Advanced Workshop on Earthquake Fault Mechanics: Theory, Simulation and Observations

ICTP, Trieste, Sept 2-14 2019

Lecture 2: fracture mechanics

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Lecture 1: earthquake dynamics from the standpoint of fracture mechanics

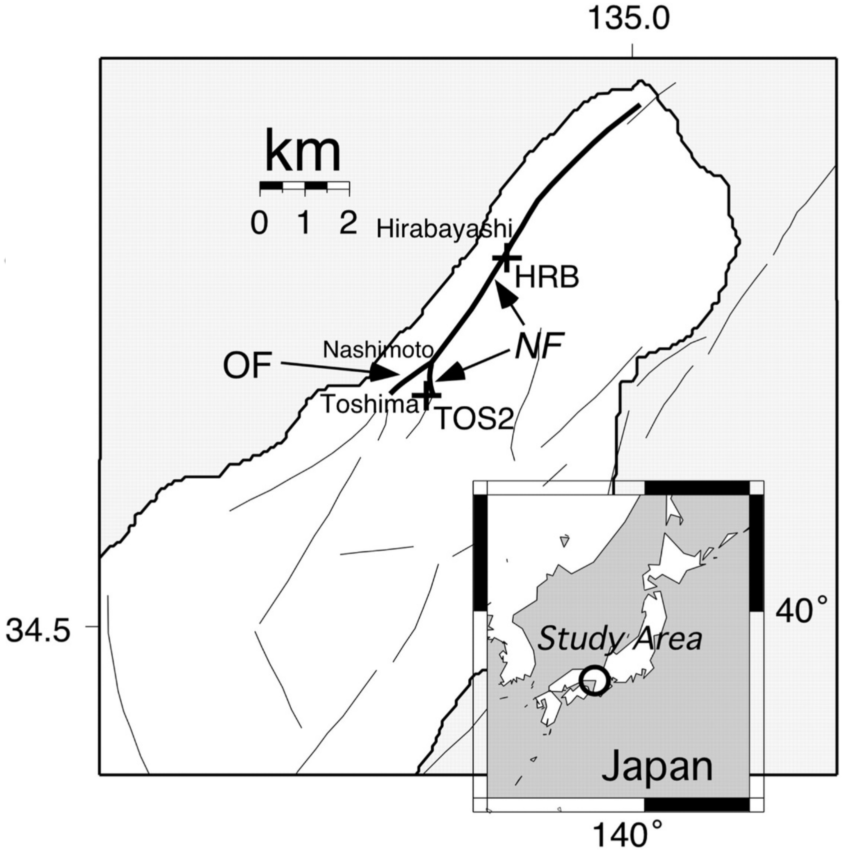
(LEFM = linear elastic fracture mechanics)

- Asymptotic crack tip fields
- Stress intensity factor K
- Energy flux to the crack tip G
- Fracture energy G_c
- → **Crack tip equation of motion**
- Implications
- Radiated energy

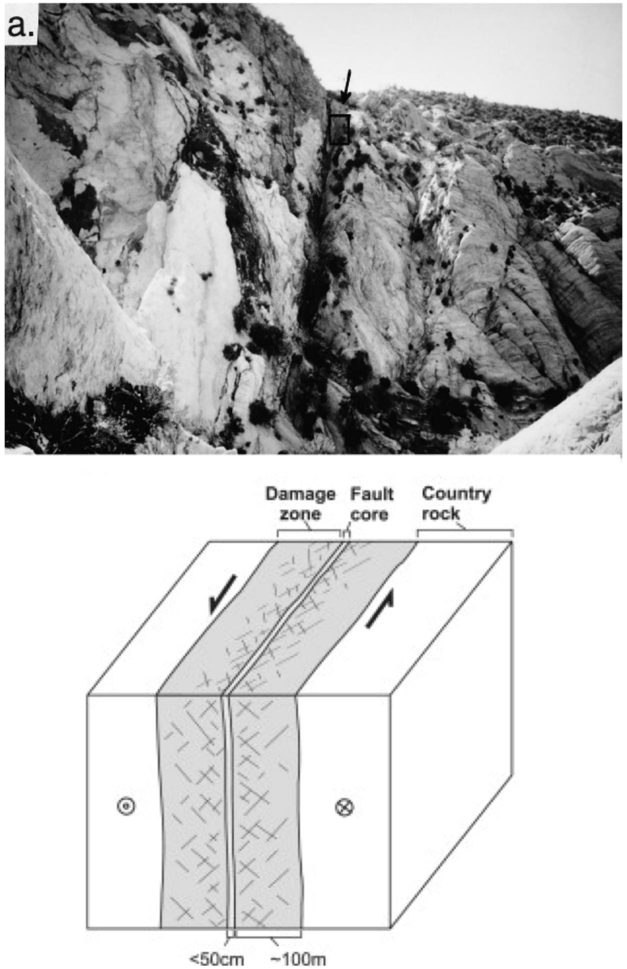
Real faults are thick …

Nojima Fault Preservation Museum

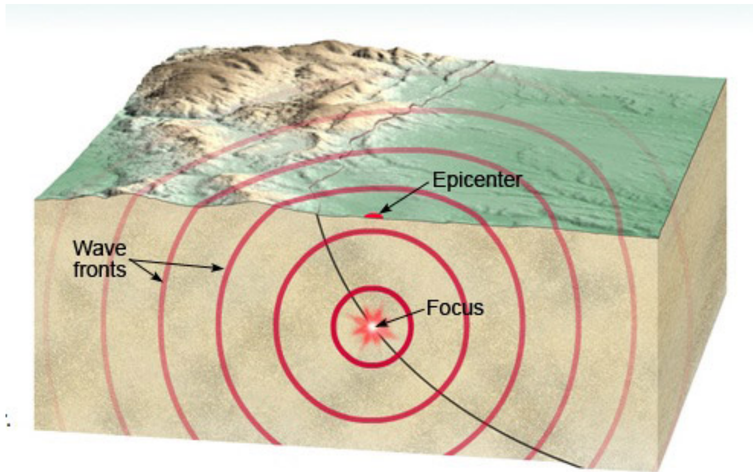
Nojima Fault, Japan



Real faults are thick …



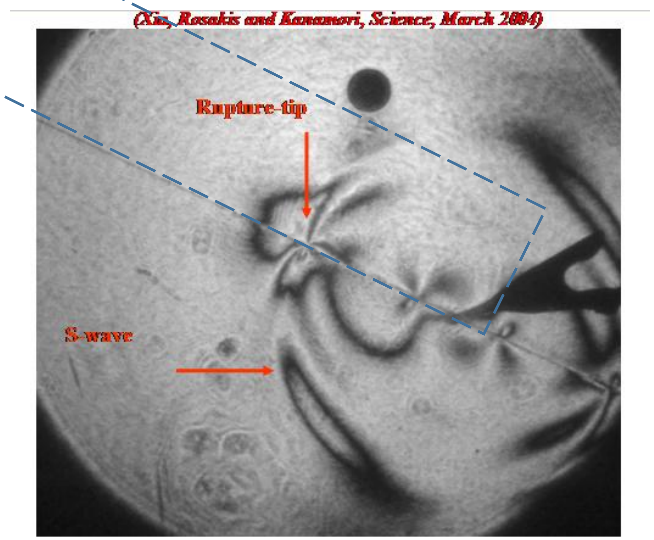
Punchbowl fault, CA
(Chester and Chester, 1998)

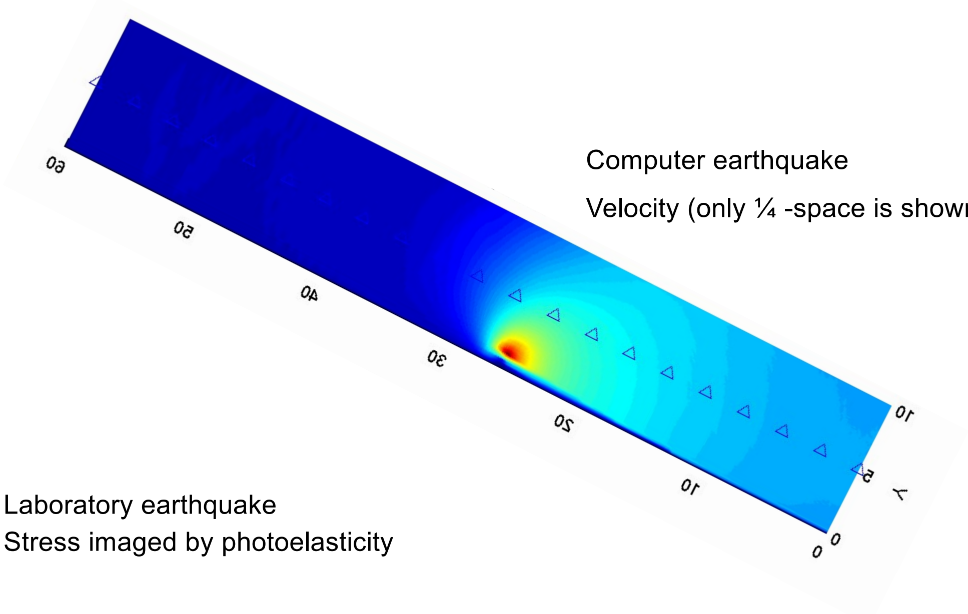


Idealized earthquake model on a thin fault

Singularities close to a crack tip

Singularities close to a crack tip





Computer earthquake

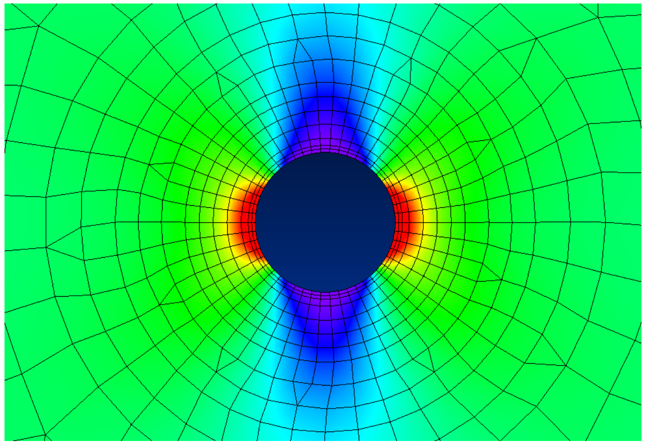
Velocity (only ¼ -space is shown)

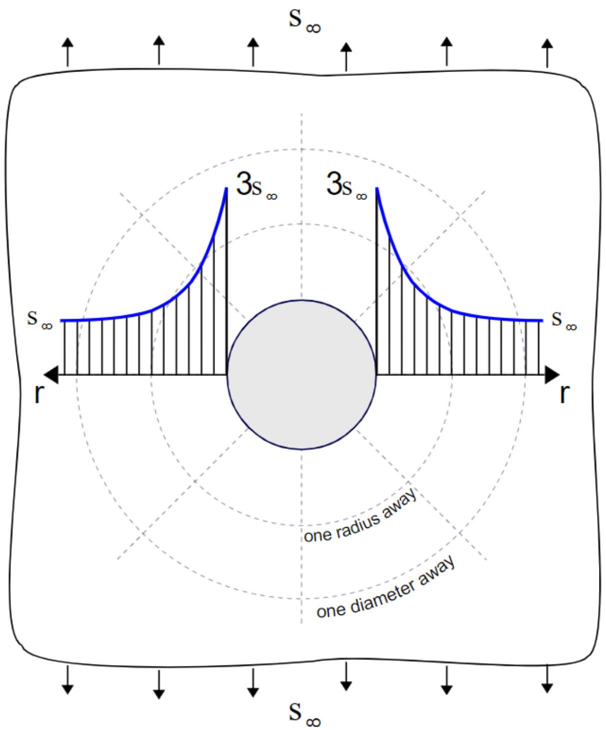
Laboratory earthquake

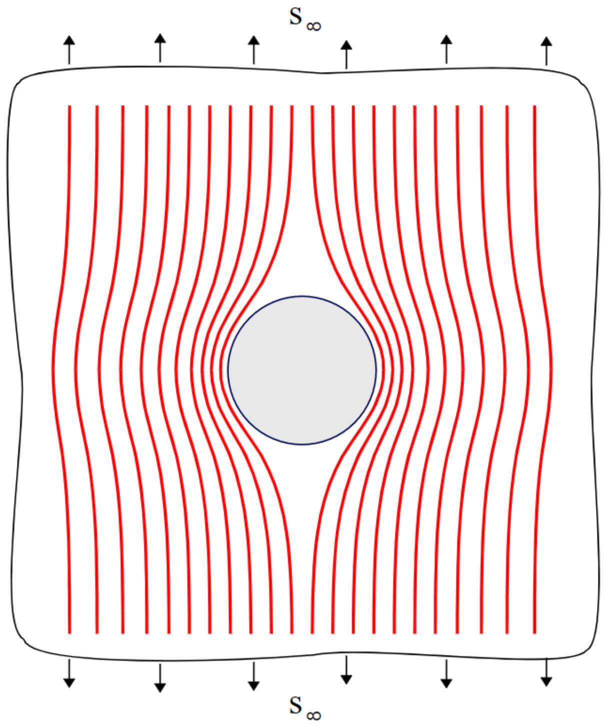
Stress imaged by photoelasticity

- Model: crack in an ideally elastic body → velocity and stress are infinite near the crack tips
- Physical model: inelastic processes occur in a **process zone**
- LEFM assumption: **small scale yielding** = the process zone is much smaller than crack and body dimensions

Circular hole

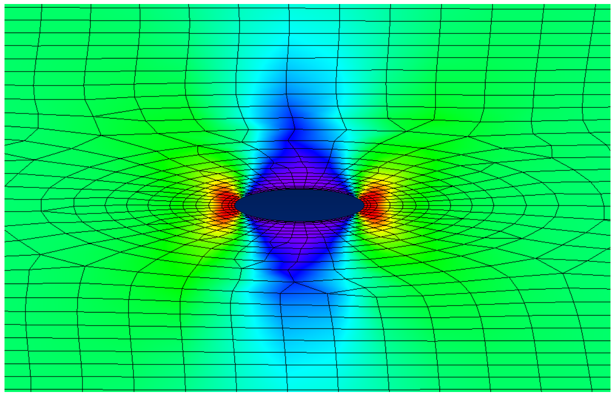


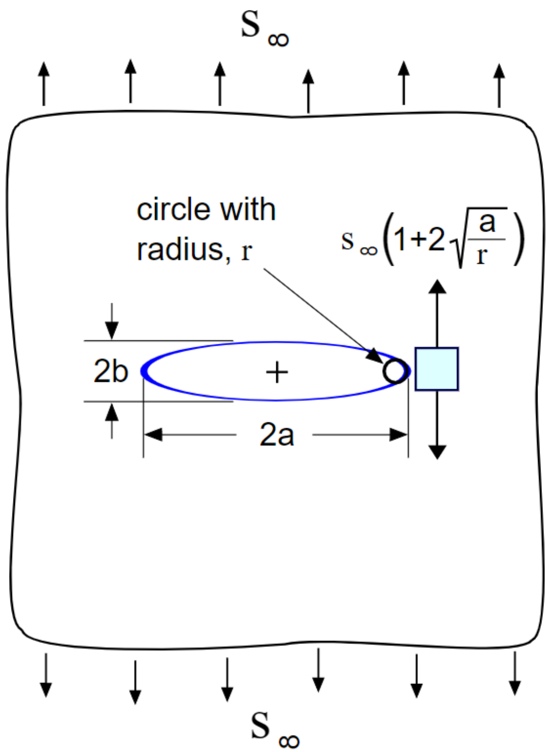


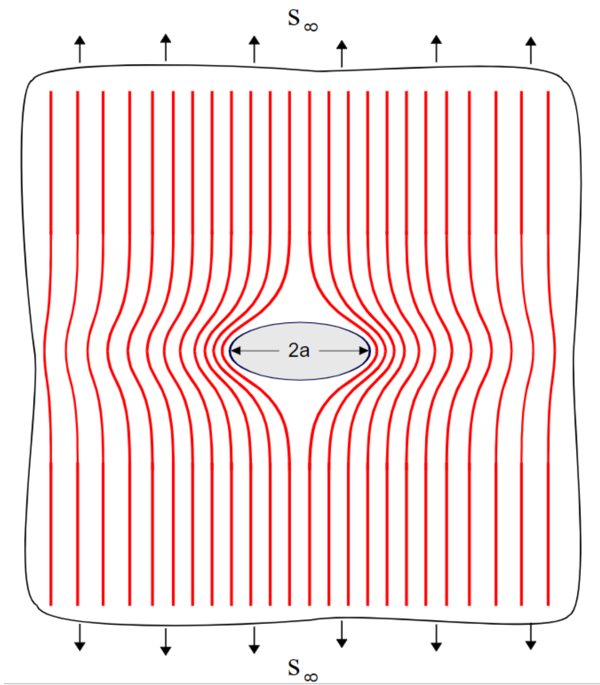


https://www.fracturemechanics.org

Elliptical hole







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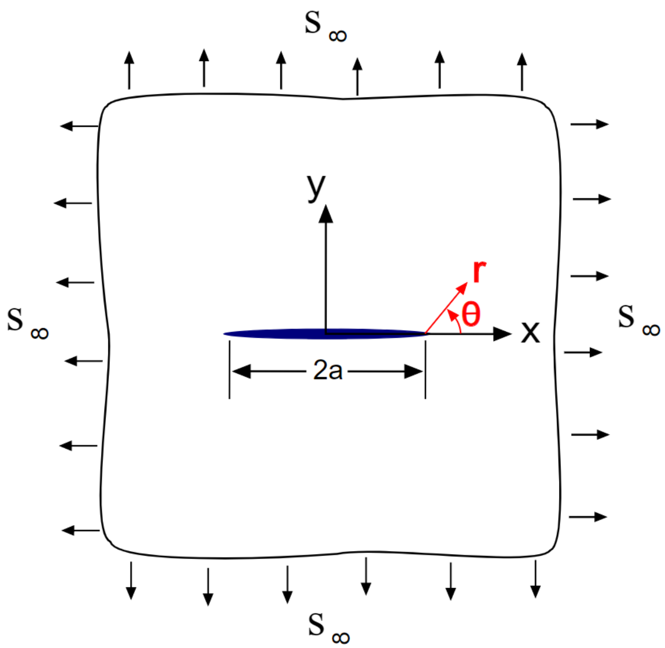
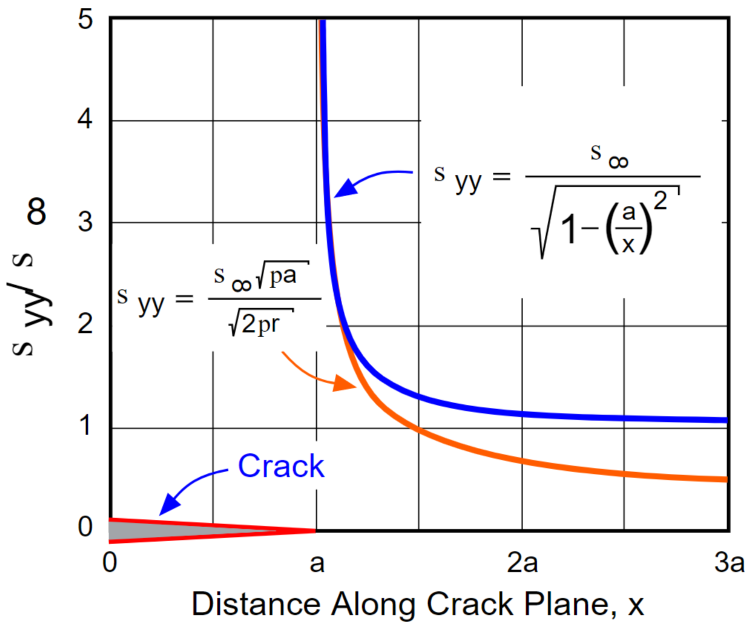
Thin crack

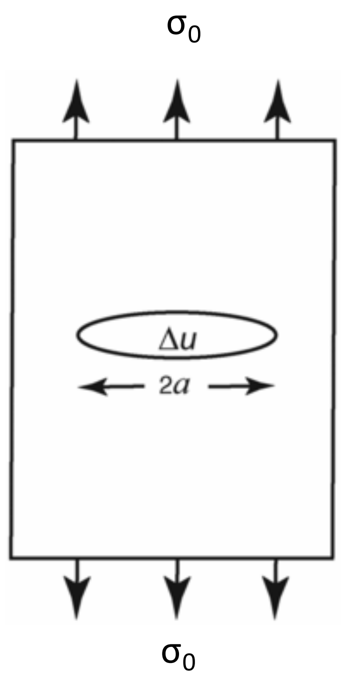


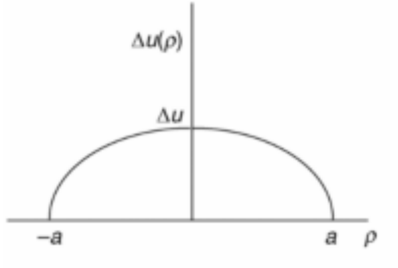


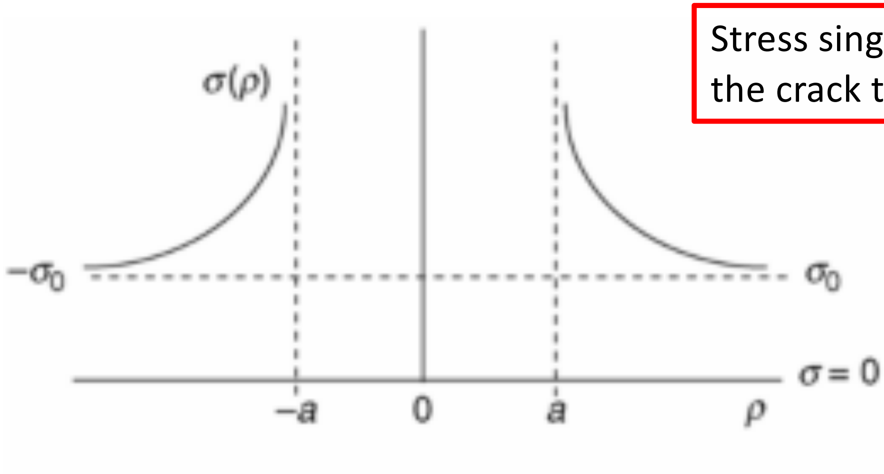
https://www.fracturemechanics.org

Cracks

Static equilibrium in a linear elastic solid with a slit and boundary conditions:
$\sigma(x) = \sigma_0$ for $|x| > a$ and
$\sigma(x) = 0$ for $|x| < a$.

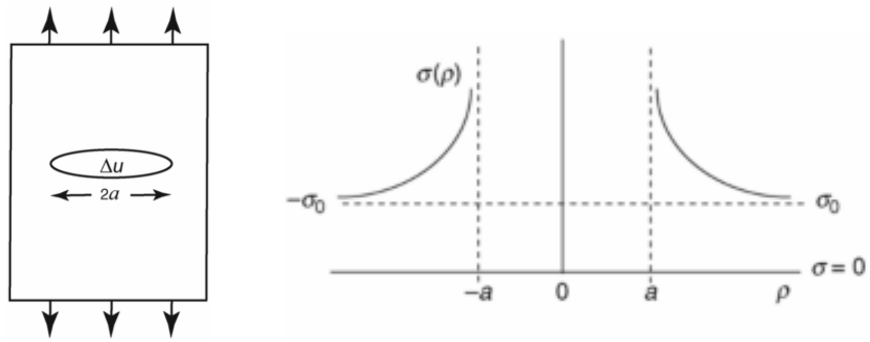






Stress singularity at the crack tips

Asymptotic stress field near crack tips



Stress singularity at the crack tips.
Asymptotic form:

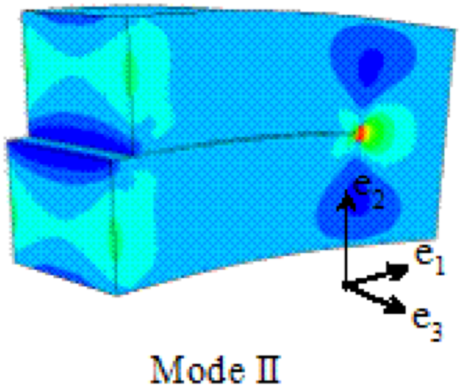
$$\sigma = \frac{K_I}{\sqrt{2\pi r}} + O(\sqrt{r})$$

where r is the distance to a crack tip,
K is the ***stress intensity factor***
and $\Delta\sigma$ the stress drop (here, σ_0 - 0)

$$K_I = \Delta\sigma\sqrt{a/2}$$

In reality, stresses are finite: singularity accommodated by inelastic deformation.

Historical comments

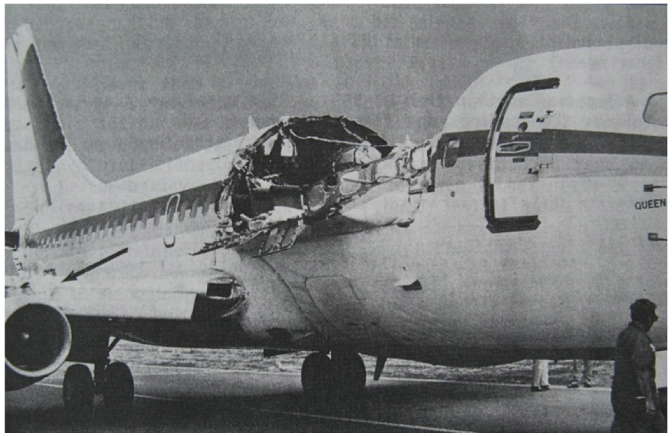


Stress concentration

$$\sigma \sim \frac{K}{\sqrt{r}}$$

Energy release rate

$$G \propto K^2$$



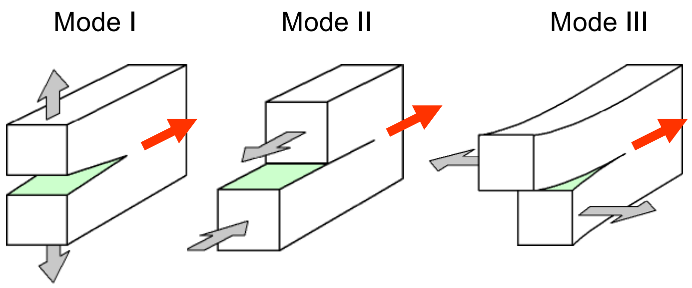
Fracture mechanics

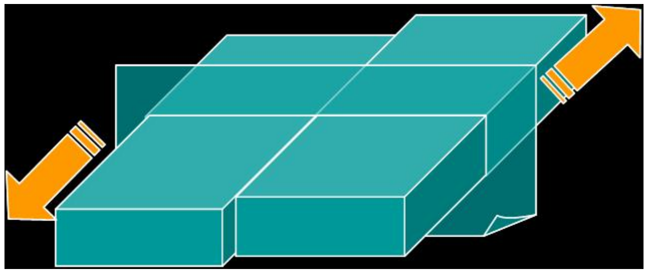
Arrest criterion based on static stress intensity factor K:

- Rupture grows dynamically if K>Kc
- Rupture stops if K=Kc

K can be computed for arbitrary rupture size
and arbitrary spatial distribution of stress drop

Fracture modes





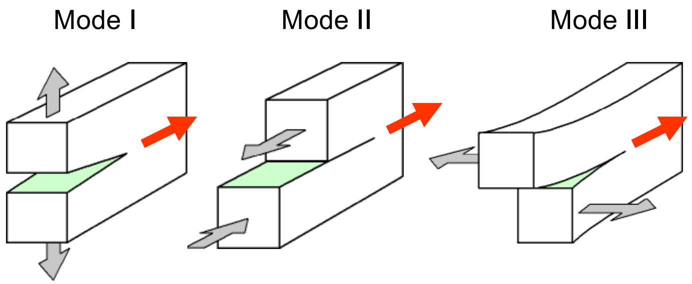
- Mode I = opening cracks
  → engineering, dykes

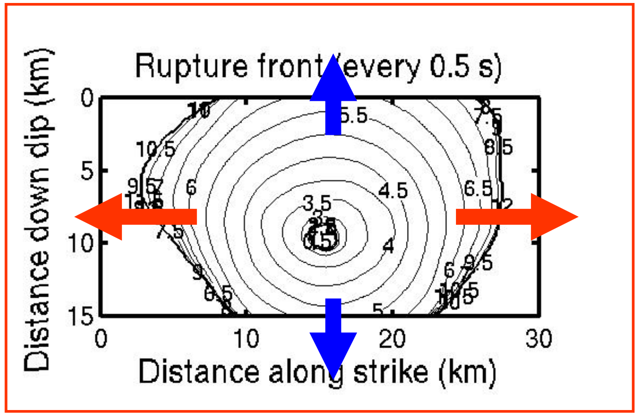
- Modes II and III = shear cracks
  → earthquakes

  - Mode II = in-plane, P-SV waves, rupture propagation // slip
    For strike-slip faults:
    - 2D: map view of depth averaged quantities

  - Mode III = anti-plane, SH waves, rupture propagation ⊥ slip
    For strike-slip faults:
    - 2D: vertical cross-section assuming invariance along strike

Fracture modes





- Mode I = opening cracks
  → engineering, dykes

- Modes II and III = shear cracks
  → earthquakes

  - Mode II = in-plane, P-SV waves, rupture propagation // slip
    For strike-slip faults:
    - 3D: horizontally propagating rupture fronts ↔

  - Mode III = anti-plane, SH waves, rupture propagation ⊥ slip
    For strike-slip faults:
    - 3D: vertically propagating fronts ↕

$$\sigma_{xz}(\rho,\phi) \approx (K_{\mathrm{II}}\cos\phi + K_{\mathrm{III}}\sin\phi)\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\rho-a}}$$

Stress singularity at the rupture front

$$\sigma = \frac{K_I}{\sqrt{2\pi r}}$$

- r = distance to the crack tip
- **K = stress intensity factor,** depends on :
  - rupture mode
  - crack and body geometry (size and shape)
  - remotely applied stress (tectonic load)
  - rupture velocity

Static stress intensity factor K_0

- Example #1: constant stress drop $\Delta\tau$ in crack of half-size *a*

$$K_{II} = \Delta\sigma\sqrt{a/2}$$

Static stress intensity factor K_0

- Example #2: non uniform stress drop in semi-infinite crack

$$K_{\mathrm{III}}(X,\ 0) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{0} \frac{\Delta\sigma(\xi)}{\sqrt{\xi}}\, d\xi$$

Dynamic stress intensity factor

In general, K depends on
- rupture velocity *v*
- stress drop $\Delta\tau$
- crack size *a*

In many useful cases it can be factored as

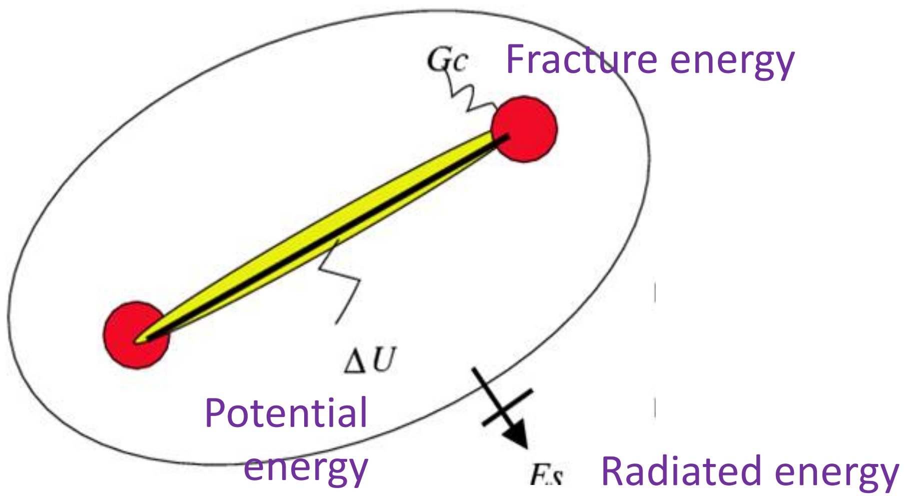
$$K_{III}(t) = \sqrt{1 - v/\beta}\, K^{*}_{III}$$

where $K^{*}(\Delta\sigma, a)$ is the *static* K value that would appear immediately after rupture arrest

and β is S-wave speed

Energy flux to the crack tip **G**

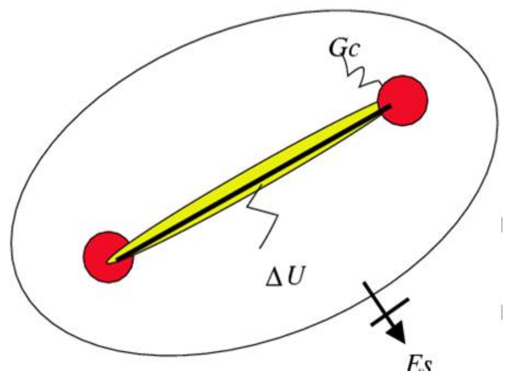
During rupture growth, energy flows into the crack tip.



Energy flux to the crack tip G

The energy flux to the tip, or energy release rate G, is related to K by:

$$G = \frac{K_{\mathrm{III}}^2}{2\mu\sqrt{1 - v^2/\beta^2}} = \sqrt{\frac{1 - v/\beta}{1 + v/\beta}}\frac{(K_{\mathrm{III}}^*)^2}{2\mu}$$



Fracture energy G_c and the crack tip equation of motion

- The energy flux G to the crack tip is dissipated in the process zone by "microscopic" inelastic processes: frictional weakening, plasticity, damage, etc

- These dissipative processes may be lumped into a single mesoscopic parameter: the **fracture energy G_c** (energy loss per unit of crack advance)

- **Griffith rupture criterion**:
  - If the crack is at rest, $G \leq G_c$
  - If the crack is propagating, $G = G_c$
    (energy balance at the crack tip)

Fracture energy G_c and the crack tip equation of motion

Griffith rupture criterion = energy balance at the crack tip during rupture growth

→ **crack tip equation of motion:**

$$G_c = \sqrt{\frac{1 - v/\beta}{1 + v/\beta}} \frac{(K_{III}^*)^2}{2\mu}$$

$$G_c = G(a, \dot{a}, \Delta\tau)$$

$$G_c \sim \sqrt{\frac{1 - \frac{\dot{a}}{\beta}}{1 + \frac{\dot{a}}{\beta}}} \; \pi a \frac{\Delta\tau^2}{2\mu} \qquad = g(\dot{a}) G_0(a)$$

Given $\Delta\tau$ and G_c, solving this ordinary differential equation gives the rupture history $a(t)$ and $\dot{a}(t)$

Graphical solution of equation of motion …

Implication #1: nucleation size

Rupture only if G=Gc
At the onset of rupture (critical equilibrium, v=0):

$$G_c = G_0(a,\Delta\tau) = \pi\ a\ \Delta\tau^2 / 2\mu$$

→ earthquake initiation requires a minimum crack size (*nucleation size*)

$$\mathbf{a_c = 2\mu\ G_c / \pi\Delta\tau^2}$$

($\mu \approx 30$ GPa, $\Delta\tau \approx 5$ MPa)
Estimates for large earthquakes $G_c \approx 10^6$ J/m^2 → $a_c \approx 1$ km
… so how can M<4 earthquakes nucleate ?!

Laboratory estimates: $G_c \approx 10^3$ J/m^2 → $a_c \approx 1$ m (M -2)

→ **G_c scaling problem**

Implication #2: limiting rupture velocity

Crack tip equation of motion:

$$\boldsymbol{G_c} \sim \sqrt{\frac{1-\frac{\dot{a}}{\beta}}{1+\frac{\dot{a}}{\beta}}}\ \boldsymbol{\pi a \frac{\Delta\tau^2}{2\mu}} \quad = \boldsymbol{g(\dot{a}) G_0(a)}$$

If $\Delta\tau$ and G_c are constant, the rupture velocity remains sub-shear

but approaches very quickly β

However, in natural and laboratory ruptures the usual range is $\dot{a} \leq 0.7\beta$!

Implication #3: rupture arrest

Rupture stops if

$$\boldsymbol{G_c > G \sim \sqrt{\frac{1-\frac{\dot{a}}{\beta}}{1+\frac{\dot{a}}{\beta}}}\ \pi a\, \Delta\tau^2/2\mu}$$

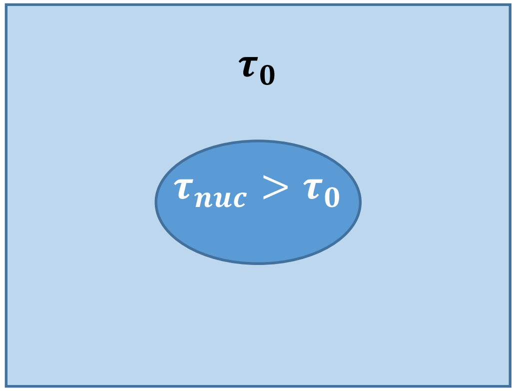
The earthquake may stop due to two effects:

- Low stress regions (negative stress drop)

  → G(a,$\Delta\tau$) decreases
- Increasing fracture energy :
  - abrupt arrest in barriers (regions of high G_c)
  - smooth arrest due to scale-dependent G_c

Rupture arrest in dynamic earthquake models

Rupture nucleated at a highly stressed patch
(area **Anuc**, background stress $\boldsymbol{\tau_0}$)

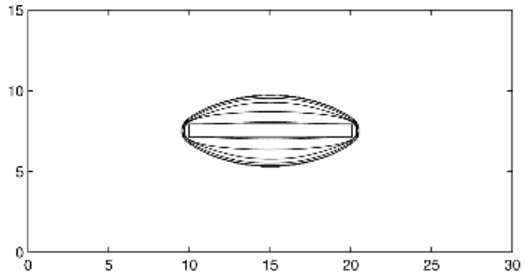


Will it stop?

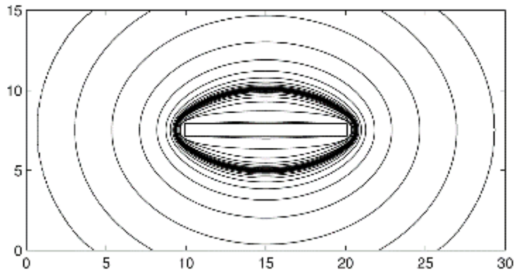
How does final rupture size depend
on nucleation size and overstress?

Rupture front plots
(rupture time contours)

Small Anuc and τ_0
→ Stopping ruptures



Large Anuc and τ_0
→ Runaway ruptures



Rupture arrest predicted by fracture mechanics theory

Fracture mechanics

Static stress concentration

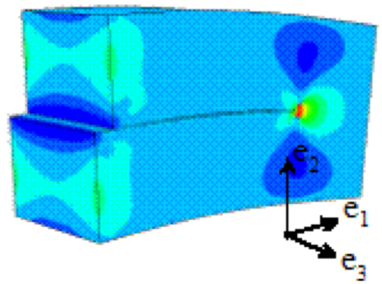
$$\sigma \sim \frac{K_0}{\sqrt{r}}$$

where Ko =static stress intensity factor

Static energy release rate

$$G_0 = K_0^2/2\mu$$

Static Griffith criterion $G_0 = G_c$ can be written as $K_0 = K_c = \sqrt{2\mu G_c}$



Rupture arrest criterion:

- Rupture grows dynamically if Ko>Kc
- Rupture stops if Ko=Kc

Ko depends on stress drop $\Delta\tau$
Ko can be computed for any spatial distribution of $\Delta\tau$

$$K_0(R) = \frac{2}{\sqrt{\pi R}} \int_0^R \frac{\Delta\tau(r)}{\sqrt{R^2 - r^2}} r dr$$

(Ripperger et al 2007, Galis et al 2014)

Rupture arrest predicted by fracture mechanics theory

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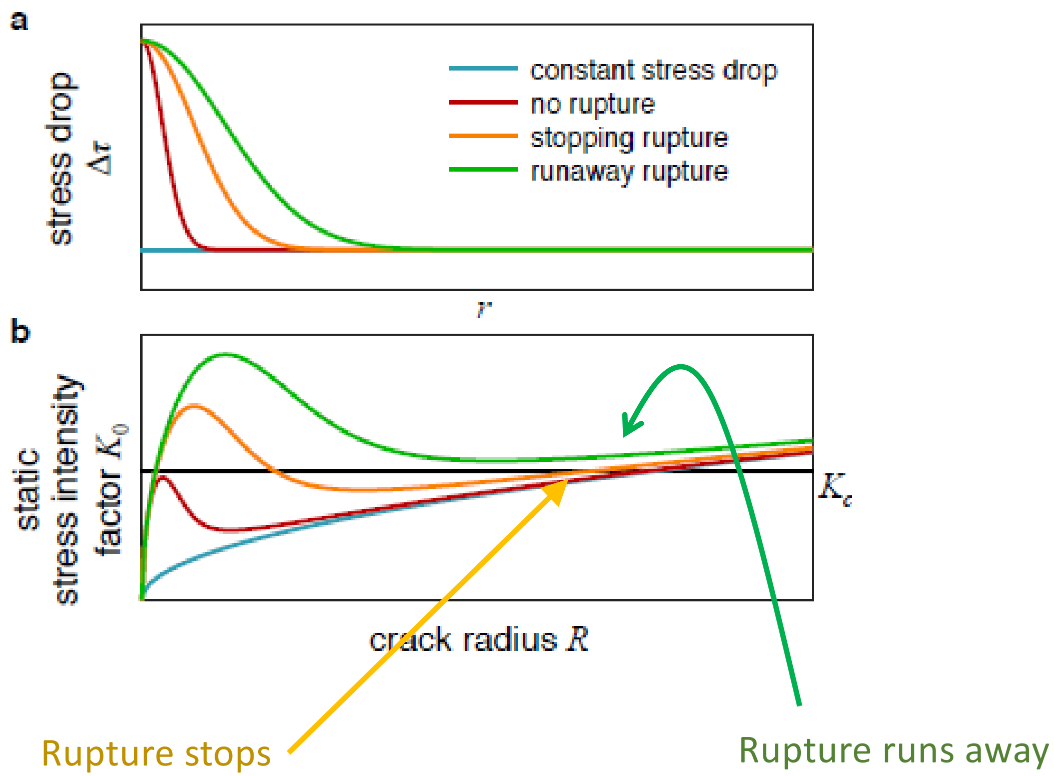
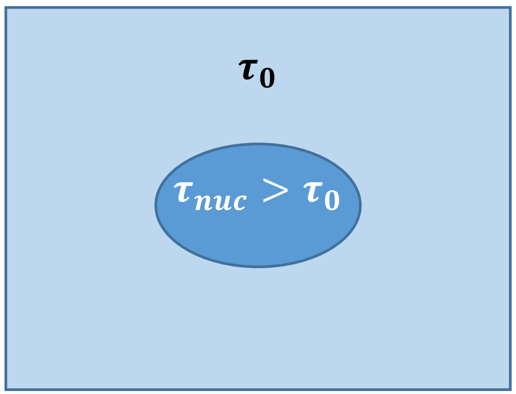


(Ripperger et al 2007, Galis et al 2014, 2017)

Rupture arrest in dynamic earthquake models is well predicted by fracture mechanics

Rupture nucleated at a highly stressed patch



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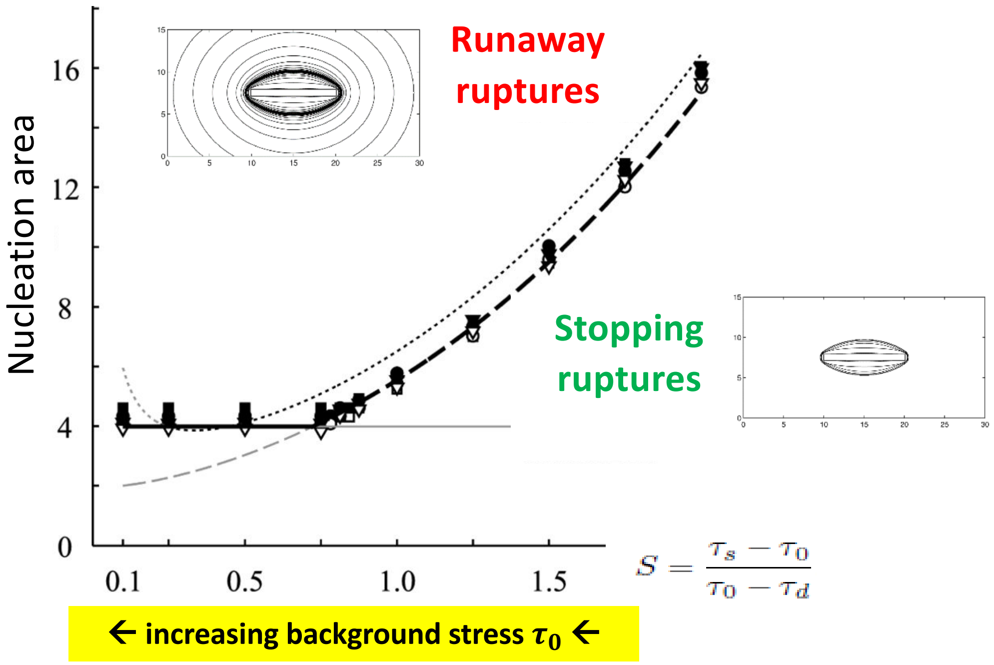
How does final rupture size depend on nucleation size and overstress?



Galis et al (2014)

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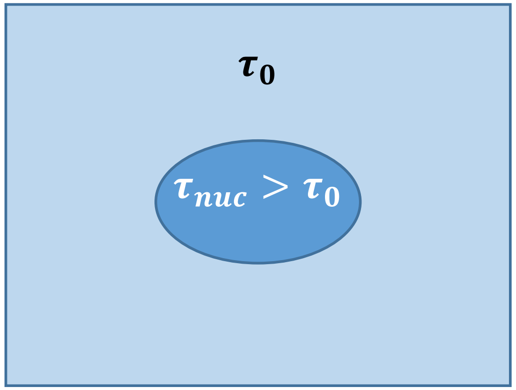

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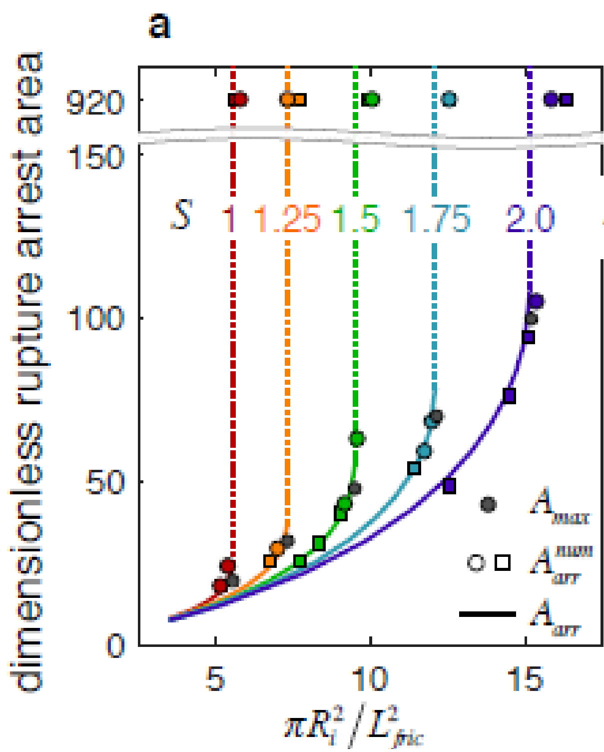
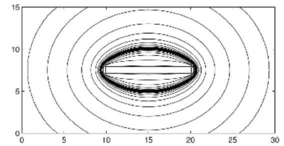


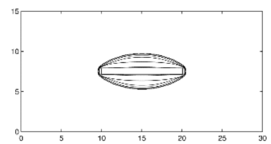
Nucleation area

$$S = \frac{\tau_s - \tau_0}{\tau_0 - \tau_d}$$

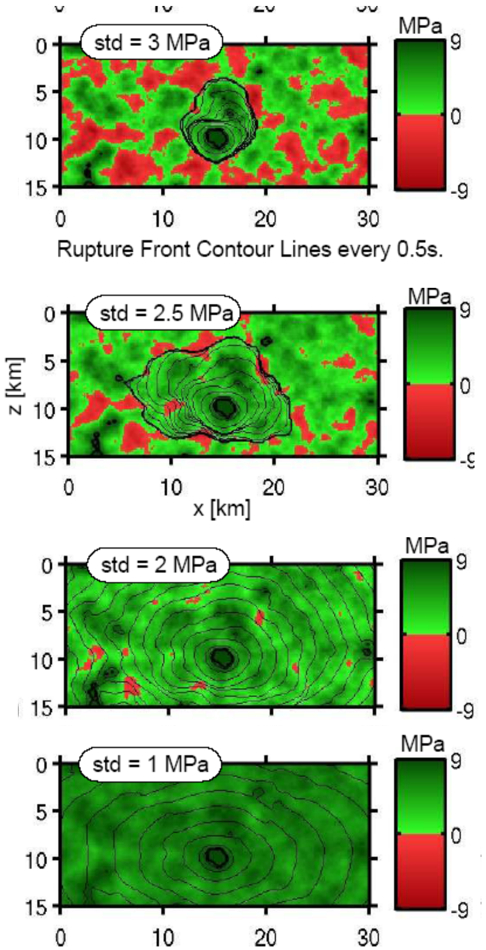
Runaway ruptures



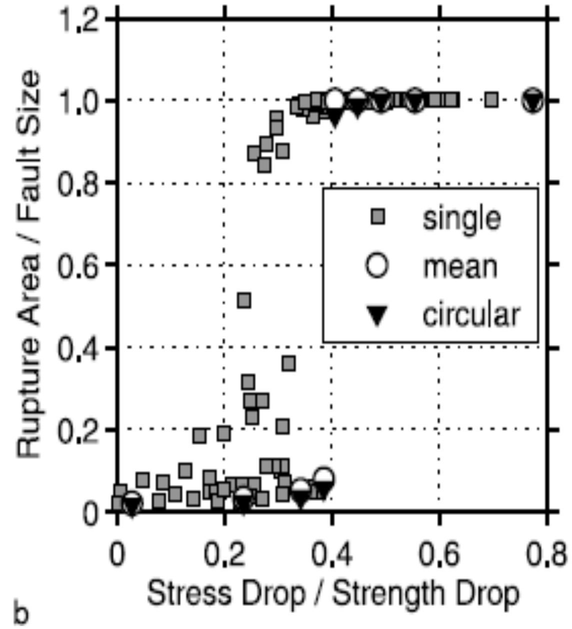
Stopping ruptures



Rupture arrest



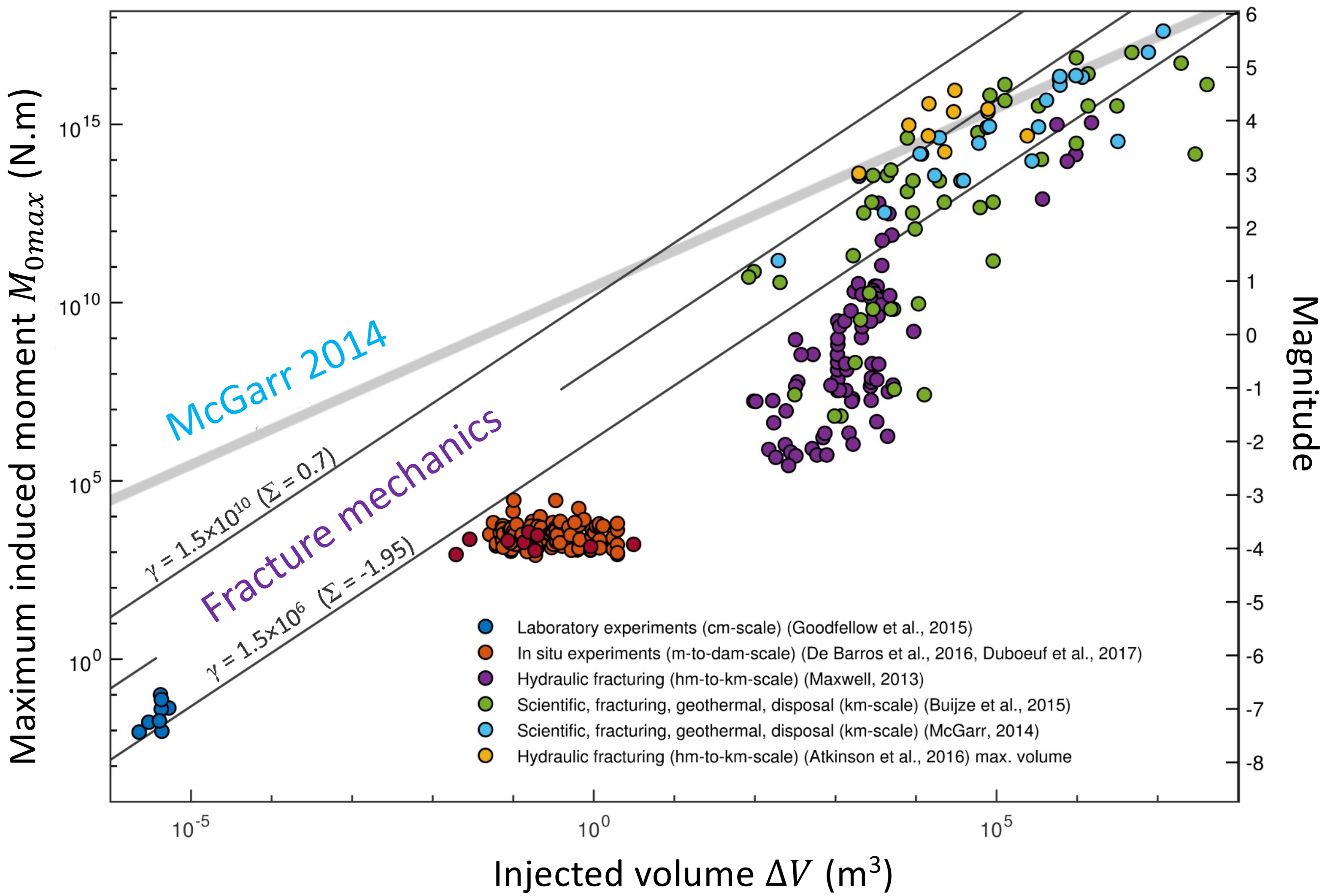
Rupture “percolation” transition



Ripperger et al (2007)

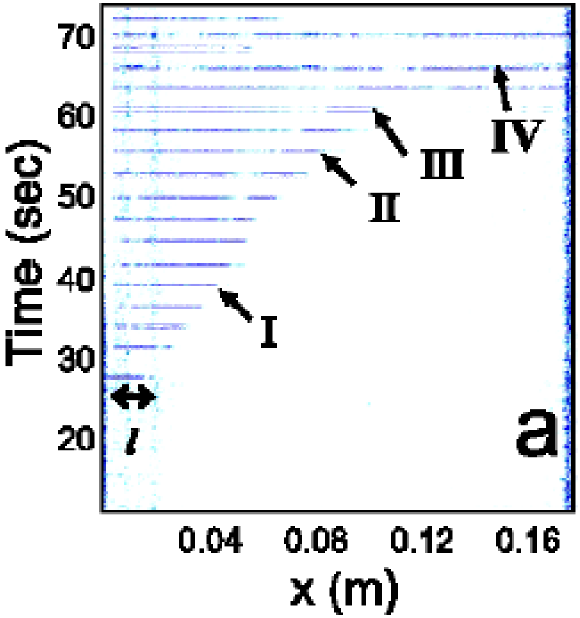
Fracture mechanics: $M_{0max} \propto \Delta V^{3/2}$

Galis et al (2017)

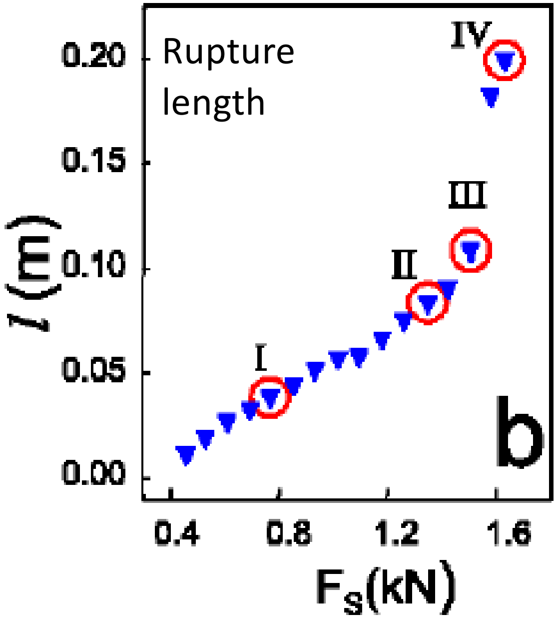


Laboratory quakes nucleated by a localized load

Rubinstein, Cohen and Fineberg (2007)

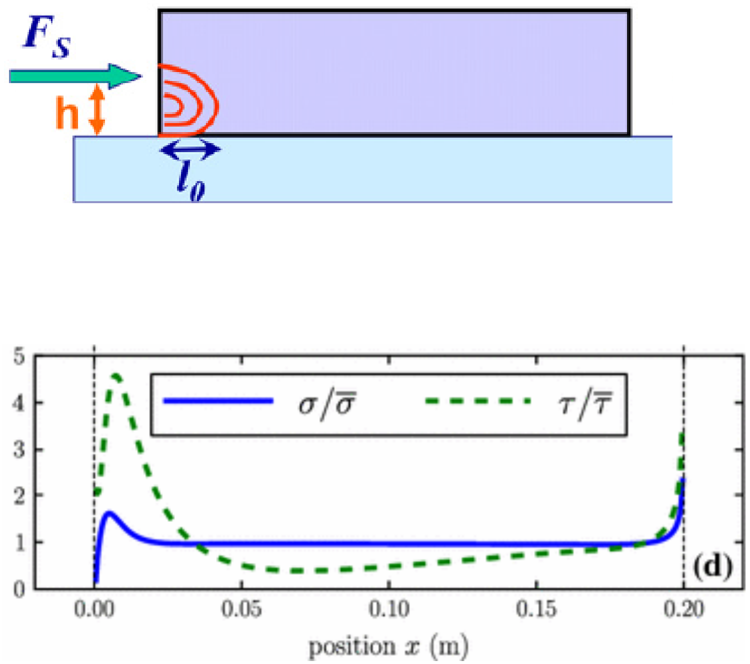


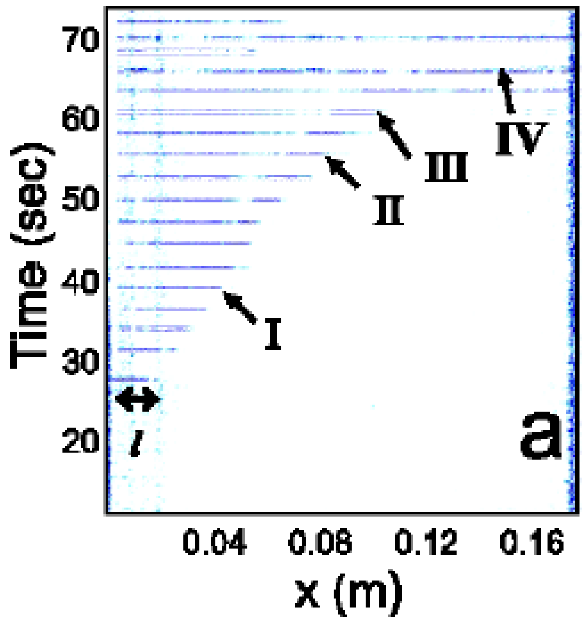
Rupture length



Loading force

Laboratory quakes nucleated by a localized load





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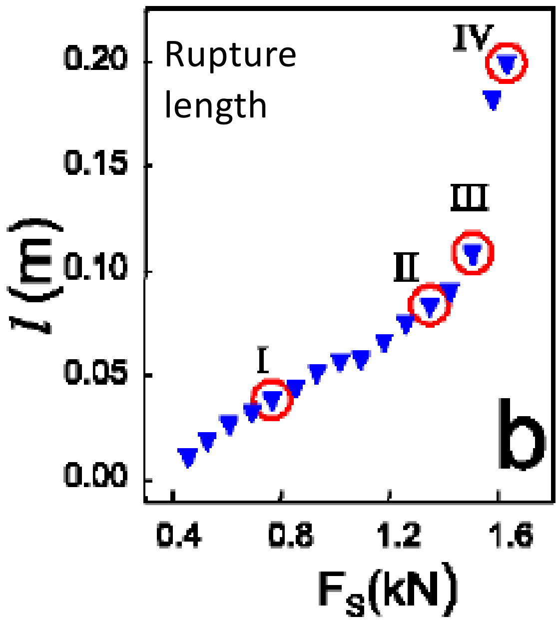
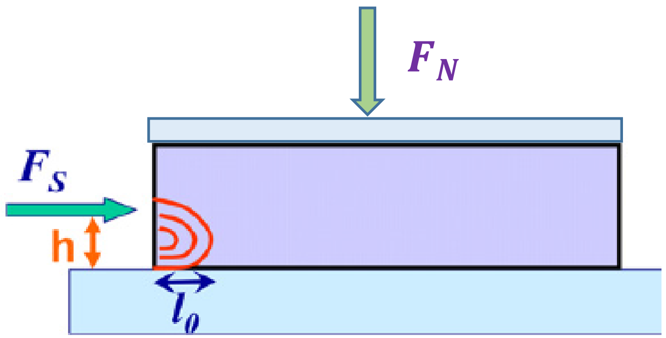
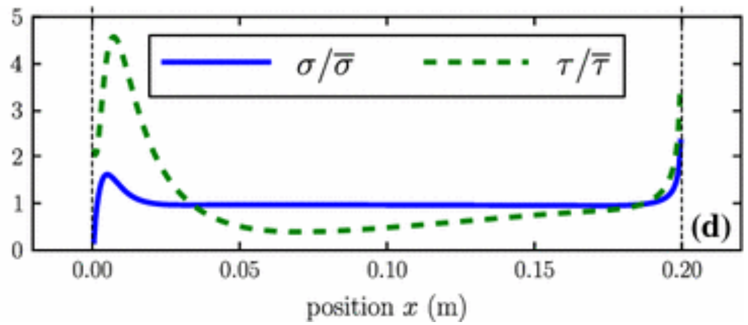


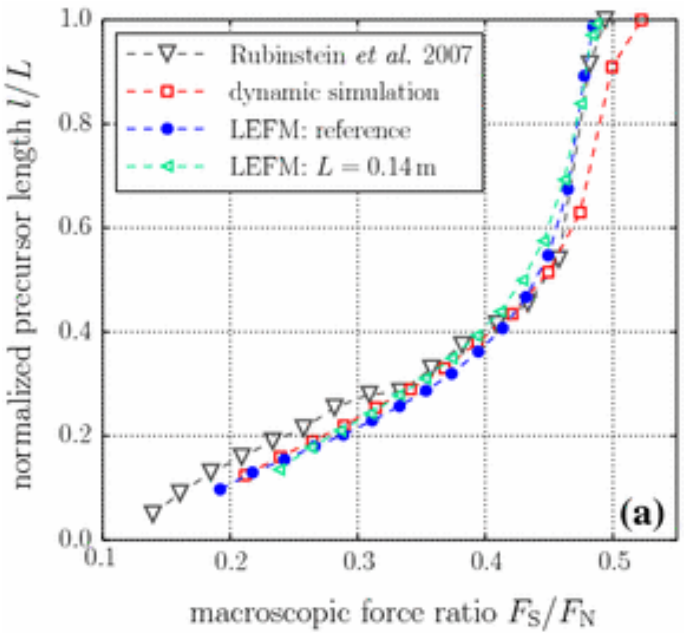
Loading force

Rubinstein, Cohen and Fineberg (2007)

Size of laboratory quakes predicted by fracture mechanics

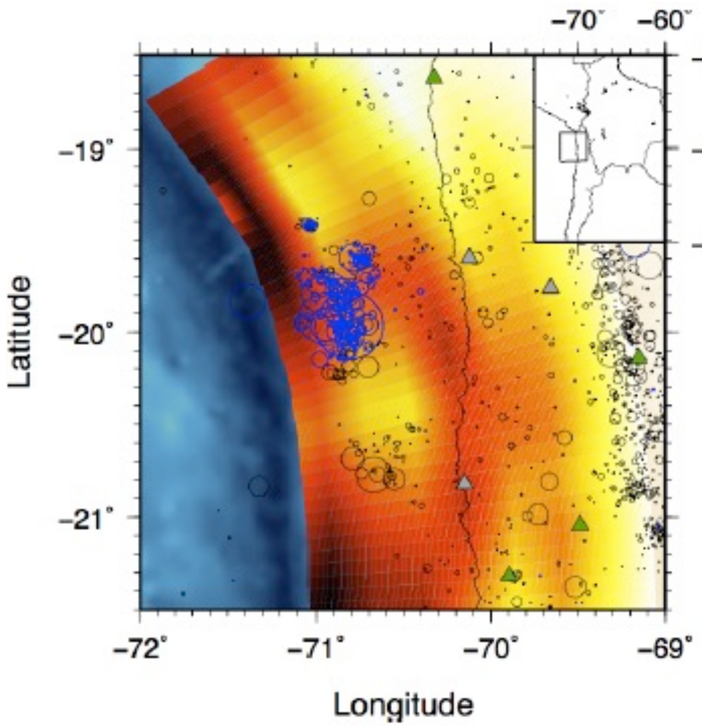


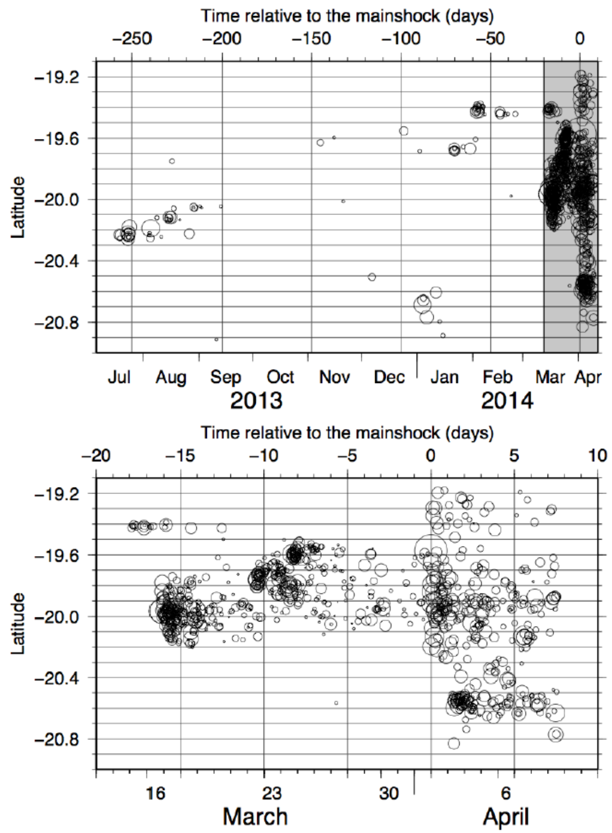




Kammer, Radiguet, Ampuero and Molinari
(Tribology Letters, 2015)

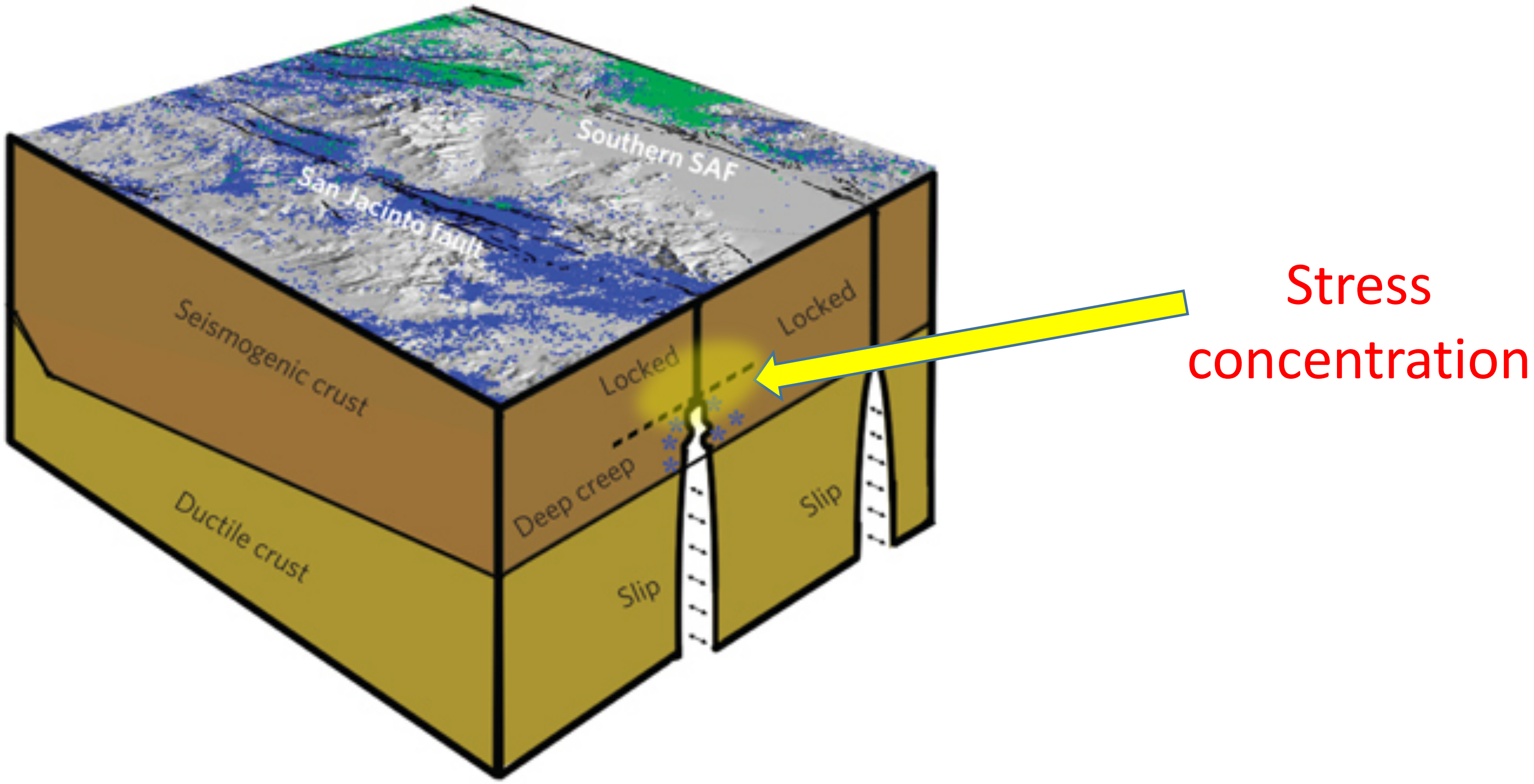
Foreshock swarms Iquique 2014



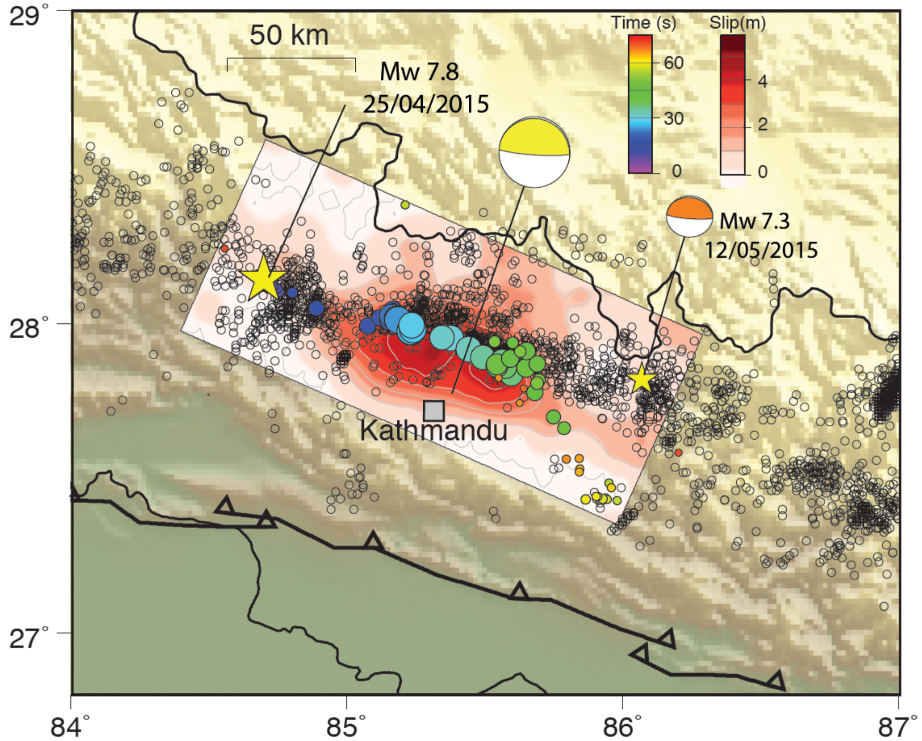


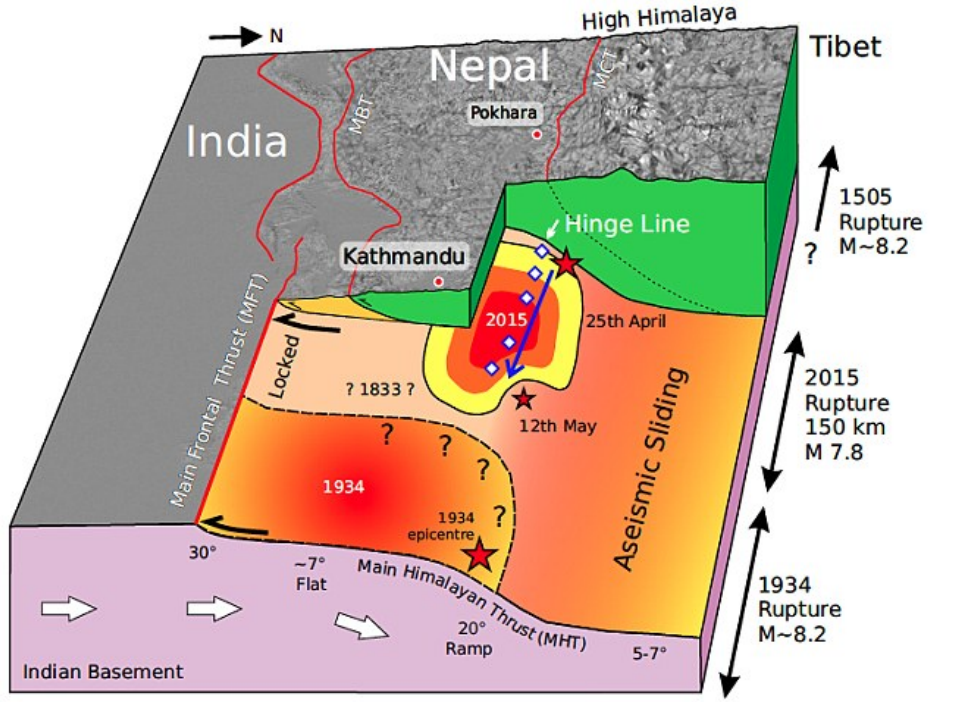
Ampuero et al (2014)

Fault loading by deep creep

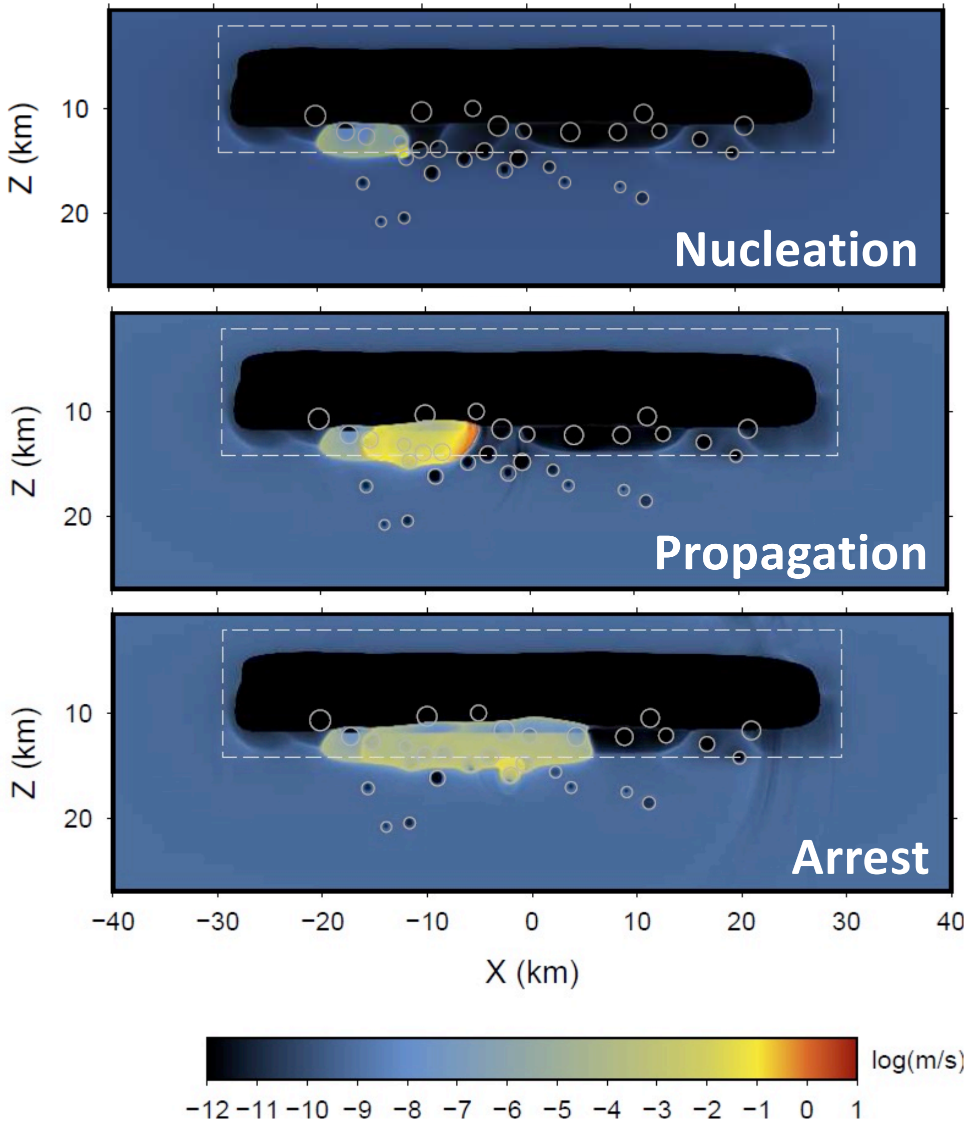


2015 Gorkha, Nepal earthquake

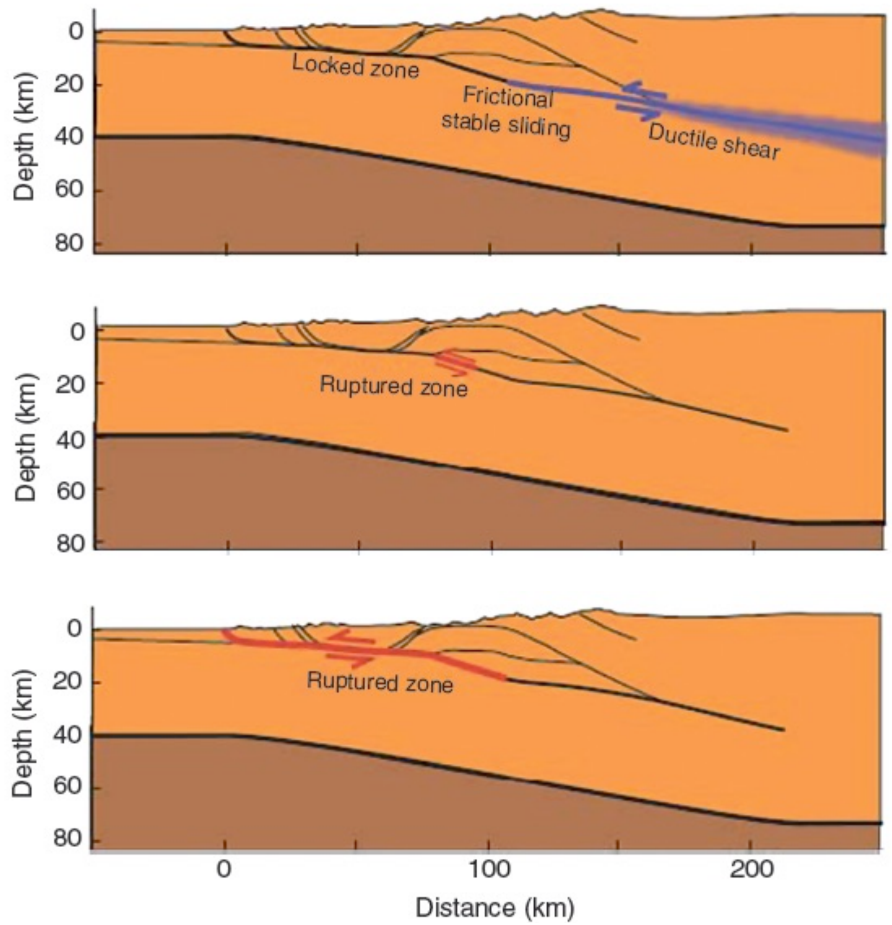




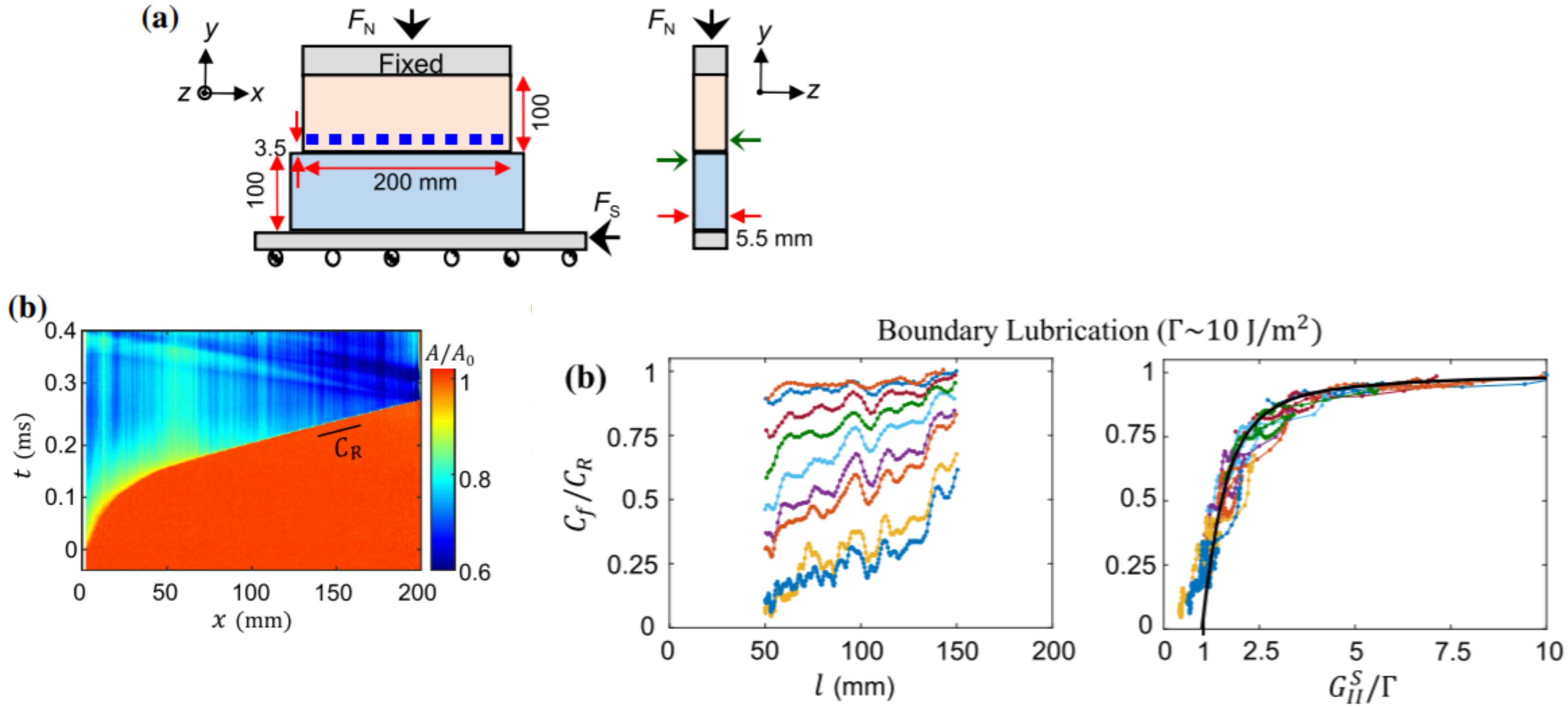
Intermediate-size event unzipping part of the lower edge of the coupled zone (Junle Jiang, Caltech)



Super-cycles: large earthquakes + smaller, deeper earthquakes in between

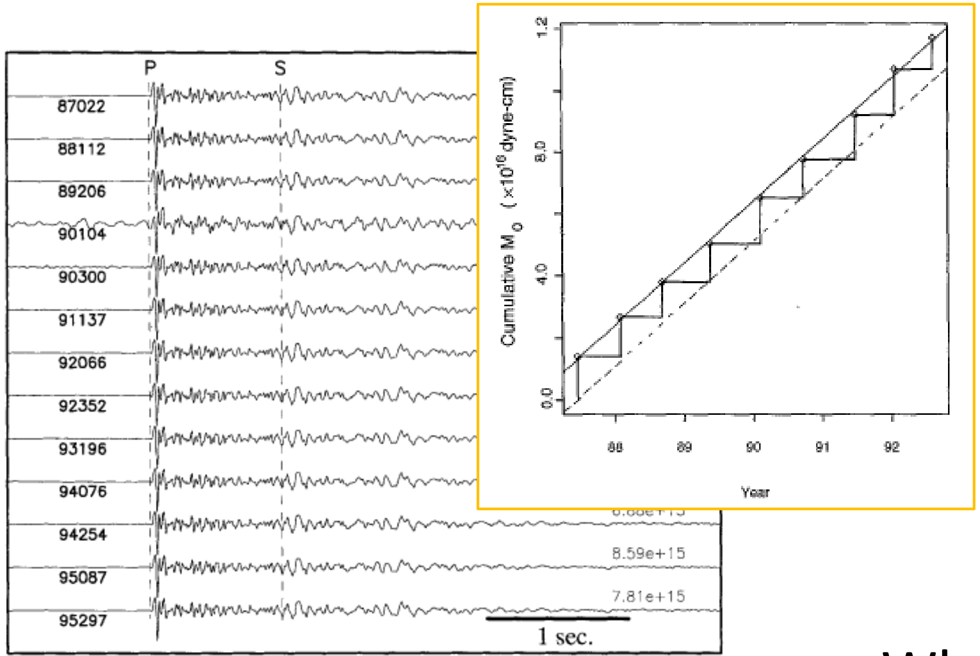


Speed of laboratory quakes



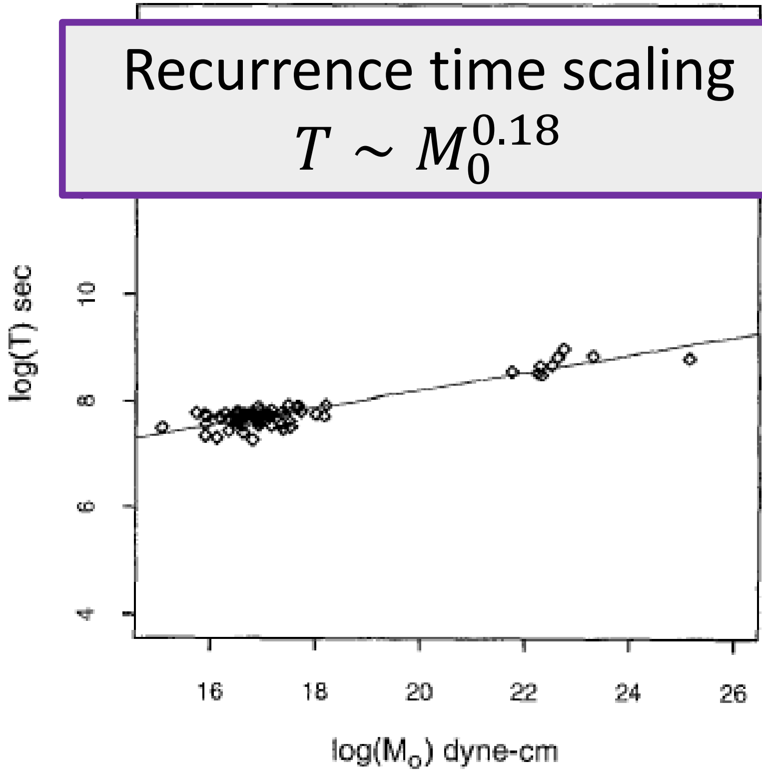
Svetlizky et al (2017)

Recurrence time scaling of repeating earthquakes



Nadeau and Johnson (1989)

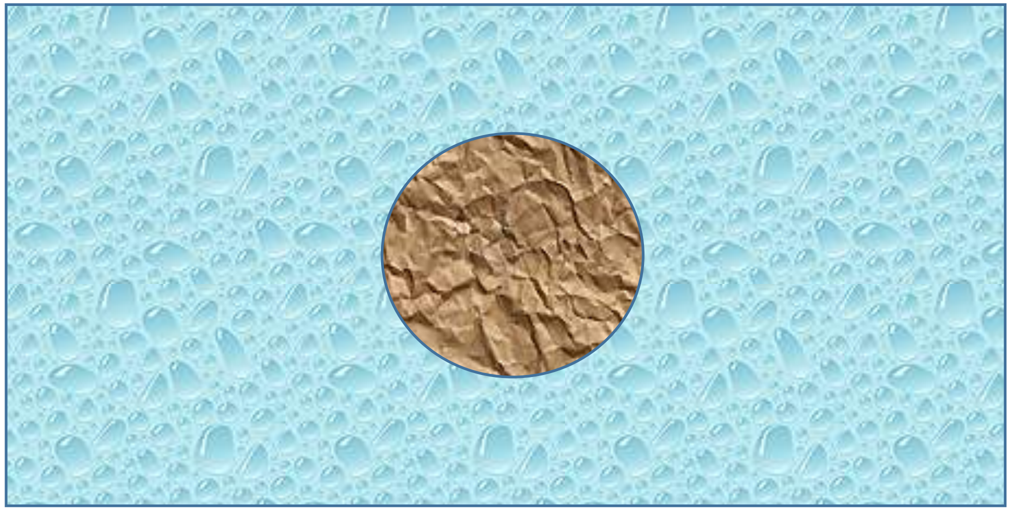
Recurrence time scaling
$T \sim M_0^{0.18}$



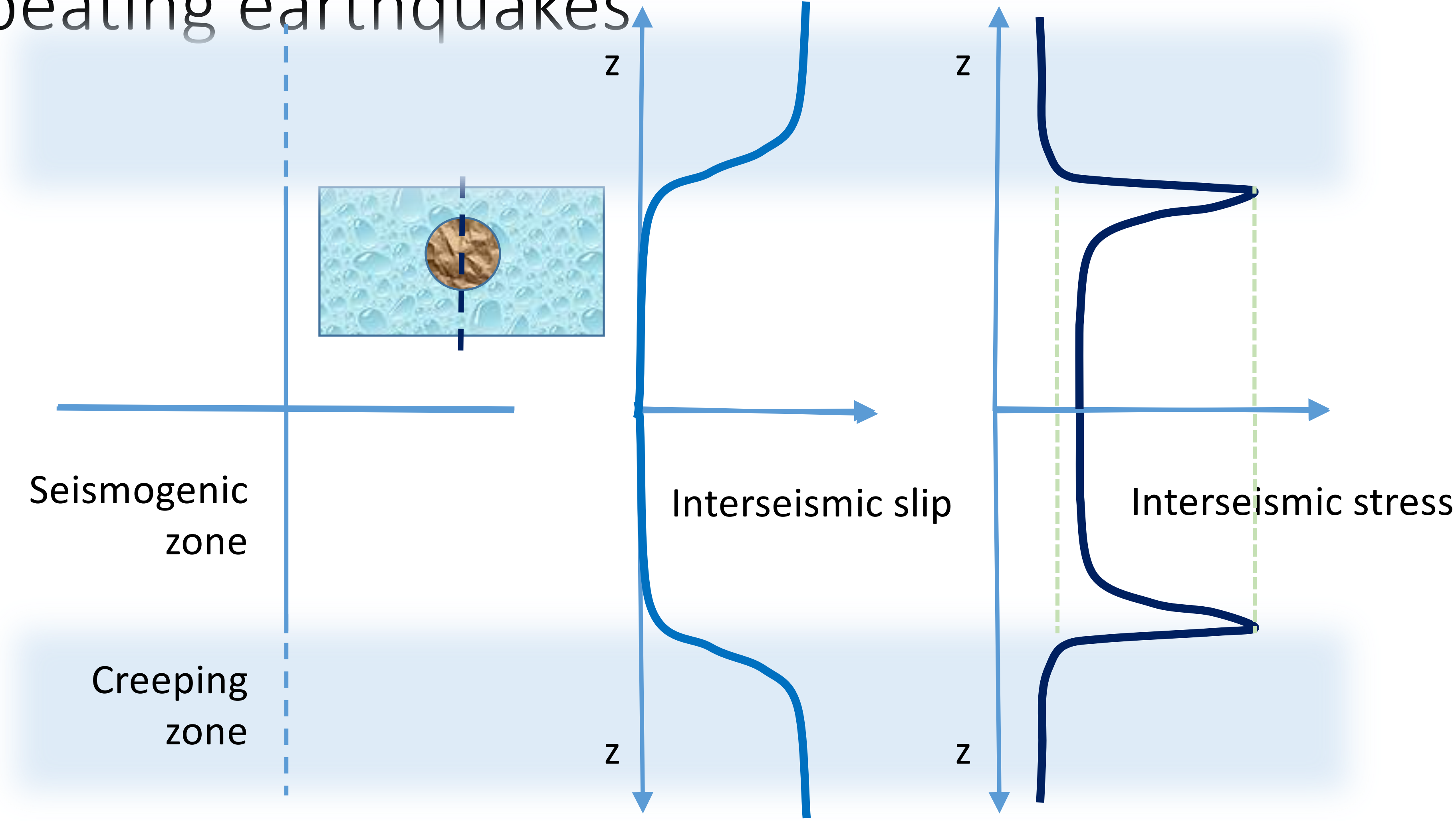
Whereas classical scaling is $T \sim M_0^{1/3}$

Repeating earthquakes

Model: a circular brittle patch (radius R) embedded in a creeping fault



Repeating earthquakes



Recurrence time scaling of repeating earthquakes

Repeating earthquake model: a circular brittle patch (radius R) embedded in a creeping fault (steady slip rate V_{creep})

From fracture mechanics, $G_c = \frac{K^2}{2\mu} \sim \frac{\Delta\tau^2 R}{2\mu}$

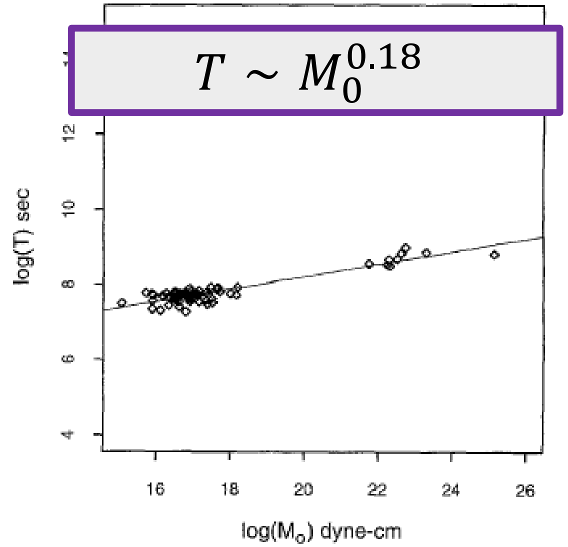
$$\Delta\tau \sim \sqrt{2\mu G_c / R}$$

From elasticity: $\Delta\tau \sim \mu D / R$

Slip budget: $D = V_{creep} T$ per event

Seismic moment: $M_0 = \mu\pi R^2 D$

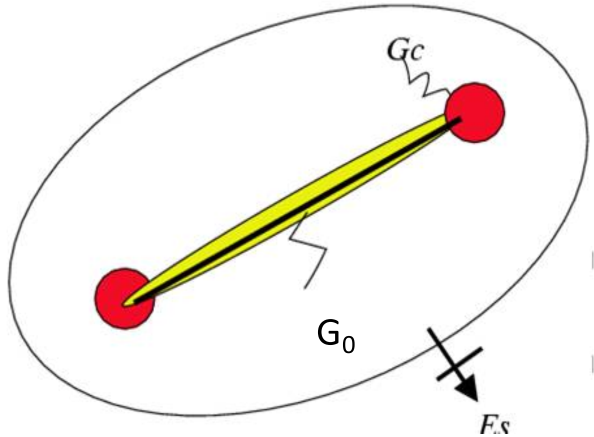
$$\rightarrow T \sim \left(\frac{2G_c}{\mu}\right)^{\frac{2}{5}} \frac{1}{V_{creep}} M_0^{\frac{1}{5}}$$



Radiated energy E_r

- Radiated energy is related to the crack tip energy flux by:

$$E_r = \int (G_0 - G_c)\, da \quad = (1-g(v)) \int G_0\, da$$

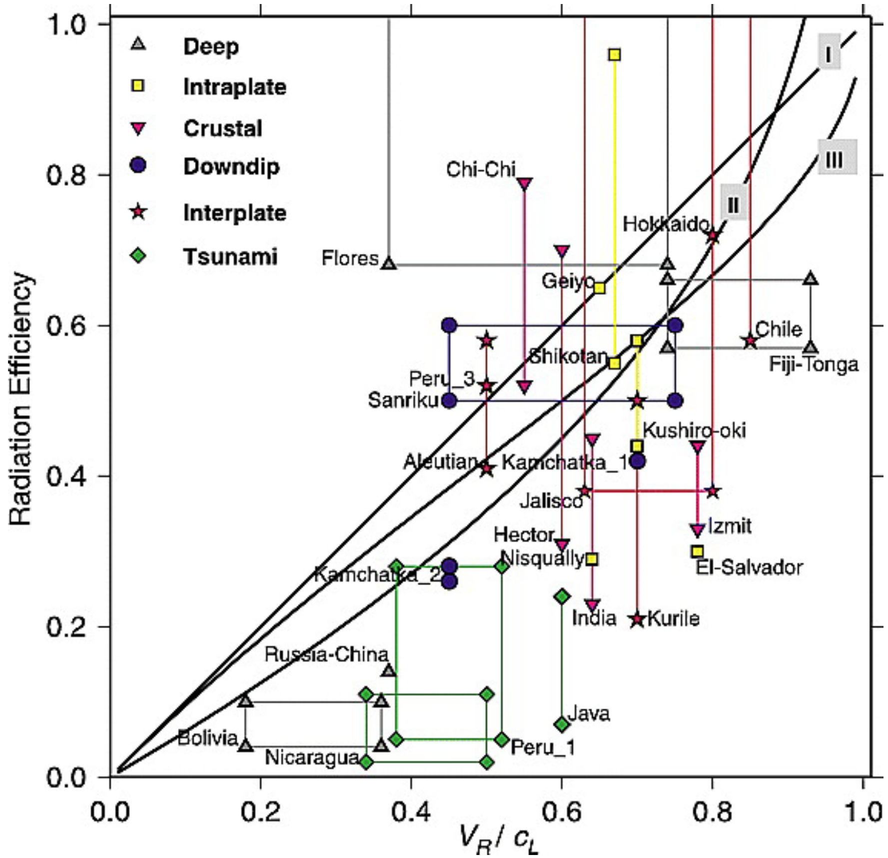


- Large rupture velocity = large E_r

  For a fast crack: $G_0 >> G_c$ → large E_r

- A crack that stops at a size not much larger than the nucleation size a_c does not have time to accelerate → low E_r

Earthquake radiation efficiency



(Venkataraman & Kanamori 2004)

High-frequency radiation

Crack tip equation of motion:

$$G_c = g(\dot{a})G_0(a)$$

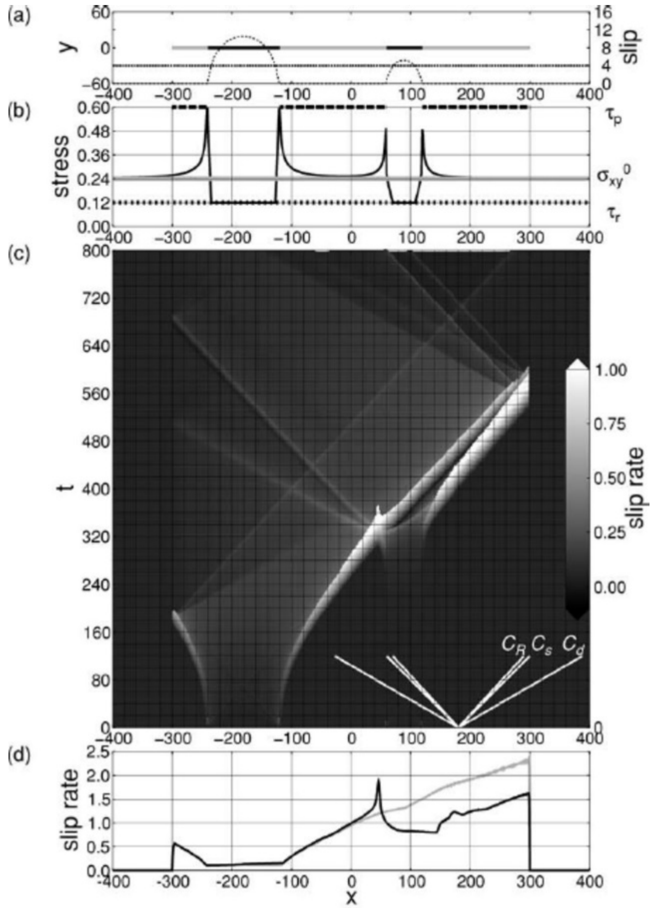
What happens if a rupture front hits a step of Gc?

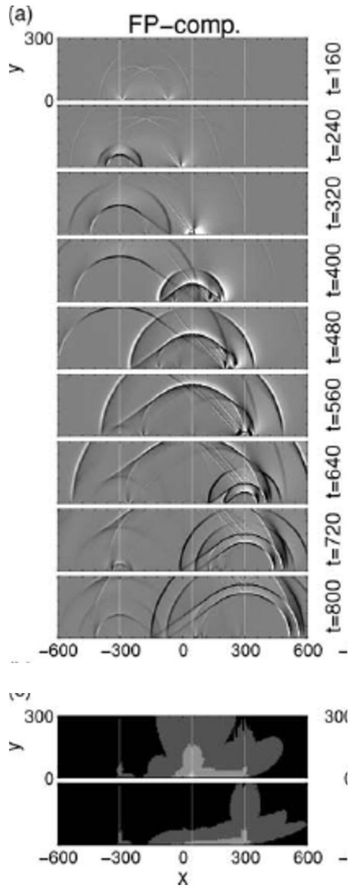
Rupture speed changes abruptly
→ high-frequency radiation

High-frequency radiation

What happens if a rupture front passes through the residual stresses left by a previous earthquake?
(a sqrt singularity)

Rupture speed changes abruptly
→ high-frequency radiation

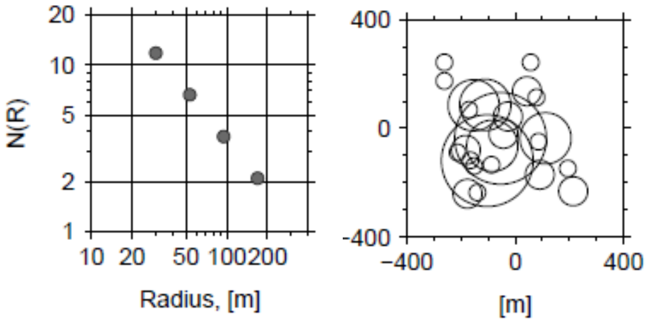


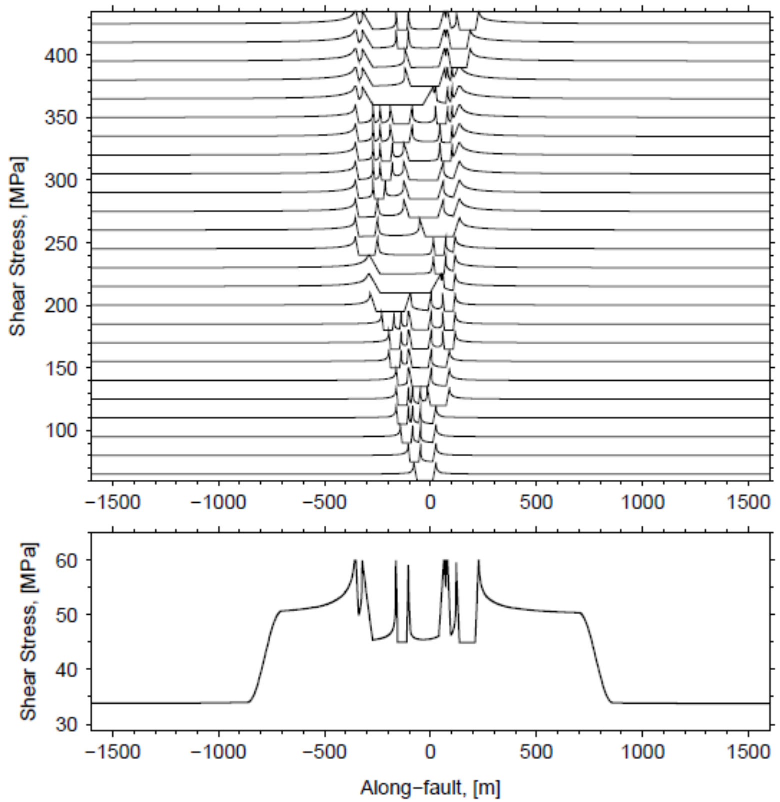


Fault-parallel ground velocity

Peak ground velocity

Kame and Uchida (2008)





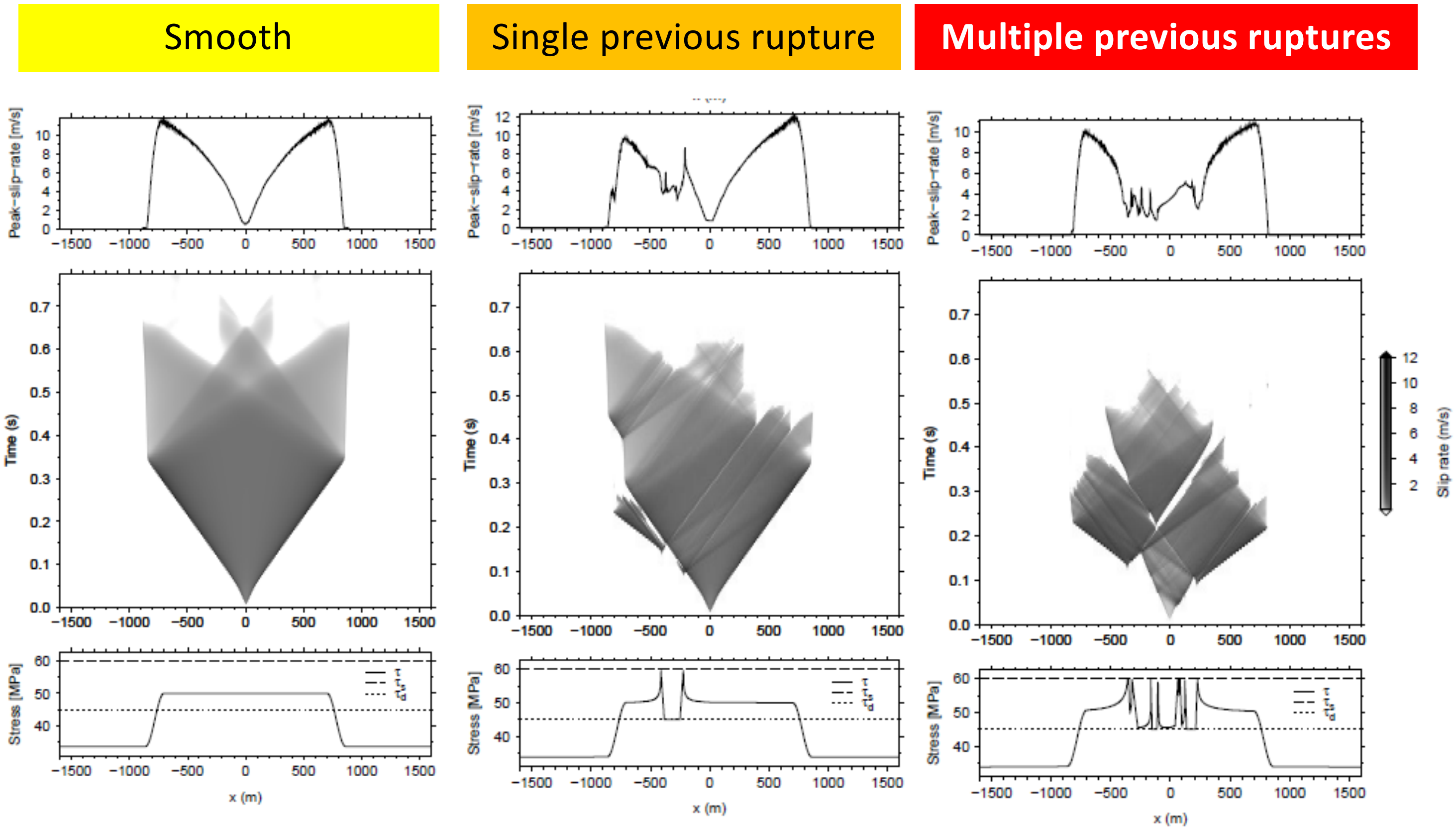
"Initial" fault stress heterogeneities result from background seismicity

Implications:

- statistical self-similarity inherited from the Gutenberg-Richter distribution
- long tail probability distribution due to the spiky nature of the residual stress concentrations at the edges of previous ruptures

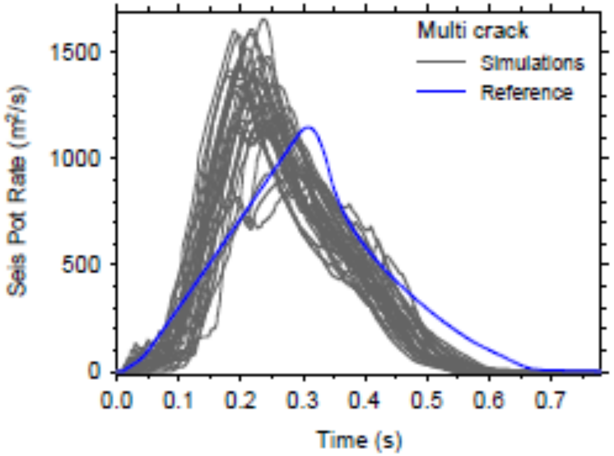
SCEC project by Ampuero, Ruiz and Mai (2008/2009)

2D dynamic ruptures with increasing level of complexity in initial stresses

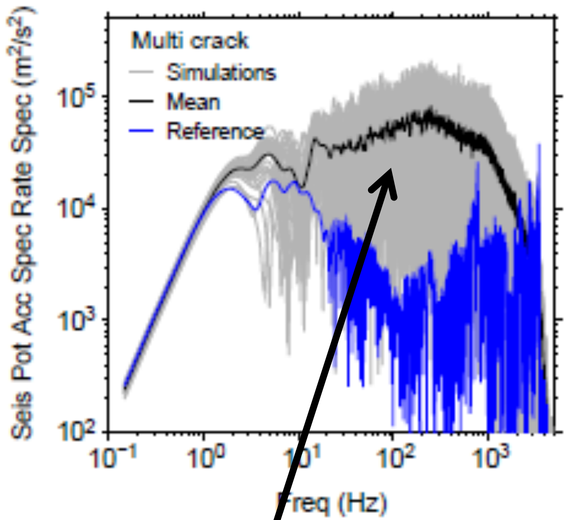


Interaction between the rupture front and the pre-existing stress concentrations radiate strong ω^{-2} phases, induce multiple-front coalescences, and produce healing fronts that encourage pulse-like rupture and heterogeneous final stresses

Far-field source time functions



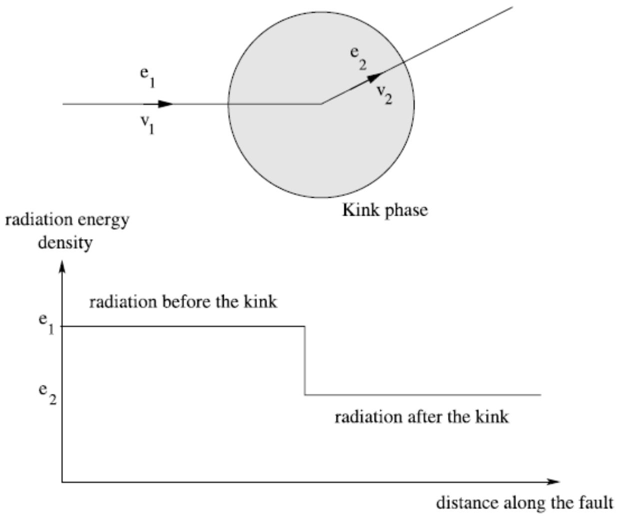
Acceleration spectra

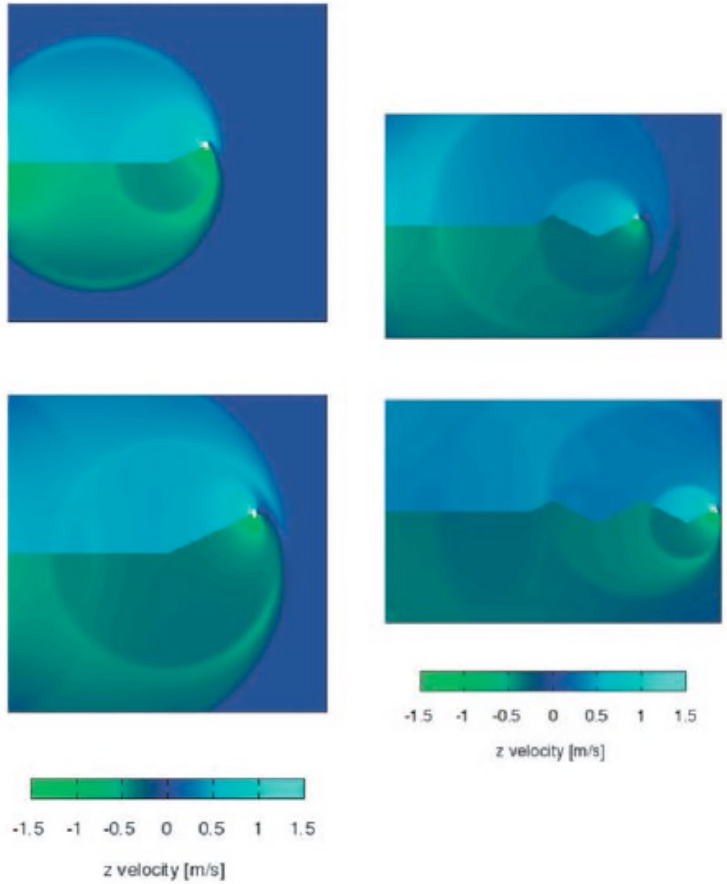


Reference rupture model with smooth arrest

Complex ruptures: enhanced high-frequency radiation

Radiation from a fault kink





Madariaga et al (2006)

Summary of Lecture 1

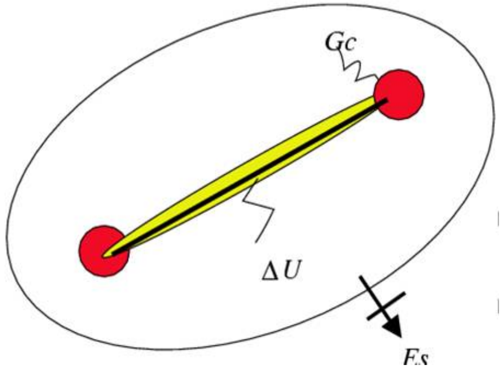
The Fracture Mechanics approach is macroscopic :

- the size of the process zone is assumed much smaller than any other dimension of the problem
- the details of the inelastic processes near the rupture front are ignored, their overall effect is accounted for by the fracture energy

G_c = energy dissipated per unit of crack advance

- the rupture criterion is based on an energy balance, governed by the singular behavior of the idealized elastic model near the crack front

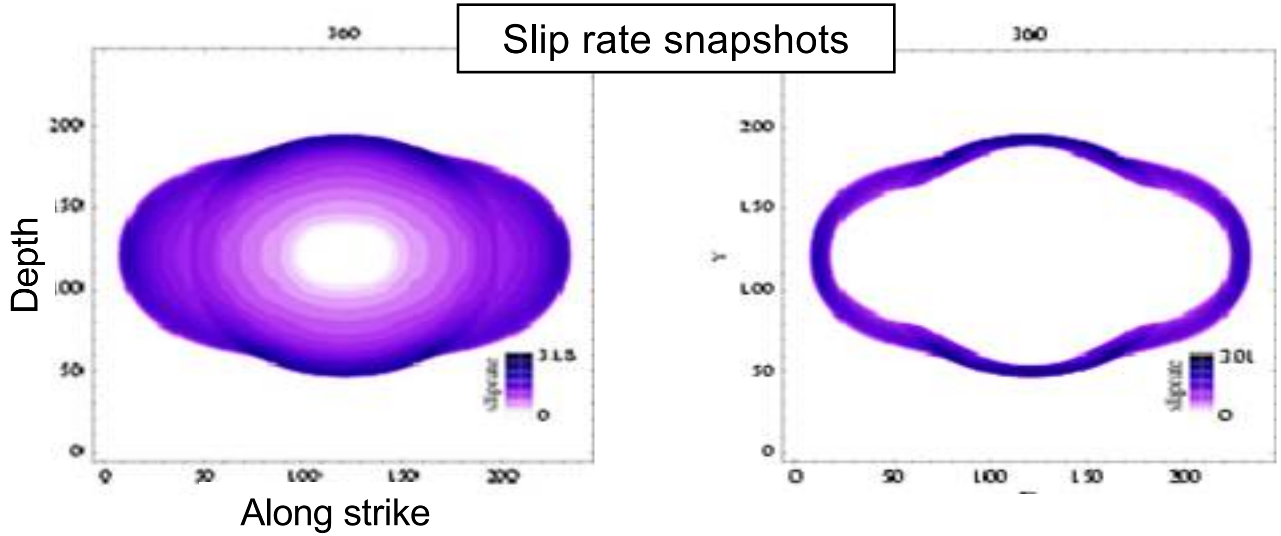
→ a **crack tip equation of motion** relates earthquake propagation parameters (size *a* and rupture velocity *v*) to physical parameters and initial conditions (G_c and stress drop $\Delta\tau$)



Crack tip equation of motion:

$$G_c \sim \sqrt{\frac{1-\frac{\dot{a}}{\beta}}{1+\frac{\dot{a}}{\beta}}}\ \pi a\, \Delta\tau^2/2\mu$$

Rupture styles: cracks and pulses

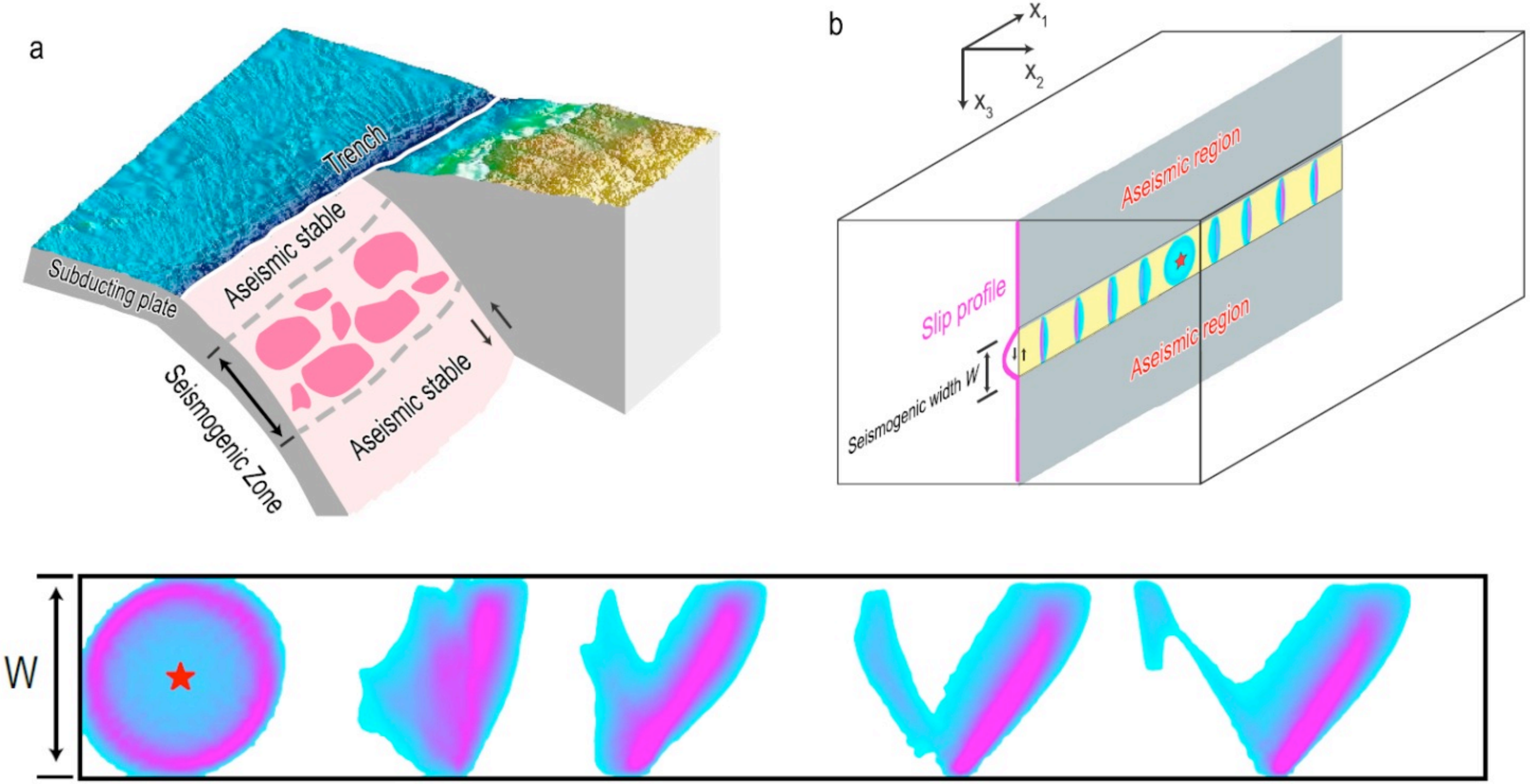


Crack : slip continues behind the rupture front, long rise time

Pulse : slip heals soon behind the rupture front, short rise time

In this lecture we focused on **cracks**.

Pulses on faults with finite seismogenic depth



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