CNRNANO
ISTITUTO NANOSCIENZE CONSIGLIO NAZIONALE DELLE RICERCHE

NEST National Enterprise for nanoScience and nanoTechnology

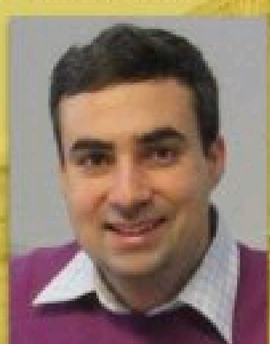
Non-local thermoelectricity & entanglement manipulation in hybrid-systems

Alessandro Braggio
CNR-Nano Pisa
NEST Labs
Scuola Normale Superiore

MIUR

FUTURO IN RICERCA

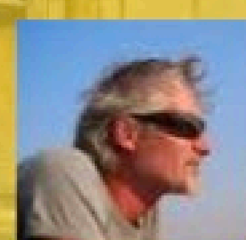
THE ROYAL SOCIETY


R.Hussein


M. Governale

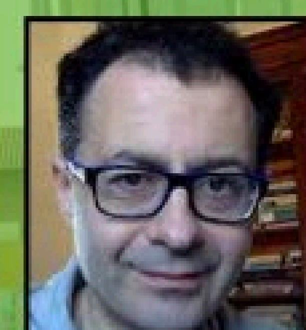
W. Belzig


S.Kohler


F. Giazotto


G.Blasi

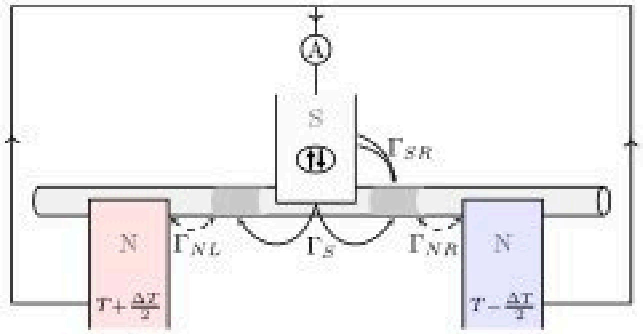

F. Taddei

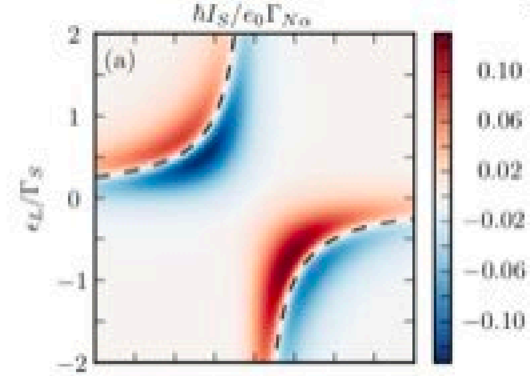

V. Giovannetti

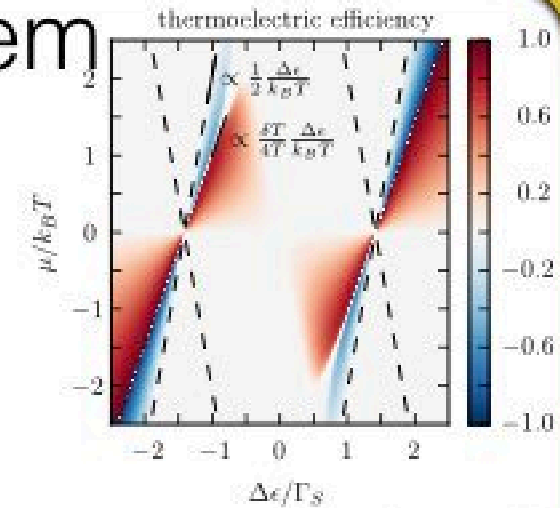
Trieste19

- Non-local thermoelectricity in hybrid nanosystem

DQD
Cooper pair splitter



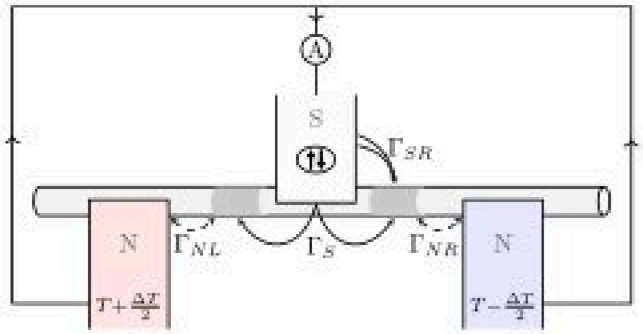


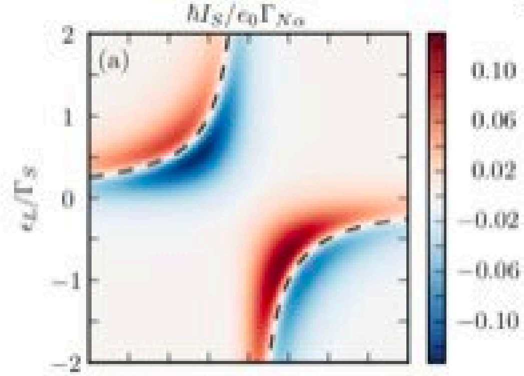


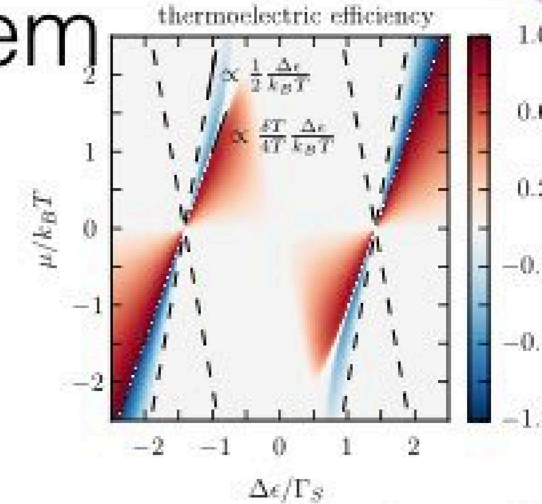
R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

- Non-local thermoelectricity in hybrid nanosystem

DQD
Cooper pair splitter



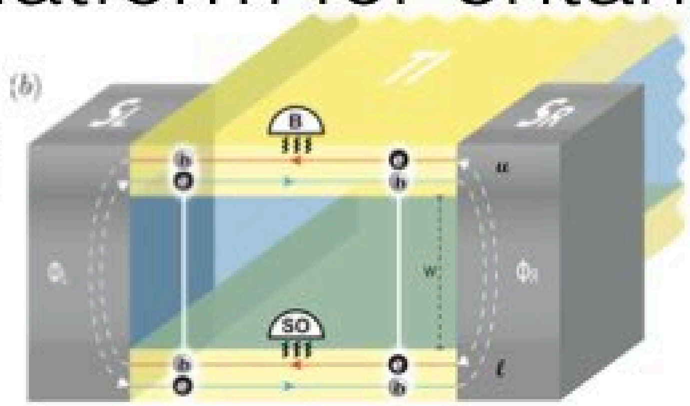


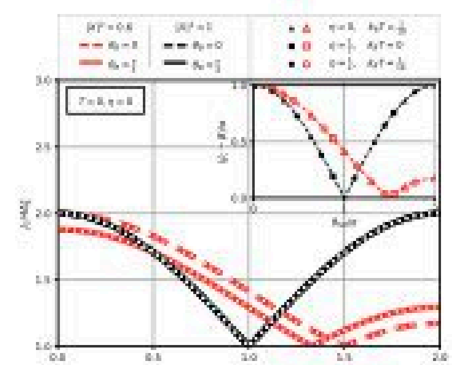


R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

- Solid state platform for entanglement manipulation

TI entanglement manipulation

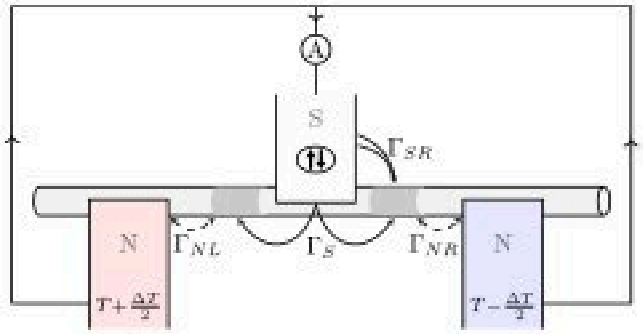


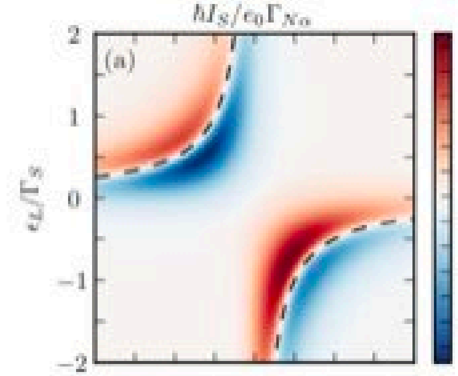


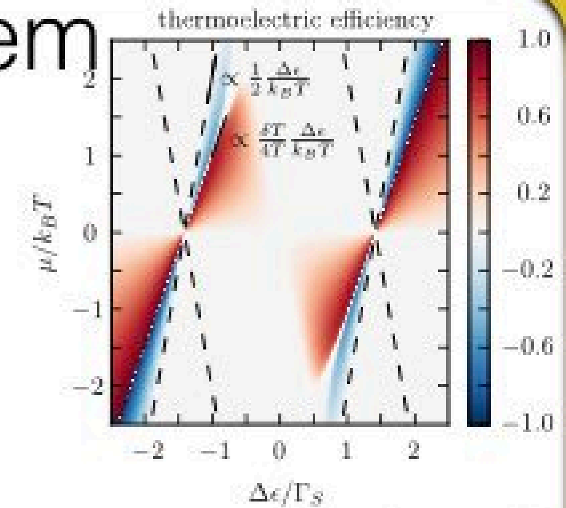
G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

- Non-local thermoelectricity in hybrid nanosystem

DQD
Cooper pair splitter



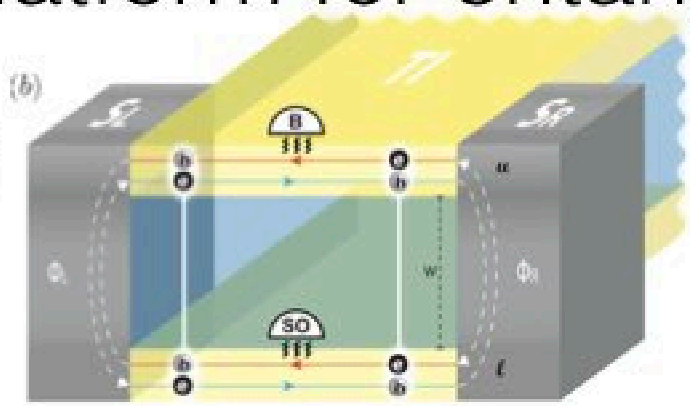


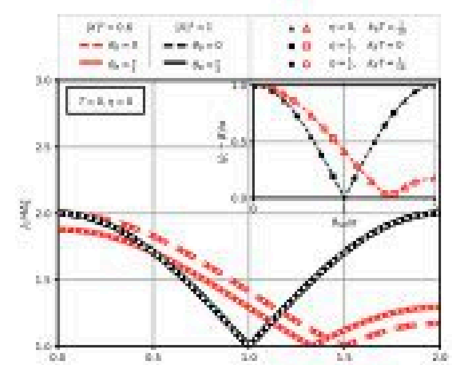


R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

- Solid state platform for entanglement manipulation

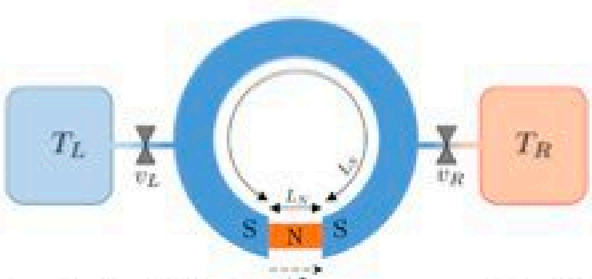
TI entanglement manipulation

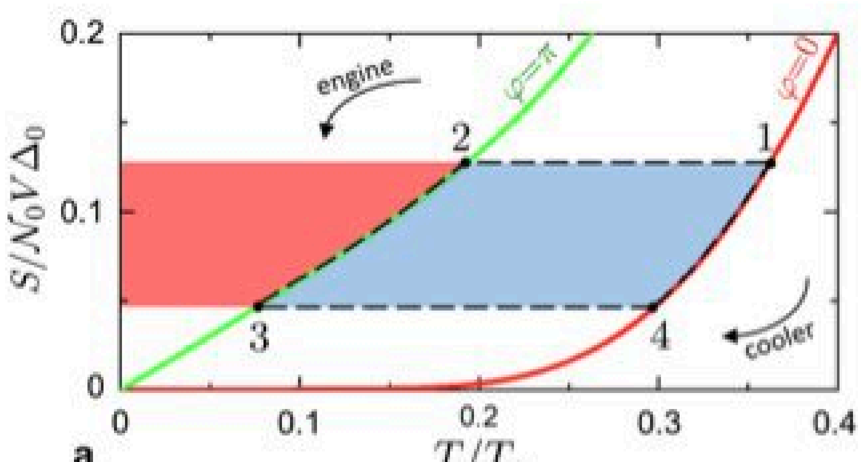


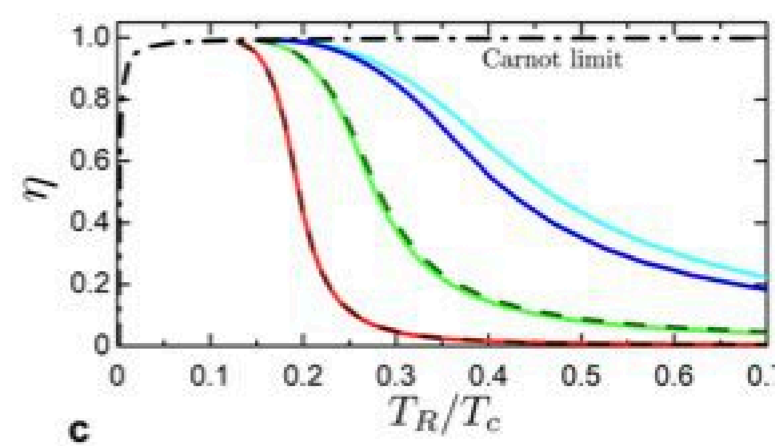


G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

- Coherent thermal machines

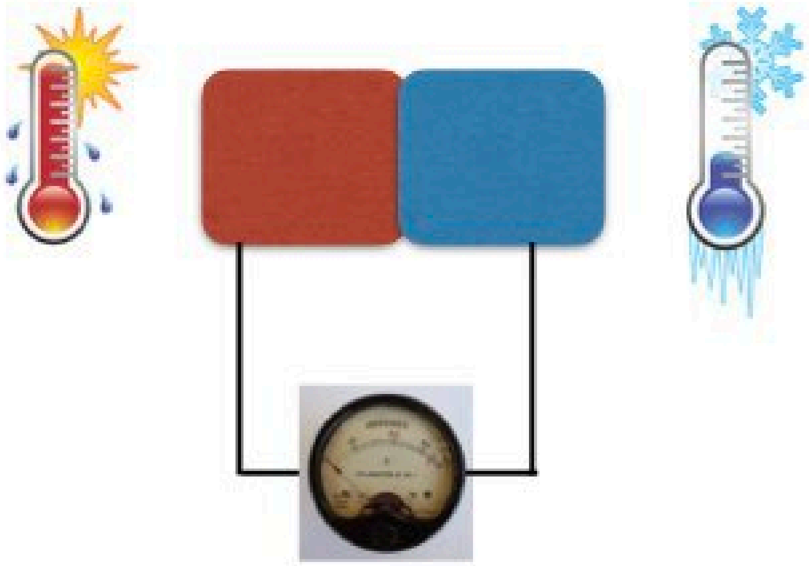






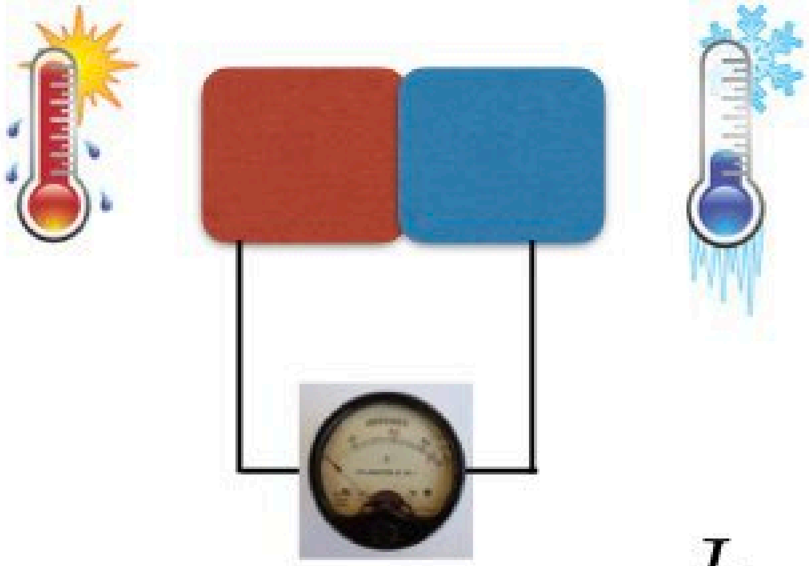
F. Vischi, M. Carrega, P. Virtanen, E. Strambini, A. Braggio and F. Giazotto, Sci. Rep. 9 3238 (2019)

Thermoelectricity



Universal property for pure and composite systems

Thermoelectricity

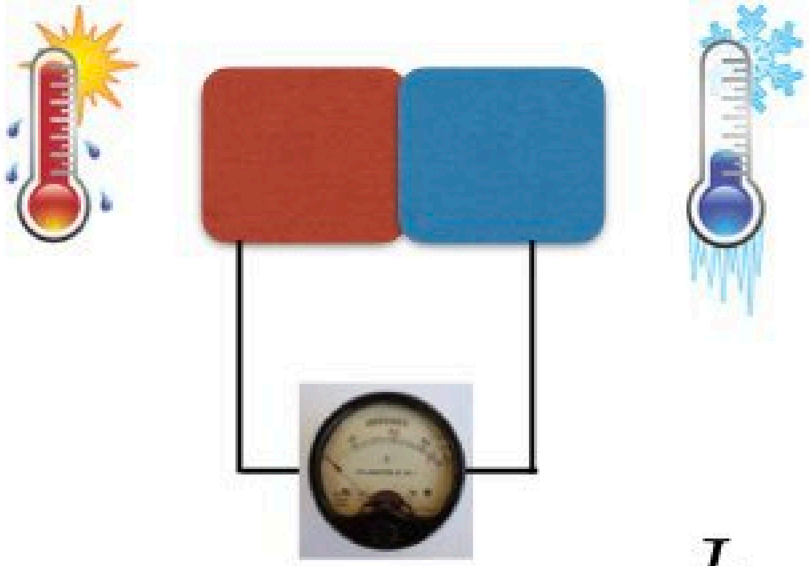


Universal property for pure and composite systems

- Quantum transport & Nanostructure Benenti's lectures

$$J_{h,\alpha} = \frac{1}{h}\int_{-\infty}^{\infty} dE(E-\mu_\alpha)\tau(E)[f_L(E)-f_R(E)]$$

Thermoelectricity



Universal property for pure and composite systems

- Quantum transport & Nanostructure

Benenti's lectures

$$J_{h,\alpha} = \frac{1}{h}\int_{-\infty}^{\infty} dE(E-\mu_\alpha)\tau(E)[f_L(E)-f_R(E)]$$

In linear regime $\delta T, \delta\mu \to 0$

$$S = \frac{1}{eT}\frac{\int_{-\infty}^{\infty} dE(E-\mu)\mathcal{T}_{LR}(E)\left[-f'(E)\right]}{\int_{-\infty}^{\infty} dE\,\mathcal{T}_{LR}(E)\left[-f'(E)\right]}, \quad (124$$

Review

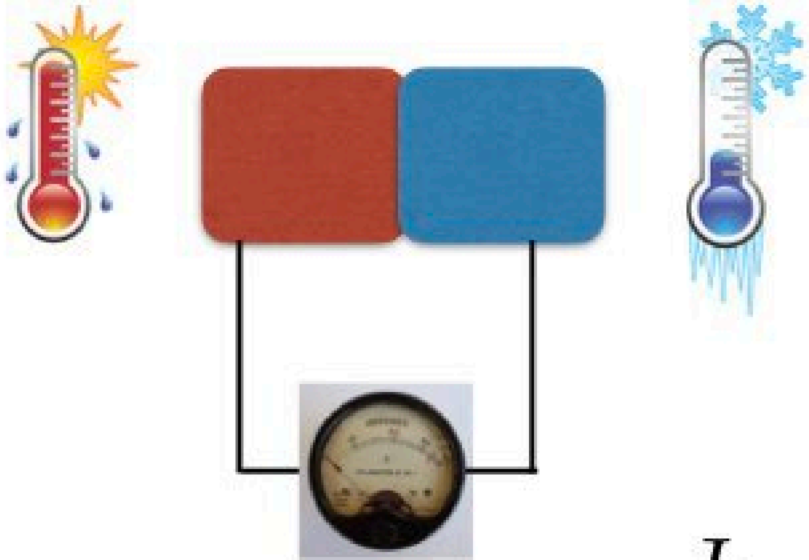
with $f'(E)$ being the derivative of the Fermi function in Eq. (111). Since $f'(E)$ is an even function of $(E-\mu)$, one sees tha
S vanishes if $\mathcal{T}_{LR}(E)$ is symmetric around μ. It is then clear that electrons and holes contribute to the thermopower wit
opposite signs and that $S = 0$ when there is particle–hole symmetry. Any system in which the symmetry is broken betwee

Fundamental aspects of steady-state conversion of heat to work at the nanoscale

Giuliano Benenti [a,b,*], Giulio Casati [a,c], Keiji Saito [d], Robert S. Whitney [e]

ELSEVIER
Contents lists available at ScienceDirect
Physics Reports
journal homepage: www.elsevier.com/locate/physrep

Thermoelectricity



Universal property for pure and composite systems

- Quantum transport & Nanostructure
  Benenti's lectures

$$J_{h,\alpha} = \frac{1}{h}\int_{-\infty}^{\infty} dE(E-\mu_\alpha)\tau(E)[f_L(E)-f_R(E)]$$

In linear regime $\delta T, \delta\mu \to 0$

$$S = \frac{1}{eT}\frac{\int_{-\infty}^{\infty} dE(E-\mu)\mathcal{T}_{LR}(E)\left[-f'(E)\right]}{\int_{-\infty}^{\infty} dE\,\mathcal{T}_{LR}(E)\left[-f'(E)\right]}, \quad (124$$

Review

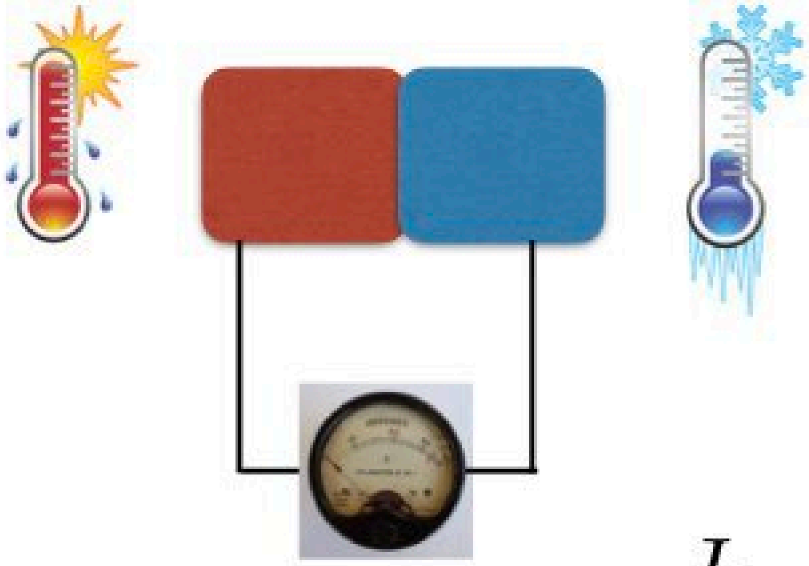
with $f'(E)$ being the derivative of the Fermi function in Eq. (111). Since $f'(E)$ is an even function of $(E-\mu)$, one sees tha S vanishes if $\mathcal{T}_{LR}(E)$ is symmetric around μ. It is then clear that electrons and holes contribute to the thermopower wit opposite signs and that $S = 0$ when there is particle–hole symmetry. Any system in which the symmetry is broken betwee

Fundamental aspects of steady-state conversion of heat to work at the nanoscale

Giuliano Benenti [a,b,*], Giulio Casati [a,c], Keiji Saito [d], Robert S. Whitney [e]

Contents lists available at ScienceDirect
Physics Reports
journal homepage: www.elsevier.com/locate/physrep
ELSEVIER

Thermoelectricity



Universal property for pure and composite systems

- Quantum transport & Nanostructure

Benenti's lectures

$$J_{h,\alpha} = \frac{1}{h}\int_{-\infty}^{\infty} dE(E-\mu_\alpha)\tau(E)[f_L(E)-f_R(E)]$$

In linear regime $\delta T, \delta\mu \to 0$

$$S = \frac{1}{eT}\frac{\int_{-\infty}^{\infty} dE(E-\mu)\mathcal{T}_{LR}(E)\left[-f'(E)\right]}{\int_{-\infty}^{\infty} dE\,\mathcal{T}_{LR}(E)\left[-f'(E)\right]}, \quad (124$$

Review

with $f'(E)$ being the derivative of the Fermi function in Eq. (111). Since $f'(E)$ is an even function of $(E-\mu)$, one sees tha S vanishes if $\mathcal{T}_{LR}(E)$ is symmetric around μ. It is then clear that electrons and holes contribute to the thermopower wit opposite signs and that $S = 0$ when there is particle–hole symmetry. Any system in which the symmetry is broken betwee

Fundamental aspects of steady-state conversion of heat to work at the nanoscale

Giuliano Benenti [a,b,*], Giulio Casati [a,c], Keiji Saito [d], Robert S. Whitney [e]

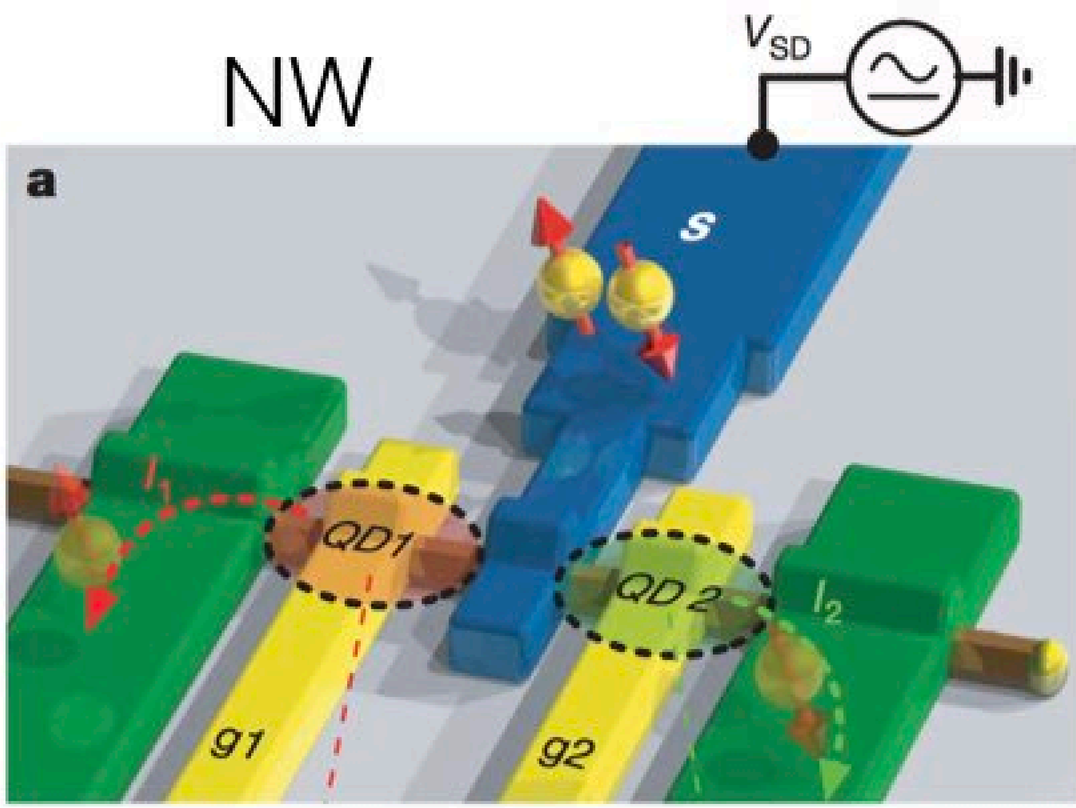
ELSEVIER
Contents lists available at ScienceDirect
Physics Reports
journal homepage: www.elsevier.com/locate/physrep

Superconductors PH-symmetry….. no thermoelectricity?

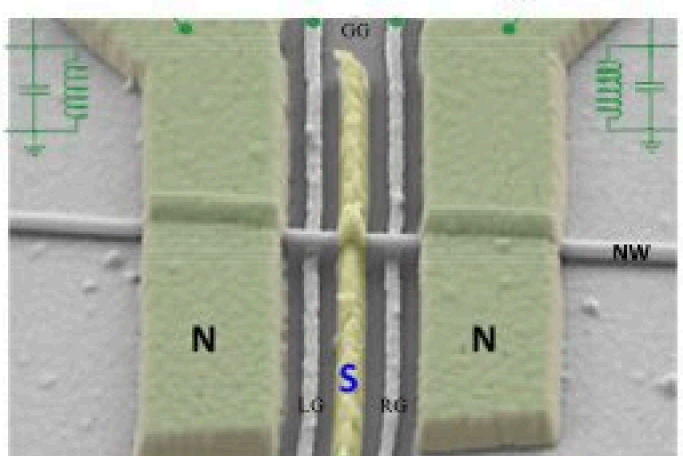
Entanglement in supercond

- Cooper pair splitters

NW

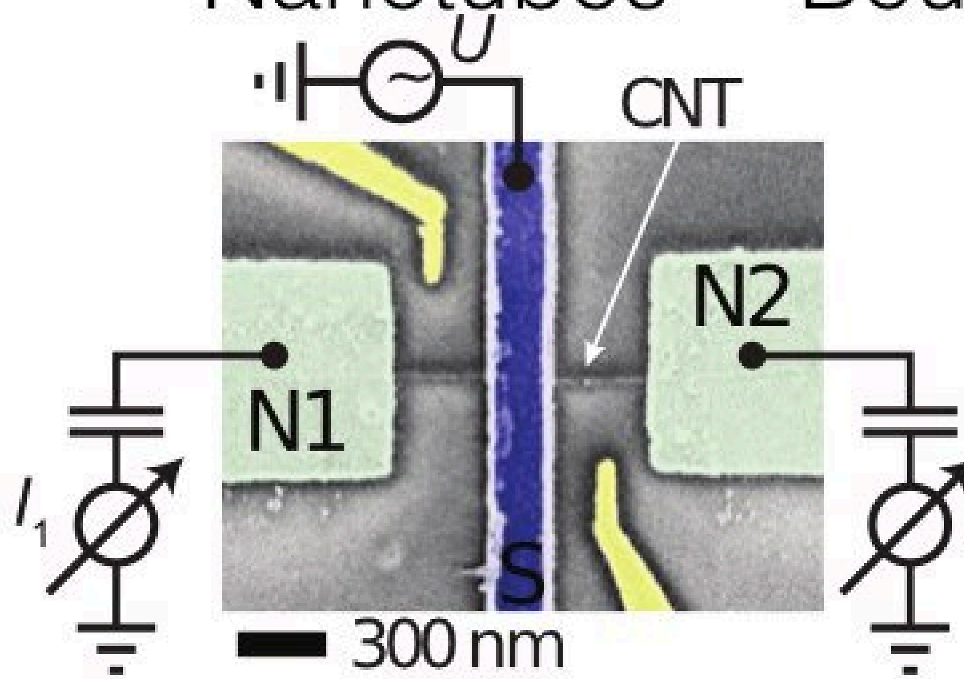


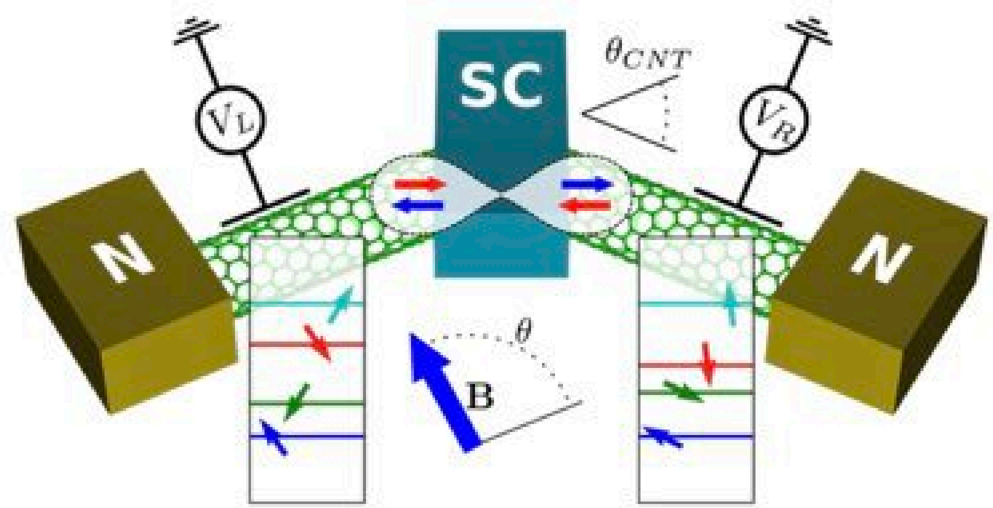
Schonenberger's group



Heiblum's group

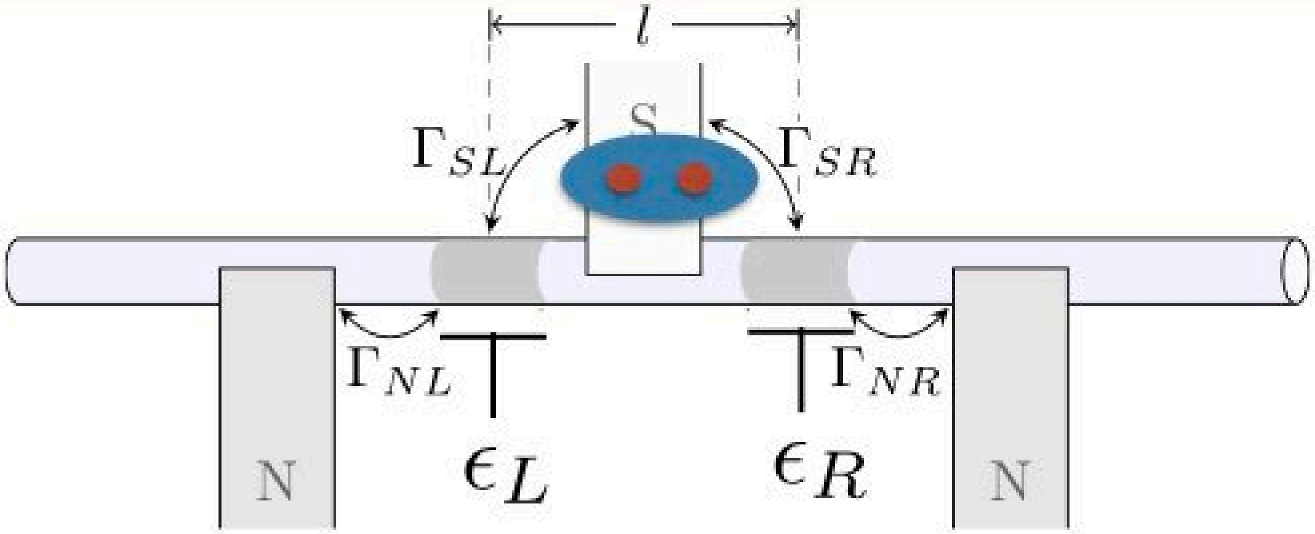
Nanotubes Bouchiat,...





Levy Yeyati's, Thierry Martin,.. groups

Double-quantum-dot CPS



- Superconductor Al

$$\xi_{Al} \approx 100 \text{ nm}$$

S-wave BCS: Spin-singlet

$$|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

- Seminconducting NW

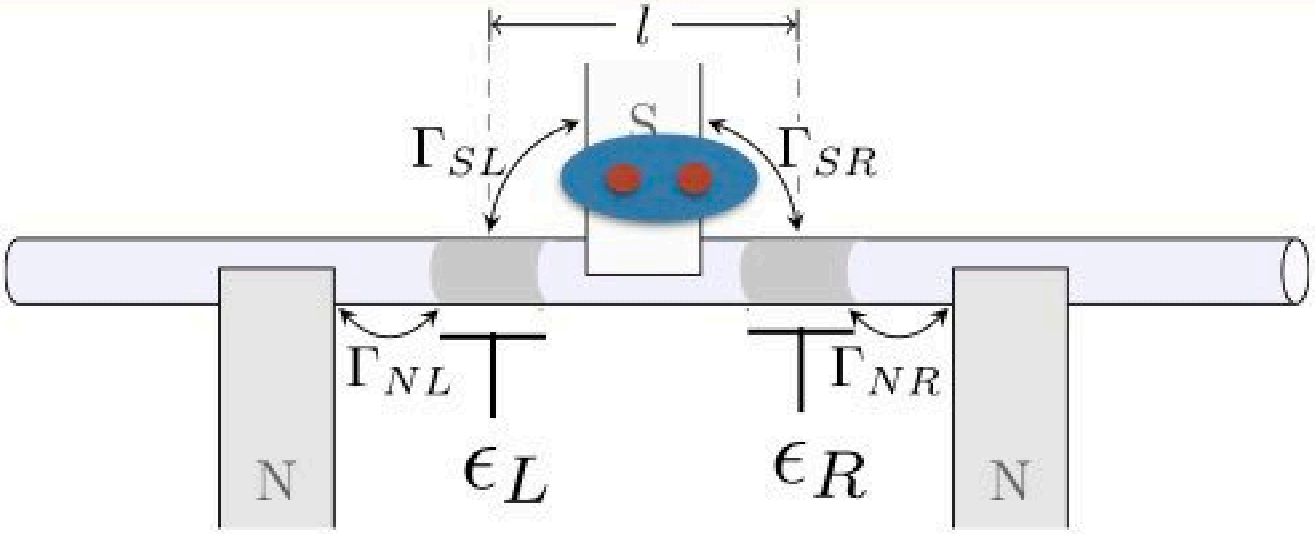
InAs,InSb

R. Hussein, L. Jaurigue, M. Governale, AB PRB'16

Γ_{SR}, Γ_{SL} Local Andreev Reflection LAR

$\Gamma_S = \sqrt{\Gamma_{SR}\Gamma_{SL}}\, e^{-l/\xi}$ Crossed Andreev Reflection CAR

Double-quantum-dot CPS



R. Hussein, L. Jaurigue, M. Governale, AB PRB'16

- Superconductor Al

$\xi_{Al} \approx 100$ nm

S-wave BCS: Spin-singlet

$|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

- Seminconducting NW

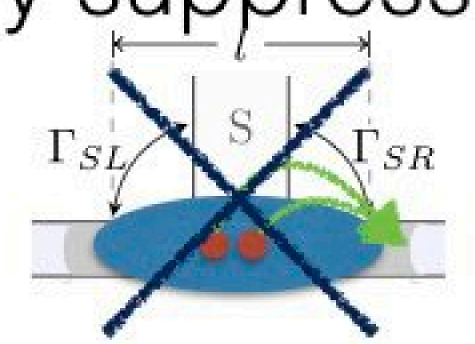
InAs,InSb

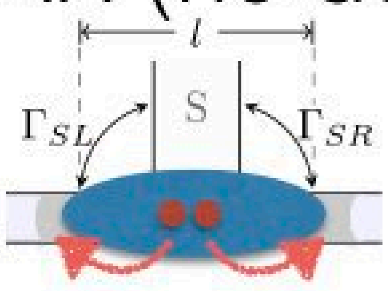
Γ_{SR}, Γ_{SL} Local Andreev Reflection LAR

$\Gamma_S = \sqrt{\Gamma_{SR}\Gamma_{SL}}\, e^{-l/\xi}$ Crossed Andreev Reflection CAR

Coulomb energy suppress LAR (no double occupation)

$U_R, U_L \to \infty$





Only CAR

Hamiltonian & Model

Hamiltonian & Model

$$H_{\mathrm{eff}} = \sum_{\alpha\sigma} \epsilon_\alpha |\alpha\sigma\rangle\langle\alpha\sigma| + \epsilon_S \frac{|S\rangle\langle S|}{2} - \frac{\Gamma_S}{\sqrt{2}} \left(|0\rangle\langle S| + |S\rangle\langle 0|\right)$$

$\Delta \to \infty$

A. V. Rozhkov, and D. P. Arovas, PRB '00 T. Meng, et al., PRB'09

AB, M. Governale, M.Pala, J. Koenig, SSComm.'11 B. Sothmann, et al., PRB'14

Tracing-outsuperconductors

Hamiltonian & Model

$$H_{\rm eff} = \sum_{\alpha\sigma} \epsilon_\alpha |\alpha\sigma\rangle\langle\alpha\sigma| + \epsilon_S \frac{|S\rangle\langle S|}{2} - \frac{\Gamma_S}{\sqrt{2}}\left(|0\rangle\langle S| + |S\rangle\langle 0|\right)$$

$\Delta \to \infty$

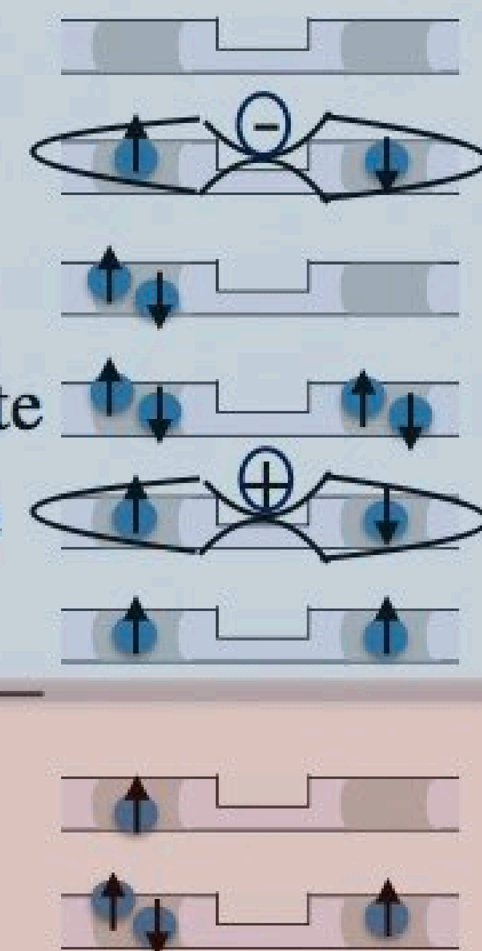
A. V. Rozhkov, and D. P. Arovas, PRB '00 T. Meng, et al., PRB'09
AB, M. Governale, M.Pala, J. Koenig, SSComm.'11 B. Sothmann, et al., PRB'14

Tracing-outsuperconductors

$$H_0 = H_{QD} + H_{S,\rm eff} = H_0^{\rm even} \oplus H_0^{\rm odd}$$

8 Even +8 Odd

			n_e
$\|0\rangle$	**Even**	empty state	0
$\|S\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} - d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})\|0\rangle$		singlet state	2
$\|d\alpha\rangle = d^\dagger_{\alpha\uparrow}d^\dagger_{\alpha\downarrow}\|0\rangle$		doubly occupied states	2
$\|dd\rangle = d^\dagger_{R\uparrow}d^\dagger_{R\downarrow}d^\dagger_{L\uparrow}d^\dagger_{L\downarrow}\|0\rangle$		quadruply occupied state	4
$\|T0\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} + d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})\|0\rangle$		unpolarized triplet state	2
$\|T\sigma\rangle = d^\dagger_{R\sigma}d^\dagger_{L\sigma}\|0\rangle$		polarized triplet states	2
$\|\alpha\sigma\rangle = d^\dagger_{\alpha\sigma}\|0\rangle$	**Odd**	singly occupied states	1
$\|t\alpha\sigma\rangle = d^\dagger_{\alpha\sigma}d^\dagger_{\bar\alpha\uparrow}d^\dagger_{\bar\alpha\downarrow}\|0\rangle$		triply occupied states	3



Hamiltonian & Model

$$H_{\text{eff}} = \sum_{\alpha\sigma} \epsilon_\alpha |\alpha\sigma\rangle\langle\alpha\sigma| + \epsilon_S \frac{|S\rangle\langle S|}{2} - \frac{\Gamma_S}{\sqrt{2}}\left(|0\rangle\langle S| + |S\rangle\langle 0|\right)$$

$\Delta \to \infty$ A. V. Rozhkov, and D. P. Arovas, PRB '00 T. Meng, et al., PRB'09

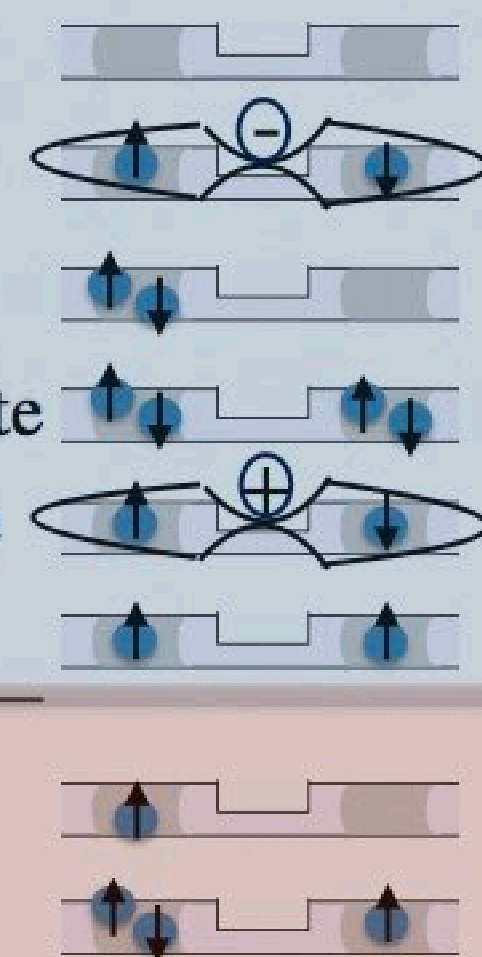
AB, M. Governale, M.Pala, J. Koenig, SSComm.'11 B. Sothmann, et al., PRB'14

Tracing-outsuperconductors

$$H_0 = H_{QD} + H_{S,\text{eff}} = H_0^{\text{even}} \oplus H_0^{\text{odd}}$$

8 Even +8 Odd

State		Description	n_e
$|0\rangle$	**Even**	empty state	0
$|S\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} - d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})|0\rangle$		singlet state	2
$|d\alpha\rangle = d^\dagger_{\alpha\uparrow}d^\dagger_{\alpha\downarrow}|0\rangle$		doubly occupied states	2
$|dd\rangle = d^\dagger_{R\uparrow}d^\dagger_{R\downarrow}d^\dagger_{L\uparrow}d^\dagger_{L\downarrow}|0\rangle$		quadruply occupied state	4
$|T0\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} + d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})|0\rangle$		unpolarized triplet state	2
$|T\sigma\rangle = d^\dagger_{R\sigma}d^\dagger_{L\sigma}|0\rangle$		polarized triplet states	2
$|\alpha\sigma\rangle = d^\dagger_{\alpha\sigma}|0\rangle$	**Odd**	singly occupied states	1
$|t\alpha\sigma\rangle = d^\dagger_{\alpha\sigma}d^\dagger_{\bar\alpha\uparrow}d^\dagger_{\bar\alpha\downarrow}|0\rangle$		triply occupied states	3



- Total Hamiltonian $H = H_{\text{eff}} + H_{\text{leads}} + \sum \left(T_{\eta\alpha} c^\dagger_{\eta k\sigma} d_{\alpha\sigma} + \text{H.c.}\right)$

Hamiltonian & Model

$$H_{\text{eff}} = \sum_{\alpha\sigma} \epsilon_\alpha |\alpha\sigma\rangle\langle\alpha\sigma| + \epsilon_S \frac{|S\rangle\langle S|}{2} - \frac{\Gamma_S}{\sqrt{2}}\left(|0\rangle\langle S| + |S\rangle\langle 0|\right)$$

$\Delta \to \infty$ A. V. Rozhkov, and D. P. Arovas, PRB '00 T. Meng, et al., PRB'09

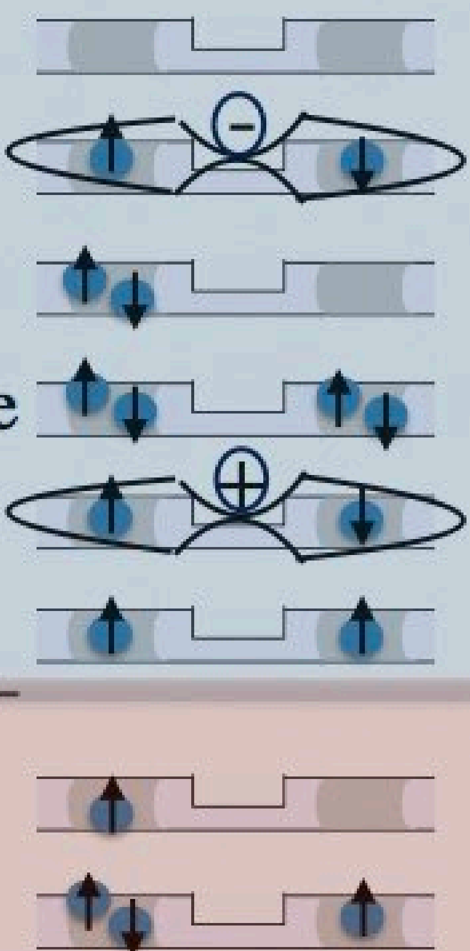
AB, M. Governale, M.Pala, J. Koenig, SSComm.'11 B. Sothmann, et al., PRB'14

Tracing-outsuperconductors

$$H_0 = H_{QD} + H_{S,\text{eff}} = H_0^{\text{even}} \oplus H_0^{\text{odd}}$$

8 Even +8 Odd

			n_e
$\lvert 0\rangle$		empty state	0
$\lvert S\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} - d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})\lvert 0\rangle$		singlet state	2
$\lvert d\alpha\rangle = d^\dagger_{\alpha\uparrow}d^\dagger_{\alpha\downarrow}\lvert 0\rangle$	**Even**	doubly occupied states	2
$\lvert dd\rangle = d^\dagger_{R\uparrow}d^\dagger_{R\downarrow}d^\dagger_{L\uparrow}d^\dagger_{L\downarrow}\lvert 0\rangle$		quadruply occupied state	4
$\lvert T0\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{R\uparrow}d^\dagger_{L\downarrow} + d^\dagger_{R\downarrow}d^\dagger_{L\uparrow})\lvert 0\rangle$		unpolarized triplet state	2
$\lvert T\sigma\rangle = d^\dagger_{R\sigma}d^\dagger_{L\sigma}\lvert 0\rangle$		polarized triplet states	2
$\lvert \alpha\sigma\rangle = d^\dagger_{\alpha\sigma}\lvert 0\rangle$	**Odd**	singly occupied states	1
$\lvert t\alpha\sigma\rangle = d^\dagger_{\alpha\sigma}d^\dagger_{\bar\alpha\uparrow}d^\dagger_{\bar\alpha\downarrow}\lvert 0\rangle$		triply occupied states	3



•T Transition between different parity sector $+\,\mathrm{H.c.})$

Transport by master eq.

- Total Hamiltonian $H = H_{\text{eff}} + H_{\text{leads}} + \sum_{k\sigma} \left(T_{\eta\alpha} c^\dagger_{\eta k\sigma} d_{\alpha\sigma} + \text{H.c.} \right)$

$T_{\eta\alpha} c^\dagger_{\eta k\sigma} d_{\alpha\sigma}$ $\quad \Gamma_N = \frac{2\pi}{\hbar} |T_{\eta\alpha}|^2 \rho_\eta$ Golden rule

Master Eq → FCS methods → Transport:curr,heat, noise,..

D. Bagrets and Yu. V. Nazarov, PRB '03; A.B., J.König, R. Fazio, PRL'06,
C. Flindt, T. Novotny, AB, M. Sassetti, A.-P. Jauho. PRL'08
C. Flindt, T. Novotny, AB, A-P. jauho PRB'10

Transport by master eq.

- Transition between different parity sector $_{\sigma} + \mathrm{H.c.})$

$k\sigma$

$T_{\eta\alpha} c^\dagger_{\eta k\sigma} d_{\alpha\sigma}$ $\quad \Gamma_N = \frac{2\pi}{\hbar} |T_{\eta\alpha}|^2 \rho_\eta$ Golden rule

Master Eq → FCS methods → Transport:curr,heat, noise,..

D. Bagrets and Yu. V. Nazarov, PRB '03; A.B., J.König, R. Fazio, PRL'06,
C. Flindt, T. Novotny, AB, M. Sassetti, A.-P. Jauho. PRL'08
C. Flindt, T. Novotny, AB, A-P. jauho PRB'10

Transport by master eq.

- T Transition between different parity sector $_{\sigma} + \mathrm{H.c.})$ $_{k\sigma}$

$T_{\eta\alpha} c^{\dagger}_{\eta k\sigma} d_{\alpha\sigma}$ $\quad \Gamma_N = \frac{2\pi}{\hbar} |T_{\eta\alpha}|^2 \rho_\eta$ Golden rule

Master Eq → FCS methods → Transport:curr,heat, noise,..

D. Bagrets and Yu. V. Nazarov, PRB ’03; A.B., J.König, R. Fazio, PRL’06,
C. Flindt, T. Novotny, AB, M. Sassetti, A.-P. Jauho. PRL’08
C. Flindt, T. Novotny, AB, A-P. jauho PRB’10

- Transport properties by Fermi golden rule $\Gamma_N \ll \Gamma_{S,\alpha}, k_B T$

$$w^{(\alpha,s)}_{a\leftarrow a'} = \sum_{\sigma} \Gamma_{N\alpha} f^{(-s)}_{\alpha}(-s\omega_{aa'}) \left| \langle a | d^{(-s)}_{\alpha\sigma} | a' \rangle \right|^2$$

Lecture note 1 Splettstoeser

R. Hussein, L. Jaurigue, M. Governale, AB PRB’16

Transport by master eq.

• T **Transition between different parity sector** $_\sigma + \mathrm{H.c.})$

$k\sigma$

$T_{\eta\alpha} c^\dagger_{\eta k\sigma} d_{\alpha\sigma}$ $\quad \Gamma_N = \frac{2\pi}{\hbar} |T_{\eta\alpha}|^2 \rho_\eta$ Golden rule

Master Eq → FCS methods → Transport:curr,heat, noise,..

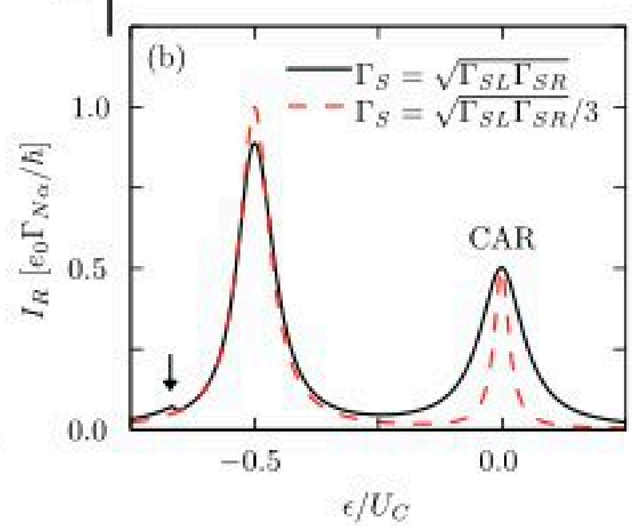
D. Bagrets and Yu. V. Nazarov, PRB '03; A.B., J.König, R. Fazio, PRL'06,
C. Flindt, T. Novotny, AB, M. Sassetti, A.-P. Jauho. PRL'08
C. Flindt, T. Novotny, AB, A-P. jauho PRB'10

• Transport properties by Fermi golden rule $\quad \Gamma_N \ll \Gamma_{S,\alpha}, k_B T$

$$w^{(\alpha,s)}_{a\leftarrow a'} = \sum_\sigma \Gamma_{N\alpha} f^{(-s)}_\alpha(-s\omega_{aa'}) \left| \langle a | d^{(-s)}_{\alpha\sigma} | a' \rangle \right|^2$$

$$I_\alpha = \frac{e_0}{\hbar} \sum_{a,a',s=\pm} s w^{(\alpha,s)}_{a\leftarrow a'} P^{\mathrm{stat}}_{a'},$$

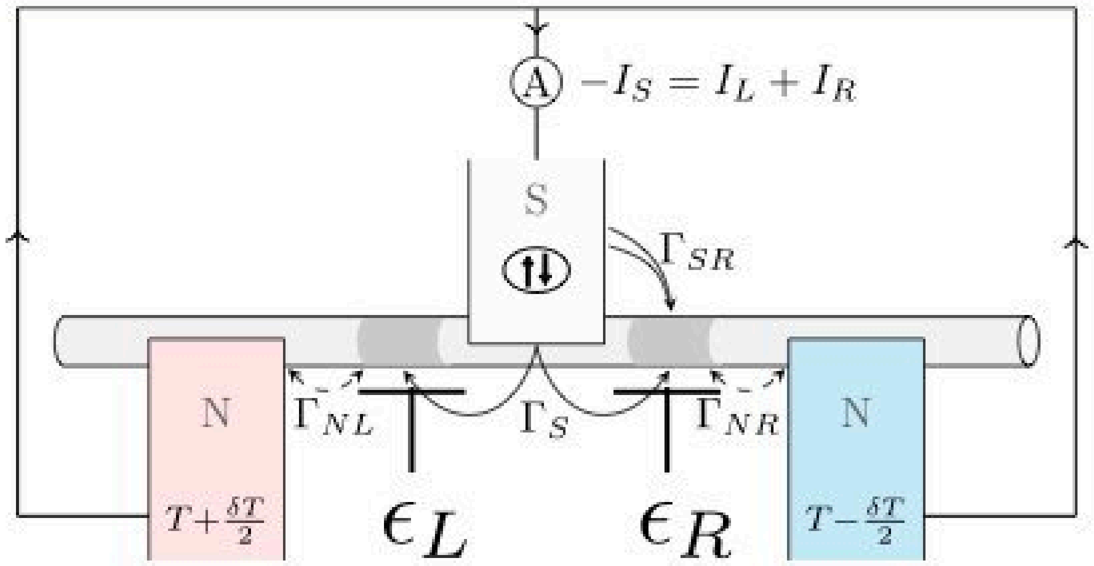
$$\dot{Q}_\alpha = -\frac{1}{\hbar} \sum_{a,a',s=\pm} (E_a - E_{a'}) w^{(\alpha,s)}_{a\leftarrow a'} P^{\mathrm{stat}}_{a'} - \frac{\mu_\alpha}{e_0} I_\alpha .$$



Lecture note 1 Splettstoeser

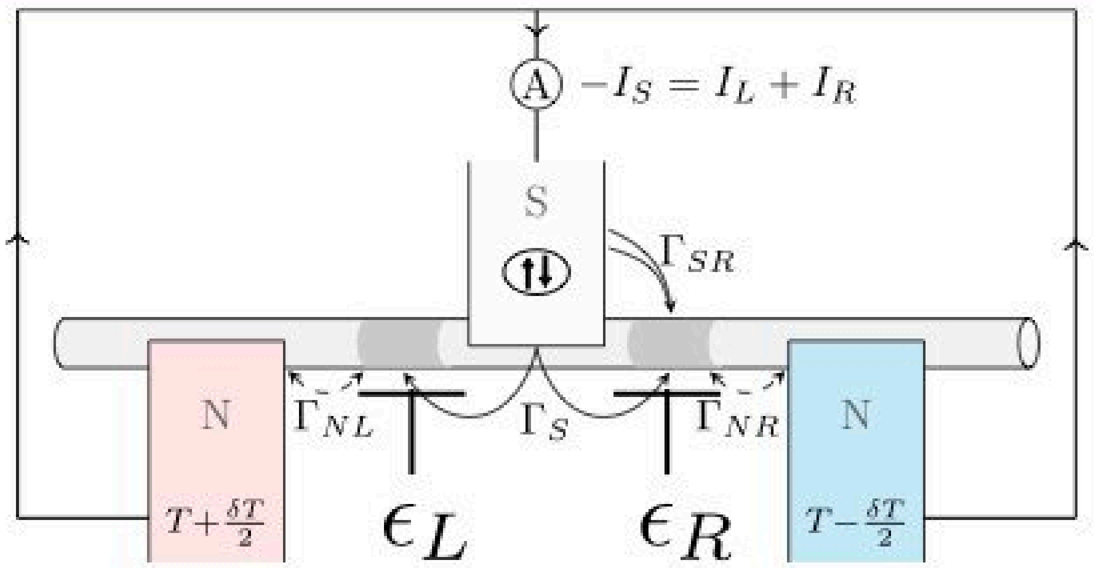
R. Hussein, L. Jaurigue, M. Governale, AB PRB'16

Non-local thermoelectricity



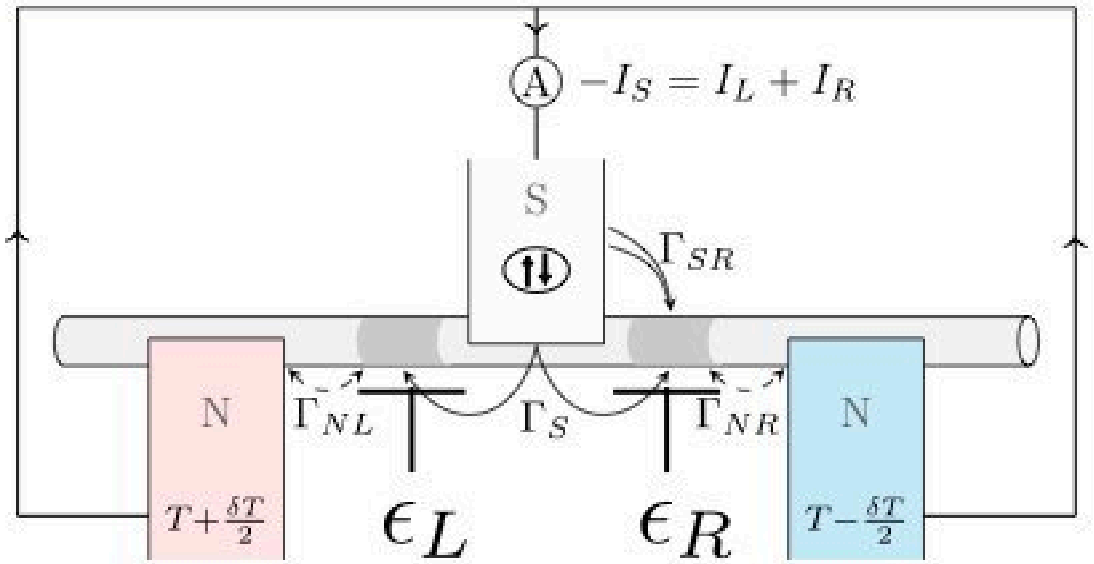
- Linear regime at CAR resonance

Non-local thermoelectricity



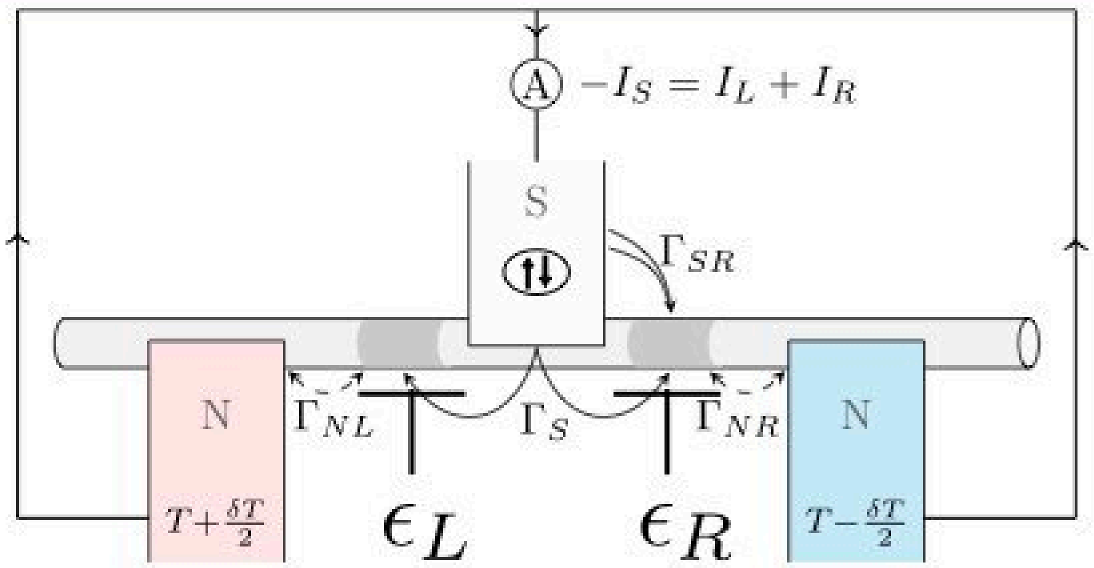
- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents

Non-local thermoelectricity



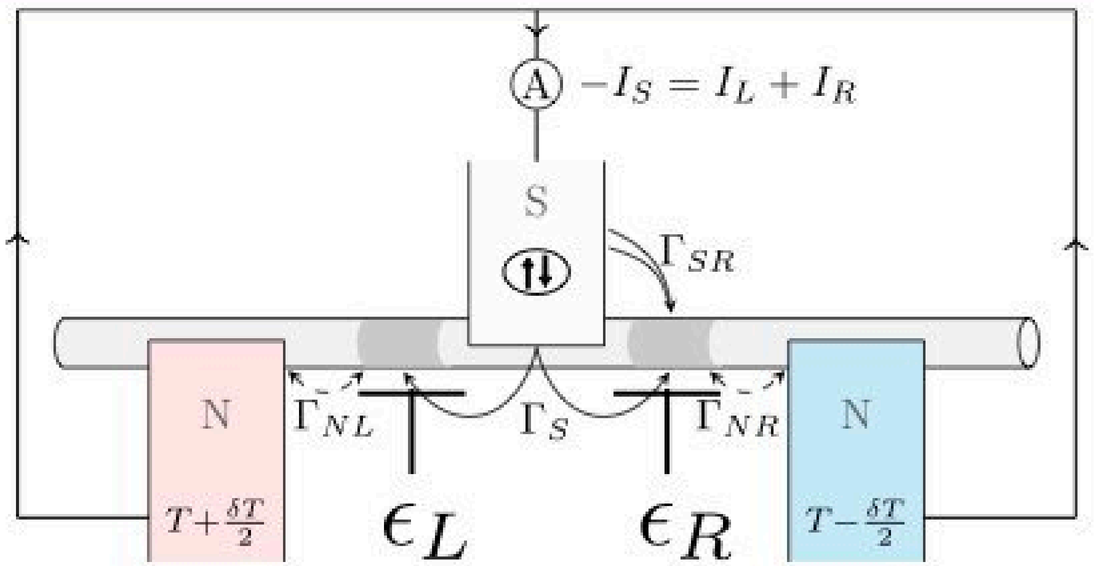
- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws

Non-local thermoelectricity



- Linear regime at CAR resonance
- 3 terminal for
  2 (heat/charge) currents
  corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws
- 4 unknown currents but:
  no heat current in superconducting lead for $\Delta \to \infty$
  no charge current between normal leads at CAR resonance

Non-local thermoelectricity



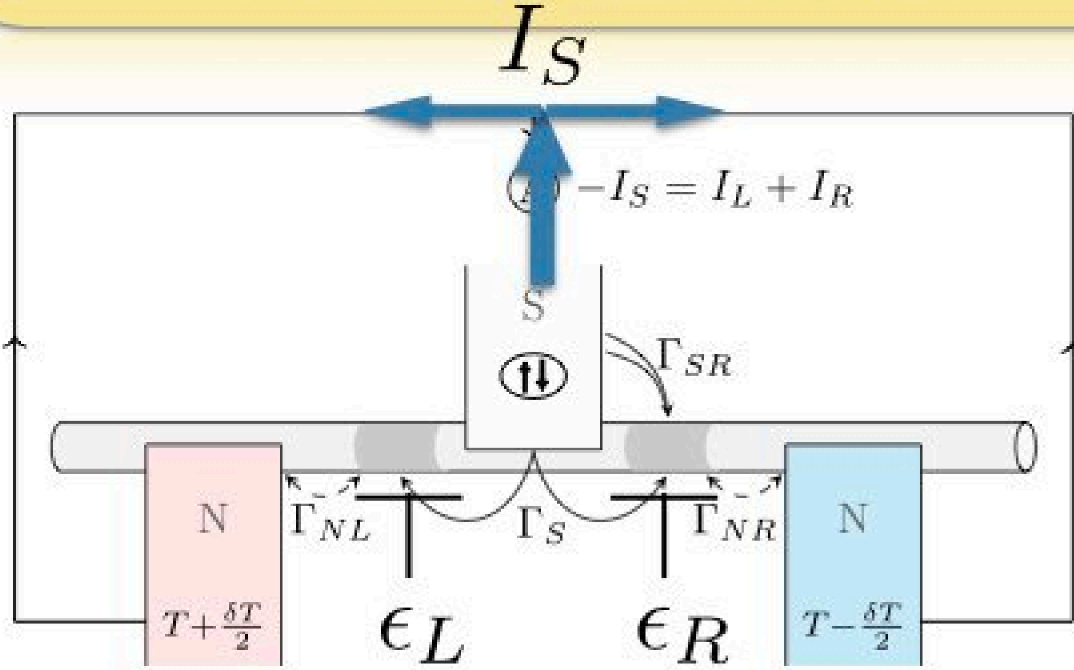
- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws

- 4 unknown currents but:
  no heat current in superconducting lead for $\Delta \to \infty$
  no charge current between normal leads at CAR resonance

- Only 2 unknown currents
  $\delta V = (\mu_L + \mu_R)/2e_0$
  $\delta T \ll T = (T_L + T_R)/2$

$$\delta I_S = L_{11}^S \delta V + L_{12}^S \delta T,$$
$$\delta \dot{Q}_R = L_{21}^R \delta V + L_{22}^R \delta T$$

Non-local thermoelectricity



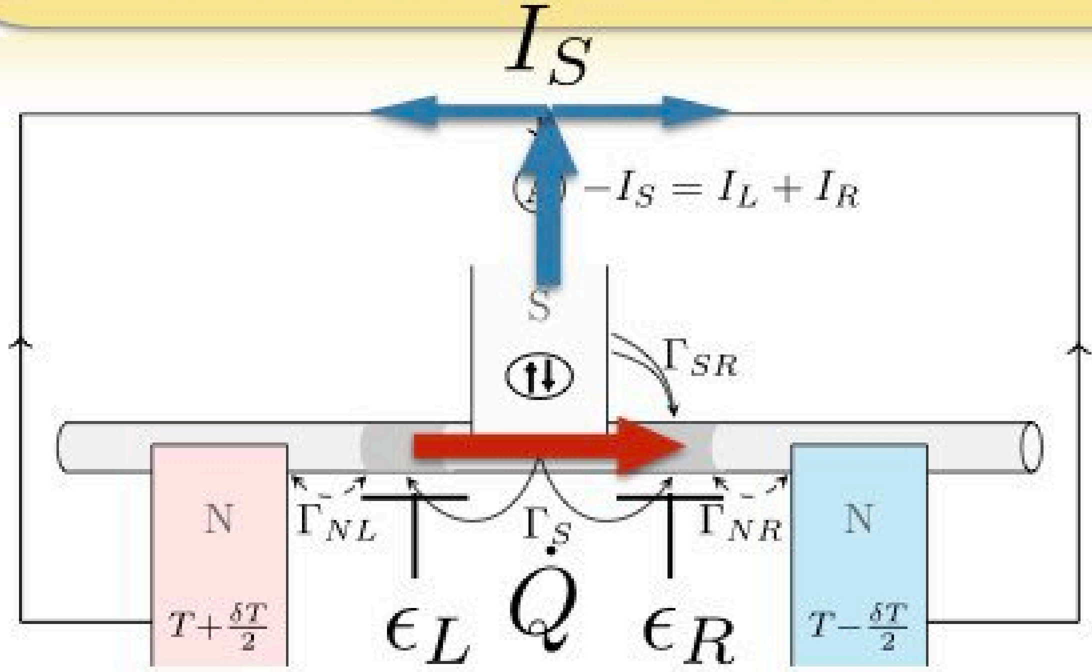
- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws

- 4 unknown currents but:
  no heat current in superconducting lead for $\Delta \to \infty$
  no charge current between normal leads at CAR resonance

- Only 2 unknown currents
  $\delta V = (\mu_L + \mu_R)/2e_0$
  $\delta T \ll T = (T_L + T_R)/2$

$$\delta I_S = L^S_{11}\delta V + L^S_{12}\delta T,$$
$$\delta \dot{Q}_R = L^R_{21}\delta V + L^R_{22}\delta T$$

Non-local thermoelectricity



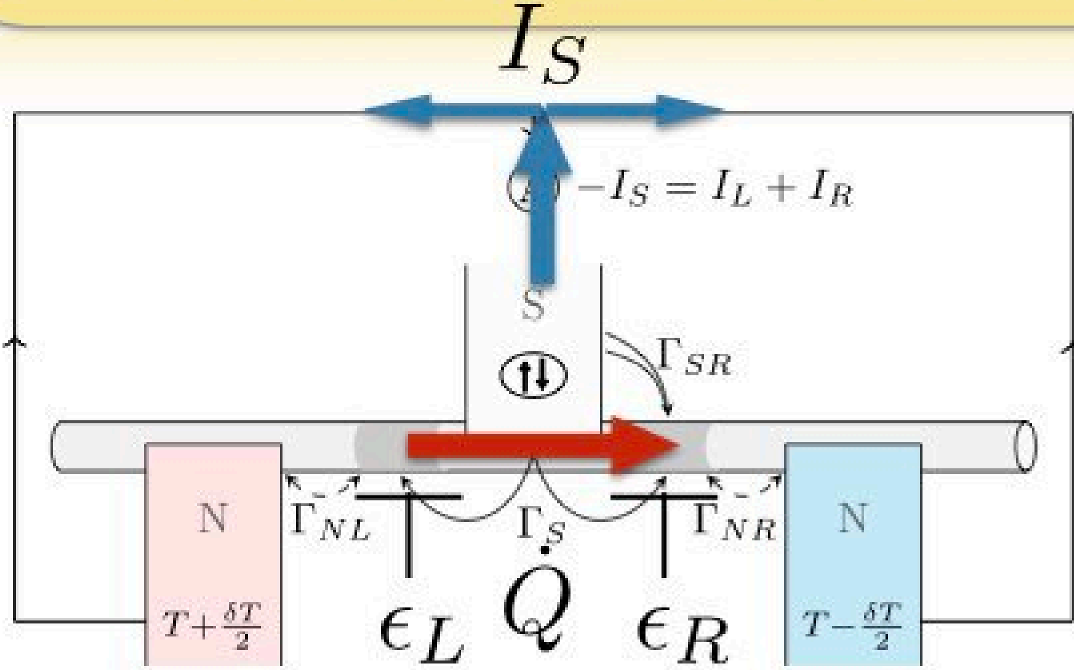
- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws

- 4 unknown currents but:
  no heat current in superconducting lead for $\Delta \to \infty$
  no charge current between normal leads at CAR resonance

- Only 2 unknown currents
  $\delta V = (\mu_L + \mu_R)/2e_0$
  $\delta T \ll T = (T_L + T_R)/2$

$$\delta I_S = L^S_{11}\delta V + L^S_{12}\delta T,$$
$$\delta \dot{Q}_R = L^R_{21}\delta V + L^R_{22}\delta T$$

Non-local thermoelectricity



- Linear regime at CAR resonance
- 3 terminal for 2 (heat/charge) currents corresponds to 3x2=6 currents
- 2 (energy/charge) cons. laws

- 4 unknown currents but:
  no heat current in superconducting lead for $\Delta \to \infty$
  no charge current between normal leads at CAR resonance

- Only 2 unknown currents
  $\delta V = (\mu_L + \mu_R)/2e_0$
  $\delta T \ll T = (T_L + T_R)/2$

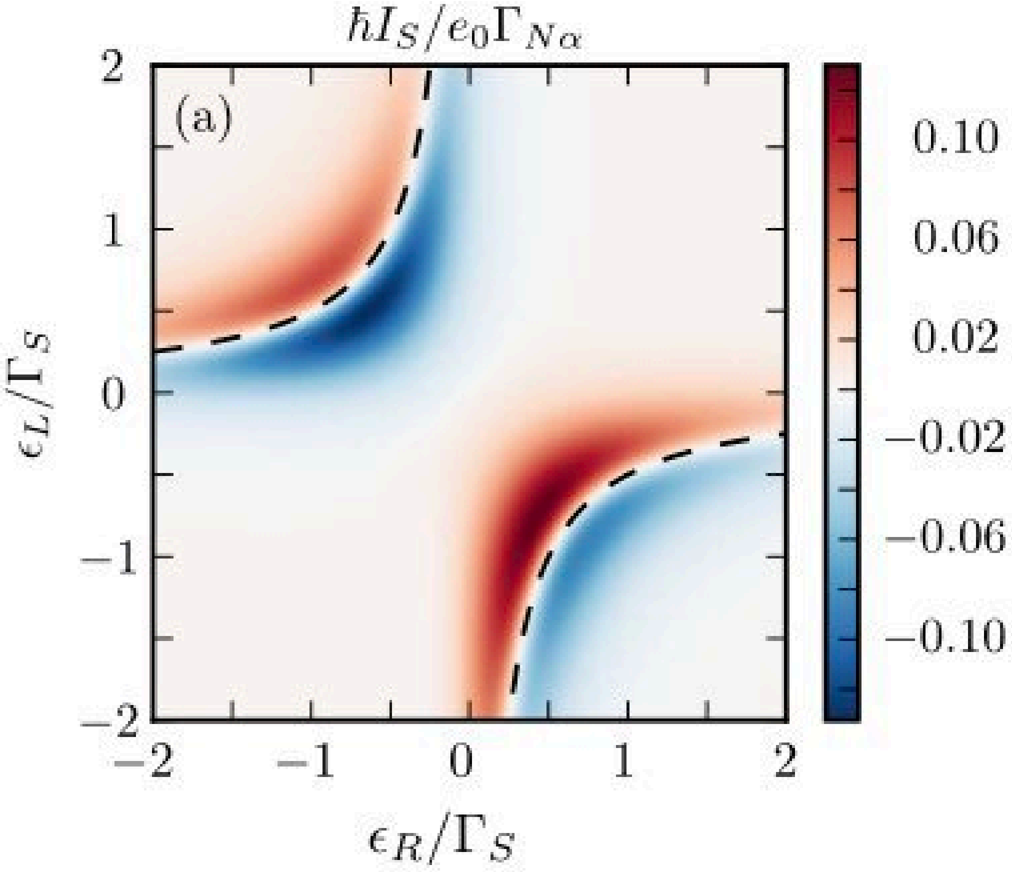
$$\delta I_S = L^S_{11}\delta V + L^S_{12}\delta T,$$
$$\delta \dot{Q}_R = L^R_{21}\delta V + L^R_{22}\delta T$$

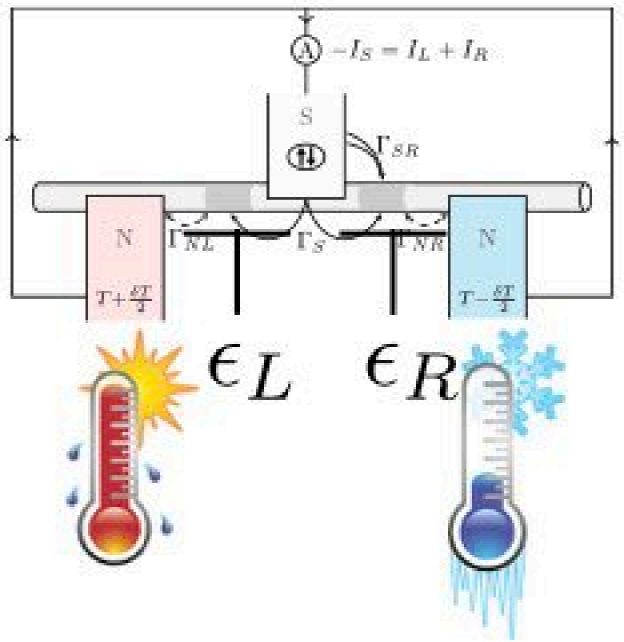
Non-local thermoelectric coeff. L^S_{12}

Non-local Seebeck coeff. $S_{NL} = -\delta V/\delta T\big|_{I_\alpha=0} = L^S_{12}/L^S_{11}$

Non-local thermoelectricity

Thermocurrent

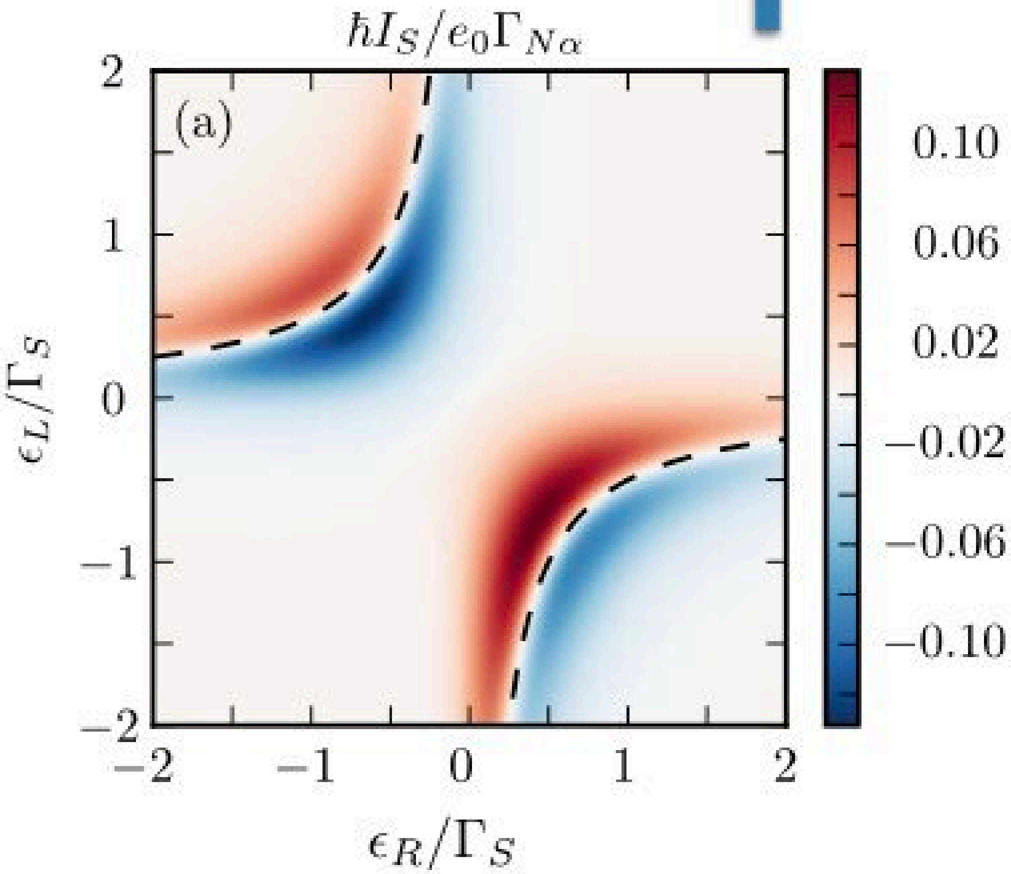


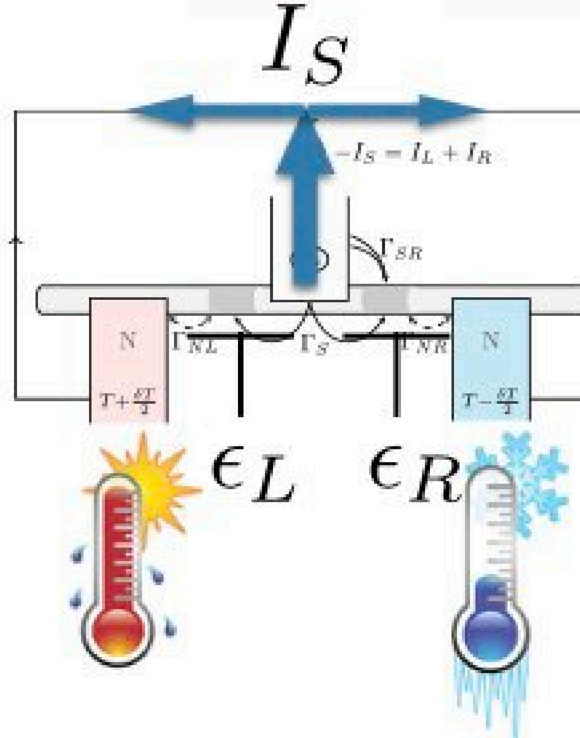


Non-local thermoelectricity

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

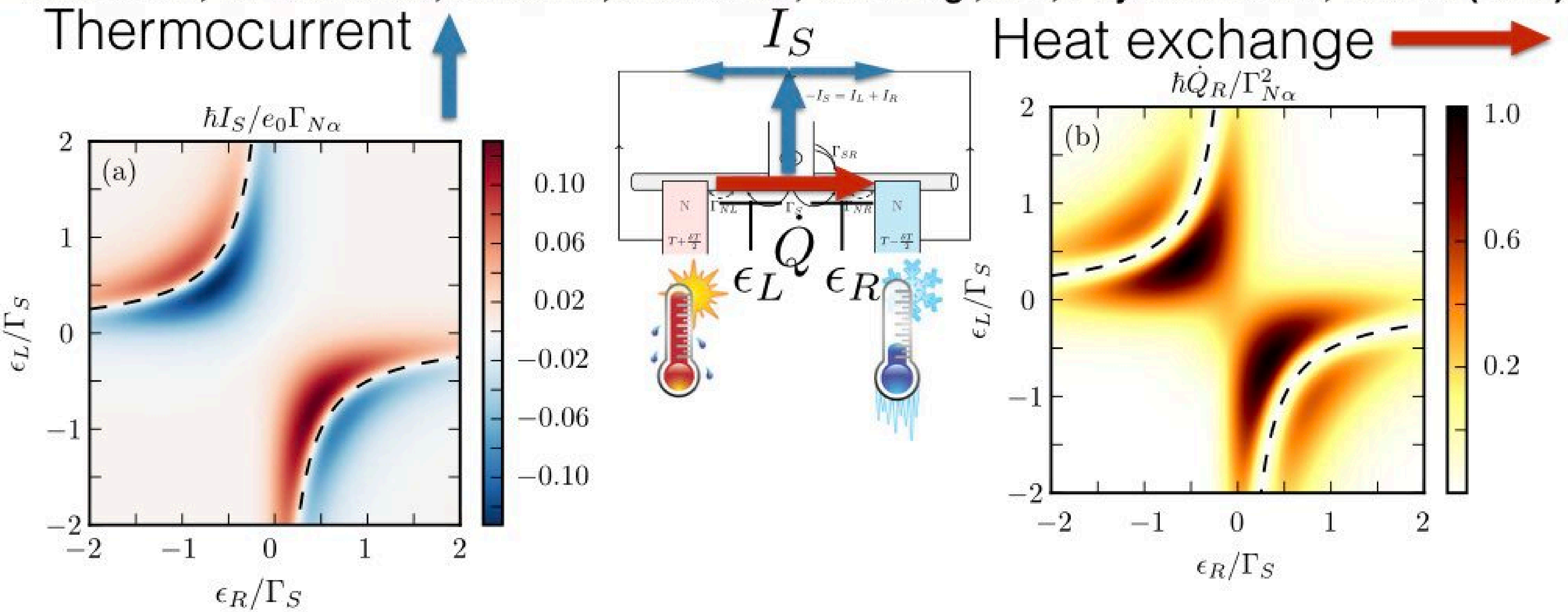
Thermocurrent





Non-local thermoelectricity

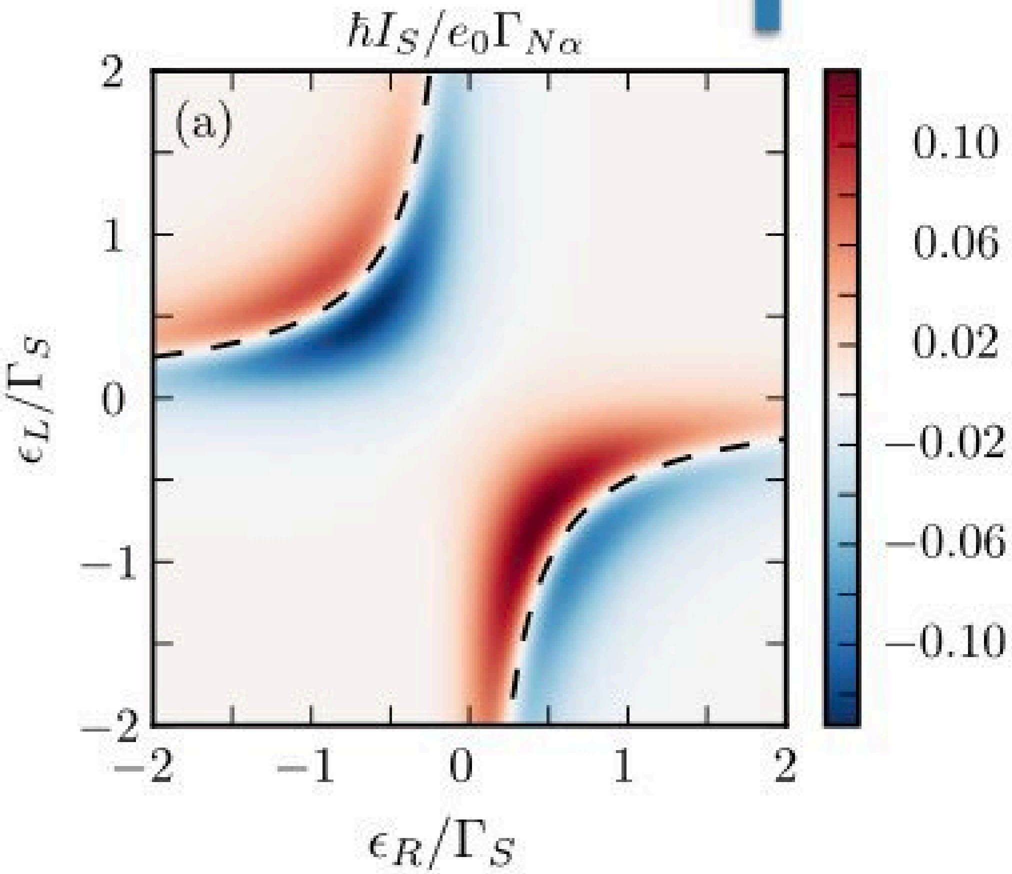
R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

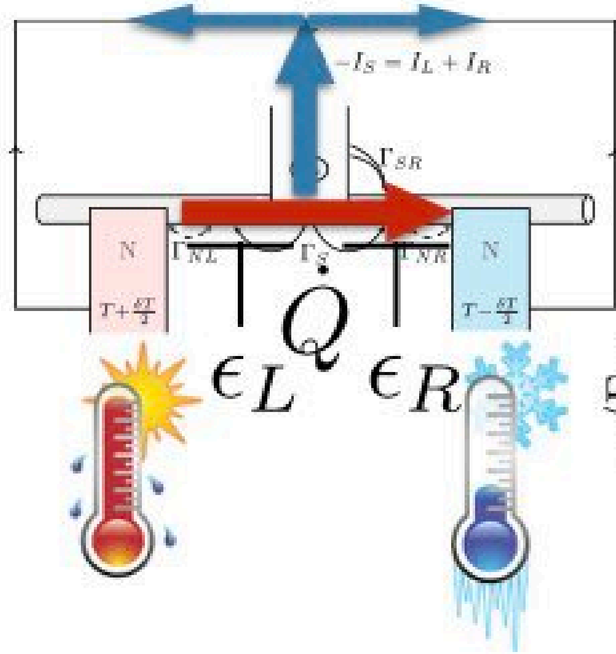


Non-local thermoelectricity

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

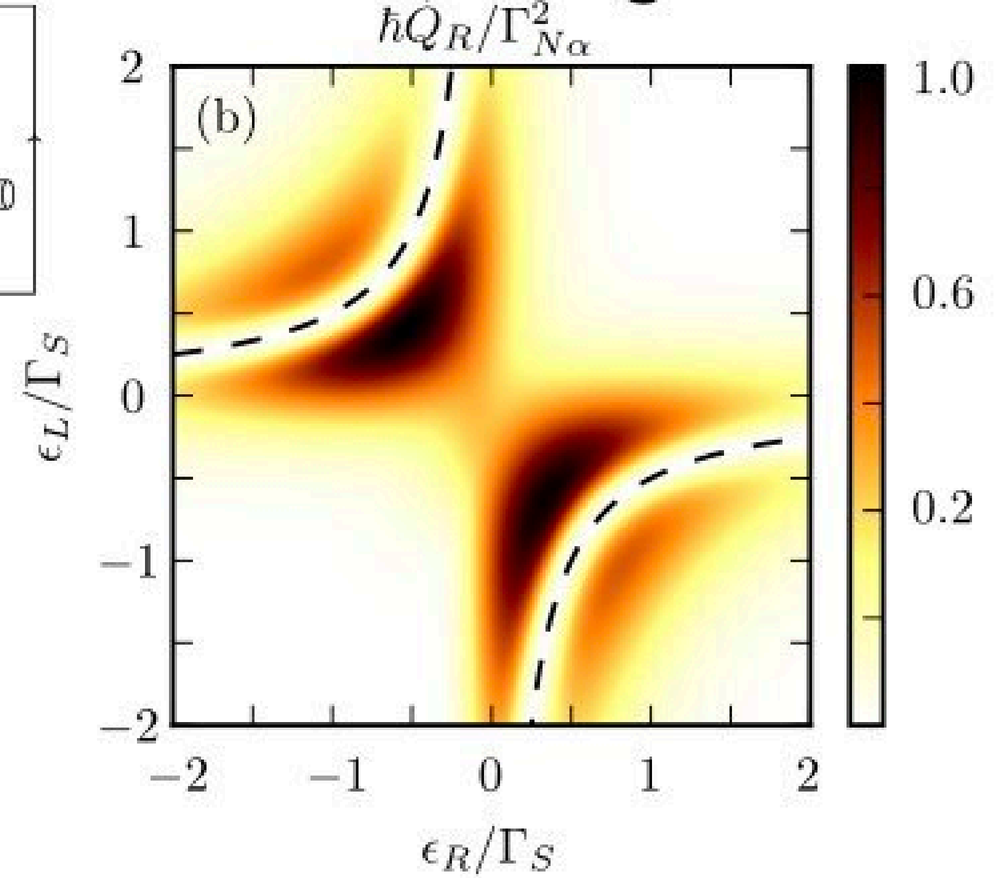
Thermocurrent





Addition Energies

Heat exchange

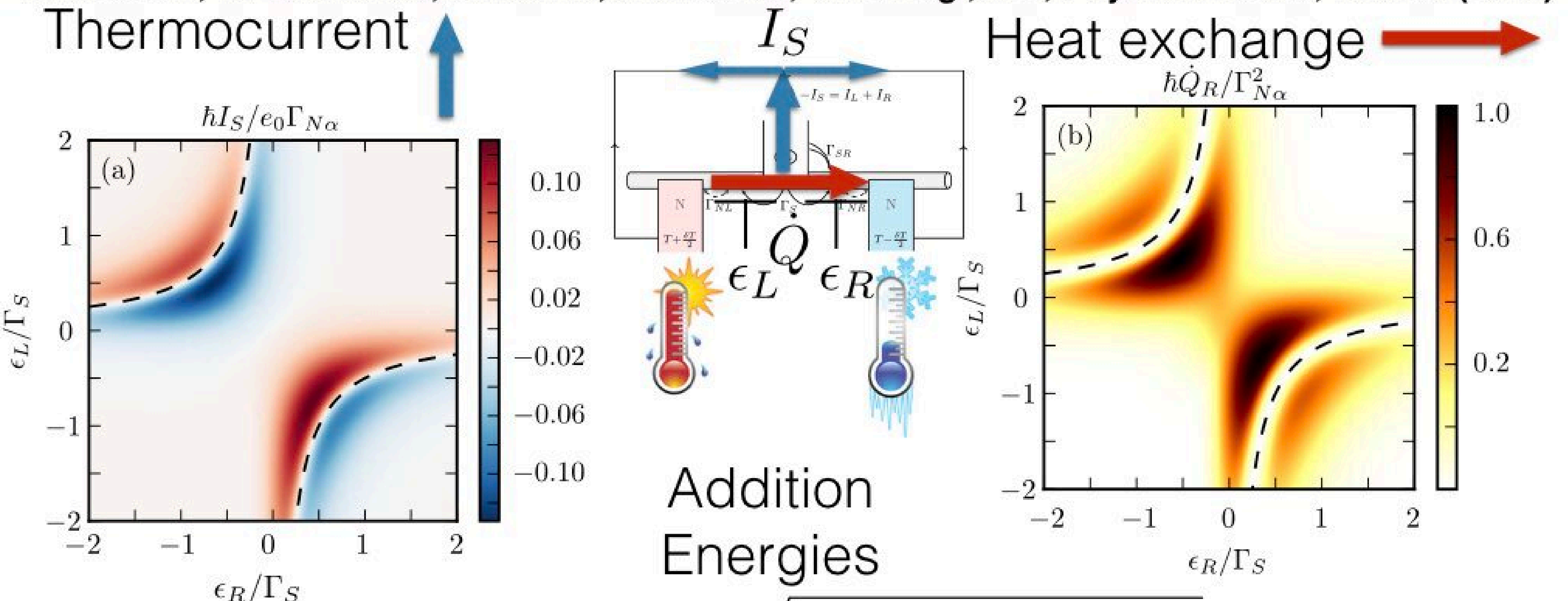


\- - - - - - $$\Delta E_\pm = \left(\epsilon_L - \epsilon_R \pm \sqrt{(\epsilon_L + \epsilon_R)^2 + 2\Gamma_S^2}\right)/2 = 0$$

- Thermoelectricity localize around the resonances

Non-local thermoelectricity

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)



$$- - - - - - \quad \Delta E_{\pm} = \left(\epsilon_L - \epsilon_R \pm \sqrt{(\epsilon_L + \epsilon_R)^2 + 2\Gamma_S^2}\right)/2 = 0$$

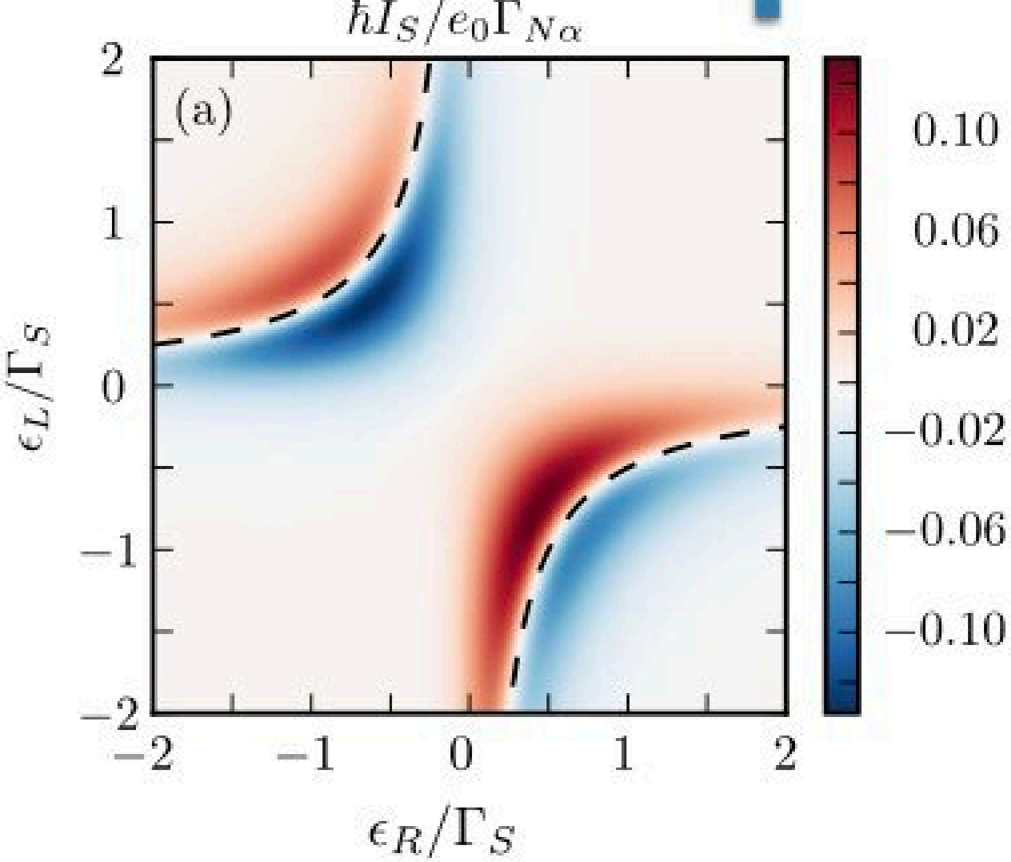
- Thermoelectricity localize around the resonances
- Sign thermoelectricity follows the qp. or qh. nature of res.

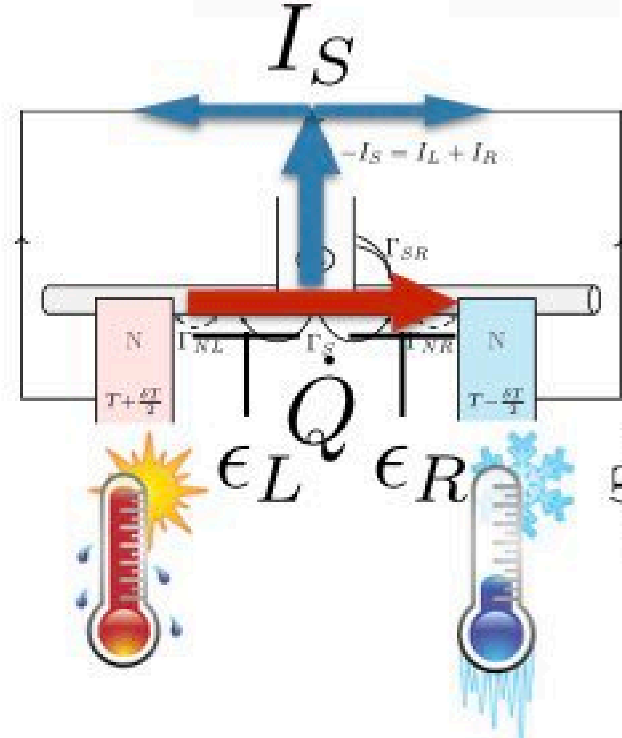
Non-local thermoelectricity

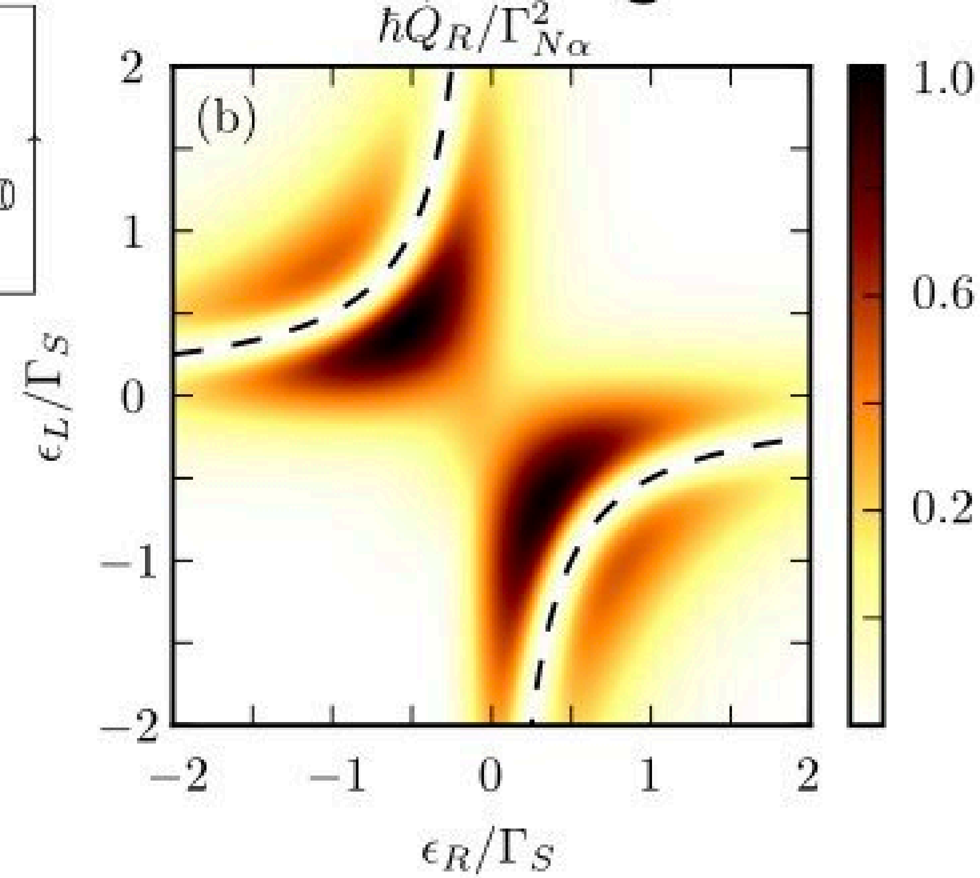
R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

Thermocurrent

Heat exchange





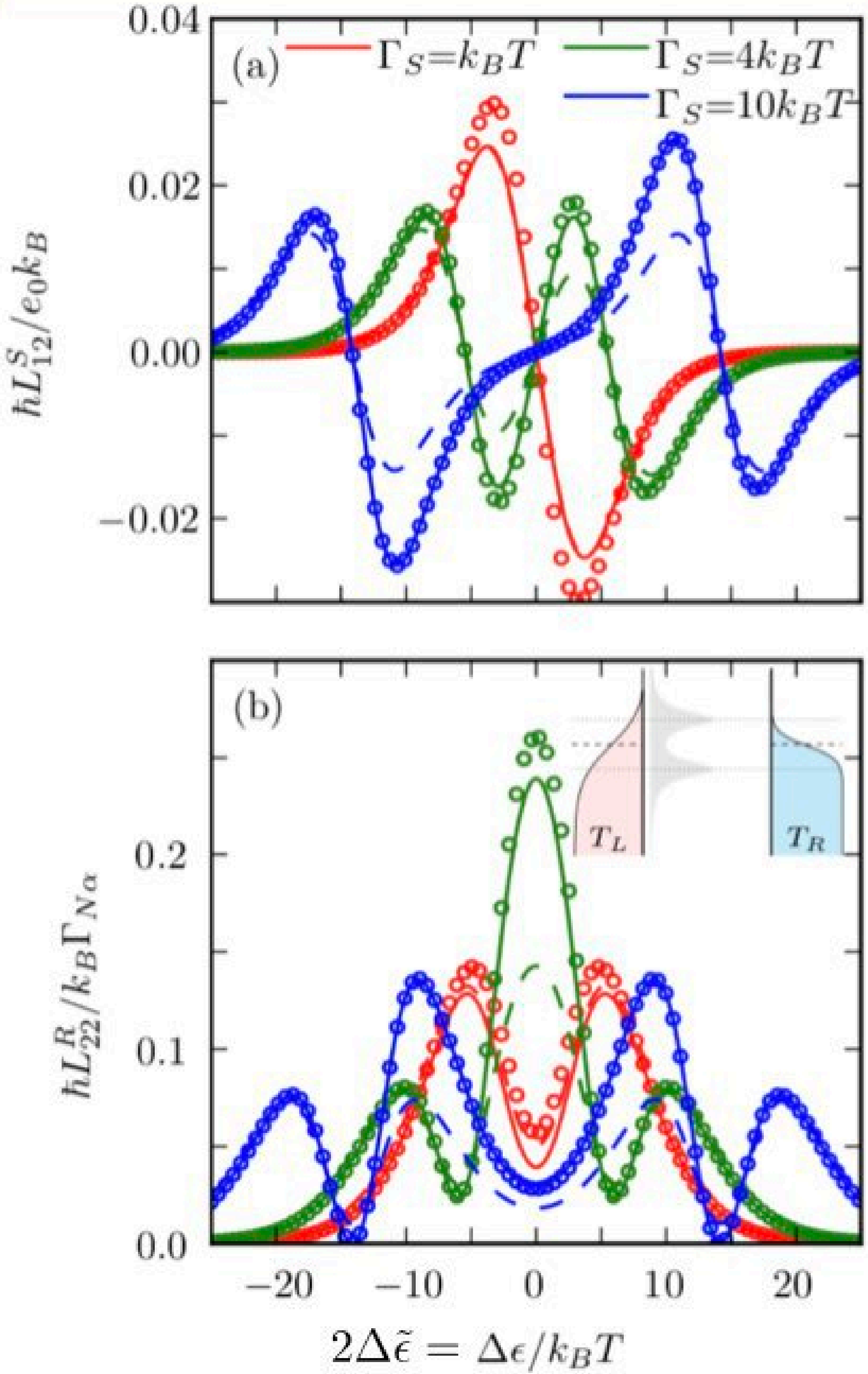


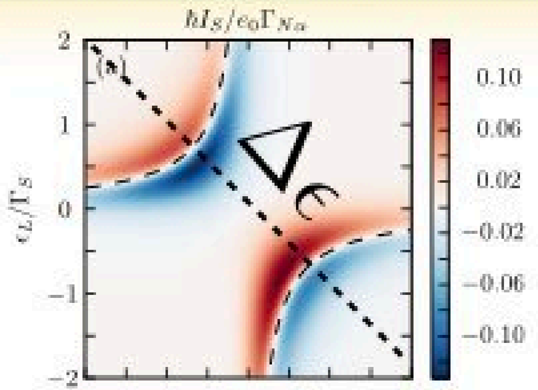
Addition Energies

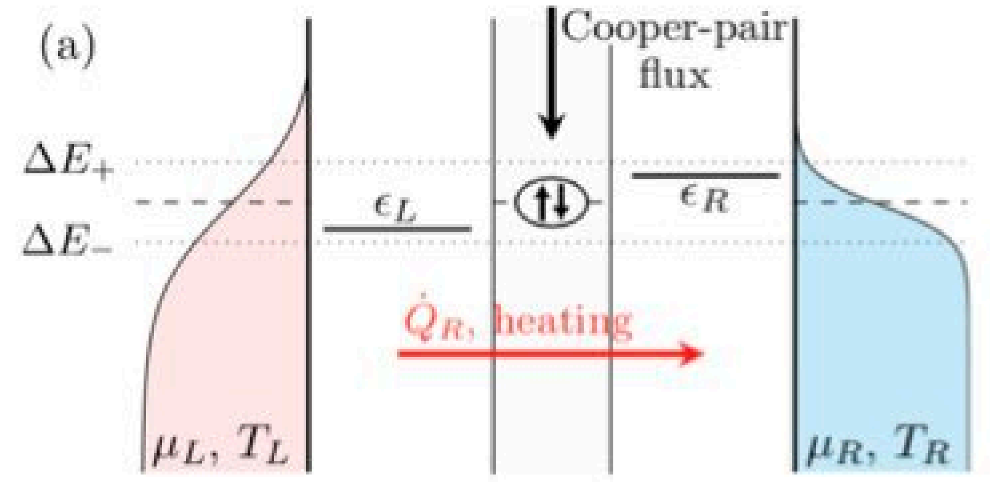
\- - - - - - $$\Delta E_\pm = \left(\epsilon_L - \epsilon_R \pm \sqrt{(\epsilon_L+\epsilon_R)^2 + 2\Gamma_S^2}\right)/2 = 0$$

- Thermoelectricity localize around the resonances
- Sign thermoelectricity follows the qp. or qh. nature of res.
- Heat localise also around res. but may also be present “in the gap” due to addictivity of the qp./qh. contributions

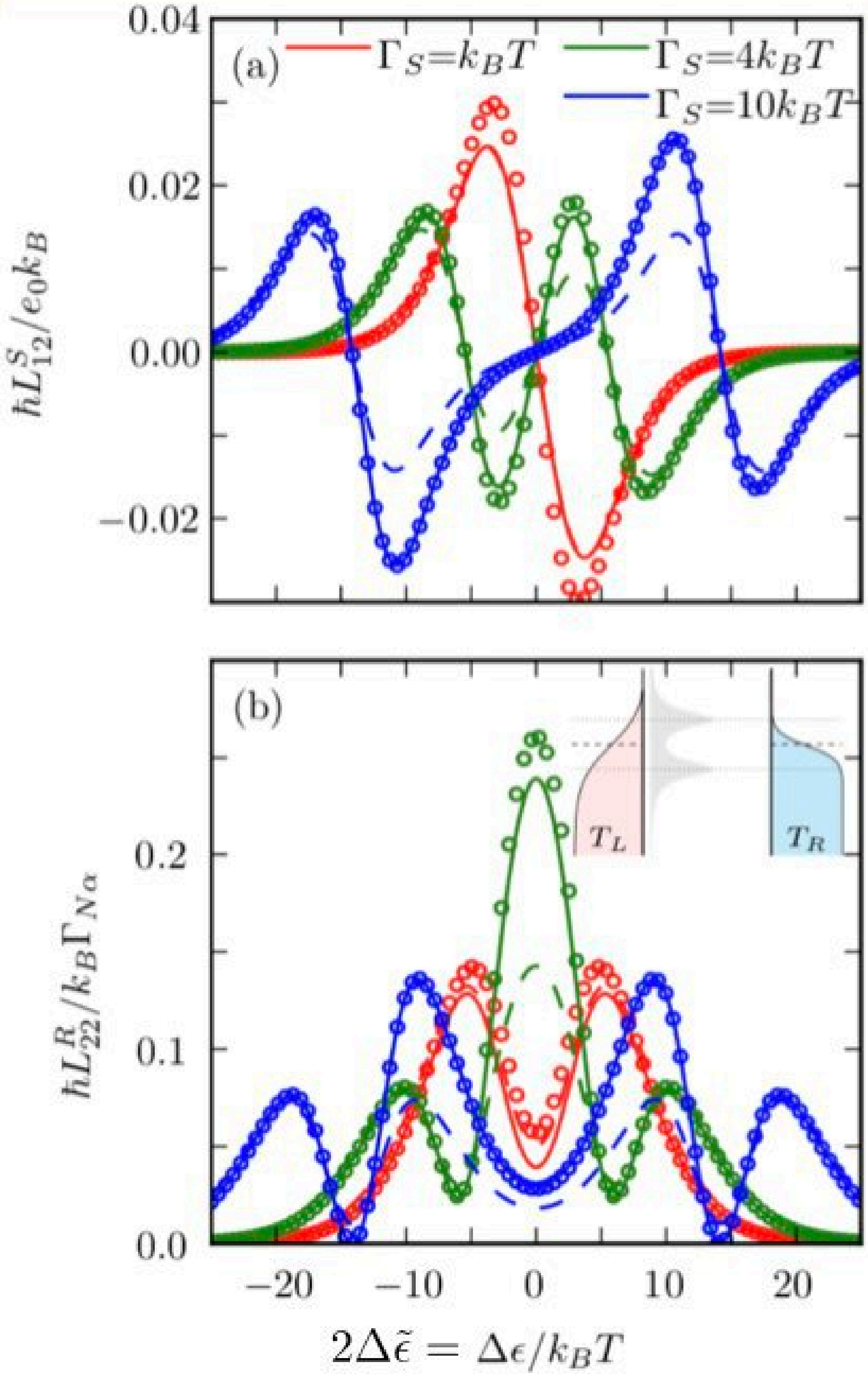
Linear coefficients

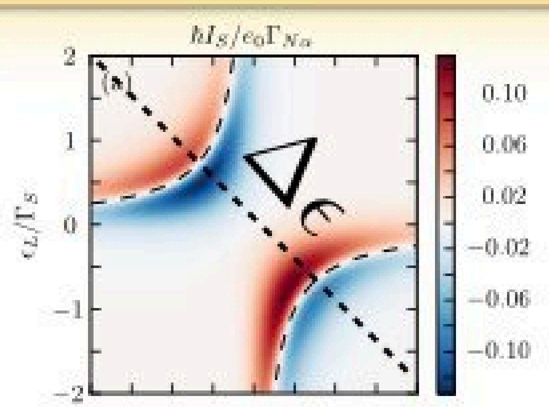


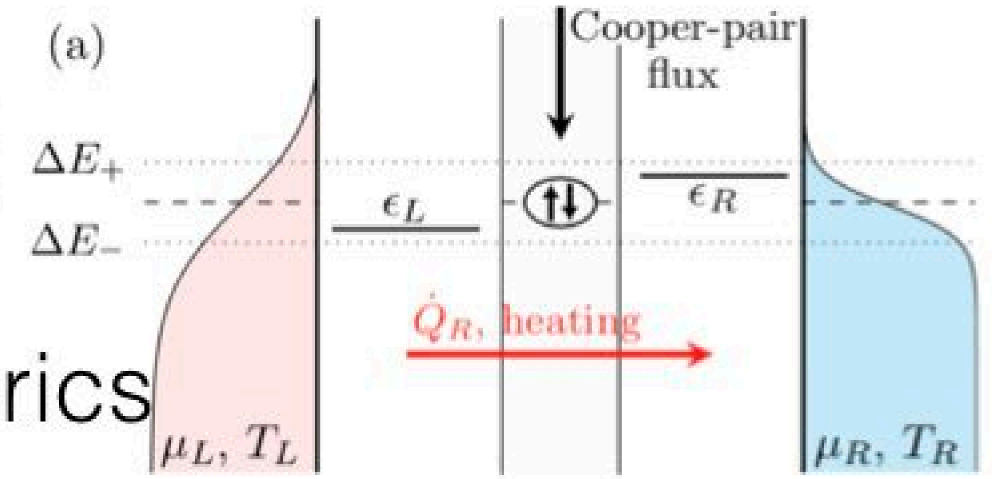




Linear coefficients

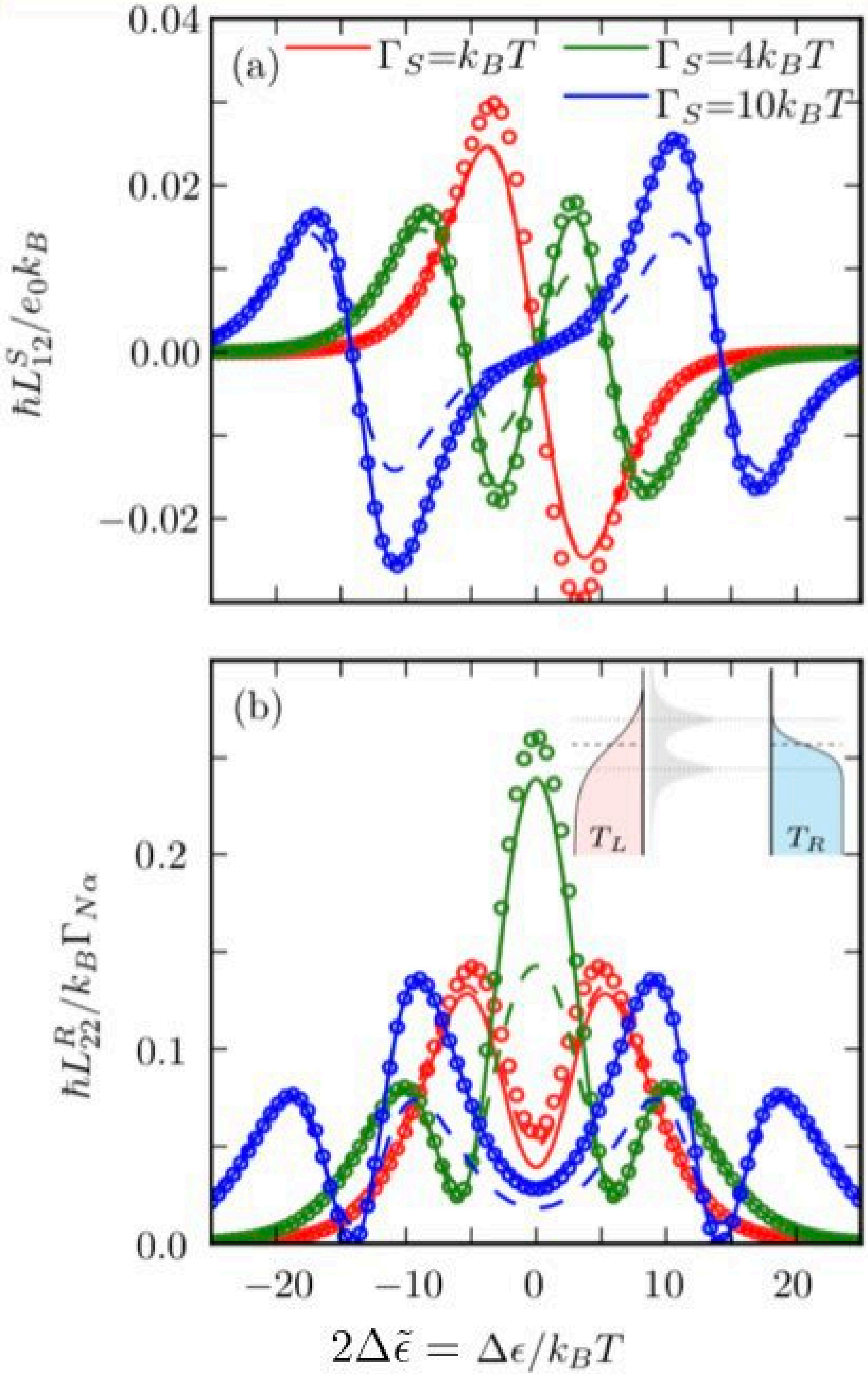


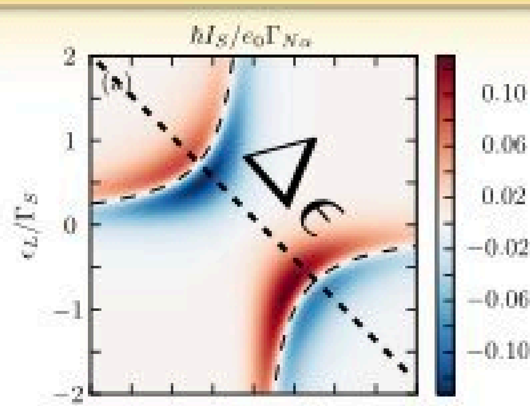


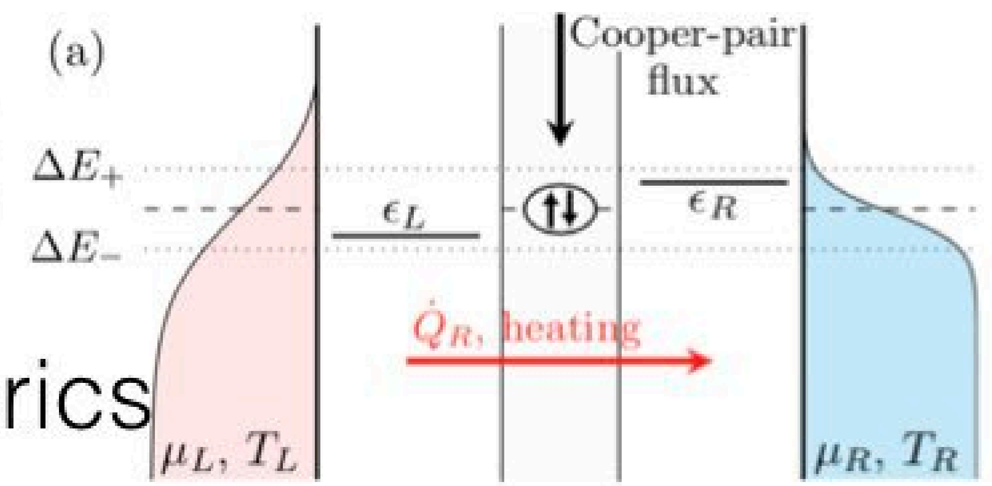


—— •Numerics

Linear coefficients

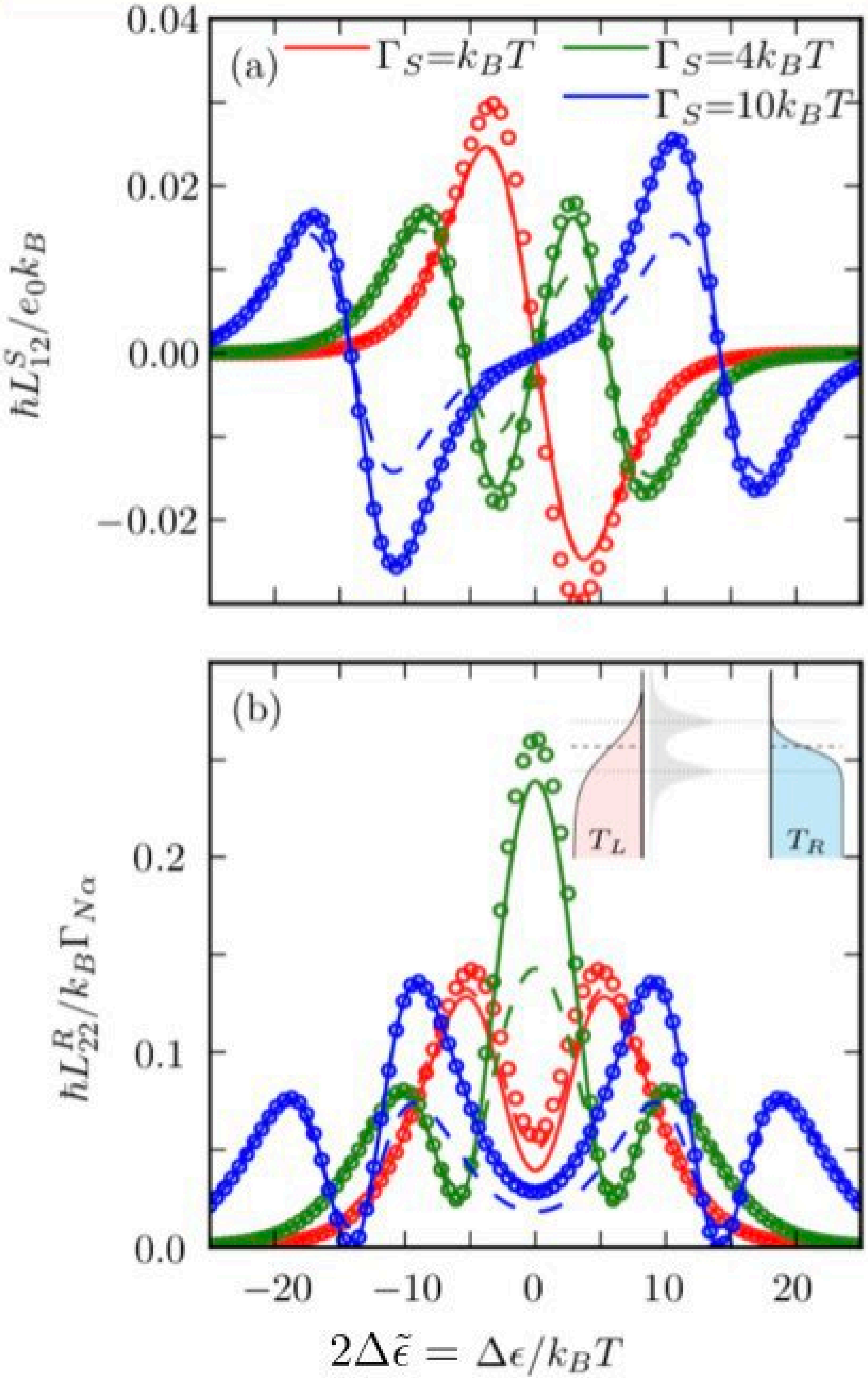


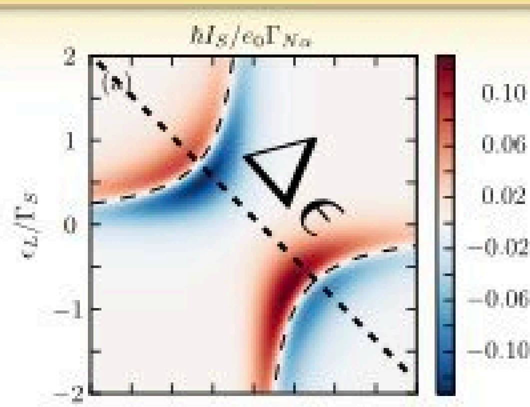


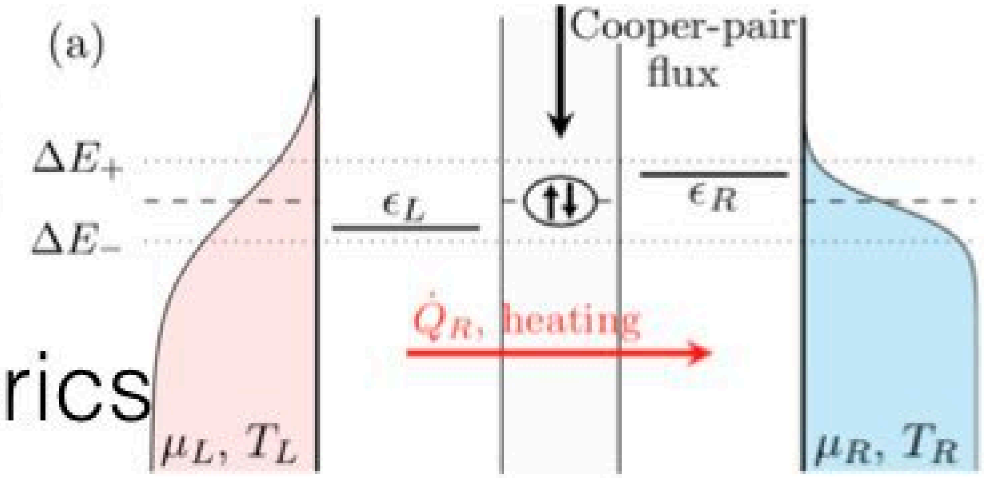


- Numerics
- Two Lorentzian resonances

Linear coefficients







- Numerics
- Two Lorentzian resonances
- Analytics(in reduced space)

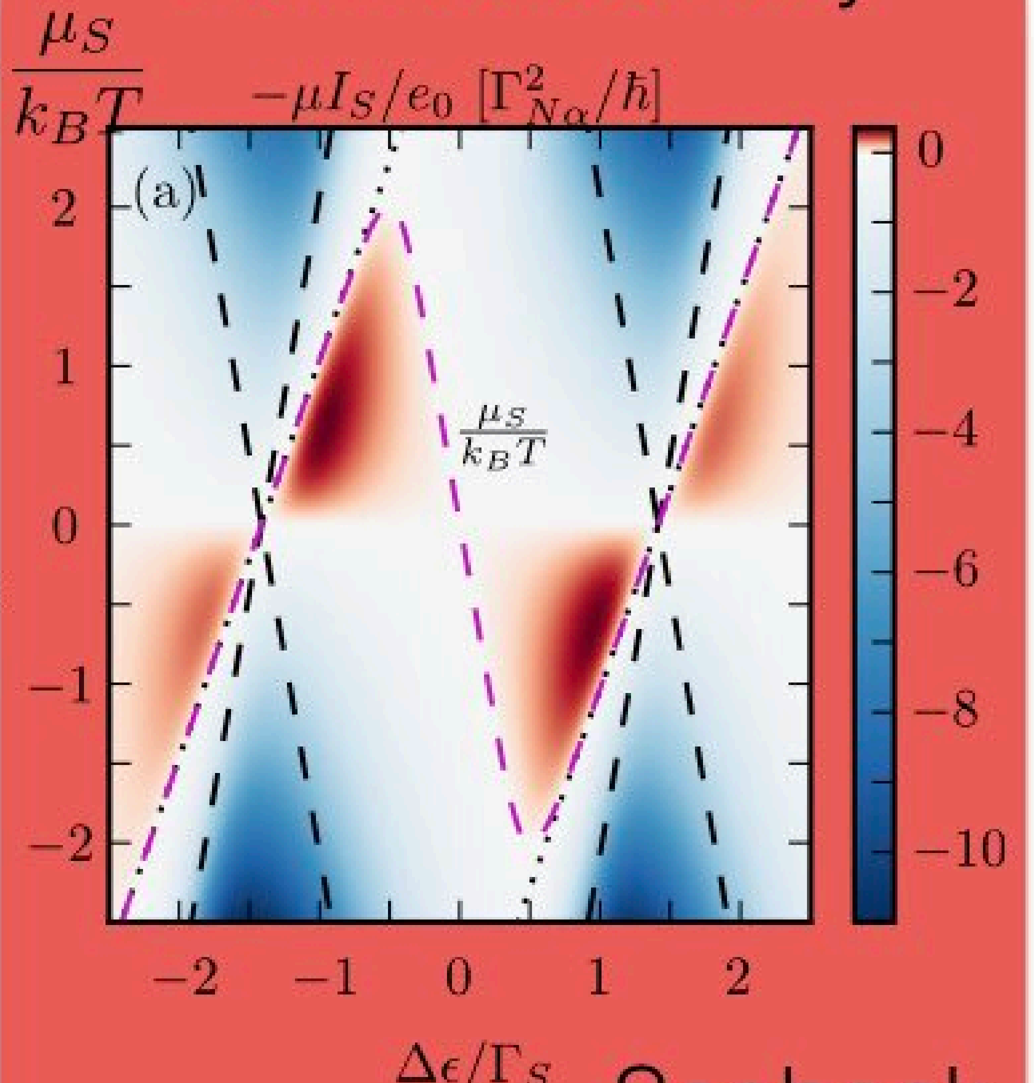
$$\frac{\hbar L^S_{12}}{e_0 k_B} = -2\frac{\Gamma_N}{k_B T} K(\Delta\tilde{\epsilon}, \Delta\tilde{\epsilon}, -\sqrt{2}\tilde{\Gamma}_S$$

$K(x, y, z)$ Universal function also for L^R_{22}

$$\tilde{\Gamma}_S = \frac{\Gamma_S}{2k_B T}$$

Non-linear behaviour

Thermoelectricity



μ_S Seebeck Voltage

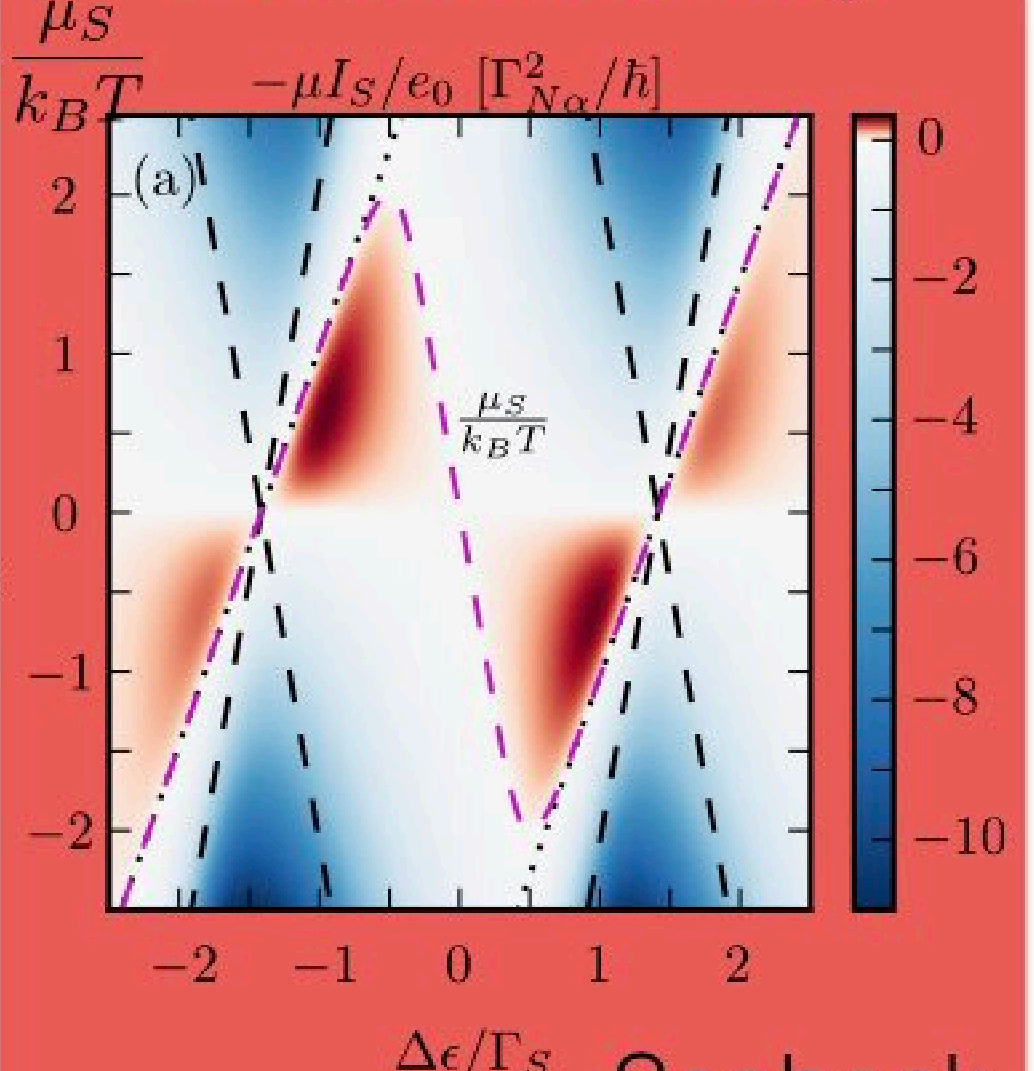
$$P = -\mu I_S > 0$$

Thermopower

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

Non-linear behaviour

Thermoelectricity

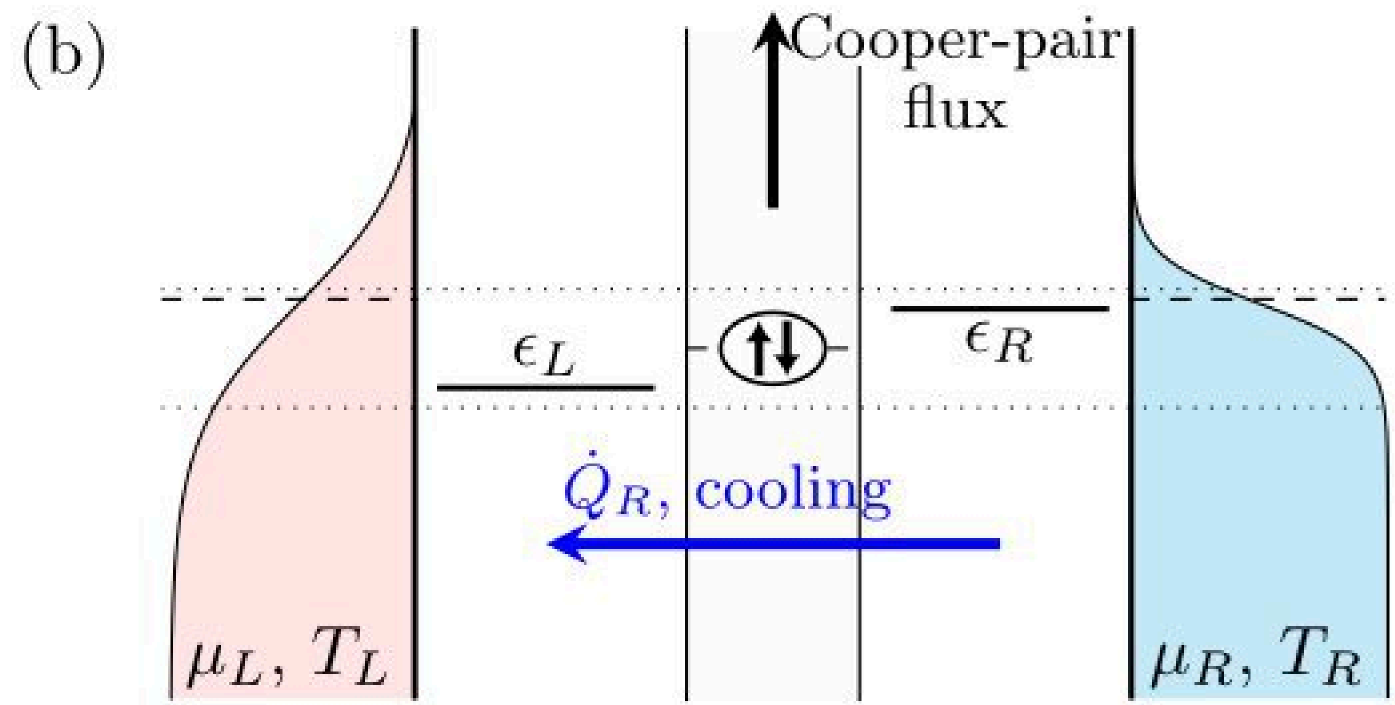


μ_S Seebeck Voltage

$P = -\mu I_S > 0$

Thermopower

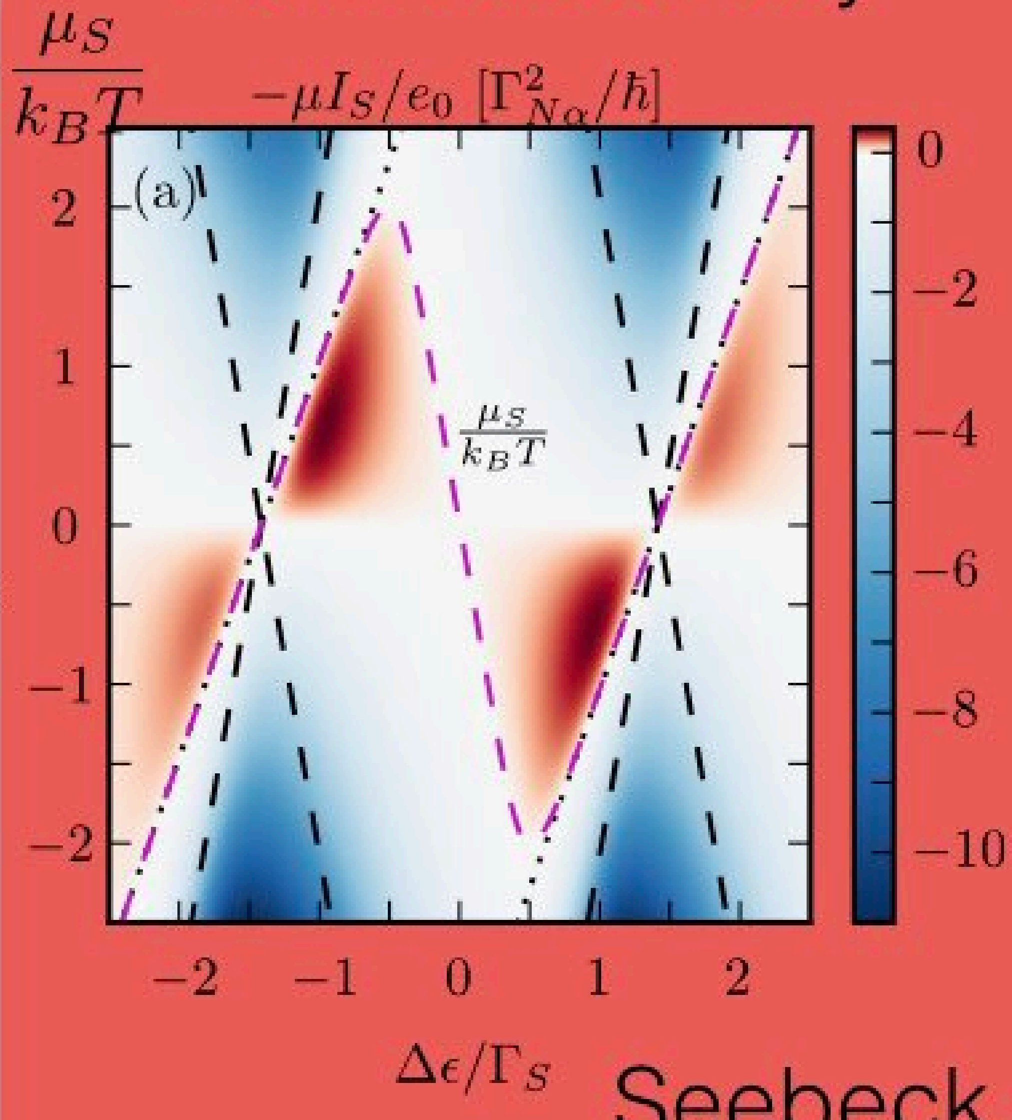
Peltier Cooling mediated by Cooper-pair-splitting



R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

Non-linear behaviour

Thermoelectricity



μ_S Seebeck Voltage

$P = -\mu I_S > 0$

Thermopower

η_C Carnot eff.

$$P > 0$$

$$\eta = \frac{1}{\eta_C} \frac{P}{|\dot{Q}_L|}$$

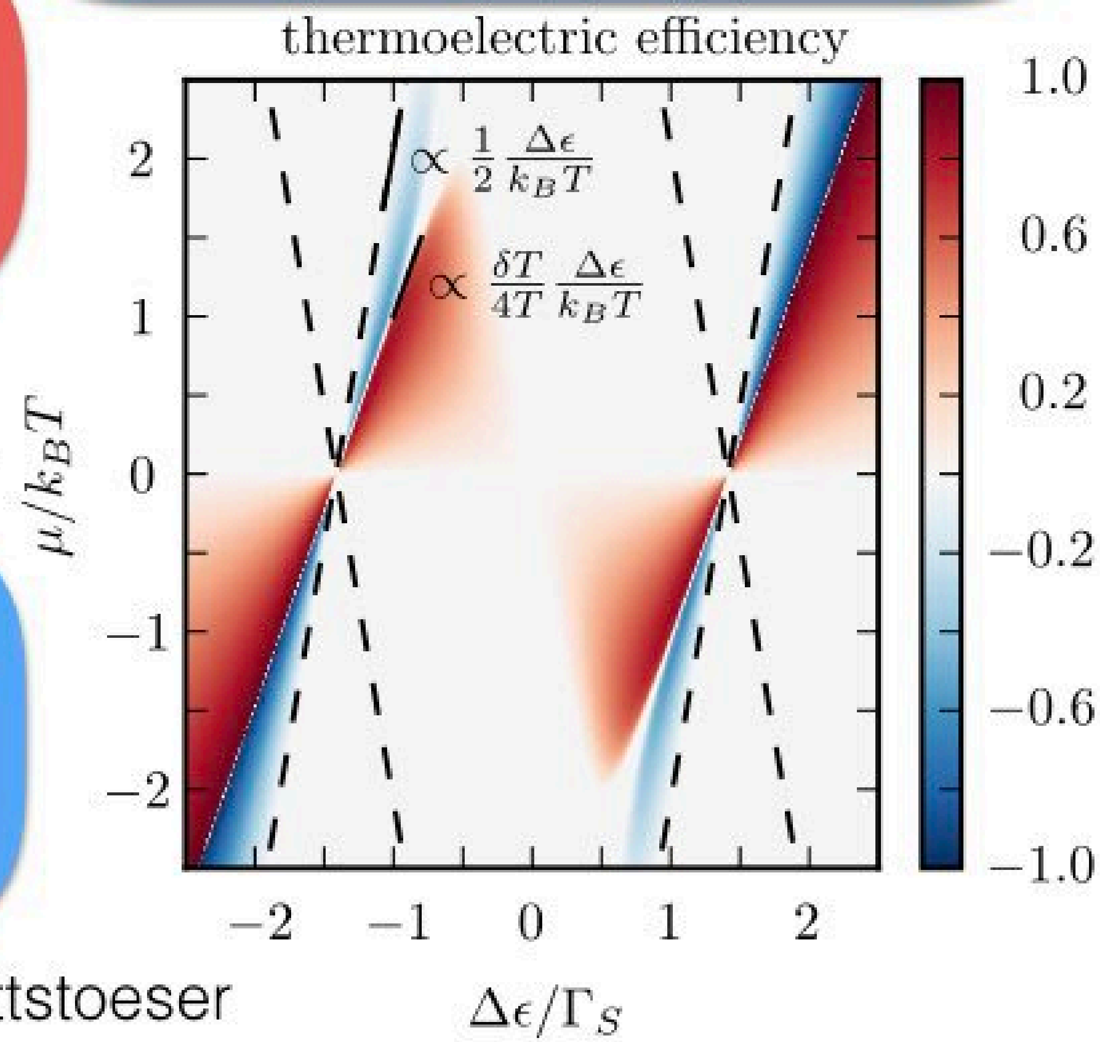
η_F Cooling eff.

$$P < 0$$

$$\eta = \frac{1}{\eta_F} \left| \frac{Q_R}{P} \right|$$

Lecture note 2 Splettstoeser

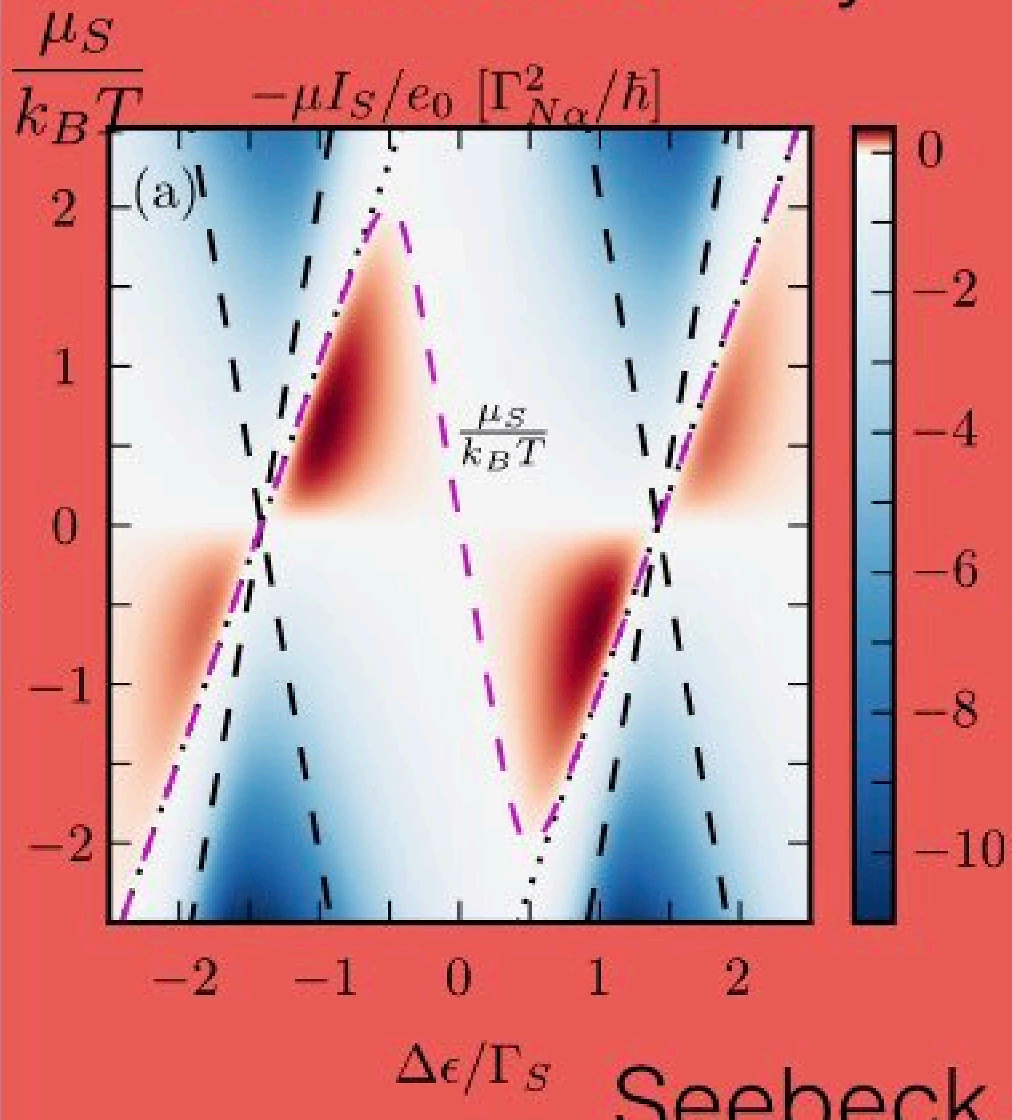
Peltier Cooling mediated by Cooper-pair-splitting



R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

Non-linear behaviour

Thermoelectricity



μ_S Seebeck Voltage

$P = -\mu I_S > 0$

Thermopower

η_C Carnot eff.

$P > 0$

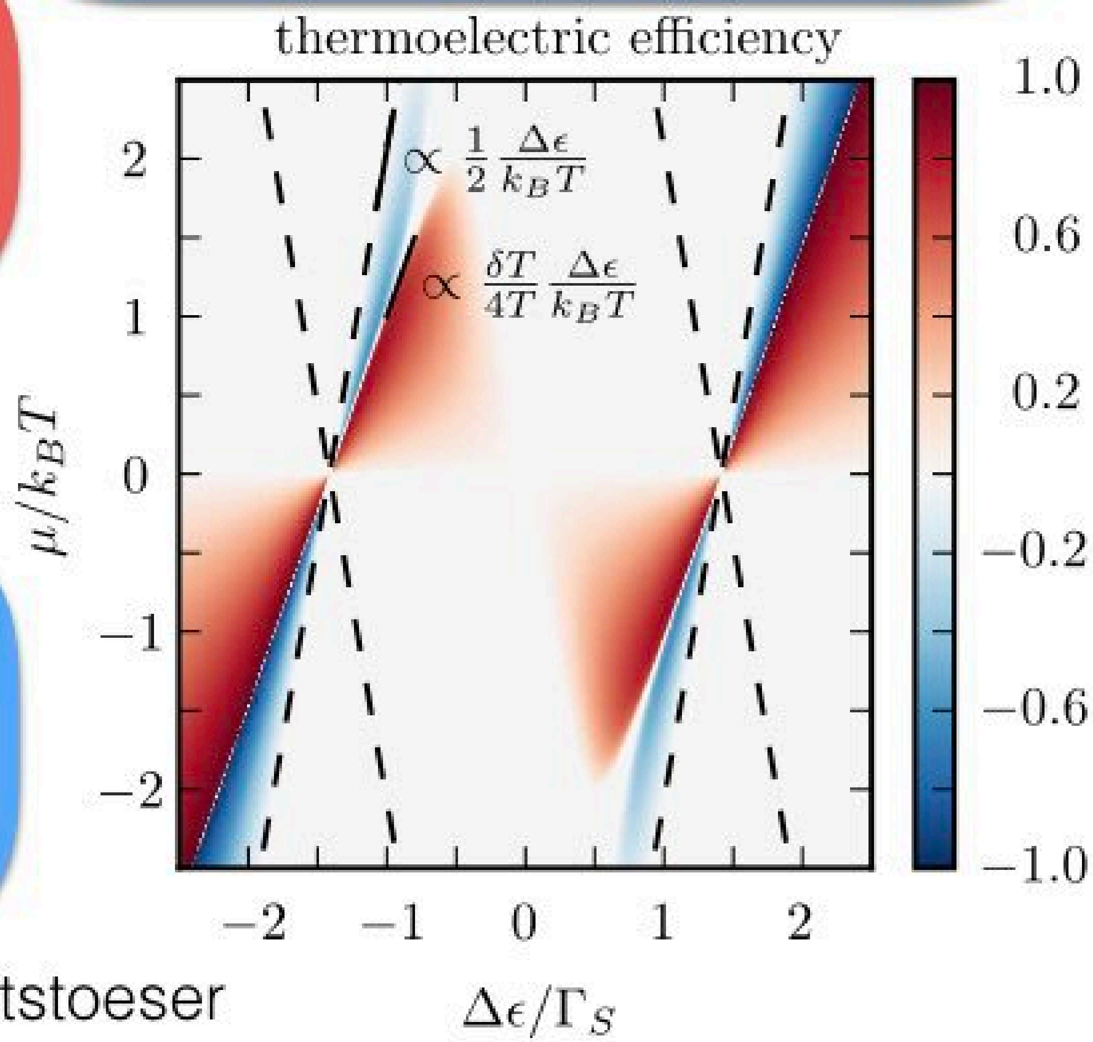
$$\eta = \frac{1}{\eta_C} \frac{P}{|\dot{Q}_L|}$$

η_F Cooling eff.

$P < 0$

$$\eta = \frac{1}{\eta_F} \left| \frac{Q_R}{P} \right|$$

Peltier Cooling mediated by Cooper-pair-splitting



Lecture note 2 Splettstoeser

- Non-local Peltier cooling
- Cooper-pairs mediate heat-exchange and also cooling!

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

Biblio and conclusion

R. Hussein, M. Governale, S. Kohler, F. Giazotto, W. Belzig , AB , Phys. Rev. B 99, 075429 (2019)

- Similar results using different approaches:

R. Sanchez, P. Burset and and A. L. Yeyati, PRB'18
N. S. Kirsanov, Z. B. Tan, D. S. Golubev, P. J. Hakonen and G.B. Lesovik PRB'19

- Interesting results in the same contest:

S. S. Pershoguba, L. I. Glazman, PRB '19
S. S. Pershoguba, L. I. Glazman, PRL '19
F. Hajiloo, F. Hassler, and J Splettstoesser, PRB'19
M. Mantovani, W. Belzig, G. Rastelli, R. Hussein,1907.04308

- Non-local coupling in a CPS generate non-local thermoelectricity
- At CAR resonance thermoelectrical effects are *only* non-local
- Heat exchange mediated by non-local Andreev processes
- Intriguing thermoelectrical effects in hybrid superconductors need further research (non-locality of Cooper pair,etc.)

Other analytical results

- Linear regime

$$\frac{\hbar L_{22}^R}{k_B\Gamma_N}=K\left(\Delta\tilde{\epsilon}^2+\frac{5\tilde{\Gamma}_S^2}{2},\Delta\tilde{\epsilon}^2+2\tilde{\Gamma}_S^2,-2\sqrt{2}\tilde{\Gamma}_S\Delta\tilde{\epsilon}\right),$$

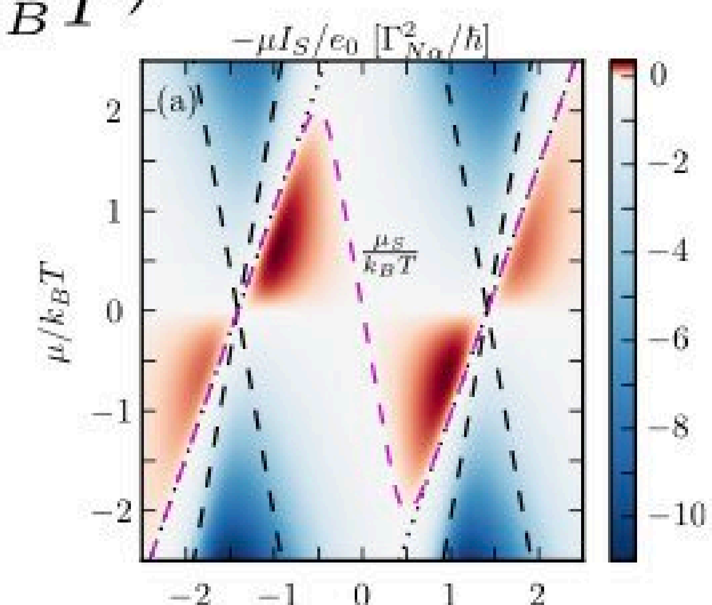
$$K(x,y,z)=\frac{x+y\cosh\tilde{\epsilon}\cosh\sqrt{2}\tilde{\Gamma}_S+z\sinh\tilde{\epsilon}\sinh\sqrt{2}\tilde{\Gamma}_S}{3(\cosh\tilde{\epsilon}+\cosh\sqrt{2}\tilde{\Gamma}_S)(2\cosh\tilde{\epsilon}+\cosh\sqrt{2}\tilde{\Gamma}_S)}$$

- Stopping voltage

$$\mu_S\approx\left[\Delta\epsilon-\frac{\sqrt{2}\Gamma_S\sinh\left(\frac{\Delta\epsilon}{2k_BT}\right)\sinh\left(\frac{\Gamma_S}{\sqrt{2}k_BT}\right)}{1+\cosh\left(\frac{\Delta\epsilon}{2k_BT}\right)\cosh\left(\frac{\Gamma_S}{\sqrt{2}k_BT}\right)}\right]\frac{\delta T}{4T}$$

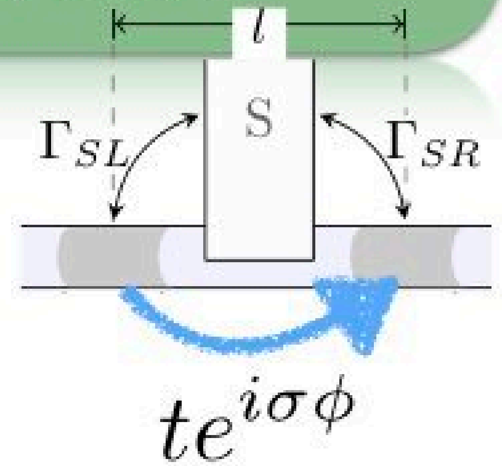
If $\Gamma_S,|\Delta\epsilon|\gg k_BT$

$$\mu_{S\pm}\approx\left(\Delta\epsilon\mp\sqrt{2}\Gamma_S\right)\frac{\delta T}{4T}$$

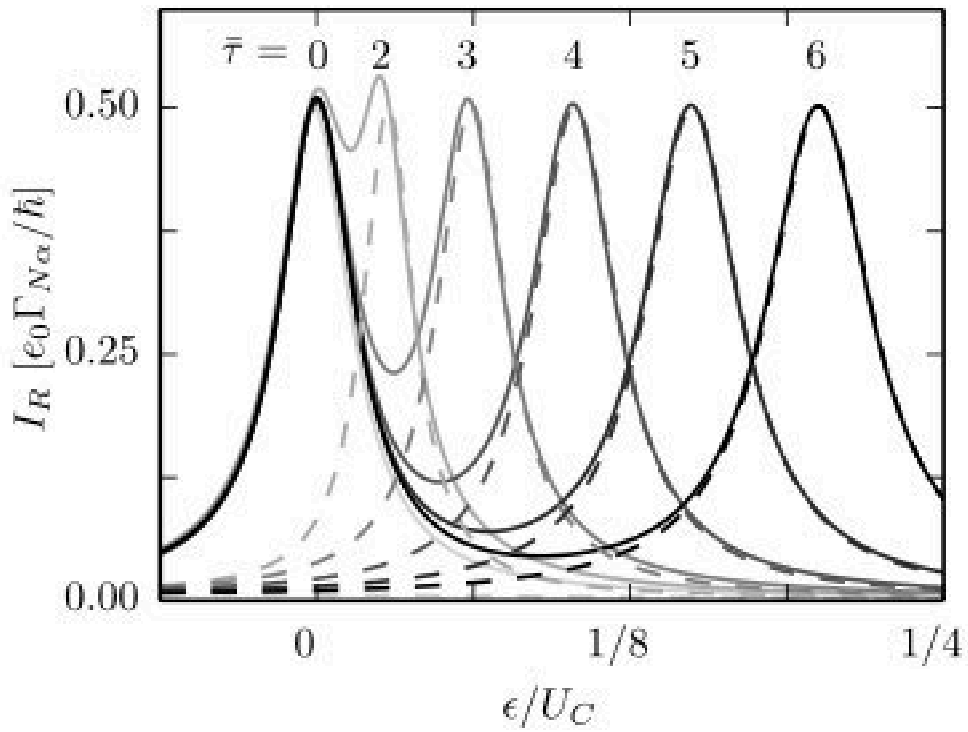


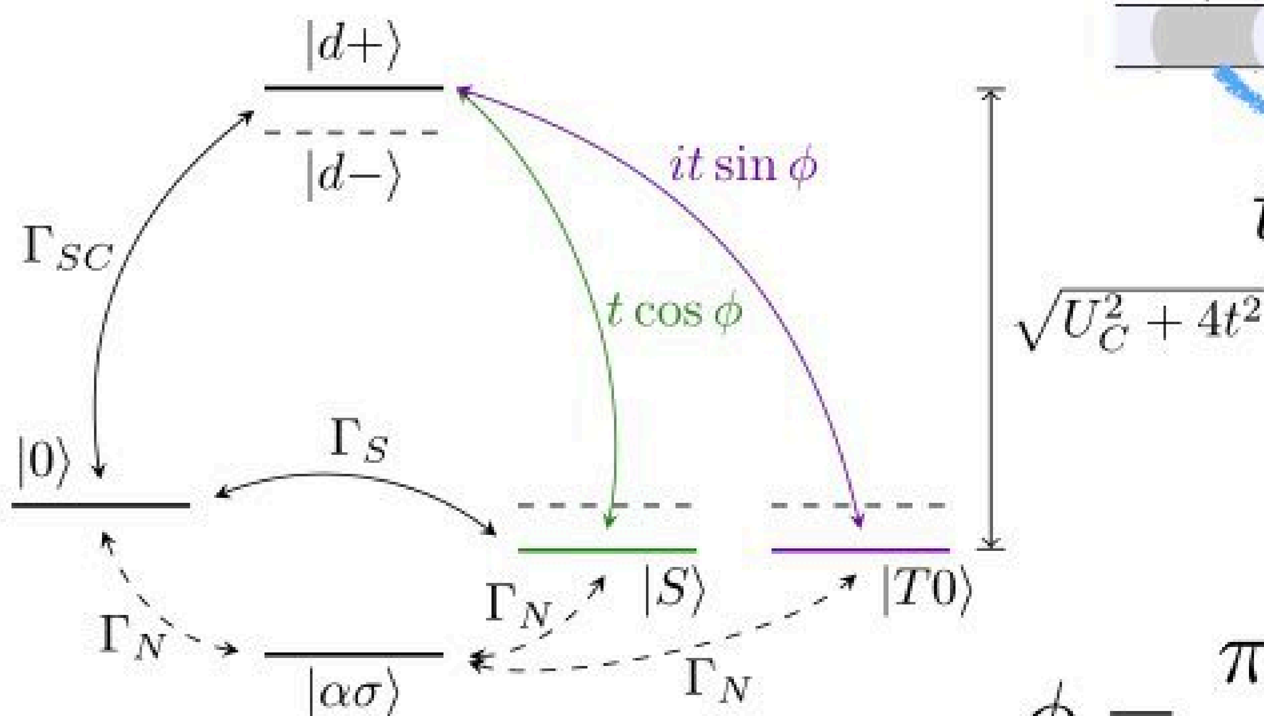
Entanglement manipulation

R. Hussein, AB, M. Governale, Phys Status Solidi'17

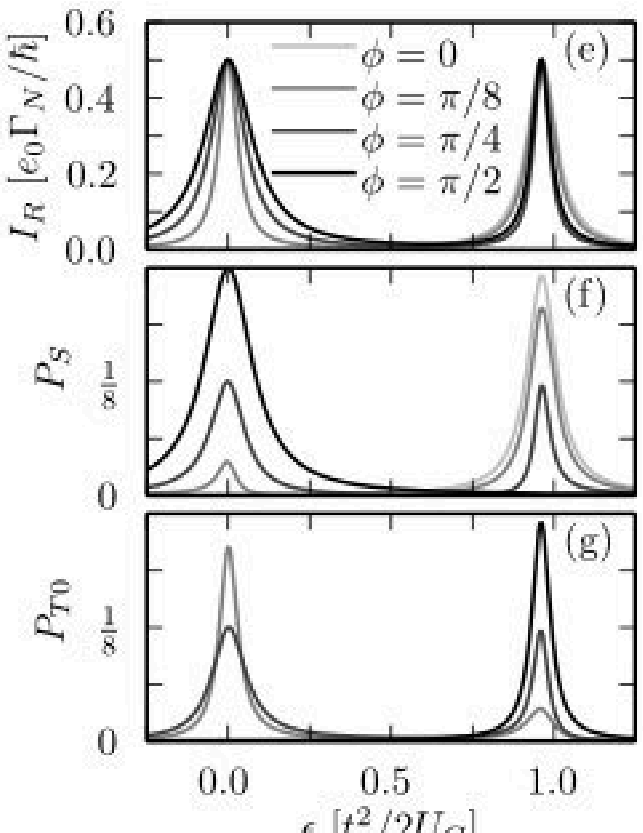


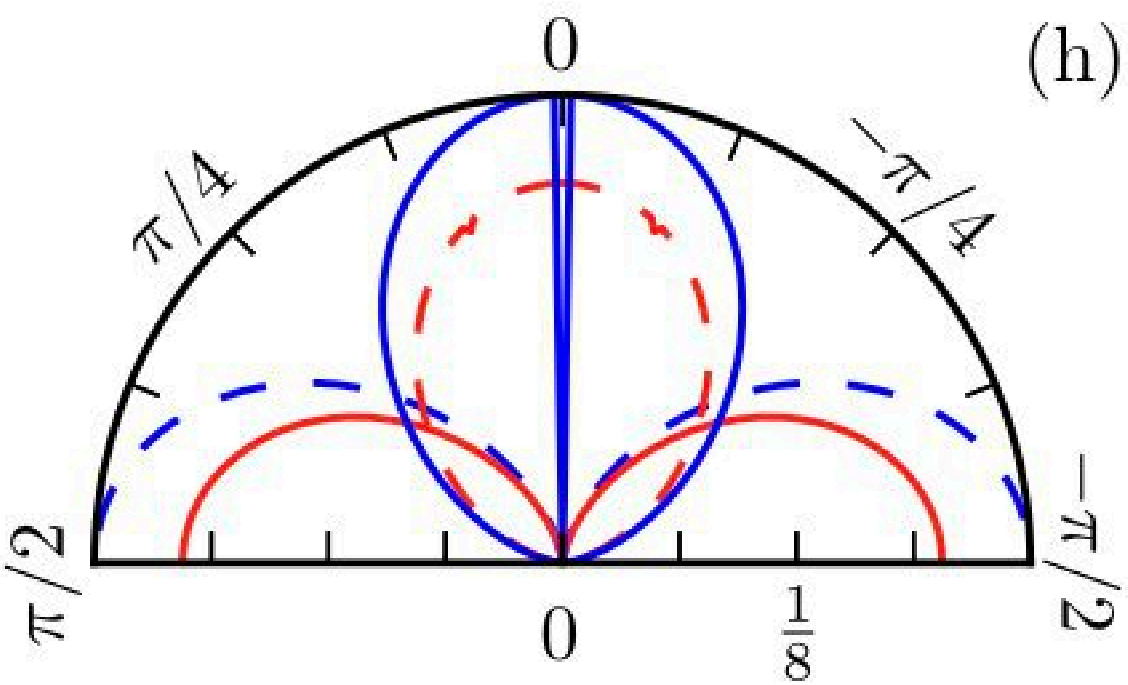
- Maximal spin-orbit





$$\phi = \frac{\pi}{2}$$



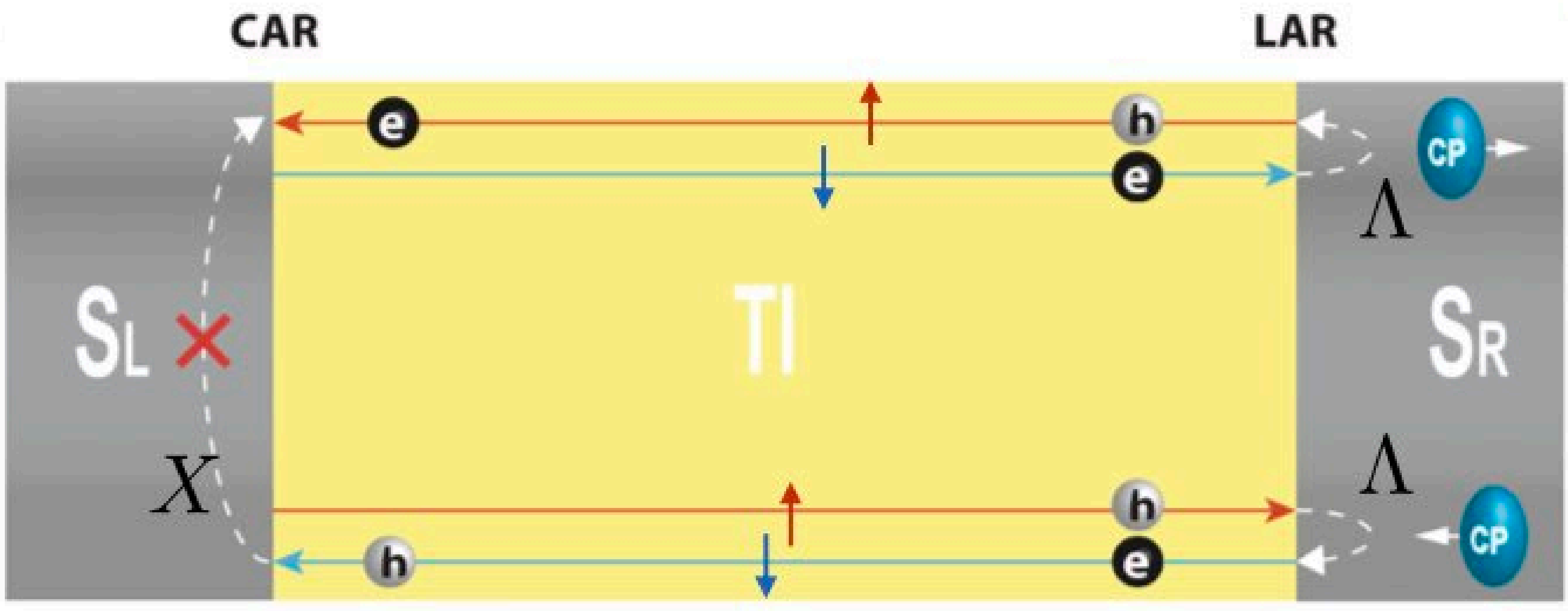


(h) Singlet to Triplet transmutation

$$|S\rangle \longrightarrow |T0\rangle$$

$$|T0\rangle \longrightarrow |S\rangle$$

Entanglement in S-TI-S



Topological Insulator

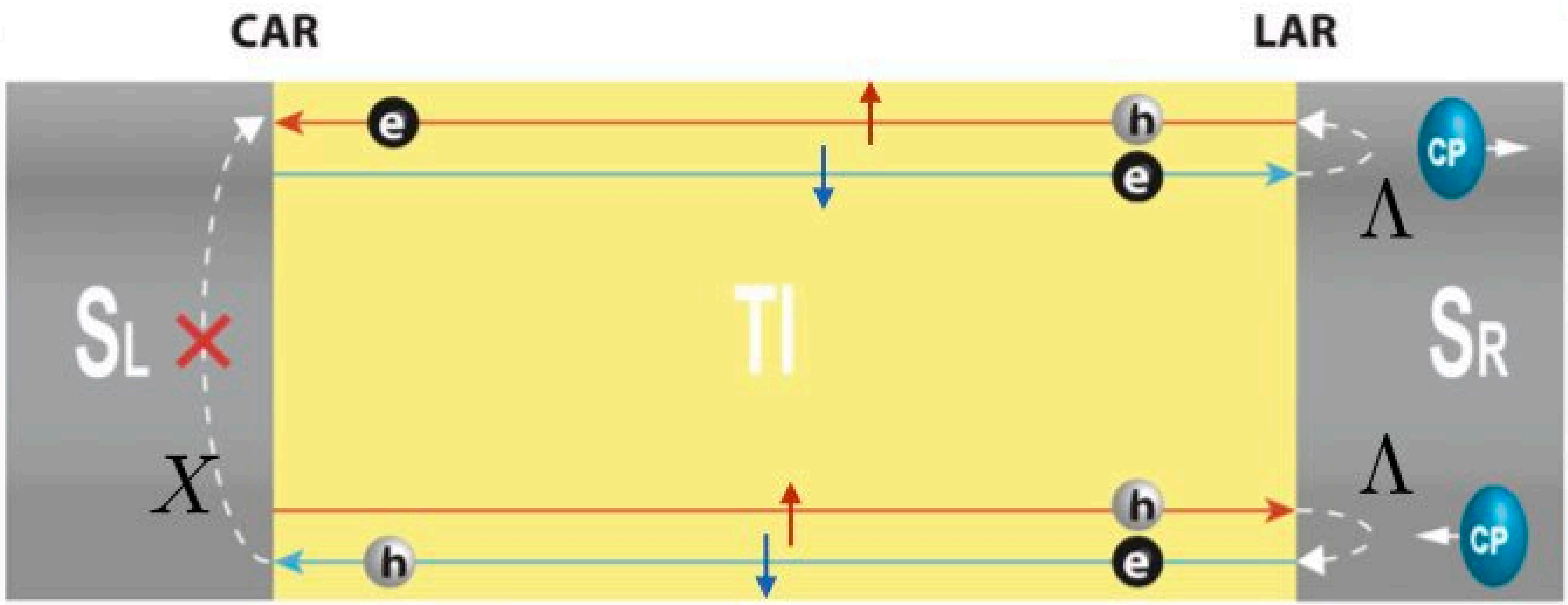
Hankiewicz's talk

Helical edge states

Andreev reflections

No CARs !

Entanglement in S-TI-S



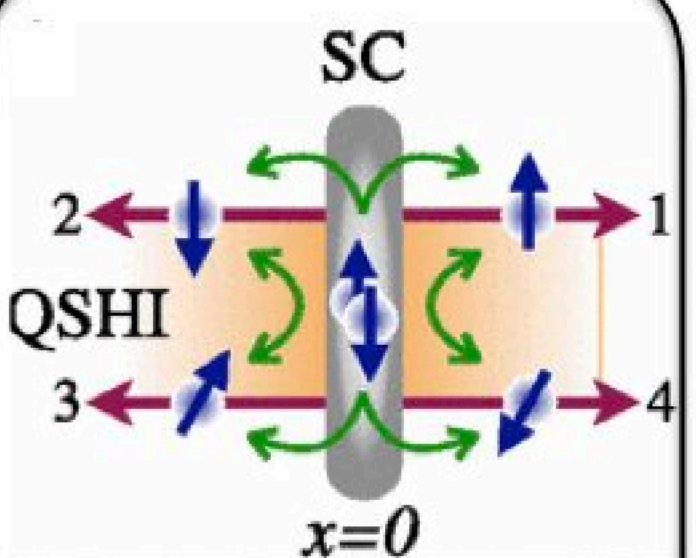
Topological Insulator

Hankiewicz's talk

Helical edge states

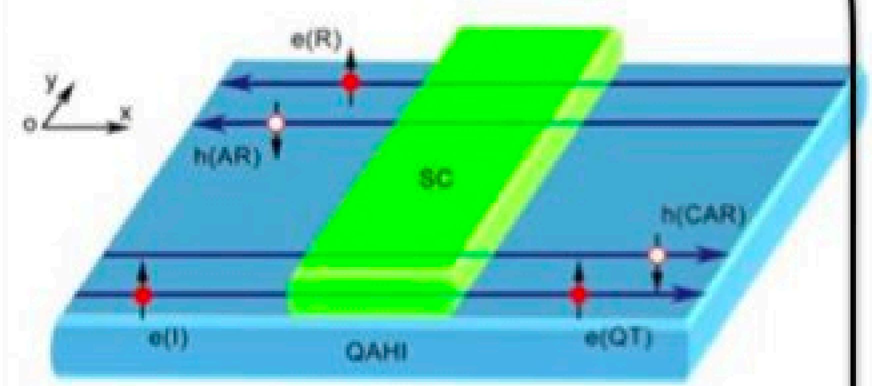
Andreev reflections

No CARs !

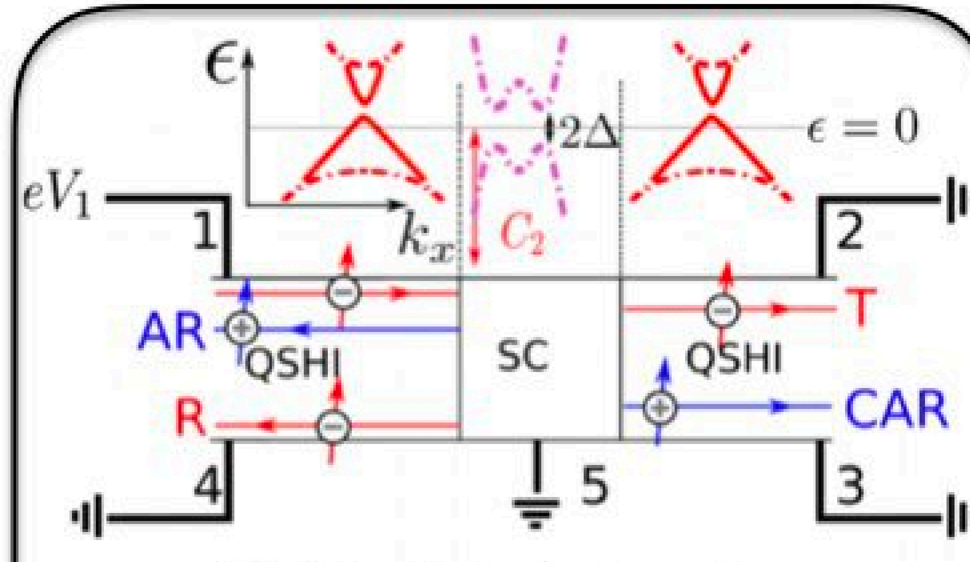


K. Sato, D. Loss, and Y. Tserkovnyak, PRL'10

K. Sato and Y. Tserkovnyak, PRB'14



Y.-T. Zhang, X. Deng, Q.-F. Sun, and Z. Qiao, Sci. Rep. '15

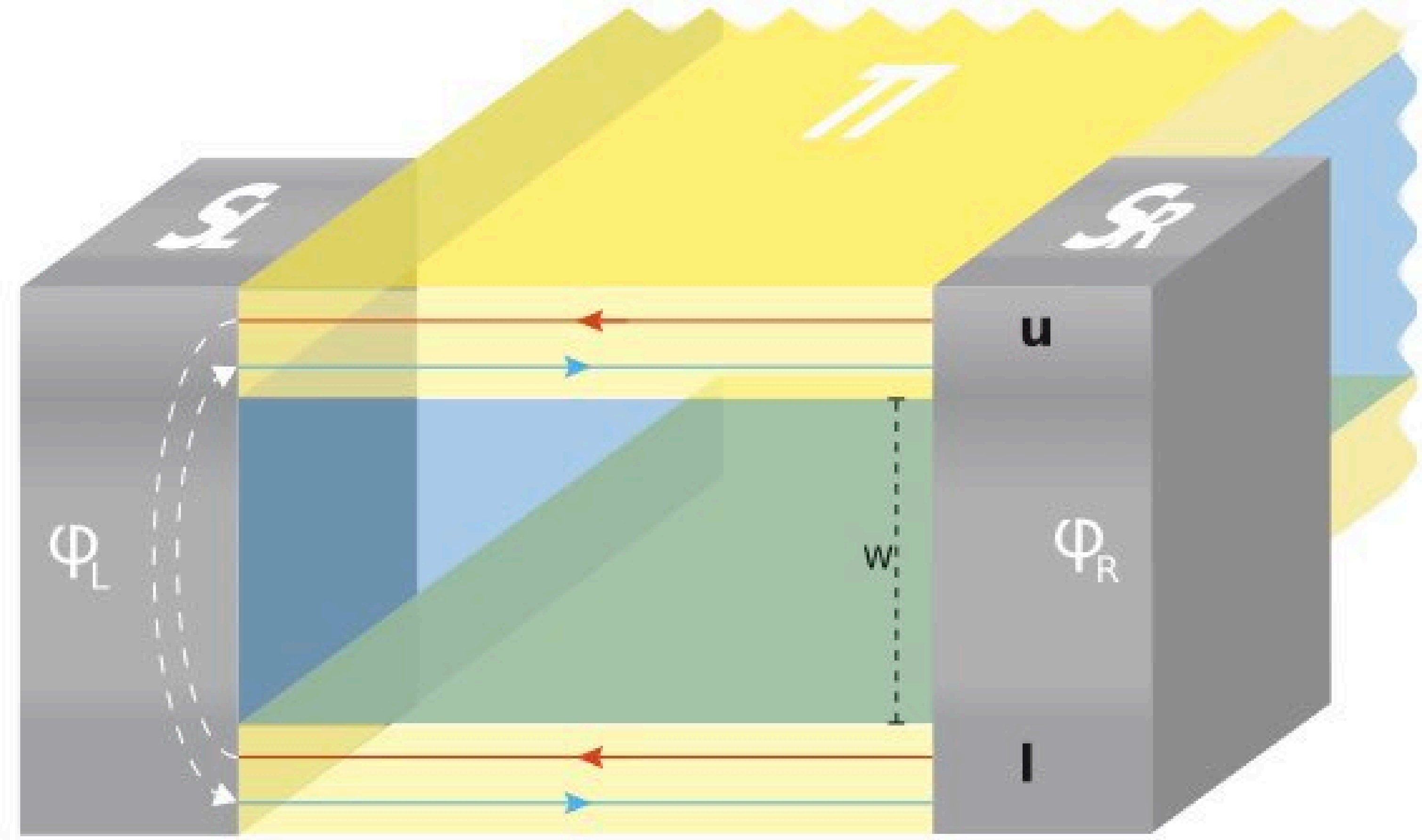


R. W. Reinthaler, P. Recher, and E. M. Hankiewicz, PRL'13

M.-S. Choi, PRB '14 ; M. Veldhorst et al. PRB '14; A. Ström et. PRB'15; J. Wang et al. PRB'15; Z. Hou et al PRB'16

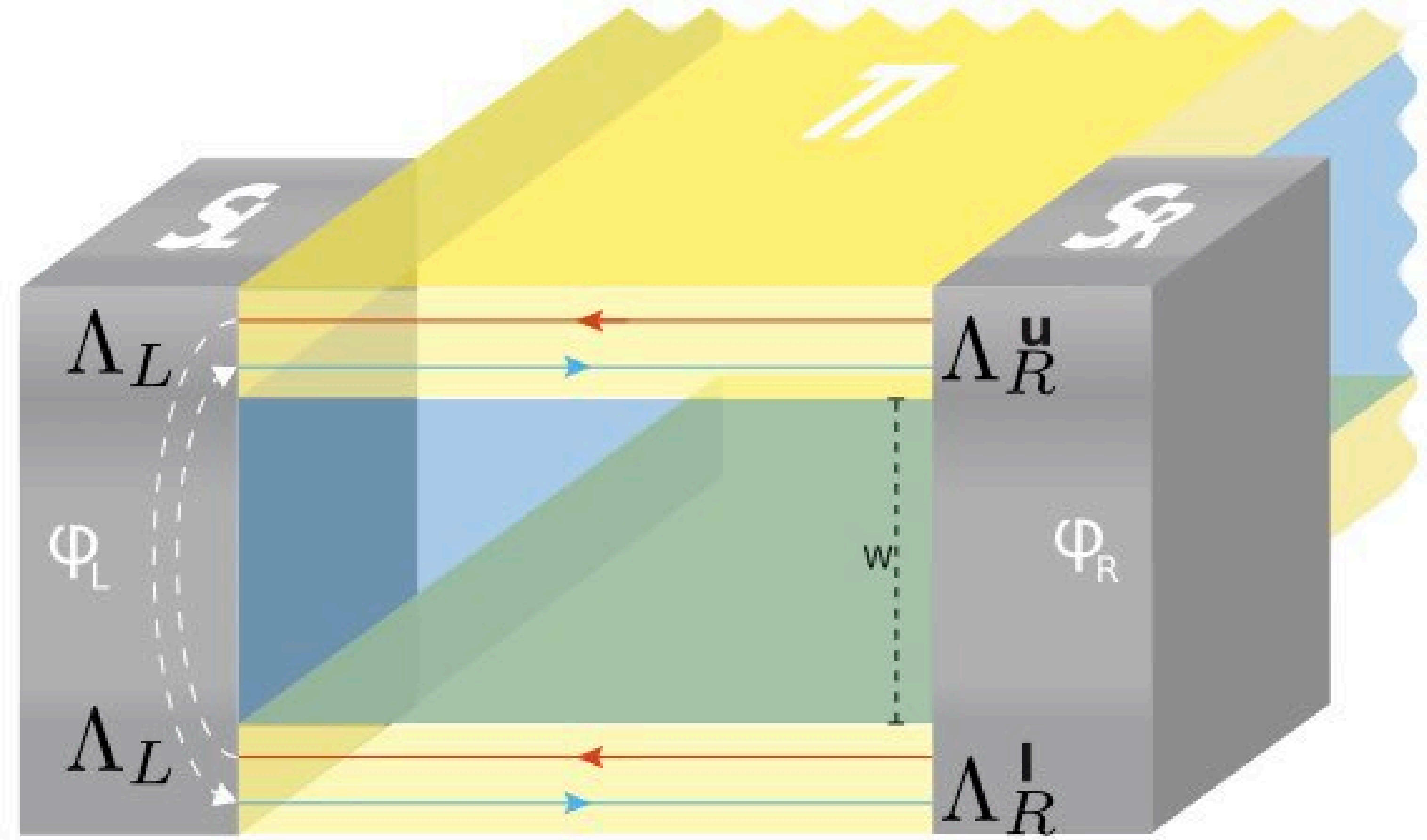
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



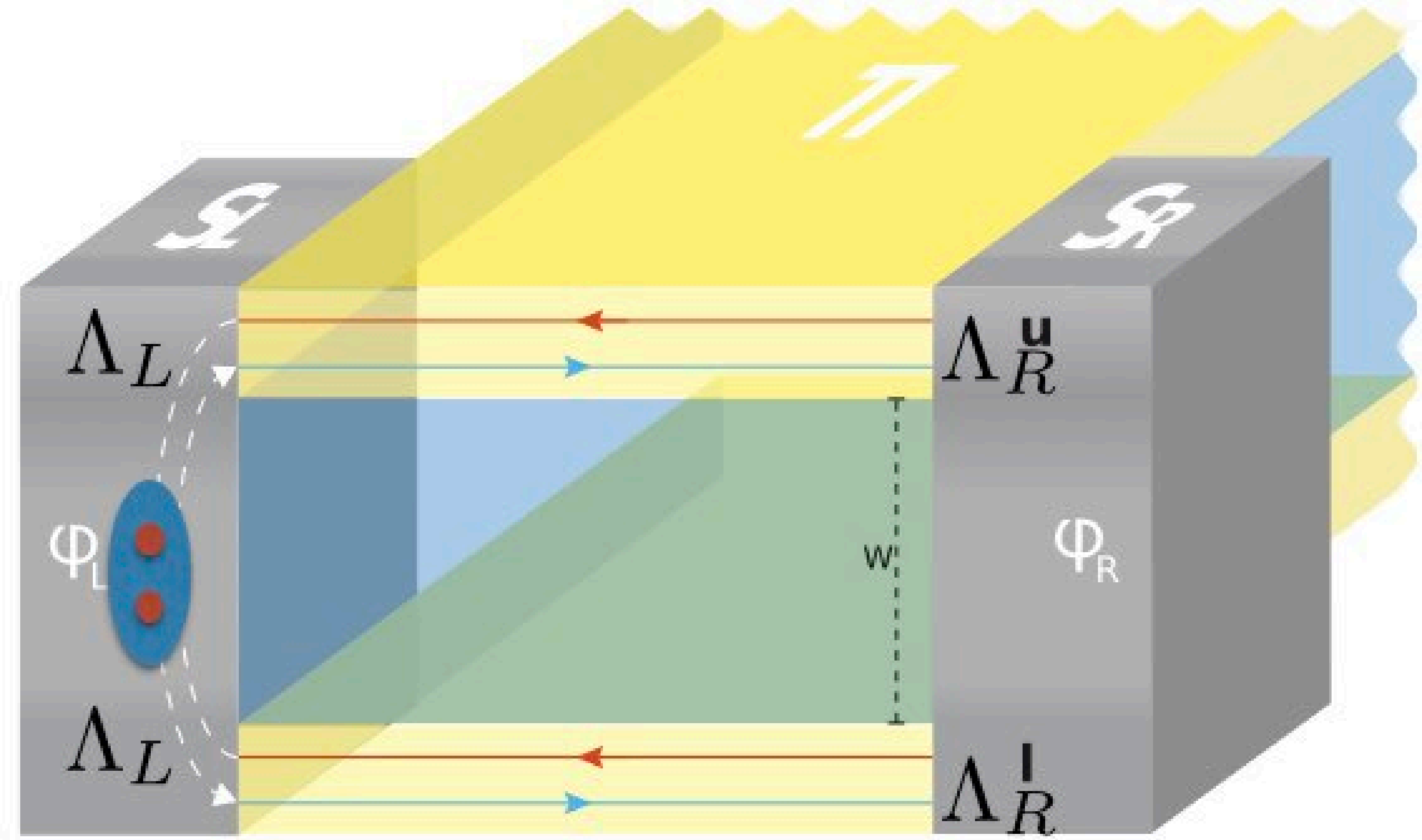
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



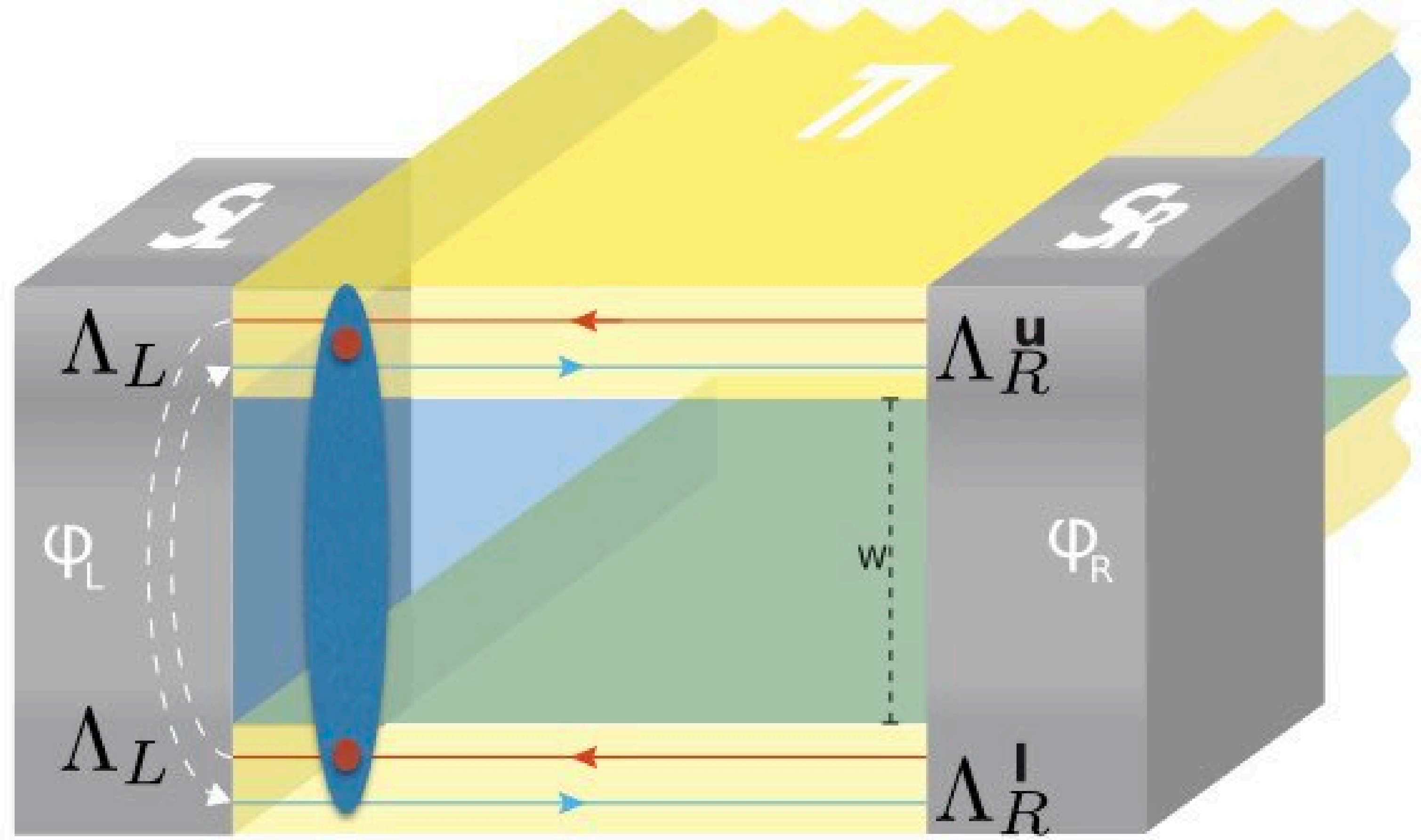
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



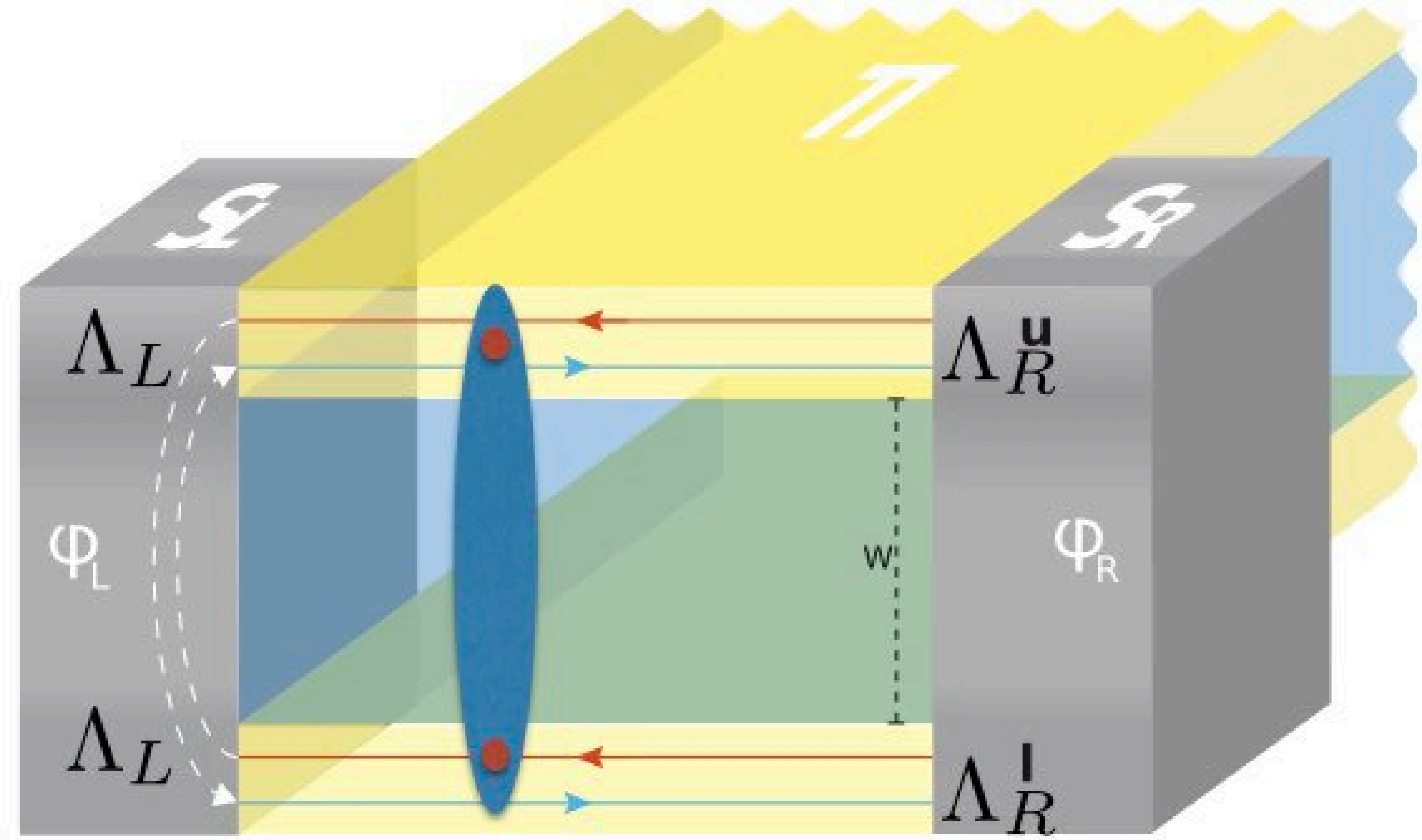
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



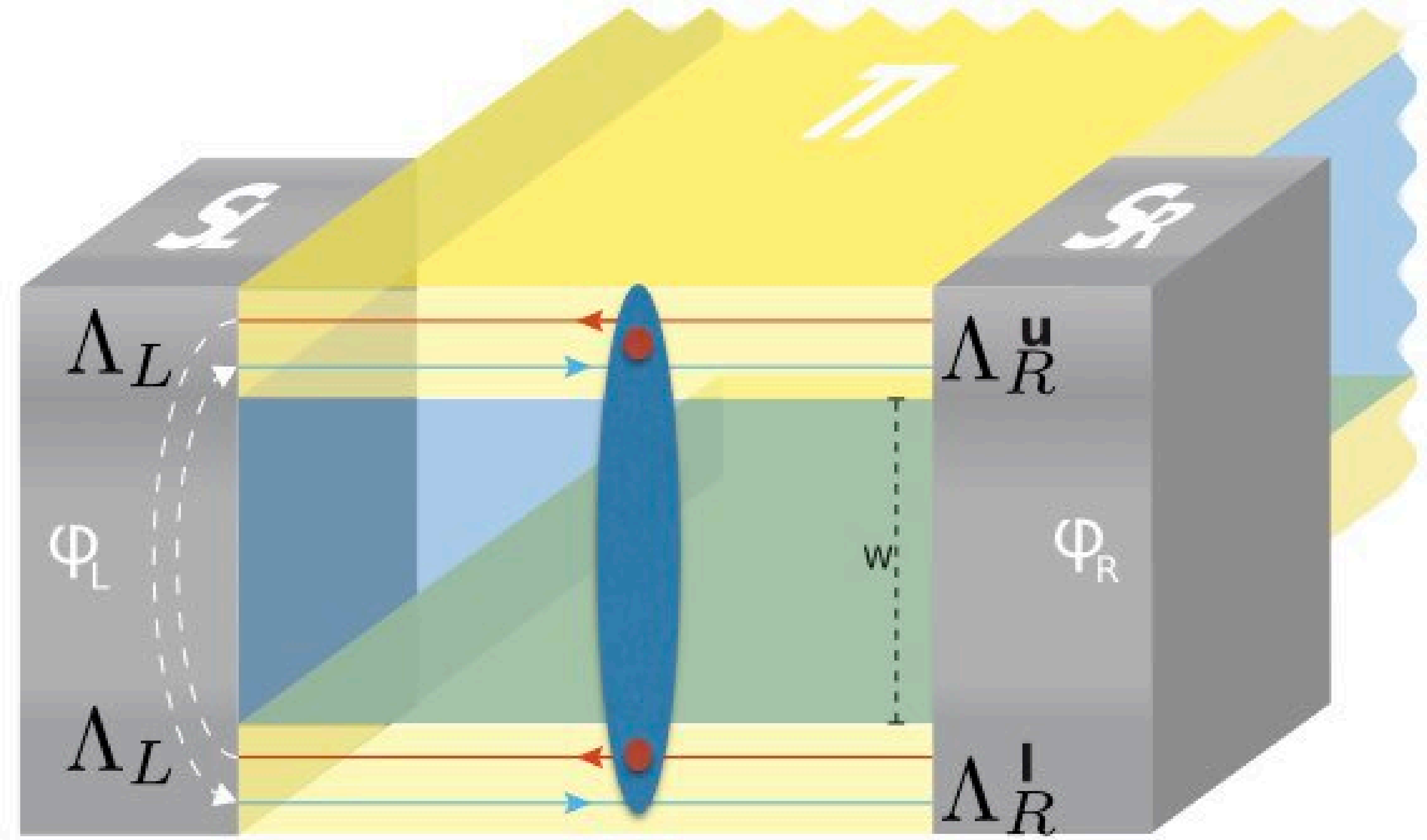
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



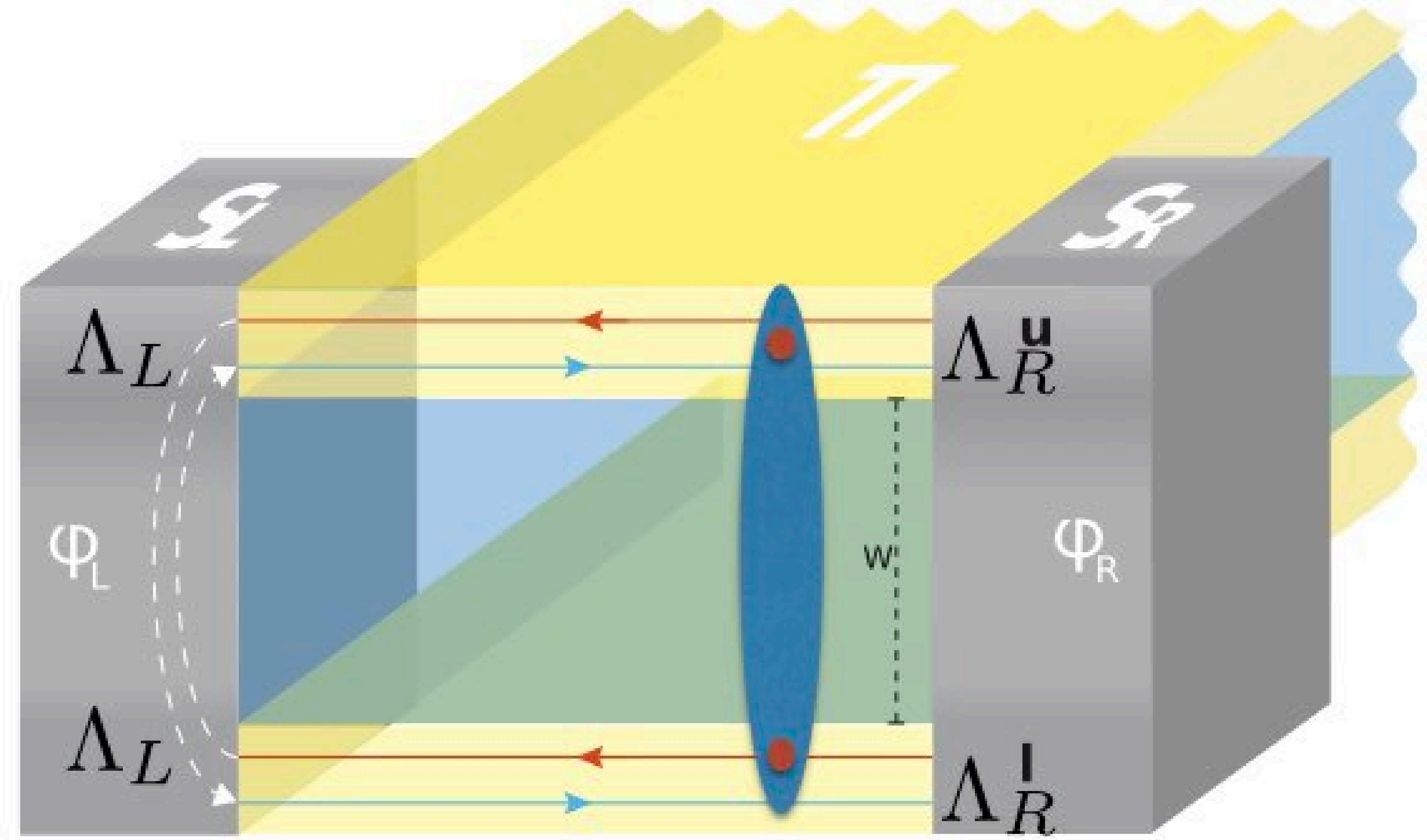
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



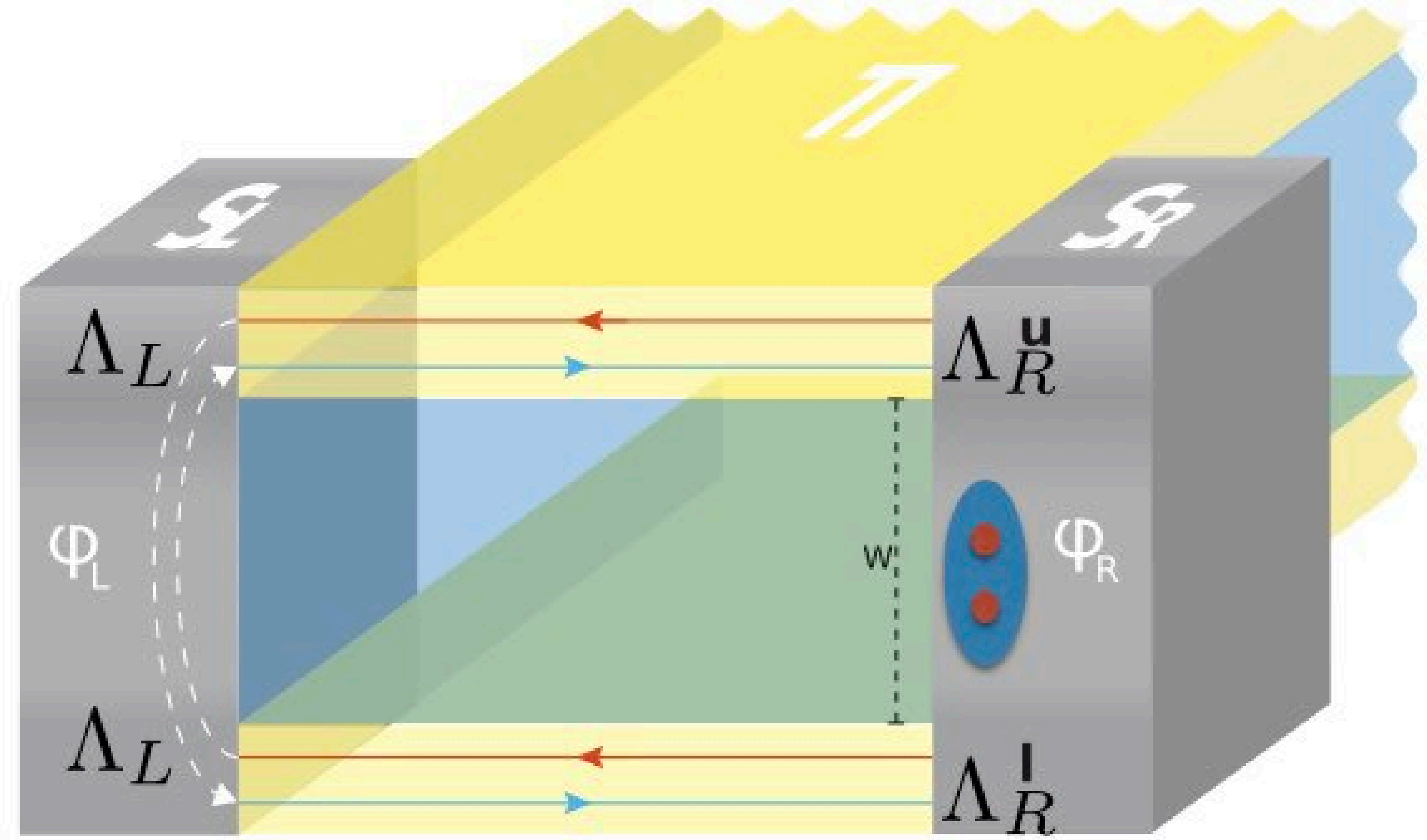
Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



Manipulation in S-TI-S JJ

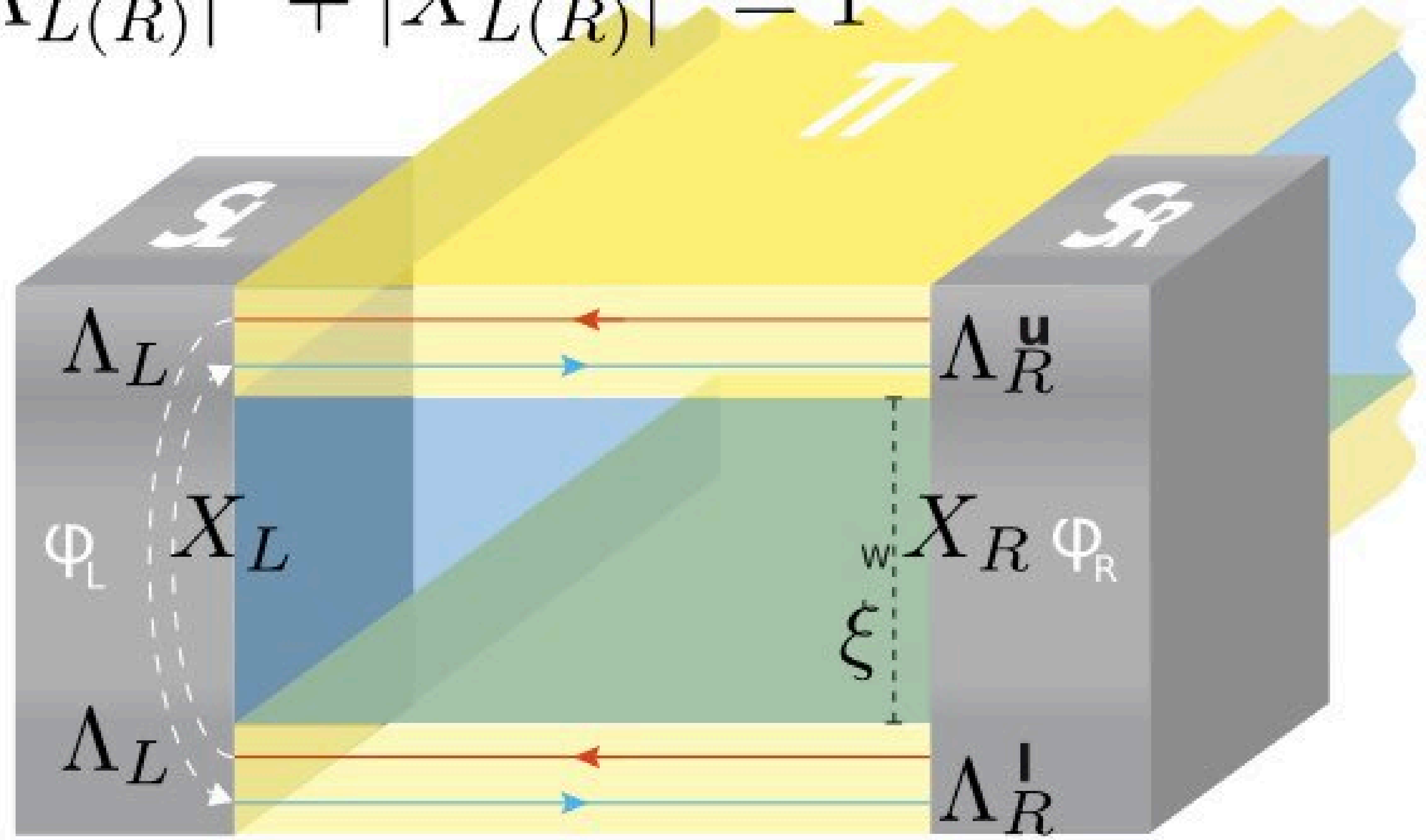
G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709



Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

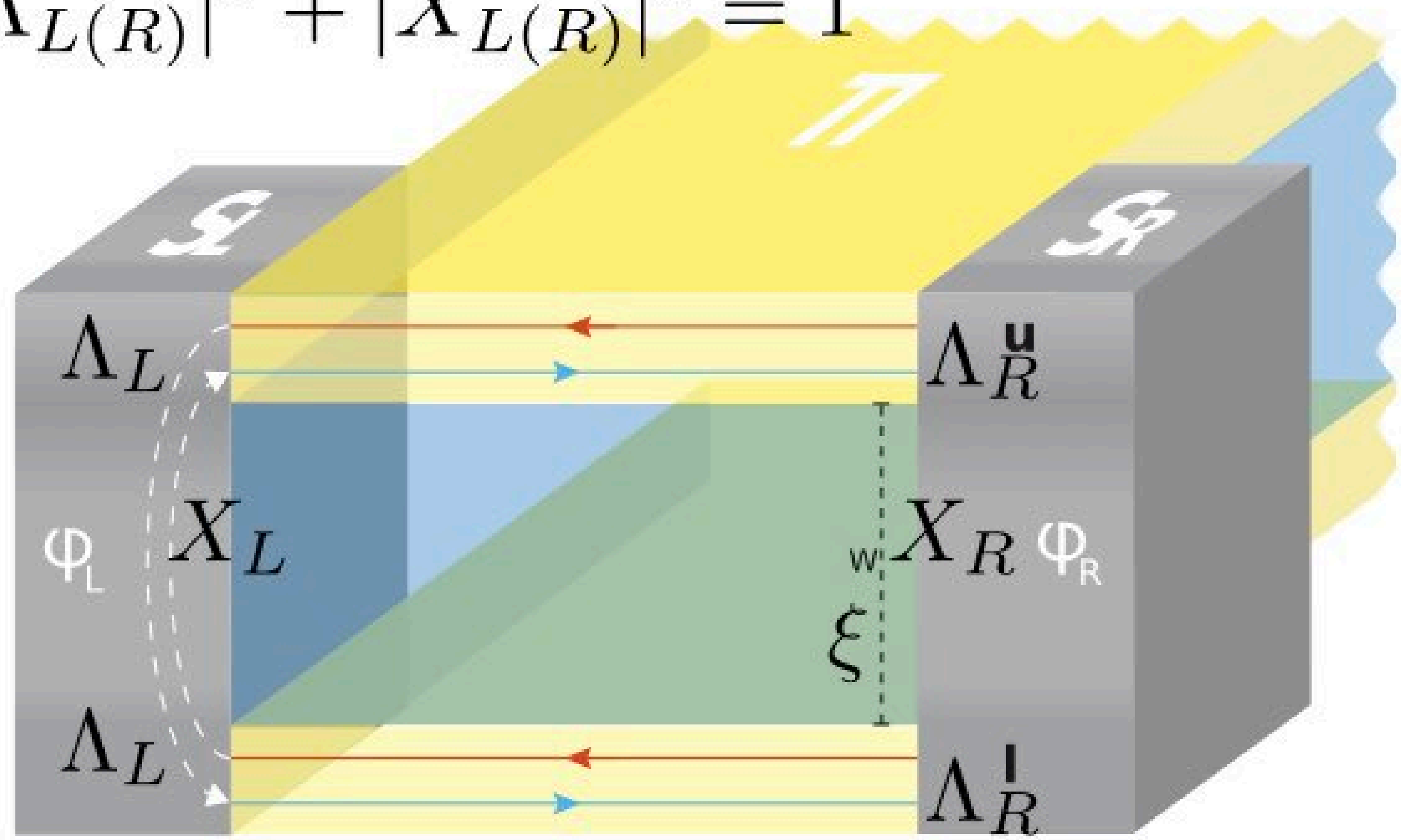
$$|\Lambda_{L(R)}|^2 + |X_{L(R)}|^2 = 1$$



Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

$$|\Lambda_{L(R)}|^2 + |X_{L(R)}|^2 = 1$$



Superposition of
LAR & CAR

$$\frac{|X|}{|\Lambda|} \propto e^{-w/\xi}$$

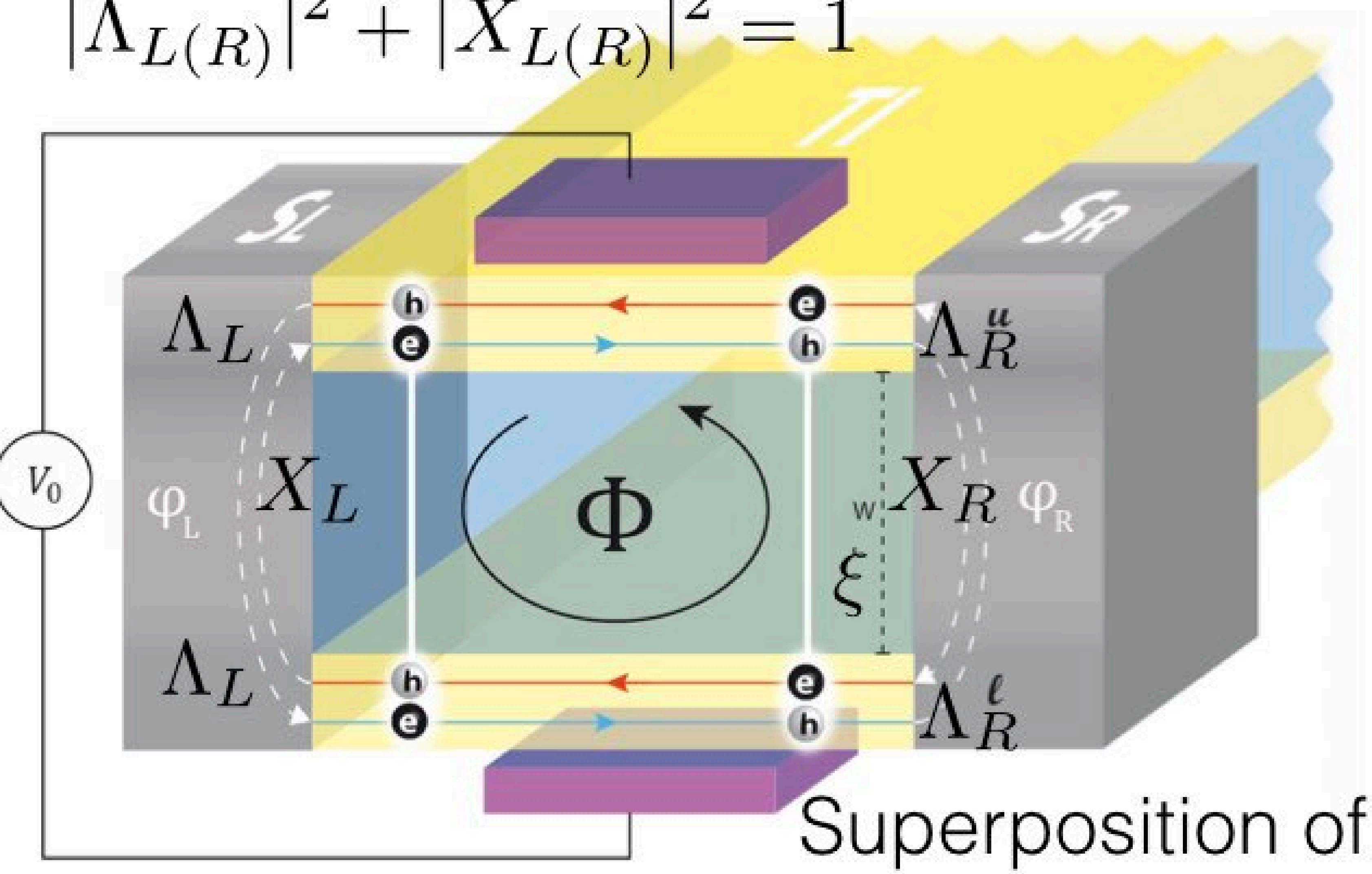
Singlet CAR

$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - |h_u^{\downarrow} e_\ell^{\uparrow}\rangle\right)$$

Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

$$|\Lambda_{L(R)}|^2 + |X_{L(R)}|^2 = 1$$



- Time-reversal Sym.

Gate Voltage V

$$\mathcal{U}_V(\theta_V) = e^{i\ \sigma_0 \theta_V/2}$$

$$\theta_V = \frac{2eVL}{\hbar v_F}$$

Dynamical phase
Xiao et al, APL '16

Superposition of
LAR & CAR

$$\frac{|X|}{|\Lambda|} \propto e^{-w/\xi}$$

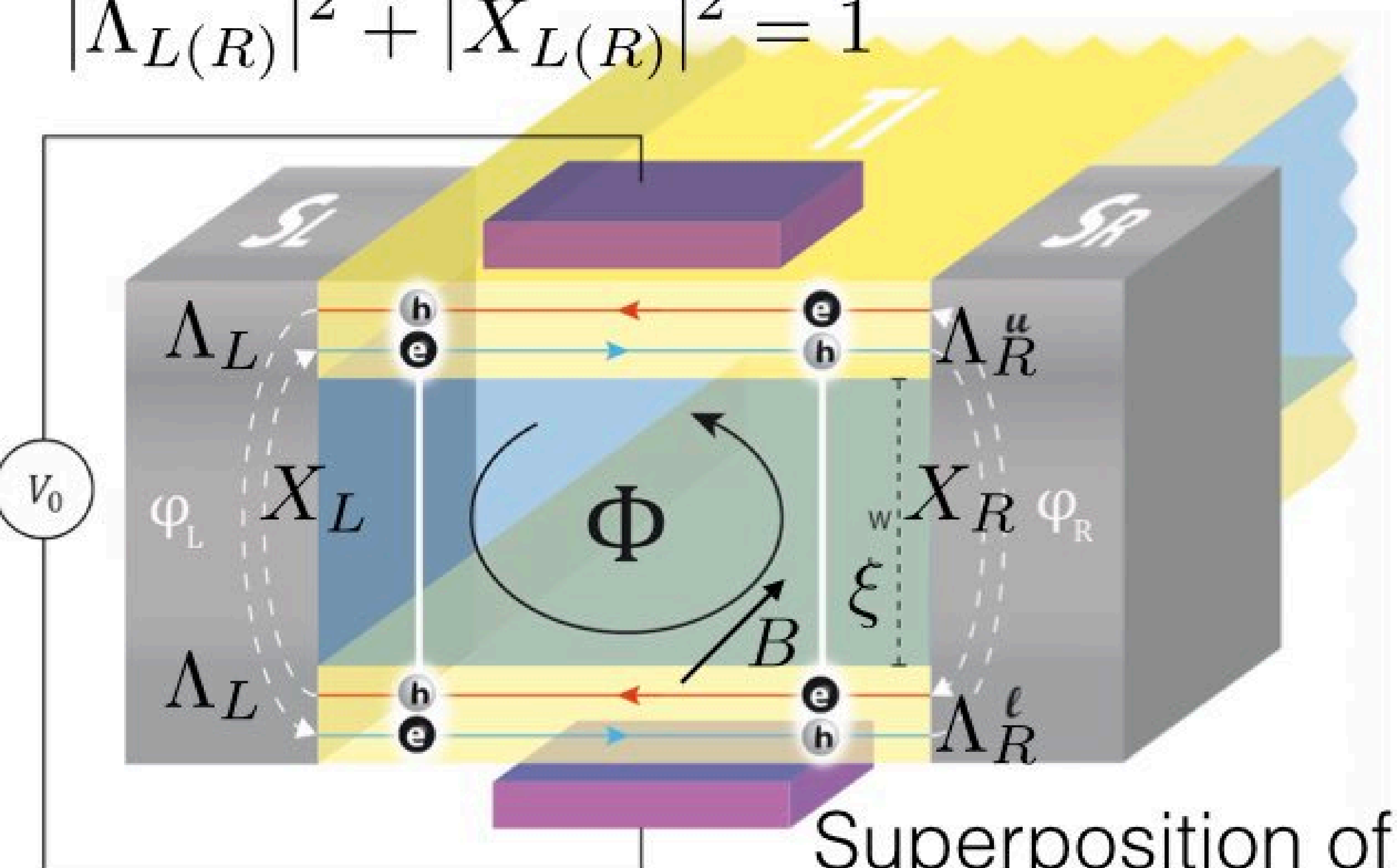
Singlet CAR

$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^\uparrow h_\ell^\downarrow\rangle - |h_u^\downarrow e_\ell^\uparrow\rangle\right)$$

Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

$$|\Lambda_{L(R)}|^2 + |X_{L(R)}|^2 = 1$$



Superposition of LAR & CAR

- Time-reversal Sym.
  Gate Voltage V

$$\mathcal{U}_{\rm V}(\theta_{\rm V}) = e^{i\ \sigma_0 \theta_{\rm V}/2}$$

$$\theta_{\rm V} = \frac{2eVL}{\hbar v_F}$$

Dynamical phase
Xiao et al, APL '16

$$\frac{|X|}{|\Lambda|} \propto e^{-w/\xi}$$

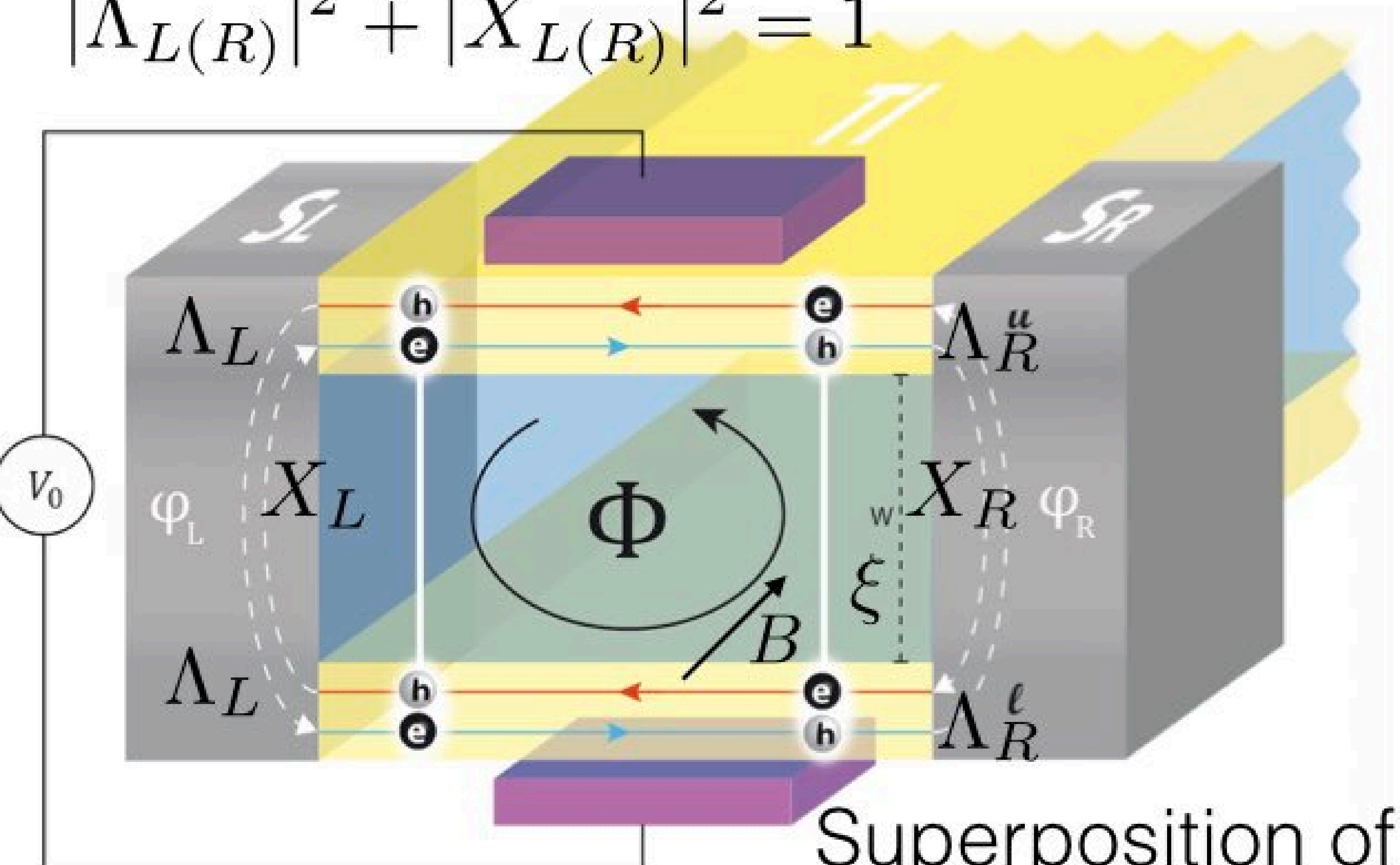
Singlet CAR

$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^\uparrow h_\ell^\downarrow\rangle - |h_u^\downarrow e_\ell^\uparrow\rangle\right)$$

Manipulation in S-TI-S JJ

G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

$$|\Lambda_{L(R)}|^2 + |X_{L(R)}|^2 = 1$$



- Time-reversal Sym.

Gate Voltage V

$$\mathcal{U}_{\mathrm{V}}(\theta_{\mathrm{V}}) = e^{i\ \sigma_0 \theta_{\mathrm{V}}/2}$$

$$\theta_{\mathrm{V}} = \frac{2eVL}{\hbar v_F}$$

Dynamical phase
Xiao et al, APL '16

Superposition of
LAR & CAR

$$\frac{|X|}{|\Lambda|} \propto e^{-w/\xi}$$

Singlet CAR

$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - |h_u^{\downarrow} e_\ell^{\uparrow}\rangle\right)$$

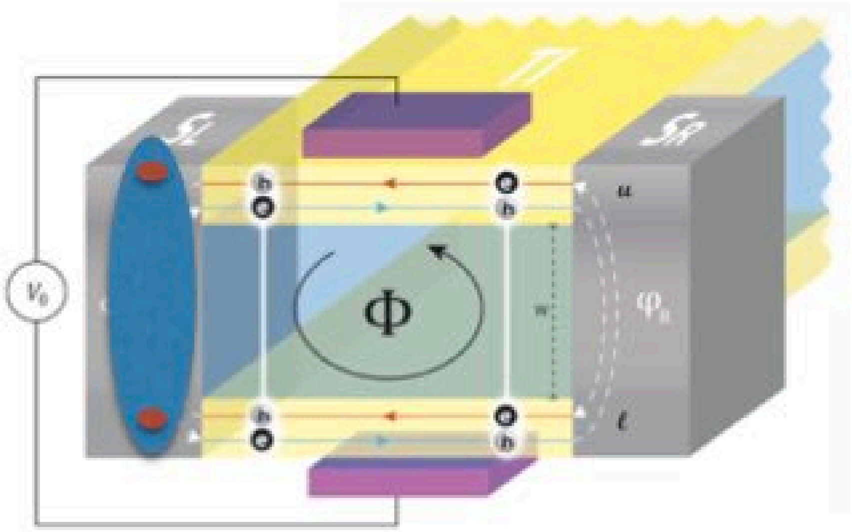
- Breaking TR $B \rightarrow \Phi$

$$\mathcal{U}_{\Phi}(\theta_{\Phi}) = e^{i\ \sigma\cdot n\ \theta_{\Phi}/2}$$

Doppler shift
Tkachov et al, PRB '15

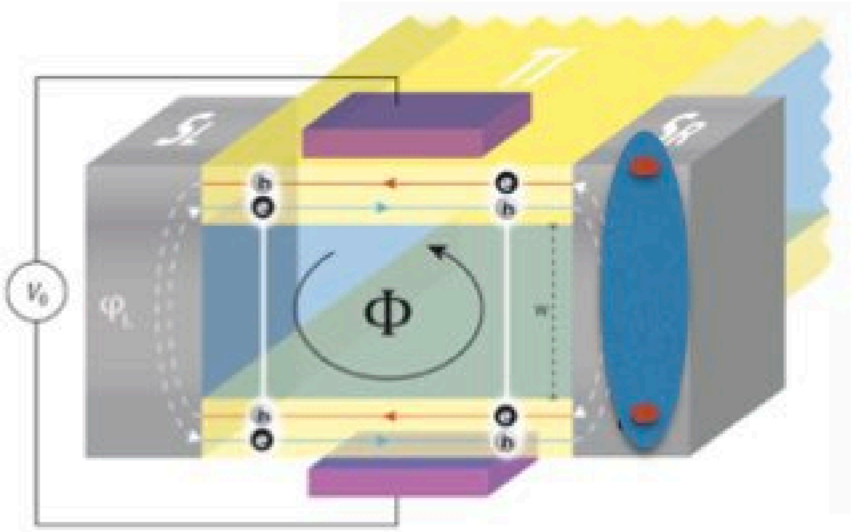
Hankiewicz's talk

Entanglement in S-TI-S



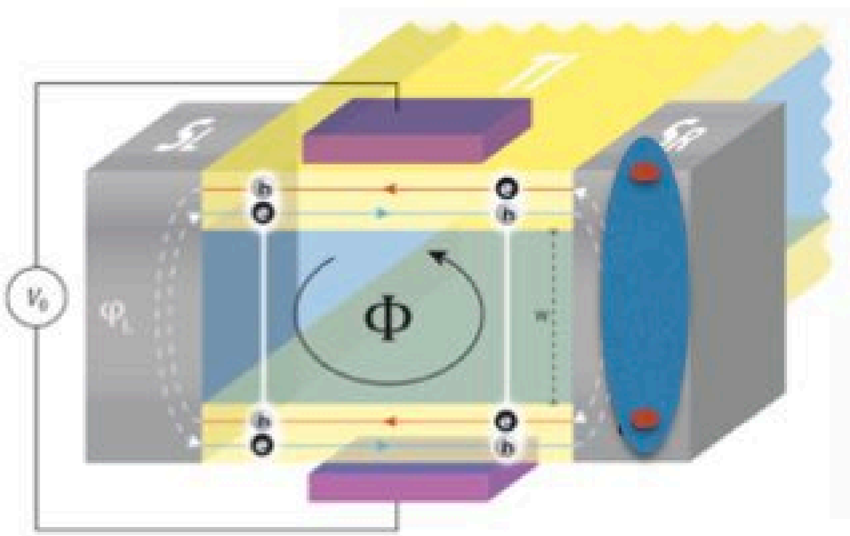
$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - |h_u^{\downarrow} e_\ell^{\uparrow}\rangle\right)$$

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - |h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

Entanglement in S-TI-S

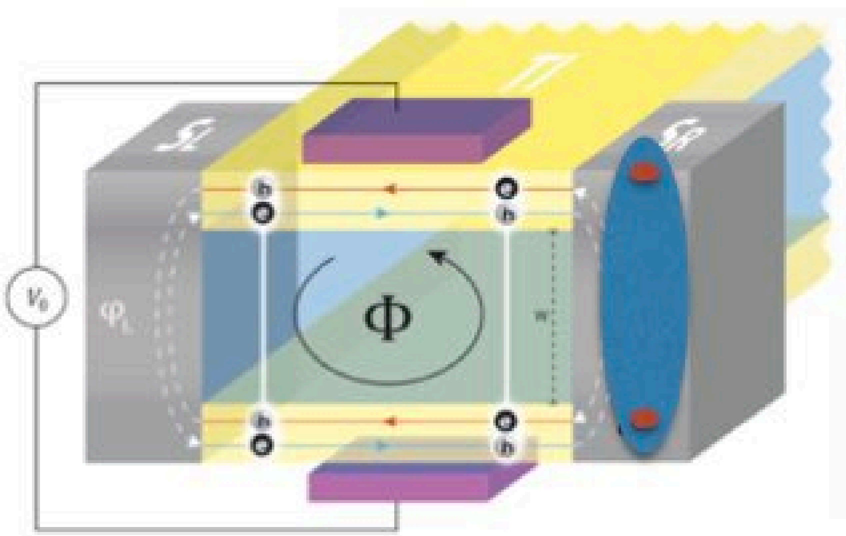


$$|C\rangle = \frac{1}{\sqrt{2}}\Big(|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - |h_u^{\downarrow} e_\ell^{\uparrow}\rangle\Big)$$

$$\mathcal{U}_\Phi \Big\downarrow \mathcal{U}_\mathrm{V}$$

$$\mathcal{U}_\Phi \mathcal{U}_V |C\rangle = \quad \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\Big(e^{-i\theta_\mathrm{V}/2}|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - e^{+i\theta_\mathrm{V}/2}|h_u^{\downarrow} e_\ell^{\uparrow}\rangle\Big)$$

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - |h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

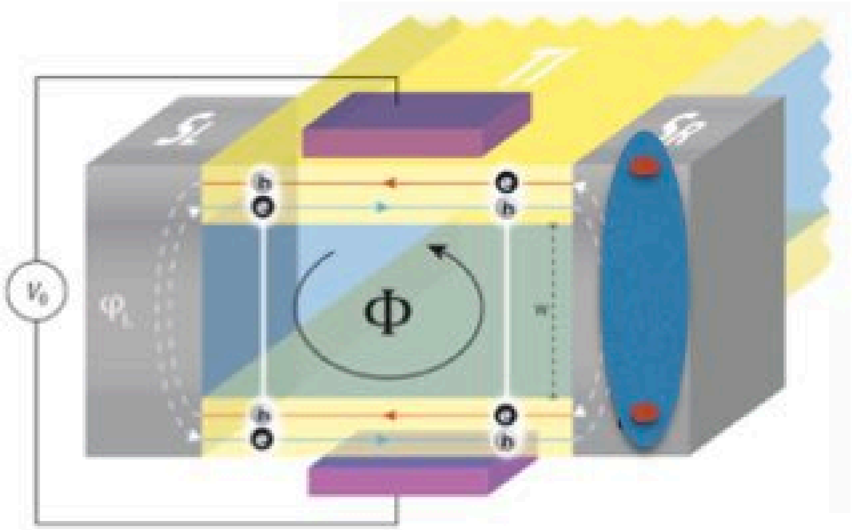
$$\Big\downarrow\ \mathcal{U}_\Phi \quad \mathcal{U}_{\mathrm{V}}$$

$$\mathcal{U}_\Phi\mathcal{U}_V|C\rangle = \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\left(e^{-i\theta_{\mathrm{V}}/2}|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - e^{+i\theta_{\mathrm{V}}/2}|h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

For $\theta_{\mathrm{V}} = \pi \quad |S\rangle \rightarrow |T0\rangle$

Singlet to Triplet transmutation

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - |h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

$\mathcal{U}_\Phi$ ⬇ $\mathcal{U}_\mathrm{V}$

$$\mathcal{U}_\Phi\mathcal{U}_V|C\rangle = \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\left(e^{-i\theta_\mathrm{V}/2}|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - e^{+i\theta_\mathrm{V}/2}|h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

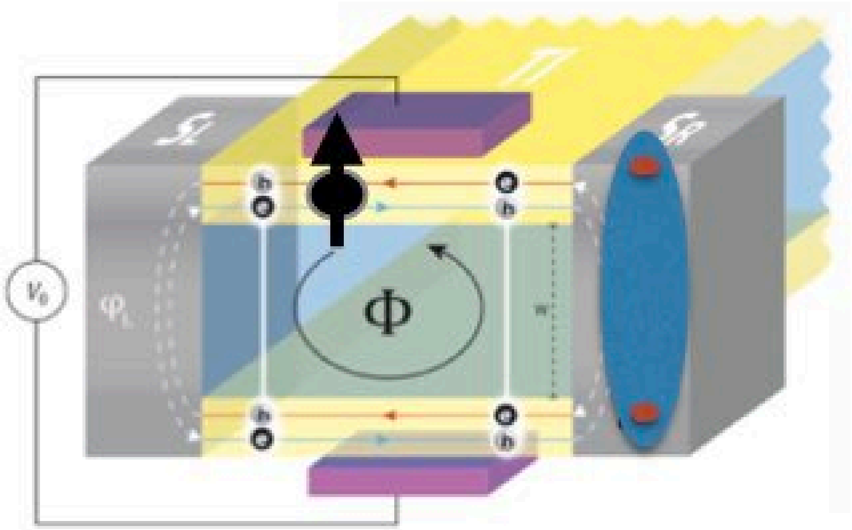
For $\theta_\mathrm{V} = \pi \quad |S\rangle \rightarrow |T0\rangle$

Singlet to Triplet transmutation

$J_c \propto |\langle C|\mathcal{U}_V(\theta_V)|C\rangle = |\cos(\theta_V/2)|$

Suppression of critical current

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - |h_u^{\downarrow} e_\ell^{\uparrow}\rangle\right)$$

$$\mathcal{U}_\Phi \quad \Big\downarrow \quad \mathcal{U}_\mathrm{V}$$

$$\mathcal{U}_\Phi \mathcal{U}_V |C\rangle = \quad \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\left(e^{-i\theta_\mathrm{V}/2}|e_u^{\uparrow} h_\ell^{\downarrow}\rangle - e^{+i\theta_\mathrm{V}/2}|h_u^{\downarrow} e_\ell^{\uparrow}\rangle\right)$$

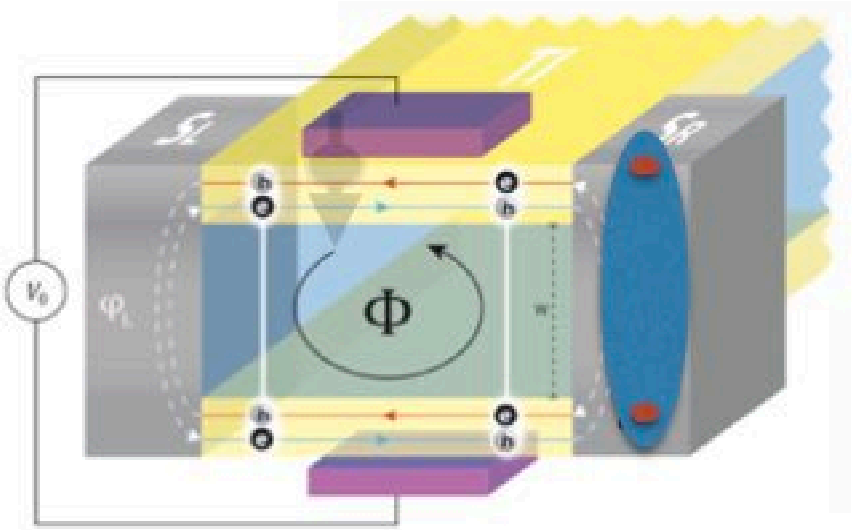
For $\theta_\mathrm{V} = \pi \quad |S\rangle \rightarrow |T0\rangle$

Singlet to Triplet transmutation

$J_c \propto |\langle C|\mathcal{U}_V(\theta_V)|C\rangle = |\cos(\theta_V/2)|$

Suppression of critical current

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - |h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

$$\mathcal{U}_\Phi \downarrow \mathcal{U}_\mathrm{V}$$

$$\mathcal{U}_\Phi\mathcal{U}_V|C\rangle = \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\left(e^{-i\theta_\mathrm{V}/2}|e_u^{\uparrow}h_\ell^{\downarrow}\rangle - e^{+i\theta_\mathrm{V}/2}|h_u^{\downarrow}e_\ell^{\uparrow}\rangle\right)$$

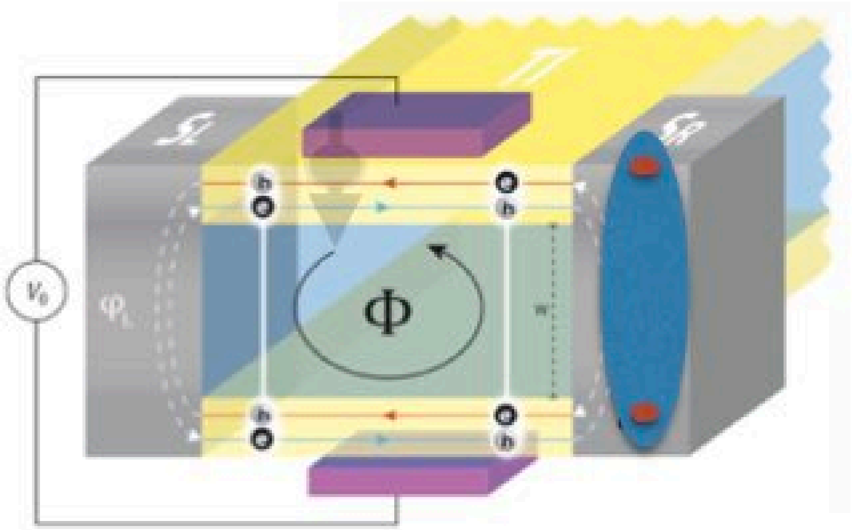
For $\theta_\mathrm{V} = \pi \quad |S\rangle \to |T0\rangle$

Singlet to Triplet transmutation

$J_c \propto |\langle C|\mathcal{U}_V(\theta_V)|C\rangle = |\cos(\theta_V/2)|$

Suppression of critical current

Entanglement in S-TI-S



$$|C\rangle = \frac{1}{\sqrt{2}}\left(|e_u^\uparrow h_\ell^\downarrow\rangle - |h_u^\downarrow e_\ell^\uparrow\rangle\right)$$

$\mathcal{U}_\Phi$ $\mathcal{U}_\mathrm{V}$

$$\mathcal{U}_\Phi \mathcal{U}_V |C\rangle = \frac{e^{i\theta_\Phi/2}}{\sqrt{2}}\left(e^{-i\theta_\mathrm{V}/2}|e_u^\uparrow h_\ell^\downarrow\rangle - e^{+i\theta_\mathrm{V}/2}|h_u^\downarrow e_\ell^\uparrow\rangle\right)$$

For $\theta_\mathrm{V} = \pi \quad |S\rangle \to |T0\rangle$

Singlet to Triplet transmutation

$J_c \propto |\langle C|\mathcal{U}_V(\theta_V)|C\rangle = |\cos(\theta_V/2)|$

Suppression of critical current

LAR singlet symmetry unaffected by V or B ! Only phase shifts

CPR Computation

$L \ll \xi$

Short junction

$$I = -\frac{2e}{\hbar}\sum_p \tanh\left[\frac{\epsilon_p}{2k_B T}\right]\frac{d\epsilon_p}{d\phi}$$

ϵ_p
ABS energies

C. W. J. Beenakker, PRL'91
C. W. J. Beenakker, in *Transport Phenomena in Mesoscopic Systems* '92
C. J. Lambert and R. Raimondi, JPCM '98

Secular equation

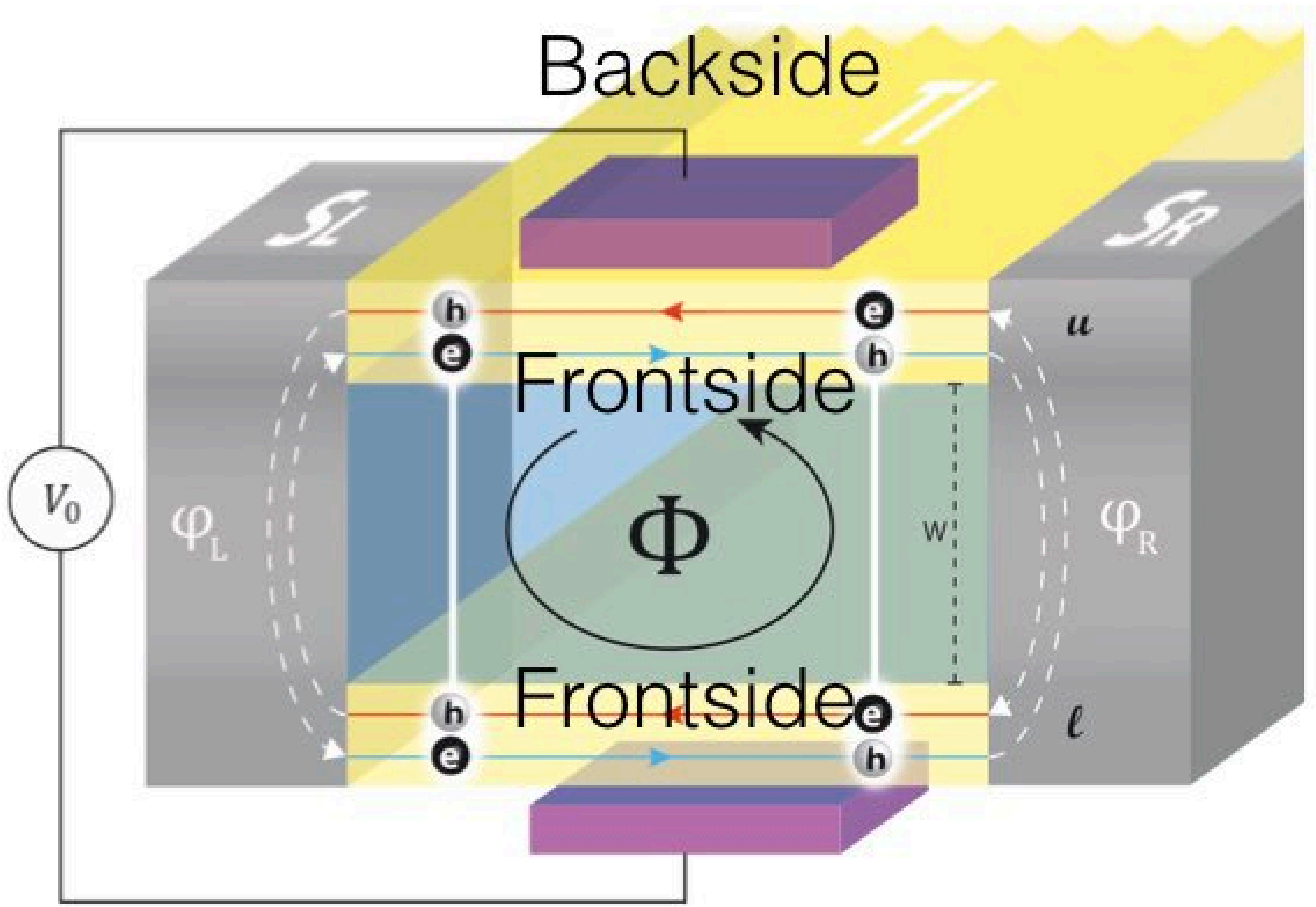
$$Det\left[e^{i\arccos(\epsilon_p/\Delta_0)}\mathbf{1} - s_A s_N\right] = 0$$

$$s_N = \begin{pmatrix} s_0 & \emptyset \\ \emptyset & s_0^* \end{pmatrix} \quad s_A = \begin{pmatrix} \emptyset & r_A \\ r_A^* & \emptyset \end{pmatrix} \quad r^* = \begin{pmatrix} r_{\mathrm{L}}^* & 0 \\ 0 & r_{\mathrm{R}}^* \end{pmatrix}$$

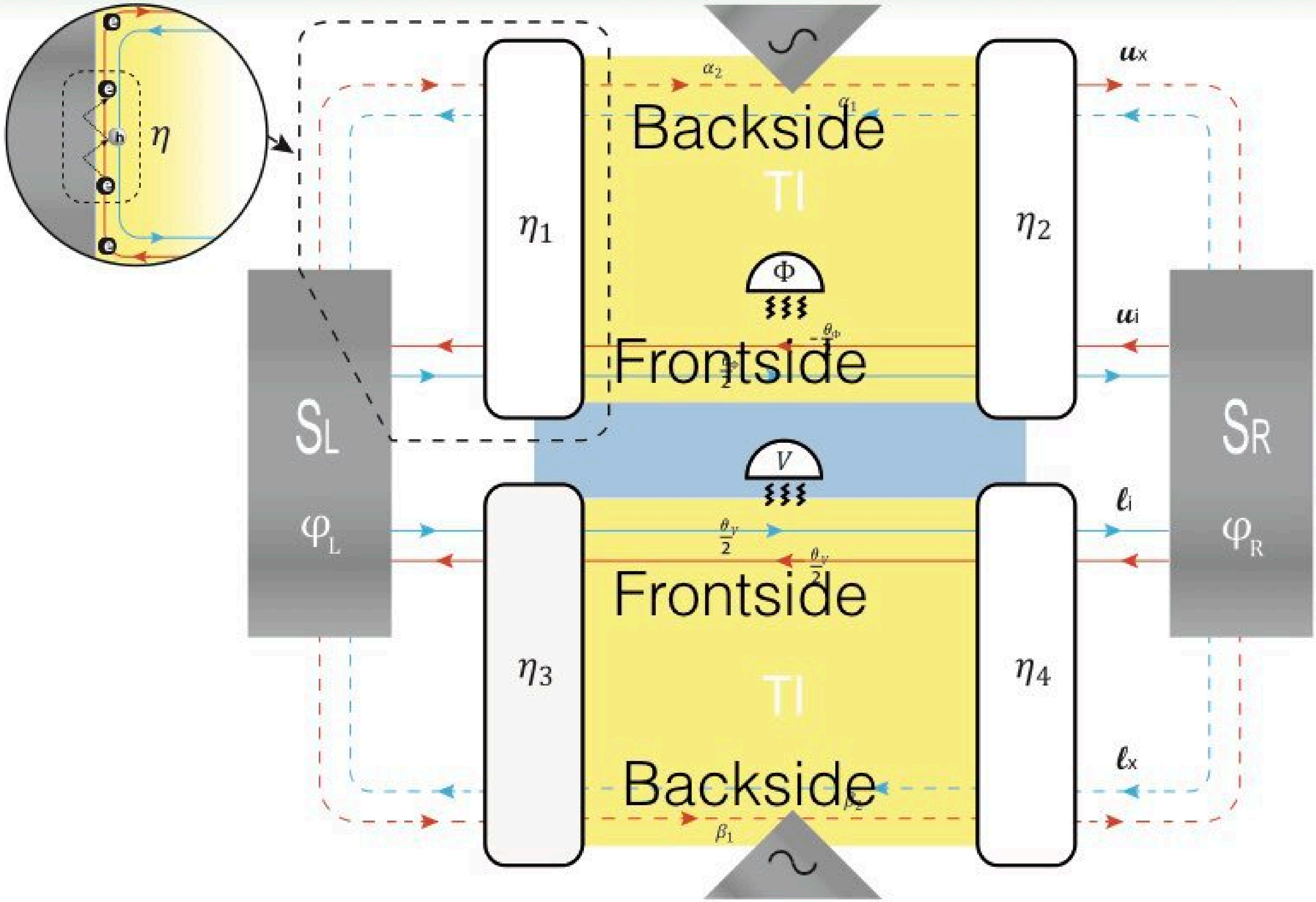
Superposition of
LAR & CAR

$$r^*_{\mathrm{L(R)}} = \begin{pmatrix} |\Lambda_{\mathrm{L(R)}}| & i|X_{\mathrm{L(R)}}| \\ i|X_{\mathrm{L(R)}}| & |\Lambda_{\mathrm{L(R)}}| \end{pmatrix} e^{i\phi_{\mathrm{L(R)}}}$$

Full Model:losses



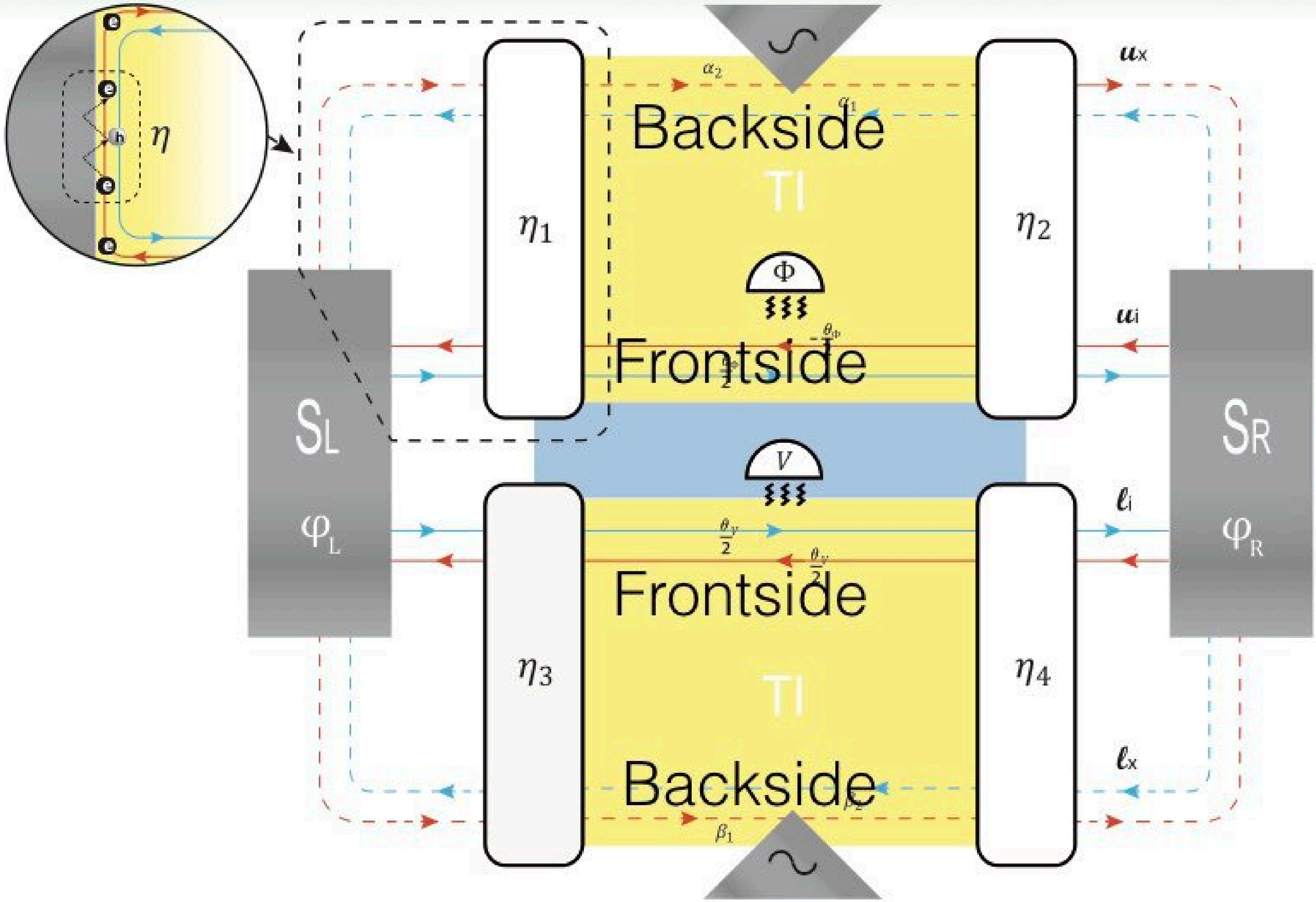
Full Model:losses



- Dephasing

$$\bar{J}(\phi,\theta_\Phi,\theta_V)=\frac{1}{(2\pi)^4}\int_0^{2\pi} d\alpha_1 d\alpha_2 d\beta_1 d\beta_2\ J(\phi,\theta_\Phi,\theta_V,\alpha_1,\alpha_2,\beta_1,\beta_2)$$

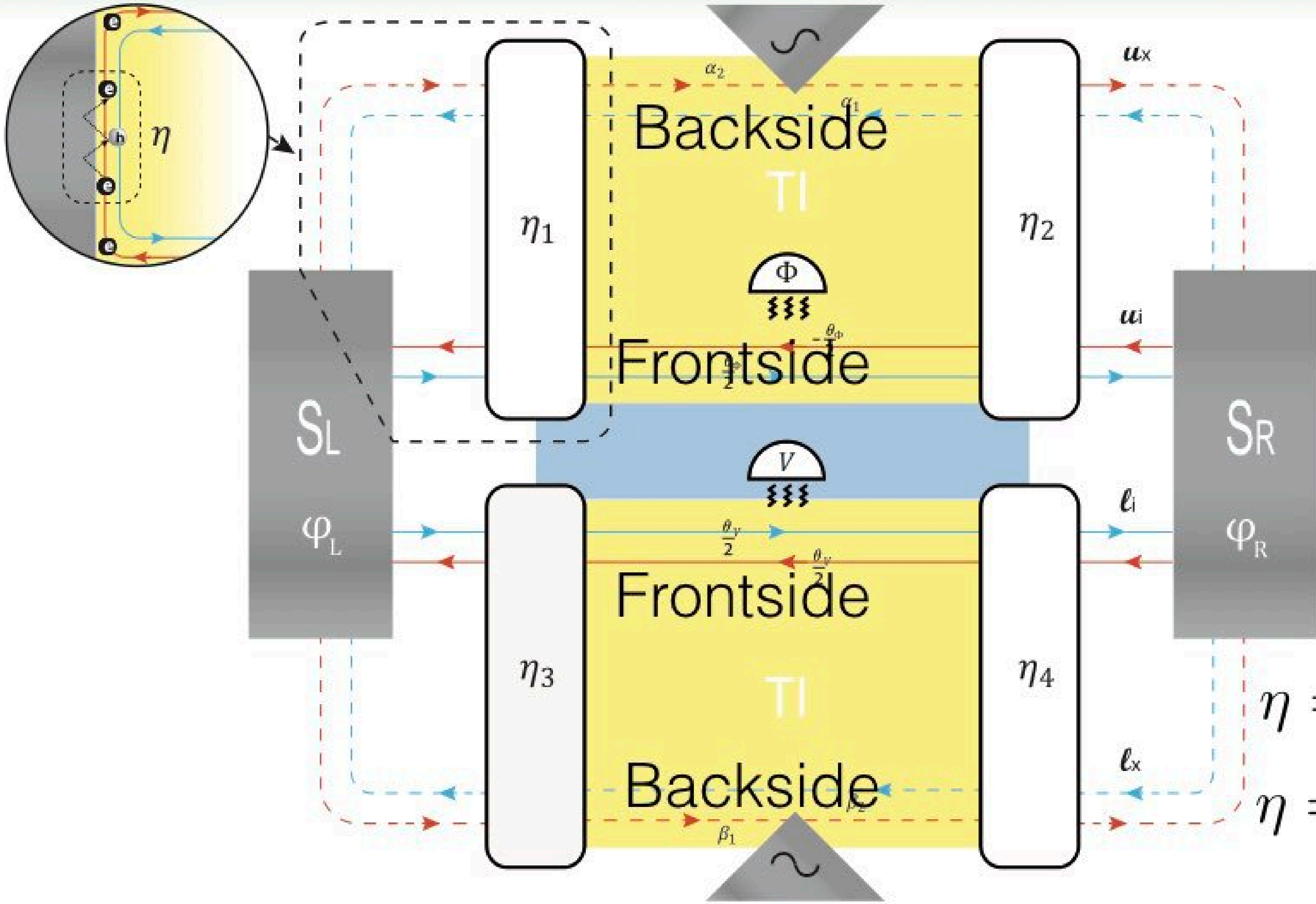
Full Model:losses



- Dephasing
- Losses in backside modes η

$$\bar{J}(\phi,\theta_\Phi,\theta_V)=\frac{1}{(2\pi)^4}\int_0^{2\pi} d\alpha_1 d\alpha_2 d\beta_1 d\beta_2\ J(\phi,\theta_\Phi,\theta_V,\alpha_1,\alpha_2,\beta_1,\beta_2)$$

Full Model:losses



- Dephasing
- Losses in backside modes η

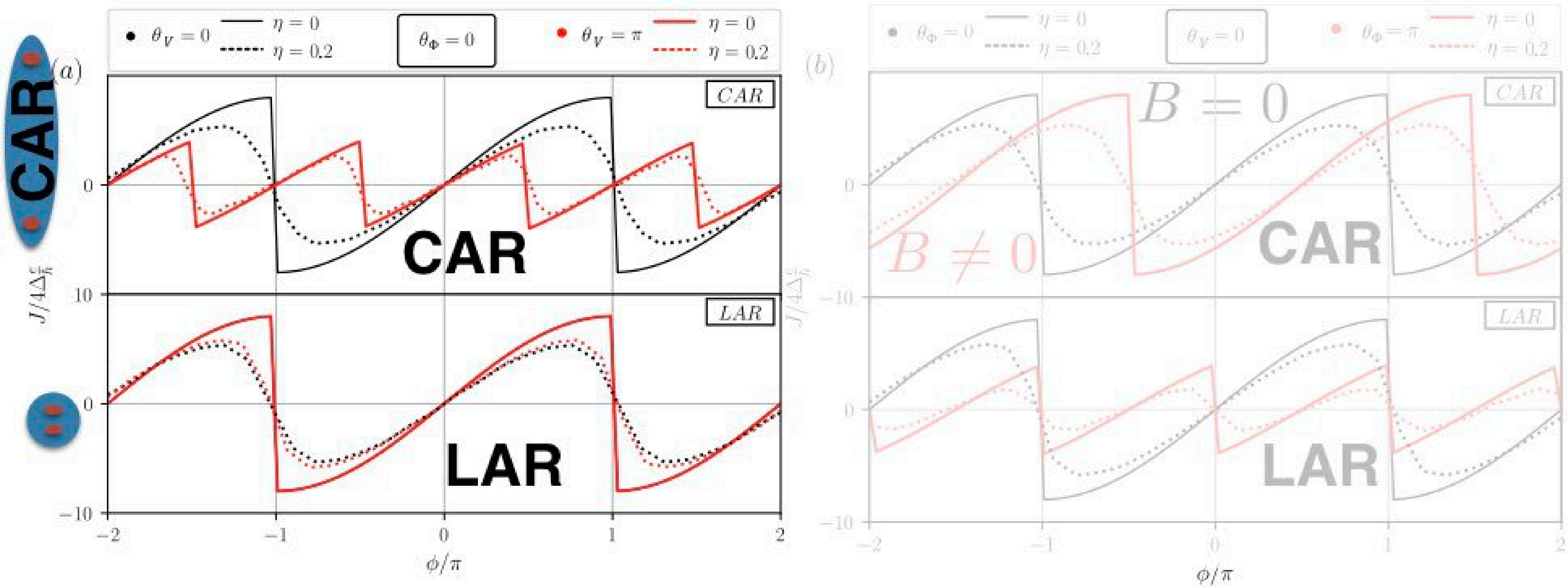
$\eta = 0$ No losses

$\eta = 1$ No Joseph current

$$\bar{J}(\phi, \theta_\Phi, \theta_V) = \frac{1}{(2\pi)^4} \int_0^{2\pi} d\alpha_1 d\alpha_2 d\beta_1 d\beta_2 \; J(\phi, \theta_\Phi, \theta_V, \alpha_1, \alpha_2, \beta_1, \beta_2)$$

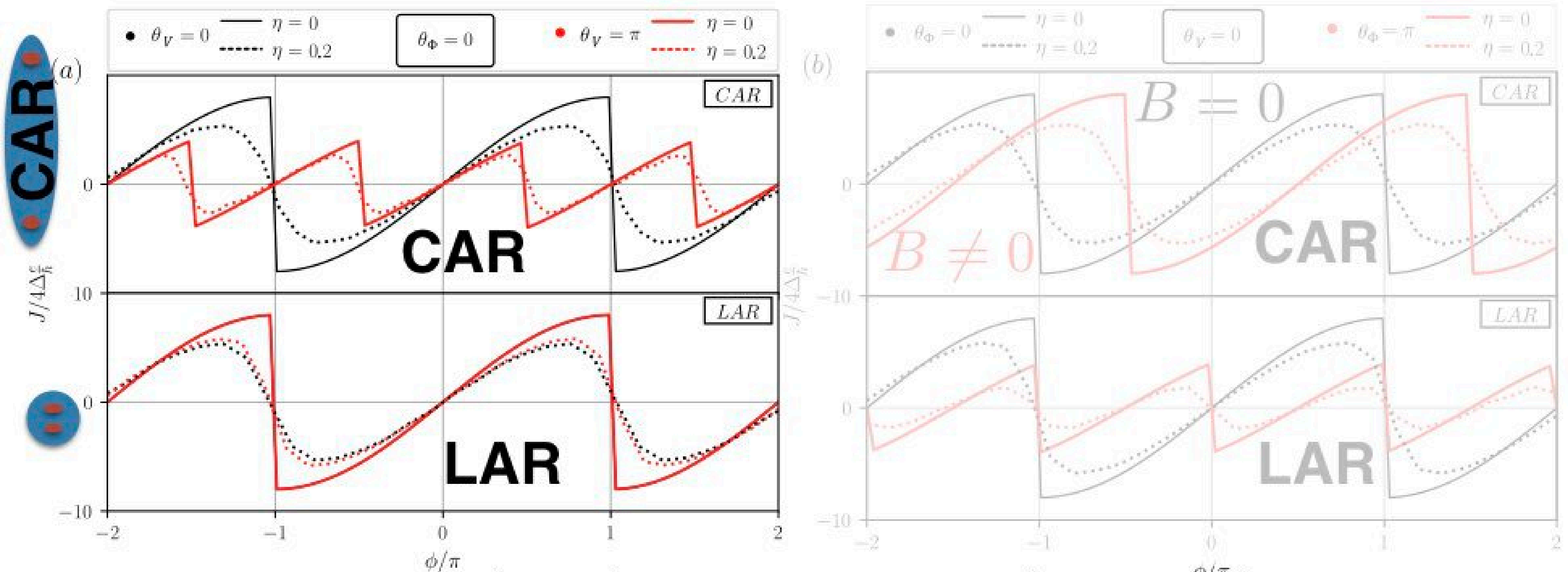
CPR of the hybrid TI JJ

Analytical ········ Numerics with losses



CPR of the hybrid TI JJ

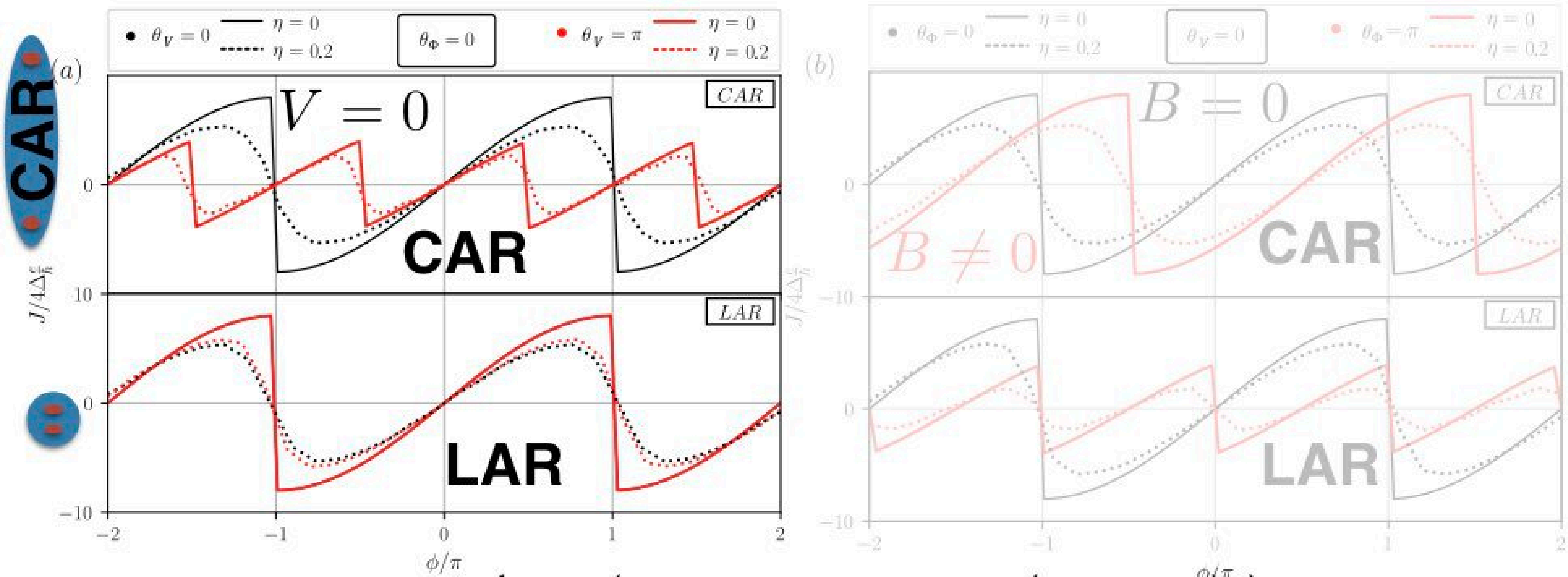
Analytical .... Numerics with losses



$$J(\phi) = 4\frac{e\Delta_0}{\hbar}\sum_{\sigma=\pm}\left\{\sin\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\times\right.$$
$$\left.\tanh\left[\frac{\Delta_0}{2k_BT}\cos\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\right]\right\}$$

CPR of the hybrid TI JJ

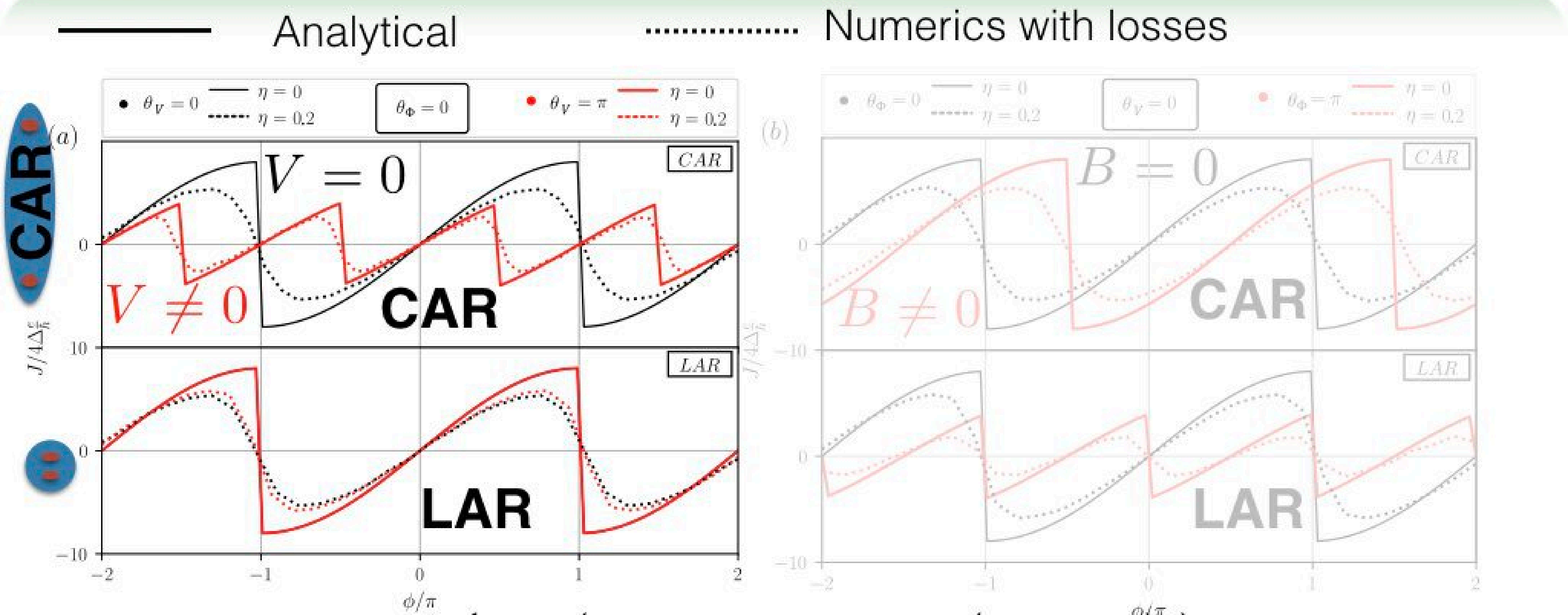
Analytical .... Numerics with losses



$$J(\phi) = 4\frac{e\Delta_0}{\hbar}\sum_{\sigma=\pm}\left\{\sin\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\times\right.$$

$$\left.\tanh\left[\frac{\Delta_0}{2k_BT}\cos\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\right]\right\}$$

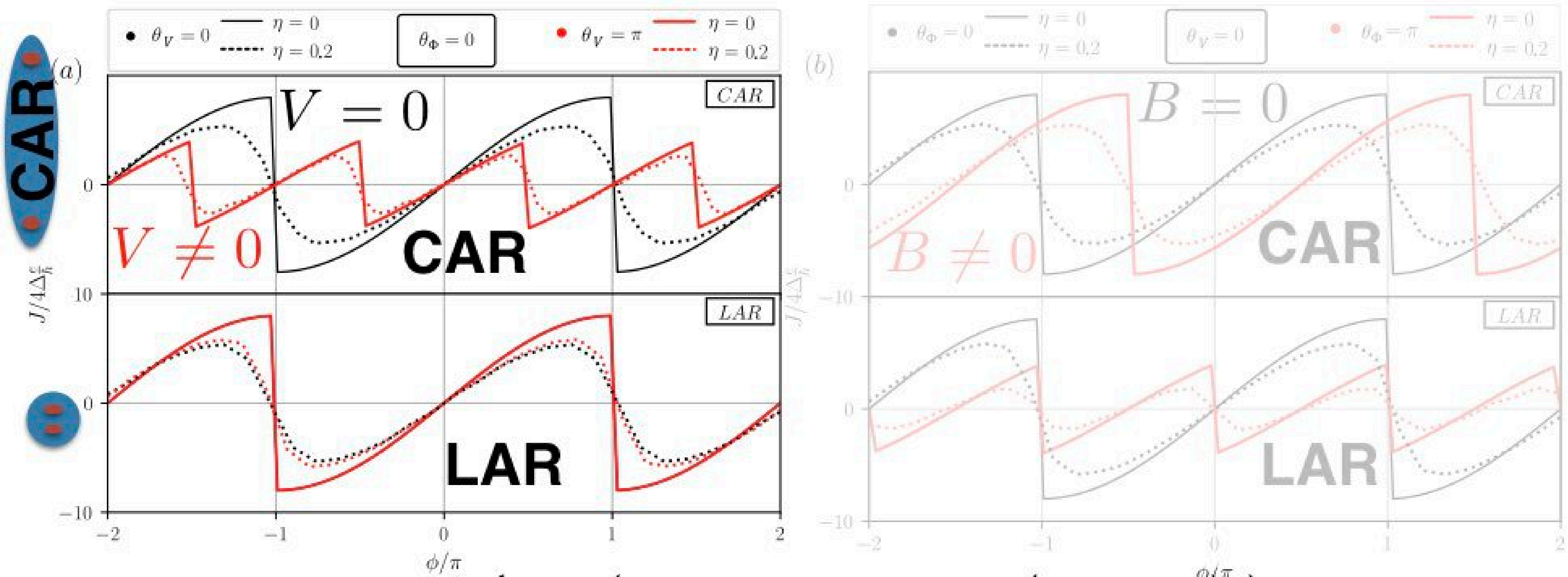
CPR of the hybrid TI JJ



$$J(\phi) = 4\frac{e\Delta_0}{\hbar}\sum_{\sigma=\pm}\left\{\sin\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\times\right.$$
$$\left.\tanh\left[\frac{\Delta_0}{2k_BT}\cos\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\right]\right\}$$

CPR of the hybrid TI JJ

Analytical ······ Numerics with losses



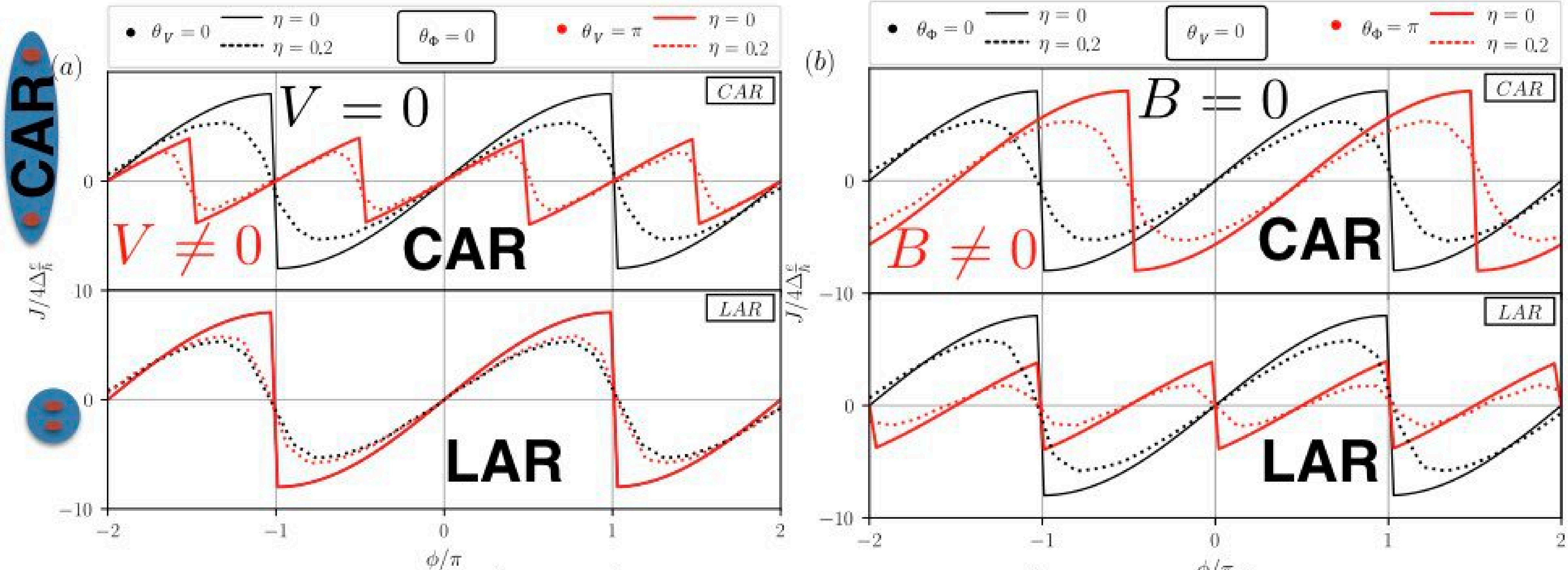
$$J(\phi) = 4\frac{e\Delta_0}{\hbar}\sum_{\sigma=\pm}\left\{\sin\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\times\right.$$

$$\left.\tanh\left[\frac{\Delta_0}{2k_BT}\cos\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\right]\right\}$$

$$\Gamma = \cos(\theta_V/2)|X_L||X_R| + \cos(\theta_\Phi/2)|\Lambda_L||\Lambda_R|$$

CPR of the hybrid TI JJ

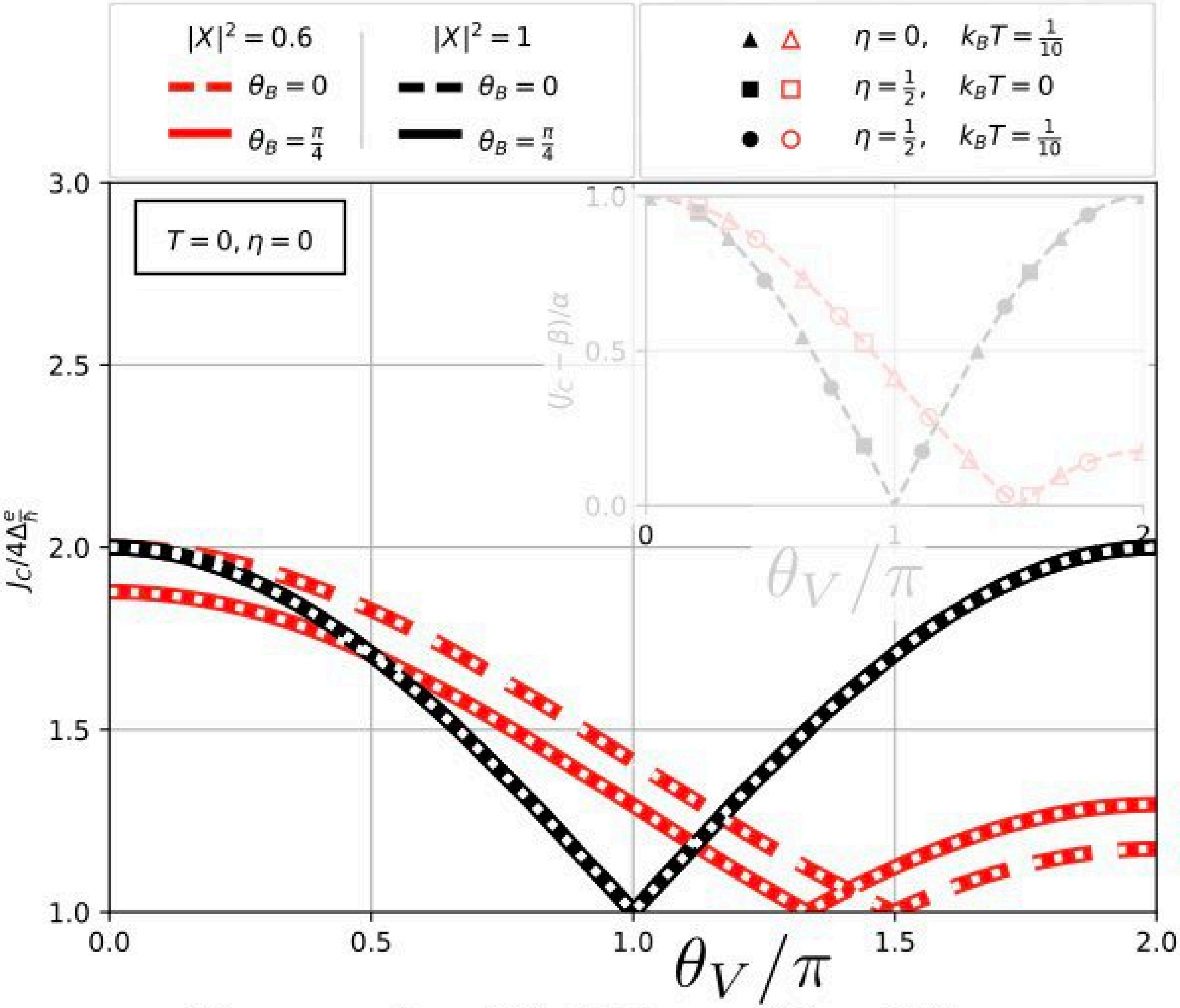
Analytical .... Numerics with losses



$$J(\phi) = 4\frac{e\Delta_0}{\hbar}\sum_{\sigma=\pm}\left\{\sin\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\times \tanh\left[\frac{\Delta_0}{2k_BT}\cos\left(\frac{\theta_\Phi}{4}+\frac{\phi}{2}+\sigma\tan^{-1}\left(\sqrt{\frac{1-\Gamma}{1+\Gamma}}\right)\right)\right]\right\}$$

$$\Gamma = \cos\left(\theta_{\mathrm{V}}/2\right)|X_L||X_R| + \cos\left(\theta_\Phi/2\right)|\Lambda_L||\Lambda_R|$$

Critical current



- $\theta_{\rm V}$ Dependence

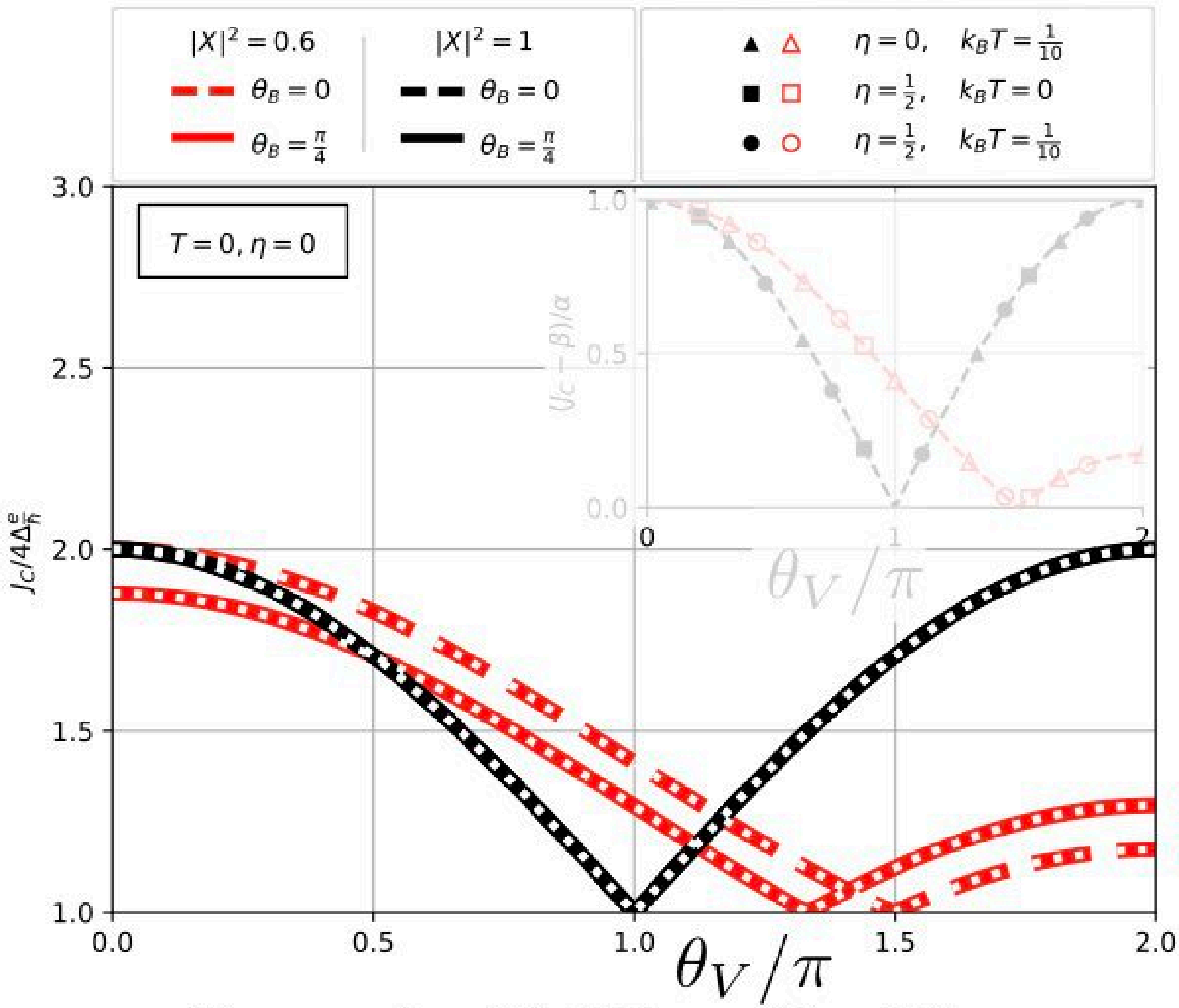
CAR are present!

$$J_{\rm c} = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$

$$\Gamma = \cos(\theta_{\rm V}/2)|X_L||X_R| + \cos(\theta_\Phi/2)|\Lambda_L||\Lambda_R|$$

G. Blasi, F.Taddei,V.Giovannetti, AB PRB '19

Critical current



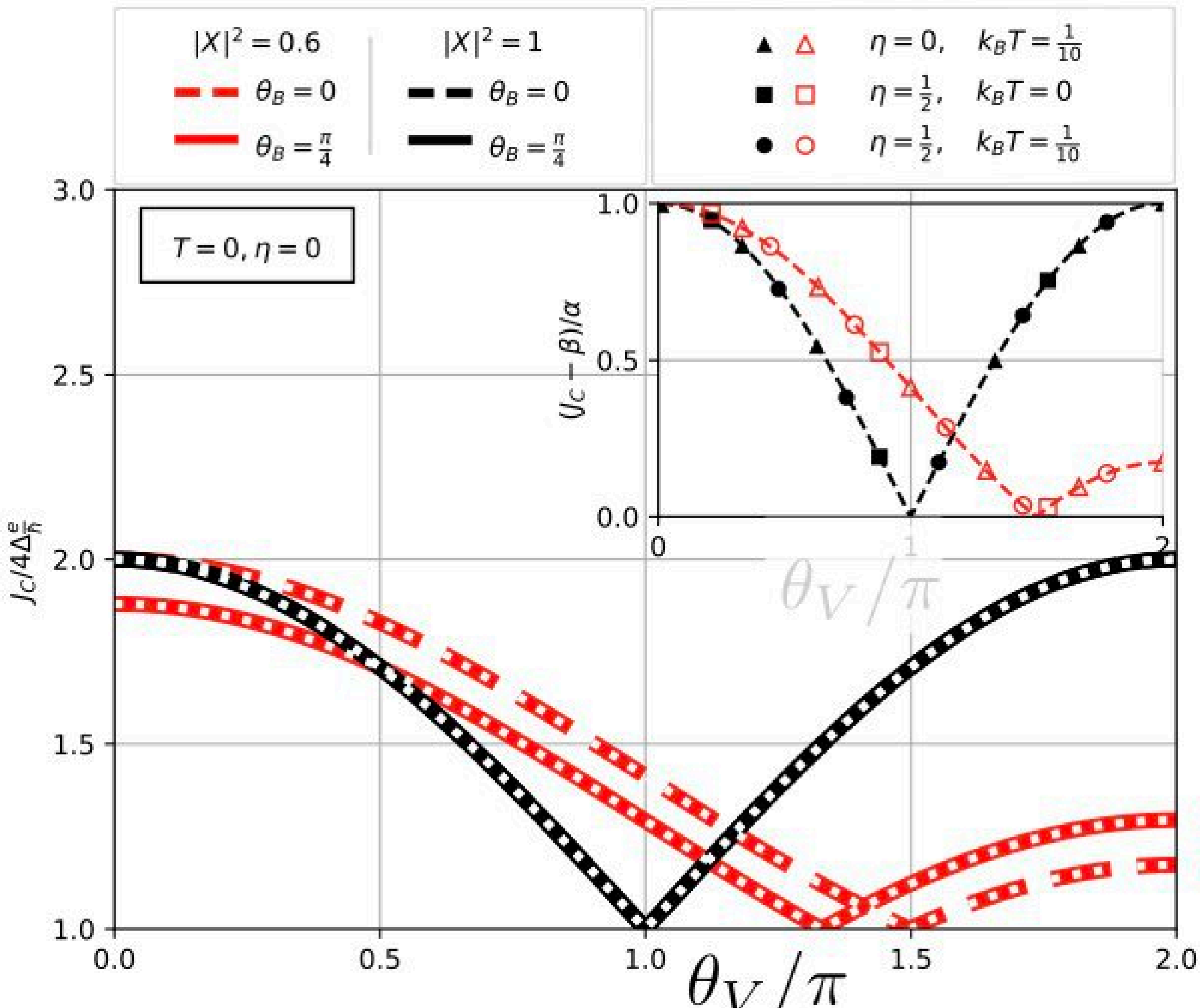
- θ_{V} Dependence CAR are present!
- Shape determines the CAR/LAR ratio

$$J_{\mathrm{c}} = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$

$$\Gamma = \cos(\theta_{\mathrm{V}}/2)|X_L||X_R| + \cos(\theta_\Phi/2)|\Lambda_L||\Lambda_R|$$

G. Blasi, F.Taddei,V.Giovannetti, AB PRB '19

Critical current



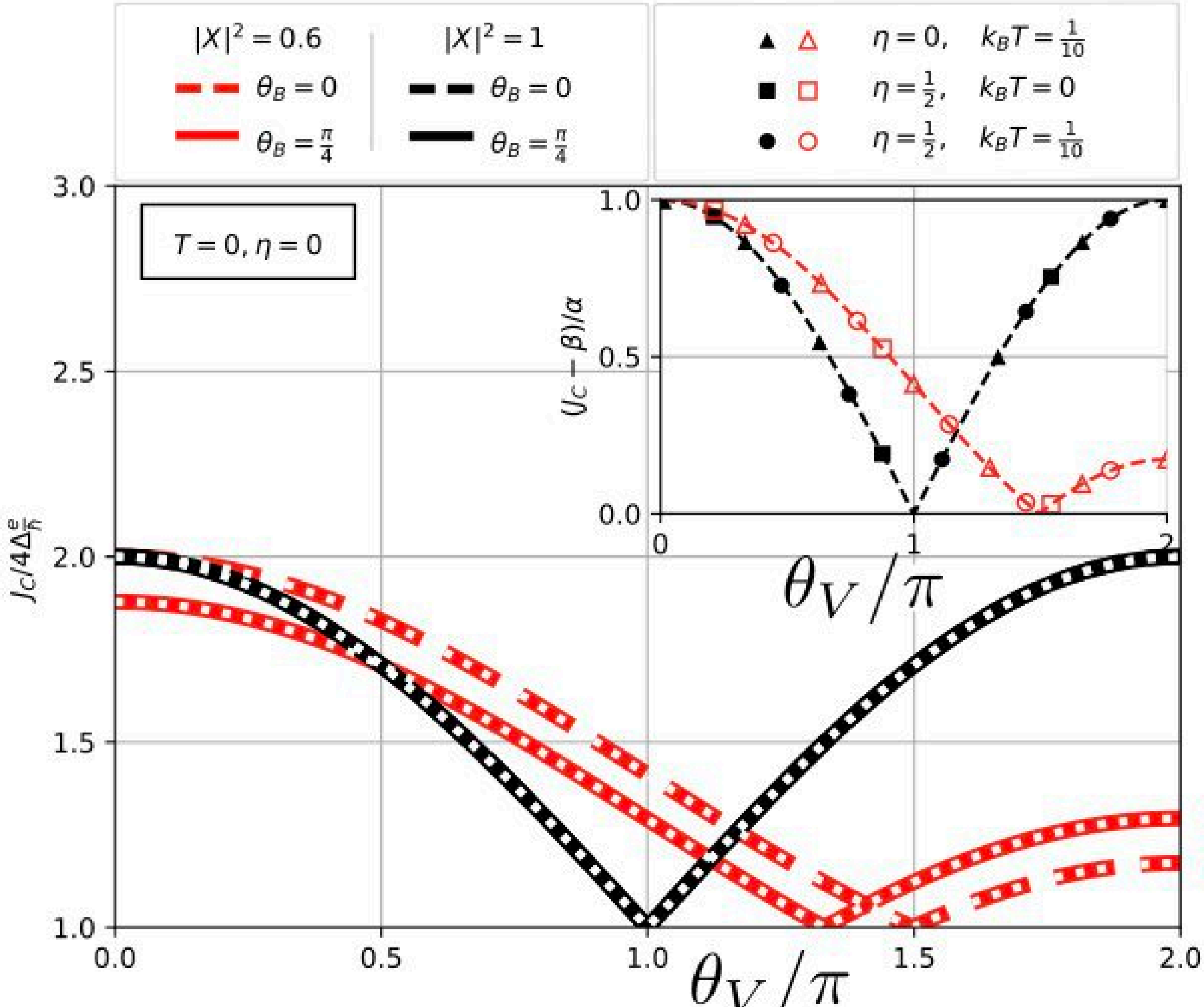
- $\theta_{\rm V}$ Dependence CAR are present!
- Shape determines the CAR/LAR ratio
- Universal independent by temperatures or losses

$$J_{\rm c} = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$

$$\Gamma = \cos(\theta_{\rm V}/2)|X_L||X_R| + \cos(\theta_\Phi/2)|\Lambda_L||\Lambda_R|$$

G. Blasi, F.Taddei,V.Giovannetti, AB PRB '19

Critical current



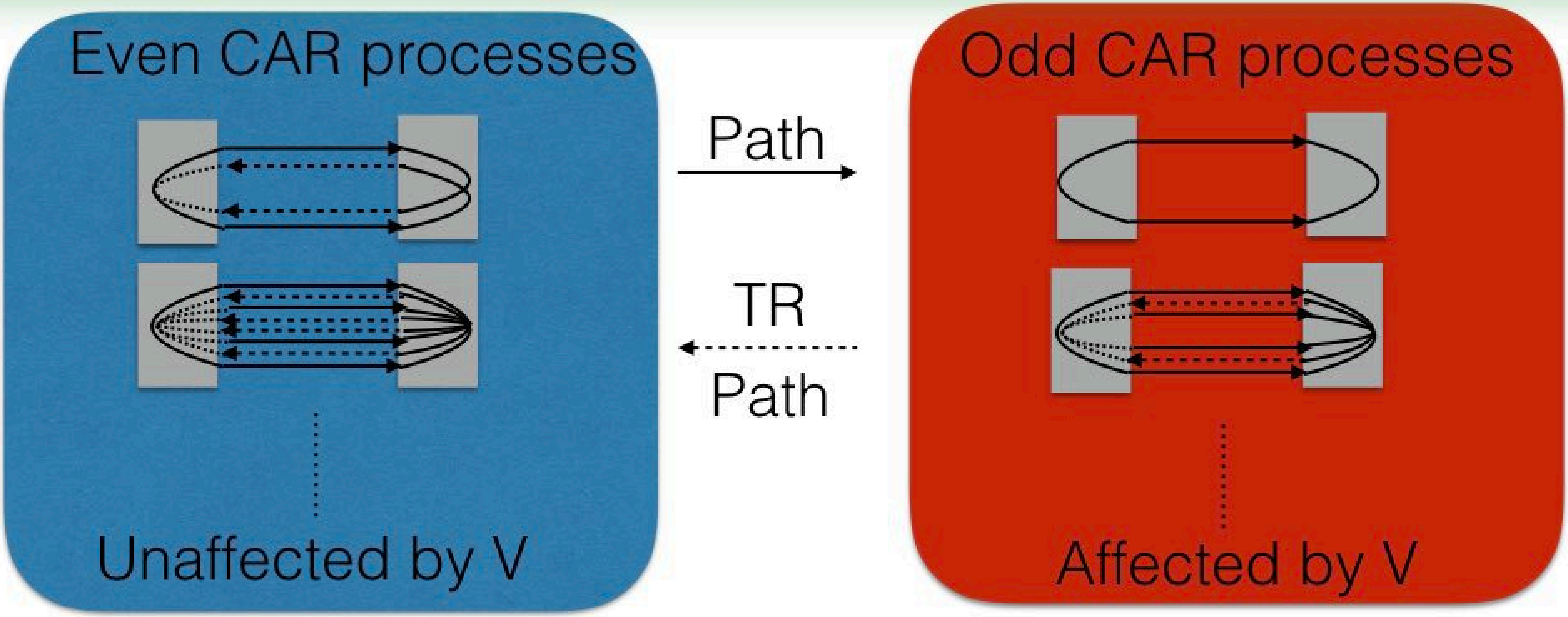
- θ_{V} Dependence
CAR are present!
- Shape determines the CAR/LAR ratio
- Universal independent by temperatures or losses
- Not complete suppression
Multiple Andreev reflections

$$J_{\mathrm{c}} = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$

$$\Gamma = \cos(\theta_{\mathrm{V}}/2)|X_L||X_R| + \cos(\theta_{\Phi}/2)|\Lambda_L||\Lambda_R|$$

G. Blasi, F.Taddei,V.Giovannetti, AB PRB '19

Multiple Andreev Reflections

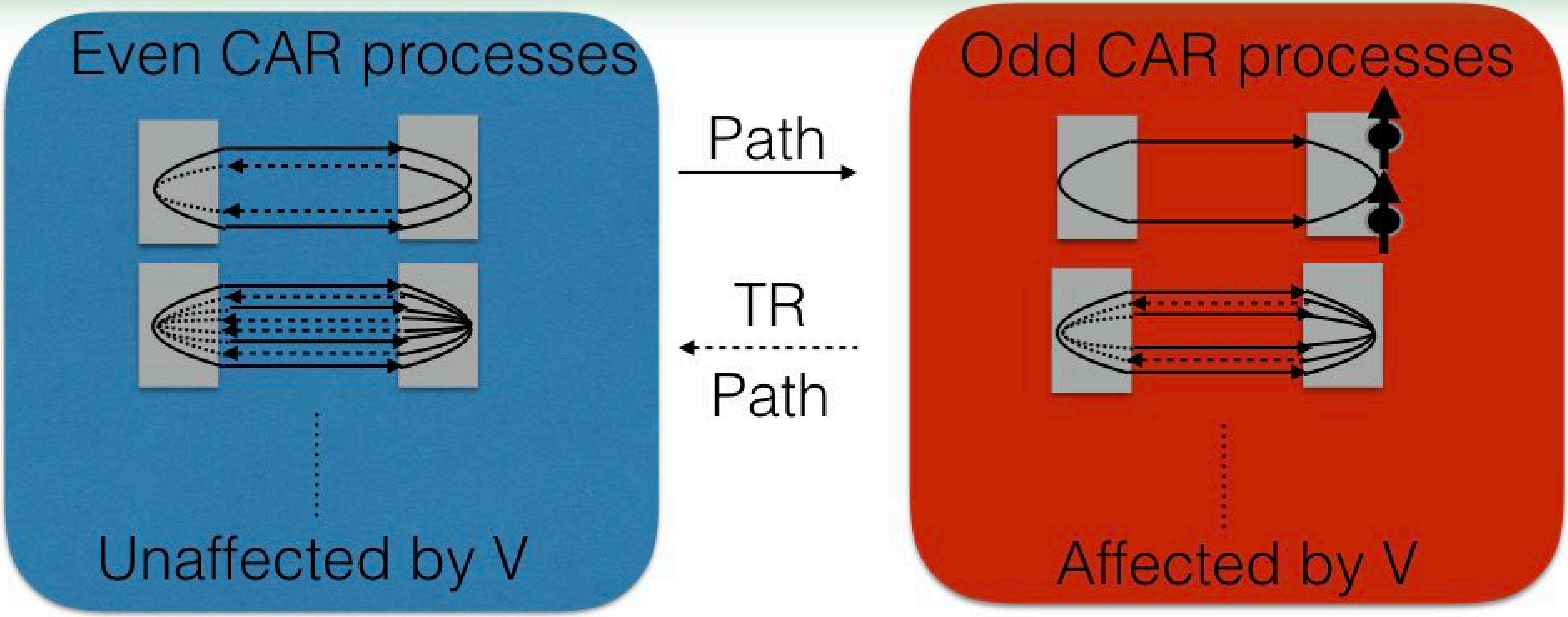


$T = \eta = 0$ $\qquad J_c = 4\frac{e\Delta_0}{\hbar}\left(1 + |\cos(\theta_{\rm V}/2)|\right)$

$$J_c = \beta(\eta, T) + \alpha(\eta, T)\,|\Gamma|$$

$\eta \approx 1$ $\qquad J_c = \frac{e\Delta_0}{\hbar}|\cos\left(\theta_{\rm V}/2\right)|(1-\eta)^4 + \mathcal{O}((1-\eta)^6)$

Multiple Andreev Reflections

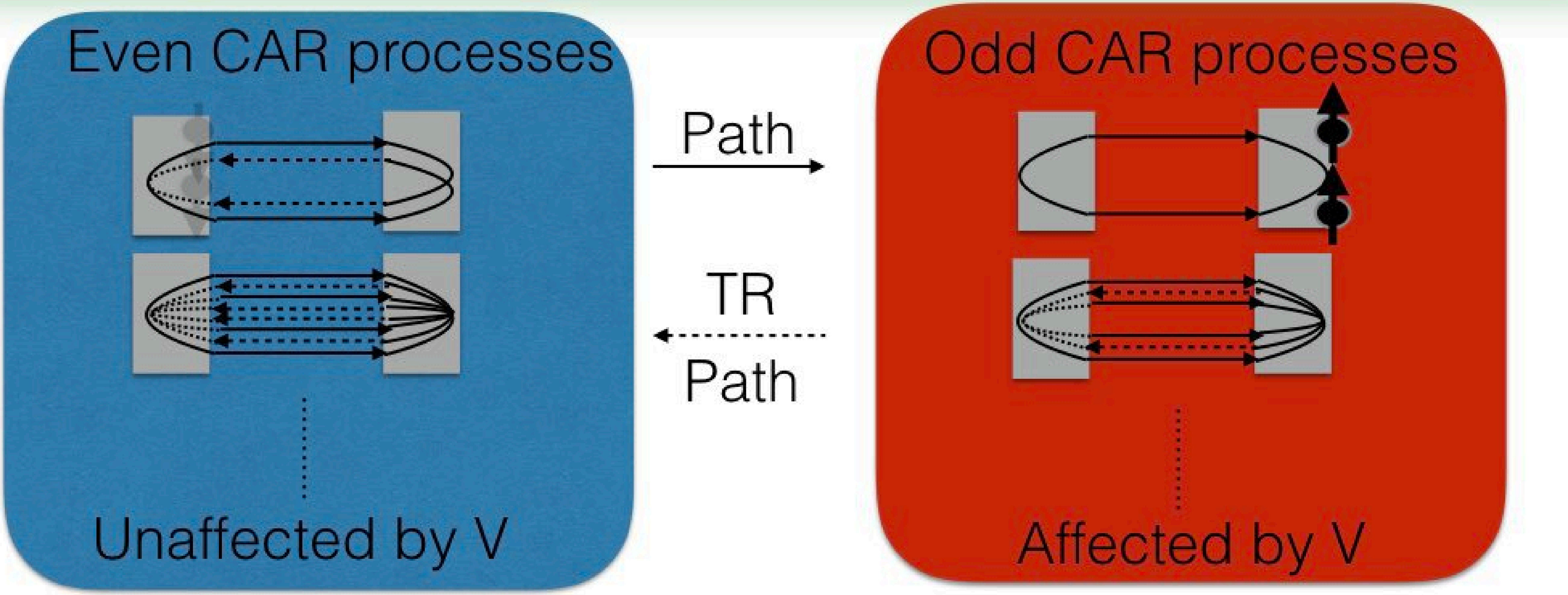


$T = \eta = 0$ $$J_c = 4\frac{e\Delta_0}{\hbar}\left(1 + |\cos(\theta_V/2)|\right)$$

$$J_c = \beta(\eta, T) + \alpha(\eta, T)\,|\Gamma|$$

$\eta \approx 1$ $$J_c = \frac{e\Delta_0}{\hbar}|\cos(\theta_V/2)|(1-\eta)^4 + \mathcal{O}((1-\eta)^6)$$

Multiple Andreev Reflections

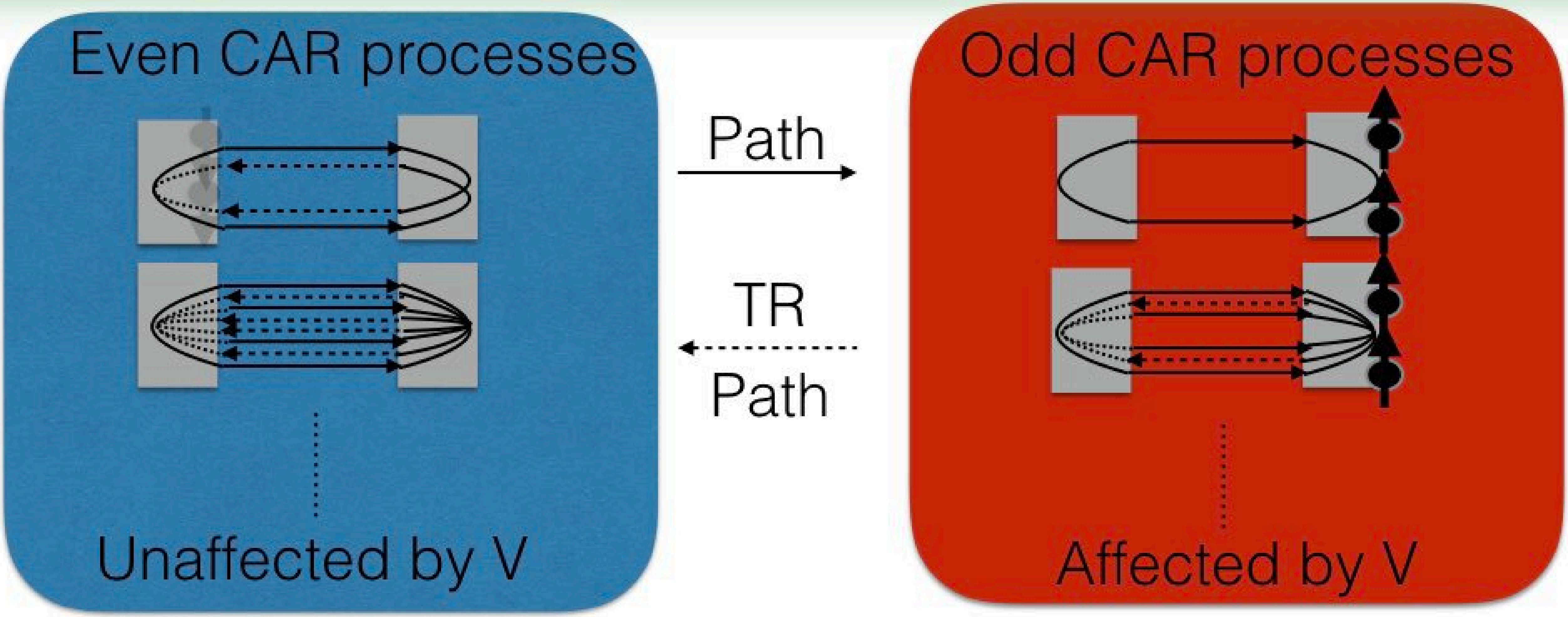


$$T = \eta = 0 \qquad J_{\mathrm{c}} = 4\frac{e\Delta_0}{\hbar}\left(1 + \left|\cos(\theta_{\mathrm{V}}/2)\right|\right)$$

$$J_{\mathrm{c}} = \beta(\eta, T) + \alpha(\eta, T)\,|\Gamma|$$

$$\eta \approx 1 \qquad J_c = \frac{e\Delta_0}{\hbar}\left|\cos\left(\theta_{\mathrm{V}}/2\right)\right|(1-\eta)^4 + \mathcal{O}((1-\eta)^6)$$

Multiple Andreev Reflections



$T = \eta = 0$

$$J_c = 4\frac{e\Delta_0}{\hbar}\left(1 + |\cos(\theta_V/2)|\right)$$

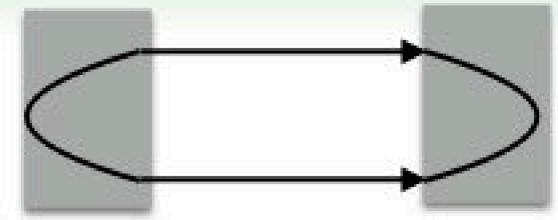
$$J_c = \beta(\eta, T) + \alpha(\eta, T)\,|\Gamma|$$

$\eta \approx 1$

$$J_c = \frac{e\Delta_0}{\hbar}|\cos(\theta_V/2)|(1-\eta)^4 + \mathcal{O}((1-\eta)^6)$$

Single-Shot Limit

$\eta \approx 1$ Electron losses at the S-TI junctions



Application of V $\theta_\Phi = 0$

$\boxed{\text{CAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\cos\left(\frac{\theta_V}{2}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6),$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6).$$

Application of B $\theta_V = 0$

$\boxed{\text{CAR}}$:

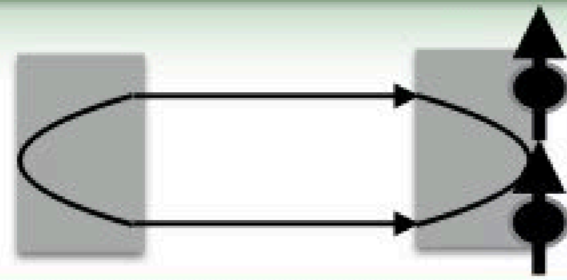
$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin\left(\phi + \frac{\theta_\Phi}{2}\right)(1-\eta)^4 + \mathcal{O}((1-\eta)^6), \tag{6a}$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\left[\sin(\phi) + \sin\left(\phi - \frac{\theta_\Phi}{2}\right)\right](1-\eta)^4 + \mathcal{O}((1-\eta)^6). \tag{6b}$$

Single-Shot Limit

$\eta \approx 1$ Electron losses at the S-TI junctions



Application of V $\theta_\Phi = 0$

$\boxed{\text{CAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\cos\left(\frac{\theta_V}{2}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6),$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6).$$

Application of B $\theta_V = 0$

$\boxed{\text{CAR}}$:

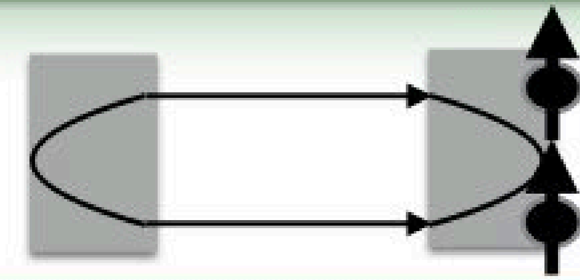
$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin\left(\phi + \frac{\theta_\Phi}{2}\right)(1-\eta)^4 + \mathcal{O}((1-\eta)^6), \tag{6a}$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\left[\sin(\phi) + \sin\left(\phi - \frac{\theta_\Phi}{2}\right)\right](1-\eta)^4 + \mathcal{O}((1-\eta)^6). \tag{6b}$$

Single-Shot Limit

$\eta \approx 1$ Electron losses at the S-TI junctions



Application of V $\theta_\Phi = 0$

$\boxed{\text{CAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\cos\left(\frac{\theta_{\rm V}}{2}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6),$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin(\phi)(1-\eta)^4 + \mathcal{O}((1-\eta)^6).$$

Application of B $\theta_{\rm V} = 0$

$\boxed{\text{CAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\sin\left(\phi + \frac{\theta_\Phi}{2}\right)(1-\eta)^4 + \mathcal{O}((1-\eta)^6), \tag{6a}$$

$\boxed{\text{LAR}}$:

$$\bar{J}(\phi) = \frac{e}{\hbar}\Delta_0 \tanh\left(\frac{\Delta_0}{2T}\right)\left[\sin(\phi) + \sin\left(\phi - \frac{\theta_\Phi}{2}\right)\right](1-\eta)^4 + \mathcal{O}((1-\eta)^6). \tag{6b}$$

Critical current: Only odd CAR reflection processes

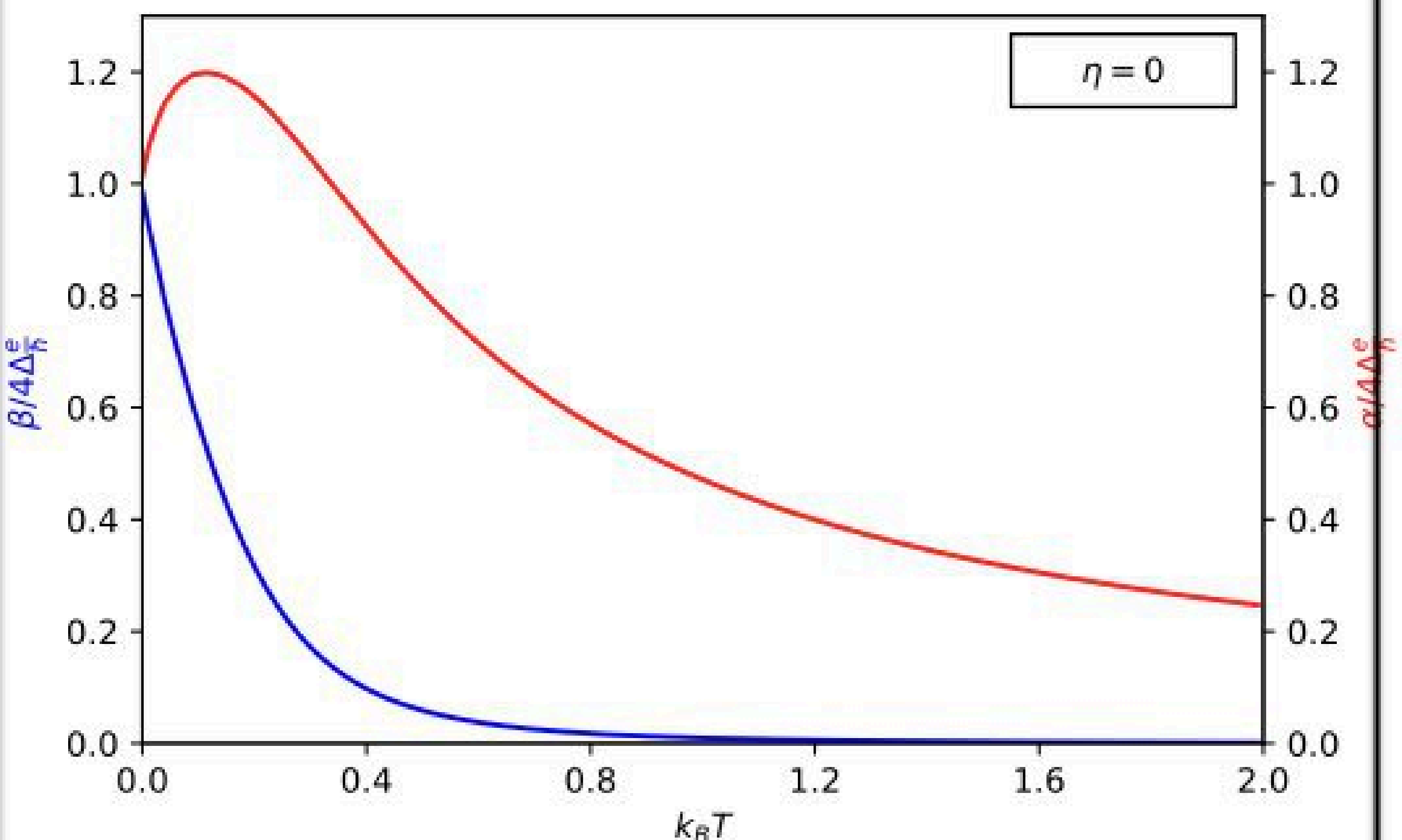
$\eta \approx 1$

$$J_c = \boxed{\frac{e\Delta_0}{\hbar}|\cos(\theta_{\rm V}/2)|(1-\eta)^4} + \mathcal{O}((1-\eta)^6)$$

Finite T and losses

Critical current

$$J_c = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$

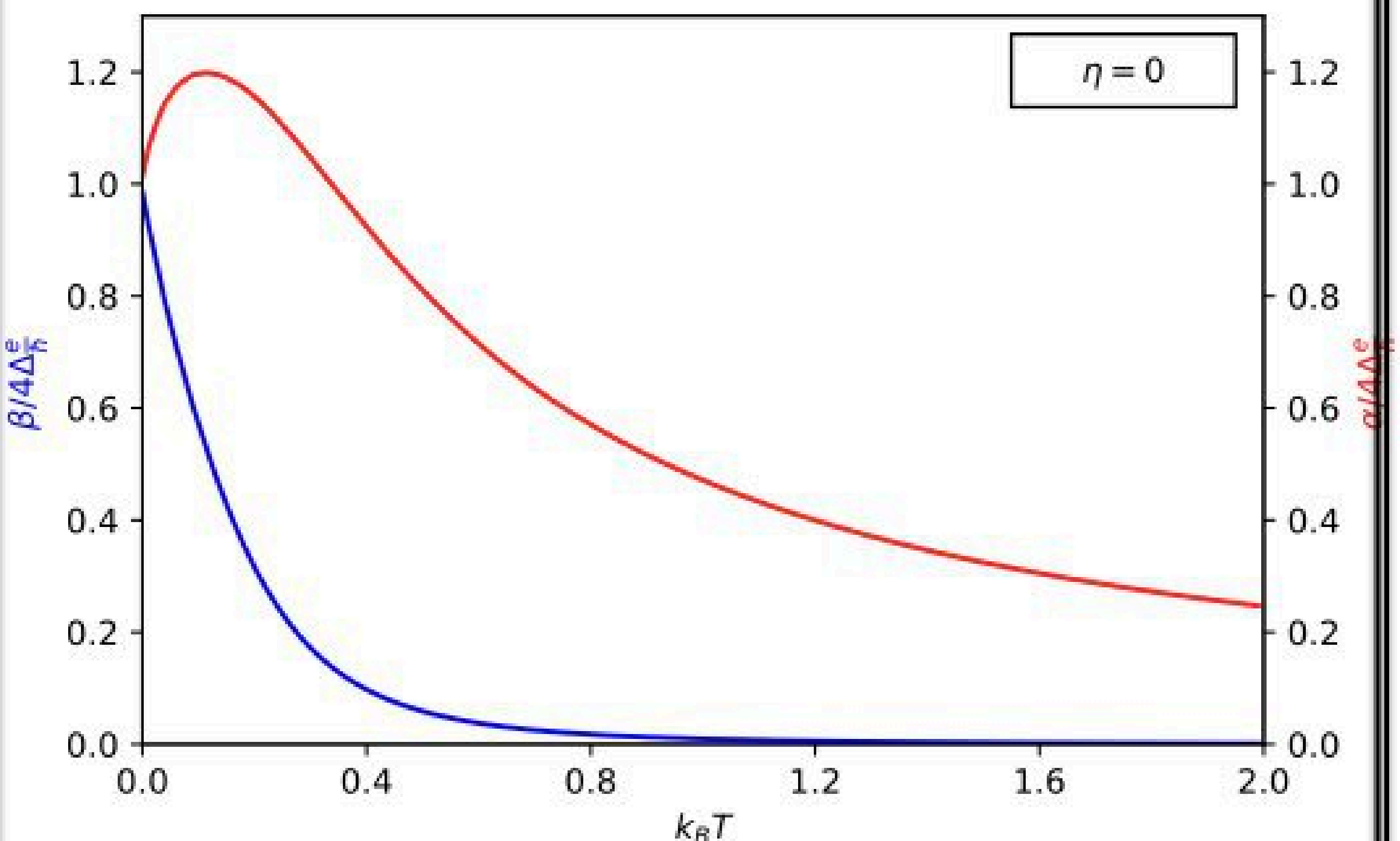


$$\begin{cases} J_c \simeq \dfrac{4e\Delta_0}{\hbar}\left(1 + |\Gamma(\theta_V, \theta_\Phi)|\right) \text{ for } k_B T \ll \Delta_0 \\ J_c \simeq \dfrac{4e\Delta_0}{\hbar}\dfrac{\Delta_0}{2k_B T}|\Gamma(\theta_V, \theta_\Phi)| \text{ for } k_B T \gg \Delta_0 \end{cases}$$

Finite T and losses

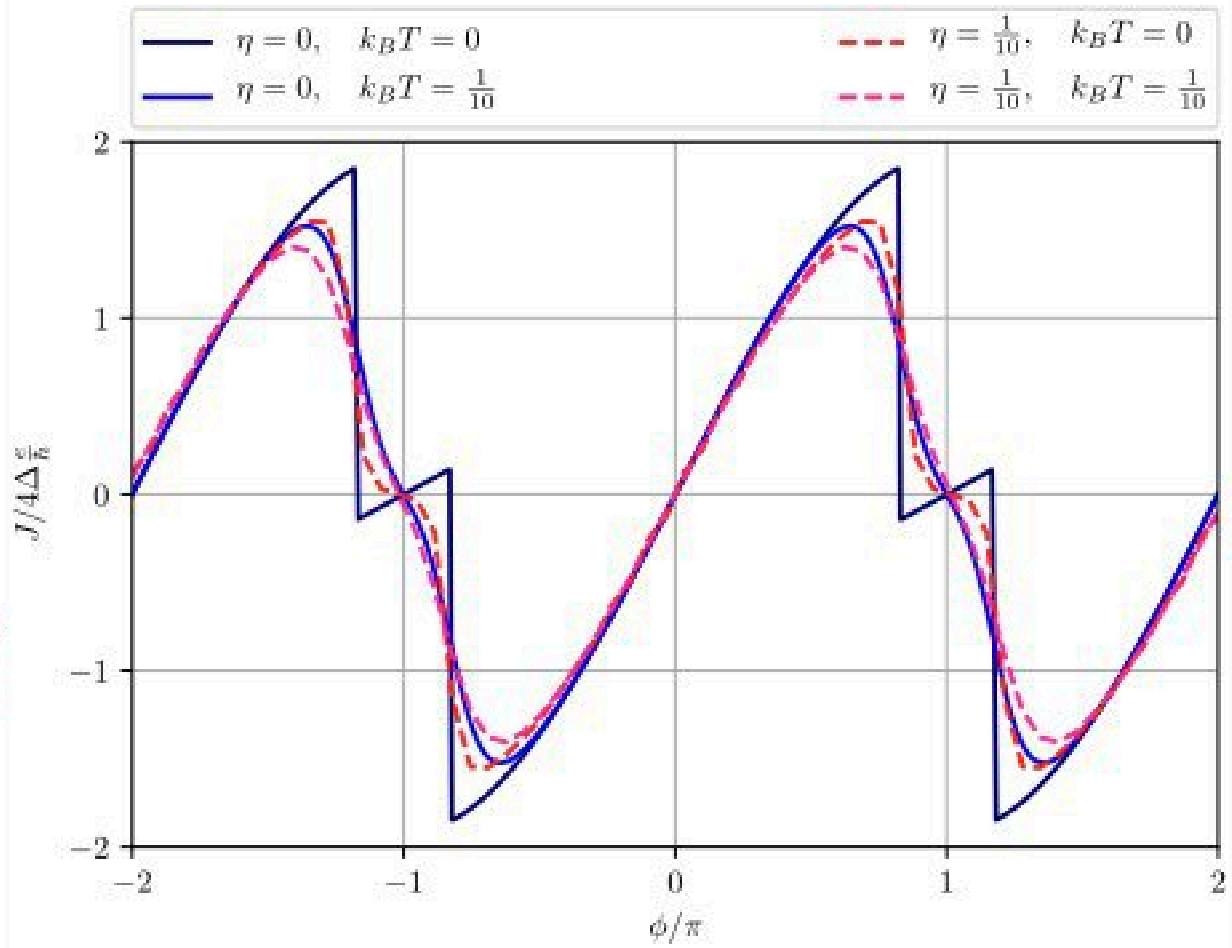
Critical current

$$J_{\mathrm{c}} = \alpha(\eta, T)\,|\Gamma| + \beta(\eta, T)$$



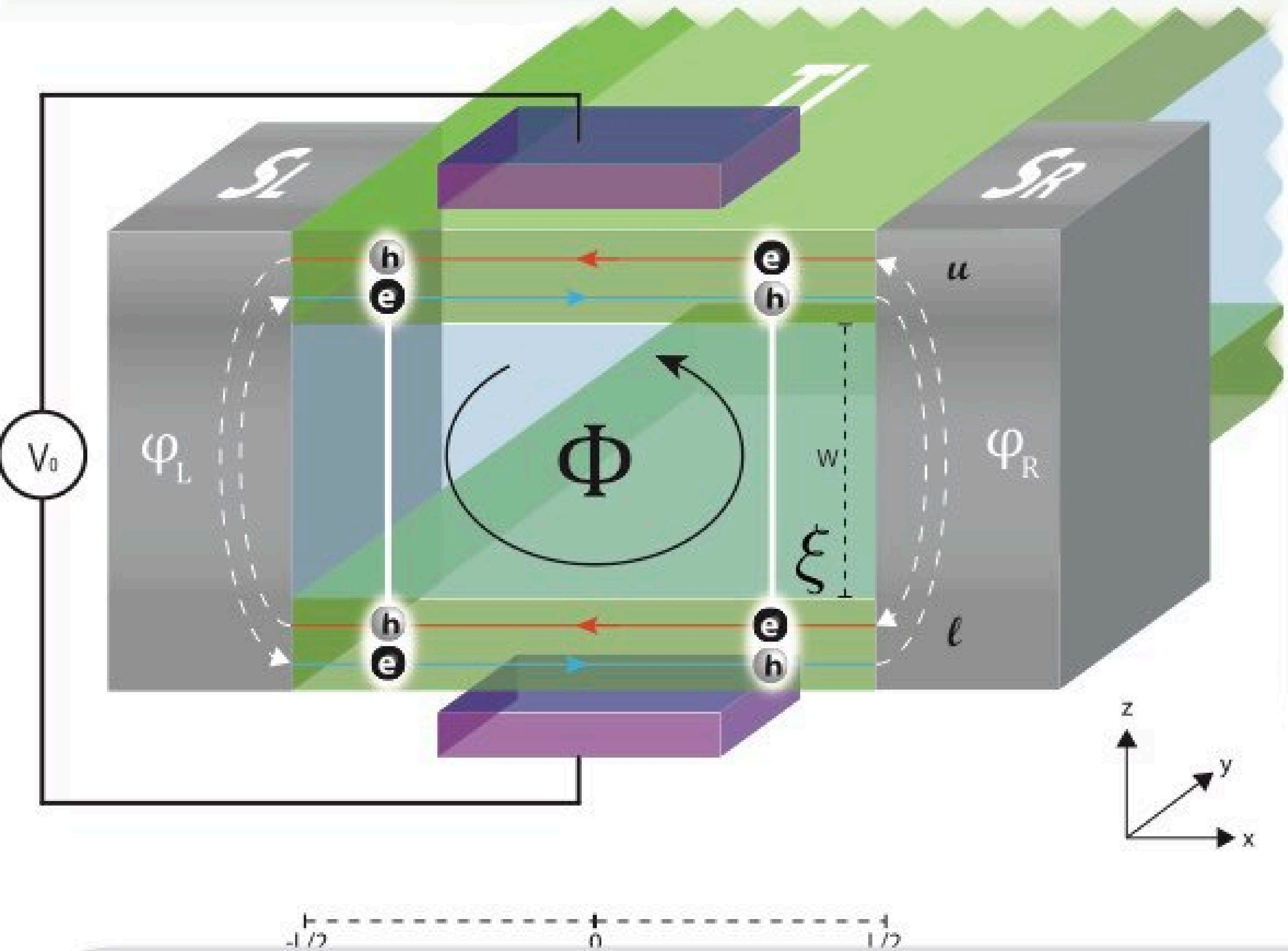
$$\begin{cases} J_{\mathrm{c}} \simeq \dfrac{4e\Delta_0}{\hbar}\left(1+|\Gamma(\theta_V,\theta_\Phi)|\right) \text{ for } k_B T \ll \Delta_0 \\ J_{\mathrm{c}} \simeq \dfrac{4e\Delta_0}{\hbar}\dfrac{\Delta_0}{2k_B T}|\Gamma(\theta_V,\theta_\Phi)| \text{ for } k_B T \gg \Delta_0 \end{cases}$$

CPR



- Temperature and losses smoothen the CPR
- Reduce the weight of higher-order MAR

Physical realisations



B

Doppler shift (breaks TRS)

$$\theta_\Phi = \pi$$
$$\theta_\Phi = 4\pi\phi/\phi_0$$

$$W \approx \xi \approx 100 \text{ nm}$$
$$L \approx \xi_{TI} \approx 600 \text{ nm}$$
$$B_y \approx 8 \text{ mT}$$

V

Local gating (TRS)

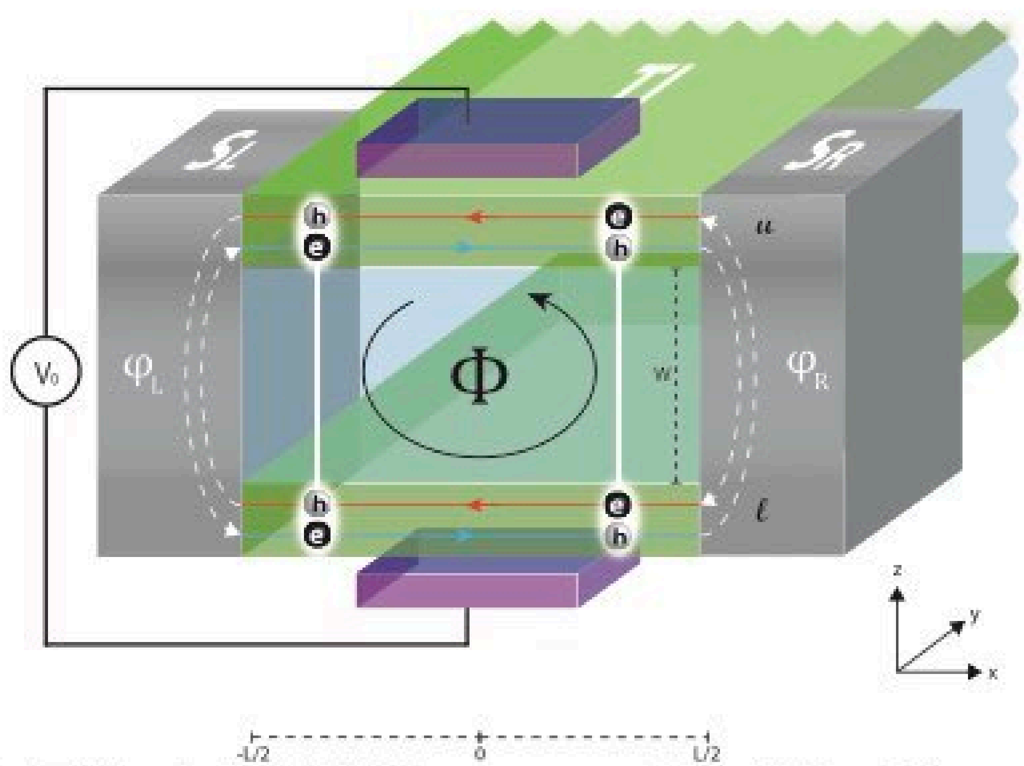
$$W \sim \xi \sim 100\,\text{nm}$$

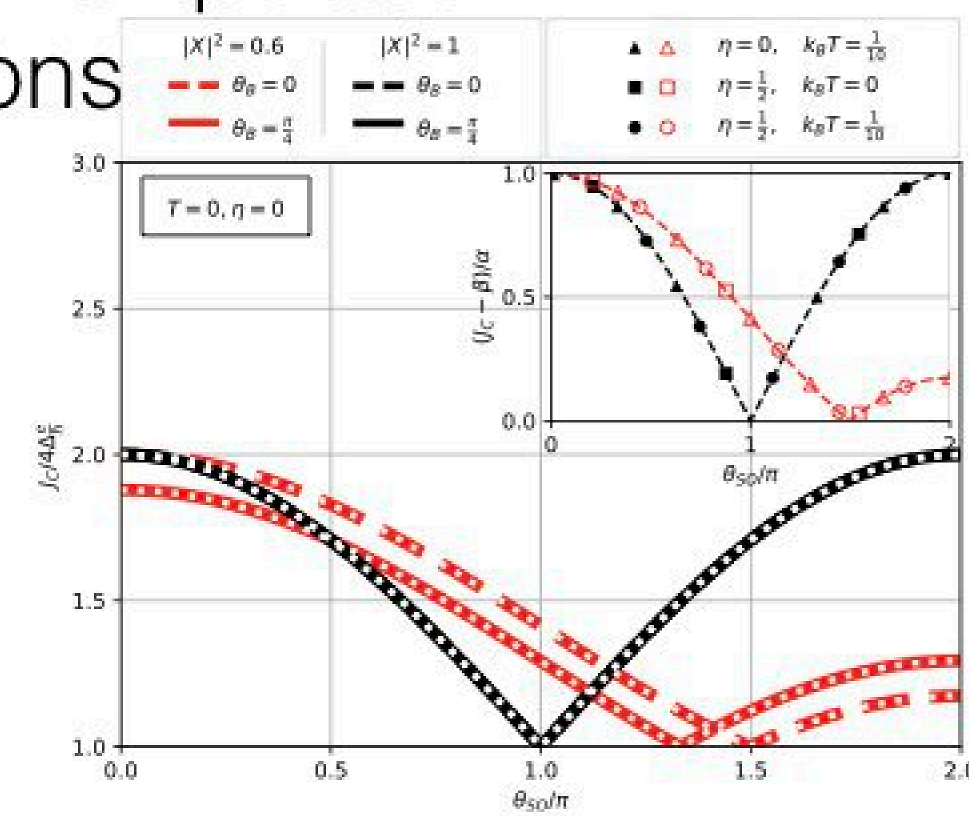
$$\theta_\text{V} = \pi$$
$$\theta_\text{V} = 2eV_0L/(\hbar v_F)$$
$$v_F = 10^5\text{m/s}$$
$$L \approx \xi_{TI} \approx 600 \text{ nm}$$

$$t = L/v_F$$
$$V_0 \approx 1.7\text{mV}$$

Conclusions & Perspectives

- CAR & LAR processes in hybrid TI Josephson junctions
- Entanglement symmetry manipulation of the Cooper pair with external potential
- CPR signature of the local vs nonlocal manipulation
- Critical current measurement of the manipulation
- Role of the multiple Andreev reflections



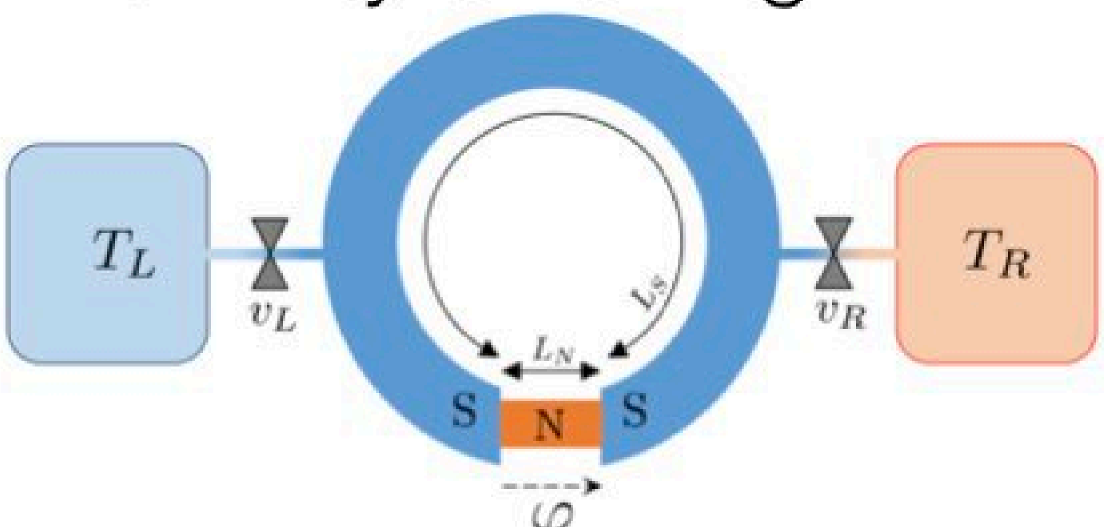


G. Blasi, F.Taddei,V.Giovannetti, AB, Phys. Rev. B 99, 064514 (2019) 1808.09709

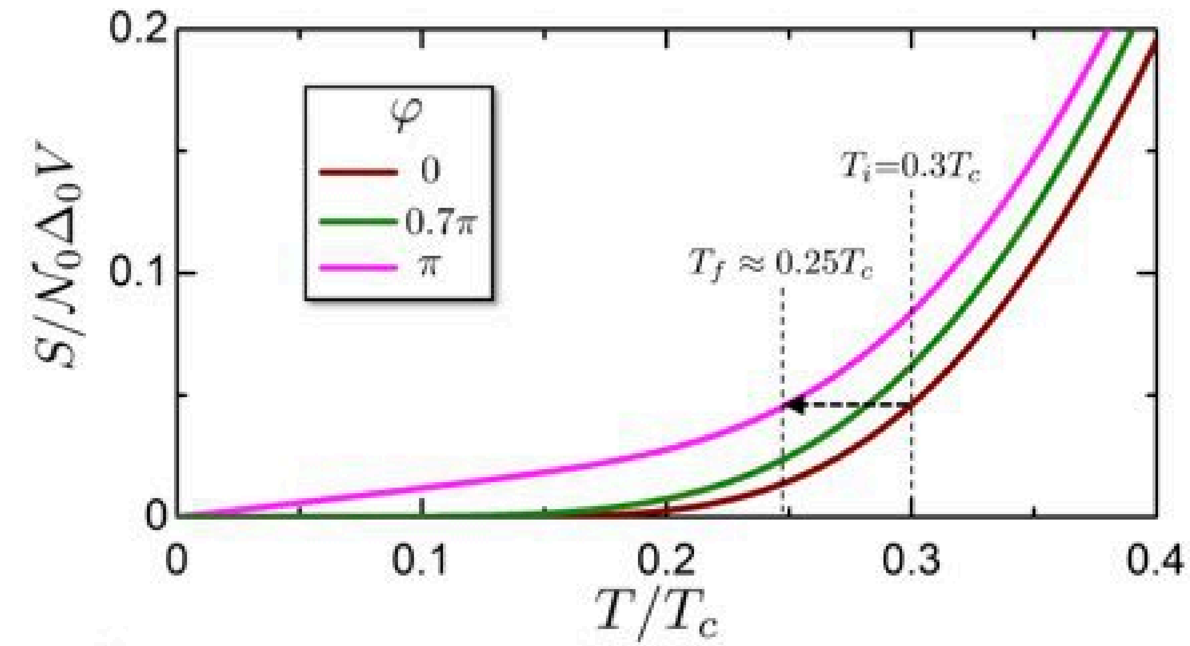
Thermodynamics cycles in Josephson junctions

F. Vischi, M. Carrega, P. Virtanen, E. Strambini, A. Braggio and F. Giazotto, Sci. Rep. **9** 3238 (2019)

Proximity SNS ring

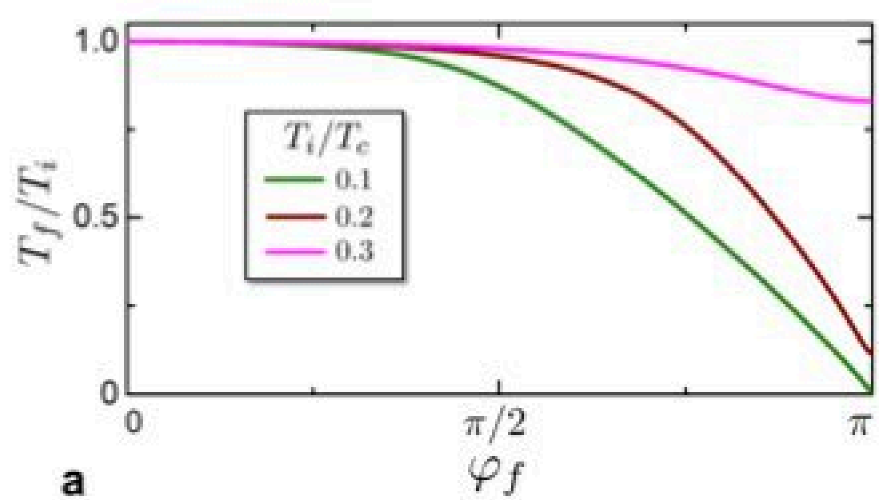


Phase dependent entropy

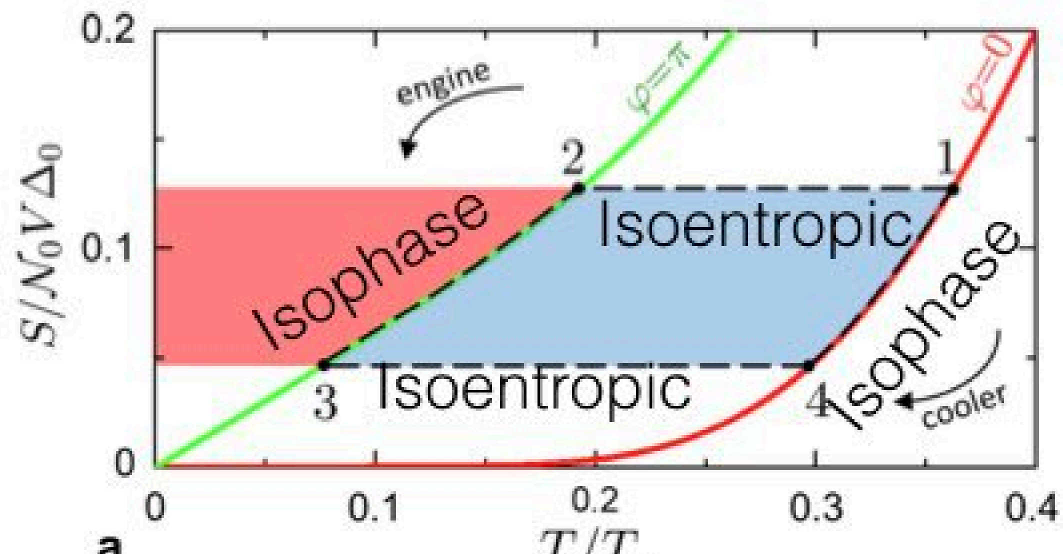


Analogous to a gas expansion

Coherent $\varphi \leftrightarrow \mathcal{V}$



Adiabatic cooling



Coherent Thermal engine cooler

CPR Computation

$L \ll \xi$

$$I = -\frac{2e}{\hbar}\sum_p \tanh\left[\frac{\epsilon_p}{2k_B T}\right]\frac{d\epsilon_p}{d\phi}$$

ϵ_p
ABS energies

C. W. J. Beenakker, PRL'91
C. W. J. Beenakker, in *Transport Phenomena in Mesoscopic Systems* '92
C. J. Lambert and R. Raimondi, JPCM '98

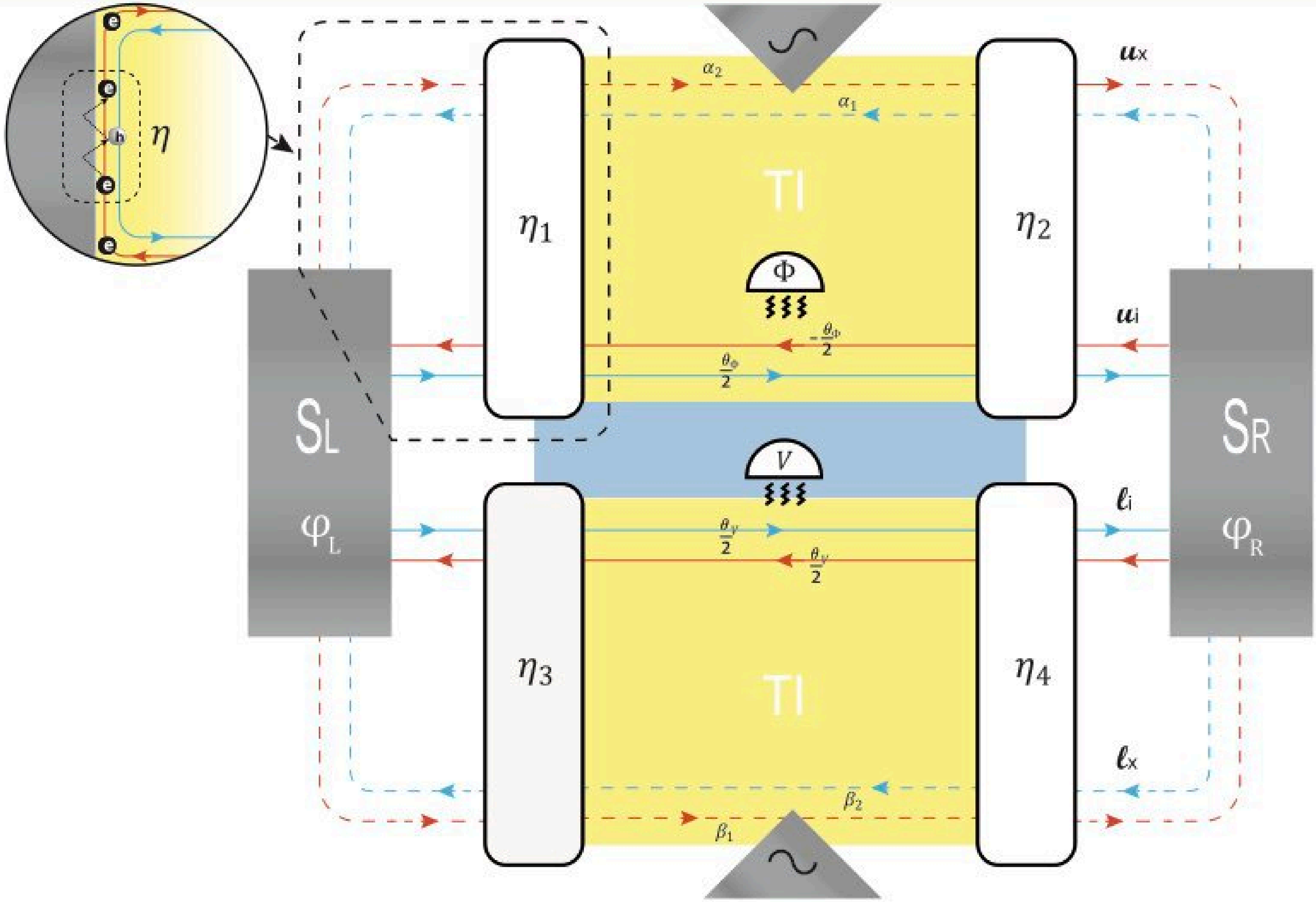
Secular equation

$$Det\left[e^{i\arccos(\epsilon_p/\Delta_0)}\mathbf{1} - s_A s_N\right] = 0$$

$$s_N = \begin{pmatrix} s_0 & \emptyset \\ \emptyset & s_0^* \end{pmatrix} \quad s_A = \begin{pmatrix} \emptyset & r_A \\ r_A^* & \emptyset \end{pmatrix} \quad r^* = \begin{pmatrix} r_{\mathrm{L}}^* & 0 \\ 0 & r_{\mathrm{R}}^* \end{pmatrix}$$

$$r_{\mathrm{L(R)}}^* = \begin{pmatrix} |\Lambda_{\mathrm{L(R)}}| & i|X_{\mathrm{L(R)}}| \\ i|X_{\mathrm{L(R)}}| & |\Lambda_{\mathrm{L(R)}}| \end{pmatrix} e^{i\phi_{\mathrm{L(R)}}}$$

Full Model



- Dephasing

$$\bar{J}(\phi,\theta_\Phi,\theta_V)=\frac{1}{(2\pi)^4}\int_0^{2\pi} d\alpha_1 d\alpha_2 d\beta_1 d\beta_2\; J(\phi,\theta_\Phi,\theta_V,\alpha_1,\alpha_2,\beta_1,\beta_2)$$

Full Model

Andreev reflections

$$
\begin{pmatrix} b^{\downarrow}_{uxL} \\ b^{\uparrow}_{uiL} \\ b^{\uparrow}_{liL} \\ b^{\downarrow}_{lxL} \\ b^{\uparrow}_{uxR} \\ b^{\downarrow}_{uiR} \\ b^{\downarrow}_{liR} \\ b^{\uparrow}_{lxR} \end{pmatrix}_{in} = \begin{pmatrix} \begin{pmatrix} |\Lambda_{Lx}| & 0 & 0 & i|X_{Lx}| \\ 0 & |\Lambda_{Li}| & i|X_{Li}| & 0 \\ 0 & i|X_{Li}| & |\Lambda_{Li}| & 0 \\ i|X_{Lx}| & 0 & 0 & |\Lambda_{Lx}| \end{pmatrix} e^{i\phi_L} & \emptyset \\ \emptyset & \begin{pmatrix} |\Lambda_{Rx}| & 0 & 0 & i|X_{Rx}| \\ 0 & |\Lambda_{Ri}| & i|X_{Ri}| & 0 \\ 0 & i|X_{Ri}| & |\Lambda_{Ri}| & 0 \\ i|X_{Rx}| & 0 & 0 & |\Lambda_{Rx}| \end{pmatrix} e^{i\phi_R} \end{pmatrix}_{r_A^*} \begin{pmatrix} c^{\uparrow}_{uxL} \\ c^{\downarrow}_{uiL} \\ c^{\downarrow}_{liL} \\ c^{\uparrow}_{lxL} \\ c^{\downarrow}_{uxR} \\ c^{\uparrow}_{uiR} \\ c^{\uparrow}_{liR} \\ c^{\downarrow}_{lxR} \end{pmatrix}_{out}
$$

Scattering matrix

$$
\begin{pmatrix} c^{\uparrow}_{uxL} \\ c^{\downarrow}_{uiL} \\ c^{\downarrow}_{liL} \\ c^{\uparrow}_{lxL} \\ c^{\downarrow}_{uxR} \\ c^{\uparrow}_{uiR} \\ c^{\uparrow}_{liR} \\ c^{\downarrow}_{lxR} \end{pmatrix}_{out} = \begin{pmatrix} 0 & A_2 & 0 & 0 & D_1 & 0 & 0 & 0 \\ A_1 & 0 & 0 & 0 & 0 & D_2 & 0 & 0 \\ 0 & 0 & 0 & A_4 & 0 & 0 & D_3 & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 & 0 & D_4 \\ C_1 & 0 & 0 & 0 & 0 & B_2 & 0 & 0 \\ 0 & C_2 & 0 & 0 & B_1 & 0 & 0 & 0 \\ 0 & 0 & C_3 & 0 & 0 & 0 & 0 & B_4 \\ 0 & 0 & 0 & C_4 & 0 & 0 & B_3 & 0 \end{pmatrix}_{s_0} \begin{pmatrix} c^{\downarrow}_{uxL} \\ c^{\uparrow}_{uiL} \\ c^{\uparrow}_{liL} \\ c^{\downarrow}_{lxL} \\ c^{\uparrow}_{uxR} \\ c^{\downarrow}_{uiR} \\ c^{\downarrow}_{liR} \\ c^{\uparrow}_{lxR} \end{pmatrix}_{in}
$$

$c^{\downarrow}_{uxL} \to c^{\downarrow}_{uiL}$:

$$
\begin{aligned}
A_1 &= r_1 + t_1 e^{i\alpha_2} r_2 e^{-i\theta_\Phi/2} t_1 + t_1 e^{i\alpha_2} \cdot r_2 e^{-i\theta_\Phi/2} r_1 e^{i\alpha_2} \cdot r_2 e^{-i\theta_\Phi/2} t_1 + \dots \\
&= r_1 + t_1^2 r_2 e^{i\alpha_2} e^{-i\theta_\Phi/2} \sum_{n=0}^{\infty} (r_2 r_1 e^{i\alpha_2} e^{-i\theta_\Phi/2})^n \\
&= r_1 + \frac{t_1^2 r_2 e^{i\alpha_2} e^{-i\theta_\Phi/2}}{1 - r_1 r_2 e^{i\alpha_2} e^{-i\theta_\Phi/2}}
\end{aligned}
$$

$A_1 = r_1 + \frac{t_1^2 r_2 e^{i\alpha_2} e^{-i\theta_\Phi/2}}{1 - r_1 r_2 e^{i\alpha_2} e^{-i\theta_\Phi/2}}$; …….. $A_3 = r_3 + \frac{t_3^2 r_4 e^{i\beta_2} e^{-i\theta_V/2}}{1 - r_3 r_4 e^{i\beta_2} e^{-i\theta_V/2}}$;

Solving secular equation numerically or …