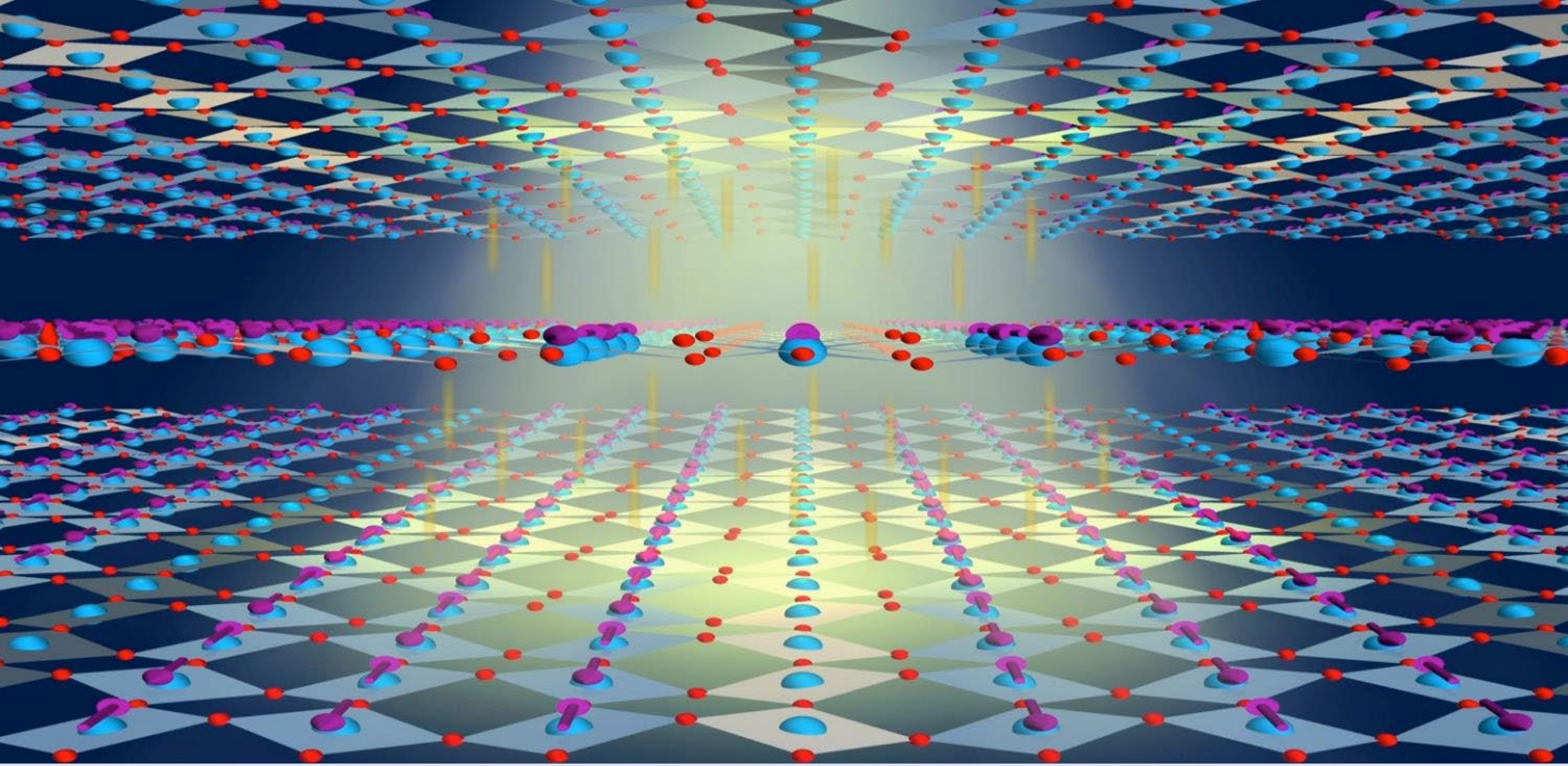
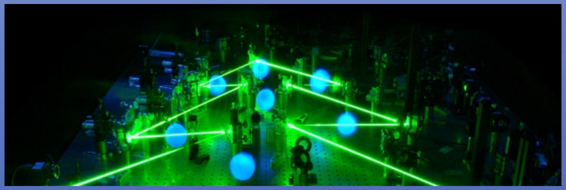


Non-equilibrium dynamics in Quantum Matter

Dieter Jaksch, University of Oxford

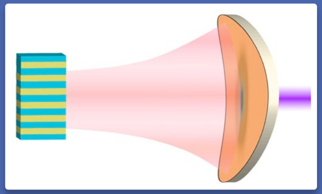
30 July 2019

Optics and Quantum Matter

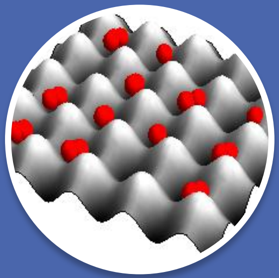


Optical control

Quantum state engeineering

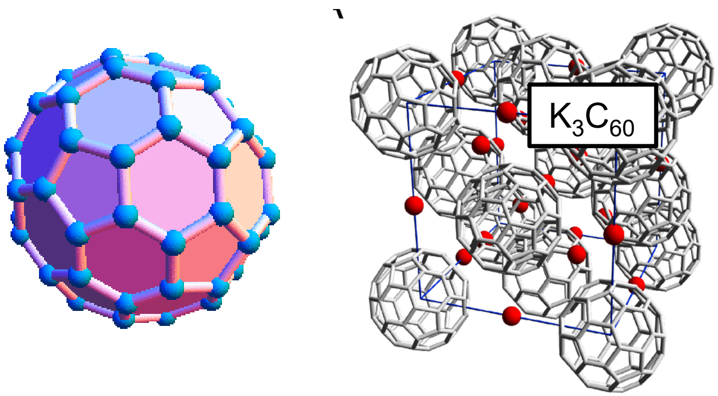


Quantum materials

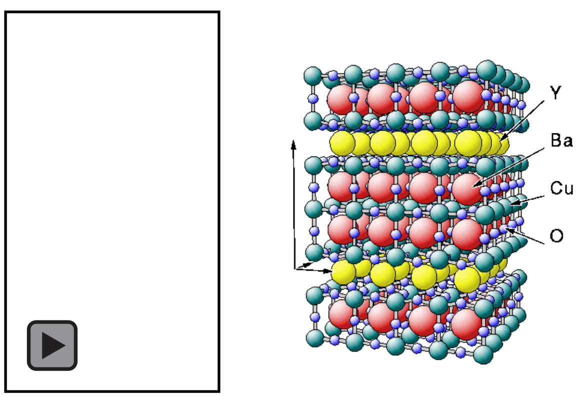


Synthetic quantum matter

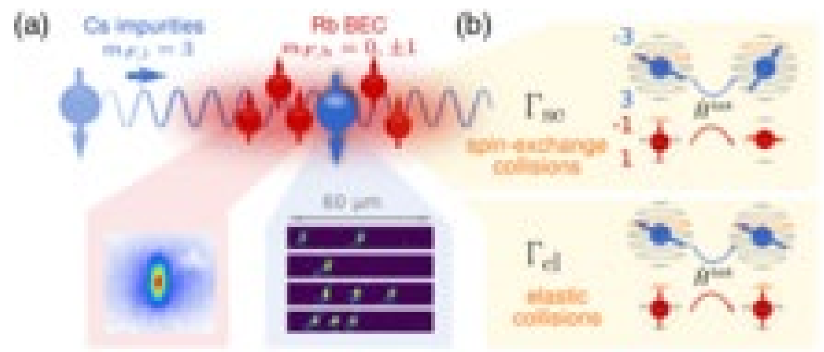
Driven and controlled quantum matter



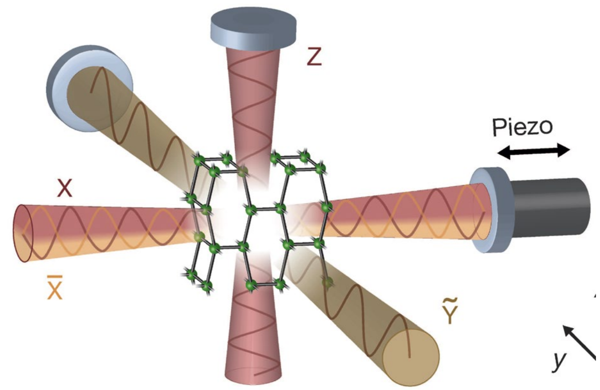
M. Mitrano, et al. Nature **530**, 461 (2016)



W. Hu. et al., Nature Mat. **13**, 705 (2014)

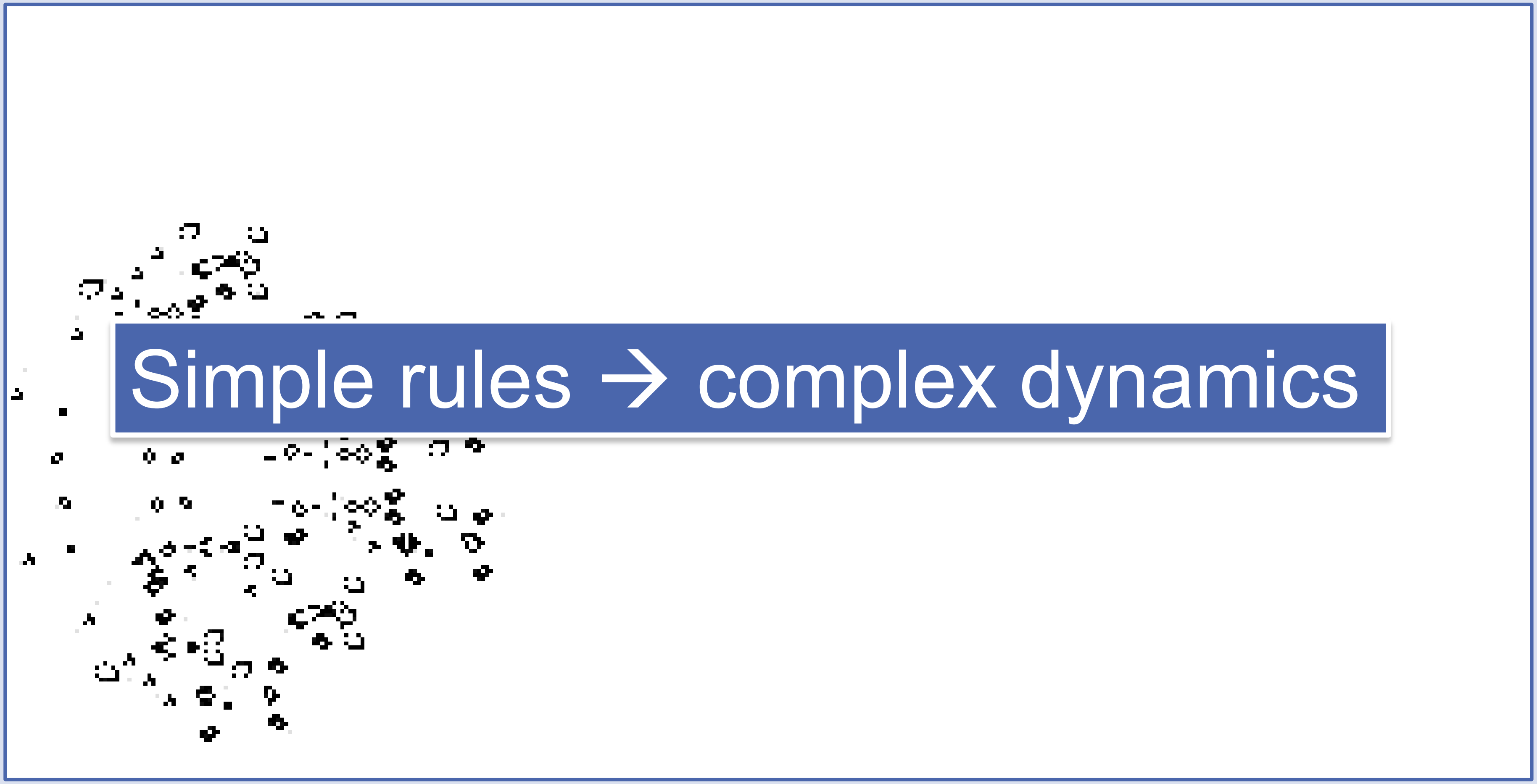


F. Schmidt et al.,Phys. Rev. Lett. **121**, 130403 (2018)



F. Goerg, et al., Nature **553**, 481 (2018)

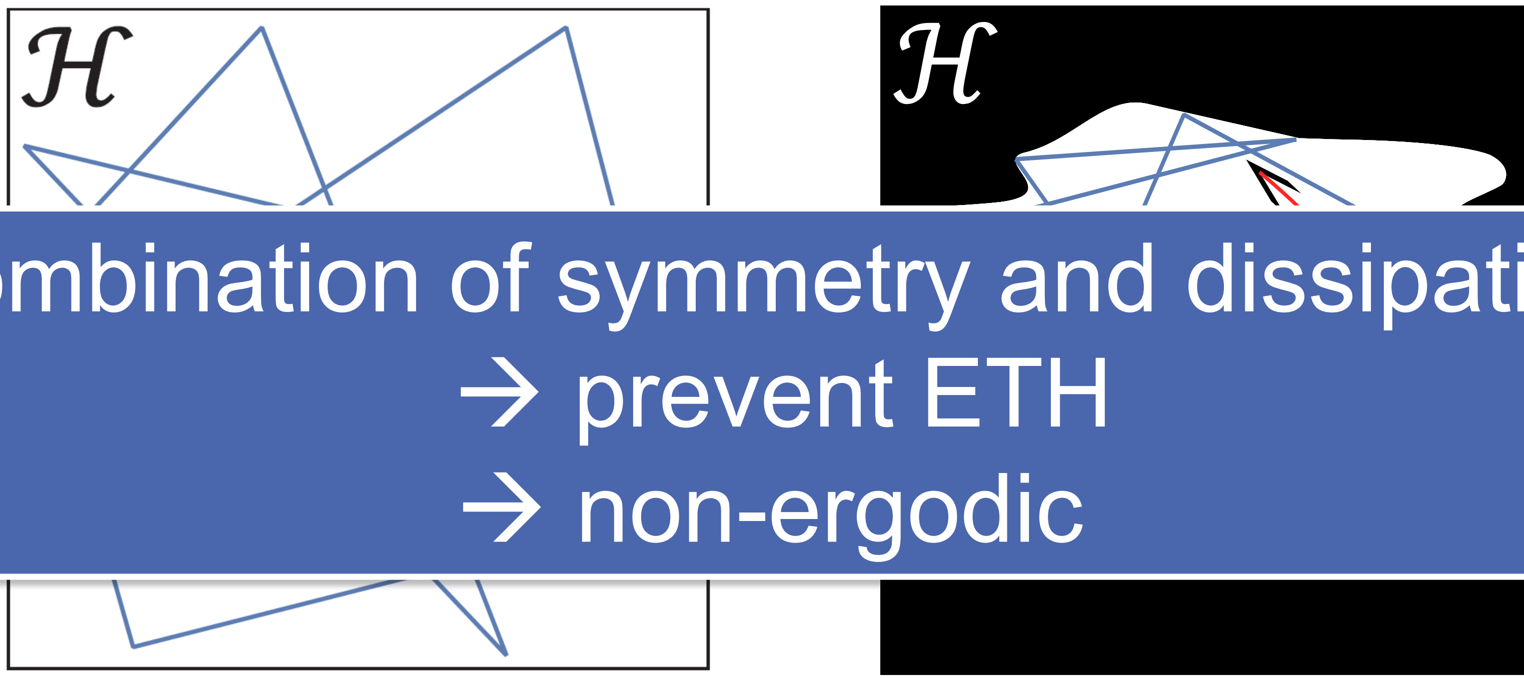
Conway's game of life



Simple rules → complex dynamics

Obtained from: http://web.stanford.edu/~cdebs/GameOfLife/

Non-stationary quantum matter



Combination of symmetry and dissipation
→ prevent ETH
→ non-ergodic

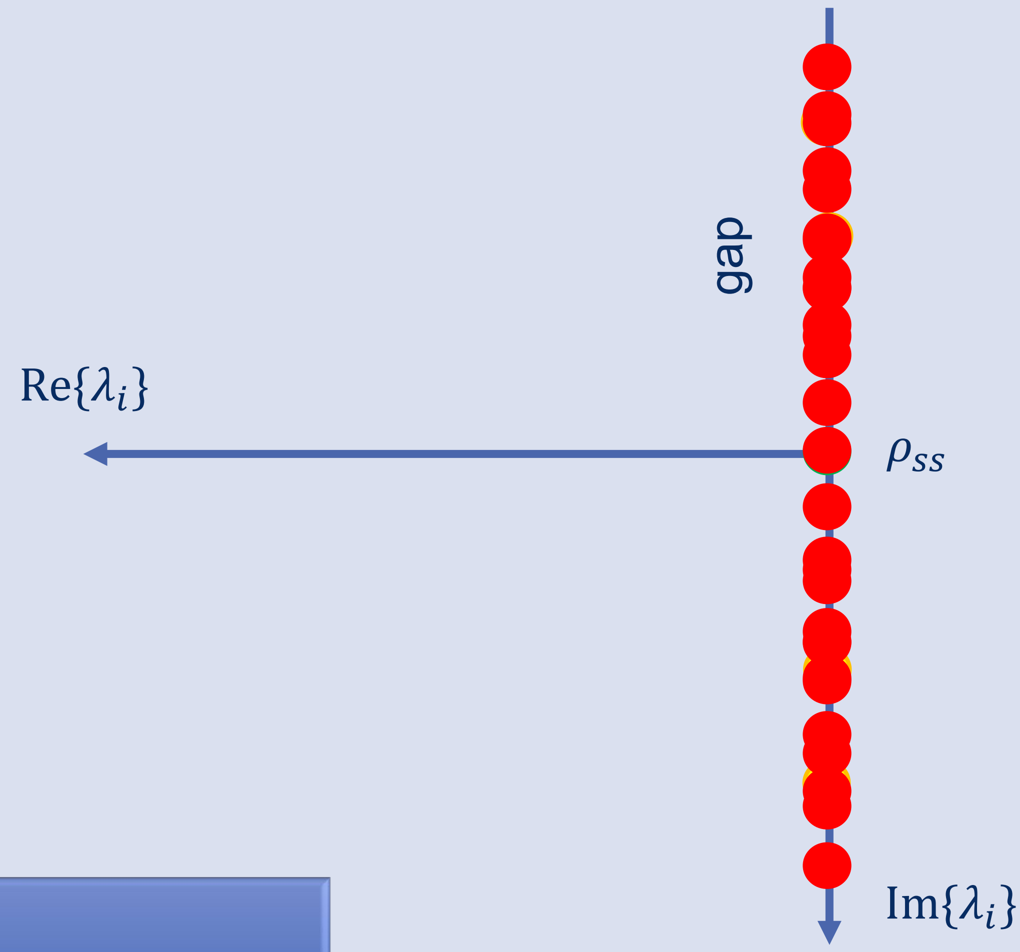
Can we engineer a many-body quantum system that shows "perpetual" non-stationary dynamics?

Eigenvalues λ_i of the dynamics

$$\rho_i = e^{\lambda_i}\rho_i$$

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H_{\mathrm{Hub}}, \rho]$$

$$+\sum_\mu \left(2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu\right)$$



Closed system
- purely imaginary λ_i
- dephasing

Open system
- negative real parts
- dissipation

Generic condition for
$[H, A] = -\lambda A$ and $[A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall\mu$

Generic condition

- If there exists a lowering operator A with the following properties

$$[H, A] = -\lambda A \quad \text{and} \qquad [A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall \mu$$

with $\lambda \neq 0$ then ρ_{nm} of the form (with integer $m, n > 0$, $m \neq n$)

$$\rho_{nm} = \mathrm{A}^n \rho_{00} (A^\dagger)^m$$

is non-stationary with $\mathcal{L}\rho_{nm} = \mathrm{i}(m-n)\lambda\rho_{nm}$

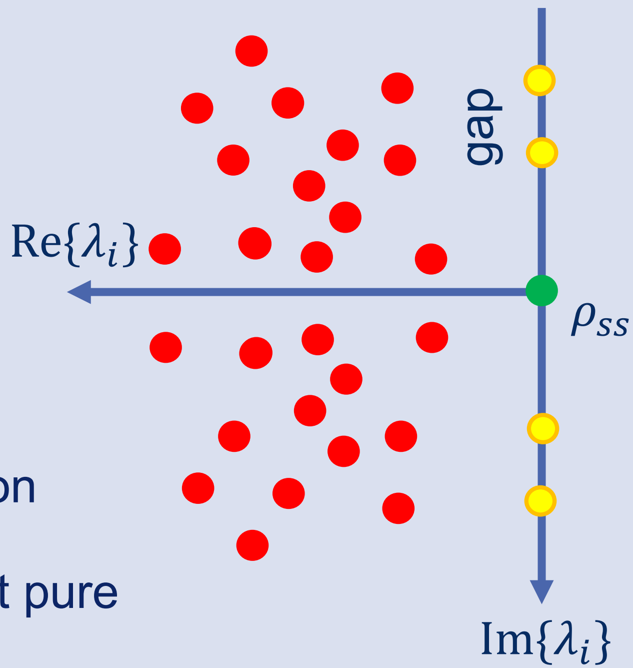
- The mixed coherences are not necessarily decoupled from the environment

$$L_\mu \rho_{nm} L_\mu^\dagger \neq 0$$

- A dark Hamiltonian $\mathfrak{H}$ describes this long-time evolution

  - $\mathfrak{H}$ is not necessarily Hermitian, eigenstates not pure

  - evolution by $\mathfrak{H}$ not decoupled from the environment



Hubbard model and spin symmetry

- Hubbard Hamiltonian on a D-dimensional bipartite lattice in magnetic field B

$$H_{\mathrm{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i} + \frac{B}{2} \sum_{i} n_{i\uparrow} - n_{i\uparrow}$$

- Spin symmetry

$$S^{z} = \sum_{j} S_{j}^{z}, \qquad S_{j}^{z} = \frac{1}{2}(n_{j,\uparrow} - n_{j,\downarrow}),$$

$$S^{+} = \sum_{j} S_{j}^{+}, \qquad S_{j}^{+} = c_{j,\uparrow}^{\dagger}\, c_{j,\downarrow}$$

$$S^{-} = \sum_{j} S_{j}^{-}, \qquad S_{j}^{-} = c_{j,\downarrow}^{\dagger}\, c_{j,\uparrow},$$

- These operators fulfil

$$[H_{\mathrm{Hub}}, S^{z}] = 0, \qquad [H_{\mathrm{Hub}}, S^{\pm}] = \pm\, B\, S^{\pm}$$

Hubbard model and η symmetry

- Hubbard Hamiltonian on a D-dimensional bipartite lattice in magnetic field B

$$H_{\mathrm{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i + \frac{B}{2} \sum_{i} n_{i\uparrow} - n_{i\downarrow}$$

- η-pairing symmetry

$$\eta^z = \frac{1}{2} \sum_{j} (n_j - 1) \,,$$

$$\eta^+ = \sum_{j} \tau(j) \eta_j^+, \quad \eta_j^+ = c_{j,\uparrow}^{\dagger} \, c_{j,\downarrow}^{\dagger},$$

$$\eta^- = \sum_{j} \tau(j) \eta_j^- \,, \qquad \eta_j^- = c_{j,\downarrow} \, c_{j,\uparrow},$$

with $\tau(j)$ a chequerboard pattern. They fulfil

$$[H_{\mathrm{Hub}}, \eta^z] = 0, \qquad [H_{\mathrm{Hub}}, \eta^{\pm}] = \pm \, 2\mu\eta^{\pm}$$

Spectrum engineering

- Starting from a generic master equation

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H_{\mathrm{Hub}}, \rho] + \sum_{\mu} (2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu)$$

Dissipation in the spin sector

$$L_\mu \propto S_\mu^z$$

off-diagonal long-range correlations between η pairs

→ **superfluid state**

Dissipation in the charge sector

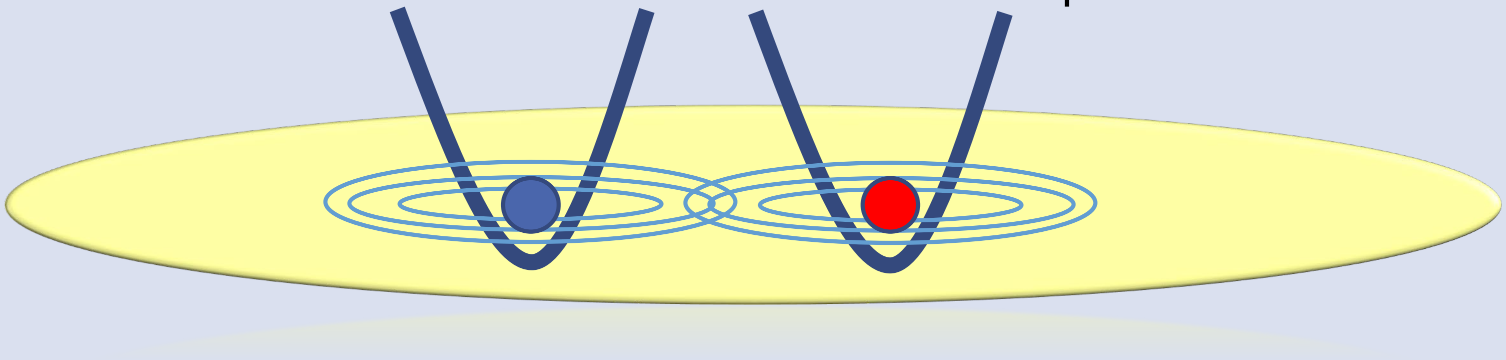
$$L_\mu \propto n_\mu$$

prevent stationarity and eigenstate thermalization

→ **non-stationarity**

B. Buca, J. Tindall, DJ, Nature Comm. **10,** 1730 (2019); J. Tindall, B. Buca, J.R. Coulthard, DJ, PRL 123, 030603 (2019)

Immerse a fermionic lattice into a BEC



- The interaction with the BEC atoms will dephase the lattice wave function locally

$$L_\mu \propto a_\uparrow \, n_{\mu\uparrow} + a_\downarrow \, n_{\mu\downarrow}$$

- If $a_\uparrow = a_\downarrow$ then we realize coupling of the bath to the density sector

$$L_\mu \propto n_\mu$$

- If $a_\uparrow = -a_\downarrow$ then we realize coupling of the bath to the spin sector

$$L_\mu \propto S_\mu^z$$

M. Bruderer and DJ, New J. Phys. **8**, 87 (2006)

Bath coupling to the spin sector, $B = 0$

- Conserved quantities are $N_\downarrow$, $N_\uparrow$, and $\eta^+\eta^-$

- We assume a stationary state of the form

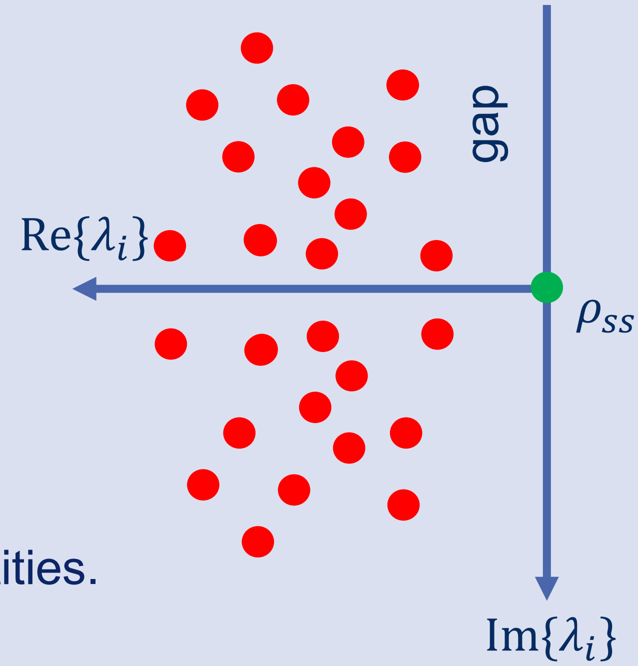
$$\rho_{ss} \propto \exp(\beta_1 N_\downarrow + \beta_2 N_\uparrow + \beta_3 \eta_+ \eta_-)$$

- i.e. at infinite temperature

- Lagrange parameters β_i fixing the conserved quantities.

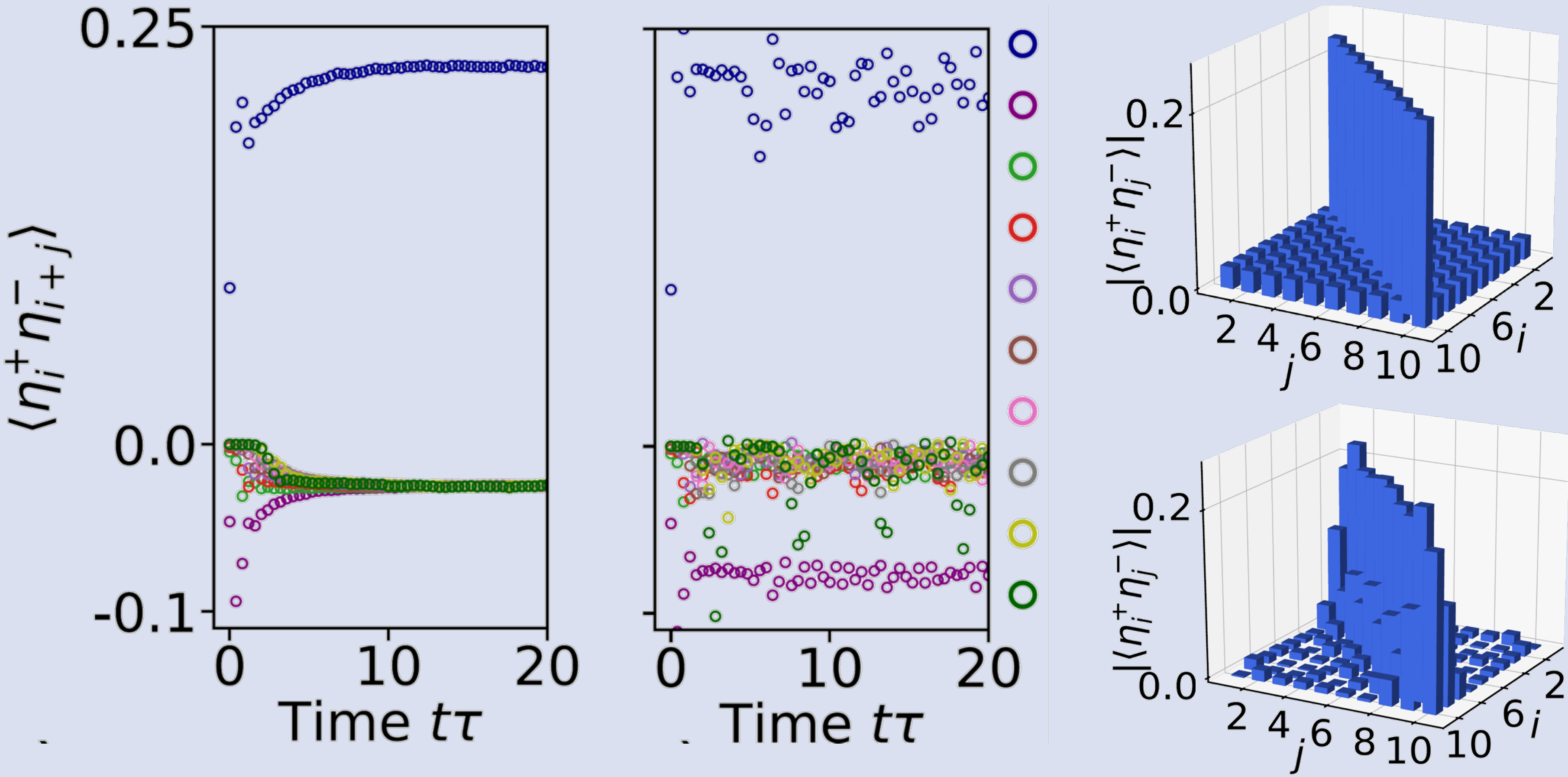
- Since in the stationary state ρ_{ss} we have $[\rho_{ss}, P_{i,j}] = 0$, where $P_{i,j}$ swaps two lattice sites we can show that

$$Tr\{\rho_{ss}, \eta_i^+ \eta_{i+j}^-\} = \text{const.}$$



J. Tindall, B. Buca, J.R. Coulthard, DJ, Phys. Rev. Lett. **123**, 030603 (2019)

A quench with and without dissipation



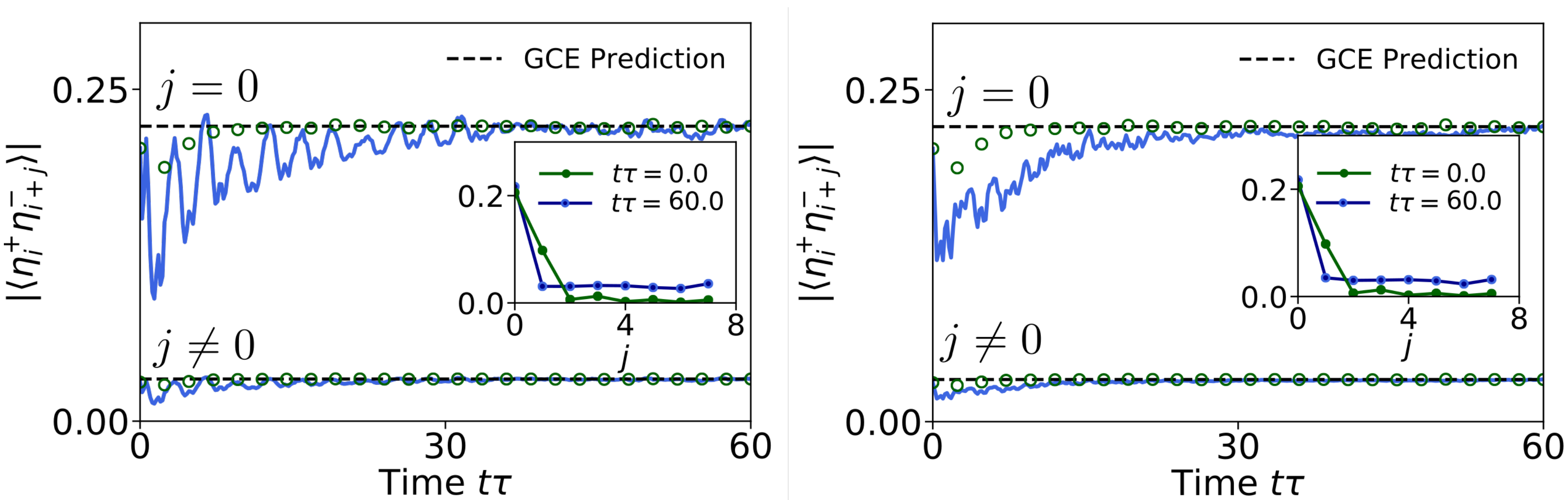
Interaction quench $U = 4\tau \rightarrow U = \tau$ for $\gamma = 0$ and for $\gamma = 2\tau$

J. Tindall, B. Buca, J.R. Coulthard, DJ, Phys. Rev. Lett. **123**, 030603 (2019)

Spin driven Hubbard model $B(t)$

- Strongly drive the isolated system in the spin sector with $B(t) = V\cos(\Omega t)$

$$H_{\mathrm{Hub}} = -t\sum_{\langle i,j\rangle,\sigma}(c_{i\sigma}^{\dagger}c_{j\sigma} + h.c.) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} - \mu\sum_{i} n_{i} + B(t)\sum_{i} f_{i}S_{i}^{z}$$



J. Tindall, B. Buca, J.R. Coulthard, DJ, Phys. Rev. Lett. **123**, 030603 (2019)

Bath coupling to the η sector

- We allow a trapping potential $\sum_i \epsilon_i n_i$

- The stationary states are given by

$$\rho_{ss} \propto \exp(\beta_0 N + \beta_1 (S^+ S^-) + \beta_2 S^z)$$

- Initial states that are superpositions of S^z contain “mixed coherences” of the form (with integer $m, n > 0$)

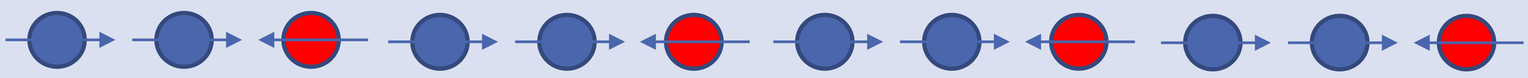
$$\rho_{nm} = (\mathrm{S}^+)^n \rho_{ss} (S^-)^m$$

- For $\mathrm{B} \neq 0$ these evolve in a non-stationary way according to

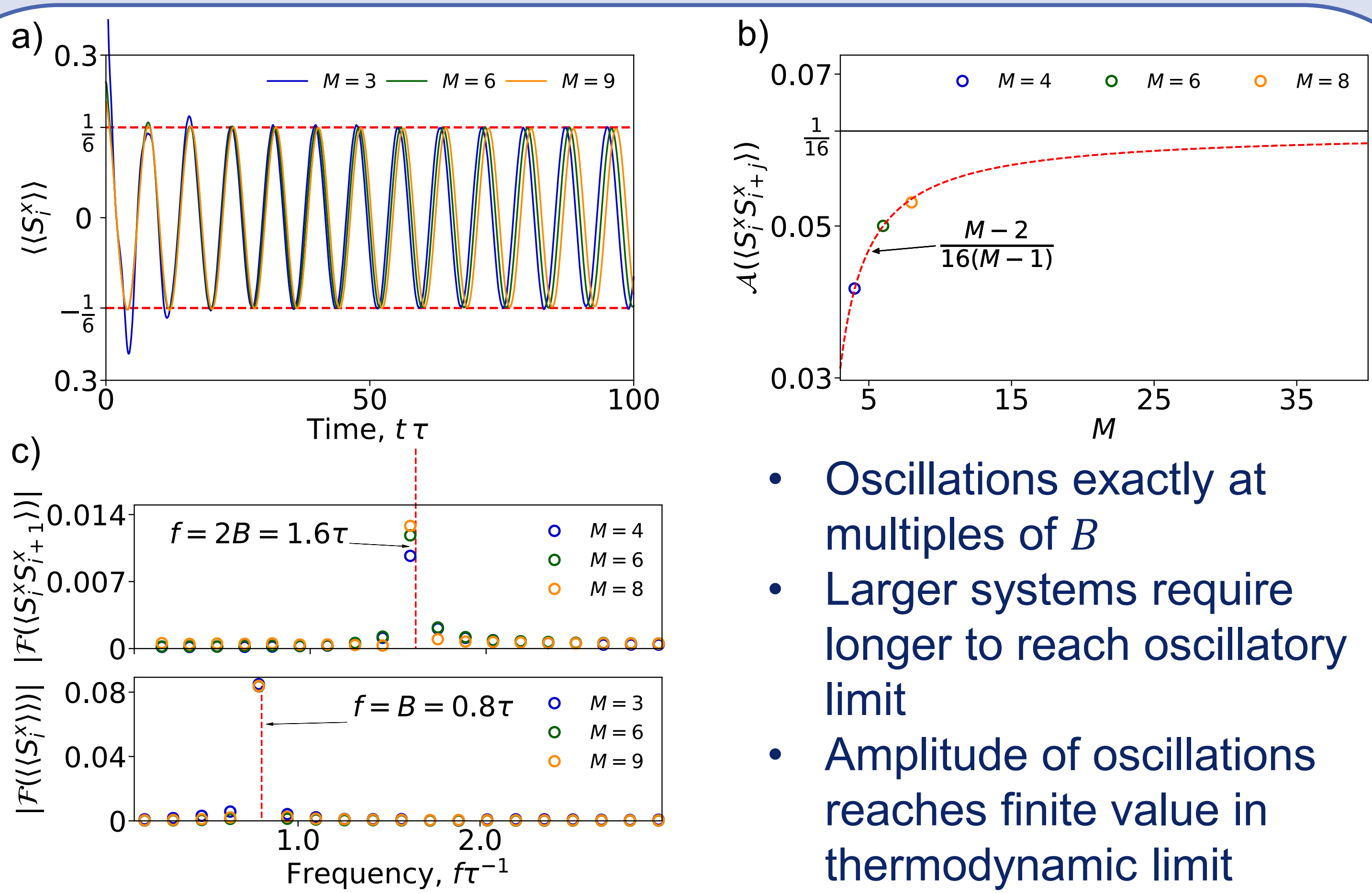
$$\mathcal{L}\rho_{nm} = \mathrm{iB}(m-n)\rho_{nm}$$

B. Buca, J. Tindall, and D. Jaksch, Nature Communications **10,** 1730 (2019).

Bulk averaged Hubbard dynamics

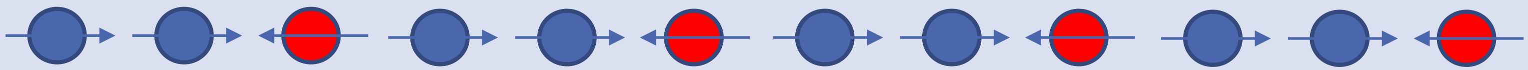


$^{1}/_{3}$



- Oscillations exactly at multiples of B
- Larger systems require longer to reach oscillatory limit
- Amplitude of oscillations reaches finite value in thermodynamic limit

Quantum trajectory simulation



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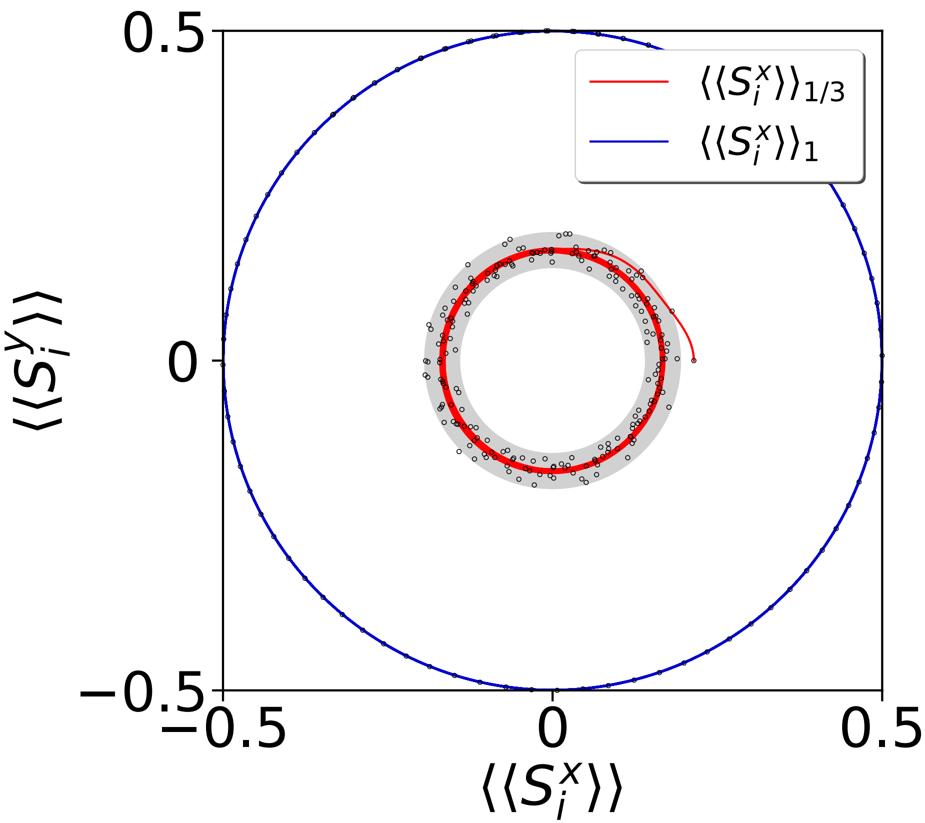
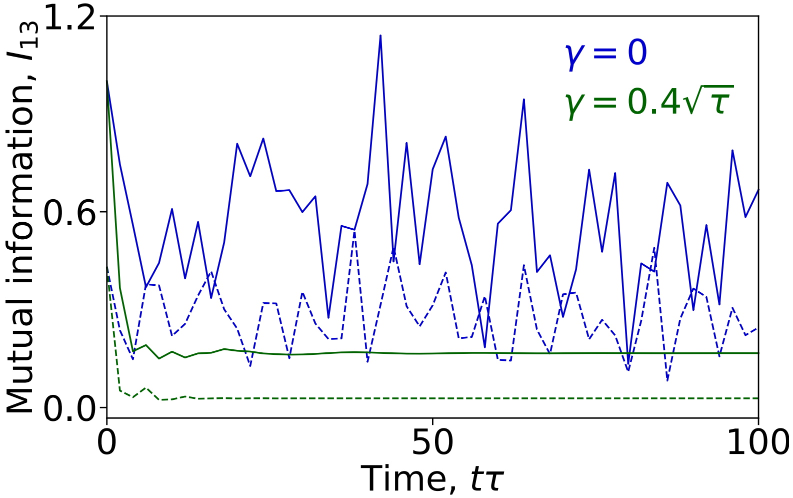
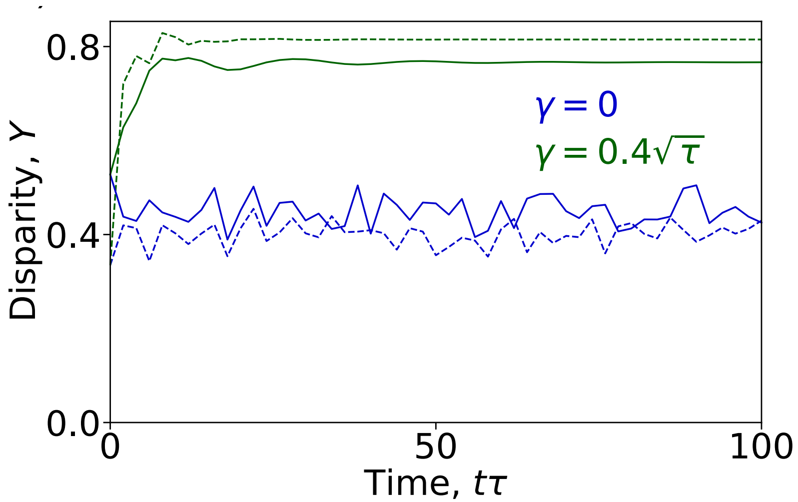


Complexity?





Mutual information:

$$I_{ij} = \frac{1}{2}\left(\mathcal{S}_i + \mathcal{S}_j - \mathcal{S}_{ij}\right)$$

Disparity:

$$Y_i = \frac{\sum_{j=1}^{M}(I_{ij})^2}{\left(\sum_{j=1}^{M} I_{ij}\right)^2}$$

Here $\mathcal{S}_i = \mathrm{tr}(\rho_i \log \rho_i)$, $\mathcal{S}_{ij} = \mathrm{tr}\left(\rho_{ij} \log \rho_{ij}\right), \rho_i = tr_{k\neq i}\, \rho$ and $\rho_{i,j} = tr_{k\neq i,j}\, \rho$

Limit cycles

- Density operator in the long-time limit

$$\lim_{t\to\infty} \rho(t) = \sum_{n\geq m} C_{n,m}\left(e^{iB(n-m)t}\rho_{nm} + h.c.\right),$$

- where the C_{nm} are determined by the initial state

- An observable $X = \prod_n X_n$ that is a product of M operators acting on individual sites will evolve according to

$$\lim_{t\to\infty} \langle X\rangle(t) = \sum_{n\geq m} D_{nm} \cos(B(m-n)t) + const.$$

- with $D_{nm} = 2C_{nm}\,\mathrm{Tr}\{\rho_{nm}X\}$

→ coherent non-decaying limit cycles of X

J. Tindall, C. Sanchez Munoz, B. Buca and DJ, arXiv:1907.12837 (2019)

Synchronization

- We consider perturbation $B \to B_n = \bar{B} + \delta B_n$ with

$$\epsilon = \frac{\overline{\delta B_n}}{\bar{B}} \ll 1$$

- and expand $\mathcal{L}$ and ρ in powers of ϵ

$$\mathcal{L} = \mathcal{L}^{(0)} + \epsilon \mathcal{L}^{(1)} + \cdots$$

$$\rho = \rho^{(0)} + \epsilon \rho^{(1)} + \cdots$$

$$\lambda = \lambda^{(0)} + \epsilon \lambda^{(1)} + \cdots$$

- For translationally invariant eigenmodes we find

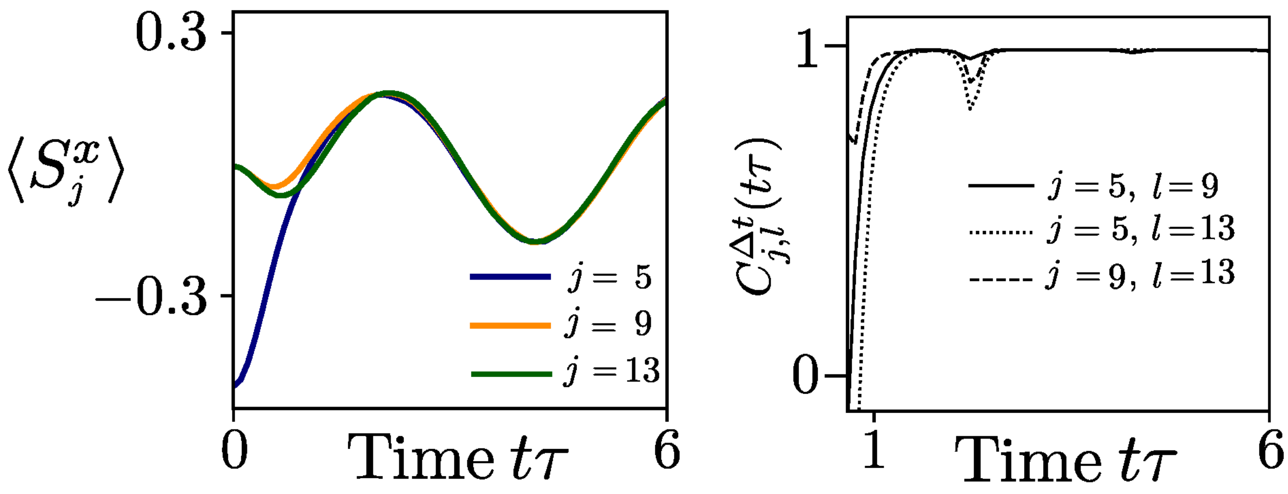
$$\lambda^{(1)} = \mathrm{Tr}\left\{\left(\rho^{(0)}\right)^{\dagger} \mathcal{L}^{(1)} \rho^{(0)}\right\} = 0$$

- and otherwise $\lambda^{(1)}$ is purely imaginary

→ limit cycles of X are stable to lowest order

J. Tindall, C. Sanchez Munoz, B. Buca and DJ, arXiv:1907.12837 (2019)

Numerical results

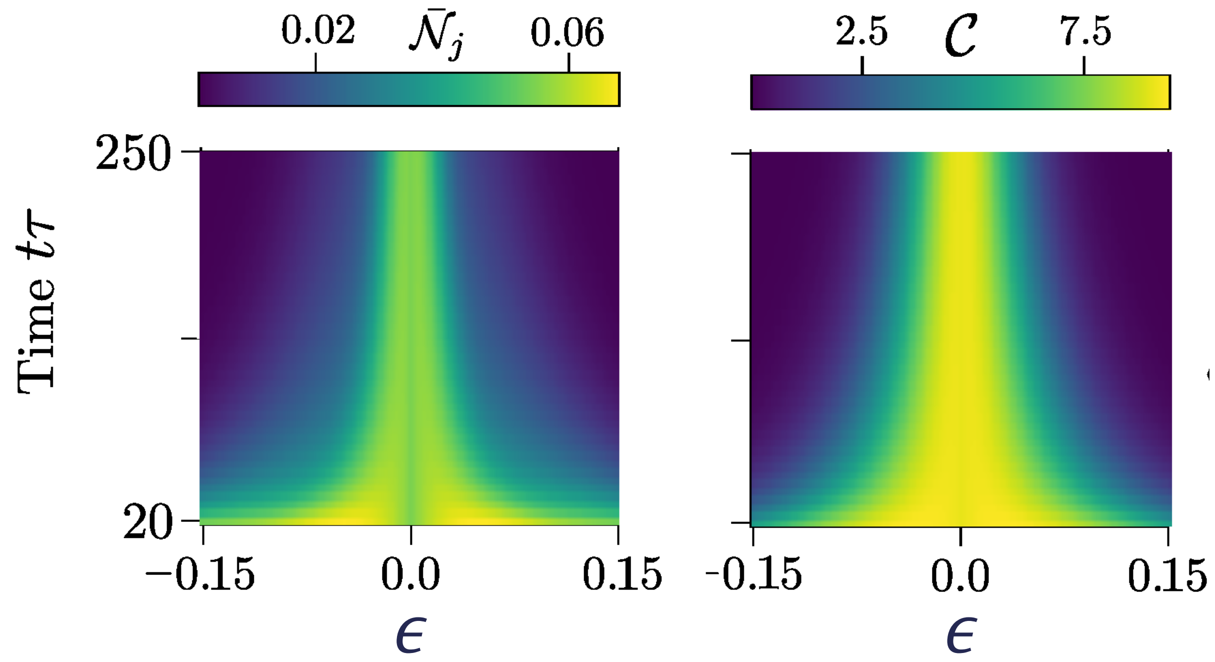


- Observables S_j^x and Pearson correlation coefficient (rolling time window)

$$C_{j,l}^{\Delta t}(\tau) = \frac{\int_\tau^{\tau+\Delta t} (S_j^x(\tau') - \bar{S}_j^x)(S_l^x(\tau') - \bar{S}_l^x)\mathrm{d}\tau'}{\sqrt{\int_\tau^{\tau+\Delta t} \left(S_j^x(\tau') - \bar{S}_j^x\right)^2 \mathrm{d}t' \int_\tau^{\tau+\Delta t} (S_l^x(\tau') - \bar{S}_l^x)^2 \mathrm{d}\tau'}}$$

with $N = 15$, $U = t$, $\bar{B} = 1.5t$, $\epsilon = 0.1t$.

Synchronization witnesses



- Negativity (one site entangled with the others)

$$\mathcal{N}_j(\rho) = \frac{1}{2}\left(\|\rho^{T_j}\| - 1\right)$$

- Off-diagonal coherences

$$C = \sum_{i \neq j} |\rho_{i,j}|$$

XXZ spin chain acting as its own bath

- 1D anisotropic Heisenberg spin ½ chain, periodic boundary conditions

$$H = \sum_j s_j^x s_{j+1}^x + s_j^y s_{j+1}^y + \Delta s_j^z s_{j+1}^z + h s_j^z$$

- Quasi-local conservation laws have been identified for this spin chain in [L. Zadnik, M. Medenjak, & T. Prosen, Nucl. Phys. B **902**, 339 (2016)] and we concentrate on $\Delta = -1/2$ where this quantity is simply

$$Y_j = s_i^x s_{i+1}^x s_{i+2}^x$$

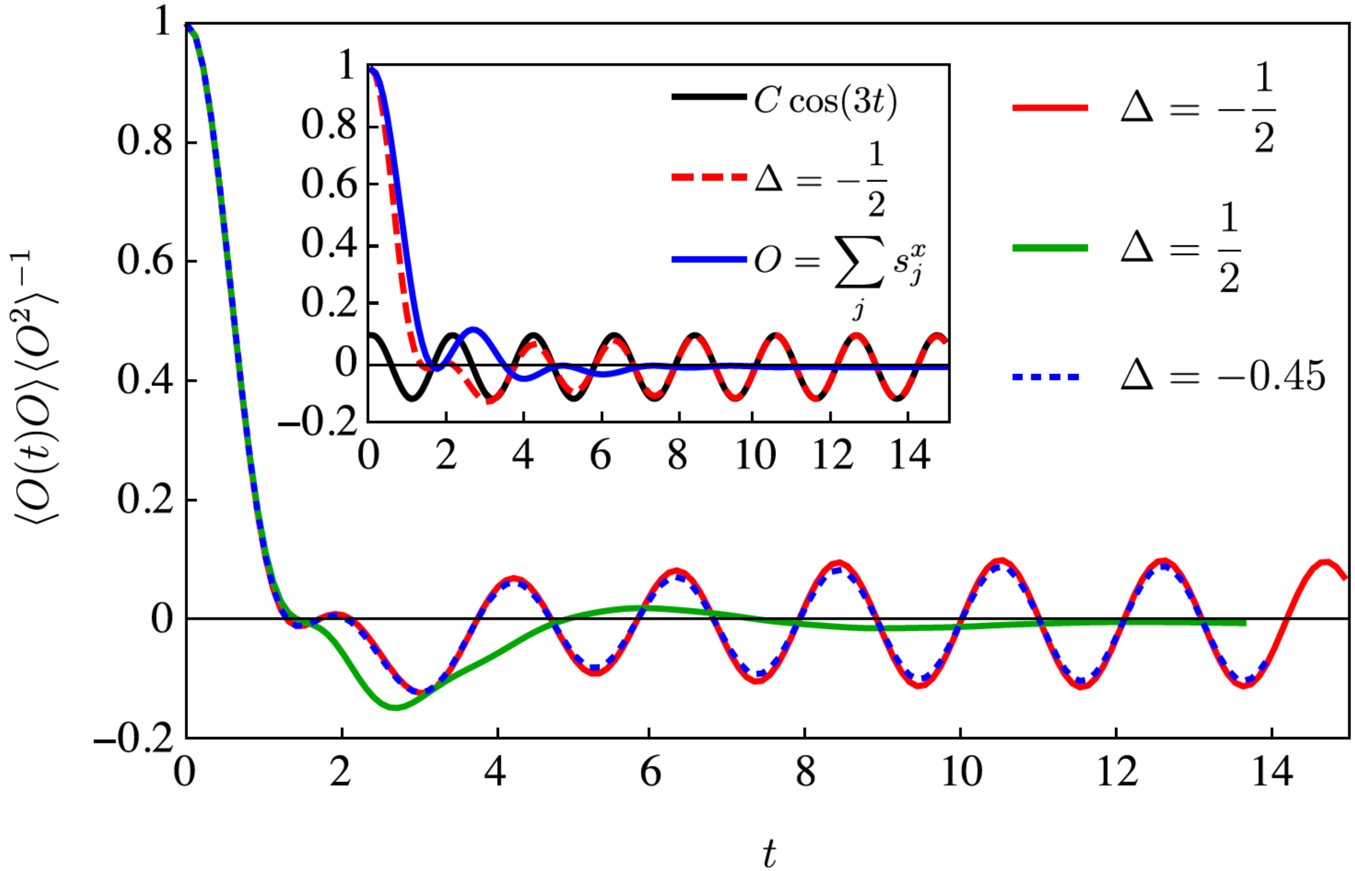
- which fulfil $[H, Y] = \omega Y$. Correlations of extensive observables $O = Y_j$ like

$$C(t) = \frac{\langle O(t) O \rangle}{\langle O^2 \rangle}$$

- are thus expected to never equilibrate within this model.

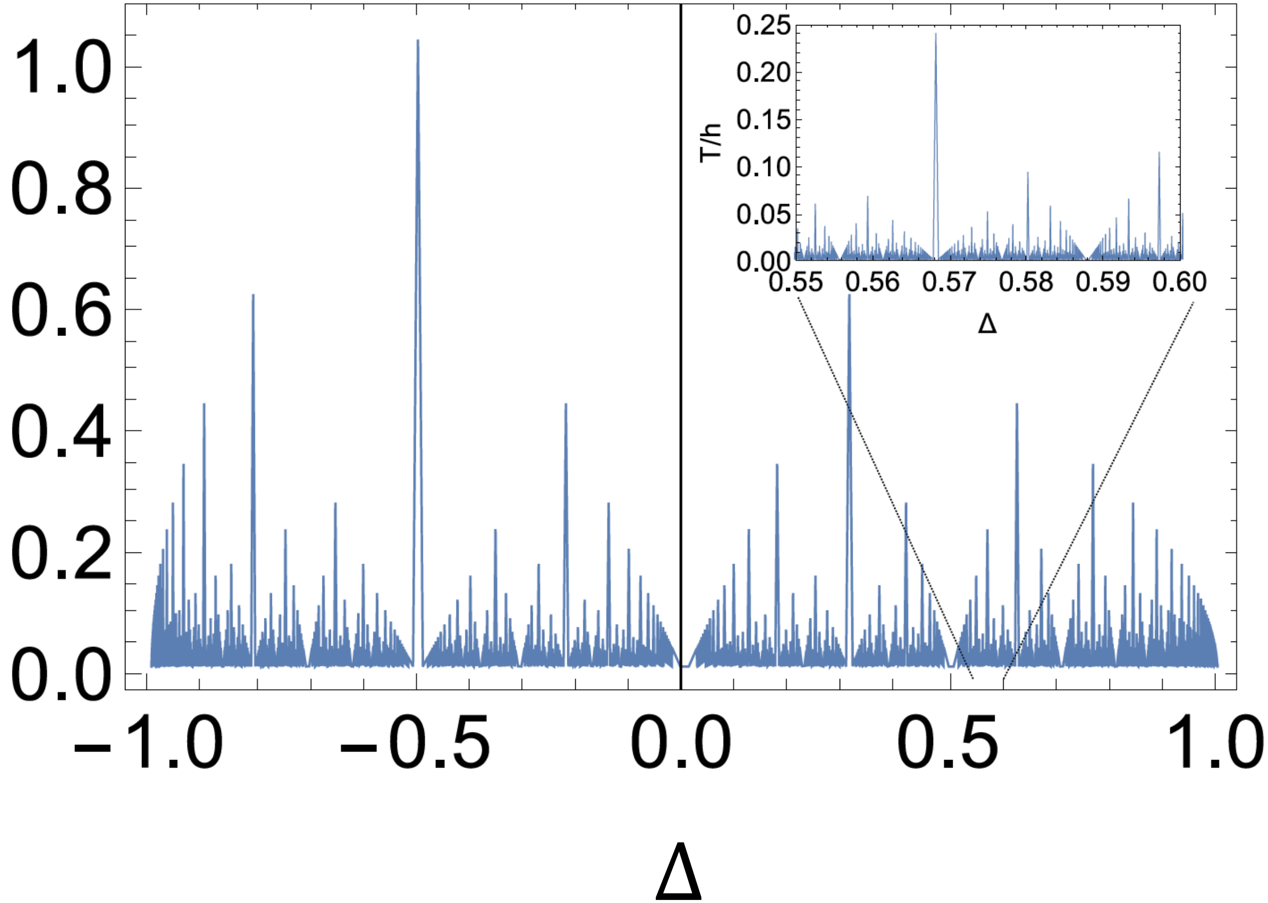
M. Medenjak, B. Buca, DJ, arXiv:1905.08266 (2019)

Persistent oscillations



M. Medenjak, B. Buca, DJ, arXiv:1905.08266 (2019)

A fractal time crystal in Δ



M. Medenjak, B. Buca, DJ, arXiv:1905.08266 (2019)

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