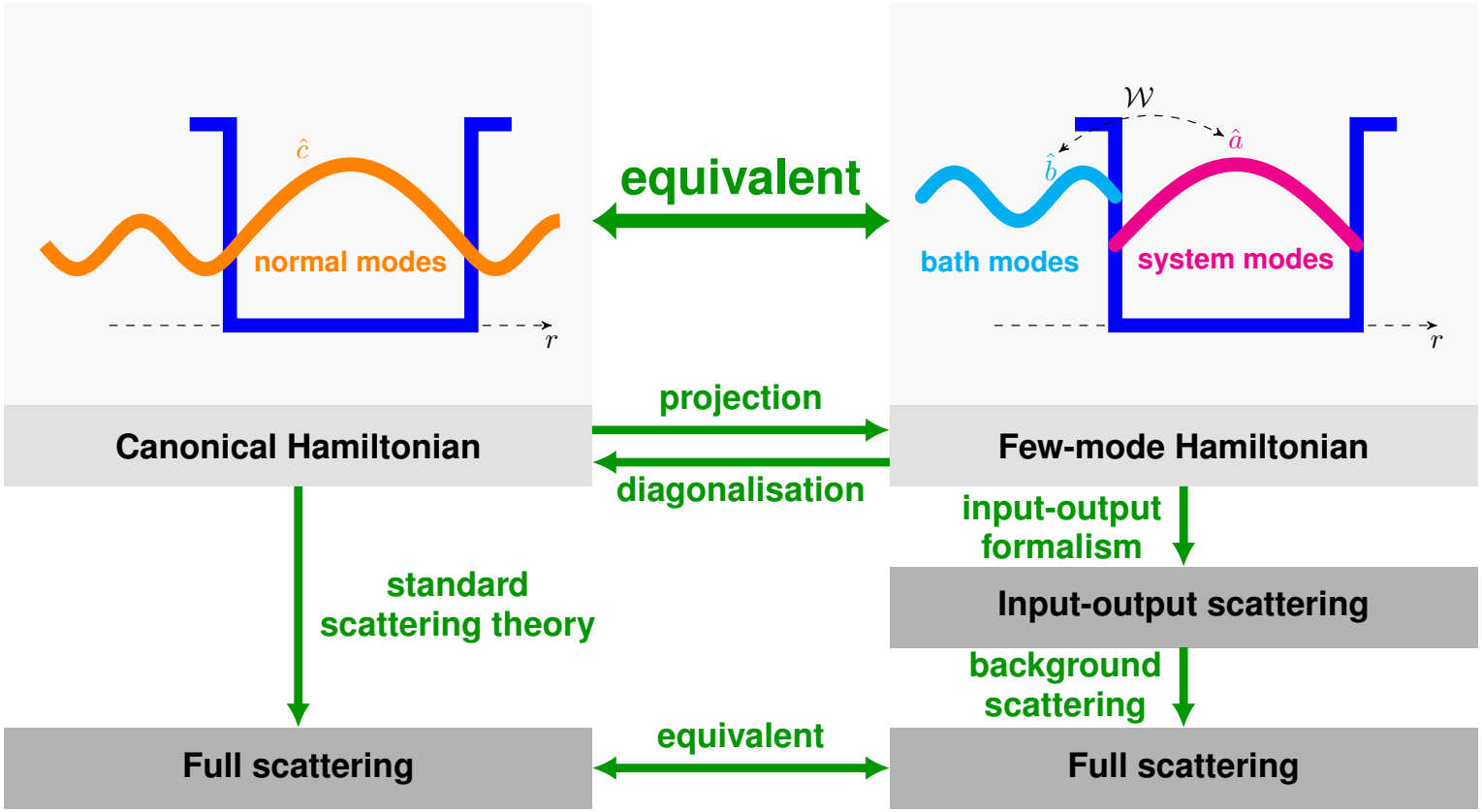




$\Rightarrow$ extract resonant dynamics

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

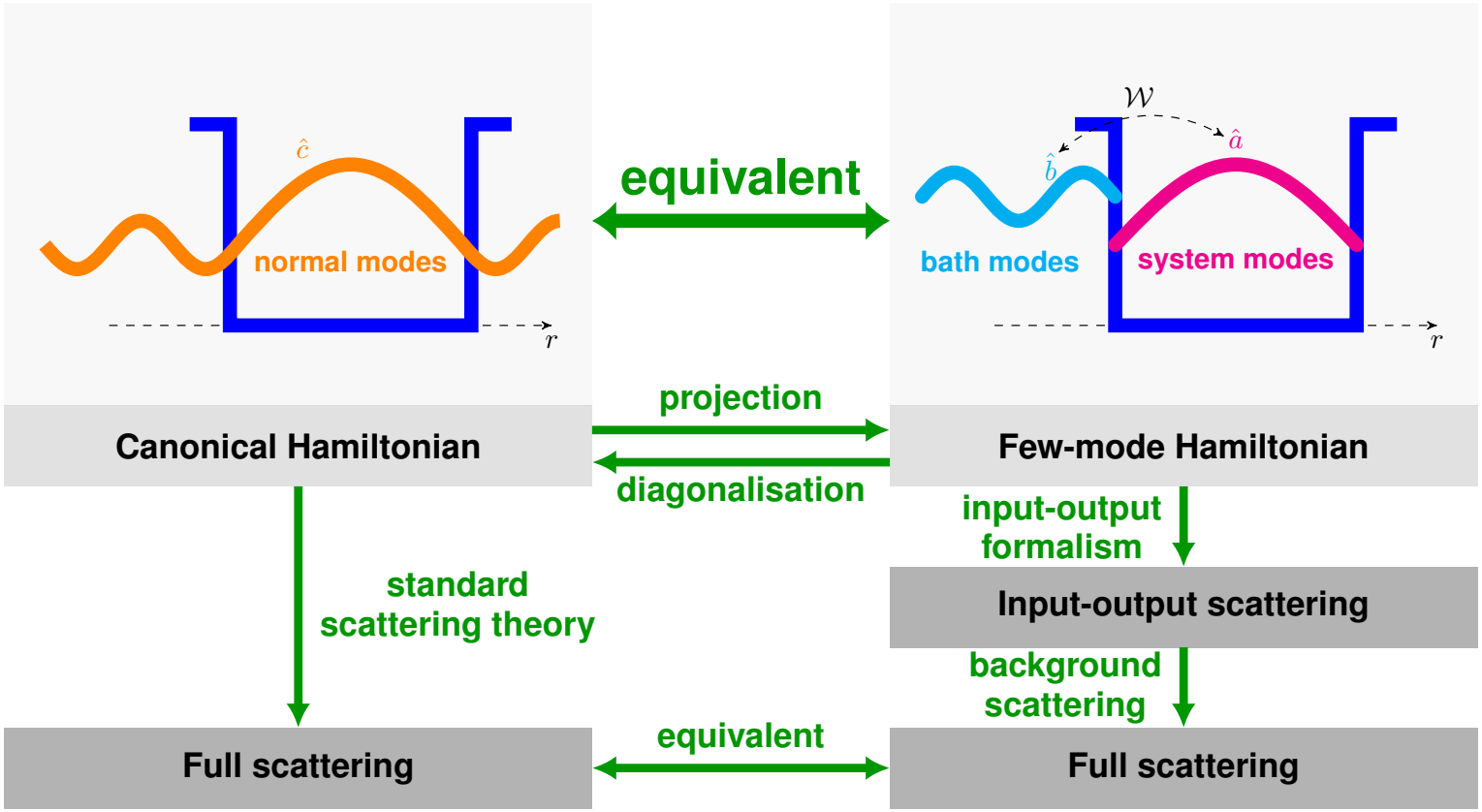
Advantages (so far)



Exact few-mode theory for the empty cavity!

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

Advantages (so far)

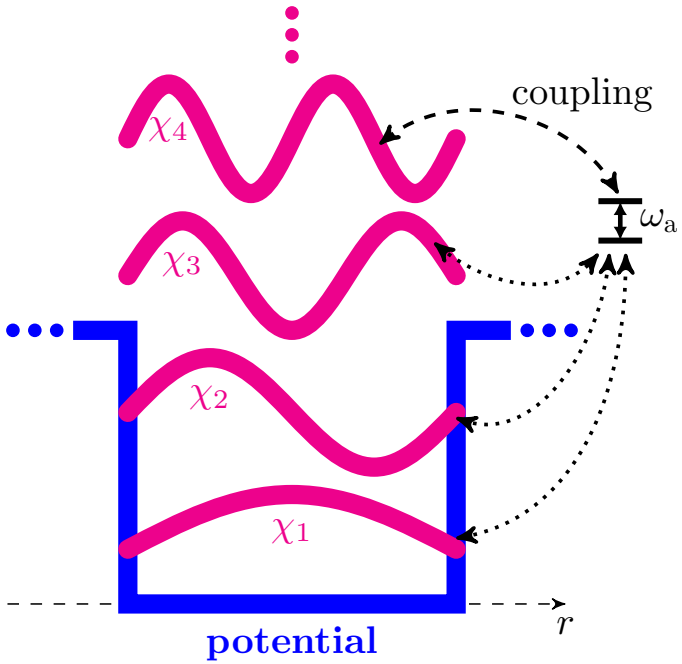


Exact few-mode theory for the empty cavity!

But what about field-matter interactions?

DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

Effective few-mode expansions



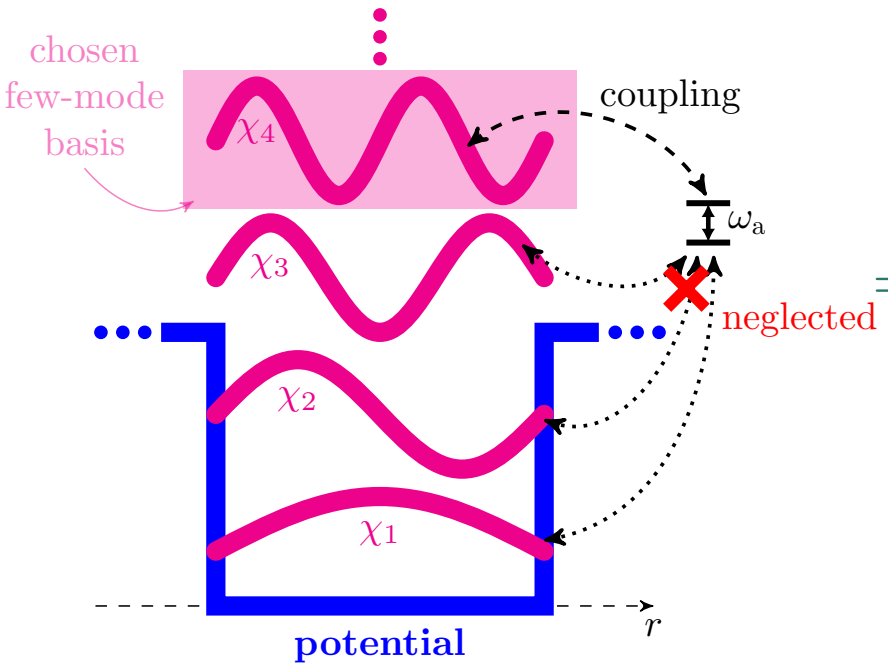
Recipe:

1. choose few-mode basis
2. perform few-mode approximation in interaction
3. include more modes if necessary

$\Rightarrow$ **Expansion scheme!**

DL & J. Evers, *submitted*

Effective few-mode expansions



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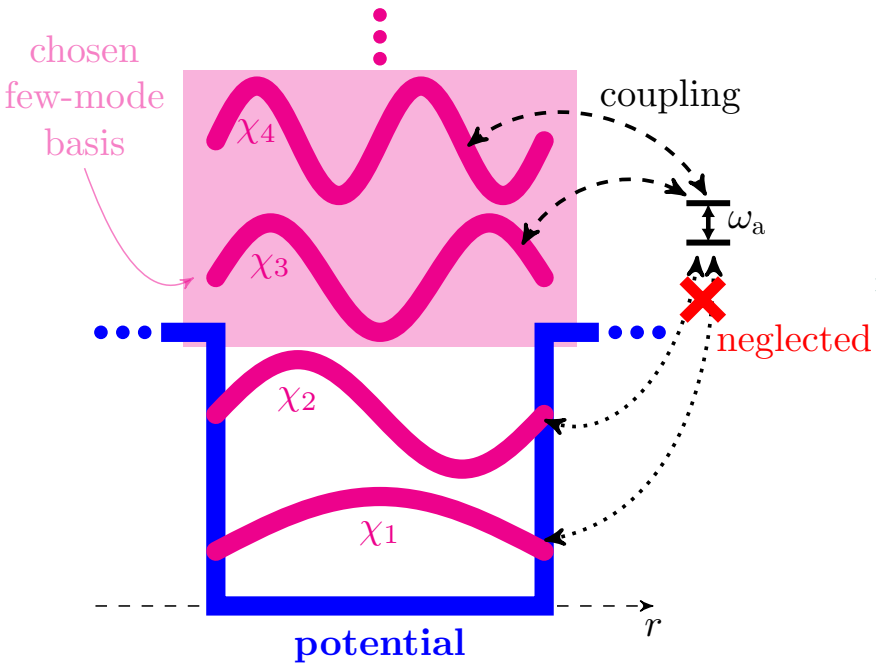

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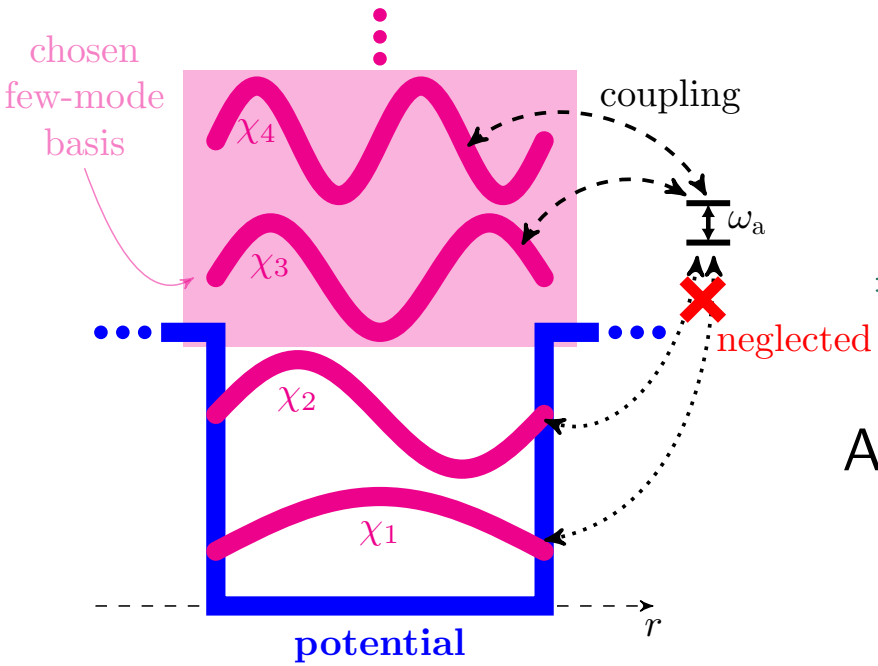

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Advantages of ab initio few-mode theory

- Empty cavity treated *exactly*
  - $\Rightarrow$ Approximate interaction only
  - $\Rightarrow$ Fully include openness, losses
- Connects to existing toolbox

DL & J. Evers, *submitted*

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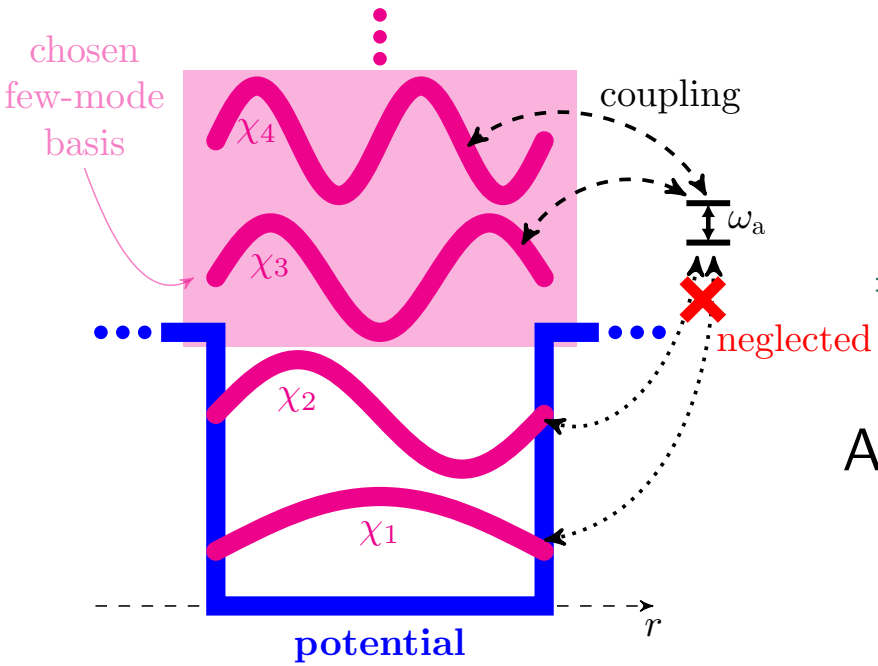

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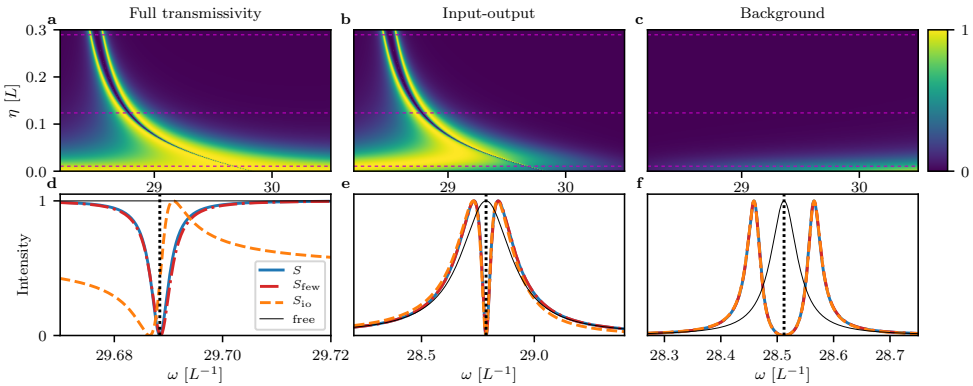
$\Rightarrow$ **New regimes accessible!**

DL & J. Evers, *submitted*

Practical aspects

- ✓ Benchmarked in linear regime
- ✓ Highly open systems

  $\Rightarrow$ S_{bg} matters!



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- ✓ Benchmarked in linear regime
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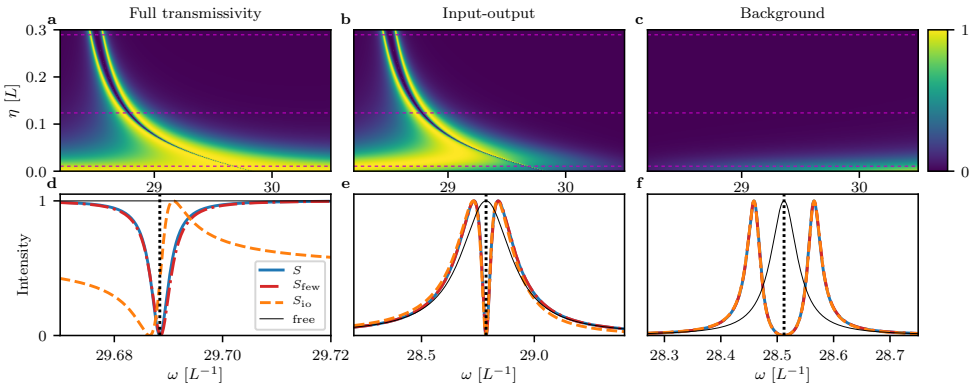
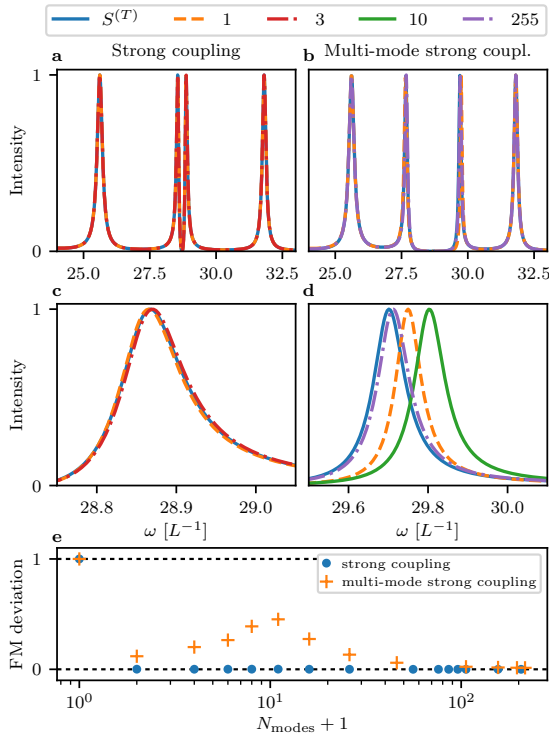
- ✓ Converging expansion





Practical aspects

- ✓ Benchmarked in linear regime
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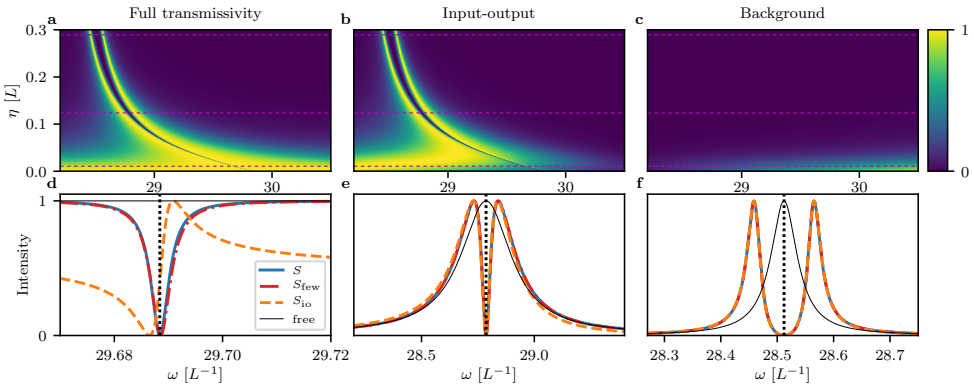
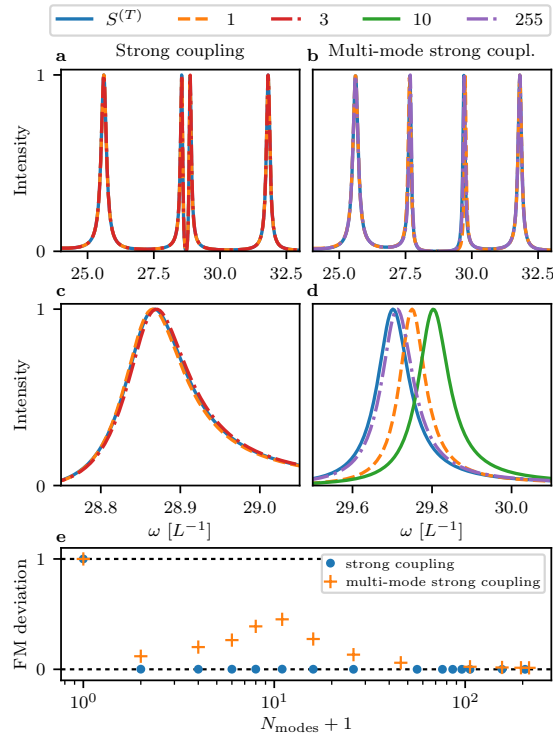
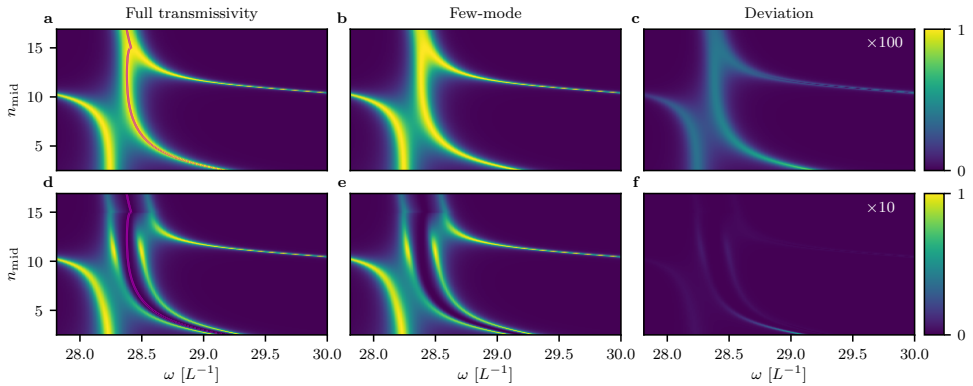
- ✓ Overlapping modes features
  - ⇒ Non-trivial bath effects!
- ✓ Ab initio calculation of quantum couplings







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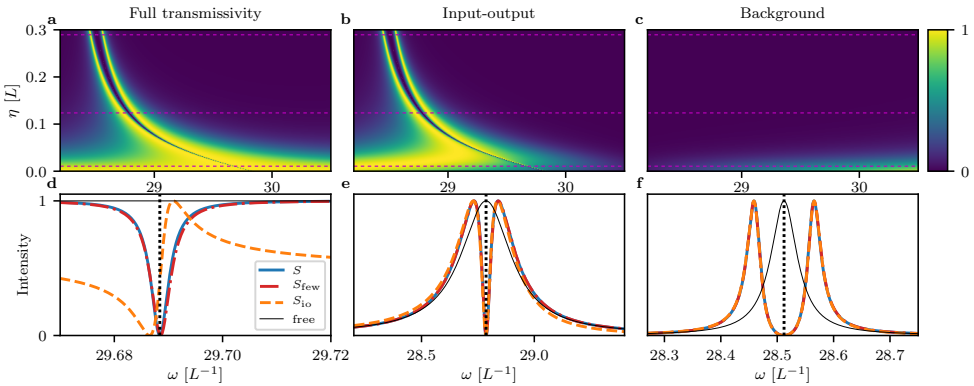
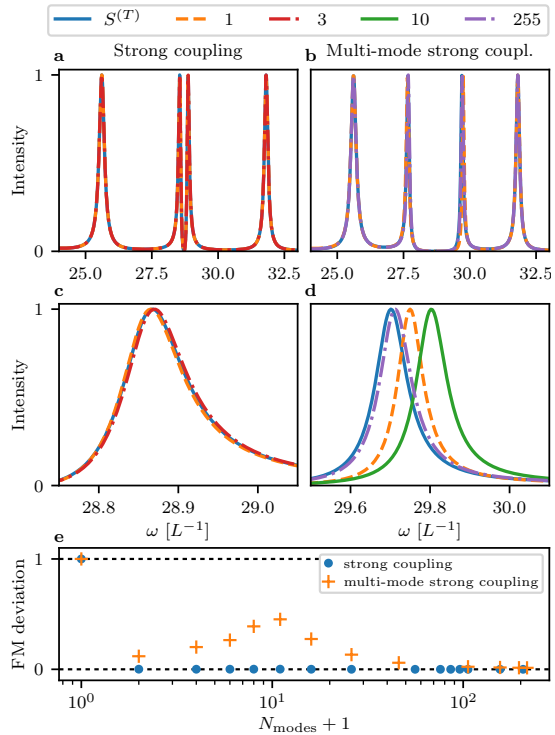
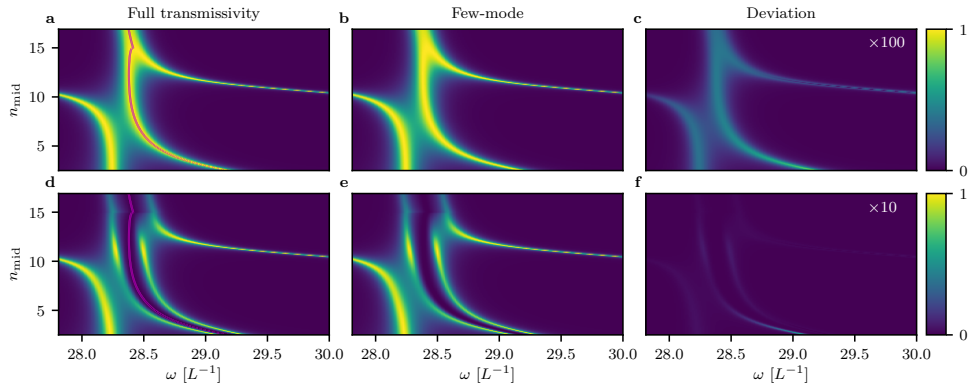
- ✓ Ab initio calculation of quantum couplings
- ? Counter-rotating loss terms[1], absorption[2]
- ? Optimal few-mode basis?







[1] Ciuti & Carusotto *Phys. Rev. A* **74**, 033811 (2006) [2] Esteban et al. *Nat. Comm.* **3**, 825 (2012)

Potential applications

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- Extreme openness, complex environments
  - Non-Hermitian photonics
  - Random lasers, complex
  - Quantum plasmonics
  - X-ray and nuclear cavity QED
  - ...
- Extreme light-matter coupling
  - Open system effects at ultra-strong coupling
  - Multi-mode strong coupling
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Pick et al. *Opt. Express* **25**, 11 (2017)
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[1] DL, K. P. Heeg, C. H. Keitel, J. Evers *in preparation*

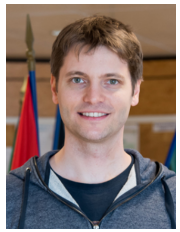
Conclusion & Outlook

- ✓ Rigorous construction of few-mode Hamiltonians
- ✓ Scattering theory via input-output formalism
  - $\Rightarrow$ Empty cavity treated exactly!
- ✓ Converging expansion scheme for interactions
- ✓ Linking ab initio theory and models in cavity QED
  - $\Rightarrow$ Access to new regimes!

- !! Explore quantum effects in X-ray cavities
- ?! Applications in extreme regimes of light-matter interactions
- ?? Applications in general quantum scattering and open systems theory?

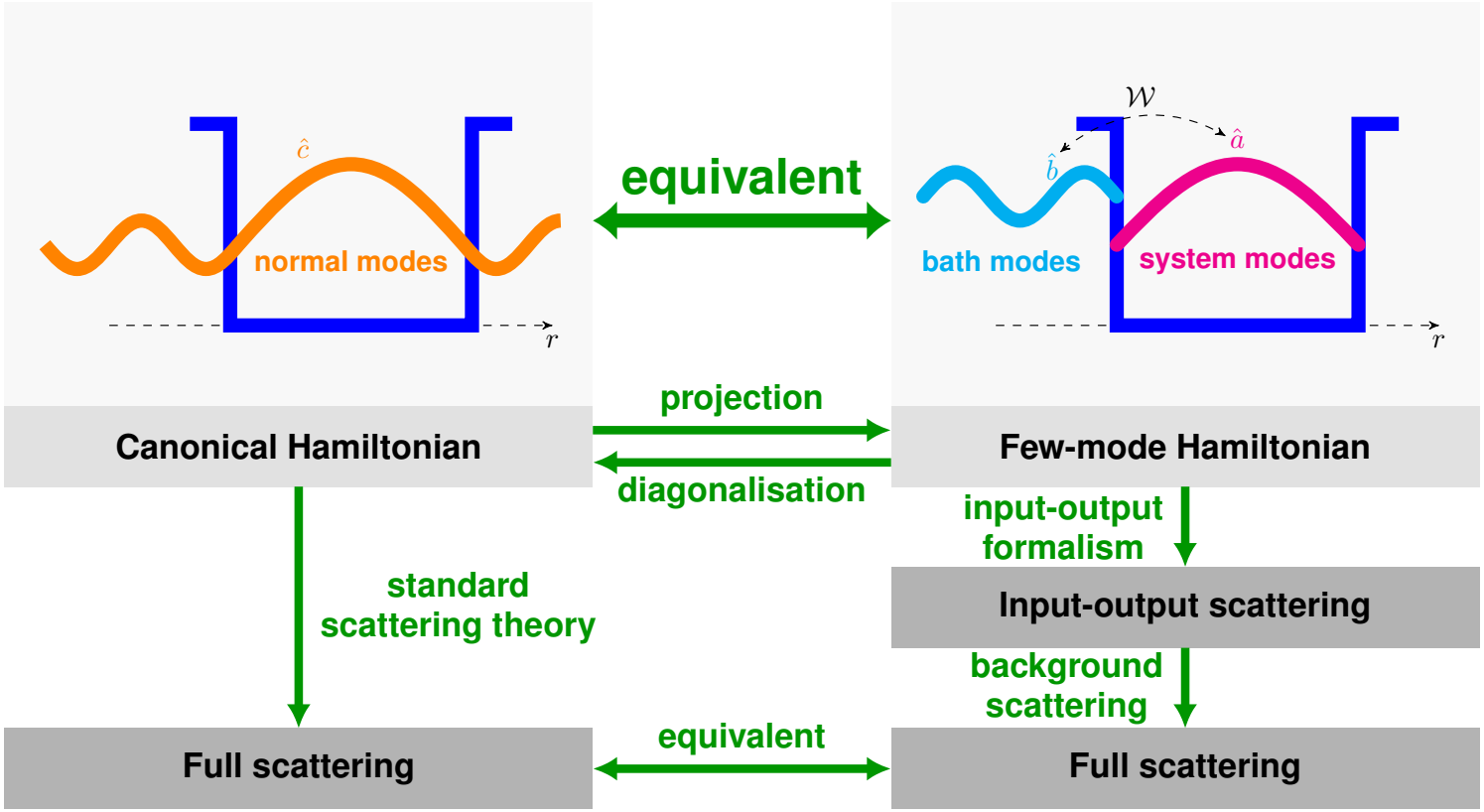
Thank you for your attention!

Jörg Evers



Kilian P. Heeg

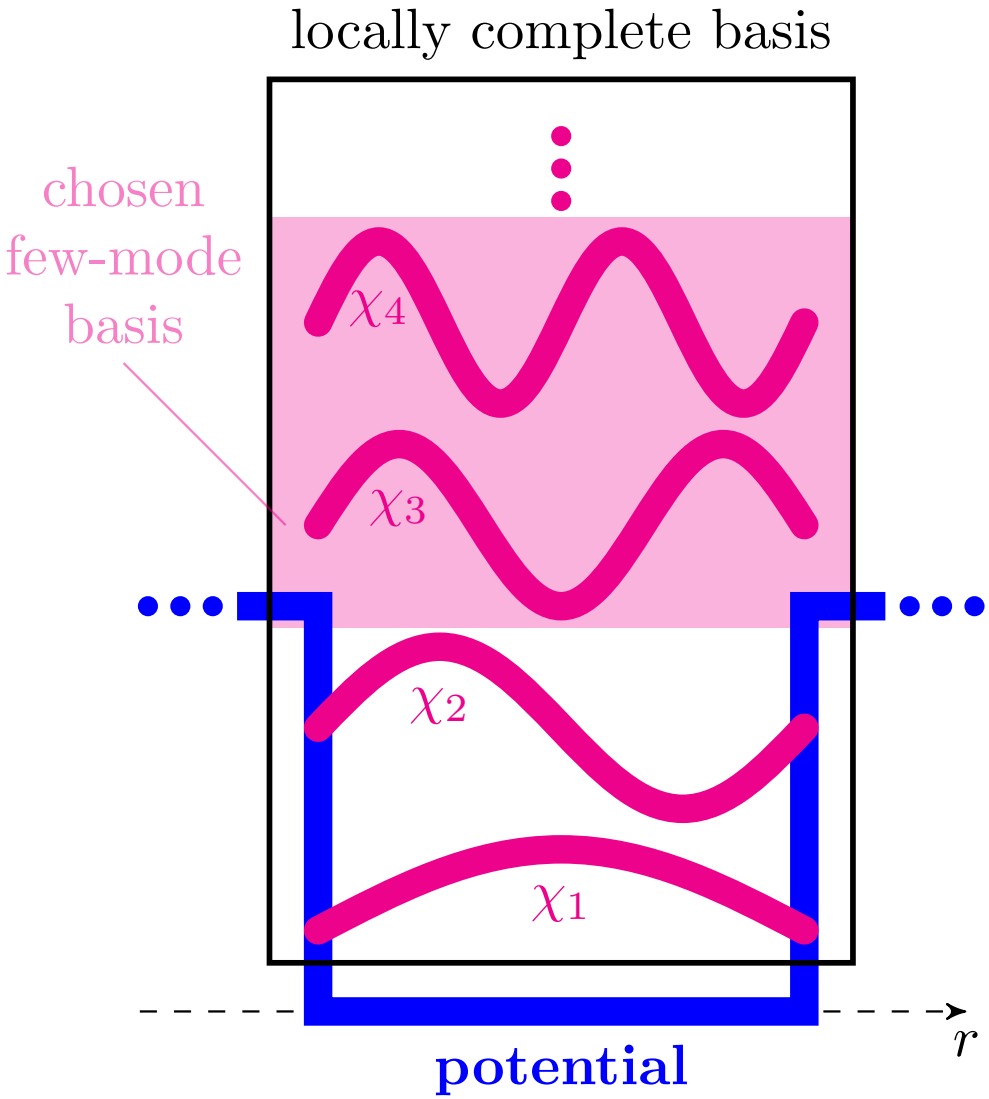
Christoph H. Keitel



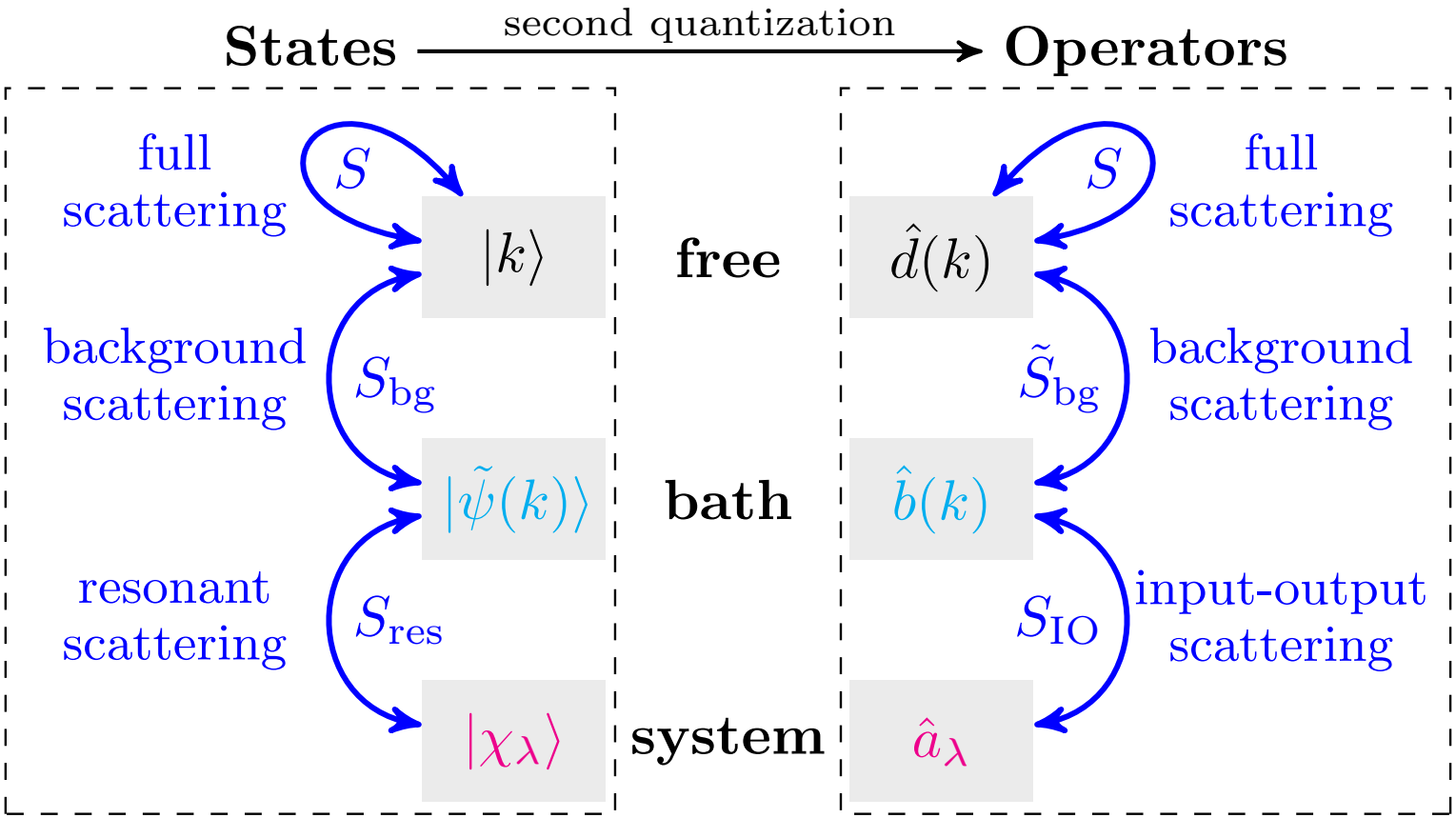
DL & J. Evers, *submitted* arXiv:1812.08556 [quant-ph]

Bonus slides

Choosing relevant modes systematically



Few-mode scattering theory



Linear dispersion theory

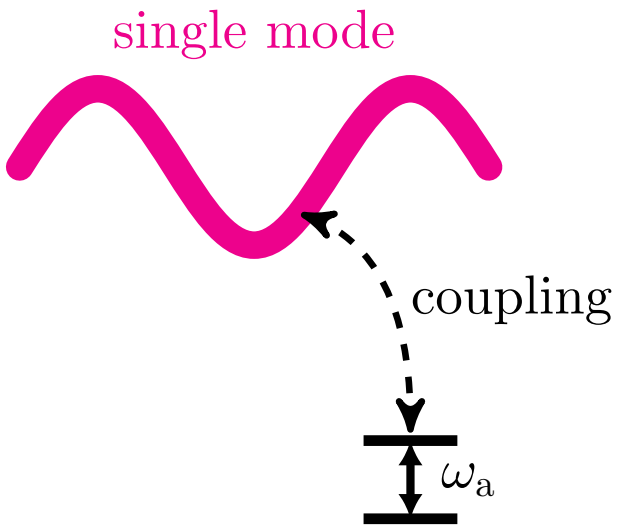
Lorentz-Lorenz formula, Clausius-Mossotti relation and others...
Summary: Basic assumption $\hat{\sigma}^- \approx -1$ (for 2-level system)

$$\frac{\partial^2}{\partial r^2} A(r,\omega) = -\omega^2 \varepsilon(r) A(r,\omega) + c_A \delta(r - r_a) A(r,\omega) + \frac{4\omega_a^3 |d|^2}{\omega^2 - \omega_a^2} \delta(r - r_a) A(r,\omega) \tag{1}$$

$$\varepsilon'(r) = \varepsilon(r) - \left(\frac{\omega_a^2}{\omega^2} \frac{4\omega_a^2 |d|^2}{\omega^2 - \omega_a^2} + \frac{c_A}{\omega^2} \right) \delta(r - r_a) \tag{2}$$

Standard references: Born & Wolf *Principles of optics* (1980)
Zhu et al. *Phys. Rev. Lett.* **64**, 2499 (1990)
X-ray: Röhlsberger *Nuclear condensed matter physics*(2004)
Ultra-strong: Malekakhlagh et al. *Phys. Rev. A* **94**, 063848 (2016)

Few-mode models in quantum optics



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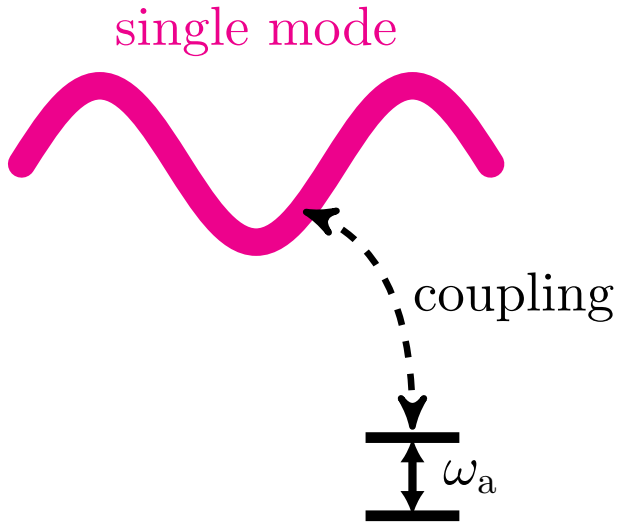

Jaynes-Cummings model

$$H = H_{\text{atom}} + H_{\text{mode}} + g\hat{a}\hat{\sigma}^{+} + h.c.$$

Jaynes & Cummings, *Proc. IEEE* **51**, 89 (1963)

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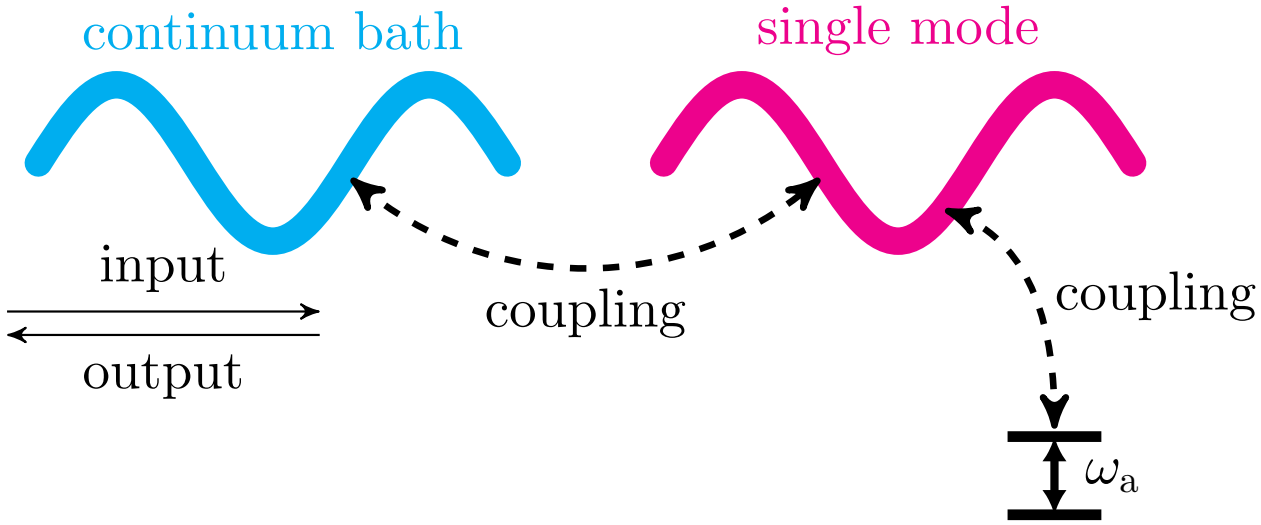

Input-output formalism

$$\hat{b}_{\text{out}} = \hat{b}_{\text{in}} + \kappa \hat{a}$$

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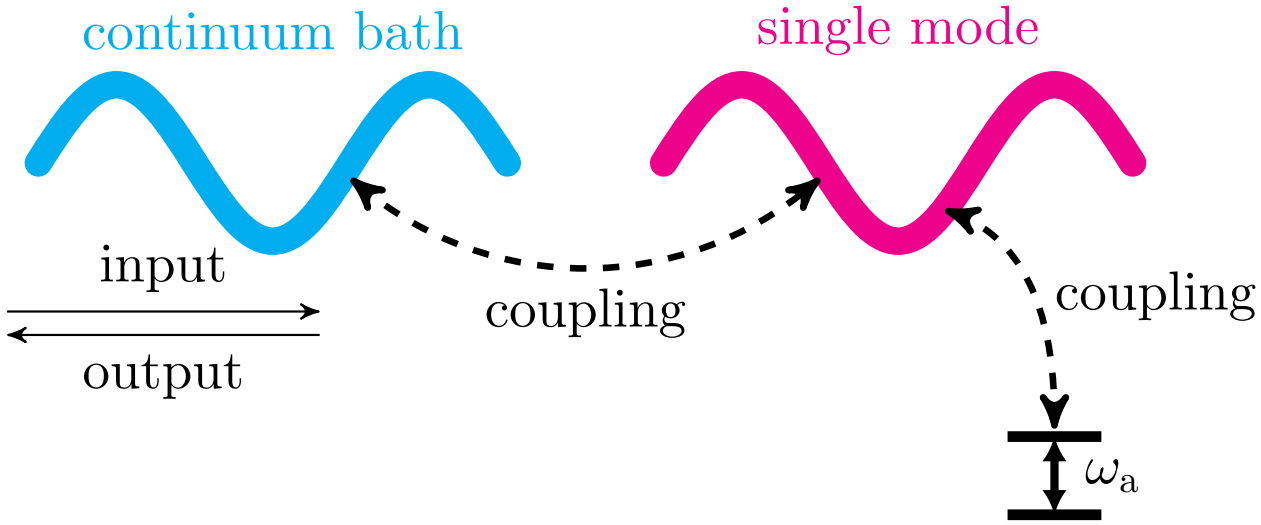

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$\Rightarrow$ Big tool box!

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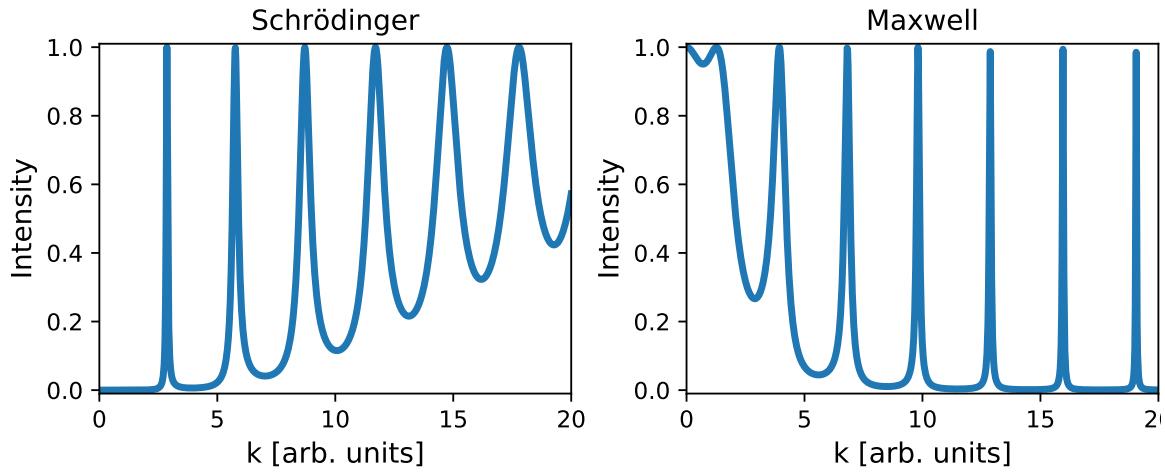
Input-output methods in general scattering theory?

Example: Schrödinger vs Maxwell Fabry-Perot



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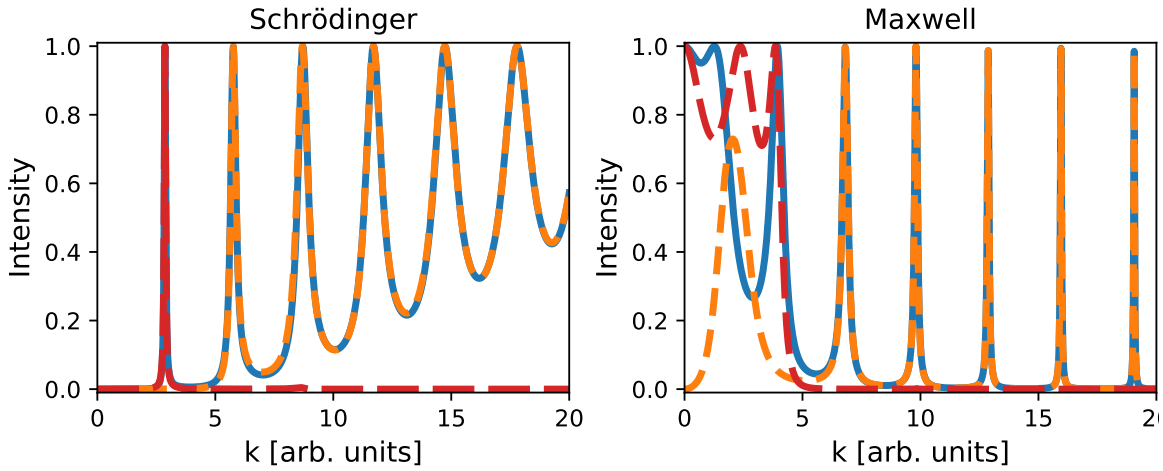

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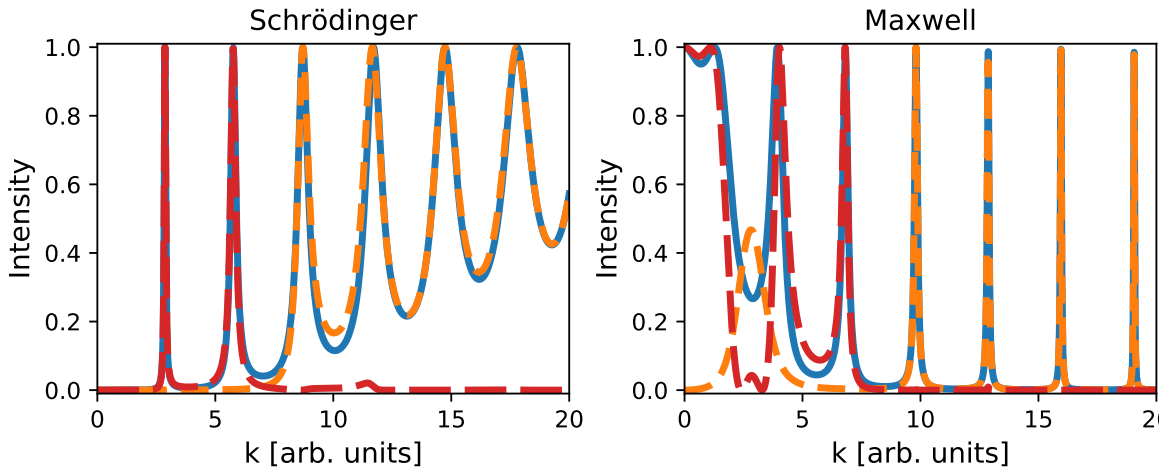

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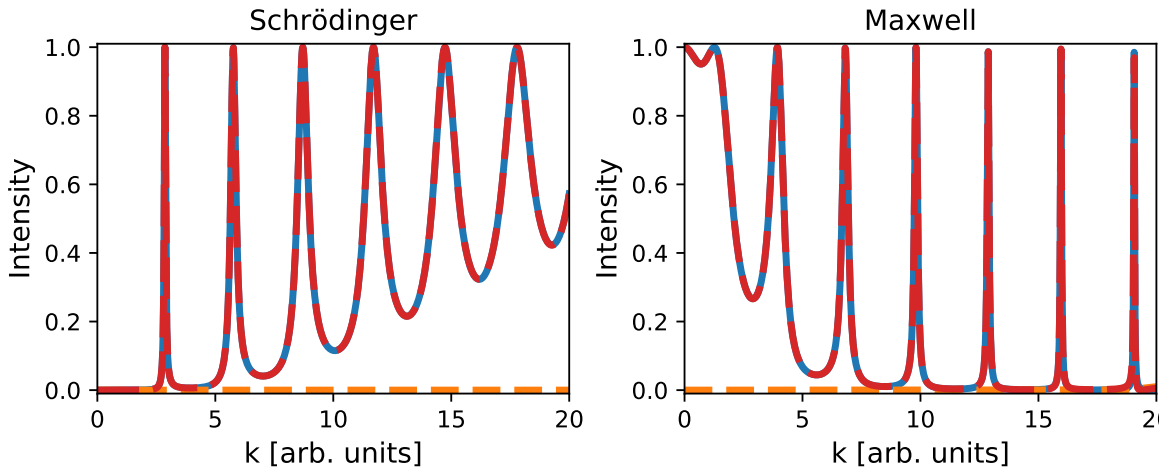

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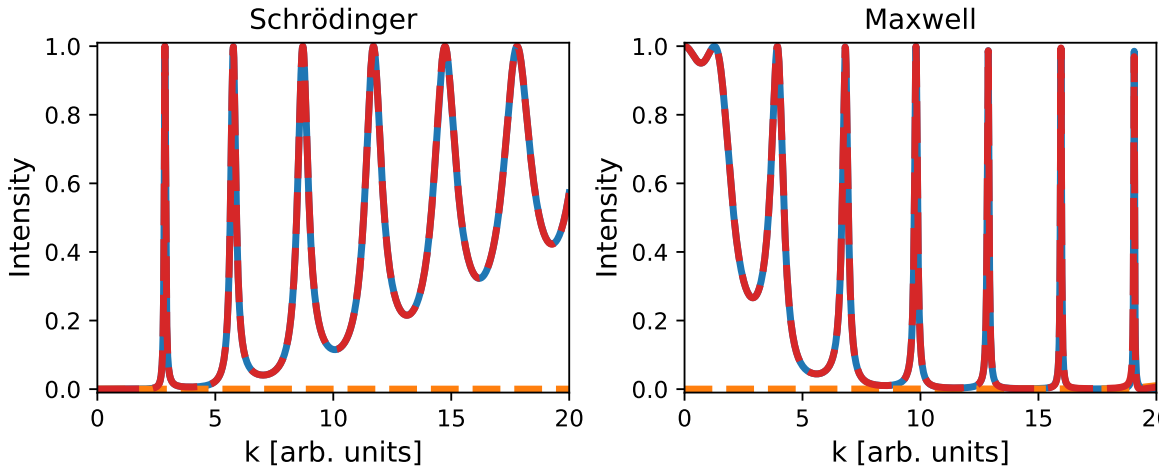
$$S(E) = \mathbb{I} - 2\pi i \mathcal{W}^\dagger \frac{1}{E - H_{\text{eff}}} \mathcal{W}$$

Weisskopf&Wigner 1930, Feshbach 1958, Fano 1961, Mahaux&Weidenmüller 1969, Dittes 2000, I. Rotter 2009 ...



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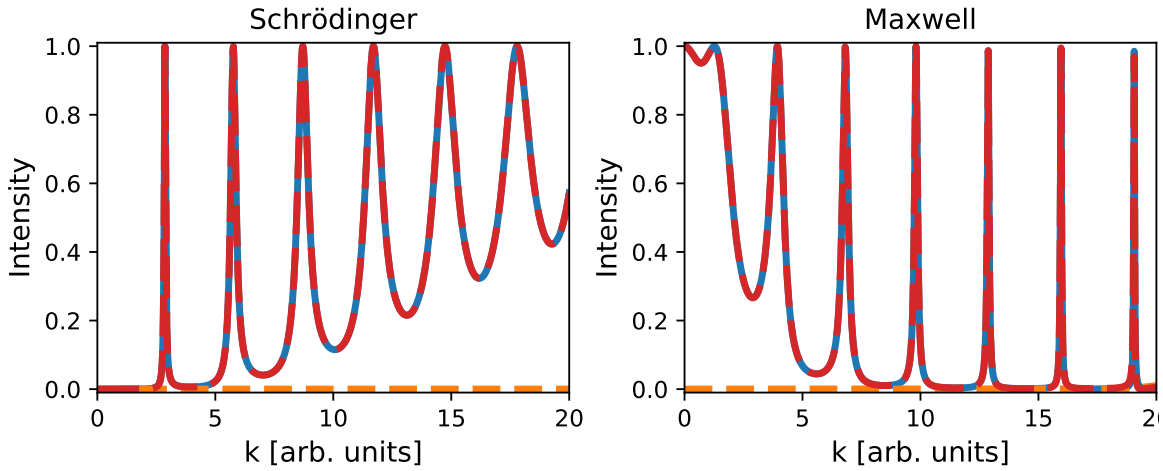

Generalized to second quantized level with interactions!

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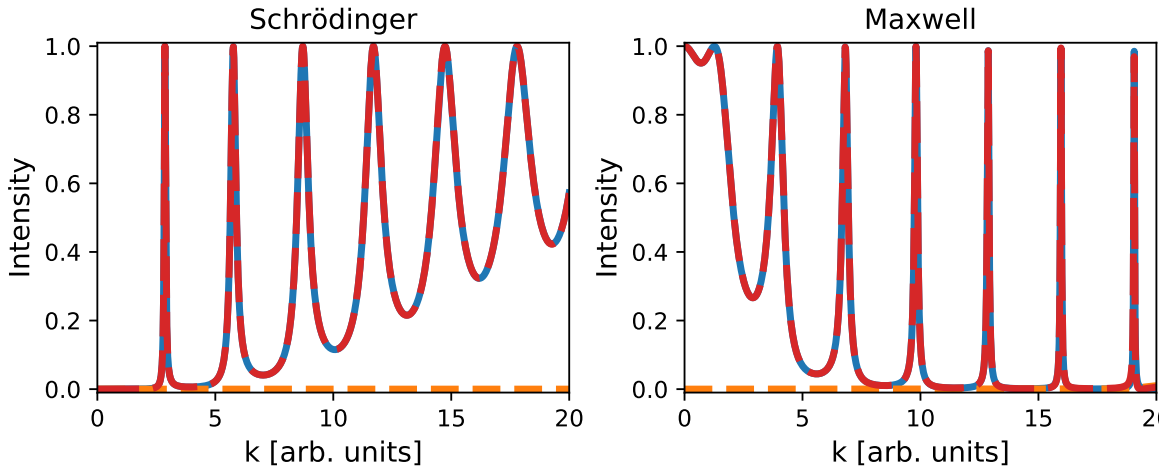
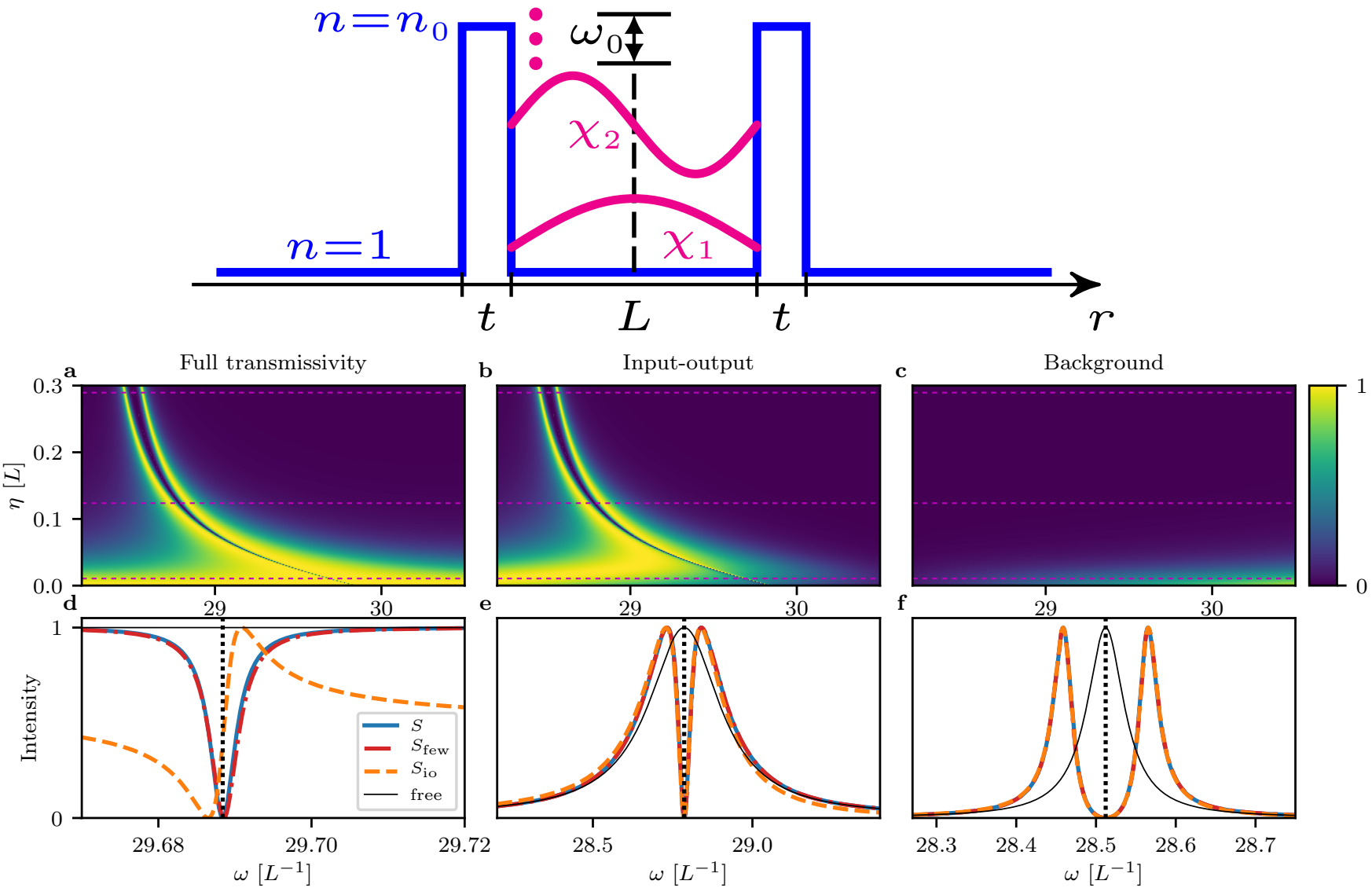


Generalized to second quantized level with interactions!

Potential applications: quantum chemistry[1], electronic transport[2], chaotic scattering[3,4], tunneling[5], random lasers[6]...

[1] Domcke, *Phys. Rev. A* **28**, 2777 (1982)
[2] Datta, *Electronic Transport in Mesoscopic Systems* (1995)
[3] Stöckmann, *Quantum Chaos* (1999)
[4] Weidenmüller&Mitchell, *Rev. Mod. Phys.* **81**, 539 (2009)
[5] Prange, *Phys. Rev.* **131**, 1083 (1963)
[6] Türeci et al, *Science* **320**, 643 (2008)

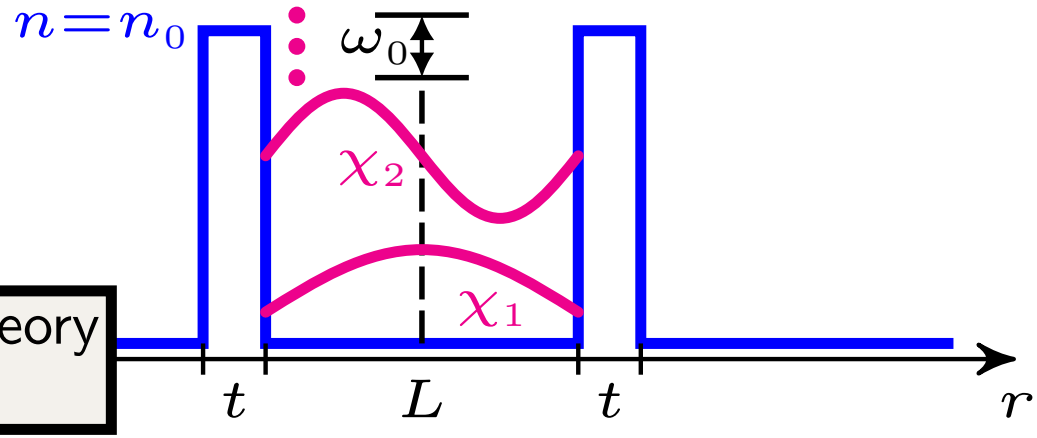
From strong coupling to free space

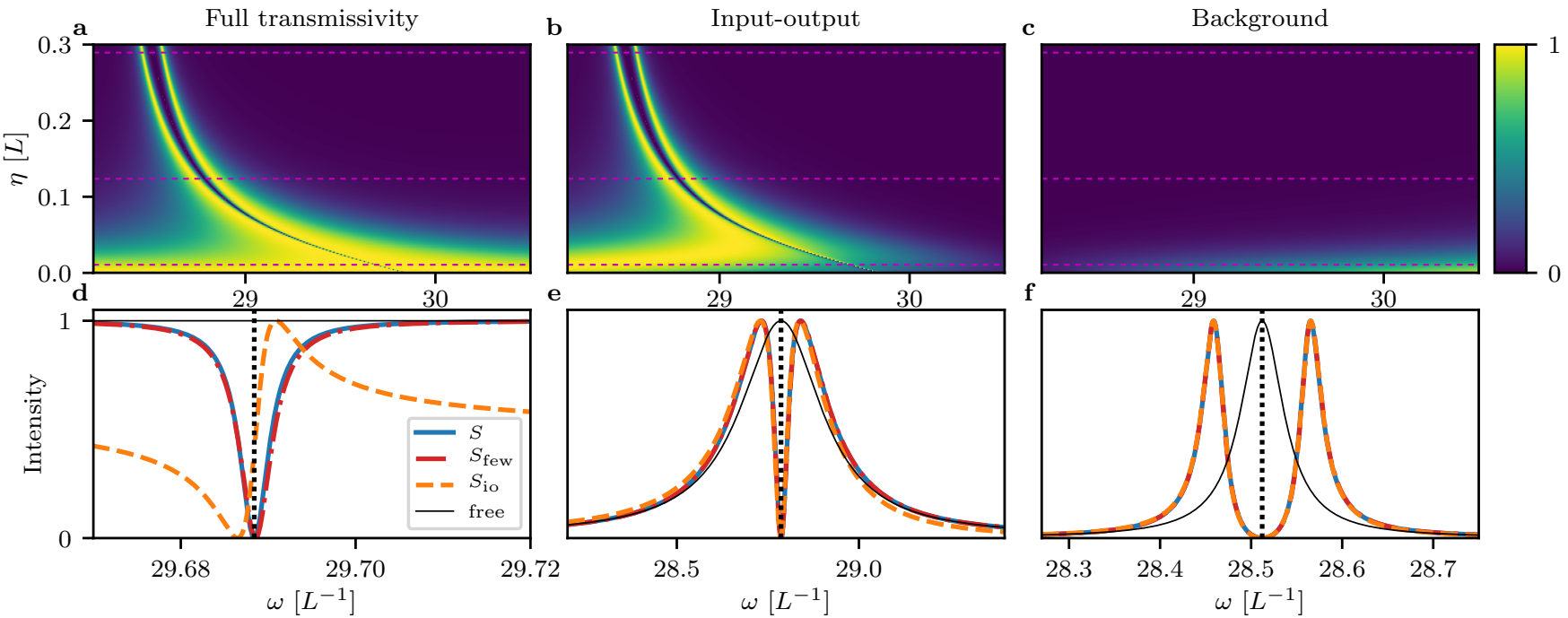


DL & J. Evers, *submitted*

From strong coupling to free space

Linear dispersion theory as benchmark!





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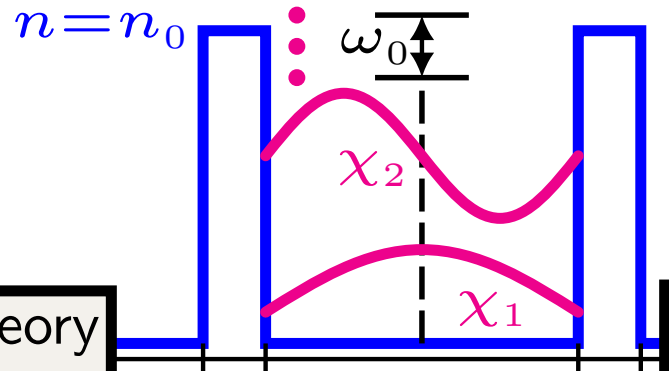
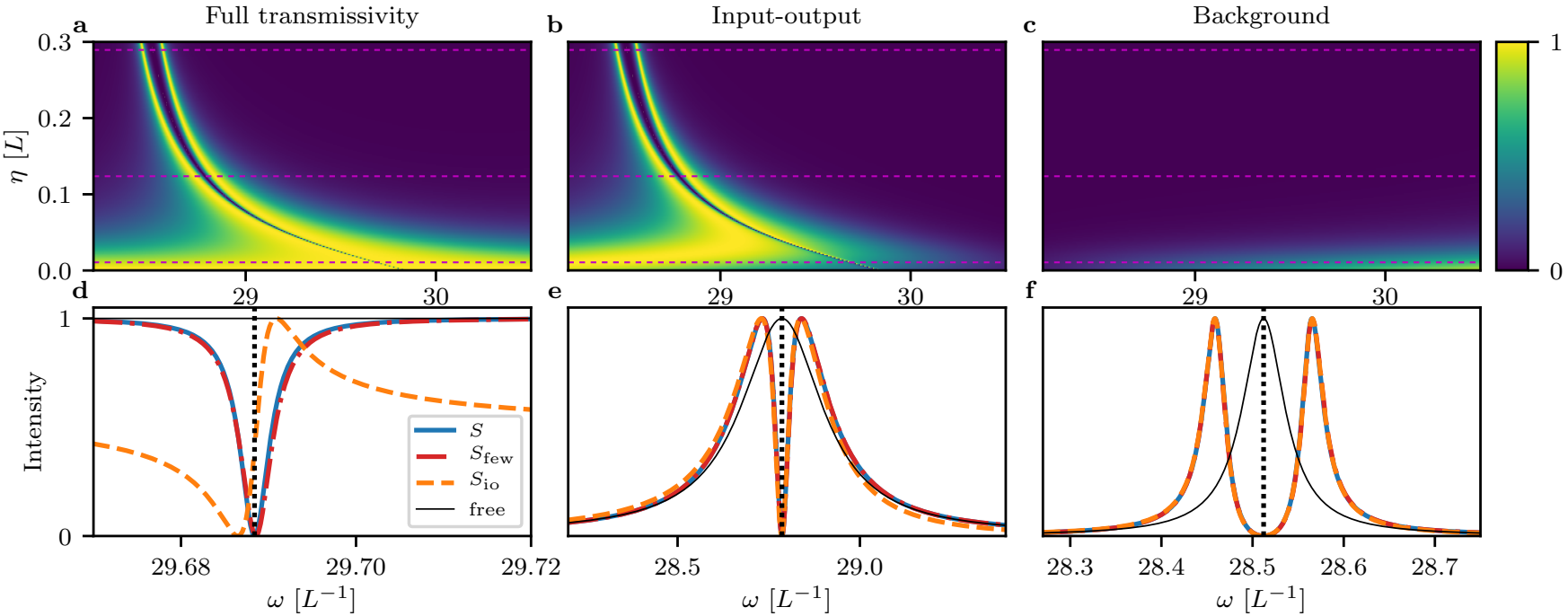


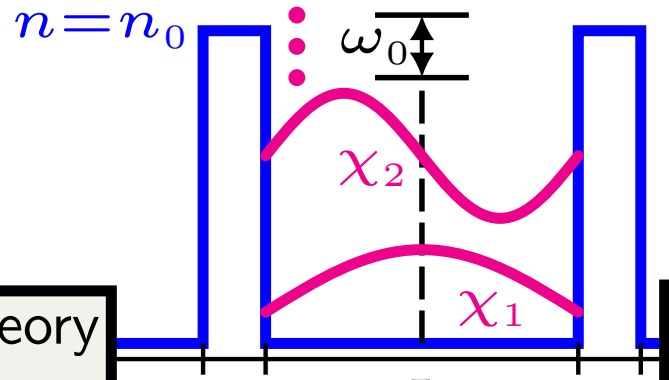
Linear dispersion theory as benchmark!

Ab initio few-mode theory captures the full range!



DL & J. Evers, *submitted*

From strong coupling to free space



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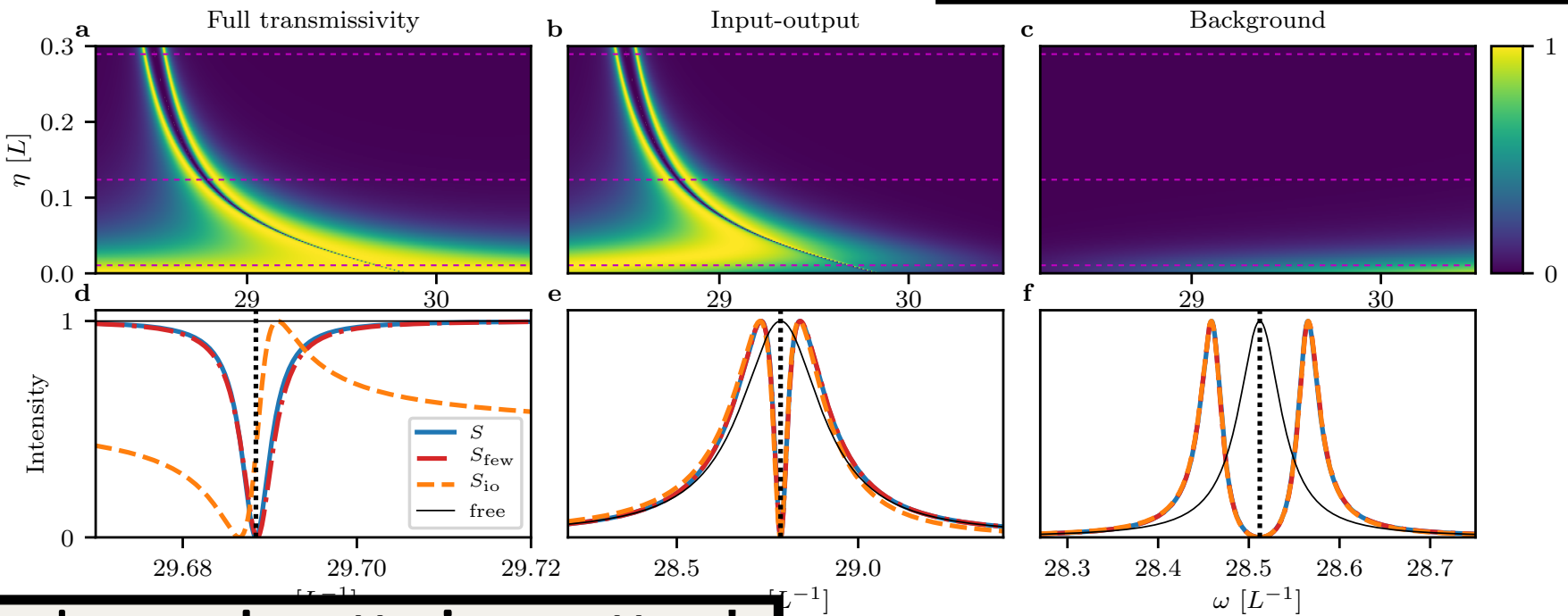
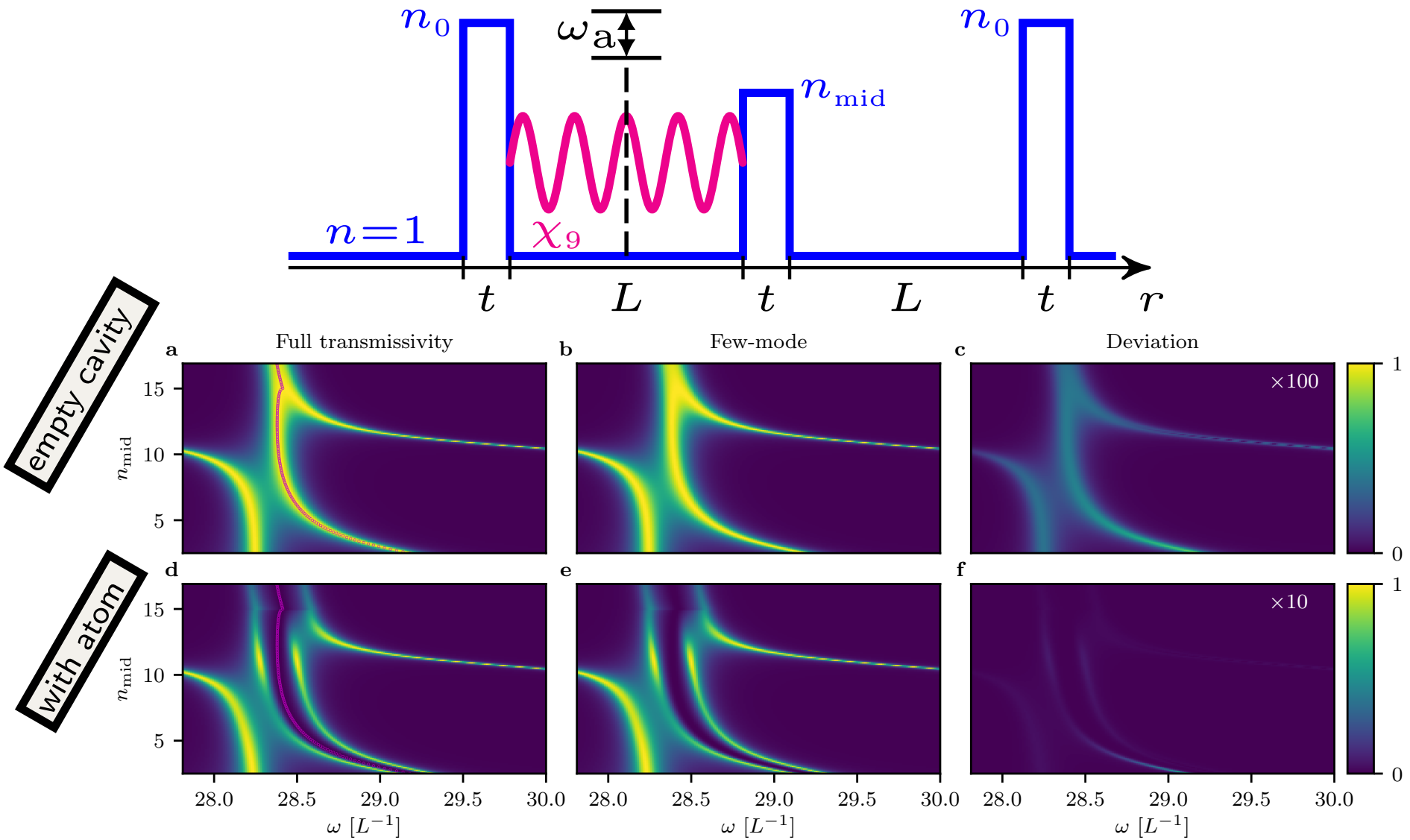


Background scattering matters!

DL & J. Evers, *submitted*

Quantum optical properties with overlapping modes



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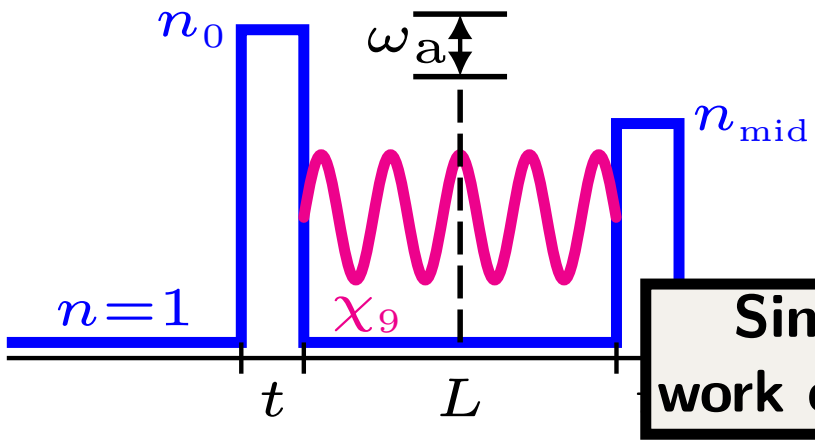
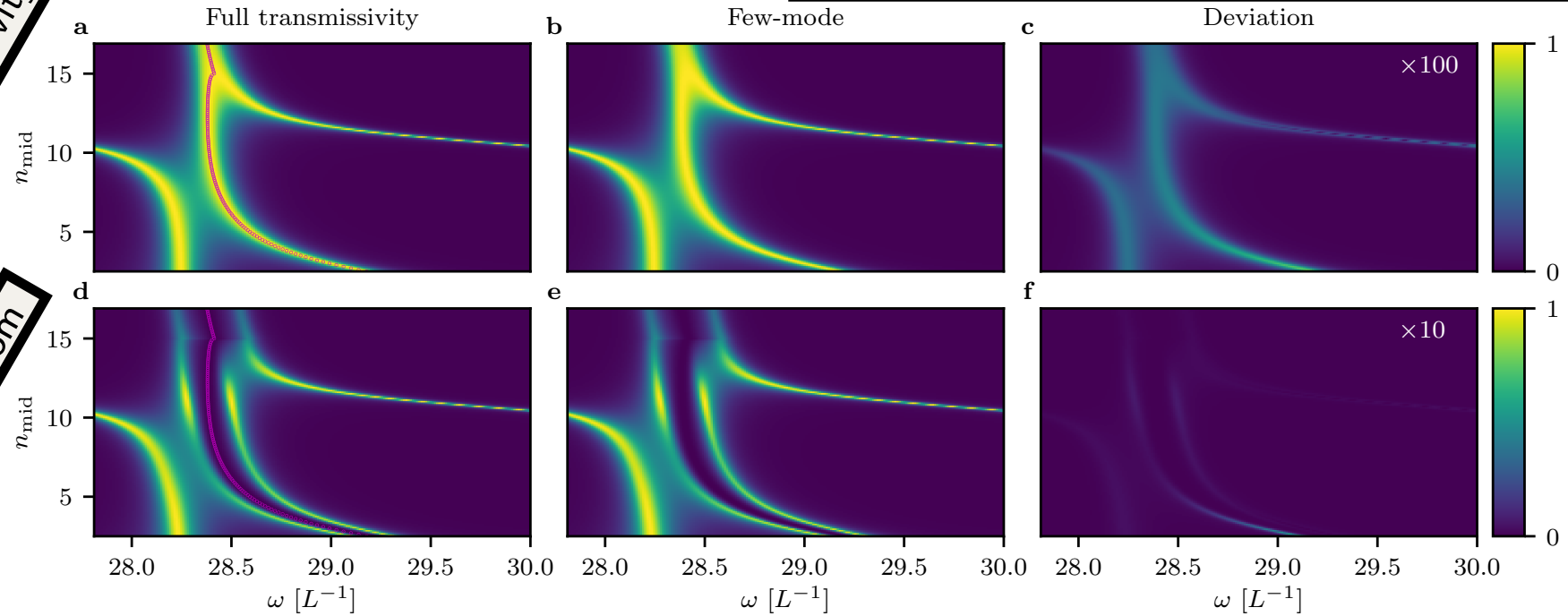


Single mode description can work even for overlapping modes!

empty cavity

with atom



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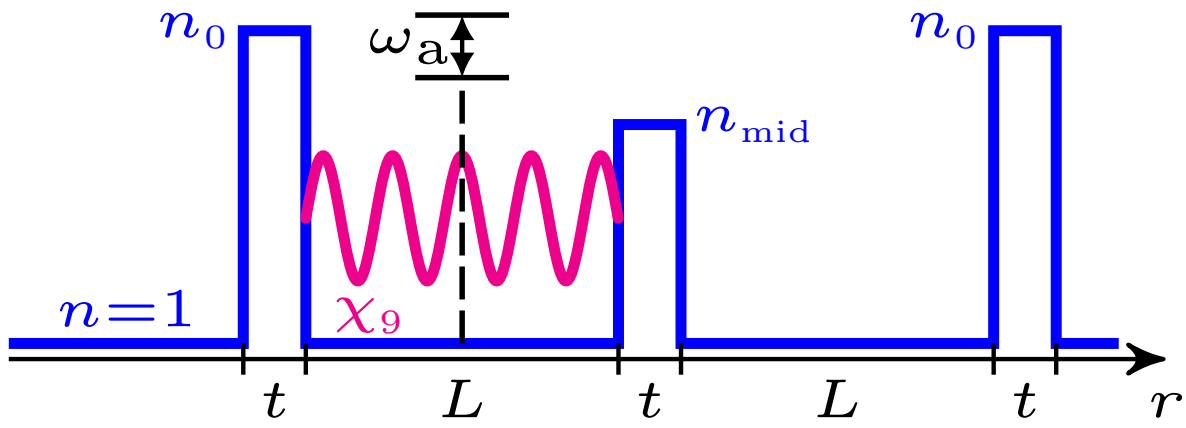

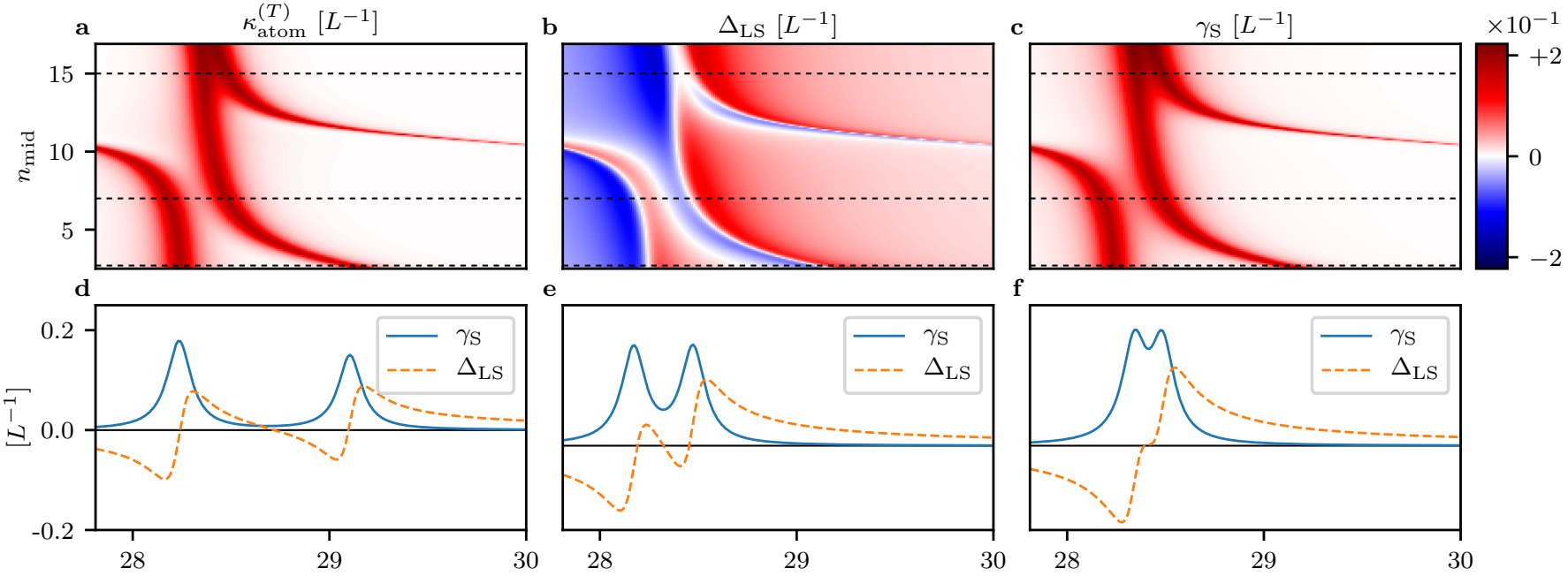

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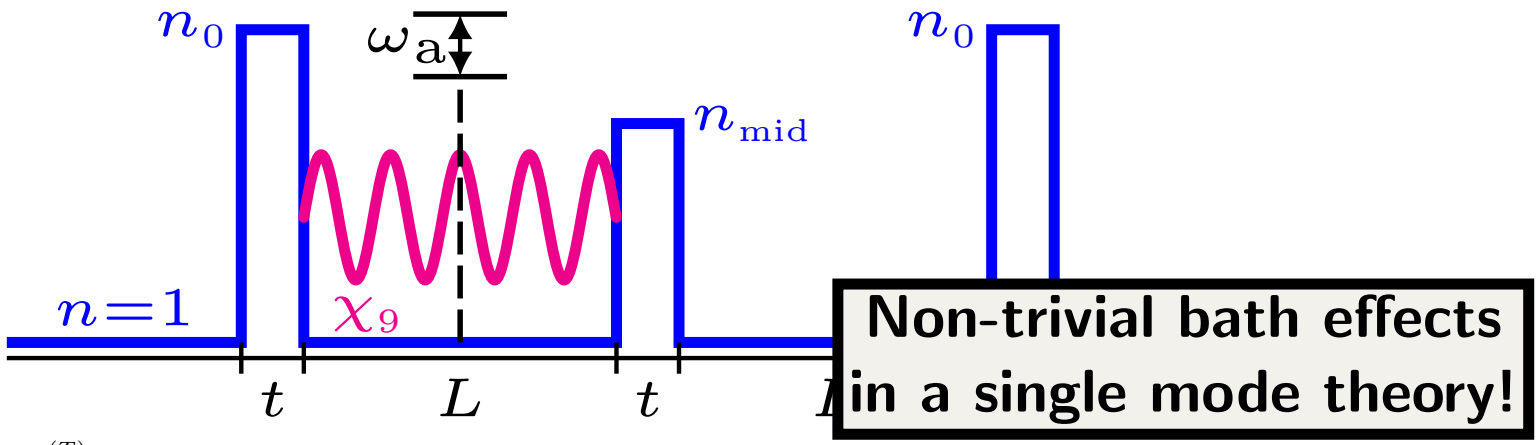
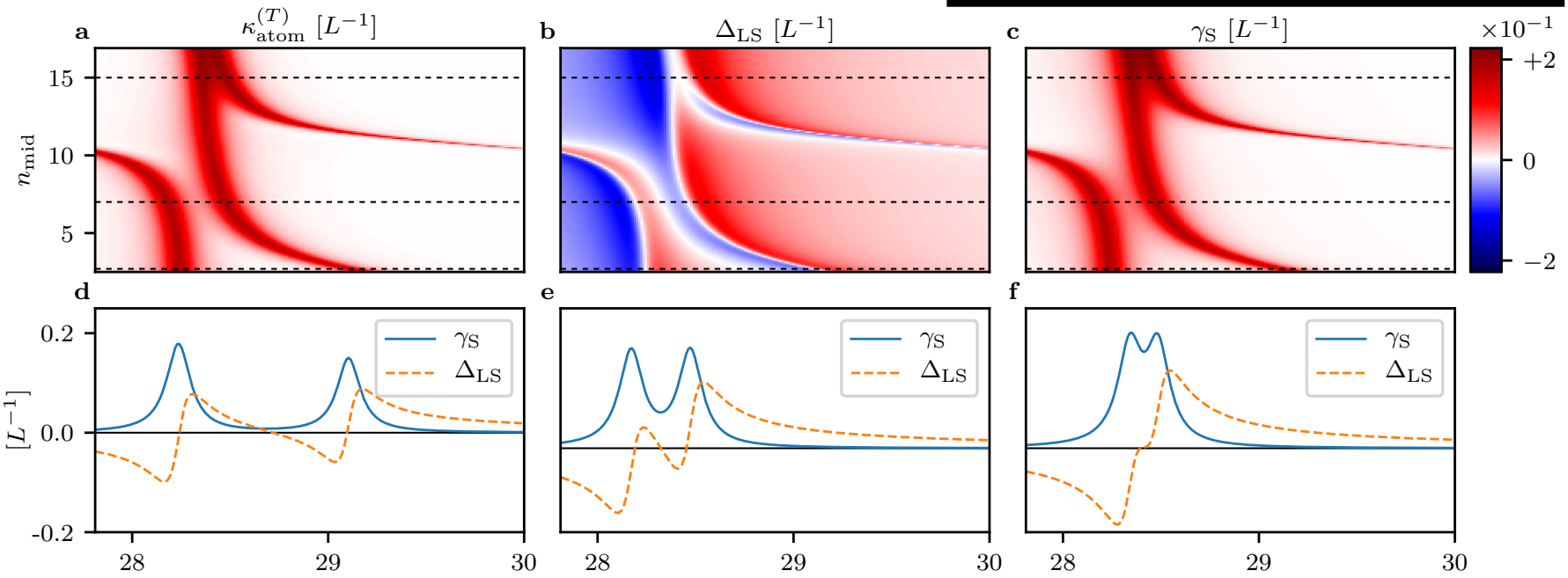


Non-trivial bath effects in a single mode theory!



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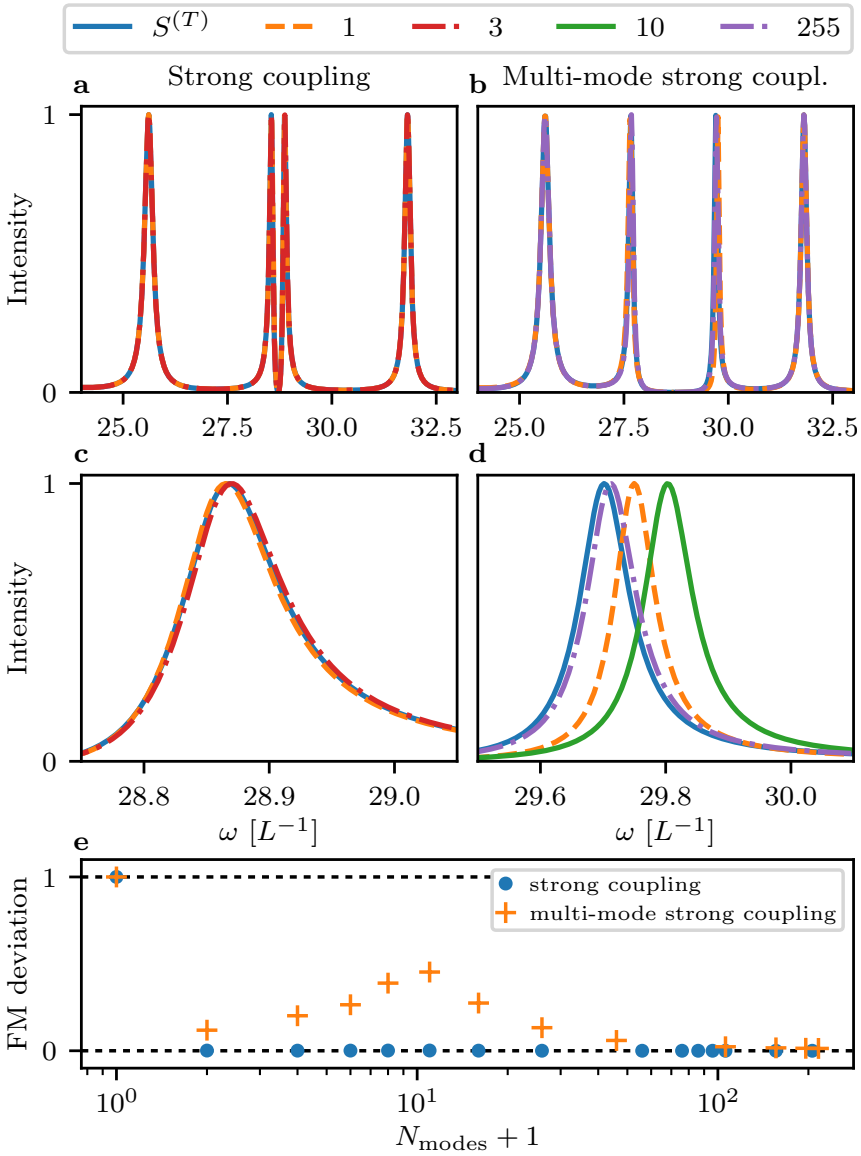
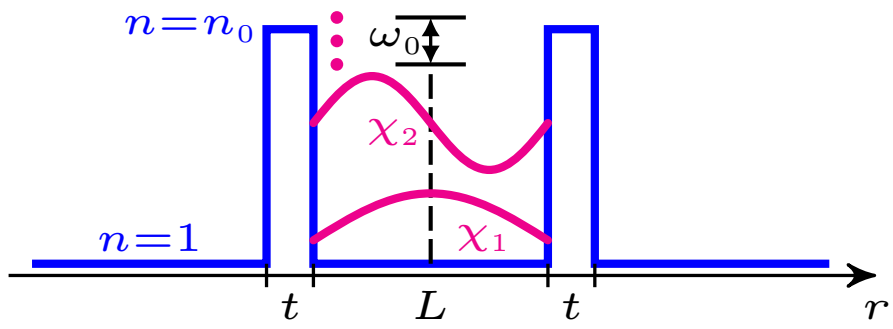
Convergence and extreme regimes





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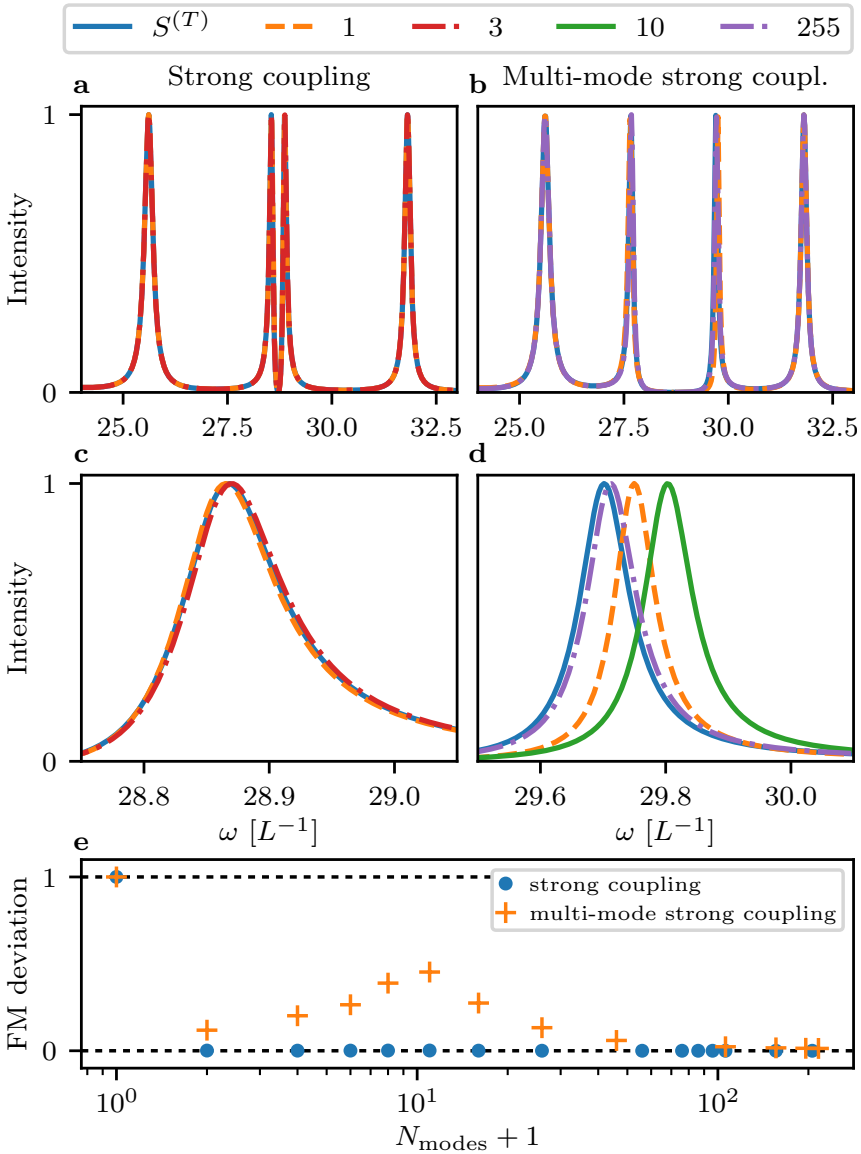
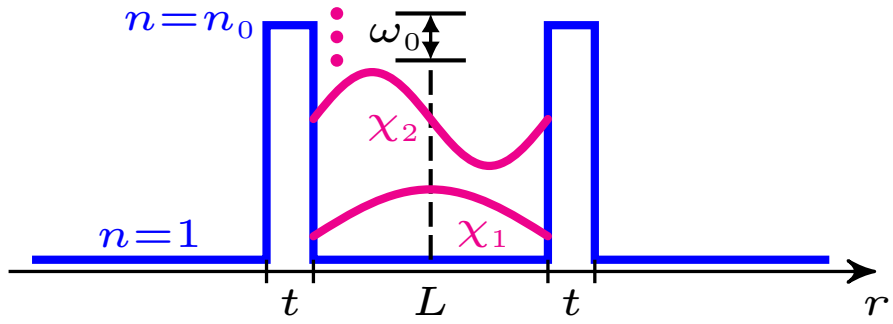


Convergence can also be shown analytically!



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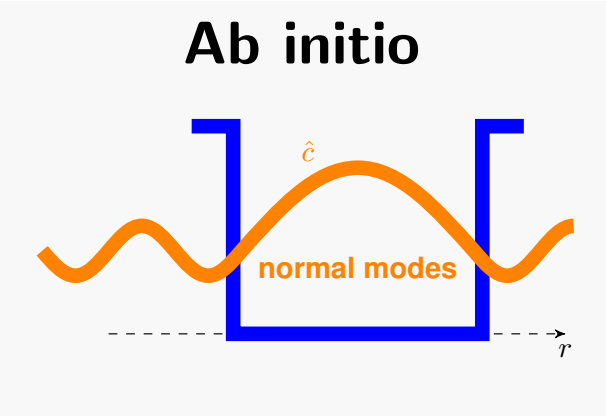
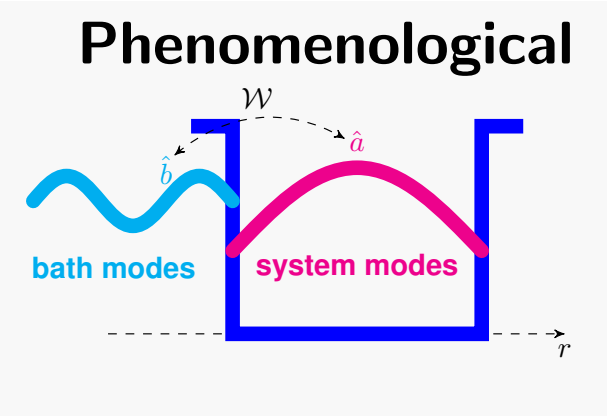
Few-mode models in cavity QED





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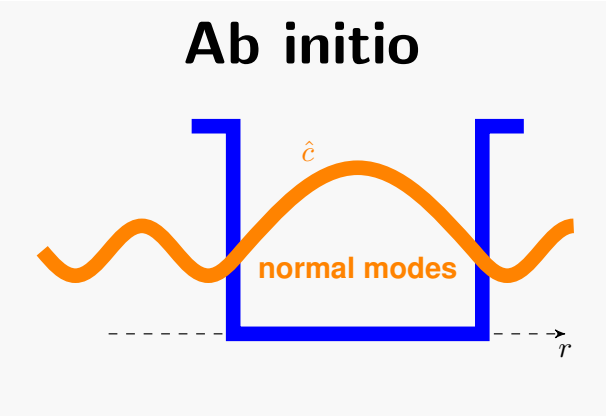

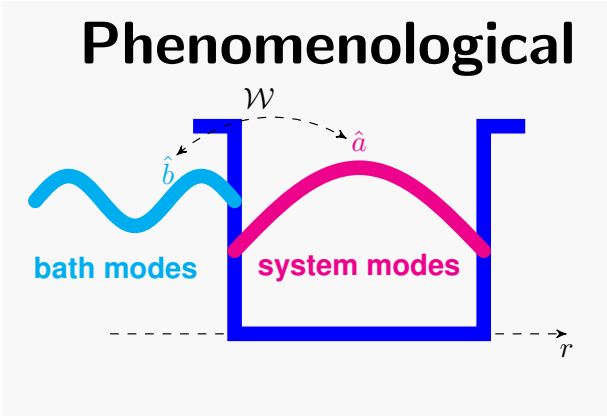

e.g. Canonical Hamiltonian

$$H_{\mathrm{cav}} = \int d\omega \hbar\omega \hat{c}^\dagger(\omega)\hat{c}(\omega)$$

$\rightarrow$ **Ab initio quantization**[1]

[1] Glauber & Lewenstein, *Phys. Rev. A* **43**, 467 (1991)

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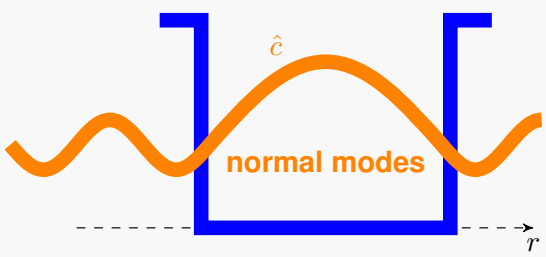

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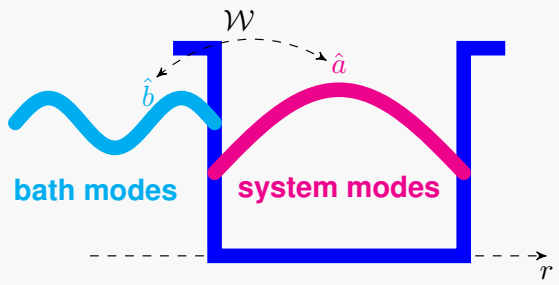

Few-mode system-bath Hamiltonian

$$H_{\mathrm{cav}} = H_{\mathrm{syst}} + H_{\mathrm{bath}} + \mathcal{W}\hat{a}\hat{b}^\dagger + h.c.$$

$\rightarrow$ **Input-output model**[2]

[1] Glauber & Lewenstein, *Phys. Rev. A* **43**, 467 (1991)
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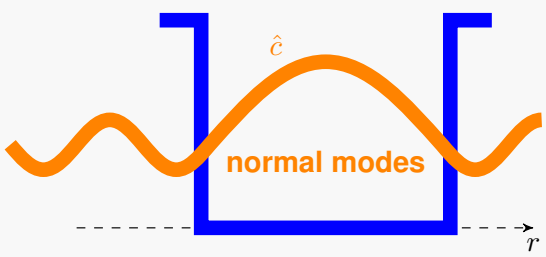

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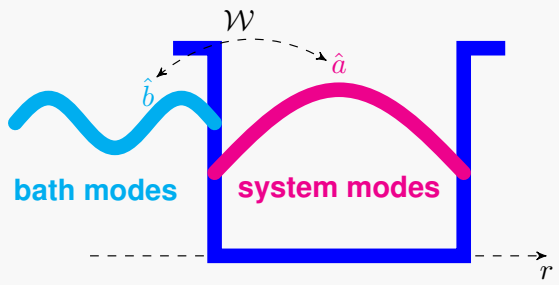

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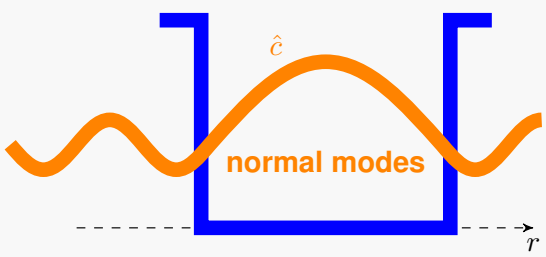

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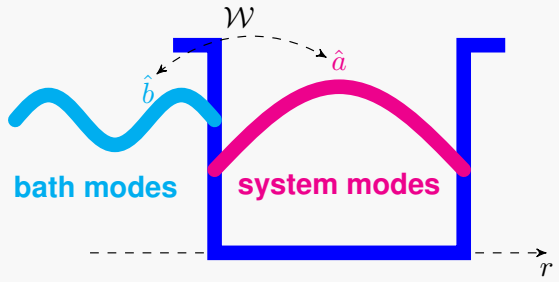

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- 🙂 Huge toolbox available
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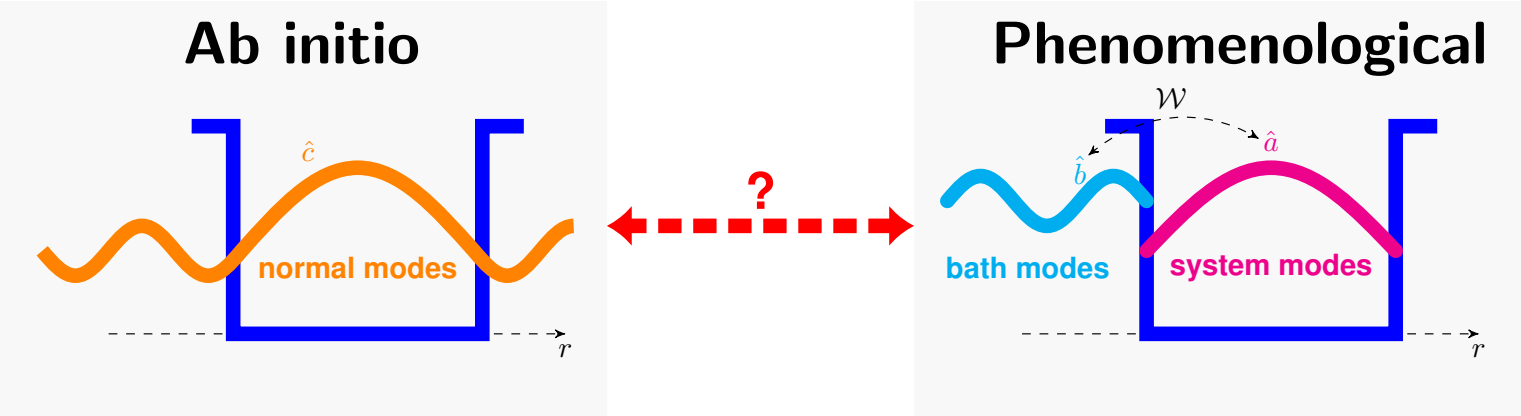

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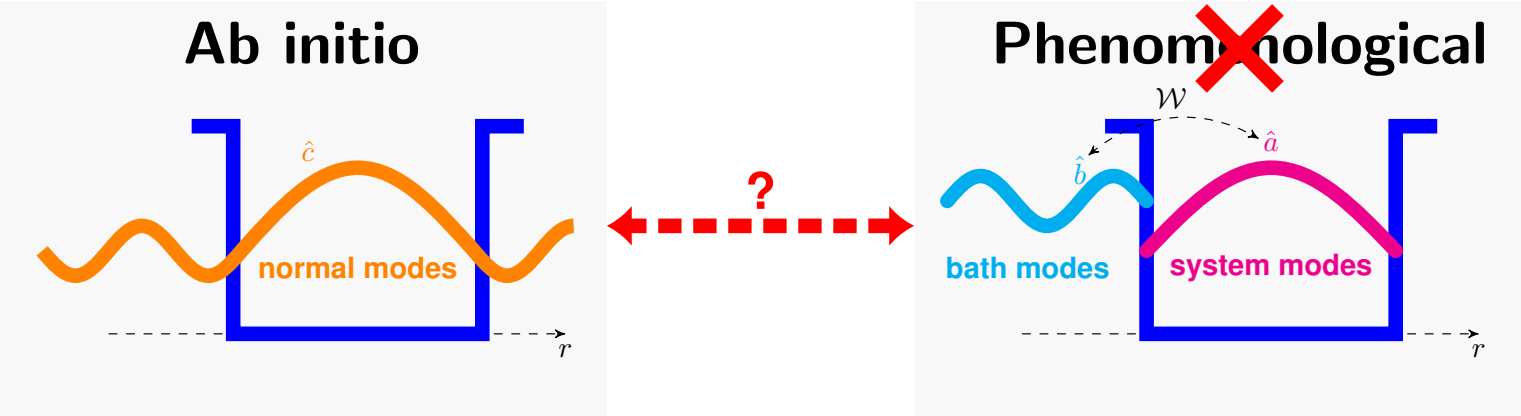


How to make

- **few-mode**

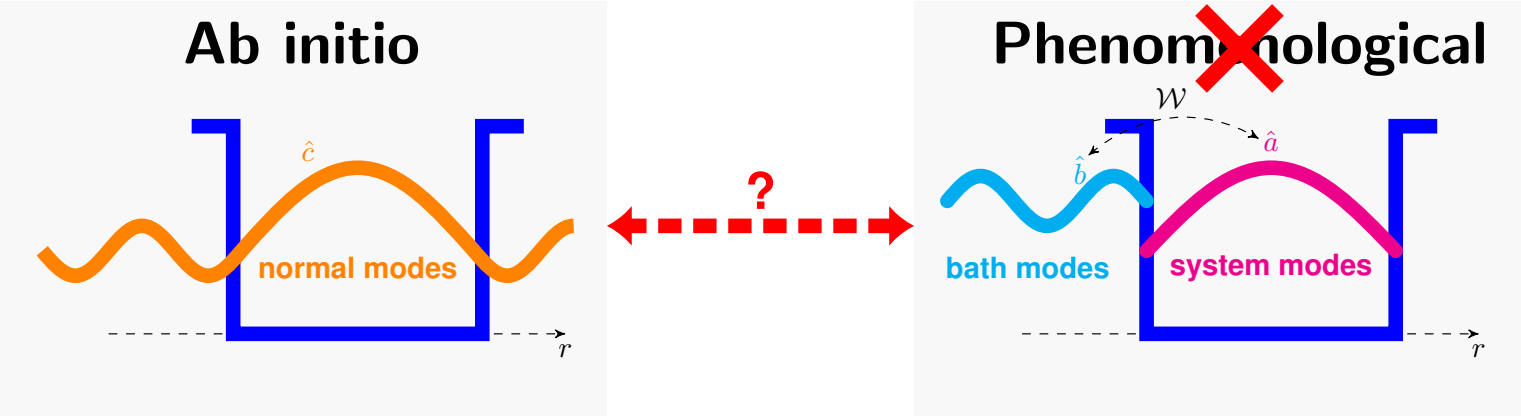
and

- **input-output**

ab initio?

?

Few-mode models in cavity QED



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⇒ **Ab initio few-mode theory**