Quantum fluctuation-dissipation relations

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College on Energy Transport and Energy Conversion in the Quantum Regime. ICTP, August 2019

Lecture 1

1. The susceptibility of a classical damped harmonic oscillator of mass m, frequency ω_0 and friction coefficient γ, perturbed by a uniform field $F(t)$, is (section 1.4):

$$\chi_{Fx}(\omega) = \frac{1}{m(\omega_0^2 - \omega^2) - i\gamma\omega}$$

a) Calculate the Green function and the response function in the time domain.

b) Find the static susceptibility and compare it with the static susceptibility of an oscillator at temperature T given by the fluctuation-dissipation theorem (section 1.2). Discuss the dependence of the static susceptibility on temperature and γ.

c) Using the FDT, calculate the static susceptibility of a quantum harmonic oscillator at temperature T. Discuss the difference between the classical and the quantum case.

2. Using time-dependent perturbation theory (your favorite version), calculate the response function $\phi_{AA}(t)$ for a quantum system with Hamiltonian $H = H_0 - f(t)A$ if the reference state is an eigenstate $|\psi_i\rangle$ of H_0: $H_0 |\psi_i\rangle = E_i |\psi_i\rangle$. Consider the particular case of a harmonic oscillator and compare the result to the one obtained in exercise 1*c)*.

Lecture 2

3. A Markov chain is *reversible*, if any trajectory $(i_1, i_2, \ldots, i_{n-1}, i_n)$ is observed with the same probability as its time reversed $(i_n, i_{n-1}, \ldots, i_2, i_1)$. Prove that detailed balance, i.e., $\mathbb{P}_{ij}\pi_j = \mathbb{P}_{ji}\pi_i$, where π_i is the stationary probability corresponding to the transition matrix $\mathbb{P}_{ji}$, is a sufficient condition for reversibility in the stationary regime. Prove that for a reversible process

$$\langle B(t_n)A(0)\rangle = \langle B(0)A(t_n)\rangle$$

and $\phi_{AB}(t) = \phi_{AB}(-t)$. Extend the discussion to quantum Markov processes.

Lecture 3

4. **Project**. Obtain the approximate response function of a classical or quantum complex network using the appropriate family of observables (section 3.4).