# Quantum information with trapped ions

- Trapped ions as qubits for quantum computing and simulation
- Qubit architectures for scalable entanglement



# Quantum thermodynamics with ions

- Quantum thermodynamics introduction
- Heat transport, Fluctuation theorems,
- Phase transitions, Heat engines
- Outlook





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## Overview



New machines

## Energy transport





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#### Transport of radial phonons via a linear ion crystal

Energy propagation Propagation of quantum correlations

#### Vibrationally assisted energy transport

Explores high dimensional Hilbert space Transport involving nonlinear interactions Understanding transport principles in light harvesting



## The ion crystal

Ca<sup>+</sup>



Qubits

Local motion

Coulomb interaction



- Excite first ion on sideband, generates spin-motion entanglement
- Motion propagates throught the crystal
- Wait-time
- Analyse if motion returned back
- Contrast of Ramsey reveals delocalization of motional excitation

## Result:



Ramm et al., NJP 16 063062 (2014) Abdelrahman et al., Nat. Comm. 8 15712 (2017)

## Light harvesting complex



#### Ishizaki, Flemming, PNAS 106 17255 (2009)

## Light harvesting complex - model



## Light harvesting complex - model

Inhomogenity inhibits the energy transfer



# Light harvesting complex - model



Environment helps fulfilling resonance condition

Vibrationally assisted energy transport

## Full Hamiltonian



Even for small phonon excitation and few ions becomes high dimensional Hilbert space

$$H_{\text{eff}}/\hbar = \sum_{i,j} \frac{J_{ij}}{2} \left( \sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ \right)$$
$$+ \sum_{i,j} \frac{K_{ij}}{2} \sigma_i^z \left( a_i + a_i^\dagger \right)$$
$$+ \sum_i \frac{\Delta_i}{2} \sigma_i^z + \sum_i \nu_i a_i^\dagger a_i$$

## Minimal system – two ions



## Measurement sequence



## Result



Gorman *et al*., PRX**8**, 011038 (2018)

## Result



Gorman *et al*., PRX**8**, 011038 (2018)

## **Result** Temperature reduced from <n>=5 to <n>=0.5



Related work with SC: Potočnik *et al.*, Nat. Comm **9**, 904 (2018)

Gorman *et al*., PRX**8**, 011038 (2018)

# Quantum thermodynamics with ions

- Quantum thermodynamics introduction
- Heat transport
- Phase transitions
- Fluctuation theorem
- Single ion refrigerator
- Heat engines
- Outlook





## Structural phase transition & defect formation

#### Germany before phase transition



#### Germany after the structural phase transition



#### 1D, 2D, 3D ion crystals

- Depends on  $\alpha = (\omega_{ax}/\omega_{rad})^2$
- Depends on the number of ions a<sub>crit</sub> = cN<sup>β</sup>

Wineland et al., J. Res. Natl. Inst. Stand. Technol. 103, 259 (1998)

Enzer et al., PRL85, 2466 (2000)



- Generate a planar Zig-Zag when  $v_{ax} < v_{rad}^y < < v_{rad}^x$
- Tune radial frequencies in y and x direction

**2D** 



#### Structural phase transition in ion crystal

U<sub>Coulomb</sub>

$$H = \sum_{i,\mu} \left( \frac{p_{i\mu}^2}{2m} + \frac{1}{2} m \omega_{\mu}^2 r_{i\mu}^2 \right) + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

U<sub>pot,harm.</sub>

$$H \approx H_0 = \hbar \omega_z \sum_n \sqrt{\gamma_n^x} a_n^{\dagger} a_n + \sqrt{\gamma_n^y} b_n^{\dagger} b_n + \sqrt{\lambda_n^z} c_n^{\dagger} c_n$$

#### Phase transition @ CP:

- One mode frequency  $\rightarrow 0$
- Large non-harmonic contributions

 $\mathsf{E}_{\mathsf{kin}}$ 

- coupled Eigen-functions
- Eigen-vectors reorder to generate new structures



#### Universal principles of defect formation

Kibble (1976)

- symmetry breaking at a second order phase transitions such that topological defects form
- may explain formation of cosmic strings or domain walls



Zurek (1985)

- Sudden quench though the critical point leads to defect formation
- experiments in solid state phys. may test theory of universal scaling

Morigi, Retzger, Plenio (2010)

 Proposal for KZ study in trapped ions crystals

Kibble, Journal of Physics A 9, 1387 (1976) Kibble, Physics Reports 67, 183 (1980)

Zurek, Nature 317, 505 (1985), DelCampo, Zurek arXiv:1310.1600, Nikoghosyan, Nigmatullin, Plenio, arXiv:1311.1543



#### Structural configuration change in ion crystals



Zigzag

Zagzig

#### Structural configuration change in ion crystals



#### Universal principles of defect formation

- System response time, thus information transfer, slows down
- At some moment, the system becomes non-adiabatic and freezes
- Relaxation time diverges / increases



#### Molecular dynamics simulations





# Experimental setup and parameters

Trap with 11 segments

Controlled by FPGA and arbitray waveform gen.

 $\omega/2\pi = 1.4$ MHz (rad.), rad. anisotropy tuned to 100 +3..5%  $\omega/2\pi = 160 - 250$ kHz (ax.)

Laser cooling / CCD observation





#### Molecular dynamics simulations



#### Experimental test of the $\beta$ =8/3 power law scaling



#### Experimental test of the $\beta$ =8/3 power law scaling

Table 1. Experimental results on the topological defect formation in ion Coulomb crystals.<sup>13–15</sup> Data was fitted to a power-law in the quench rate  $\tau_Q$  of the form  $n \propto \tau_Q^{-\alpha}$ .

Group	Number of ions	Kink number	Fitted exponent $\alpha$
Mainz University <sup>14</sup>	16	$\{0,1\}$	$2.68\pm0.06$
$PTB^{15}$	$29\pm2$	$\{0,1\}$	$2.7\pm0.3$
Simon Fraser University <sup>13</sup>	$42\pm1$	$\{0,1\}$	2.1 - 3.1
0.05 Offset kink formation		Ulm et al, Na	at. Com. 4, 2290 (201:
J. Mod. Phys. A 29, 30018 (2014)	Pyka et al, N Ejtemaee, P	lat. Com. 4, 2291 (20 RA 87, 051401 (2013)	
	10 10 25		
4 5 6 7	8 9 10 (dω <sub>ax</sub> /dt)  <sub>cp</sub> (10 <sup>7</sup> /s	20 5 <sup>2</sup> )	30

# Experimental testing of fluctuation theorem at the quantum limit

Jarzynski, PRL 78, 2690 (1997) Crooks, PRE 60, 2721 (1999)

> Liphardt, et al., Sci. 296 (2002) 1832

Huber et al., PRL **101**, 070403 (2008)

- Work distribution measured with RNA
- Proposal for a test of Jarzynski equ. with a single ion
- Experimental realization work distribution measued

An et al., Nat. Phys.11, 193 (2015)







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## Single molecule streching

#### Liphardt, et al., Sci. 296 (2002) 1832



### Single molecule streching

Attach RNA to glass bead of laser tweezer unfold/refold single RNA molecule

Crooks fluctuation theorem:

$$\frac{P(-W)}{P(+W)} = \exp^{-W/k_B T}$$

Verify Crooks fluctuation theorem experimentally

Crooks, Phys. Rev. E 60(1999) 2721 Liphardt, et al., Sci. 296 (2002) 1832





#### quantum Jarzynski equality

 $\Delta F = -k_B T \ln \langle e^{-W/k_B T} \rangle$ 

 $\langle e^{-W/k_BT} \rangle = \int dW e^{-W/k_BT} P(W)$ 

Jarzynski, Phys. Rev. Lett. 78 (1997) 2690

free energy difference

average exponented work


## Non-equilibrium phonon States in a Paul trap

quantum work probabilty

**Proposed exp. Scheme:** 

- Start with thermal state n=0... ~ 10
- 2) Determine E<sup>0</sup>
- 3) Act (non-adiabatically)

on trap potential

4) Determine E<sup>t</sup>



Non-equilibrium phonon states



### Work probability distribution

$$P(W) = \sum_{m,n} \delta[W - (E_m^{\tau} - E_n^{0})] P_{m,n}^{\tau} P_n^{0}$$



#### **Provide Work – Displacement Operation**



#### $\sigma_x$ Dependent Displacement Operation

P. C. Haljan et al., Phys. Rev. Lett. 94, 153602 (2005).

P. J. Lee et al., Journal of Optics B 7, S371 (2005).

$$H_{bsb} = \frac{\eta\Omega}{2} \left( a^{\Box}\sigma^{+} + a\sigma^{-} \right)$$
$$H_{rsb} = \frac{\eta\Omega}{2} \left( a^{\Box}\sigma^{-} + a\sigma^{+} \right)$$

$$H_{bsb} + H_{rsb} = \frac{\eta \Omega}{2} (a^{\Box} + a) \sigma_x$$

#### **Pure Displacement Operation**





#### **Final State Measurements – Fitting Methods**



#### **Final State Measurements – Intermediate Work**



#### **Final State Measurements – Intermediate Work**





## Proposals for engines

Maser Scovil et al, PRL 2, 262 (1959)

Three Level System Geva et al., J Chem Phys (1996)

#### Quantum Thermodynamics

Gemmer et al, Springer, Lect Notes 784 (2009),





opto-Mechanical

(2014)

Zhang et al., PRL 112, 150602



Quantum dot Esposito et al., PRE 81, 041106 (2010)

## Heat engines

- single-ion **Otto heat** engine classical operation
- **autonomous** heat engine study phase stability
- absorption refrigerator
- **spin-driven** heat engine in the quantum regime quantum motion
- *future*: multi-ion crystal **quantum heat** engine





## **Classical heat engines**

heat





RESERVOIR

hot

PISTON

SYSTEM

Heat

Engine

mechanical

work

heat

James Watt (1783):  $\eta \cong 5 - 7\%$ Modern power plats:  $\eta \cong 30\%$ 

RESERVOIR

cold





James Watt



Sadi Carnot

$$\eta = \frac{\text{Work produced}}{\text{Heat absorbed}} = \frac{W}{Q_H} \le 1 - \frac{T_C}{T_H} = 1 - \frac{\beta_H}{\beta_C}$$

## Single ion heat engine

J. Roßnagel, et al. "A single-atom heat engine", Sci. 352, 325 (2016)

selected as one of the top ten breakthroughs in physics in the year 2016 by IOP Physics World

## The working principle – single ion HE



To reach reach large axial amplitudes of movement

- strong radial confinement
- weak axial confinement

# Setting the reservoir temperature by radial excitation and cooling



## Stroboscopic motion measurements



Princeton Instruments ICCD:

- 8 ns gate time
- 10 MHz frame reate



## Working principle and results

 $P = 3.4 \times 10^{-22} \text{ J/s}$ 

 $\eta = 0.28\%$ 



J. Roßnagel, et al. "A single-atom heat engine", Sci. 352, 325 (2016)

## Heat engine efficiency



# Stability of autonomous machine

Selected for a Viewpoint in Physics PHYSICAL REVIEW X 7, 031022 (2017)

Autonomous Quantum Clocks: Does Thermodynamics Limit Our Ability to Measure Time?

Paul Erker, 12 Mark T. Mitchison, 34 Ralph Silva, 5 Mischa P. Woods, 67 Nicolas Brunner, 5 and Marcus Huber8



Prediction:

Accuracy of ticking increases with heat consumption and with entropy production



#### Our system – the phonon laser





- Blue- and red-detuned beams near dipole transition can lead to autonomous harmonic motion
- Damping and excitation balance during each motional cycle
- Beams are in resonance with trapped ion at separate times: 2ω periodicity in photon emission rates

First demonstration by Udem group MPQ Munich: K. Vahala et al., Nat. Phys. **5**, 682 (2009)

#### Stable operating point





#### Phase stability





Recoils occur at velocity return points: Inherent phase stability!

#### Refrigerator







Dzmitry Matsukevich



Refrigerator: cools cold bath by work

Absorption Refrigerator: Driven by heat instead of work

## via **trilinear** Hamiltonian $\hat{H} = \hbar \xi (\hat{a}_h^{\dagger} \hat{a}_w \hat{a}_c + \hat{a}_h \hat{a}_w^{\dagger} \hat{a}_c^{\dagger})$ Dzmitry **Matsukevich** HOT $\omega_h = \omega_w + \omega_c$ $\omega_h = \sqrt{29/5}\omega_z$ COLD WORK $\omega_w = \sqrt{\omega_z^2 - \omega_x^2}$ $\omega_{w} = \sqrt{\omega_{x}^{2} - 12\omega_{z}^{2}/5}$

#### **Refrigerator with trapped ions**

Harmonic oscillators interacting





Maslennikov et al. Nat. Comm. 10, 202 (2019)

#### Equilibrium









#### **Fridge operation**

# The higher the work mode phonon number, the colder the cold mode



Maslennikov et al. Nat. Comm. 10, 202 (2019)

# Spin driven heat engine in the quantum limit

"A spin heat engine coupled to a harmonicoscillator flywheel", Phys. Rev. Lett. 2019 in press, arXiv:1808.02390

## Heat-Engine Operation in the Quantum Regime

Generic heat engine	Implementation with a trapped <sup>40</sup> Ca <sup>+</sup> ion
Working medium	Spin of the valence electron: $ \uparrow\rangle$ , $ \downarrow\rangle$
Thermal baths	Controlling the spin by optical pumping
Gearing mechanism	Spin-dependent optical dipole force
Storage for delivered work	Axial oscillation: $ 0\rangle$ , $ 1\rangle$ , $ 2\rangle$ ,



### **Spins Thermodynamics**



## Controlling the Spins Thermodynamics



Function	Cooling	Heating
Polarisation	circular	linear
Duration	180 ns	130 ns
Excitation $(p_{\uparrow})$	0.13	0.30
Temperature	0.4 mK	0.7 mK
Period ( = axial oscillation)		740 ns



### Heat-Engine Operation



Lindenfels et al., PRL (2019), arXiv 1808.02390

Schmiegelow et al., PRL 116, 033002 (2016)

## Single-ion operation



## Single-ion operation and analysis



- Red SB excitation: all motiotal state, except |n=0> transferred to ↑
- Measurement of Q-function:

$$\mathcal{Q}(\alpha, \alpha^*) = \frac{1}{\pi} \langle 0 | \hat{D}^{\dagger}(\alpha) \hat{\rho} \hat{D}(\alpha) | 0 \rangle$$

Lv, et al, Phys. Rev. A 95, 043813 (2017)

## Measured Qfunction

- starting from |n=0>
- Q-funct. modelled as dispaced ( $\beta$ ) squeezed ( $\zeta$ ) thermal ( $\bar{n}$ ) distribution





## Analysis of the heat engine function

- Reconstruct a density matrix from experimentally determined set {β,ζ,n}
- Determine work E
- Determine HE-ergotropy W

$$\mathcal{W} = \hbar \omega_t |\beta|^2 + \hbar \omega_t \sinh^2(|\zeta|)(2\bar{n}+1)$$
$$E = \mathcal{W} + \hbar \omega_t \bar{n}.$$

- Determine relative energy fluctuations ∆E/E
- Thermal and spin-projection noise contributions

Lindenfels et al., PRL (2019), arXiv 1808.02390



#### experimental / theory heat engine collaboration



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## Future plans



## Realize and analyze engine with full quantum control over working fluid and reservoirs



Goals:

- Investigate the role of multi-particle quantum entanglement in heat engines
- Study close connection between quantum error correction, quantum computing and heat engines