# quantum measurement as a thermodynamic Resource

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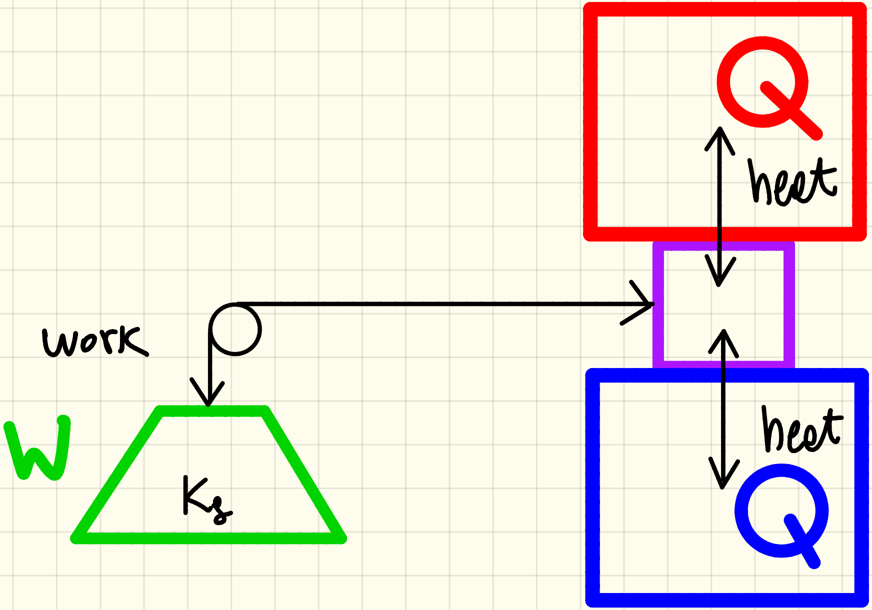

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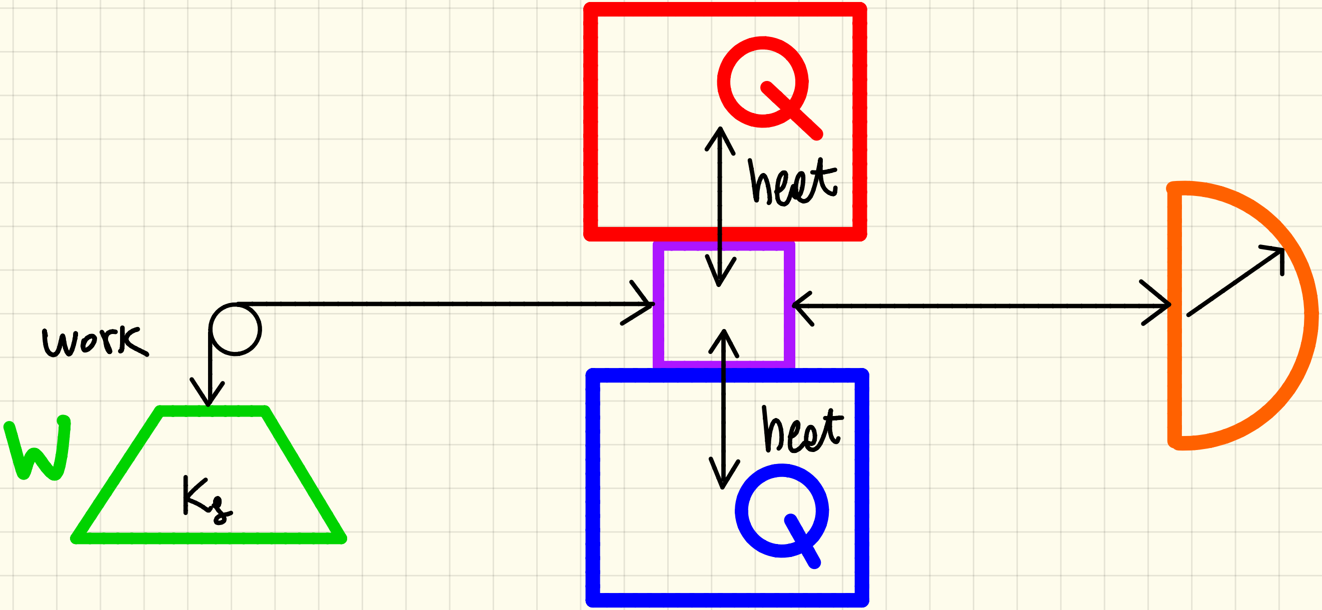

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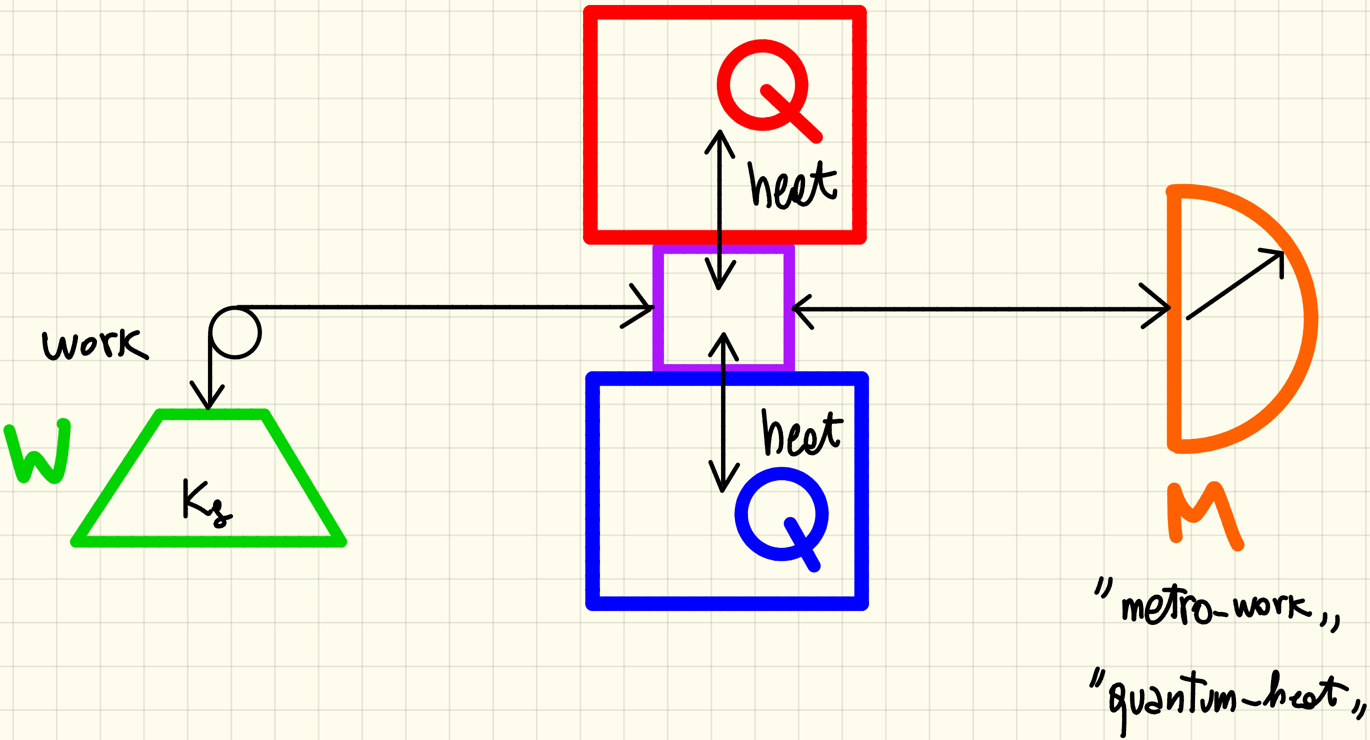



Elouard et al., npj-quantum info, 3 (2017)

EUROPHYSICS LETTERS 15 August 2004

*Europhys. Lett.*, **67** (4), pp. 565–571 (2004)

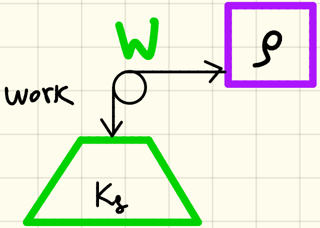
DOI: 10.1209/epl/i2004-10101-2

# Maximal work extraction from finite quantum systems

(via unitary evolution)

A. E. ALLAHVERDYAN[1,2], R. BALIAN[3] and TH. M. NIEUWENHUIZEN[1]

**ERGOTROPY** $\tilde{W}$




$$\rho \to U \rho U^{\dagger}$$

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

PAPER: No subject

arXiv 1905.10262

# Maximal energy extraction via quantum measurement

Andrea Solfanelli[1,2], Lorenzo Buffoni[1,3],
Alessandro Cuccoli[1,2] and Michele Campisi[1,2]

METROTROPY

$\mathcal{M}$

M

$$\rho \to \sum_k \Pi_k \rho \Pi_k$$

# ERGOTROPY $\mathcal{W}$

$[H, \rho] = 0$

$$E = \Sigma\, E_k r_k$$

$$\rho' = U \rho\, U^{+}$$

$$E' = \Sigma_k E_k \langle k | U \rho U^{+} | k \rangle$$

$$= \Sigma_k E_k P_{ke} r_e$$

$$= \mathbf{E}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{r}$$

$$P_{ke} = |\langle k | U | e \rangle|^2$$

$$\mathcal{W} = \max (E - E')$$

$$= E - \min E'$$

$\mathbf{P}$ is uni-stochastic

$\Downarrow$

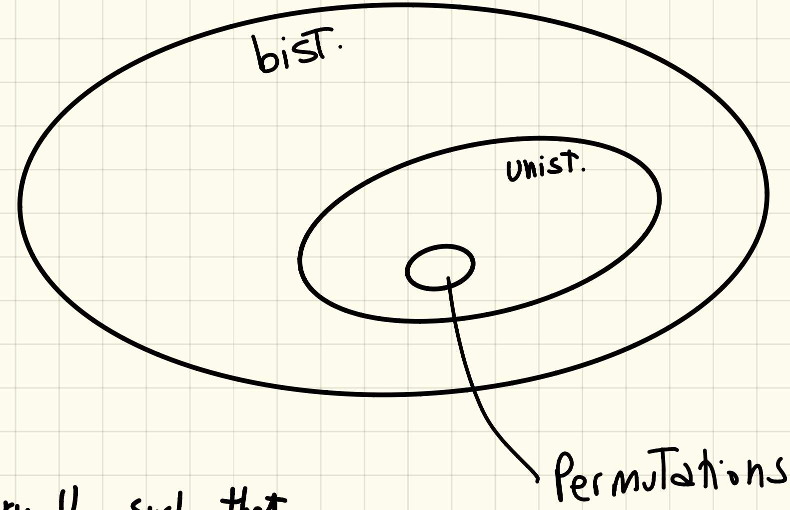
$\mathbf{P}$ is doubly stochastic

**Birkhoff theorem**

$$\mathbf{P} = \Sigma_i \lambda_i \sigma_i$$

$\sigma_i \leftarrow$ permutations

$$\min \longrightarrow \mathbf{P} = \sigma_w$$




given a permutation matrix $\sigma$

you can always find a unitary $U$ such that

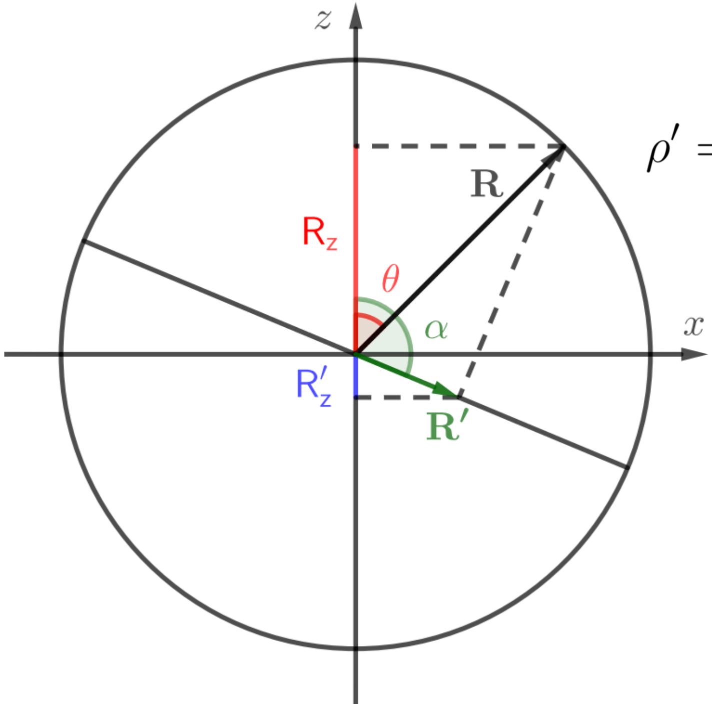
$$\sigma_{ij} = |\langle i | U | j \rangle|^2$$

e.g.

$$\sigma = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad U = \begin{pmatrix} 0 & 0 & e^{i\alpha} \\ e^{i\beta} & 0 & 0 \\ 0 & e^{i\gamma} & 0 \end{pmatrix}$$

# ERGOTROPY vs METROTROPY

Two level system $\quad H = \sigma_z$




$$\rho = (\mathbb{1} + \mathbf{R} \cdot \boldsymbol{\sigma})/2$$

$$\rho' = \frac{\mathbb{1} + (\mathbf{R} \cdot \mathbf{n})\mathbf{n} \cdot \boldsymbol{\sigma}}{2} = \frac{\mathbb{1} + \mathbf{R}' \cdot \boldsymbol{\sigma}}{2}$$

$$\mathcal{W} = (|\vec{R}| + R_z)\, b_z$$

$$\mathcal{M} = \frac{\mathcal{W}}{2}$$

# METROTROPY $\mathcal{M}$

$[H, \rho] = 0$

$$\rho' = \sum_k \pi_k \rho \pi_k$$

$\pi_k = |\psi_k\rangle\langle\psi_k|$

$$\begin{aligned}
E' &= \sum_{lk} E_l \langle l|\psi_k\rangle\langle\psi_k|\rho|\psi_k\rangle\langle\psi_k|l\rangle \\
&= \sum_{lkn} E_l \langle l|\psi_k\rangle\langle\psi_k|n\rangle\langle n|\psi_k\rangle\langle\psi_k|l\rangle r_n \\
&= \sum_{lkn} E_l |\langle l|\psi_k\rangle|^2 |\langle n|\psi_k\rangle|^2 r_n \\
&= \mathbf{E}^T \cdot \mathbf{P}^T \cdot \mathbf{P} \cdot \mathbf{r}
\end{aligned}$$

$$\mathcal{M} = \max(E - E') = E - \min E'$$

$P_{ek} = |\langle e|\psi_k\rangle|^2 = |\langle e|U|k\rangle|^2$

1) Use Birkhoff theorem

to find

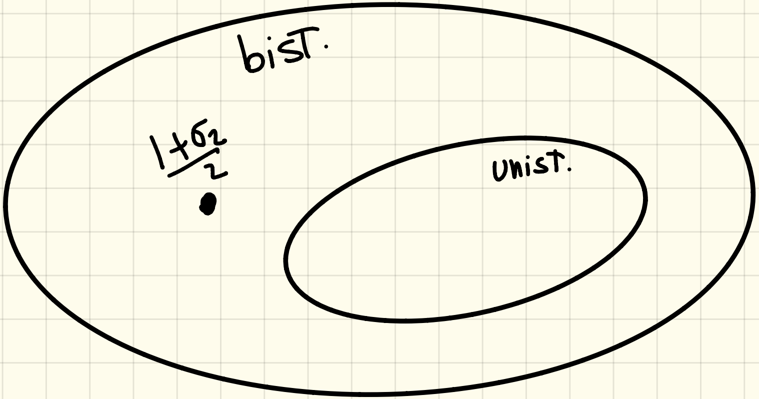
$$\min_{\mathcal{B}_n} E^T \cdot P^T \cdot P \cdot r$$

$\mathcal{B}_n$ = set of Bistochastic matrices

$$P = \frac{1 + \sigma_2}{2}$$

$$\sigma_2 = \operatorname{argmin} E^T \cdot \frac{\sigma + \sigma^T}{2} \cdot r$$

2) Not all bistocastic matrices are unistochastic !




example

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \leftarrow \text{not unistochastic}$$

$$\downarrow$$

$$\begin{pmatrix} |1\rangle & |2\rangle & |3\rangle \\ |2\rangle & |3\rangle & |1\rangle \end{pmatrix}$$

# Theorem

(Au-Yeung, Linear algebra its appl. 150 (1991))

Any convex combination of permutation matrices which is unistochastic, is such that it involves only permutations that are pairwise complementary.

Two N ×N matrices A and B are said to be complementary if, for any $1 \leq i, j, h, k \leq N$, $A_{ij} = A_{hk} = B_{ik} = 1$ implies $B_{hj} = 1$.

Any permutation that is complementary to the identity is symmetric

Is any convex combination involving only complementary permutations unistochastic?

OPEN QUESTION

YES: up to $N \leq 15$!

Au-Yeung, Linear algebra its appl. <u>150</u> (1991)

# STATEMENT

the minimum of $\mathbf{E}^T \cdot \mathbf{P}^T \cdot \mathbf{P} \cdot \mathbf{r}$ over all

convex combinations containing only pairwise permutations is achieved by

$$\mathbf{P} = (\mathbb{1} + \boldsymbol{\sigma}_{\mathcal{M}})/2$$

$$\boldsymbol{\sigma}_{\mathcal{M}} \doteq \arg\min_{\boldsymbol{\sigma} \in \mathcal{I}_N} \mathbf{E}^T \cdot \boldsymbol{\sigma} \cdot \mathbf{r}\,.$$

$I_n$ = Set of symmetric permutations

(A. Solfanelli et al
arXiv 1905.10262)

Symmetric permutation

$$\sigma_M = \begin{pmatrix} |a\rangle & |b\rangle & |c\rangle & |d\rangle & \cdots & |x\rangle \\ |b\rangle & |a\rangle & |d\rangle & |c\rangle & \cdots & |x\rangle \end{pmatrix}$$

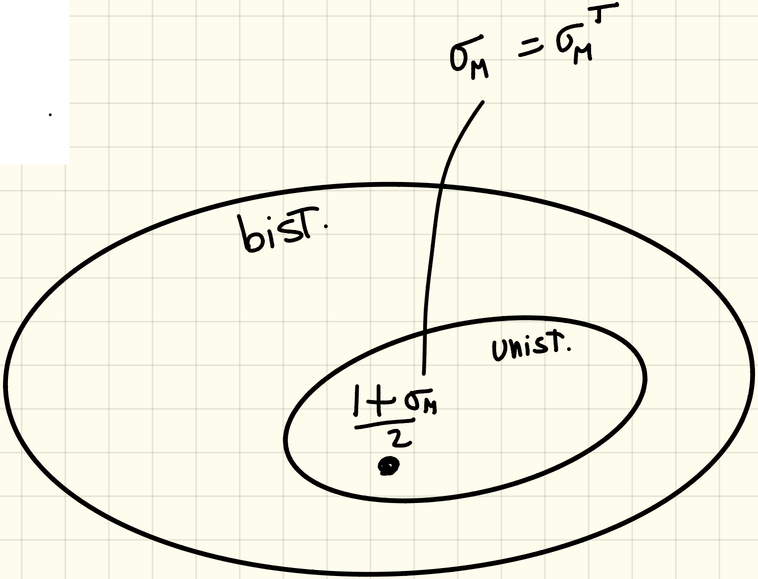
$$U|a\rangle = \frac{|a\rangle + |b\rangle}{\sqrt{2}} \qquad U|b\rangle = \frac{|a\rangle - |b\rangle}{\sqrt{2}}$$

$$U|c\rangle = \frac{|c\rangle + |d\rangle}{\sqrt{2}} \qquad U|d\rangle = \frac{|c\rangle - |d\rangle}{\sqrt{2}}$$

$$\vdots$$

$$U|x\rangle = |x\rangle, \qquad x \neq a, b, c, d, \ldots$$

$$P_{ij} = |\langle i|U|j\rangle|^2$$



# Theorem

the minimum of $\mathbf{E}^T \cdot \mathbf{P}^T \cdot \mathbf{P} \cdot \mathbf{r}$ over all **UNISTOCHASTIC MATRICES** is achieved by

$$\mathbf{P} = (\mathbb{1} + \boldsymbol{\sigma}_{\mathcal{M}})/2$$

$$\boldsymbol{\sigma}_{\mathcal{M}} \doteq \arg\min_{\boldsymbol{\sigma} \in \mathcal{I}_N} \mathbf{E}^T \cdot \boldsymbol{\sigma} \cdot \mathbf{r} \,.$$

$I_n$ = Set of symmetric permutations

(A. Solfanelli et al
arXiv 1905.1026)

example 4-level system

$$\sigma_M = \begin{pmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & & & \\ & 1/2 & 1/2 & \\ & 1/2 & 1/2 & \\ & & & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & & & \\ & 1/\sqrt{2} & 1/\sqrt{2} & \\ & 1/\sqrt{2} & -1/\sqrt{2} & \\ & & & 1 \end{pmatrix}$$

## Corollaries

- $$\mathcal{M} = \frac{E - \min_{I_n} E^T \cdot \sigma \cdot r}{2}$$

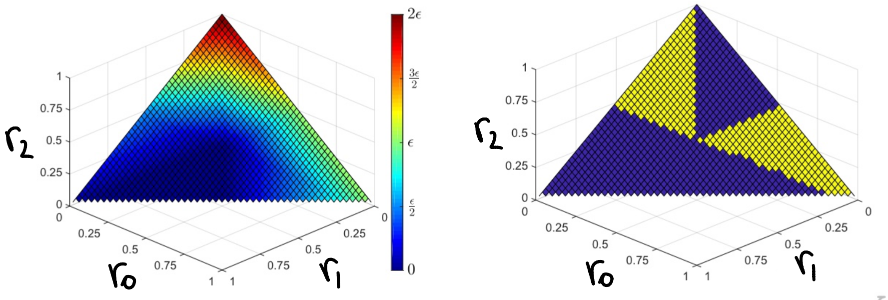
- if $\sigma_w$ is symmetric, then $\mathcal{M} = \frac{W}{2}$

- $\mathcal{M} \leq \frac{W}{2}$ $\quad$ $([H, \rho] = 0)$

- $\mathcal{M} \leq W$ $\quad$ (in general)

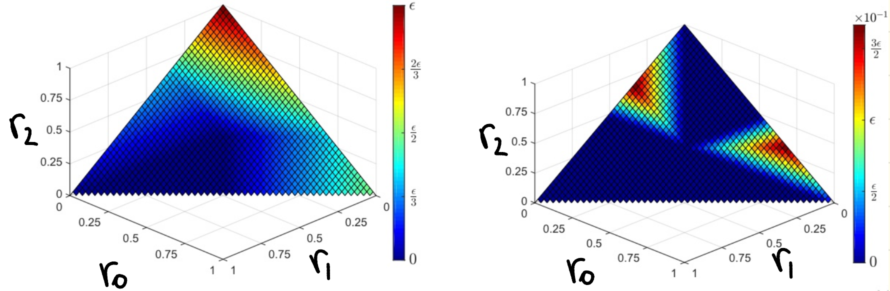
## Conjecture

- $\mathcal{M} \leq \frac{W}{2}$ $\quad$ (in general) three level system $\quad \mathbf{E} = \varepsilon \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} r_0 \\ r_1 \\ r_2 \end{pmatrix}$




(a) (b)

Figure 1: Panel a): Ergotropy, $\mathcal{W}$. Panel b): Symmetricity of the ergotropy permutation $\sigma_{\mathcal{W}}$. Blue denotes symmetric $\sigma_{\mathcal{W}}$ , yellow denotes non-symmetric $\sigma_{\mathcal{W}}$.




(a) (b)

Figure 2: Panel (a): Metrotropy $\mathcal{M}$. Panel b): $\mathcal{W}/2 - \mathcal{M}$.

PHYSICAL REVIEW LETTERS **122,** 070603 (2019)

Editors' Suggestion

Featured in Physics

# Quantum Measurement Cooling

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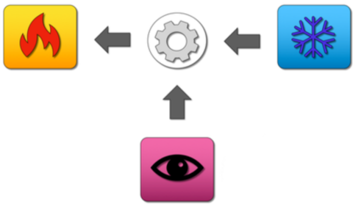
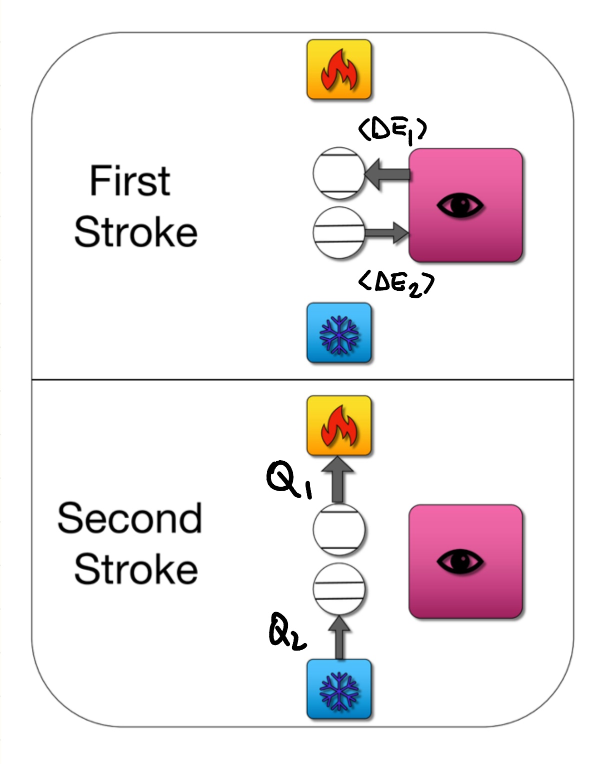
## Synopsis: Refrigeration by Quantum Measurements

February 21, 2019

**A proposed noise-tolerant approach to quantum refrigeration eliminates the need for feedback control by exploiting the invasiveness of quantum measurements.**



L. Buffoni *et al.*, Phys. Rev. Lett. (2019)




$$\frac{w_1}{2}\sigma_1^2 \qquad \frac{w_2}{2}\sigma_2^2$$

$$H = H_1 + H_2 + V(t)$$

$$\rho = \frac{e^{-\beta_1 H_1}}{Z_1} \otimes \frac{e^{-\beta_2 H_2}}{Z_2}$$

$$\boxed{\rho \to \rho' = \sum_k \Pi_k \, \rho \, \Pi_k}$$

$$\mathrm{Tr}_i \; H_i (\rho_i' - \rho_i) = \langle \Delta E_i \rangle = Q_i$$

$$\langle M \rangle = \langle \Delta E \rangle = \langle \Delta E_1 \rangle + \langle \Delta E_2 \rangle$$

$$= Q_1 + Q_2$$

$$\boxed{\beta_1 Q_1 + \beta_2 Q_2 = D[\rho'_1 \| \rho_1] + D[\rho'_2 \| \rho_2] + I_{1/2}[\rho'] + \Delta S \geq 0}$$

$$D[\rho'_1 \| \rho_1] \geq 0 \qquad D[\rho'_2 \| \rho_2] \geq 0 \qquad \underbrace{I_{1/2}[\rho'] \geq 0} \qquad \underbrace{\Delta S \geq 0}$$

$$I_{1/2}[\rho'] = D[\rho' \| \rho'_1 \otimes \rho'_2] \geq 0$$

$$\rho' = \sum_k \Pi_k \rho \Pi_k$$

is unital !

$$\Downarrow$$

$$\Delta S \geq 0$$

$$D[\rho \| \sigma] = \mathrm{Tr}\left(\rho \ln \rho - \rho \ln \sigma\right)$$

$$I_{1/2}[\rho'] = \sum_i S[\rho'_i] - S[\rho_t]$$

$$S[\rho] = -\mathrm{Tr}\, \rho \ln \rho$$

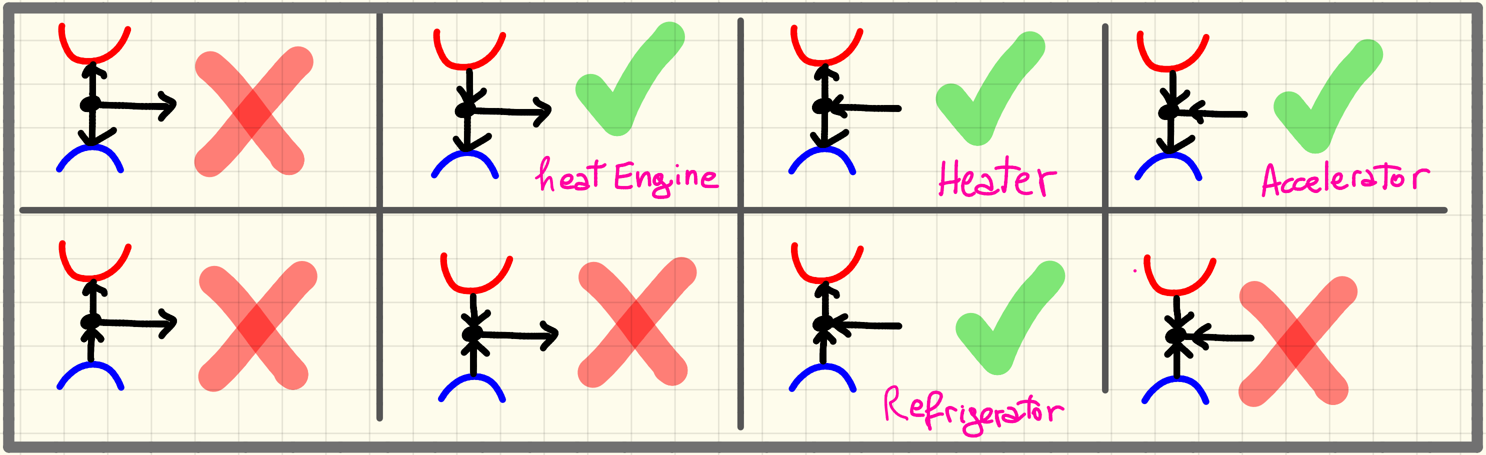
$$\Delta S = S[\rho_t] - S[\rho_0]$$

$$\beta_1 Q_1 + \beta_2 Q_2 \geqslant 0$$

$$\underset{\substack{\parallel \\ \langle M \rangle}}{\langle \Delta E \rangle} = Q_1 + Q_2$$

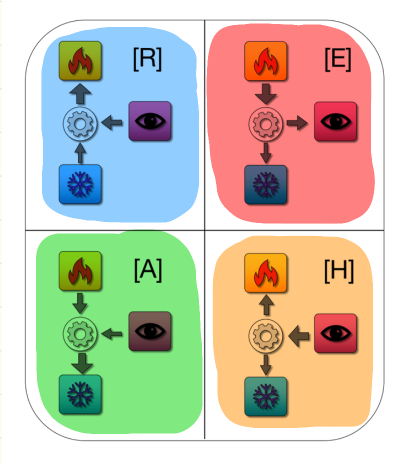
A. Solfanelli

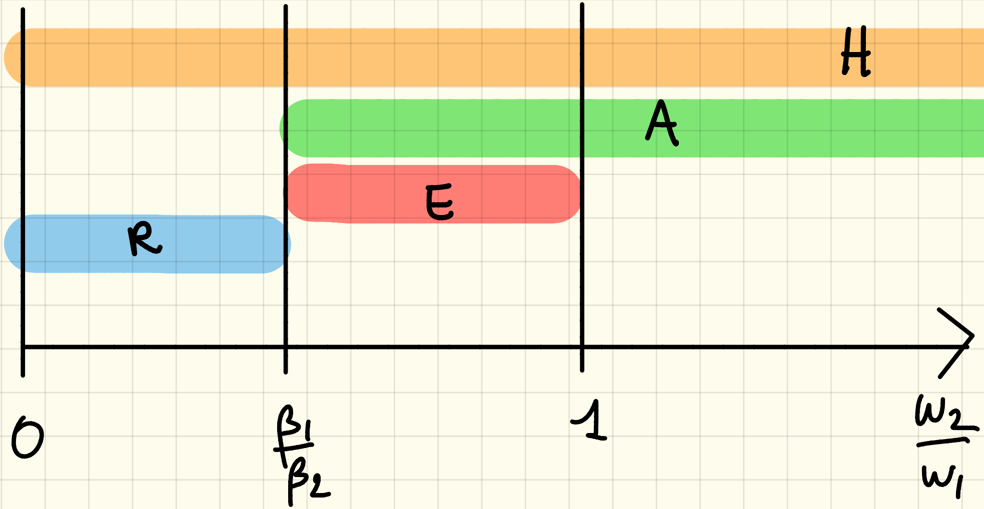
B.Sc. thesis UNIFI



# Results

1



## Results

2

$$\begin{cases} |\psi_1^*\rangle = |\uparrow\uparrow\rangle \\ |\psi_2^*\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \\ |\psi_3^*\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \\ |\psi_4^*\rangle = |\downarrow\downarrow\rangle \end{cases}$$

maximises

$\eta^{[R]}$, $-Q_2$ in [R]-range

$\eta^{[E]}$, $\langle \Delta E \rangle$ in [E]-range

$$\langle \Delta E_{1,2} \rangle = \frac{\pm\omega_i}{2}\left(\frac{1}{1+e^{\beta_1\omega_1}} - \frac{1}{1+e^{\beta_2\omega_2}}\right)$$

$$\eta^{[R]} = \frac{1}{\frac{w_1}{w_2} - 1}$$

$$\eta^{[E]} = 1 - \frac{w_2}{w_1}$$

## Results

3

Maximal efficiency and $\begin{cases} \text{metro-work output [E]} \\ \text{heat extraction [R]} \end{cases}$

can be achieved with higher rank projectors

e.g.:

$$q_1 = |\psi_1^*\rangle\langle\psi_1^*| + |\psi_2^*\rangle\langle\psi_2^*|$$

$$q_2 = |\psi_3^*\rangle\langle\psi_3^*| + |\psi_4^*\rangle\langle\psi_4^*|$$

## Results

(4) Generally, for two qudits

In order to realize anything other than [H], some of the measurement projectors must be entangled

( cannot be written in factorised form )

(4.1) However, extremal efficiency occurs when

$$\rho' = \sum p'_n |n\rangle\langle n|$$

$$|n\rangle = |n_1\rangle|n_2\rangle$$

## Results

5

Let $|\psi_K\rangle = U|K\rangle$

Pick U randomly from the invariant SU(2N) measure

then

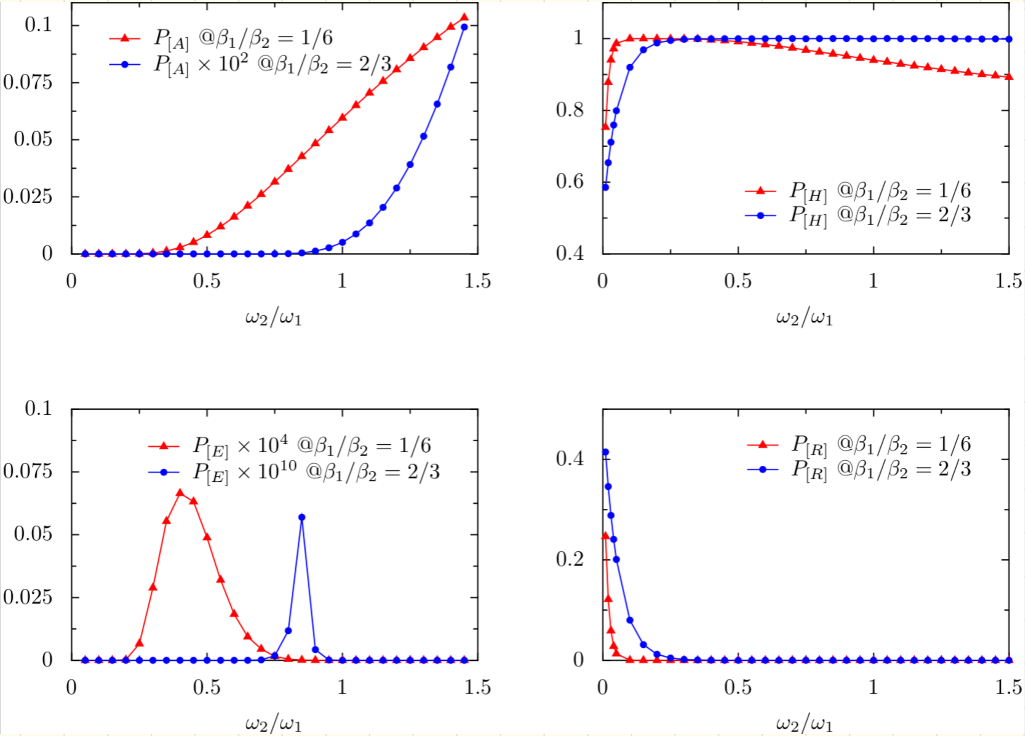
$$\overline{\langle \Delta E_i \rangle} \geq 0 \Rightarrow [H]$$

$$\left( \overline{f} = \int_{SU(2N)} dm \, f \right)$$

Results

6

# Monte Carlo Sampling of SU(4)



Experiment....

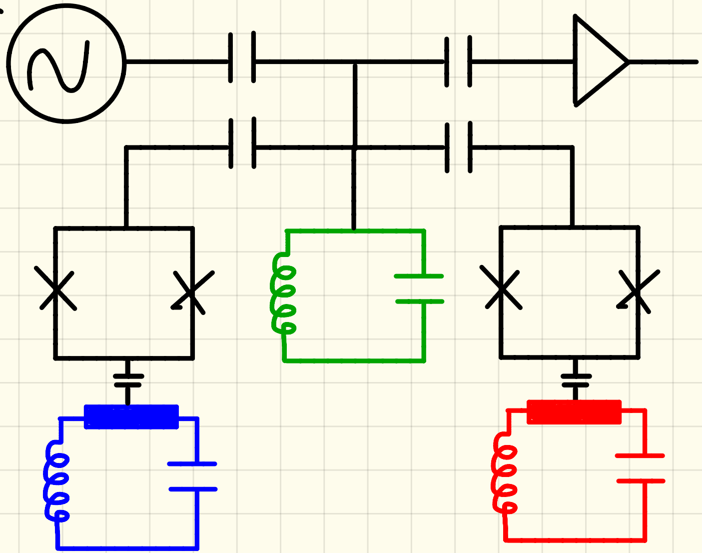
circuit QED + circuit QTD

$\downarrow$

circuit Quantum Thermo Dynamics $\rightarrow$ Pekola, Giazotto....

$$\rho' = \sum_k \Pi_k \, \rho \, \Pi_k$$
$$= \sum_k U P_k U^\dagger \rho \, U P_k U^\dagger$$

Filipp et al., PRL 102 200402 (2009)



Ronzani et al Nat phys 14 991 (2018)

thank you