Quantum dynamics under strong driving: Counter-rotating hybridized rotating wave method

Zhiguo Lü(吕智国)

Collaborators Prof. Hang Zheng, Prof. H.S. Goan, Dr. YiYing Yan

Key Laboratory of Artificial Structures and Quantum Control (Ministry of Education),
School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, China

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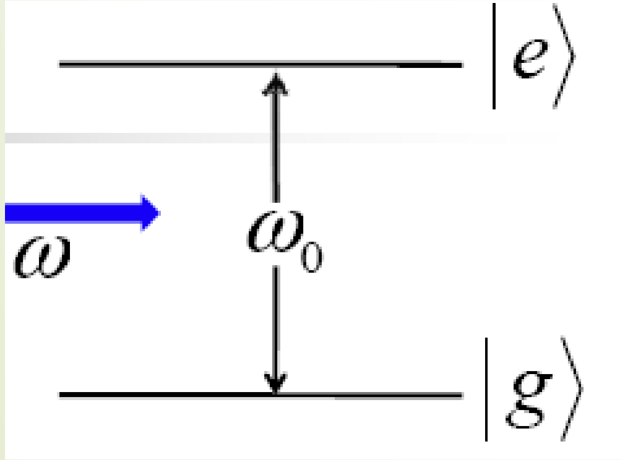
Outline

- **Background**
- **Our method：CHRW method**
- **Bloch-Siegert shift**
- **Driven quantum dynamics**
- **Fluorescence and absorption spectra**
- **Summary**

Motivation

- The rotating wave approximation of the system-field interactions is a wide-used treatment under serious conditions.



Semiclassical Rabi model

$$\begin{aligned} H(t) &= \frac{1}{2}\omega_0\sigma_z + \frac{A}{2}\cos(\omega t)\sigma_x \\ &= \frac{1}{2}\omega_0\sigma_z + \frac{A}{4}(e^{i\omega t}\sigma_- + e^{-i\omega t}\sigma_+) \\ &\quad + \frac{A}{4}(e^{-i\omega t}\sigma_- + e^{i\omega t}\sigma_+), \end{aligned}$$

$$\sigma_+ = \frac{\sigma_x + i\sigma_y}{2} \qquad \sigma_- = \frac{\sigma_x + i\sigma_y}{2}$$

Background

- The physics of the ultrastrong- and deep-strong coupling regimes of light-matter interaction may be realized through state-of-the -art technology such as circuit QED, flux qubit etc.

A/ω_0	ω_0(GHz)	Experiment
0.1,...,0.3	10	Ref.[a]
0.1,...,0.3	40	Ref.[b]
0.25,...,1	4.8	Ref.[c]

(a)W.D. Oliver, Y. Yu, et. al, Science 310, 1653(2005).
(b)C.M.Wilson, G.Johansson, et. al, Phys. Rev. B. 81, 024520(2010).
(c) F. Yoshihara,Y.Nakamura, et.al, Phys. Rev. B. 89, 020503(R)(2014).
(d) F. Yoshihara, T. Fuse, et. al, Nat. Phys. 13, 44 (2017).

- The analytical method could provide a clear picture to understand the physics of strongly driven systems.

Background

- Driven two-level system

NMR $$H_{\mathrm{NMR}}(t) = -\vec{\mu} \cdot \overrightarrow{B(t)} = -\frac{1}{2}\hbar g[\sigma_x B_x(t) + \sigma_y B_y(t) + \sigma_z B_z(t)]$$

Rabi model $$H_{\mathrm{A}}(t) = \frac{1}{2}\omega_0 \sigma_z + \Omega \cos(\omega t)\sigma_x$$

Tunneling TLS $$H_{\mathrm{TLS}}(t) = -\frac{1}{2}\Delta\sigma_x - \frac{1}{2}[\varepsilon_0 + \epsilon(t)]\sigma_z$$

M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998)

Background

- Rabi rotating wave approximation

$$H(t) = \frac{1}{2}\omega_0\sigma_z + \frac{A}{2}(\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}) + \frac{A}{2}(\sigma_+ e^{i\omega t} + \sigma_- e^{-i\omega t})$$

- Floquet theory $i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle$

$$|\psi_\alpha(t)\rangle = |u_\alpha(t)\rangle e^{-i\varepsilon_\alpha t}, \quad |u_\alpha(t)\rangle = |u_\alpha(t+T)\rangle$$

$$\mathcal{H}_F|u_\alpha(t)\rangle = \varepsilon_\alpha|u_\alpha(t)\rangle$$

Floquet Hamiltonian $\mathcal{H}_F = H(t) - i\partial_t$

M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998)

Our plan

1. **CHRW approach for semiclassical Rabi models, which is as simple as the usual RWA approach but the results are in good agreement with the numerical calculations.**
2. **We studied the quantum dynamics of the semiclassical Rabi model without and with static bias, the Bloch-Siegert shift.**
3. **Because of the simple math structure, the approach my be used for more complex systems.**

Unitary transformation

- Driven two-level system $H(t) = \frac{\omega_0}{2}\sigma_z + \frac{A}{2}\cos(\omega t)\sigma_x$

- Unitary transformation $UU^\dagger = U^\dagger U = 1 \; U = e^S \quad S^\dagger = -S$

$$e^{S(t)}\left[H(t) - i\frac{d}{dt}\right][e^{-S(t)}e^{S(t)}|\Psi(t)\rangle] \equiv \left[H'(t) - i\frac{d}{dt}\right]|\Psi'(t)\rangle = 0$$

$$H'(t) = e^{S(t)}H(t)e^{-S(t)} - ie^{S(t)}\frac{d}{dt}e^{-S(t)}$$

$$S(t) = i\frac{A}{2\omega}\xi\sin(\omega t)\sigma_x$$

$$H'(t) = \frac{1}{2}\omega_0 \left\{ \cos\left[\frac{A}{\omega}\xi \sin(\omega t)\right]\sigma_z + \sin\left[\frac{A}{\omega}\xi\sin(\omega t)\right]\sigma_y \right\} + \frac{A}{2}(1-\xi)\cos(\omega t)\sigma_x$$

Using $$\exp\left[i\frac{A}{\omega}\xi\sin(\omega t)\right] = \sum_{n=-\infty}^{\infty} J_n\left(\frac{A}{\omega}\xi\right)\exp(in\omega t)$$

Jacobi-Anger relations

$$H'_0 = \frac{1}{2}\omega_0 J_0\left(\frac{A}{\omega}\xi\right)\sigma_z,$$

$$H'_1(t) = \frac{A}{2}(1-\xi)\cos(\omega t)\sigma_x + \omega_0 J_1\left(\frac{A}{\omega}\xi\right)\sin(\omega t)\sigma_y,$$

$$H'_2(t) = \omega_0 \sum_{n=1}^{\infty} J_{2n}\left(\frac{A}{\omega}\xi\right)\cos(2n\omega t)\sigma_z + \omega_0 \sum_{n=1}^{\infty} J_{2n+1}\left(\frac{A}{\omega}\xi\right)\sin[(2n+1)\omega t]\sigma_y,$$

Zhiguo Lü, and Hang Zheng, Effects of counterrotating interaction on driven tunneling dynamics: coherent destruction of tunneling and Bloch-Siegert shift, **Phy.Rev.A** 86, 023831(2012)

The parameter ξ

$$J_1\left(\frac{A}{\omega}\xi\right)\omega_0 = \frac{A}{2}(1-\xi) \equiv \frac{\tilde{A}}{4}$$

The Hamiltonian

$$H_{\mathrm{CHRW}}(t) = \frac{1}{2}J_0\left(\frac{A}{\omega}\xi\right)\omega_0\sigma_z + \frac{\tilde{A}}{4}(e^{-i\omega t}\sigma_+ + e^{i\omega t}\sigma_-)$$

$$\tilde{H}_{\mathrm{CHRW}} = R(t)H_{\mathrm{CHRW}}(t)R^\dagger(t) - R(t)i\frac{d}{dt}R^\dagger(t) = \frac{\tilde{\Delta}}{2}\sigma_z + \frac{\tilde{A}}{4}\sigma_x$$

$$R(t) = \exp\left(\frac{i}{2}\omega t\sigma_z\right)$$

Rabi frequency $\tilde{\Omega}_R = \sqrt{\tilde{\Delta}^2 + \frac{\tilde{A}^2}{4}}$

Modified detuning $\tilde{\Delta} = J_0\left(\frac{A}{\omega}\xi\right)\omega_0 - \omega$

Diagonalization of the CHRW Hamiltonian

Its eigenstates and corresponding eigenenergies are given as follows

$$|\widetilde{\mp}\rangle = \sin\theta|\mp\rangle \pm \cos\theta|\pm\rangle,$$
$$E_{\pm} = \pm\frac{1}{2}\sqrt{\tilde{\Delta}^2 + \tilde{A}^2/4} \equiv \pm\frac{1}{2}\tilde{\Omega}_R,$$

$$\sigma_z|\pm\rangle = \pm|\pm\rangle$$

$$\theta = \arctan\left[2(\tilde{\Omega}_R - \tilde{\Delta})/\tilde{A}\right]$$

Time evolution operator : $\mathrm{U_S(t) = e^{-S(t)}R^{\dagger}(t)\exp(-i\tilde{H}_{CHRW}t)}$

Provided that the initial states of TLS is $|\psi_{\pm}(0)\rangle = |\widetilde{\pm}\rangle$, we have the final state $|\psi_{\pm}(t)\rangle = U_S(t)|\psi_{\pm}(0)\rangle \equiv e^{-i\varepsilon_{\pm n}t}|u_{\pm n}(t)\rangle$, where

Floquet mode $$|u_{\pm n}(t)\rangle = e^{i(n+1/2)\omega_l t}e^{-S(t)}R^{\dagger}(t)|\widetilde{\pm}\rangle,$$

quasienergies $$\varepsilon_{\pm n} = (\omega_l \pm \tilde{\Omega}_R)/2 + n\omega_l.$$

Our renormalized scheme

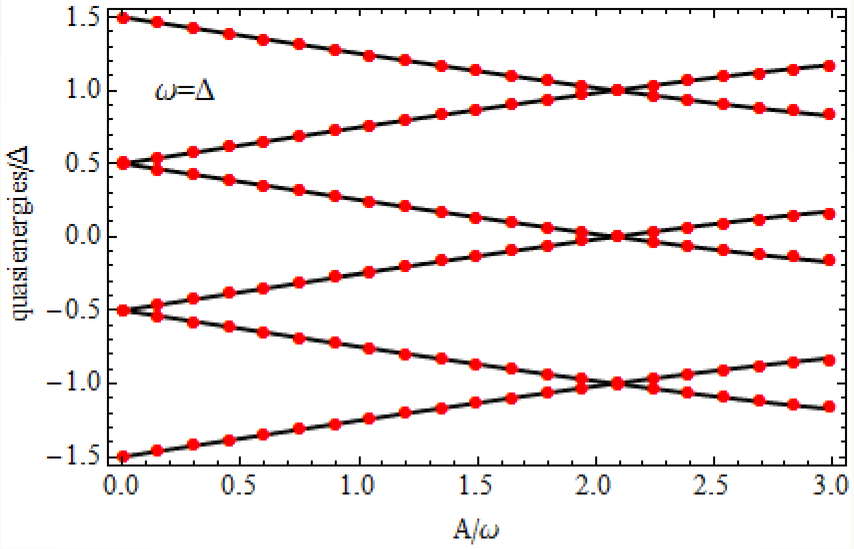
$$1. A \to \tilde{A} = 2A(1-\xi)$$

$$2. \omega_0 \to \tilde{\omega}_0 = \omega_0 J_0\left(\frac{A}{\omega}\xi\right)$$

$$3. \Omega_R = \sqrt{(\omega-\omega_0)^2 + A^2/4} \to \tilde{\Omega}_R = \sqrt{(\omega-\tilde{\omega}_0)^2 + \tilde{A}^2/4}$$

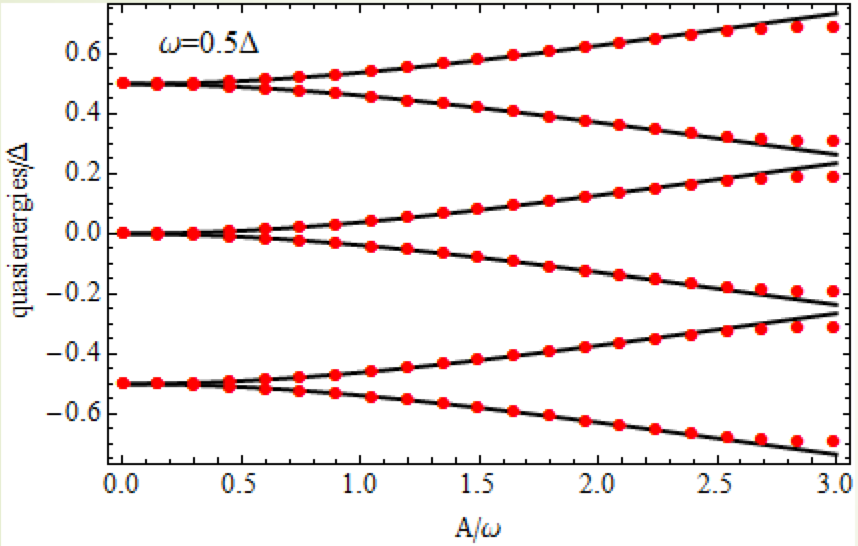
Checks:quasienergy

$\omega = \omega_0$

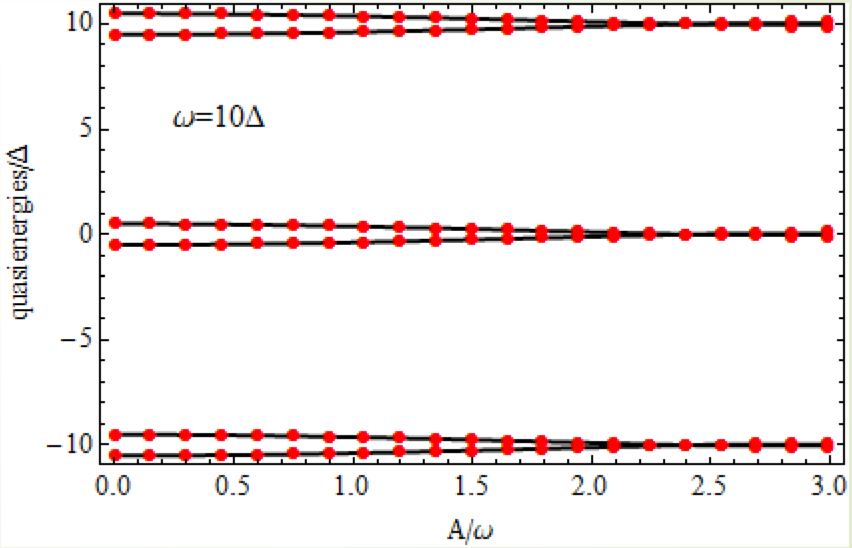


Solid line: CHRW
Red dot: numerical

$\omega = 0.5\omega_0$



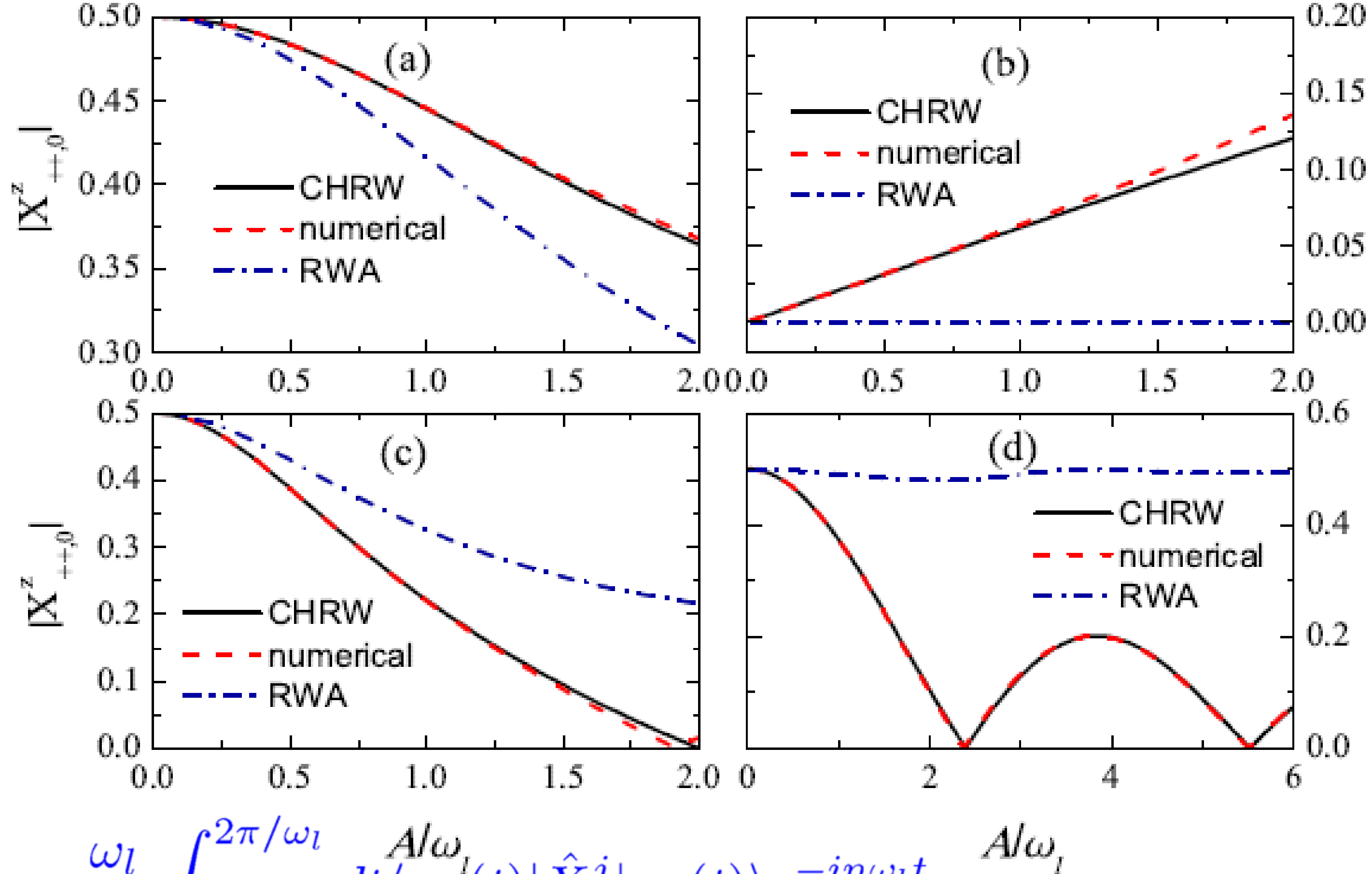
$\omega = 10\omega_0$



Solid line: CHRW
Red dashed line: numerical
Blue line:RWA

Checks: transition element

(a) $\omega_l = 0.5\omega_0$, (b) $\omega_l = \omega_0$, (c) $\omega_l = 1.5\omega_0$, and (d) $\omega_l = 5\omega_0$.



$$X^j_{\alpha\beta,n} = \frac{\omega_l}{2\pi}\int_0^{2\pi/\omega_l} dt\langle u_\alpha(t)|\hat{X}^j|u_\beta(t)\rangle e^{-in\omega_l t}, \quad (1)$$

$$\Delta_{\alpha\beta,n} = \varepsilon_\alpha - \varepsilon_\beta + n\omega_l. \quad (2)$$

Rabi frequency

$$\Omega_{\mathrm{RWA}}^2 = (\omega-\omega_0)^2 + A^2/4$$

$$0 = \frac{\partial \Omega_{\mathrm{RWA}}^2}{\partial \omega_0} \qquad \omega_{\mathrm{Res}} = \omega_0$$

$$\tilde{\Omega}_{\mathrm{R}}^2 = \left(\omega - \omega_0 J_0\left(\frac{A}{\omega}\xi\right)\right)^2 + A^2(1-\xi)^2$$

$$\tilde{\Omega}_{\mathrm{R}}^2 \approx (\omega-\omega_0)^2 + \frac{\omega_0 A^2}{2(\omega+\omega_0)} - \frac{\omega_0 A^4}{32(\omega+\omega_0)^3}$$

It is the same as the exact one to fourth order of *A*

$$\frac{\partial}{\partial \omega_0}\tilde{\Omega}_{\mathrm{R}}^2 = 0, \qquad \omega_{\mathrm{Res-CHRW}} = \omega_0 + \frac{A^2}{16\omega_0} + \frac{A^4}{4\times 256\,\omega_0^3}$$

Bloch-Siegert shift

MARCH 15, 1940 PHYSICAL REVIEW VOLUME 57

Magnetic Resonance for Nonrotating Fields

F. BLOCH AND A. SIEGERT*
Department of Physics, Stanford University, Stanford University, California
(Received January 19, 1940)

A treatment of the magnetic resonance is given for a particle with spin $\frac{1}{2}$ in a constant field H_0 and under the action of an arbitrary alternating field with circular frequency ω perpendicular to H_0. A method of finding a solution, valid at any time, is given which converges the better the smaller the deviations from a rotating field or the larger H_0. It is shown that in the lowest order correction the shape of the resonance curve is unchanged but that it is shifted by a percentage amount $H_1^2/16\,H_0^2$ where H_1 is the effective amplitude of the oscillating field. This also involves a correction in the values of the magnetic moments thus obtained towards smaller values which however in all practical cases is negligibly small.

Resonance condition

$$\frac{\partial}{\partial \omega_0} \tilde{\Omega}_R^2 = 0,$$

Bloch-Siegert shift

Bloch & Siegert, PR57, 522 (1940)

Definition $\delta\omega_{\mathrm{BS}} = \omega_{\mathrm{res}} - \omega_0$

The resonance frequency is defined as the frequency at which the transition probability Pup averaged is a maximum.

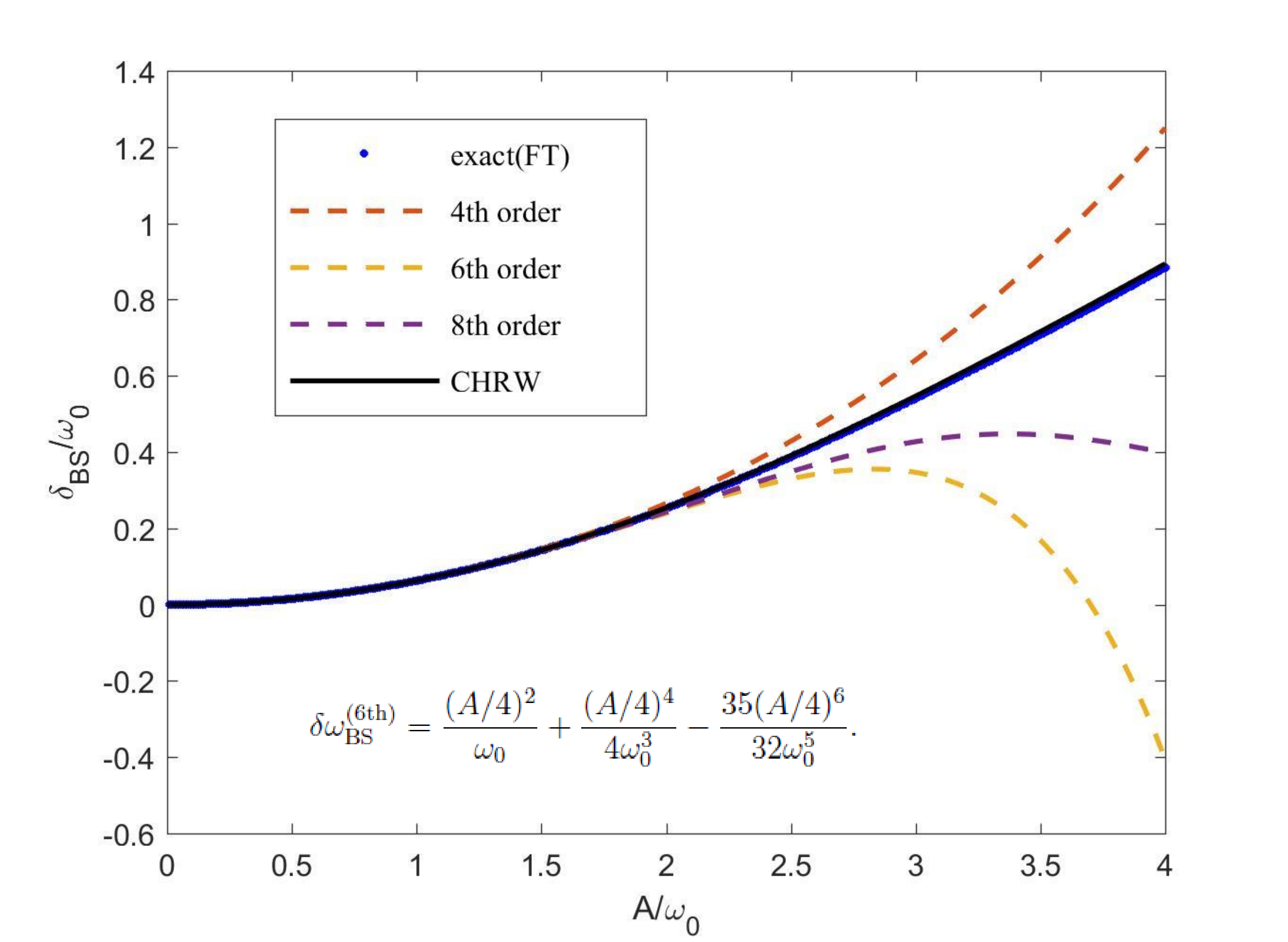
Resonance condition

$$\frac{\partial\tilde{\Omega}_R^2}{\partial\omega_0} = 2\left[\omega_0 J_0\left(\frac{A}{\omega}\xi\right) - \omega\right]\left[J_0\left(\frac{A}{\omega}\xi\right) - \omega_0\frac{A}{\omega}\right.$$
$$\left.\times J_1\left(\frac{A}{\omega}\xi\right)\frac{\partial\xi}{\partial\omega_0}\right] - 2A^2(1-\xi)\frac{\partial\xi}{\partial\omega_0} = 0$$

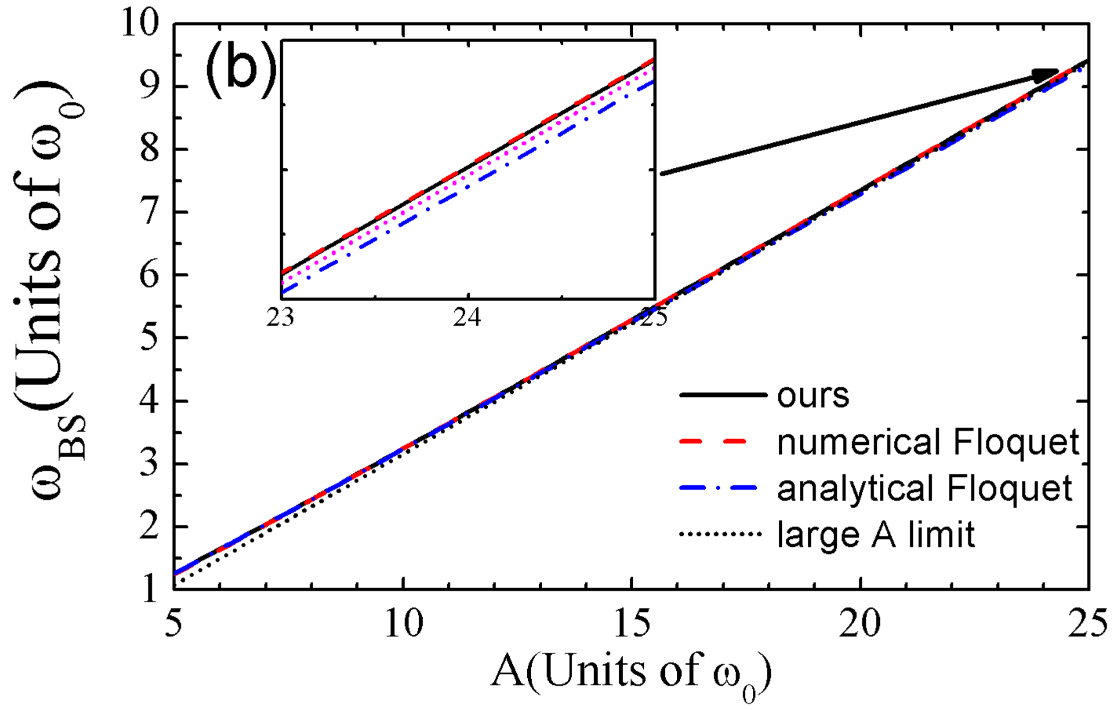
$$\frac{\partial\xi}{\partial\omega_0} = -\frac{2\omega J_1\left(\frac{A}{\omega}\xi\right)}{A\left\{\omega + \omega_0\left[J_0\left(\frac{A}{\omega}\xi\right) - J_2\left(\frac{A}{\omega}\xi\right)\right]\right\}}$$

$$J_1\left(\frac{A}{\omega}\xi\right)\omega_0 = \frac{A}{2}(1-\xi) \equiv \frac{\tilde{A}}{4}$$

Bloch-Siegert shift



Bloch-Siegert shift



$$\omega = \omega_0 + \frac{\omega A^2}{4(\omega+\omega_0)^2} + \frac{(2\omega_0-\omega)A^4}{64(\omega+\omega_0)^4} + \frac{(9\omega^5 - 126\omega^4\omega_0 + 82\omega^3\omega_0^2 + 42\omega^2\omega_0^3 - 23\omega\omega_0^4 - 8\omega_0^5)A^6}{256(\omega+\omega_0)^6(9\omega^2-\omega_0^2)^2}$$

$$A/\omega_0 \to \infty \qquad \omega_{\mathrm{res}} = A/2.404826.$$

J. H. Shirley, Phys. Rev. **138**, B979 (1965).

Compared with SC flux qubit:

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **89**, 020503(R) (2014)

Flux qubit noise spectroscopy using Rabi oscillations under strong driving conditions

Fumiki Yoshihara,[1,*] Yasunobu Nakamura,[1,2] Fei Yan,[3] Simon Gustavsson,[4] Jonas Bylander,[4,5]
William D. Oliver,[4,6] and Jaw-Shen Tsai[1,7]

[1]*Center for Emergent Matter Science (CEMS), RIKEN, Wako, Saitama 351-0198, Japan*
[2]*Research Center for Advanced Science and Technology (RCAST), The University of Tokyo, Komaba, Meguro-ku, Tokyo 153-8904, Japan*
[3]*Department of Nuclear Science and Engineering, Massachusetts Institute of Technology (MIT), Cambridge, Massachusetts 02139, USA*
[4]*Research Laboratory of Electronics, MIT, Cambridge, Massachusetts 02139, USA*
[5]*Department of Microtechnology and Nanoscience, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden*
[6]*MIT Lincoln Laboratory, 244 Wood Street, Lexington, Massachusetts 02420, USA*
[7]*NEC Smart Energy Research Laboratories, Tsukuba, Ibaraki 305-8501, Japan*

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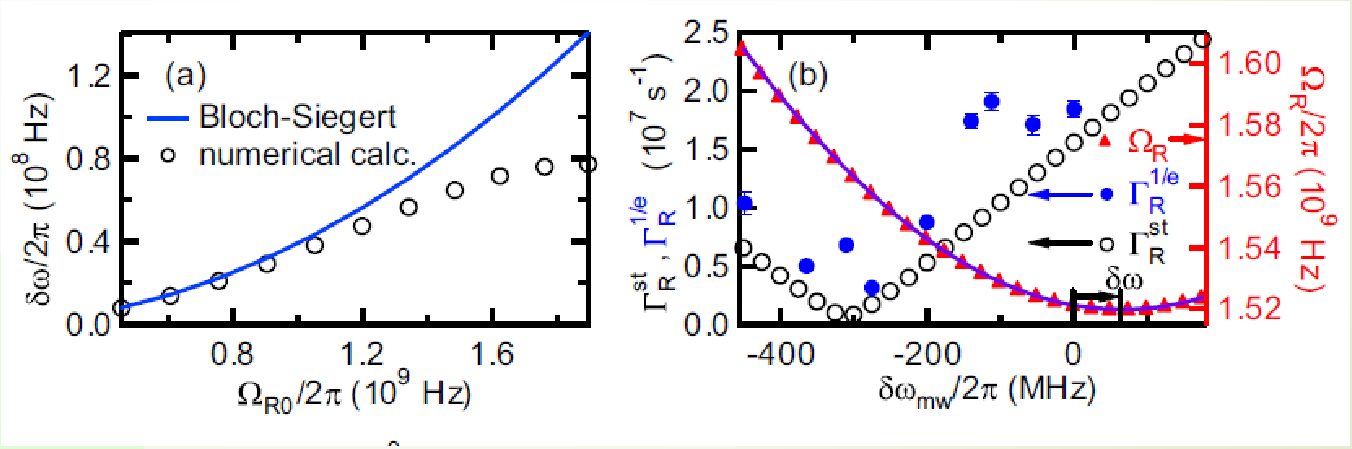


$$\Delta/2\pi = 4.869\text{GHz}$$

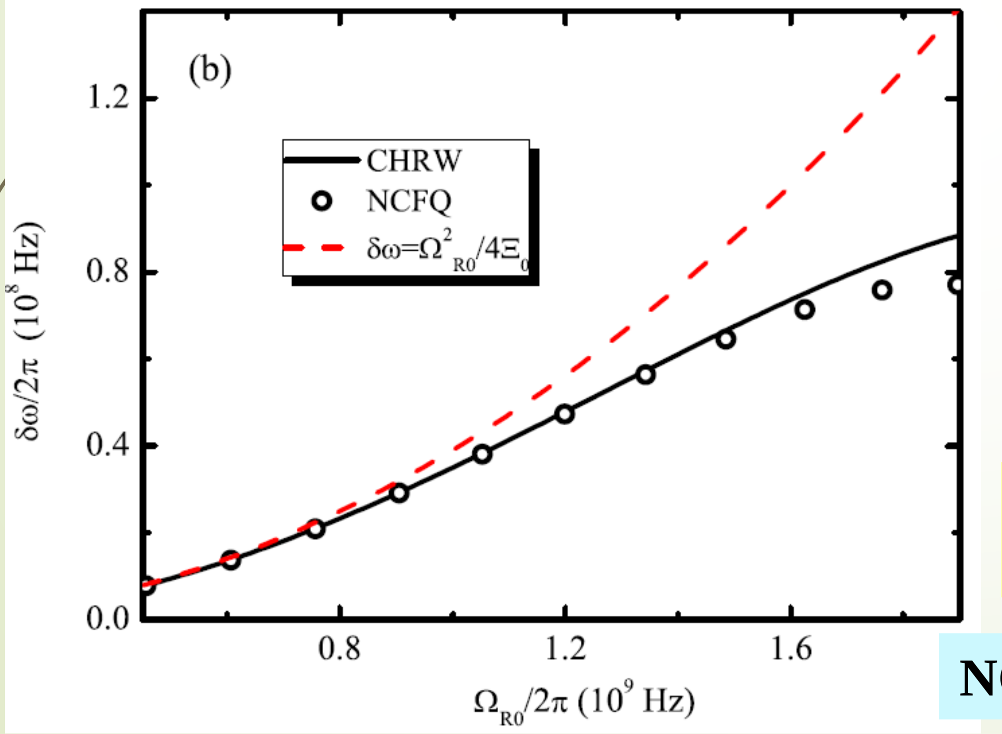
$$\varepsilon/2\pi = 4.154\text{GHz}$$

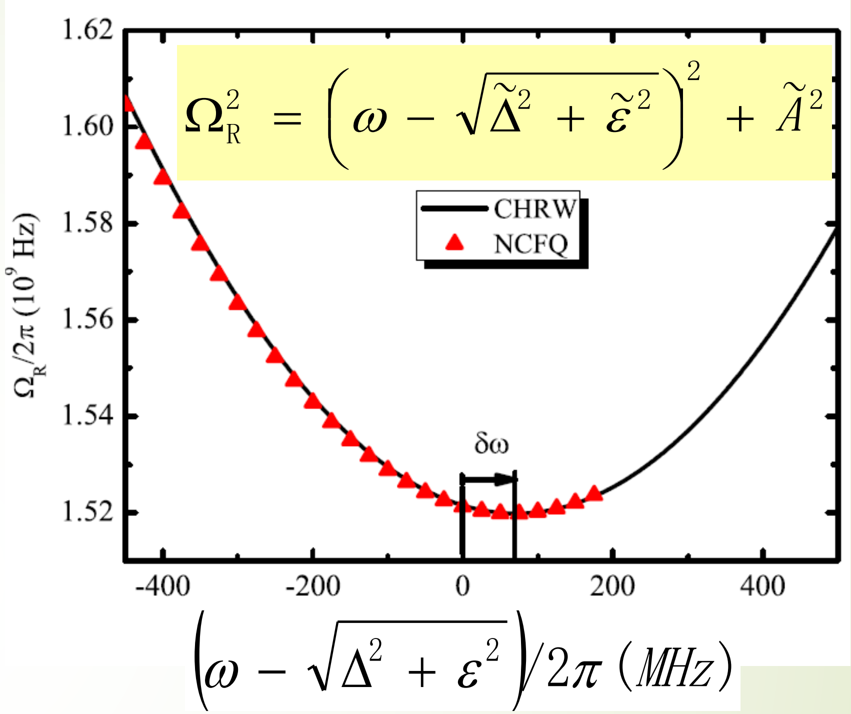
$$\sqrt{\Delta^2 + \varepsilon^2} / 2\pi = 6.400\text{GHz}$$

Our results

$$\Omega_R^2 = \left(\omega - \sqrt{\Delta^2 + \varepsilon^2} - \delta\omega\right)^2 + \Omega_{R0}^2$$

$$\Omega_{R0}^2 = \frac{1}{4}\frac{A^2\Delta^2}{\Delta^2 + \varepsilon^2}$$





$$\delta\omega_{BS} = \frac{A^2\Delta^2}{16\left(\Delta^2 + \varepsilon^2\right)^{3/2}}$$

$$\Omega_R^2 = \left(\omega - \sqrt{\tilde{\Delta}^2 + \tilde{\varepsilon}^2}\right)^2 + \tilde{A}^2$$

NCFQ: PRB89, 020503(R)(2014)

Quantum dynamics

What we dropped:

$$\sin(n\omega t),\ \text{odd} \quad n \geq 3$$

$$\cos(n\omega t),\ \text{even} \quad n \geq 2$$

What we calculate:

$$\sigma_z(t) = \langle \psi(t) | \sigma_z | \psi(t) \rangle = \langle \psi'(t) | e^{S(t)} \sigma_z e^{-S(t)} | \psi'(t) \rangle$$

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle \longrightarrow i\frac{d}{dt}|\psi'(t)\rangle = H'(t)|\psi'(t)\rangle$$

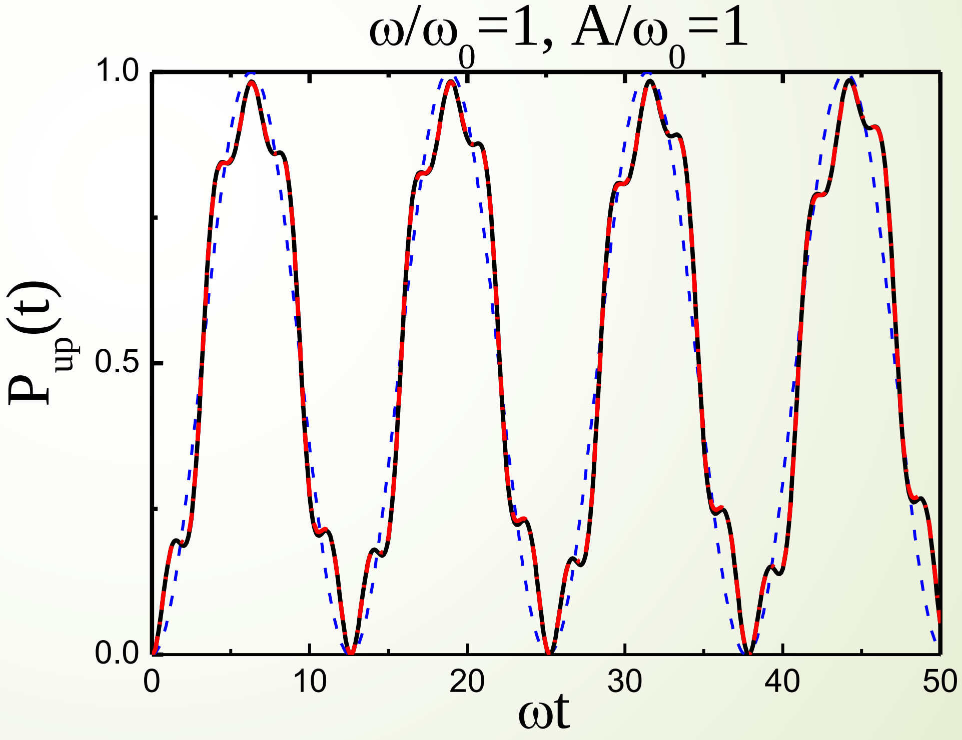
Quantum dynamics

$$P_{up}(t) = \frac{1}{2}(1 + \sigma_z(t))$$

Black: ours (CHRW)

Red: exactly numerical

Blue: RWA



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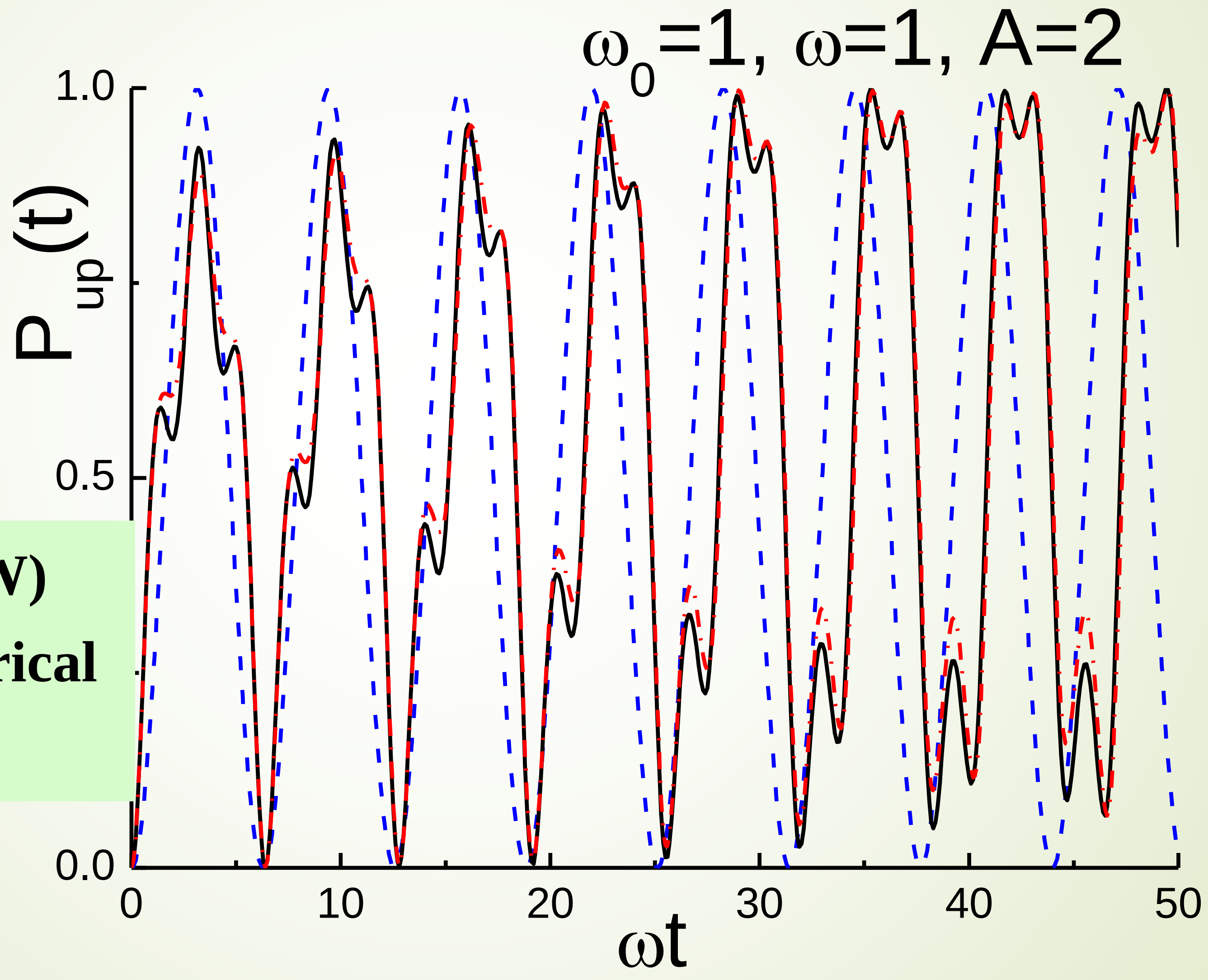

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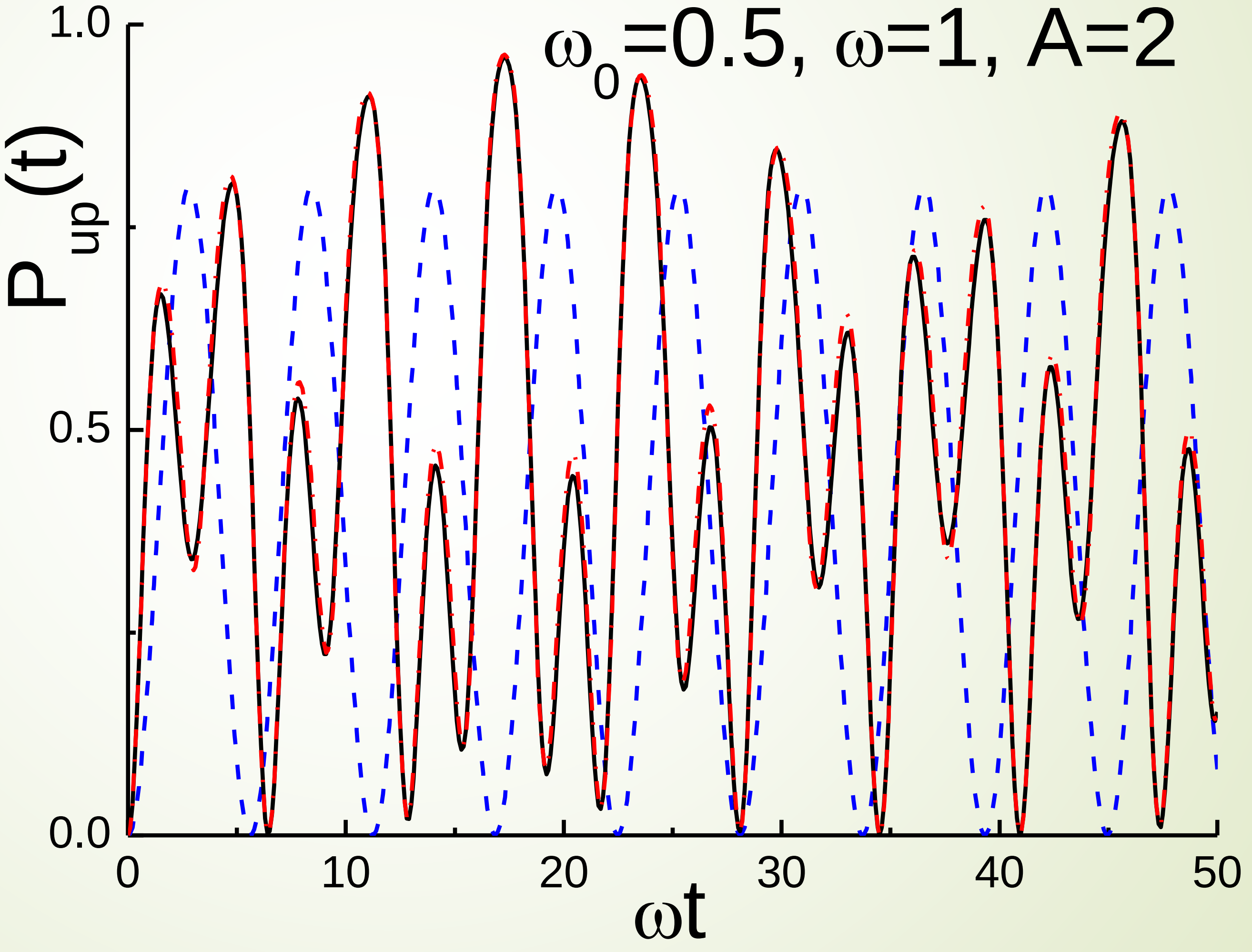

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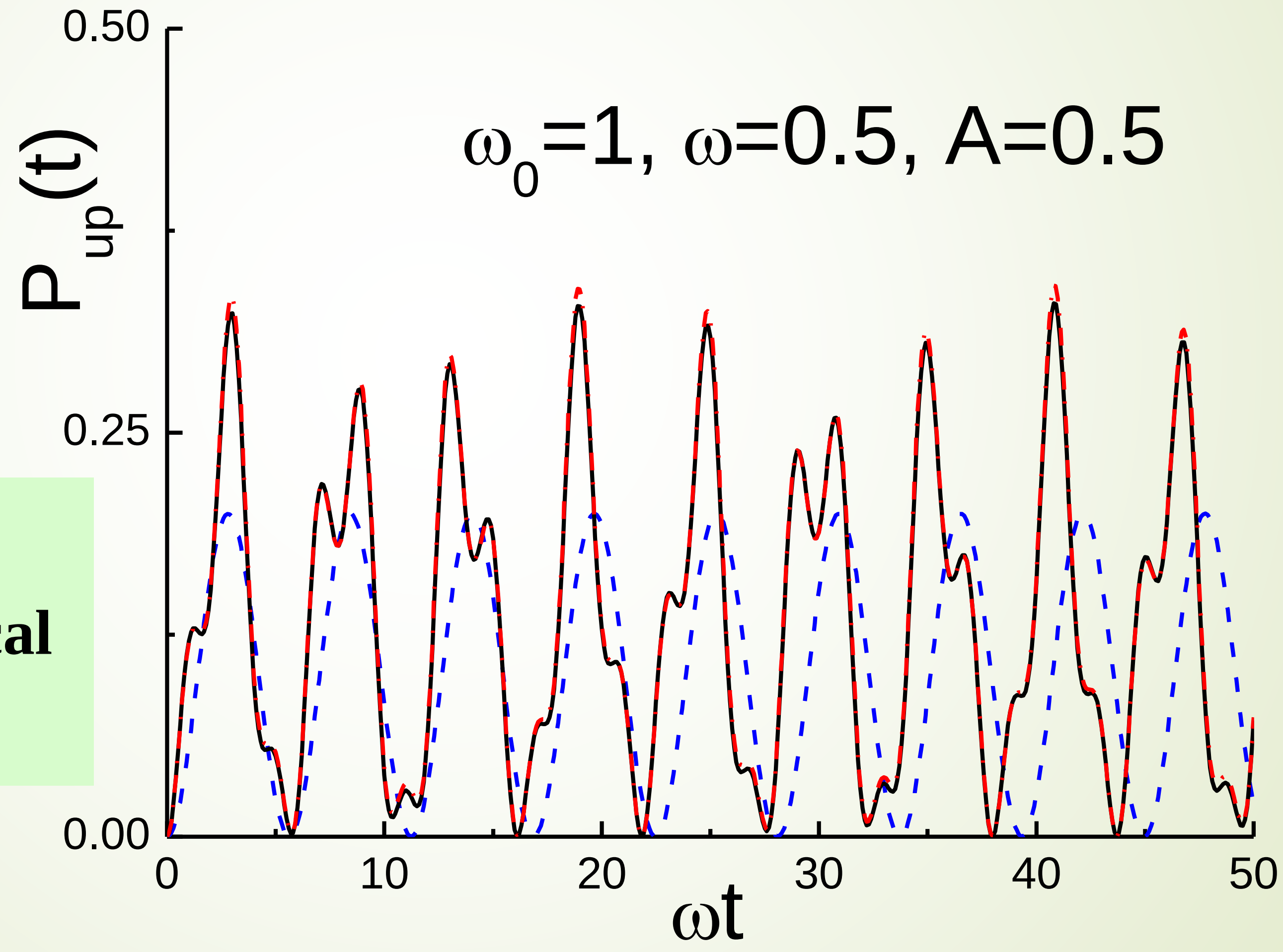

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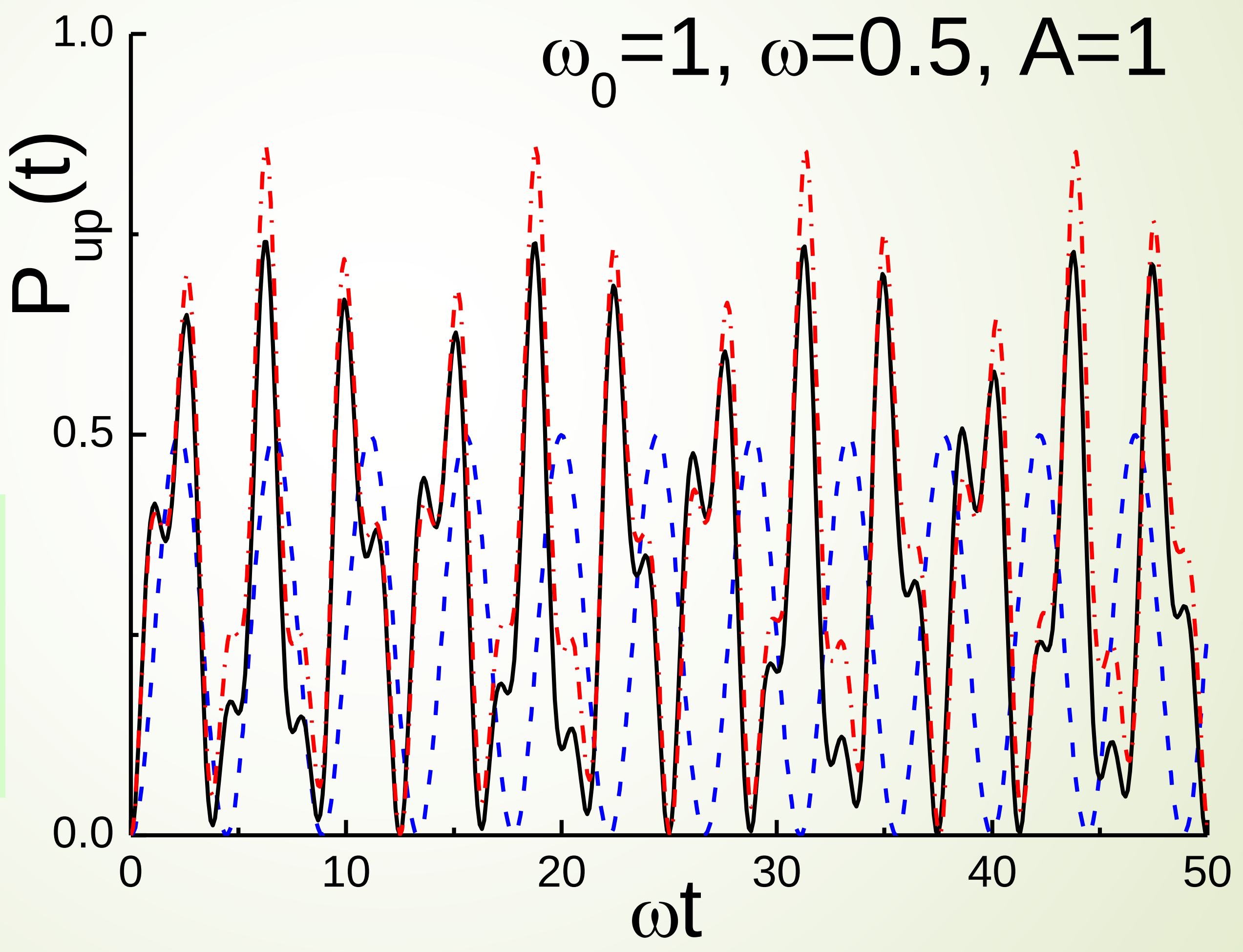


Black: ours

Red: exact numerical

Blue: RWA

Driven tunneling dynamics

$$P_{up}(t)=\frac{1}{2}(1+\sigma_z(t))$$

$$H=-\frac{\Delta}{2}\sigma_x-\frac{\varepsilon(t)}{2}\sigma_z=-\frac{\Delta}{2}\sigma_x-\frac{\varepsilon}{2}\sigma_z-\frac{A\cos(\omega t)}{2}\sigma_z$$

$$=-\frac{\Delta}{2}\sigma_x-\frac{\varepsilon}{2}\sigma_z-\frac{A}{2}\left(e^{i\omega t}\sigma_-+e^{-i\omega t}\sigma_+\right)-\frac{A}{2}\left(e^{i\omega t}\sigma_++e^{-i\omega t}\sigma_-\right)$$

$$\sigma_\pm=\left(\sigma_z\pm i\sigma_y\right)/2$$

PHYSICAL REVIEW A **81**, 022117 (2010)

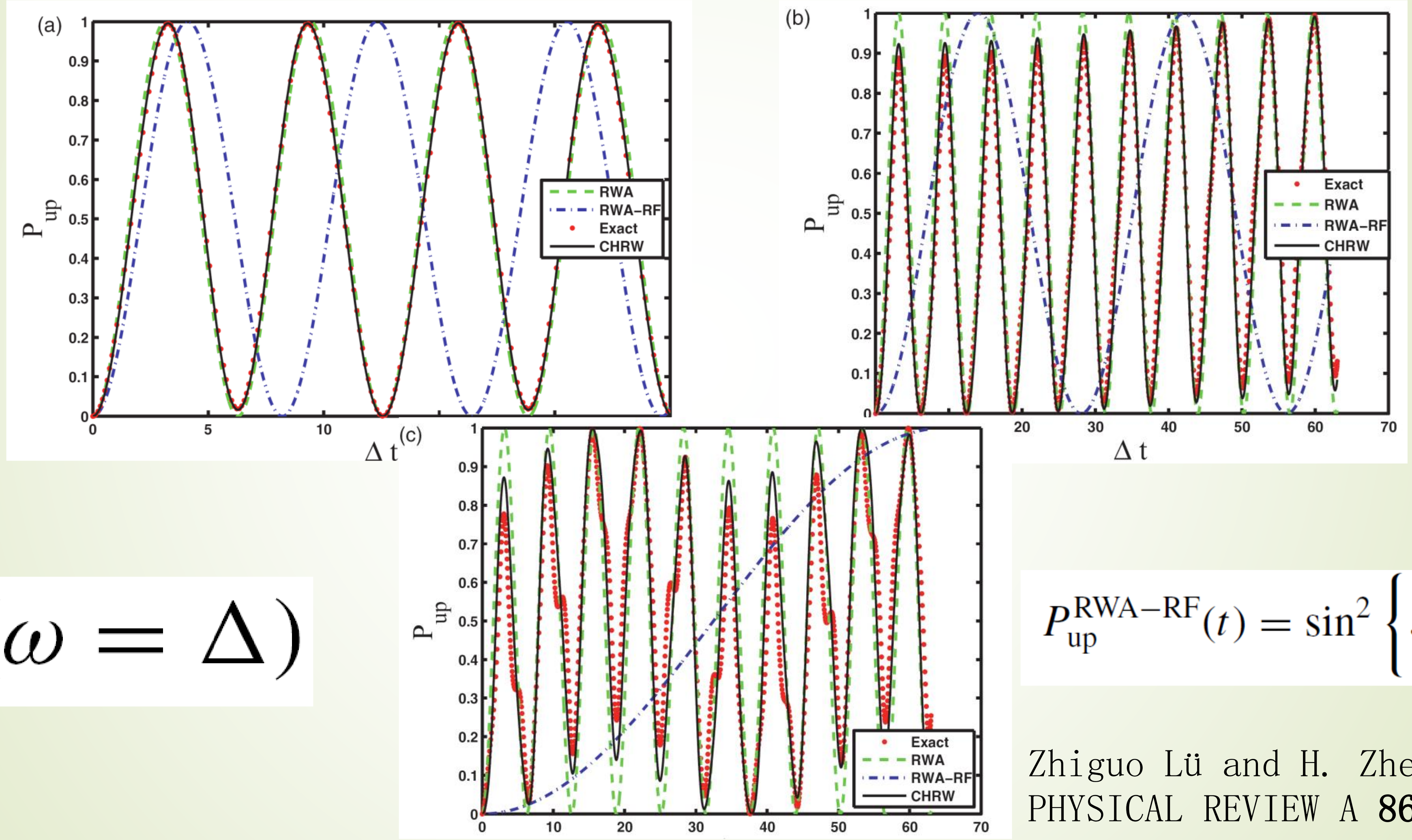
Dissipative two-level system under strong ac driving: A combination of Floquet and Van Vleck perturbation theory

Johannes Hausinger* and Milena Grifoni
Institut für Theoretische Physik, Universität Regensburg, DE-93040 Regensburg, Germany

In a first step, we neglect environmental effects on the driven TLS and consider the Hamiltonian

$$H_{\rm TLS}(t)=-\frac{\hbar}{2}\left[\Delta\sigma_x+(\varepsilon+A\cos\omega t)\sigma_z\right]. \qquad (1)$$

Resonance and near Resonance $\varepsilon = 0$



$$(\omega = \Delta)$$

$$P_{\mathrm{up}}^{\mathrm{RWA-RF}}(t) = \sin^2\left\{J_0\left(\frac{A}{\omega}\right)\frac{\Delta t}{2}\right\}$$

Zhiguo Lü and H. Zheng,
PHYSICAL REVIEW A **86**, 023831 (2012)

FIG. 1. (Color online) $P_{\mathrm{up}}(t)$ as a function of dimensionless time Δt for $A/\omega = 1,2$, and $A/\omega = 2.5$ in the on-resonance case

Resonance and near Resonance $\varepsilon = 0$

$(\omega \sim \Delta)$

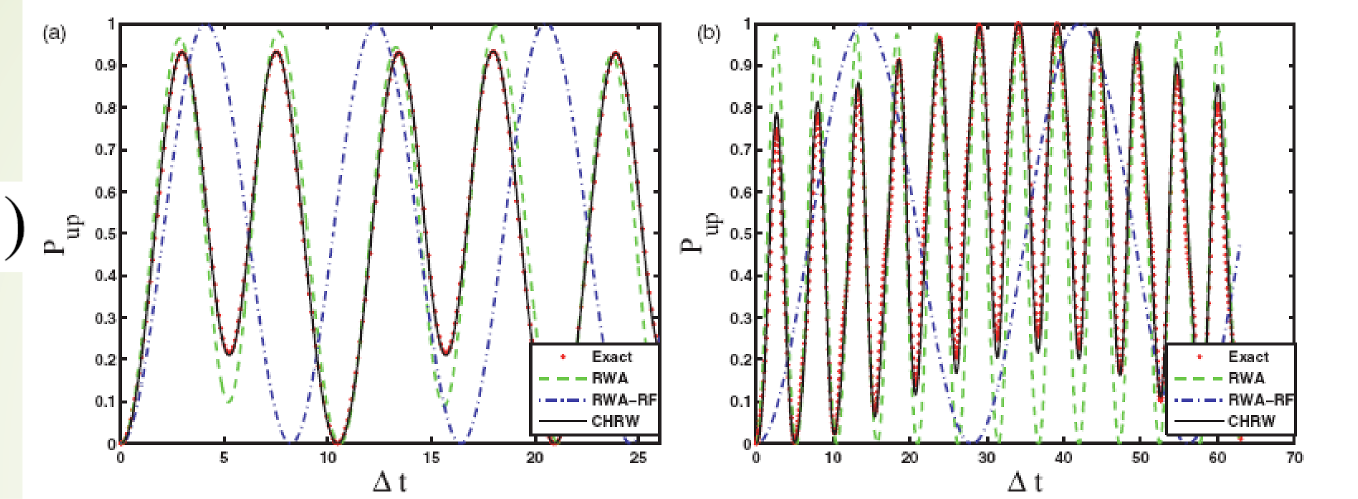


FIG. 2. (Color online) $P_{\mathrm{up}}(t)$ as a function of dimensionless time Δt for $A/\omega = 1$ and $A/\omega = 2$ in the near-resonance case $(\omega/\Delta = 1.2)$

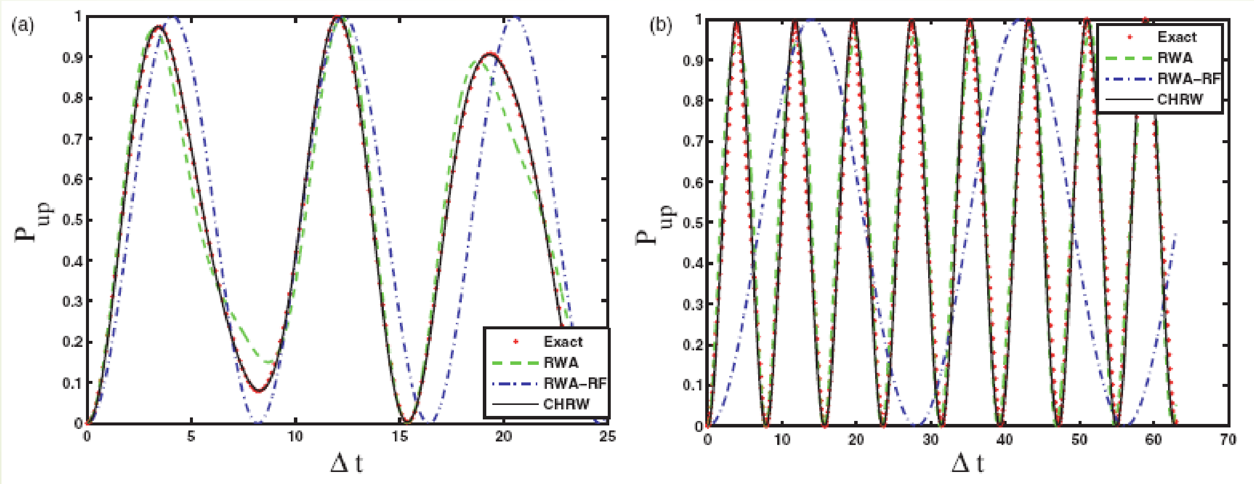
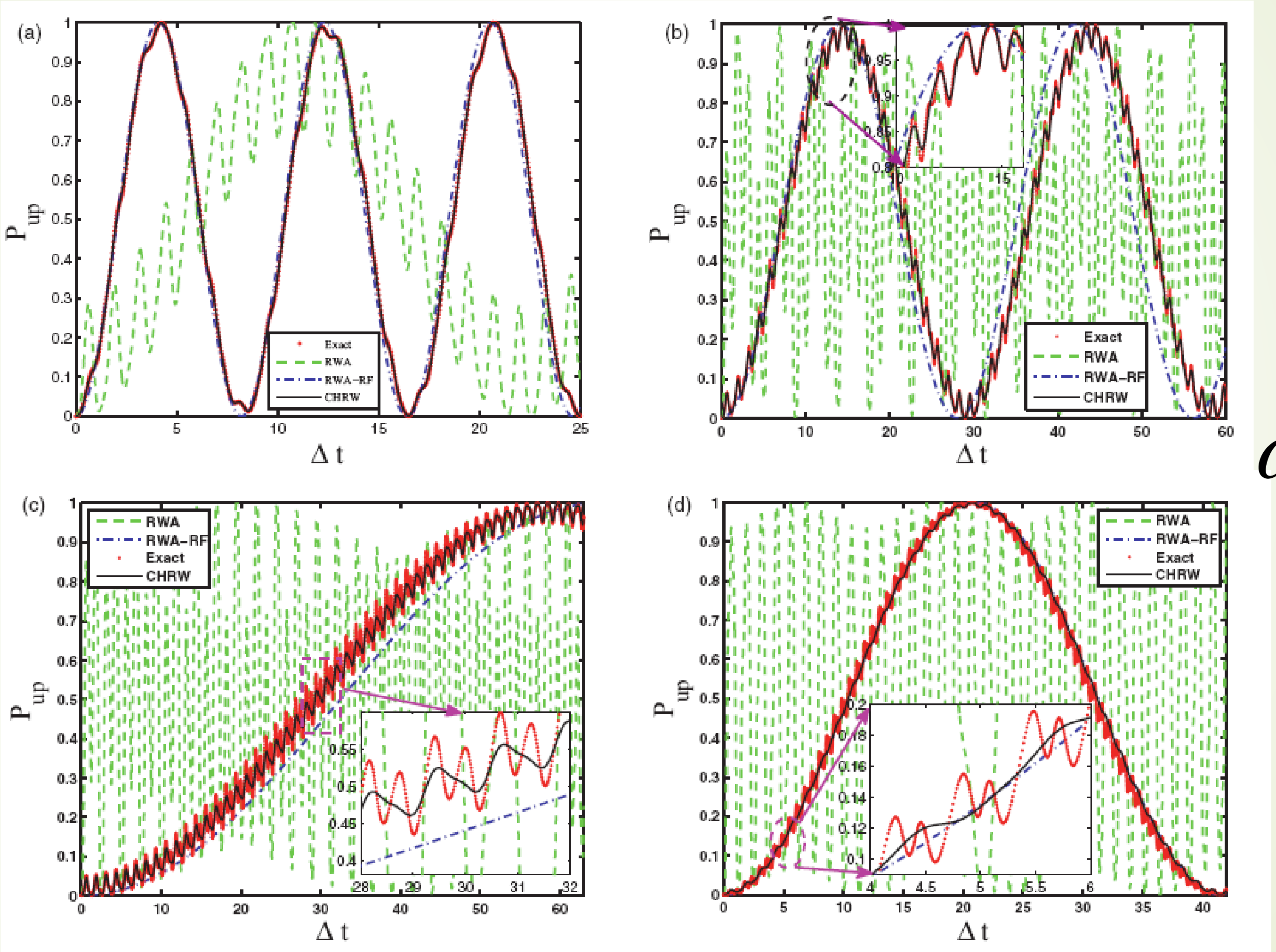


FIG. 3. (Color online) $P_{\mathrm{up}}(t)$ as a function of dimensionless time Δt for $A/\omega = 1$, and $A/\omega = 2$ in the near-resonance case $(\omega/\Delta = 0.8)$

Far off-resonance and CDT

$\varepsilon = 0$



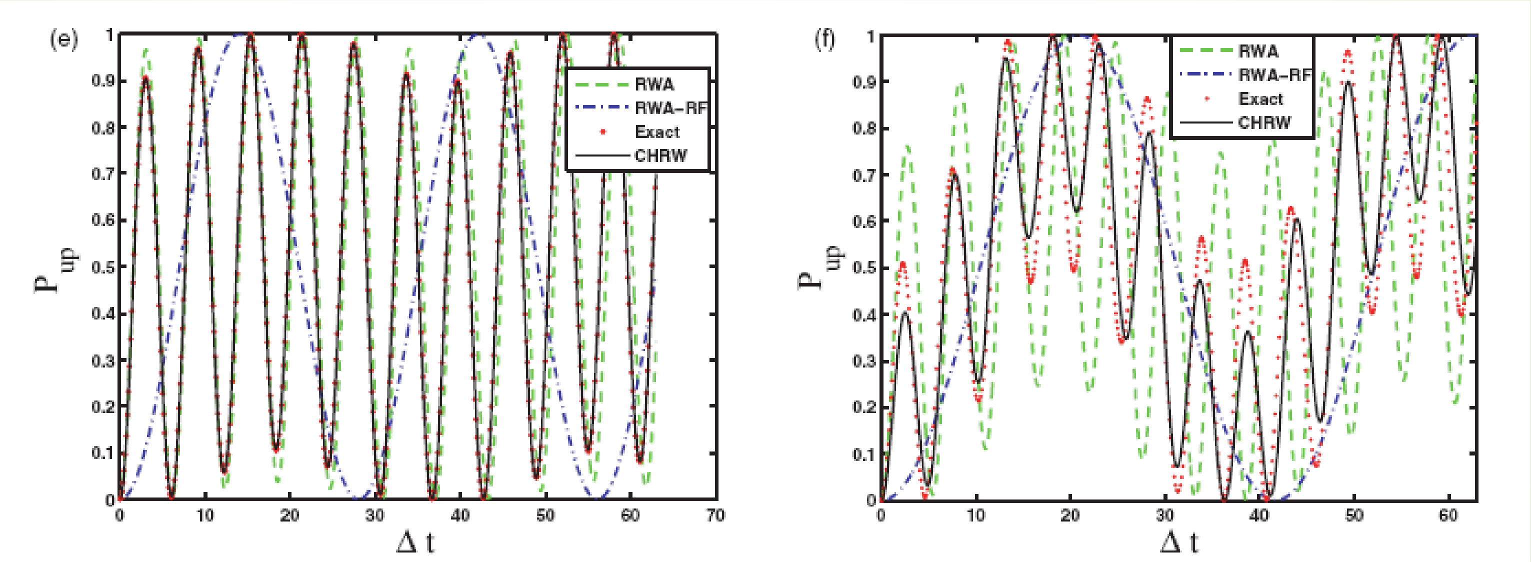
$$\omega = 5\Delta$$

FIG. 4. (Color online) $P_{\rm up}(t)$ as a function of dimensionless time Δt for $A/\omega = 1, 2, 2.5$, and $A/\omega = 6$ in the far off-resonance case

Far off-resonance and CDT

$\varepsilon = 0$

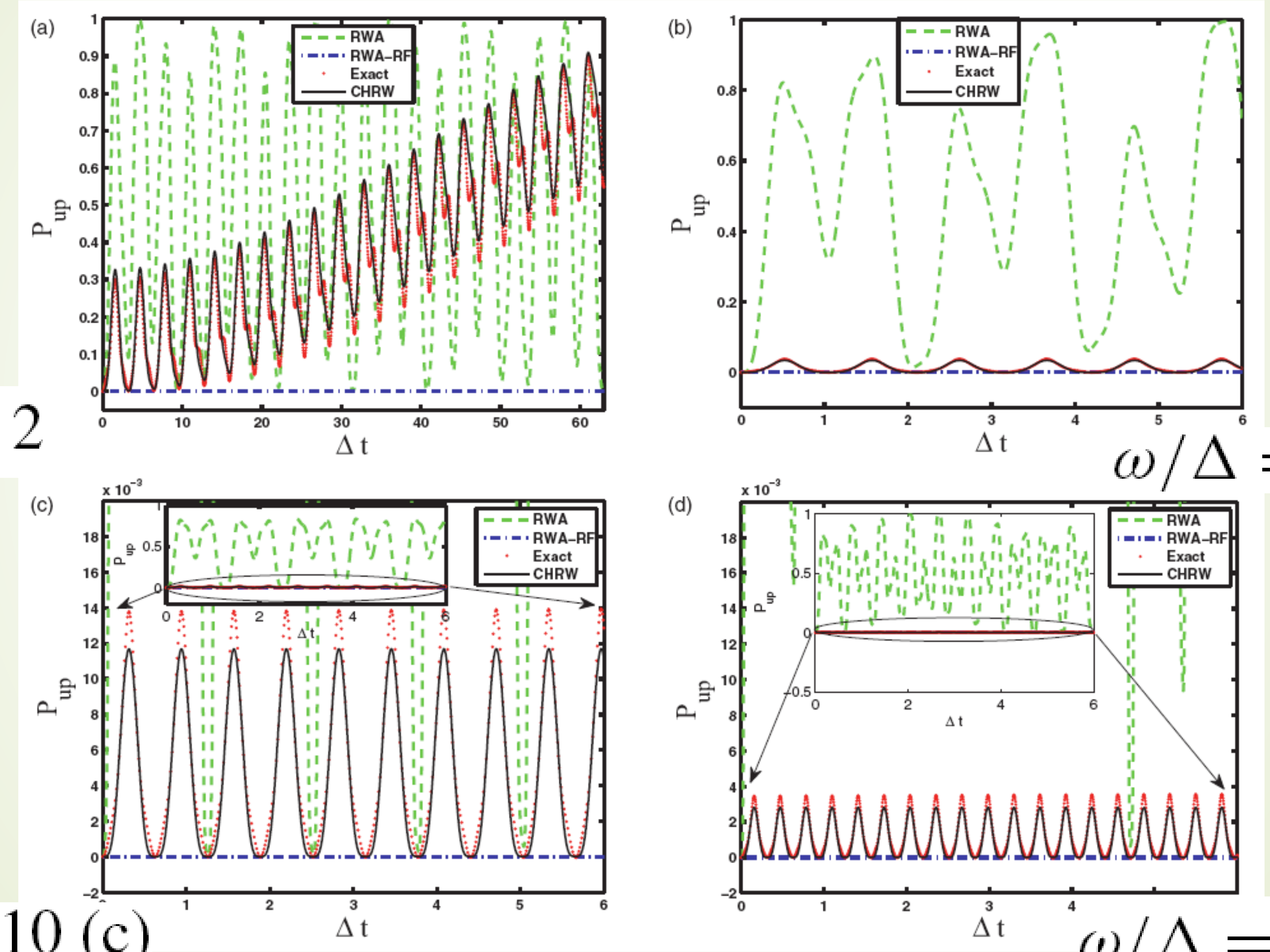
The dynamics of the other far off-resonance case $6\omega = \Delta$ is shown



$(A/\omega = 2)$

$(A/\omega = 6)$

Far off-resonance and CDT $\varepsilon = 0$



$\omega/\Delta = 2$

$\omega/\Delta = 6$ (b)

$\omega/\Delta = 10$ (c)

$\omega/\Delta = 20$ (d)

FIG. 5. (Color online) $P_{up}(t)$ as a function of dimensionless time Δt with $A/\omega = 2.4048 (J_0(2.4048) \simeq 0)$ for different driving frequencies

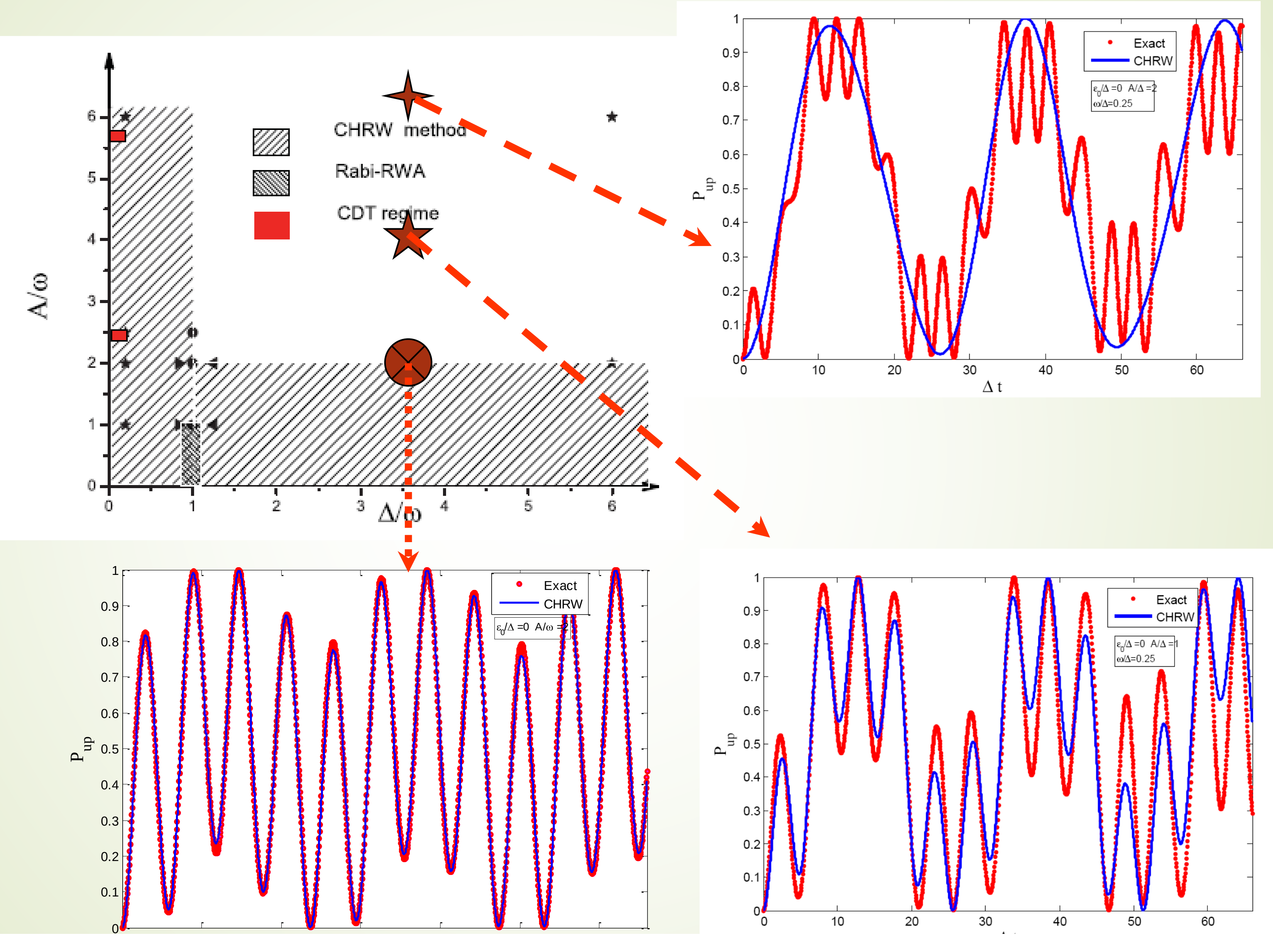


FIG. 6. (Color online) Regions of validity for different RWAs

Bias-modulated case

$$S(t) = -\mathrm{i}\frac{A}{2\omega}\sin(\omega t)\{\xi\sigma_z + \zeta\sigma_x\}$$

In a first step, we neglect environmental effects on the driven TLS and consider the Hamiltonian

$$H_{\mathrm{TLS}}(t) = -\frac{\hbar}{2}[\Delta\sigma_x + (\varepsilon + A\cos\omega t)\sigma_z]. \qquad (1)$$

$$H_{\mathrm{CHRW}}(t) = \frac{1}{2}\sqrt{\tilde{\Delta}^2 + \tilde{\varepsilon}^2}\sigma_z + \frac{\tilde{A}}{2}\left(e^{i\omega t}\sigma_- + e^{-i\omega t}\sigma_+\right)$$

$$\tilde{\Delta} = \Delta - \xi[1 - J_0(Z)](\Delta\xi - \varepsilon\zeta)/X^2$$

$$\tilde{\varepsilon} = \varepsilon + \zeta[1 - J_0(Z)](\Delta\xi - \varepsilon\zeta)/X^2$$

$$\tilde{A} = 2(\Delta\xi - \varepsilon\zeta)J_1(Z)/X, \quad X = \sqrt{\xi^2 + \zeta^2}$$

$$Z = A\sqrt{\xi^2 + \zeta^2}/\omega$$

$$J_c = (1 - J_0(Z) - J_2(Z))/X^2$$

$$A\left[\tilde{\Delta}(1 - \xi - \zeta^2 J_c) + \tilde{\varepsilon}\zeta(1 - \xi J_c)\right]/\sqrt{\tilde{\Delta}^2 + \tilde{\varepsilon}^2} - \tilde{A} = 0$$

$$\tilde{\varepsilon}(1 - \xi - \zeta^2 J_c) - \tilde{\Delta}\zeta(1 - \xi J_c) = 0$$

Zhiguo Lü, Yiying Yan, Hsi-Sheng Goan and Hang Zheng, Bias-modulated dynamics of a strongly driven two-level system, **Phys. Rev.A** 93, 033803(2016)

Bias modulated dynamics: Resonace $\omega = \Xi = \sqrt{\Delta^2 + \varepsilon^2}$

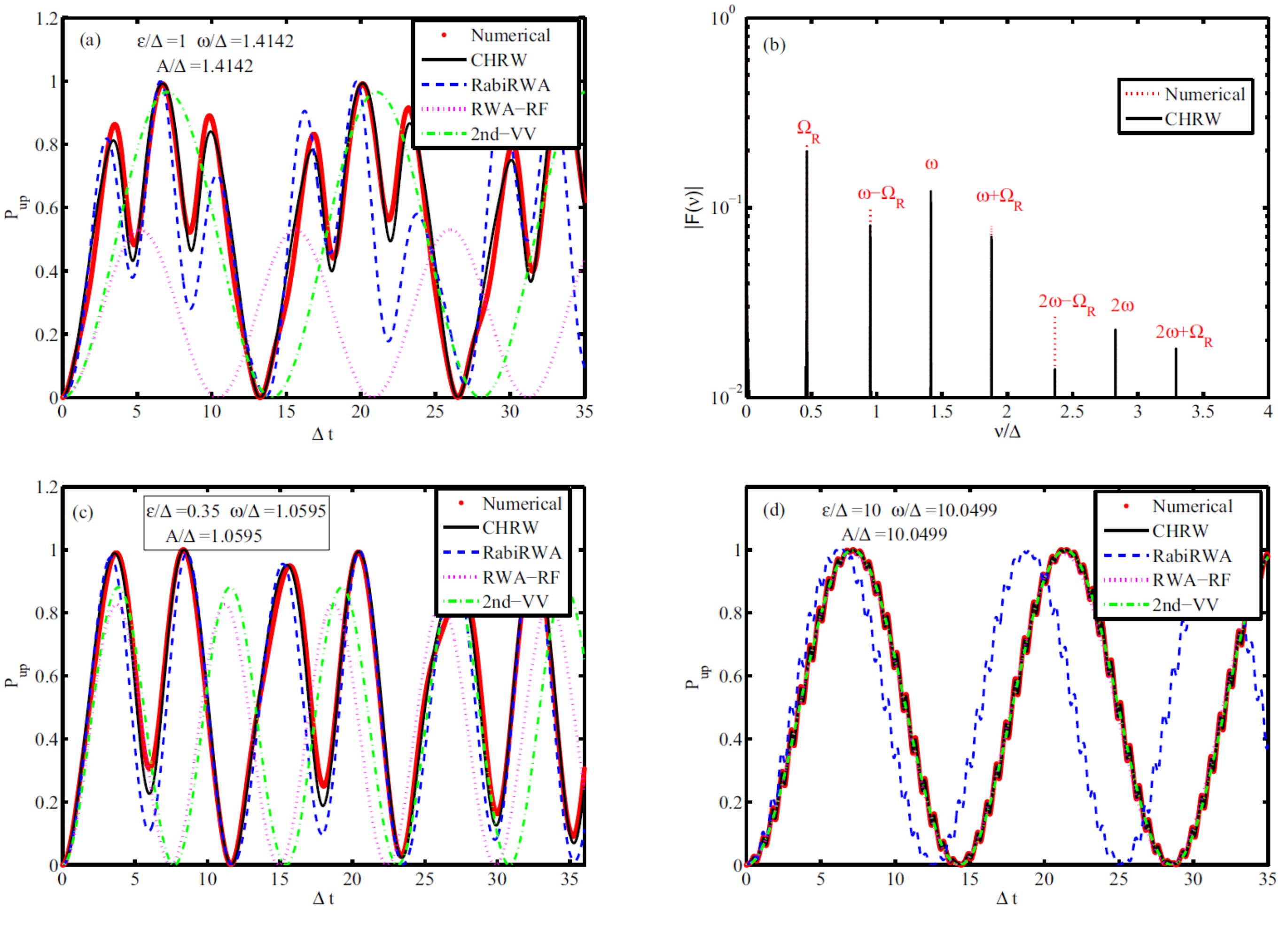


FIG. 1. Time evolutions of $P_{\mathrm{up}}(t) = \langle \frac{\sigma_z(t)+1}{2} \rangle$ as a function of Δt for different values of the bias (a) $\epsilon/\Delta = 1.0$, (c) 0.35, and (d) 10 in the on-resonance case ($\omega = \Xi_0 = \sqrt{\Delta^2 + \epsilon^2}$). The driving strength A is set to be $A/\Xi_0 = 1$. The Fourier transform $F(\nu)$ of $P_{\mathrm{up}}(t)$ in (a) is shown in (b) with a discrete set of frequency components.

Biased modulated dynamics: near resonace

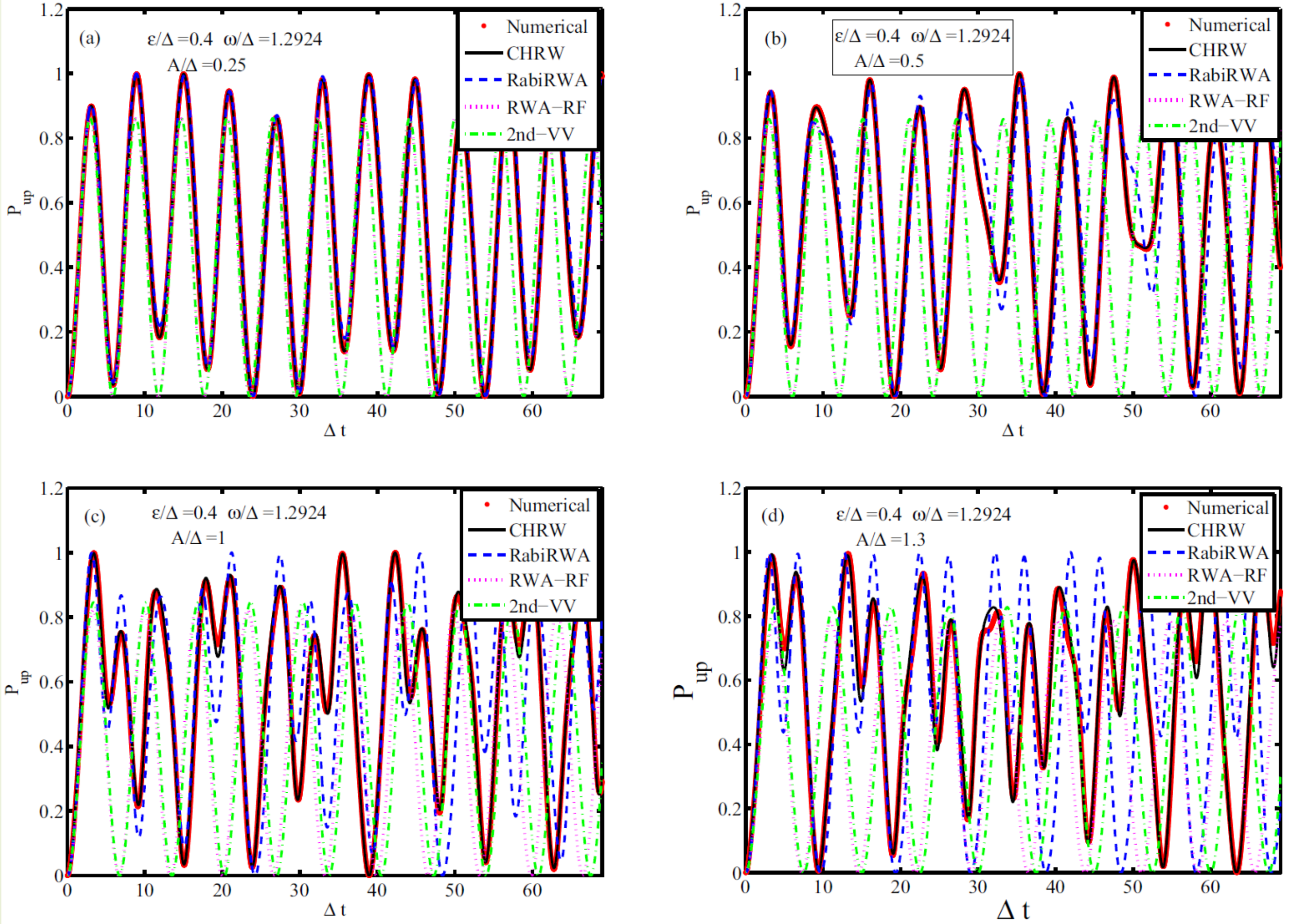


FIG. 2. Time evolutions of $P_{\rm up}(t) = \langle \frac{\sigma_z(t)+1}{2} \rangle$ as a function of Δt for different values of the driving strength (a) $A/\Delta = 0.25$, (b) 0.5, (c) 1.0, and (d) 1.3 in the near-resonance case ($\omega = 1.2\Xi_0 = 1.2924\Delta$). The bias ϵ is set to be a fixed value of $\epsilon/\Delta = 0.4$.

$$\omega = 1.2\Xi$$

Biased modulated dynamcis: far off-resonace

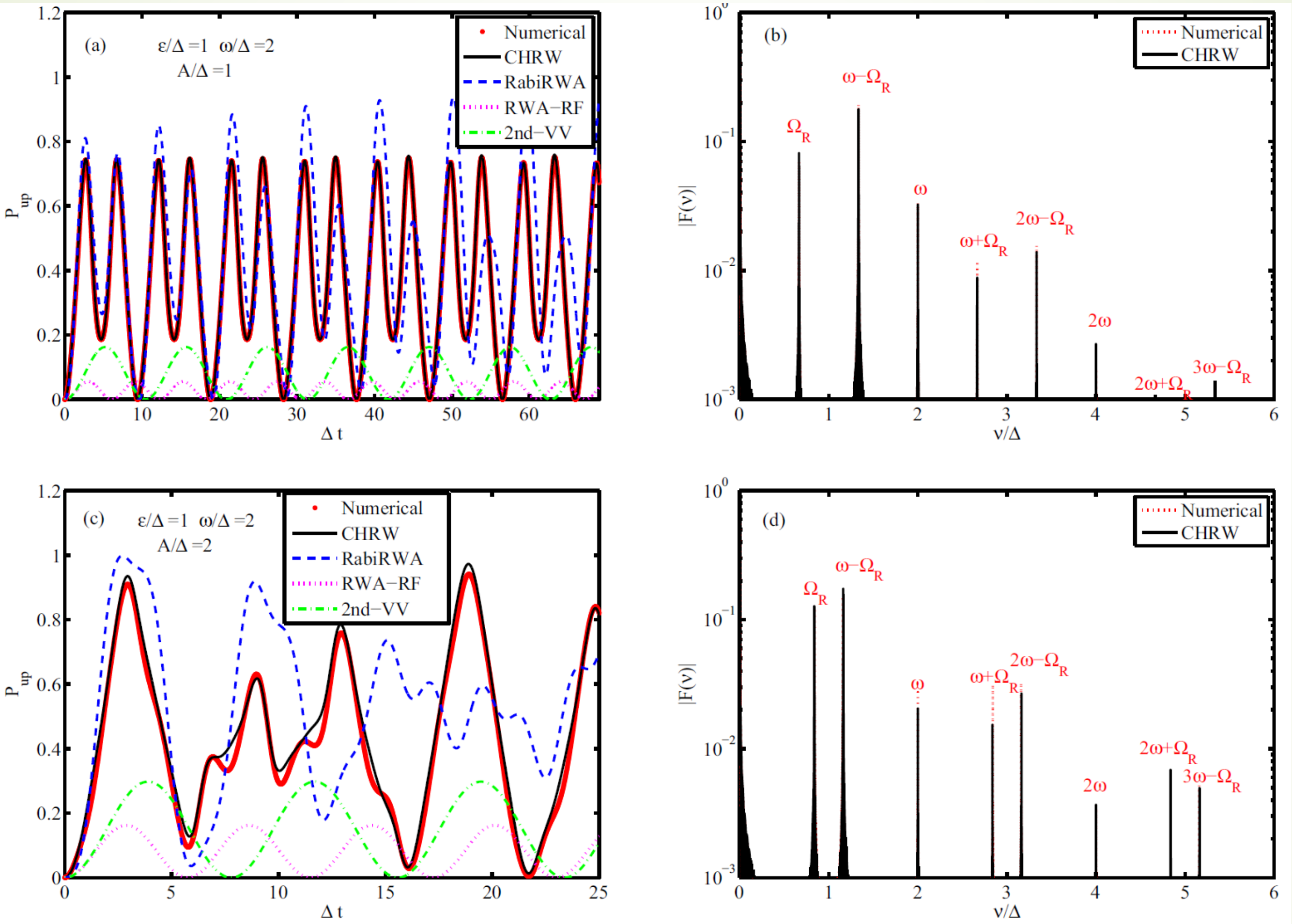


FIG. 3. Time evolutions of $P_{\rm up}(t) = \langle \frac{\sigma_z(t)+1}{2} \rangle$ as a function of Δt for different values of the driving strength (a) $A/\Delta = 1$, and (c) $A/\Delta = 2$ in the off-resonance case ($\omega = 2\Delta > \Xi_0$). The corresponding Fourier transform $F(\nu)$ of $P_{\rm up}(t)$ in (a) and (c) is shown in (b) and (d), respectively. The bias ϵ is set to be $\epsilon/\Delta = 1$.

Bias-modulated dynamics

Off-resonance

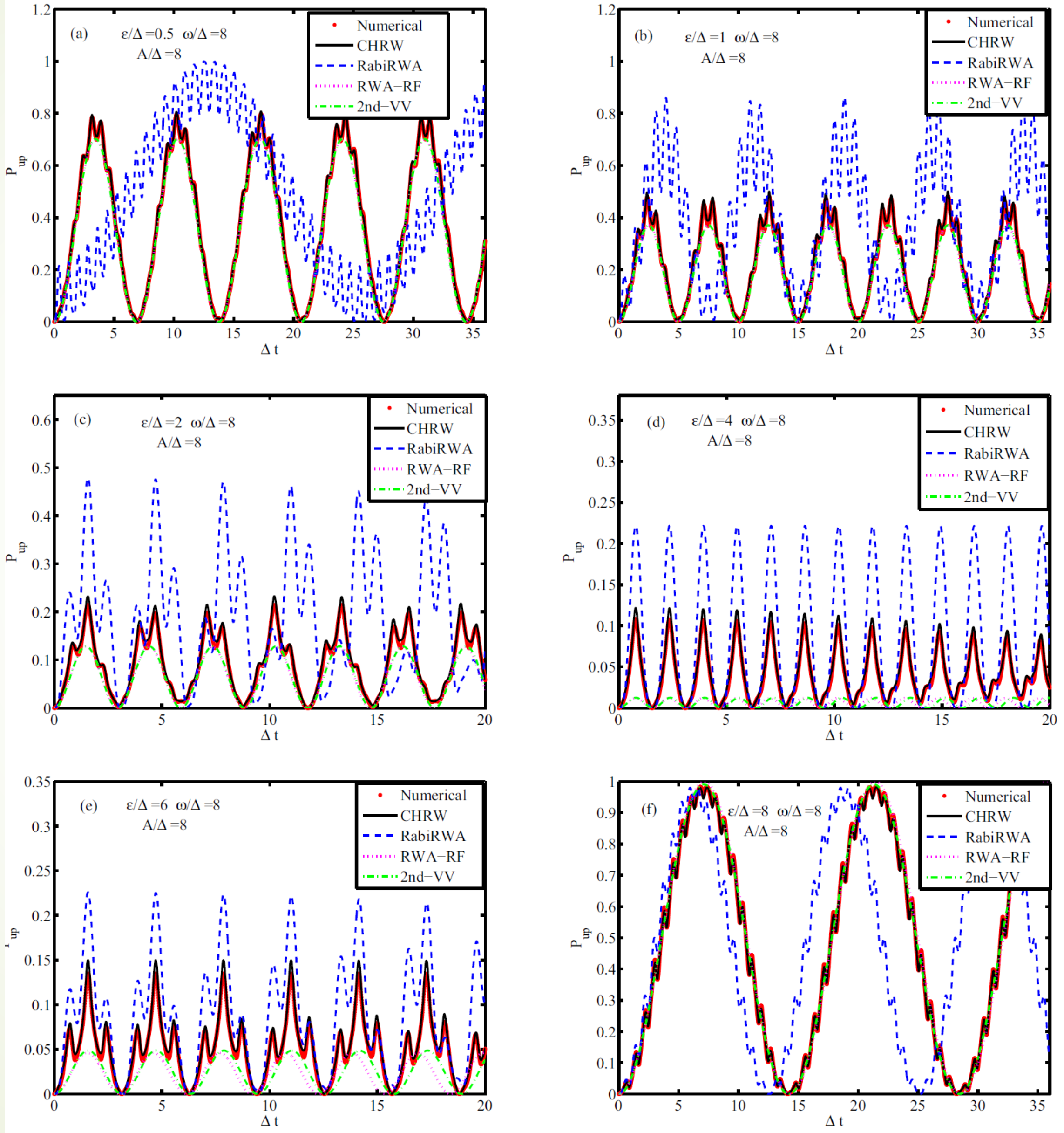


FIG. 4. Time evolutions of $P_{\mathrm{up}}(t) = \langle \frac{\sigma_z(t)+1}{2} \rangle$ as a function of Δt for different values of the bias (a) $\epsilon/\Delta = 0.5$, (b) 1, (c) 2, (d) 4, (e) 6, and) 8 in the off-resonance case ($\omega = 8\Delta \neq \Xi_0$). The driving strength A is set to be $A/\omega = 1$ ($A/\Delta = 8$).

Fluorescence spectrum

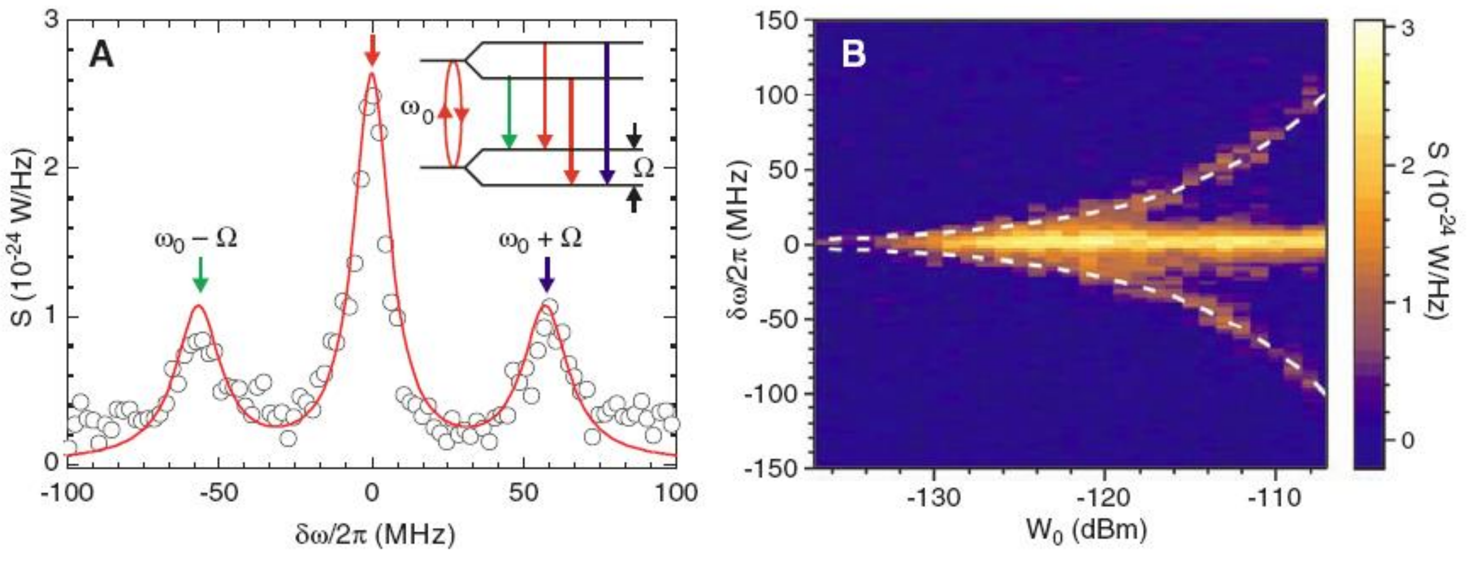
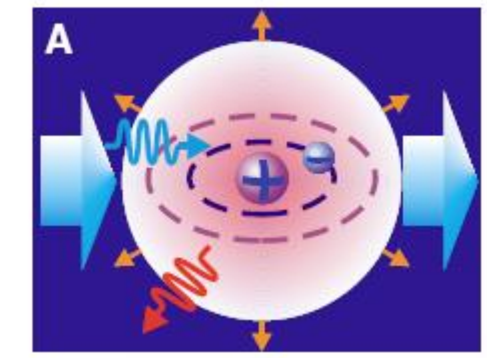


Fig. 3. Resonance fluorescence triplet: spectrum of inelastically scattered radiation. (**A**) Linear frequency spectral density [$S = 2\pi S(\omega)$] of emission power under a resonant drive with the Rabi frequency of $\Omega/2\pi = 57$ MHz corresponding to the incident microwave power of $W_0 = -112$ dBm or 6.3×10^{-15} W. Experimental data are shown by the open circles. The red solid curve is the emission calculated from Eq. 3 with no fitting parameters. A schematic of the triplet transitions in the dressed-state picture is presented in the … use of strong driving, and transiti… Ω (marked by colored arrows), giv… e fluorescence emission spectrum as a function of the driving power. The dashed white lines indicate the calculated position of the side peaks shifted by $\pm\Omega/2\pi$ from the main resonance. The split peak was used for calibration of the field amplitude at the atom.



artificial atom

O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Yu. A.Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Science **327**, 840 (2010).

Effects of CR interaction on Fluorescence

Our model

$$H(t)=\frac{1}{2}\omega_0\sigma_z + A\cos(\omega_L t)\sigma_x + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger + b_k)\sigma_x$$

$$\begin{aligned} H'_{\text{eff}}(t) &= \frac{1}{2}\omega_0 J_0\left(\frac{2A}{\omega_L}\xi\right)\sigma_z + \omega_0 J_1\left(\frac{2A}{\omega_L}\xi\right)\sin(\omega_L t)\sigma_y \\ &\quad + A(1-\xi)\cos(\omega_L t)\sigma_x + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+), \end{aligned}$$

$$\begin{aligned} H'_2(t) &= \omega_0\sum_{n=1}^{\infty} J_{2n+1}\left(\frac{2A}{\omega_L}\zeta\right)\sin\left[(2n+1)\omega_L t\right]\sigma_y \\ &\quad + \omega_0\sum_{n=1}^{\infty} J_{2n}\left(\frac{2A}{\omega_L}\xi\right)\cos(2n\omega_L t)\sigma_z + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-), \end{aligned}$$

Y. Yan, **Zhiguo Lü**, and H. Zheng, Effects of counter-rotating-wave terms of the driving field on the spectrum of resonance fluorescence, **Phys. Rev.** A88， 053821(2013).

Using $R(t) = \exp\left[i\omega_L t\left(\frac{1}{2}\sigma_z + \sum_k b_k^\dagger b_k\right)\right]$

$$\begin{aligned}\tilde{H} &= R(t)H'(t)R^\dagger(t) - iR(t)\frac{d}{dt}R^\dagger(t) \\ &= \frac{1}{2}(\tilde{\delta}\sigma_z + \tilde{\Omega}\sigma_x) + \sum_k(\omega_k - \omega_L)b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+),\end{aligned}$$

Born-Markov master equation

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}_S(t) &= -i[\tilde{H}_{0S}, \tilde{\rho}_S(t)] - \int_0^\infty d\tau \frac{1}{4}\sum_k g_k^2 \\ &\times\left\{e^{-i(\omega_k-\omega_L)\tau}[\sigma_+, e^{-i\tilde{H}_{0S}\tau}\sigma_- e^{i\tilde{H}_{0S}\tau}\tilde{\rho}_S(t)]\right. \\ &\left.-e^{i(\omega_k-\omega_L)\tau}[\sigma_-, \tilde{\rho}_S(t)e^{-i\tilde{H}_{0S}\tau}\sigma_+ e^{i\tilde{H}_{0S}\tau}]\right\}.\end{aligned}$$

Resonance Fluorescence

Definition

$$S(\omega) = \frac{1}{2\pi} \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \int_0^T dt' g^{(1)}(t,t') e^{-i\omega(t-t')},$$

Two time correlation function

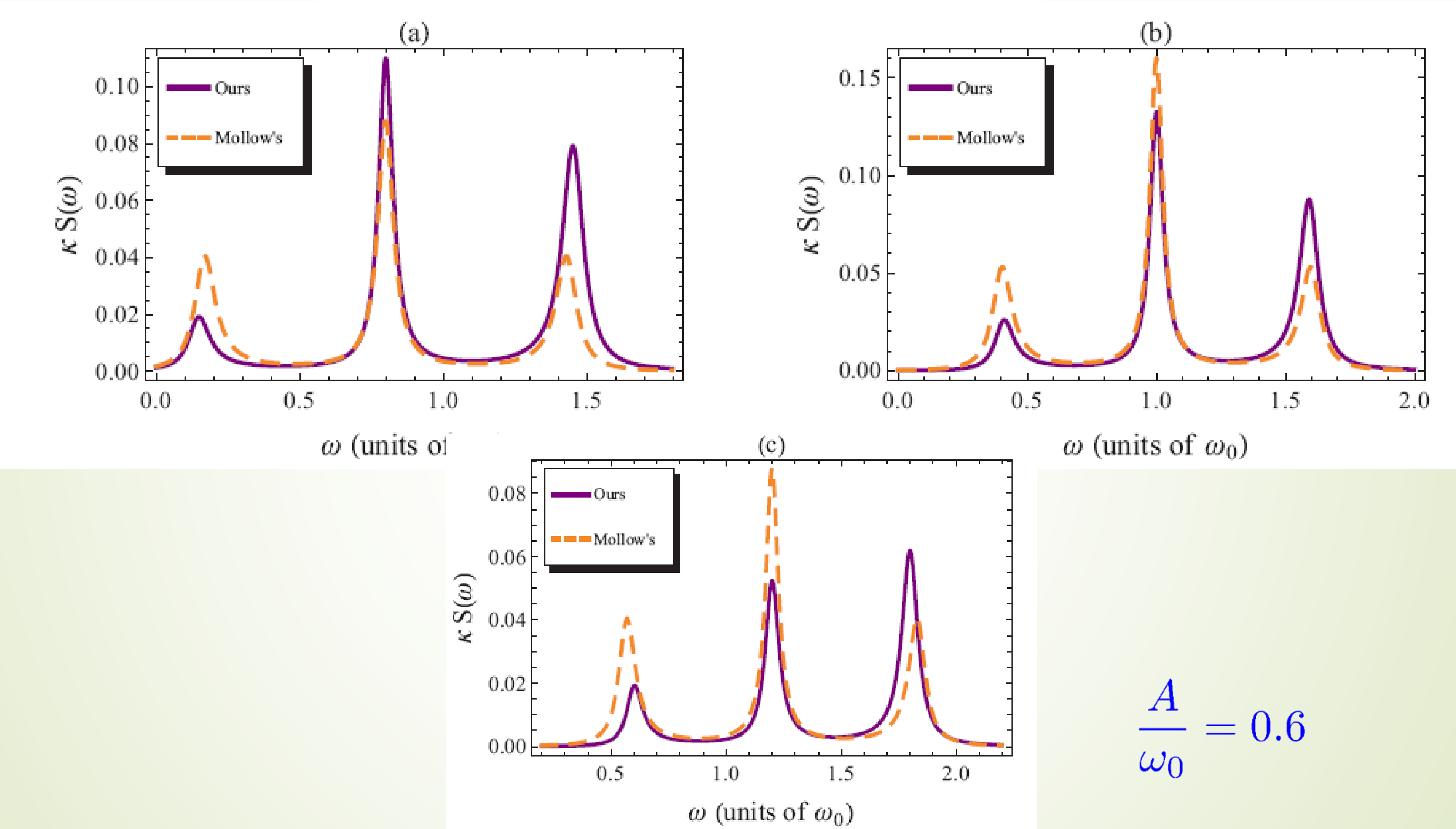
$$g^{(1)}(t,t') = \langle U^\dagger(t)\sigma_+ U(t) U^\dagger(t')\sigma_- U(t')\rangle$$

In the transformed frame

$$\begin{aligned} g^{(1)}(t,t') \;&=\; \mathrm{Tr}\Big[R(t)\, e^{S(t)}\sigma_+ e^{-S(t)} R^\dagger\,(t)\,\tilde{U}\,(\tau)\,R\,(t')\,e^{S(t')}\sigma_- e^{-S(t')} \\ &\times R^\dagger\,(t')\,\tilde{\rho}_{\mathrm{ss}} \otimes \rho_B \tilde{U}^\dagger\,(\tau)\Big]. \qquad \tau = t - t' \end{aligned}$$

B. R. Mollow, Phys. Rev. **188**, 1969 (1969).

Results (i) the asymmetry of the sidebands with respect to the central peak,



$$\frac{A}{\omega_0} = 0.6$$

D. E. Browne and C. H. Keitel, J. Mod. Opt. 47, 1307 (2000)

Yiying Yan, Zhiguo Lü, and Hang Zheng, PHYSICAL REVIEW A 88, 053821 (2013)

(ii) The generation of the higher-order Mollow triplets at $n\omega_L, \; n = 3, 5, \ldots$

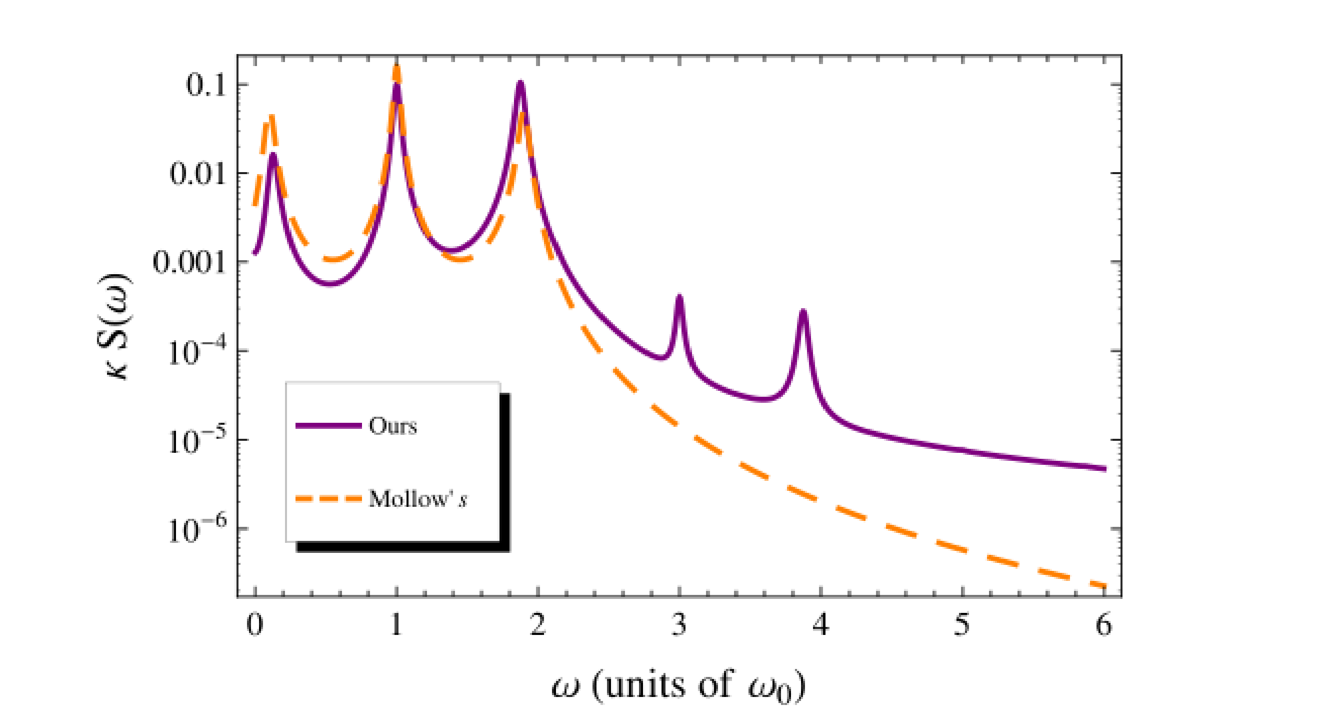
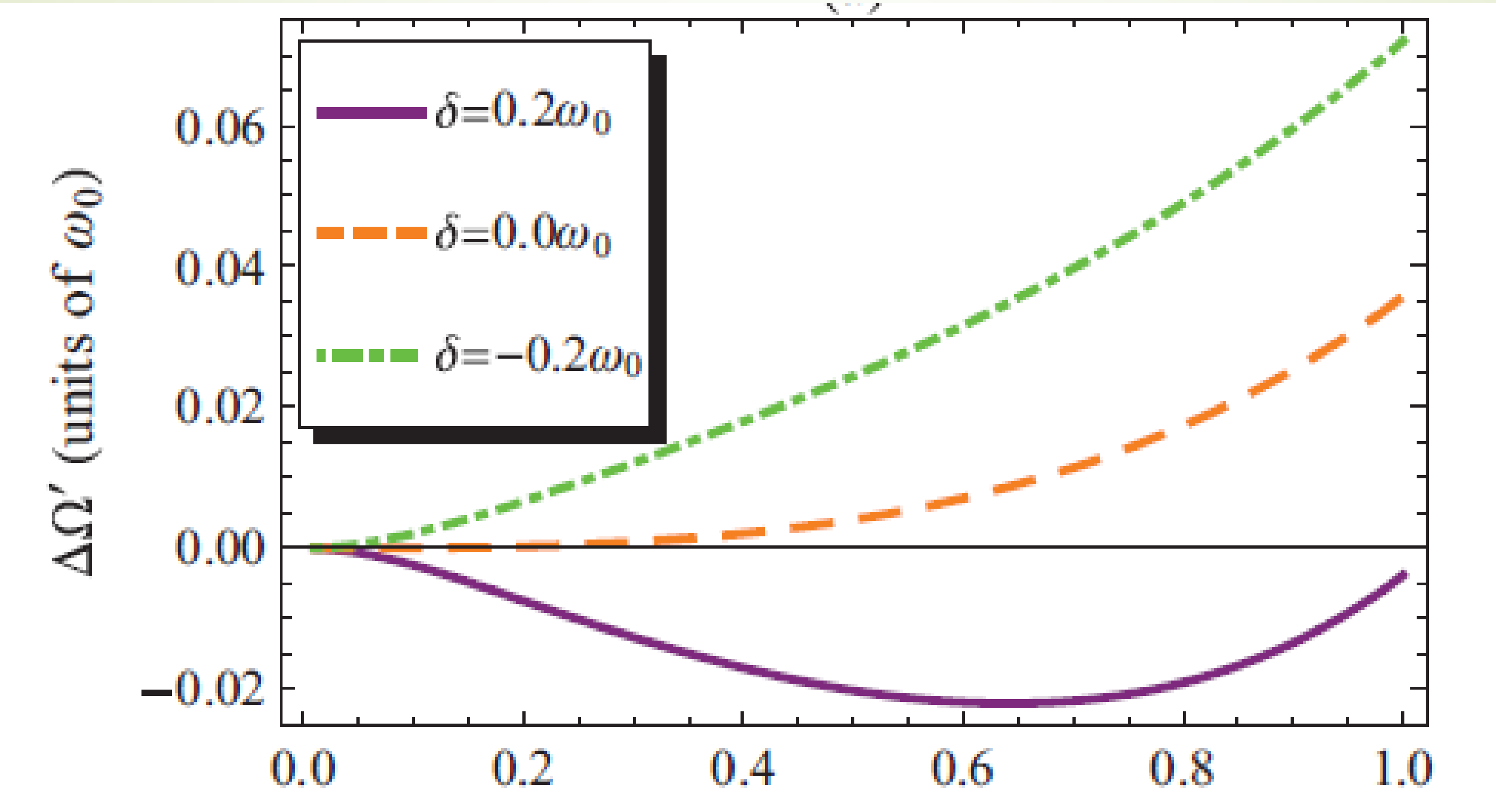


FIG. 4. (Color online) The fluorescence spectrum obtained by our method (solid line) and Mollow's method (dashed line) on a logarithmic scale for $\Omega = 20\kappa$, $\Omega/\omega_0 = 0.9$, and $\omega_L = \omega_0$. The inset shows the spectrum obtained in Ref. [14] with the same parameters.

(iii) Shifts of the sidebands



A ($unitsof\omega_0$)

$$\Delta\Omega = \Omega_{R-RWA} - \tilde{\Omega}_R$$

probe-pump spectrum

$$H_{\rm tot}(t) = H(t) + H_p(t)$$

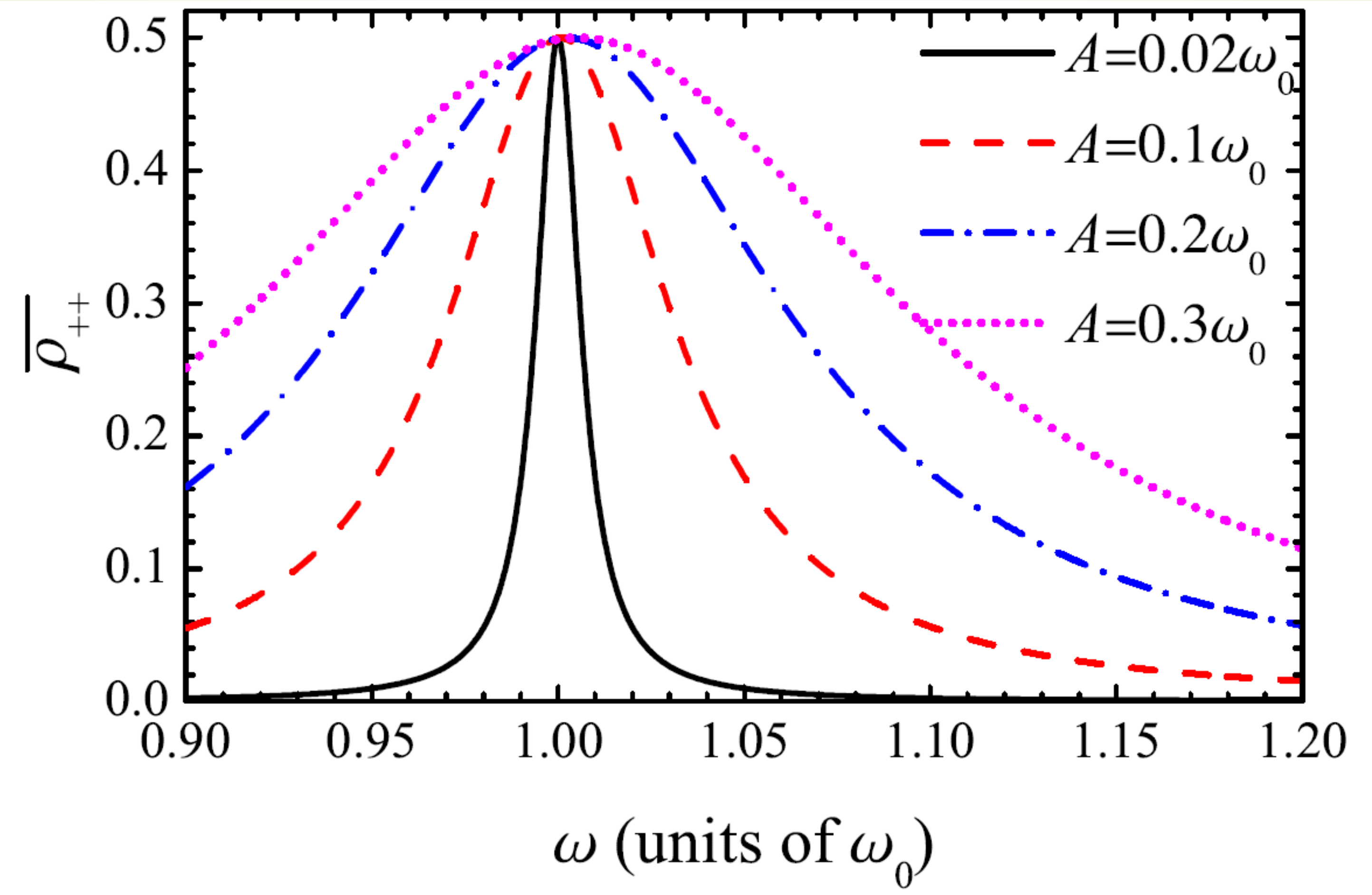
$$H_p(t) = \Omega_p(\sigma_+ e^{-i\nu t} + \sigma_- e^{i\nu t})$$

$$A(\nu) = 2|\Omega_p|^2 \mathrm{Re} \int_0^\infty d\tau \mathcal{A}(\tau) e^{i\nu\tau}$$

$$\mathcal{A}(\tau) = \lim_{t\to\infty} \langle [U^\dagger(t+\tau)\sigma_- U(t+\tau), U^\dagger(t)\sigma_+ U(t)] \rangle$$

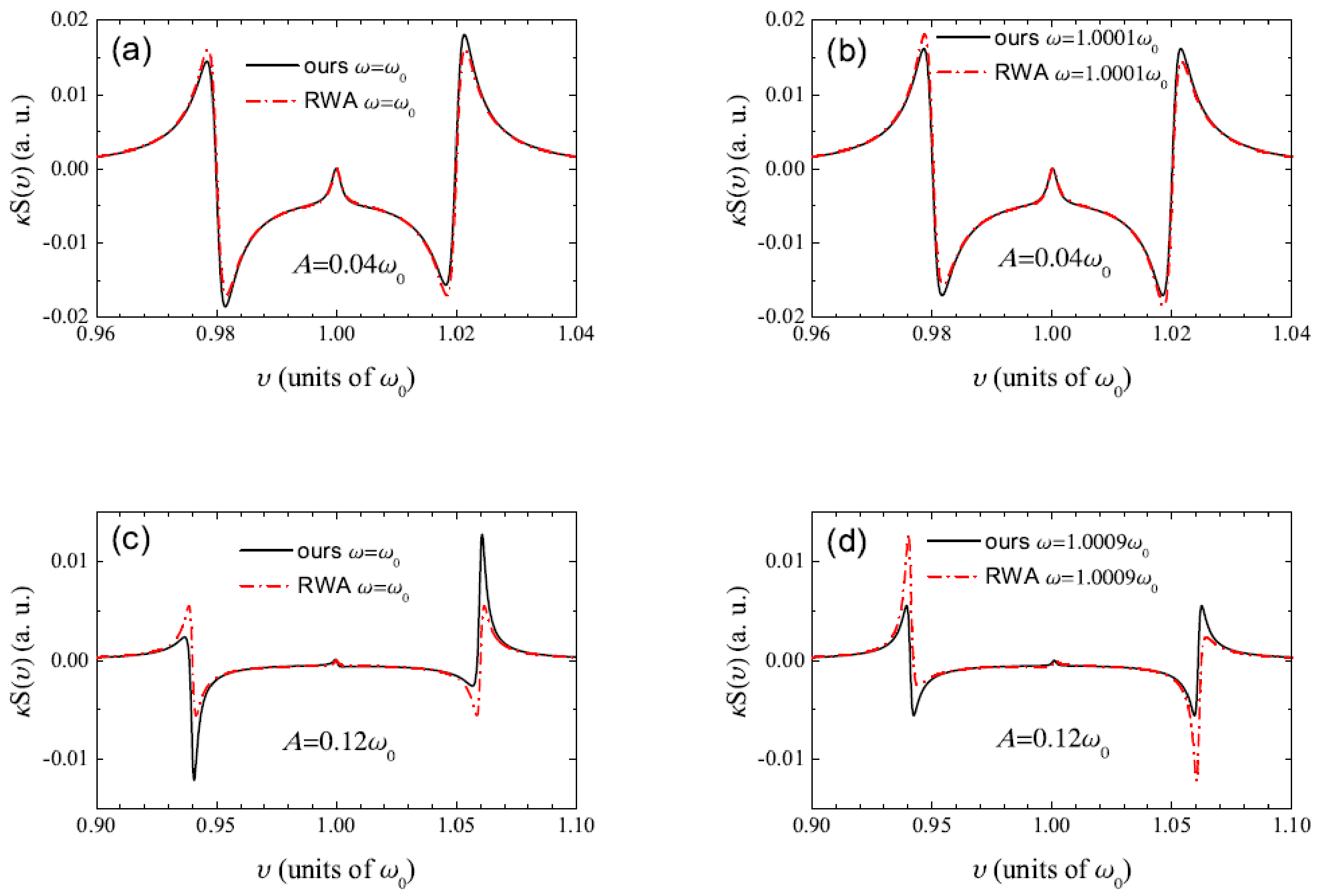
B. R. Mollow, Phys. Rev. A **5**, 2217 (1972).

Population of the upper state



The time-averaged population as a function of driving frequency

Yinying Yan, **Zhiguo Lü***, and H. Zheng, Bloch-Siegert shift of Rabi model, Phys. Rev. A91, 053834(2015).


(a)
ours ω=ω0
RWA ω=ω0
A=0.04ω0
κS(υ) (a. u.)
υ (units of ω0)
(b)
ours ω=1.0001ω0
RWA ω=1.0001ω0
A=0.04ω0
κS(υ) (a. u.)
υ (units of ω0)
(c)
ours ω=ω0
RWA ω=ω0
A=0.12ω0
κS(υ) (a. u.)
υ (units of ω0)
(d)
ours ω=1.0009ω0
RWA ω=1.0009ω0
A=0.12ω0
κS(υ) (a. u.)
υ (units of ω0)

Applications:

Multi-chromatic field

$$H(t) = \frac{1}{2}[\omega_0 + \Omega_z \cos(\omega_z t)]\sigma_z + \Omega_x \cos(\omega_x t)\sigma_x$$

superconducting circuits

Bloch-Siegert shift

Observation of the Bloch-Siegert shift in a driven quantum-to-classical transition, Phys. Rev. B **96**, 020501(R)

Observation of the Bloch-Siegert Shift in a Qubit-Oscillator System in the Ultrastrong Coupling Regime, Phys. Rev. Lett. **105**, 237001

Mooij

Driven dissipative system

Dynamical effects of counter-rotating couplings on interference between driving and dissipation Phys. Rev. A**90**, 053850 (2014)

Summary

- (i) We investigate the dynamics of a driven two-level system (classical Rabi model) using the counter-rotating hybridized rotating-wave method (CHRW), which is a simple method based on a unitary transformation. Since the CHRW approach is mathematically simple as well as tractable and physically clear, it may be extended to some complicated problems where it is difficult to do a numerical study.
- (ii)We calculate the Bloch-Siegert (BS) shift over the entire driving-strength range. we demonstrate the signatures caused by the BS shift by monitoring the excited-state population and the probe-pump spectrum under the experiment accessible conditions.
- (iii) We investigate the fluorescence spectrum of a two-level system driven by a monochromatic classical field by the Born-Markovian master equation based on a unitary transformation We find the main effects of counter-rotating-wave terms of the driving on spectral features of the fluorescence.

Thank you!

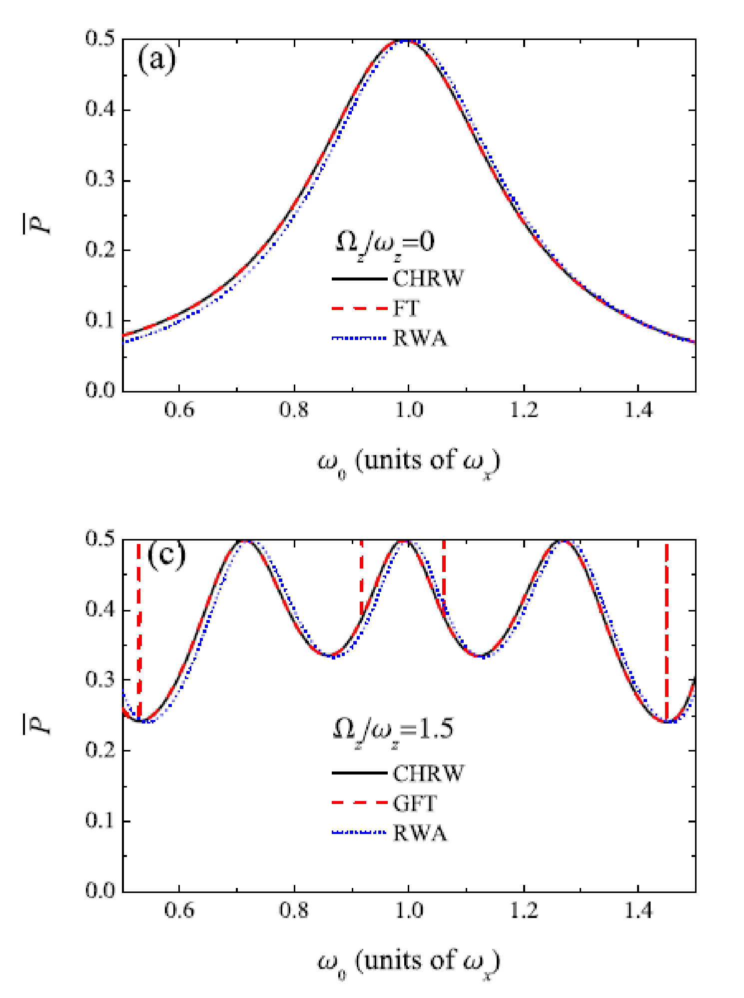
The tenth order of *A*

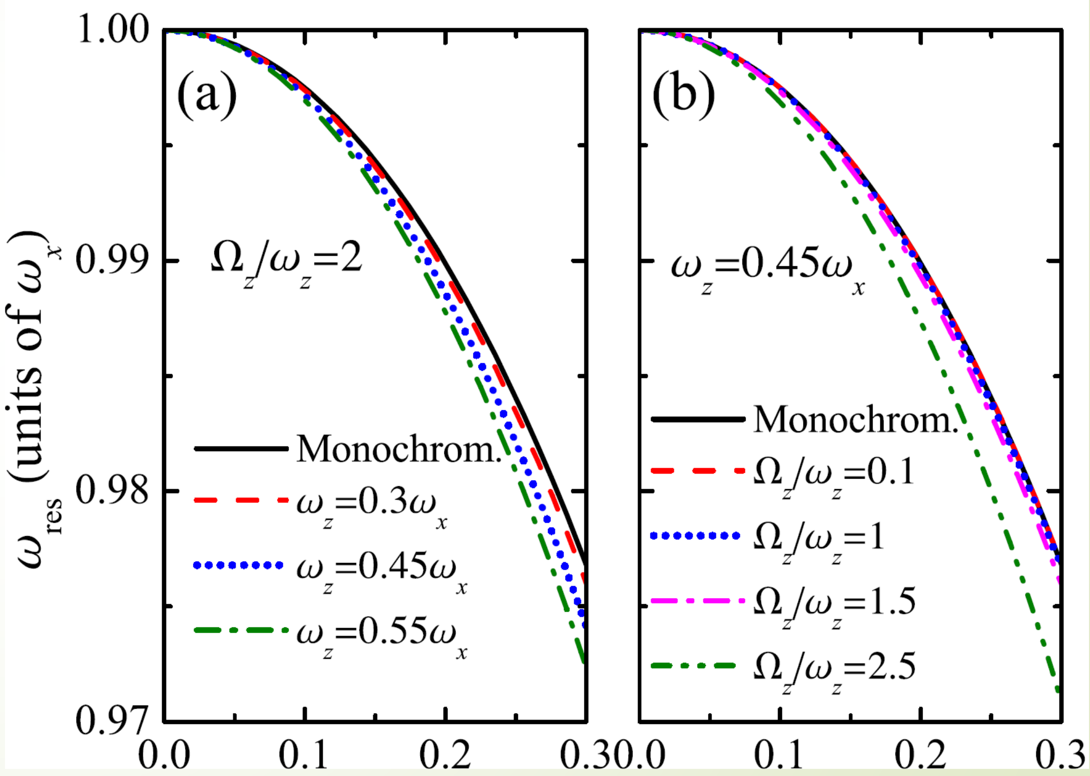
$$\xi = \frac{\omega}{\Delta+\omega} + \frac{A^2\Delta\omega}{8(\Delta+\omega)^4} + \frac{A^4\Delta(8\Delta-\omega)\omega}{192(\Delta+\omega)^7} + \frac{A^6\Delta\omega\left(169\Delta^2-46\Delta\omega+\omega^2\right)}{9216(\Delta+\omega)^{10}} + \frac{A^8\Delta\omega\left(6799\Delta^3-2903\Delta^2\omega+197\Delta\omega^2-\omega^3\right)}{737280(\Delta+\omega)^{13}} + \frac{A^{10}\Delta\omega\left(443821\Delta^4-259736\Delta^3\omega+32766\Delta^2\omega^2-776\Delta\omega^3+\omega^4\right)}{88473600(\Delta+\omega)^{16}}$$

$${\Omega_\mathrm{R}}^2 = (\Delta-\omega)^2 + \frac{\Delta A^2}{2(\Delta+\omega)} - \frac{\Delta A^4}{32(\Delta+\omega)^3} + \frac{\Delta\left(-13\Delta^2-3\Delta\omega+\omega^2\right)A^6}{1152(\Delta+\omega)^7} - \frac{\left(\Delta\left(381\Delta^3-77\Delta^2\omega-25\Delta\omega^2+\omega^3\right)\right)A^8}{73728(\Delta+\omega)^{10}} + \frac{\Delta\left(-19616\Delta^4+10153\Delta^3\omega-45\Delta^2\omega^2-113\Delta\omega^3+\omega^4\right)A^{10}}{7372800(\Delta+\omega)^{13}}$$

Enhancement of BS shift

$$H(t) = \frac{1}{2}[\omega_0 + \Omega_z \cos(\omega_z t)]\sigma_z + \Omega_x \cos(\omega_x t)\sigma_x$$

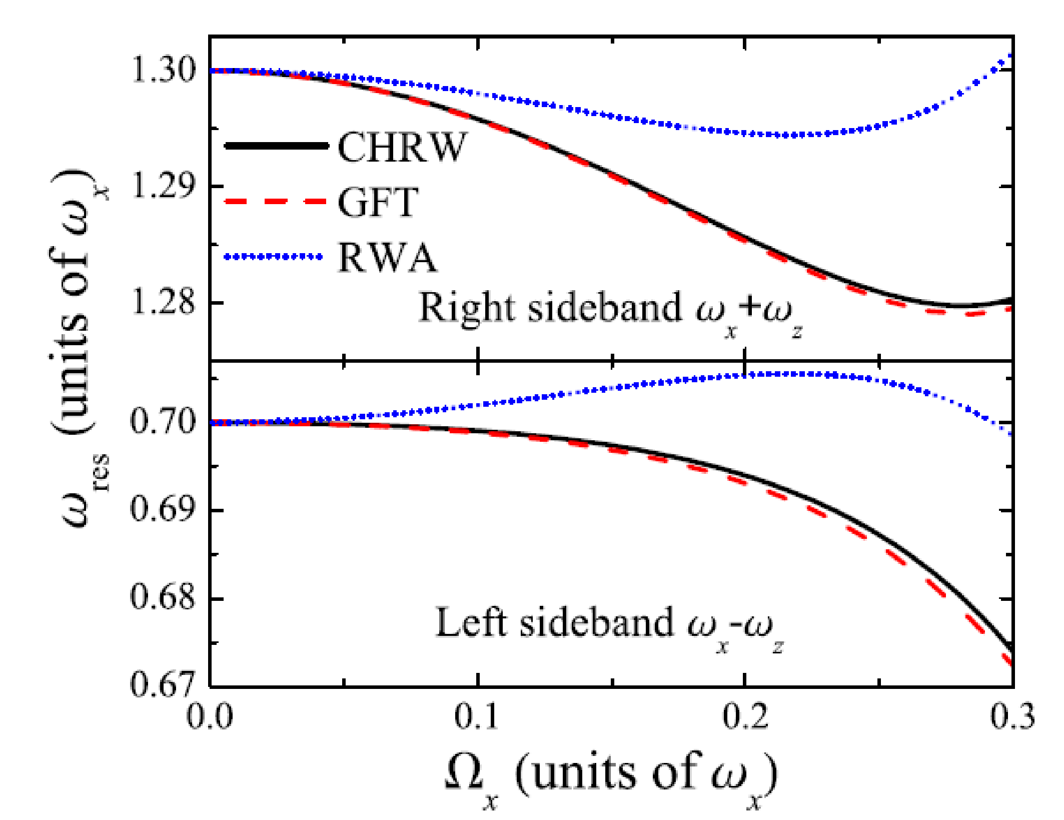




Effects of counter-rotating couplings of the Rabi model with frequency modulation, **Phys. Rev. A 96**, 033802 (2017).

Enhancement of BS shift

$$H(t) = \frac{1}{2}[\omega_0 + \Omega_z \cos(\omega_z t)]\sigma_z + \Omega_x \cos(\omega_x t)\sigma_x$$



$\omega_z = 0.3\omega_x$ and $\Omega_z/\omega_z = 2$.

Effects of counter-rotating couplings of the Rabi model with frequency modulation, **Phys. Rev. A 96**, 033802 (2017).

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