The stability of frictional sliding and the spectrum of fault slip behavior from slow to fast stick-slip

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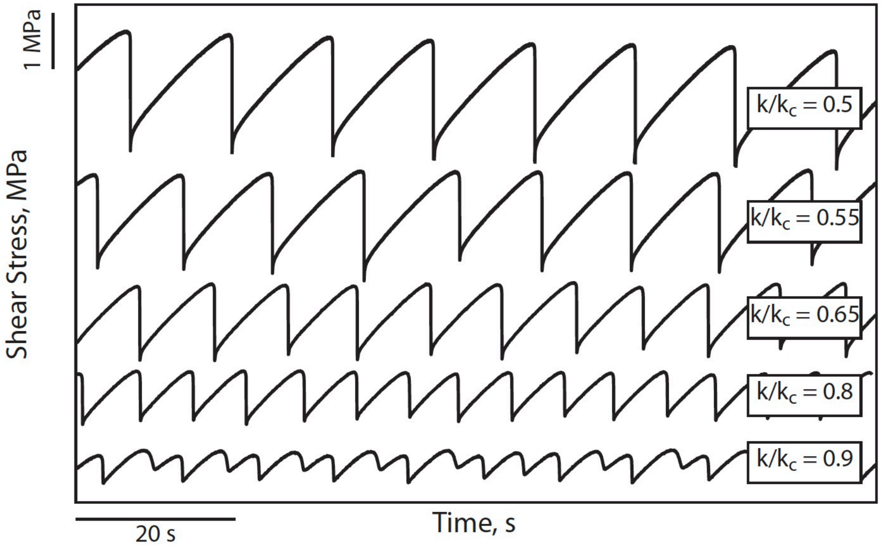
SAPIENZA UNIVERSITÀ DI ROMA

Advanced Workshop on Earthquake Fault Mechanics: Theory, Simulation and Observations"
ICTP, Trieste 02 September 2019 - 14 September 2019

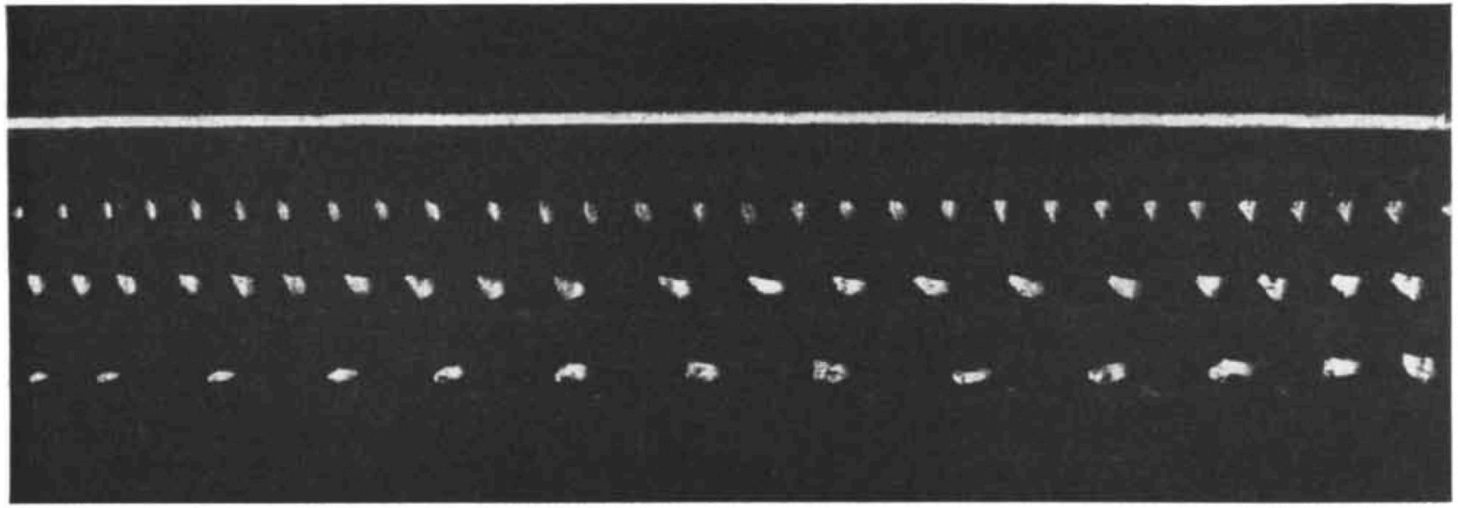


Stick and Slip

When two substances rub against each other, they frequently stick and then slip. The phenomenon accounts for the squeak of bearings, the music of violins and many other sounds of our daily experience

by Ernest Rabinowicz

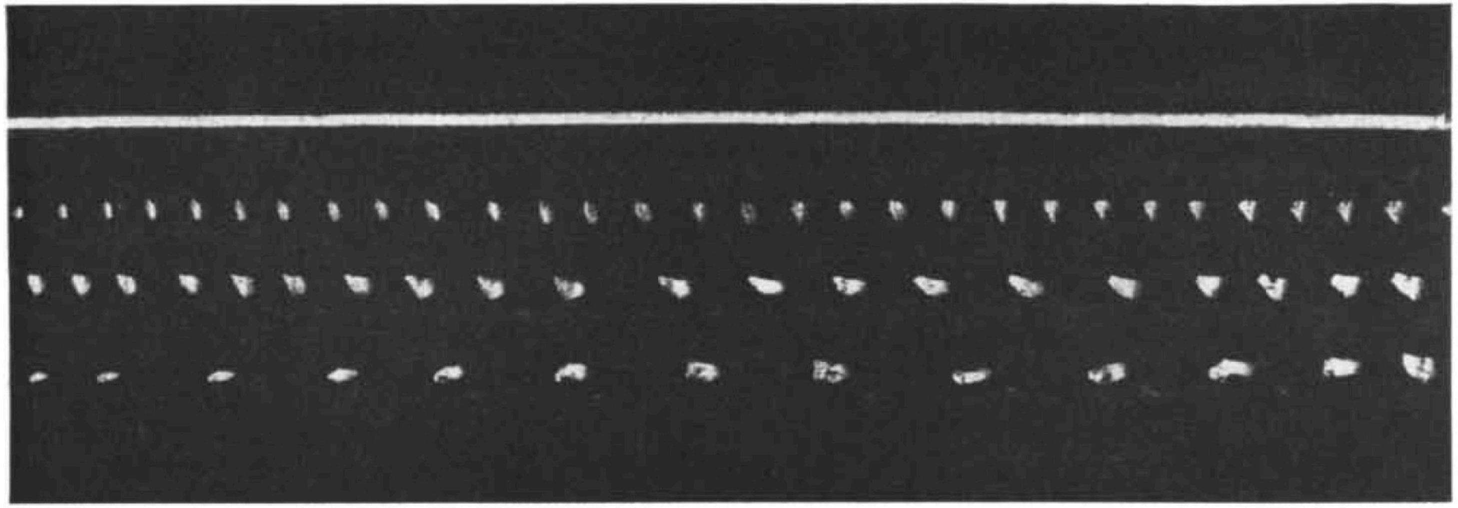
1956



CHALK MARKS on a blackboard demonstrate stick-slip. The top mark was made by a piece of chalk held at an acute angle to the direction of motion; the marks below it, by pieces of chalk held at an obtuse angle to this direction. In the latter marks the chalk stuck to the blackboard, then slipped, then stuck again and so on. The more tightly the chalk is held, the smaller the distance of slip.

Outline:

- Theoretical framework for stick-slip motion (1D spring-slider system)
- Spring-slider system (hands on)
- Slow earthquakes and spectrum of fault slip behavior – a laboratory approach



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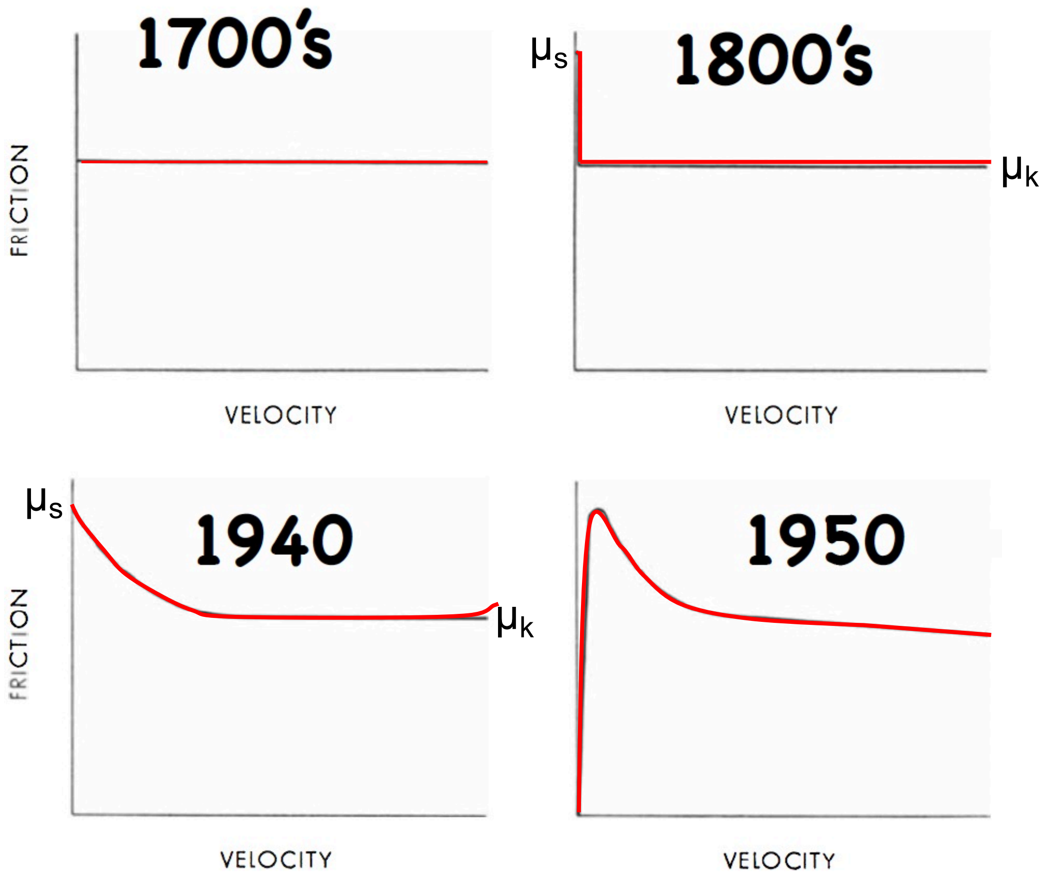
Brief historical summary of our understanding of friction



EVOLUTION OF THE FRICTION CONCEPT is illustrated. In the late 18th century it was thought that the coefficient of friction remained constant as the relative velocity of the sliding substances was increased (*upper left*). In the early 19th century it was postulated that there were two kinds of friction: static and kinetic (*upper right*). Friction was greatest when two substances were moved from a state of rest, and fell off immediately when they began to slide. Around 1940 it was shown that friction fell off gradually with the increase of velocity (*lower left*). Today it is known that friction first increases with velocity and then falls off (*lower right*). When the changing relationship between friction and velocity has the slope to the left of the peak in this curve, substances slide steadily. When it has the form of the steeper part of the slope to the right of the peak, stick-slip occurs.

Rabinowicz, 1956

Stability of steady frictional slipping

Assume:

- Frictional stress τ at constant normal stress (σ_n)
- That the frictional stress τ is only dependent on slip rate
- One degree-of-freedom elastic system

Individuate two stability regimes:

$$\frac{d\tau(v)}{dv} > 0$$ Stable sliding

$$\frac{d\tau(v)}{dv} < 0$$ Unstable sliding

Where we can describe the function $\tau(v)$ in terms of $\mu_s > \mu_k$

Rice and Ruina, 1983

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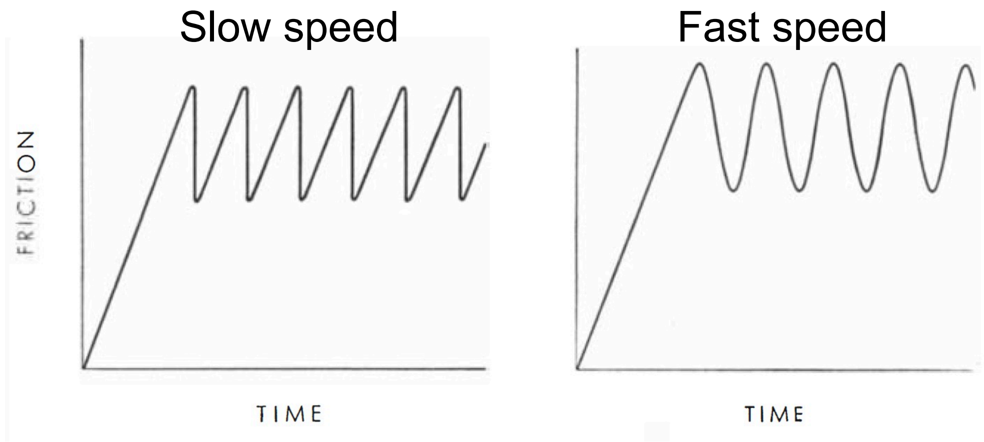


Where we can describe the function $\tau(v)$ in terms of $\mu_s > \mu_k$

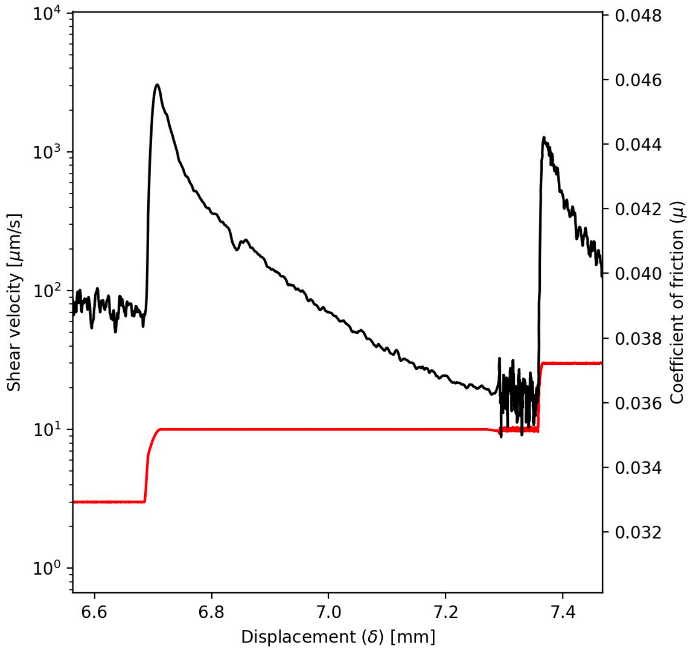
Stability of steady frictional slipping

However, there is a problem !!

How can we observe, experimentally, stable sliding even if $\dfrac{d\tau(v)}{dv} < 0$?

Velocity weakening

Friction decreases with increasing velocity, setting the stage for an instability.

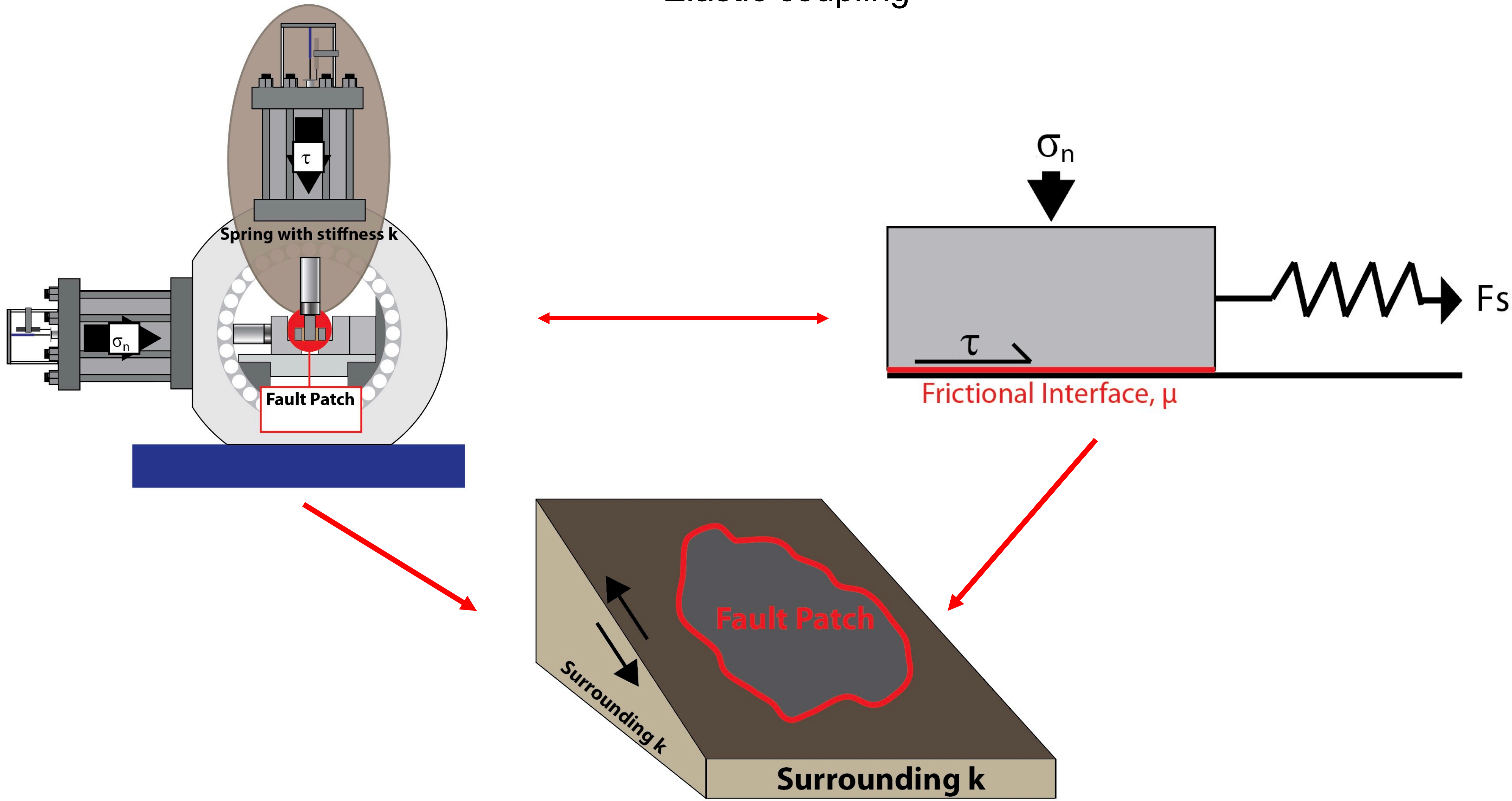


For high stiffness experimental apparatuses we observe that in response to a velocity jump friction can decrease with the absence of stick-slip motion.

This observation implies that velocity weakening is a necessary but not sufficient condition for unstable sliding.

Rate- and state- constitutive equations and 1D spring slider model

Elastic coupling



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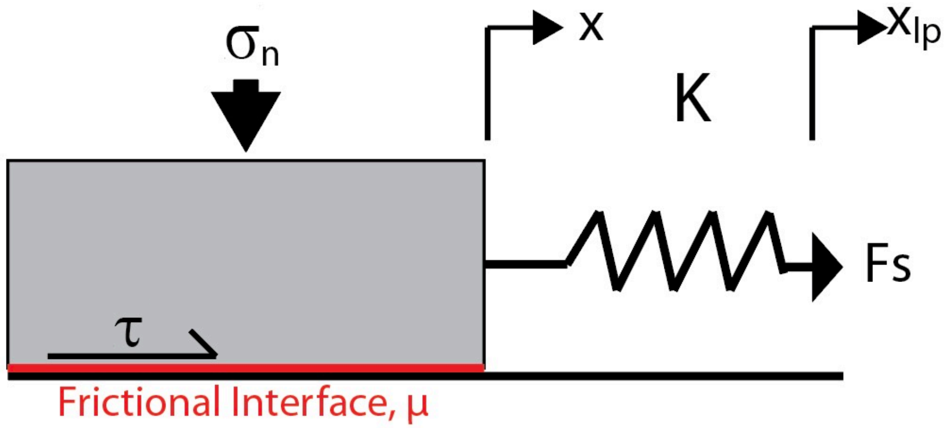


$$Fs = m\ddot{x} + \mu N$$

Equation of motion during sliding

$$Fs = K(x_{lp} - x)$$

$m\ddot{x}$ Inertia that at slow speed can be neglected

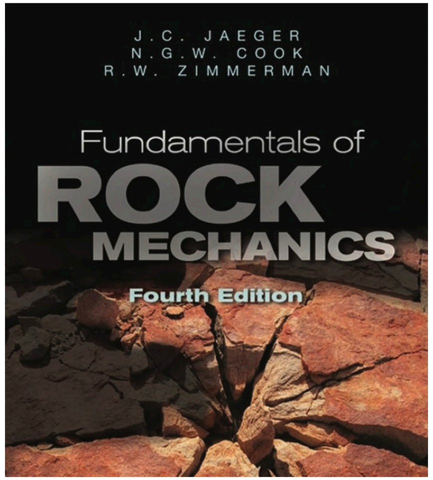
$$K(x_{lp} - x) = \mu N$$

Differentiating in time

$$\frac{d\mu}{dt} = k(v_{lp} - v)$$

For a full description of this problem:

Johnson and Scholz, 1976 JGR
Burridge and Knopoff, 1967
Mora and Place, 1994



Rate- and state- constitutive equations and 1D spring slider model

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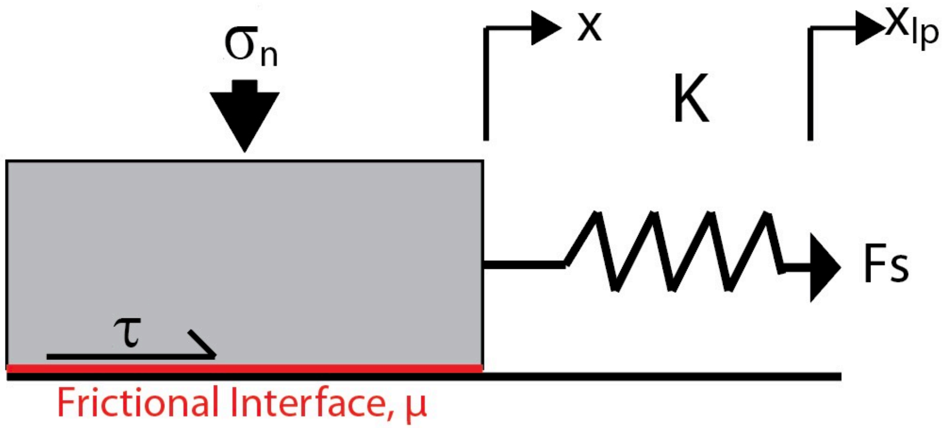

$Fs = m\ddot{x} + \mu N$ Equation of motion during sliding

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Differentiating in time

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$$\begin{cases} \dfrac{\tau(\theta,v)}{\sigma_n} = \mu_0 + a\ln\left(\dfrac{v}{v_0}\right) + b\ln\left(\dfrac{v_0\theta}{D_c}\right) & (1) \text{ Friction law} \\ \dfrac{d\theta}{dt} = 1 - \left(\dfrac{v\theta}{D_c}\right) & (2) \text{ Evolution law} \\ \dfrac{d\mu}{dt} = k(v_{lp} - v) & (3) \text{ Elastic coupling} \end{cases}$$

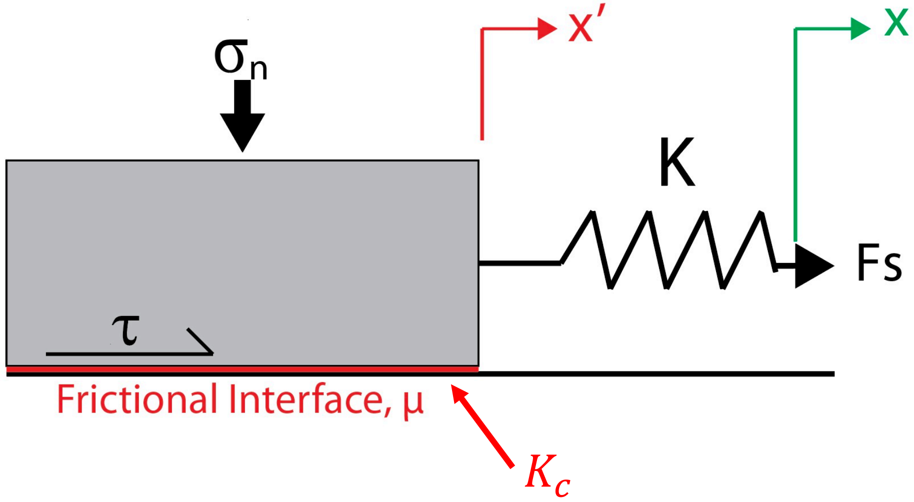
Solving (1) for the slider velocity

$$v = v_o exp\left[\frac{\mu - \mu_0 - bln\left(\frac{v_0\theta}{D_c}\right)}{a}\right]$$

Substituting in (3)

$$\begin{cases} \dfrac{d\mu}{dt} = k\left\{v_{lp} - v_o exp\left[\dfrac{\mu - \mu_0 - bln\left(\frac{v_0\theta}{D_c}\right)}{a}\right]\right\} \\ \dfrac{d\theta}{dt} = 1 - \left(\dfrac{v\theta}{D_c}\right) \end{cases}$$

Rate- and state- constitutive equations and 1D spring slider model



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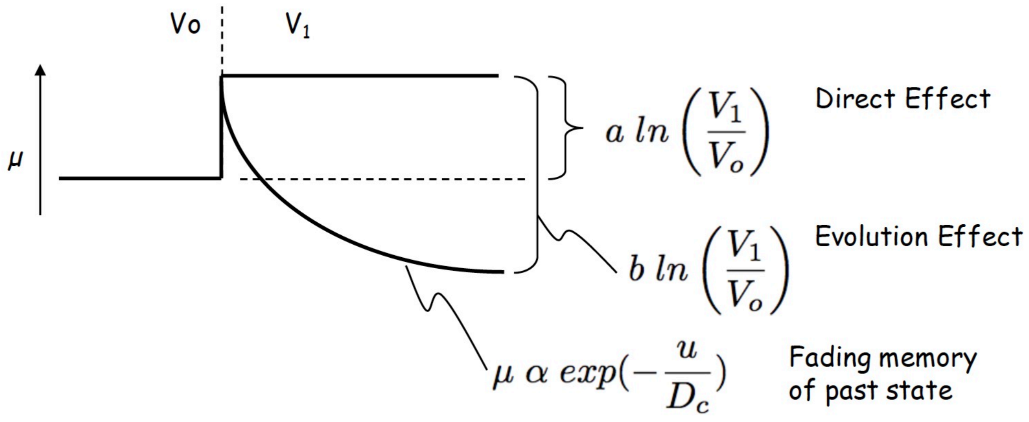
Elastic coupling

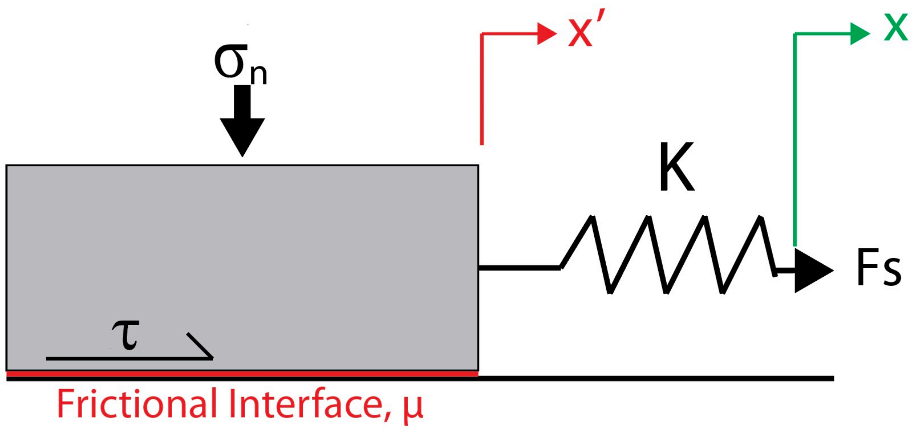
Stability of one-degree of freedom elastic system
Considering a single decay process
(Rice and Ruina, 1983; Gu et al., 1984; Roy and Marone 1996))

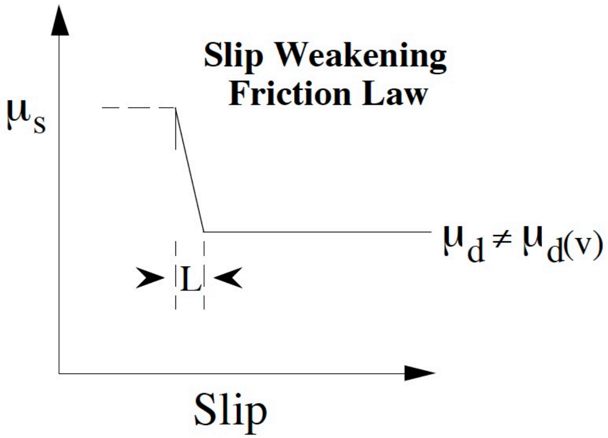


Critical fault rheologic stiffness
(i.e. rate of frictional weakening)

$$K_c = \frac{\sigma_{n(b-a)}}{D_c}\left[1 + \frac{mv_0^2}{\sigma_n a D_c}\right]$$

Rate- and state- constitutive equations and 1D spring slider model





$$K_c = \frac{\sigma_n(\mu_s - \mu_d)}{L}$$

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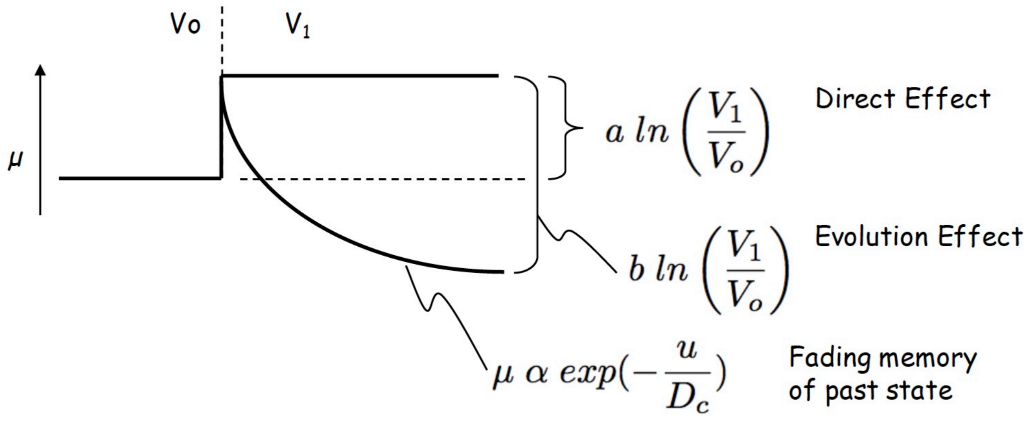

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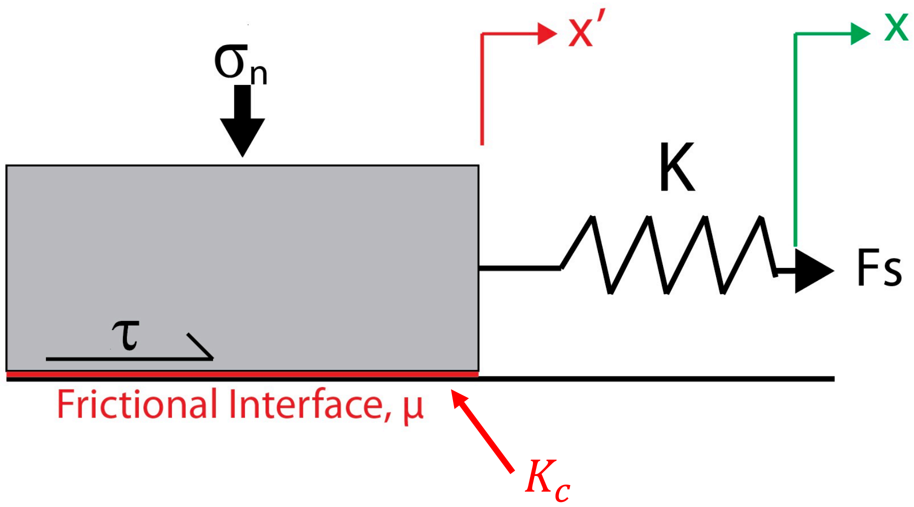
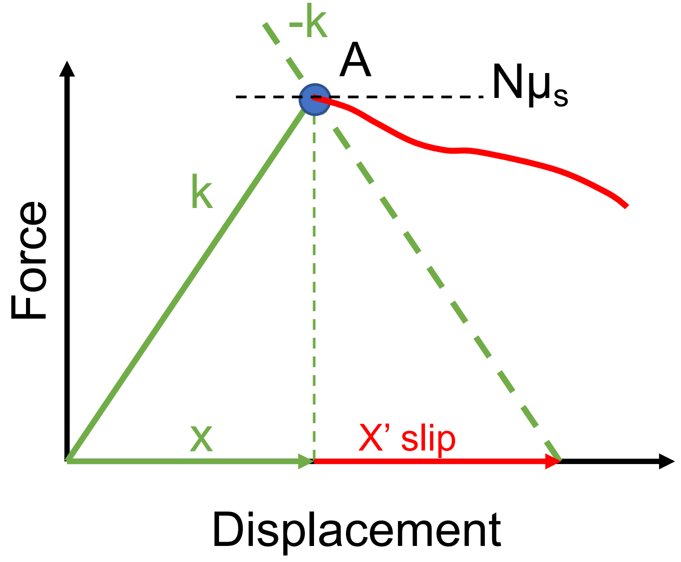
Rate- and state- constitutive equations and 1D spring slider model





fundamental requirement for instability
(a-b) < 0 Velocity weakening

(1) $k/k_c > 1$ Stable sliding

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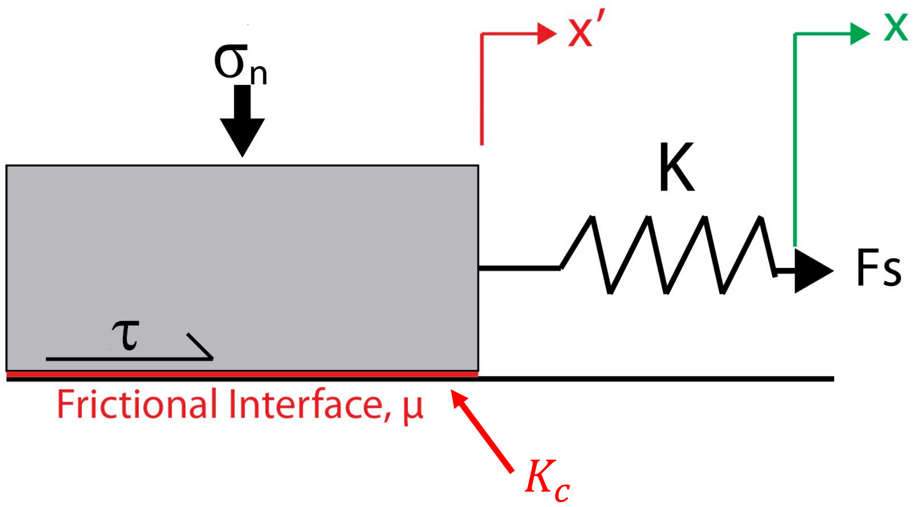
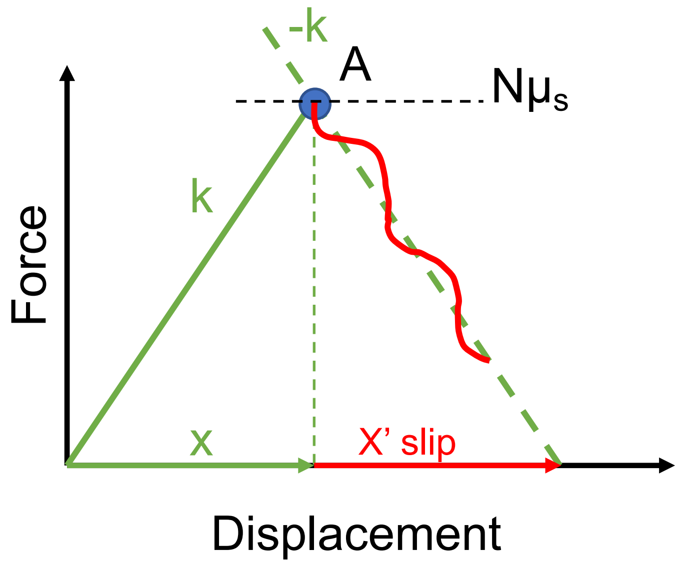
Rate- and state- constitutive equations and 1D spring slider model





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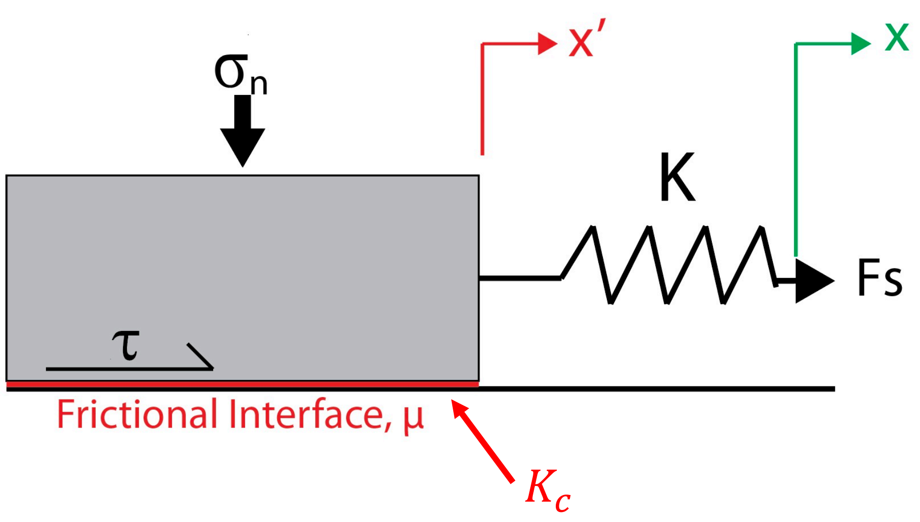
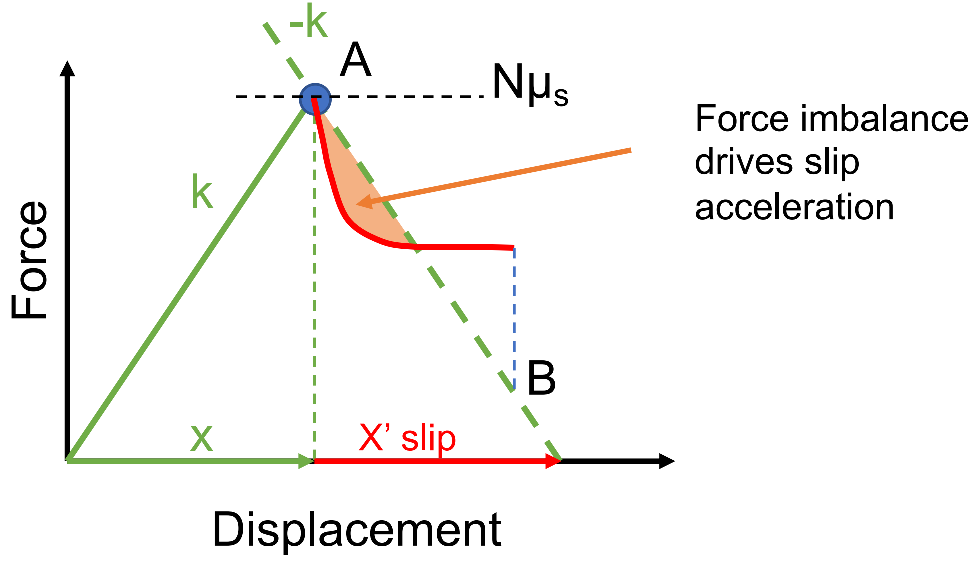
(2) k/k_c ~ 1 Conditional Stability

Rate- and state- constitutive equations and 1D spring slider model





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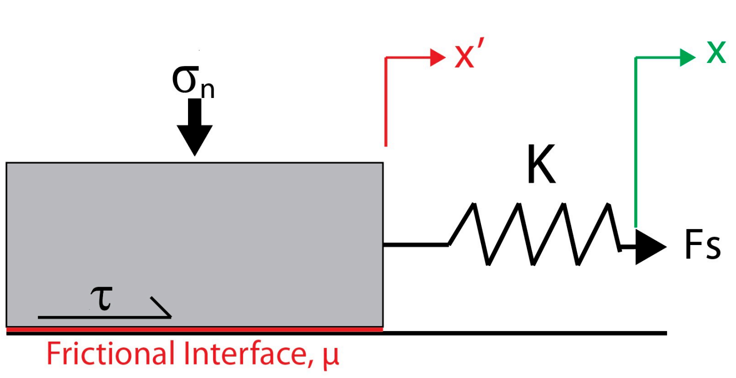
(3) $k/k_c < 1$ Unstable sliding

Rate- and state- constitutive equations and 1D spring slider model



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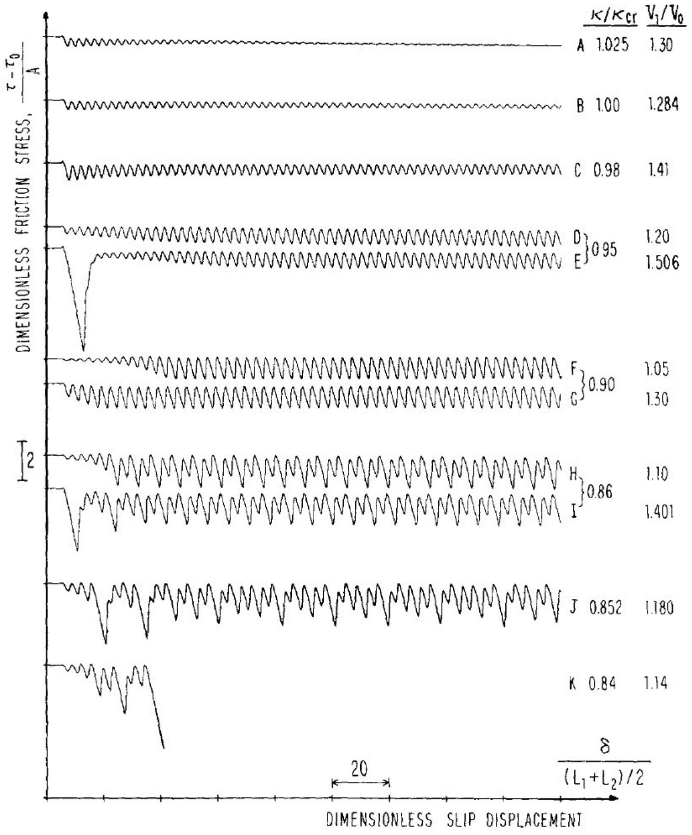
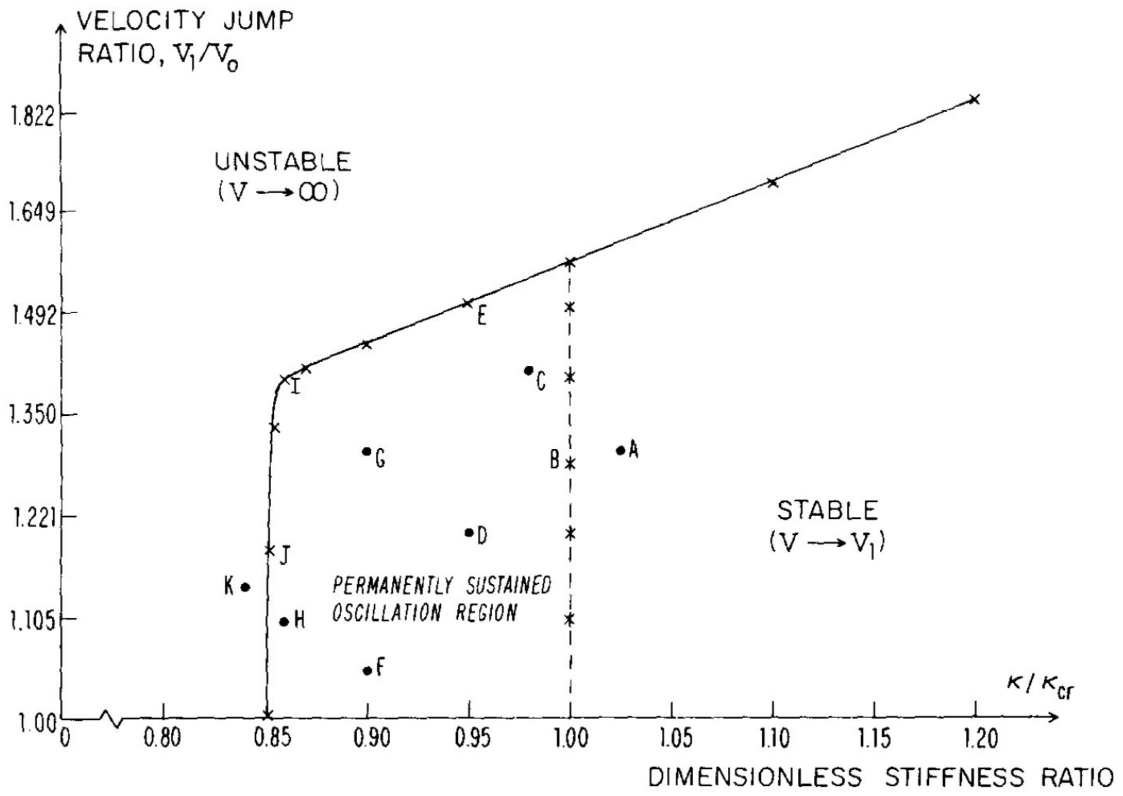




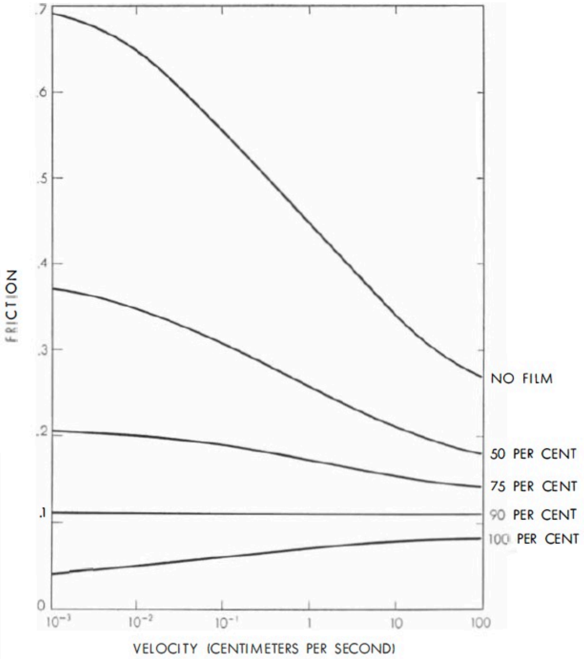
Gu et al., 1984

down, in other cases speeding up. For example, a car's tires squeal if it rounds a corner rapidly but not if the turn is slow; on the other hand, a door that creaks when opened slowly may be silent when swung rapidly. Secondly, we may reduce the stored energy (*e.g.*, in the spring) whose intermittent release is responsible for stick-slip. Stiffening the spring will accomplish this end; similarly, stiffening a toolholder will make the tool cut more smoothly. Or we may damp the stored energy by immersing some part of the vibrating system in a bath of viscous oil.

The third and most common method is to lubricate the sliding surfaces. A lubricant forms a soft film which has far less frictional resistance than a metal's surface. The problem here is to maintain the film over the whole interface. As the surfaces slide, the lubricant is gradually worn off, so that parts of the metal surfaces come into contact with each other. So long as the lubricant coverage is 90 per cent or better, stick-slip cannot occur. But when coverage has fallen to 75 per cent, stick-slip becomes very possible [*see chart below*]. At this stage its squeaky protest is a boon, for it serves as a warning that the lubricant must be replenished. The quality of the lubricant is important; some poor lubricants never give even 90 per cent coverage, no matter how much is applied.

External factors, such as humidity, also may play a part. Squeaks in an automobile are apt to be silenced on a wet day—and, perversely, almost invariably when the car is taken to a garage to have the squeaks located and removed. Demonstrations of stick-slip during public lectures are likewise hazardous undertakings.

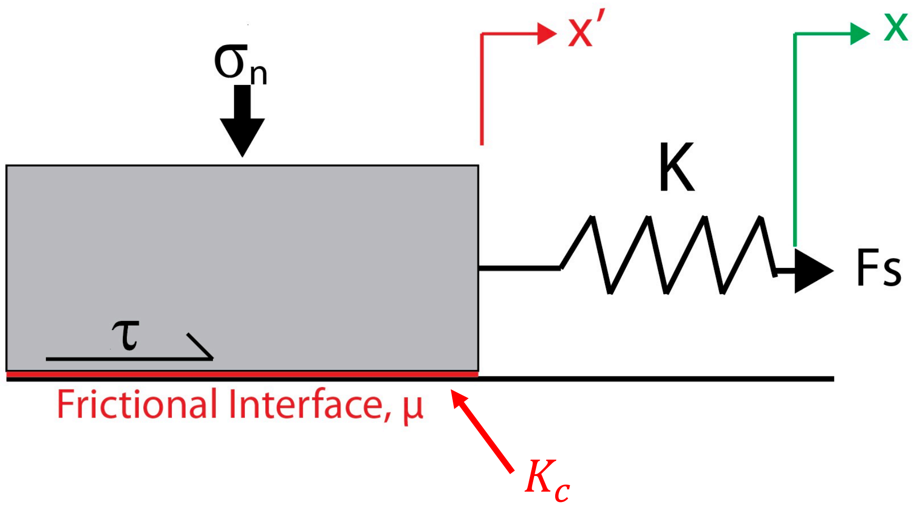
Friction in a machine brings a train of unhappy events. The sliding surfaces

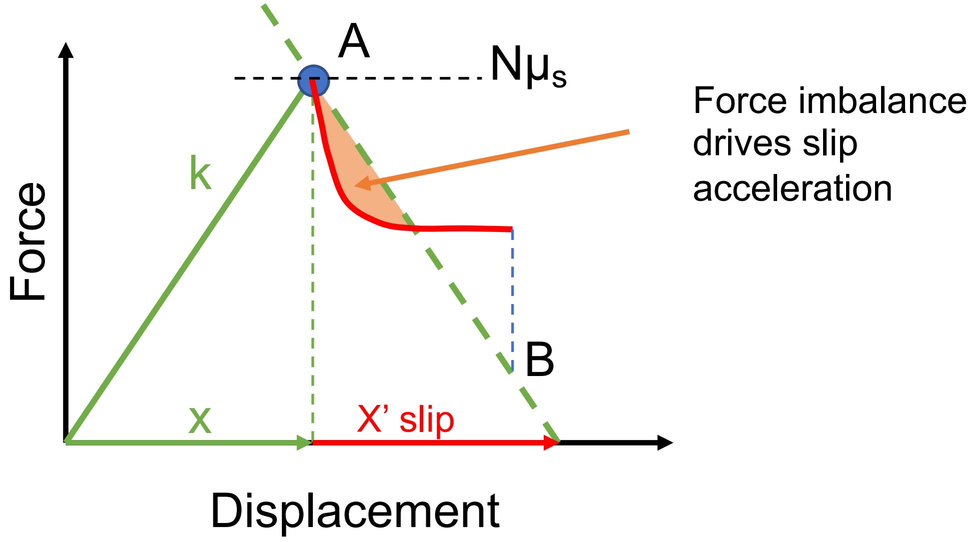


LUBRICATED SURFACES may be subject to stick-slip. This chart represents one piece of steel slid over another with a film of lubricant between them. When the lubricant is first applied, it covers 100 per cent of the area between the two surfaces. This area is steadily reduced as the surfaces are rubbed together. When 90 per cent of the film remains, the curve is still almost horizontal and no stick-slip occurs. When only 75 per cent remains, the slope of curve is down (*see curve at lower right on page 112*) and stick-slip can begin.

External factors, such as humidity, also may play a part. Squeaks in an automobile are apt to be silenced on a wet day—and, perversely, almost invariably when the car is taken to a garage to have the squeaks located and removed. Demonstrations of stick-slip during public lectures are likewise hazardous undertakings.

Nonetheless I will attempt it, so please have low expectations.

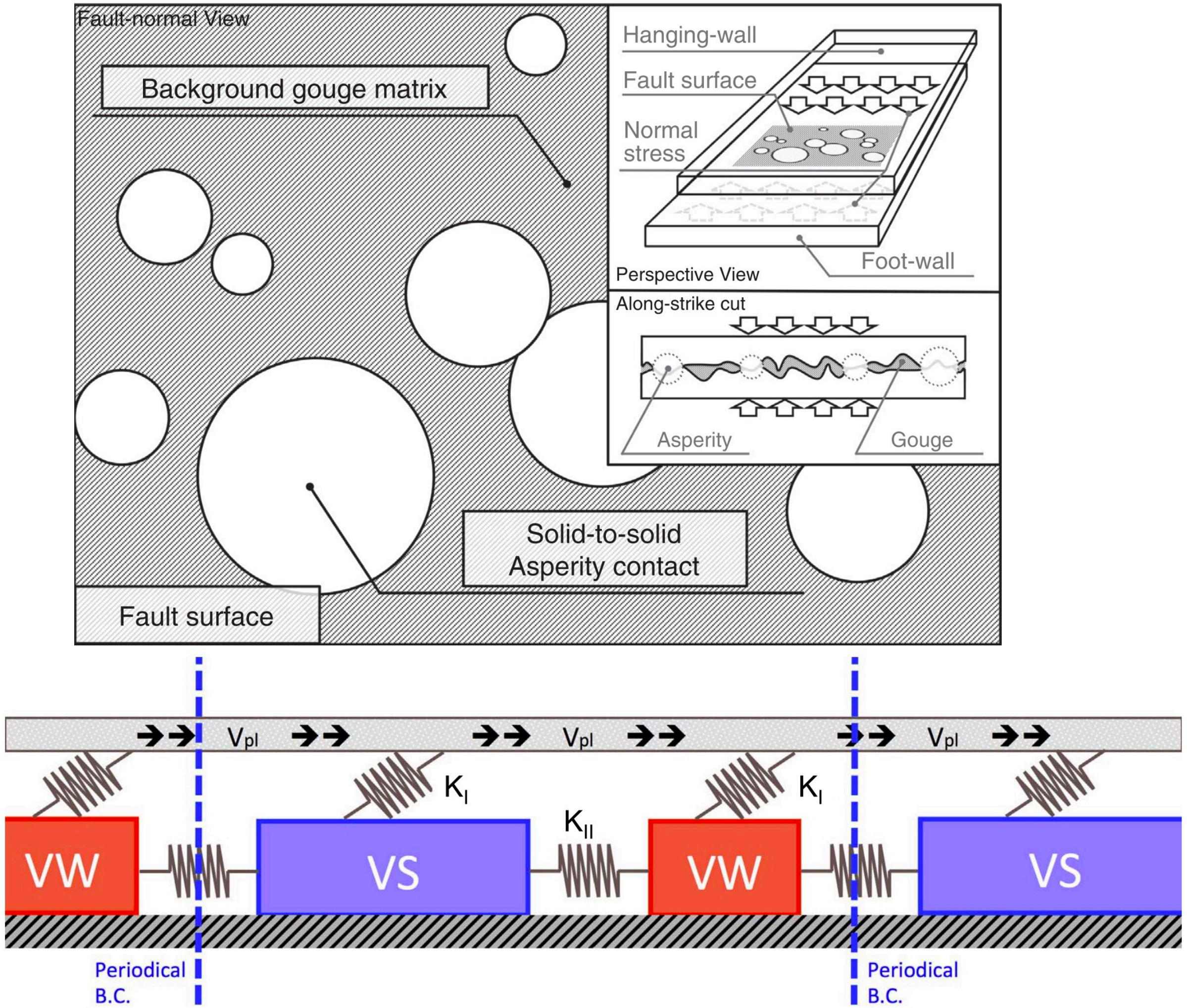




Experiments:

- We will attempt at reproducing fault slip from stable sliding to fast stick slip
- We will use three springs (which one is the third?)
- We will measure and record the displacement of the the block to analyze this transition

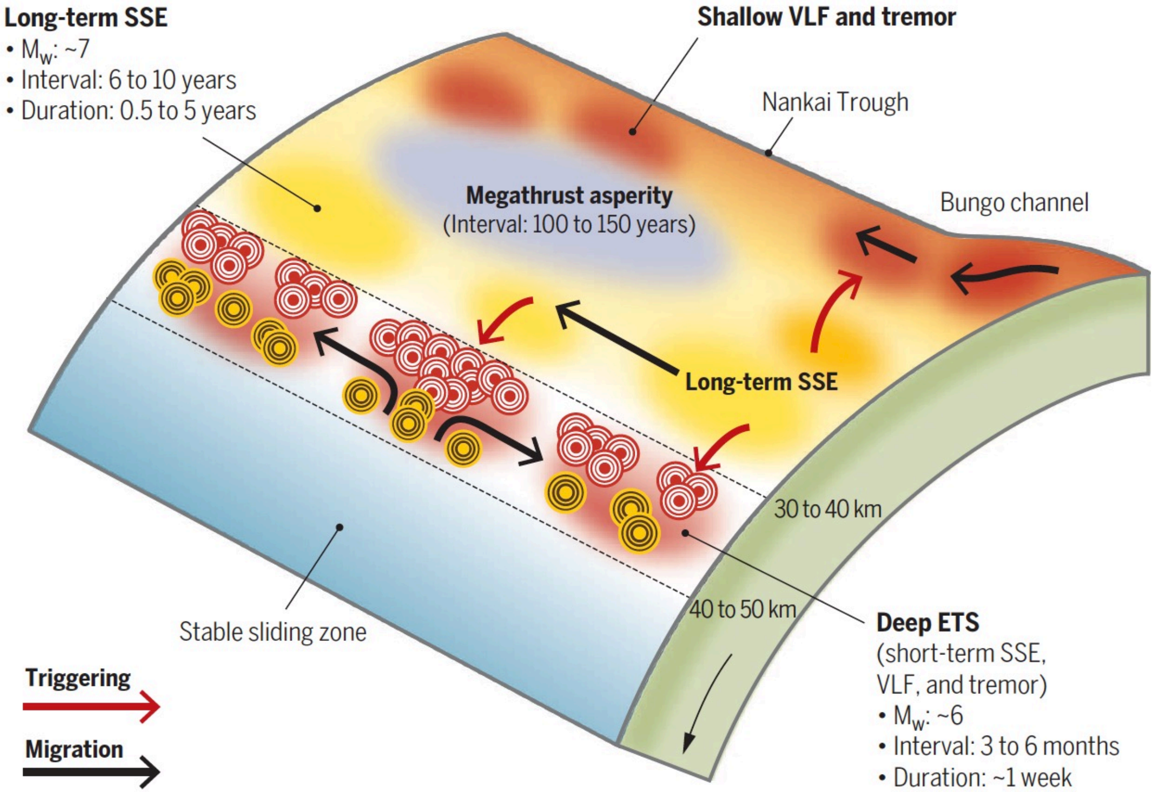
However, faults are heterogeneous systems and the analysis we have seen so far put the basis for more comprehensive models set up.

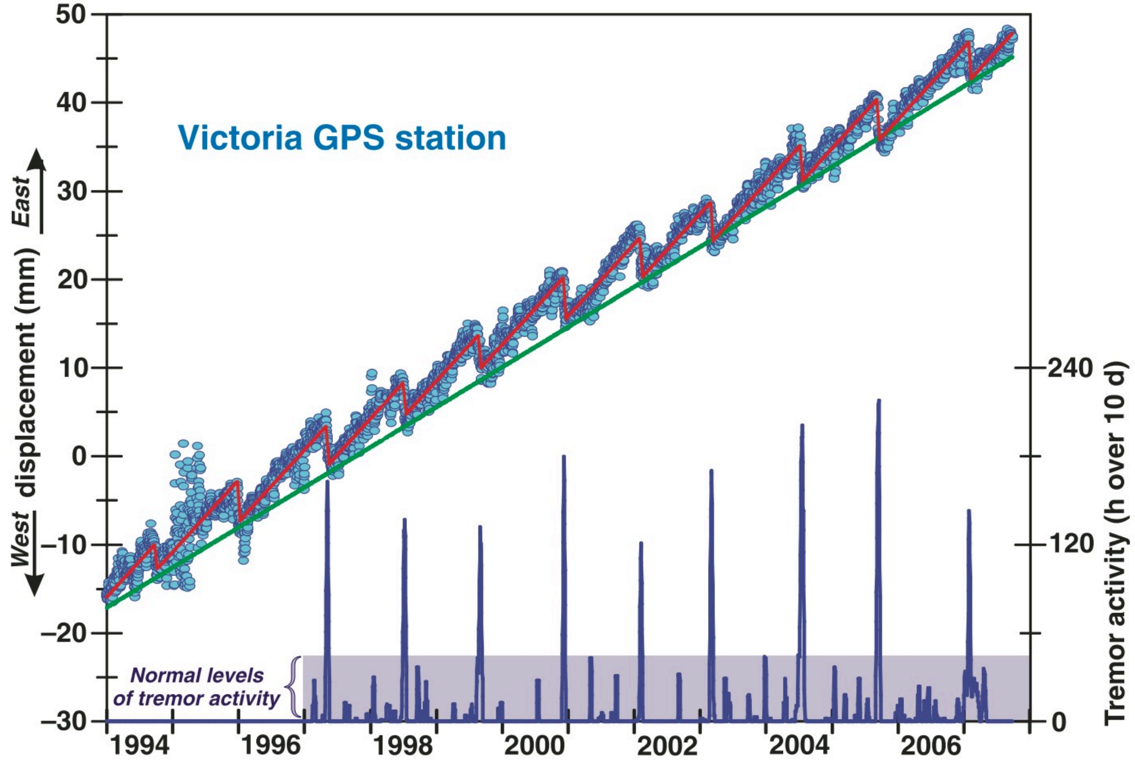


Luo and Ampuero, 2017; Skarbek et al., 2012

From aseismic creep to dynamic stick-slip passing through slow earthquakes

Slow Slip Events (SSE) play a fundamental role in the slip budget of faults.





Obara and Kato, 2016 Science

Rogers and Dragert, 2003 *Science*

From aseismic creep to dynamic stick-slip passing through slow earthquakes

Fault slip occurs via a spectrum of behaviors

Seismic (fast dynamic)

Slow (quasi-dynamic)

Aseismic (creep)

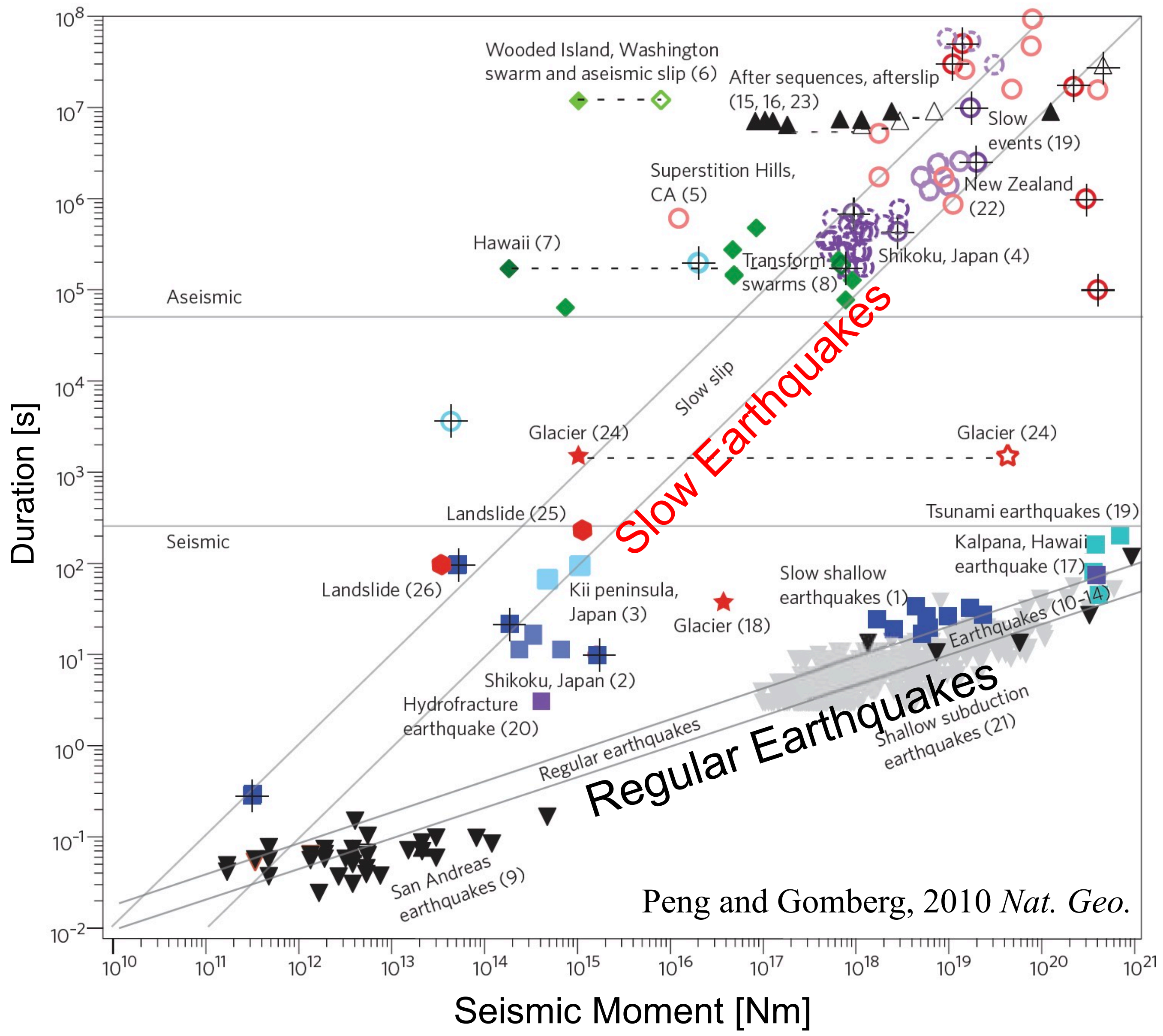
changing the way fault slip behavior is understood.

Scaling relationships:

$M_0 \sim T^3$ for regular earthquakes

$M_0 \sim T$ for slow earthquakes (?)

(Ide, 2007)



From aseismic creep to dynamic stick-slip passing through slow earthquakes

Mechanism(s): Why are they slow?

- Dilatancy hardening in areas of high pore fluid pressure (e.g., Segall et al., 2010 JGR)
- Designer friction law (i.e. rate dependence of friction rate dependence) (e.g., Rubin, 2008 JGR)
- Dehydration Reactions (e.g., Brantut et al., 2011)
- Slow frictional stick-slip near the stability boundary

They could represent a quasi-dynamic frictional instability

The fault zone energy release rate equals the frictional weakening rate resulting in a quasi-dynamic stress drop (Self-driven instability)

Rate dependence of the critical rheologic stiffness kc

Complex behavior near the stability boundary

From aseismic creep to dynamic stick-slip passing through slow earthquakes

We modify the shear loading stiffness in order to match kc via:

- A spring in series with the vertical loading piston (k)
- By changing the normal stress

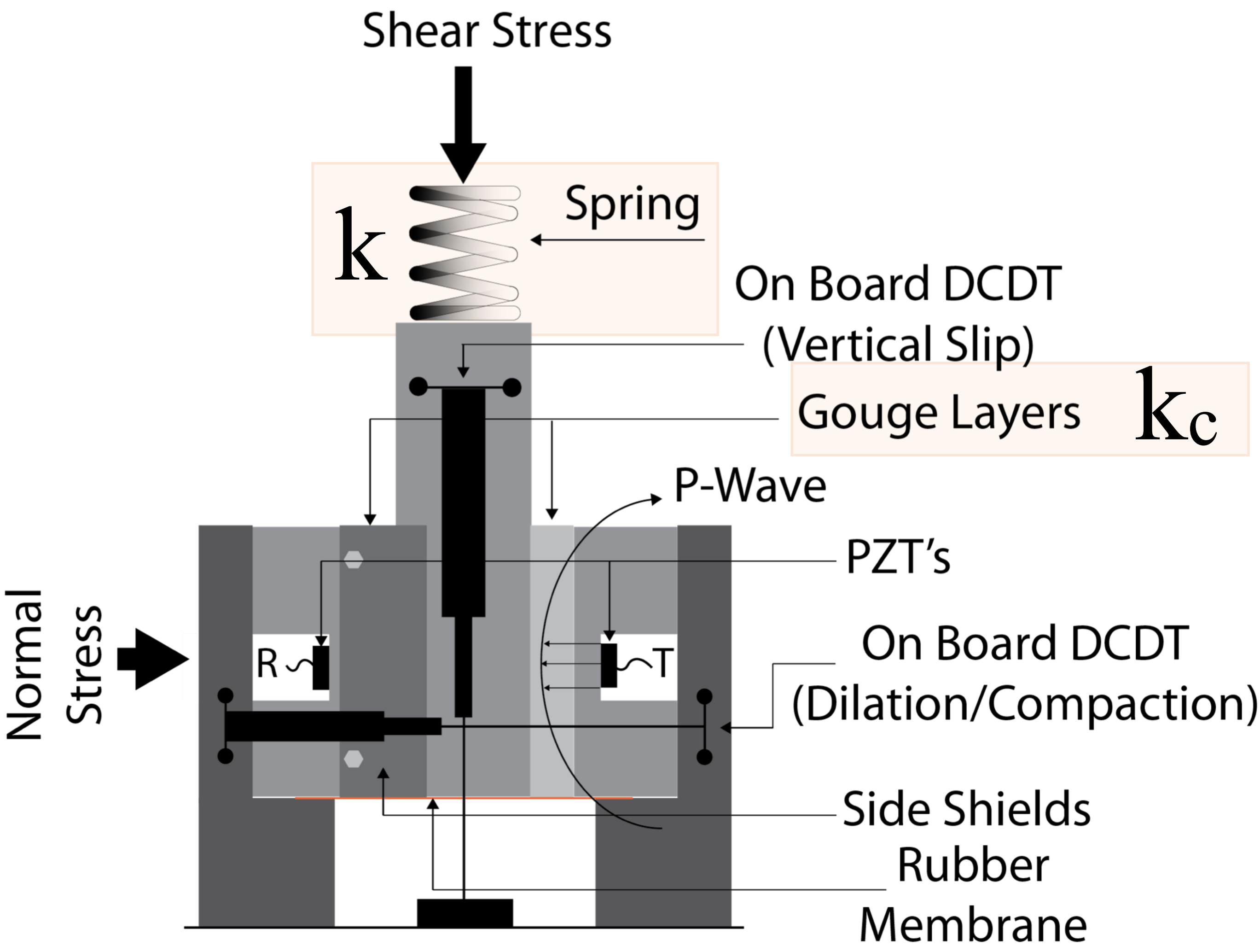
$$k' = \frac{k}{\sigma_n}$$

Surrounding stiffness

$$k_c = \frac{(b-a)}{D_c}$$

Critical rheologic stiffness

Double-direct shear configuration



Scuderi et al., 2016 *NatGeosc;* Leeman et al., 2016 *NatComm*

From aseismic creep to dynamic stick-slip passing through slow earthquakes

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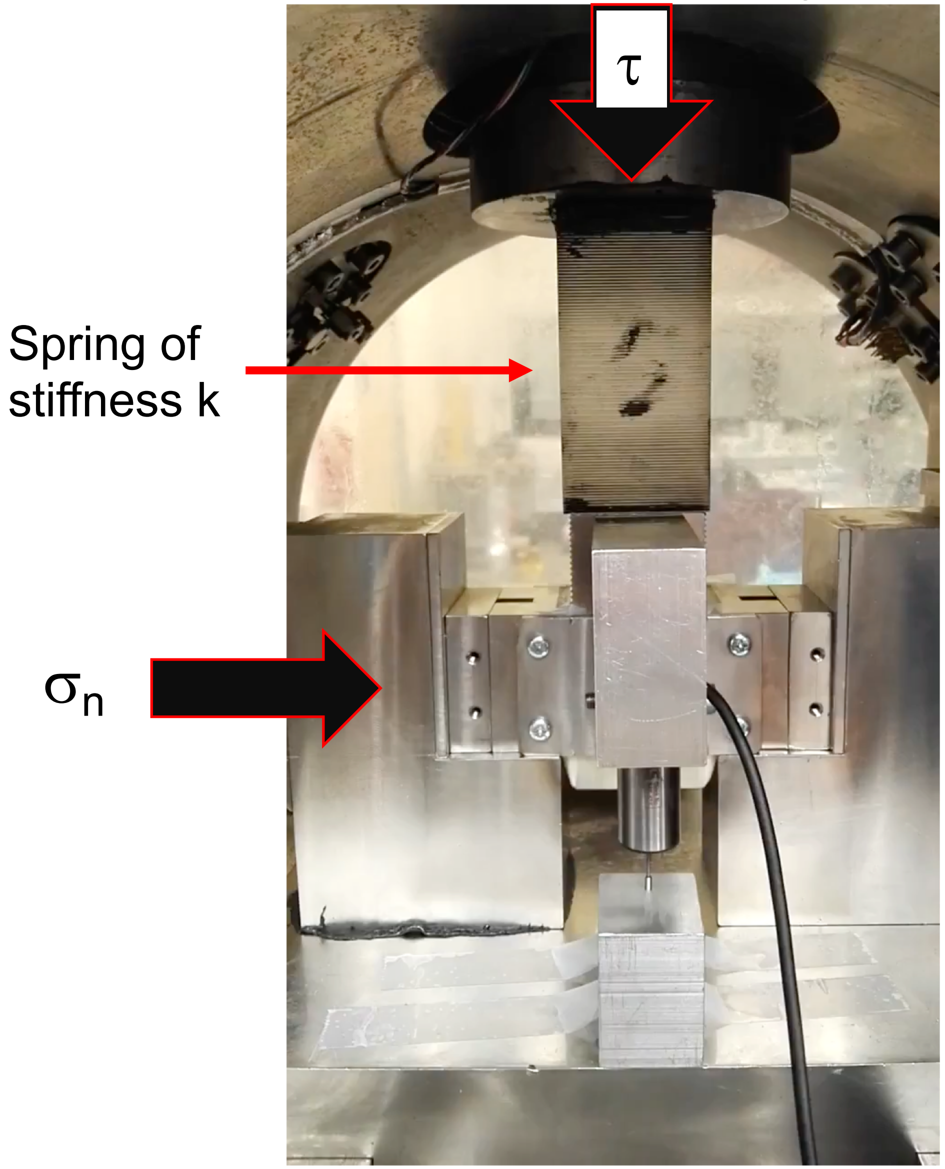
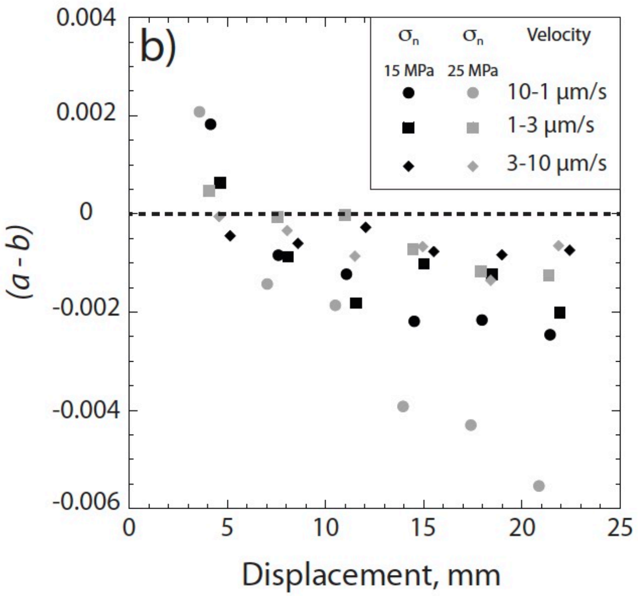
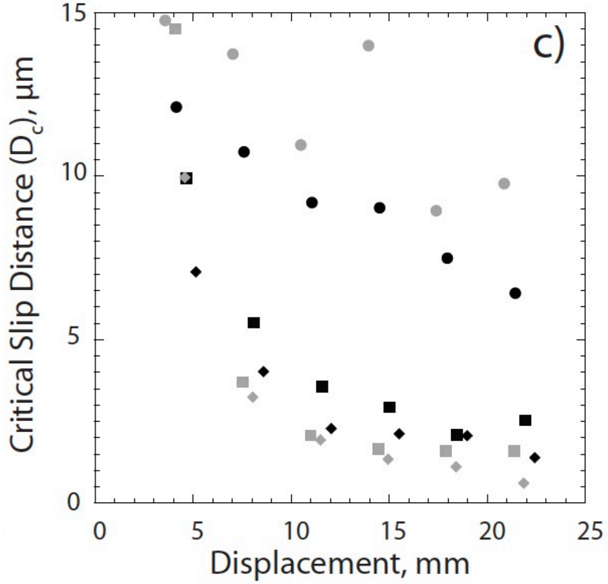


Scuderi et al., 2016 *Nature Geoscience*

From aseismic creep to dynamic stick-slip passing through slow earthquakes





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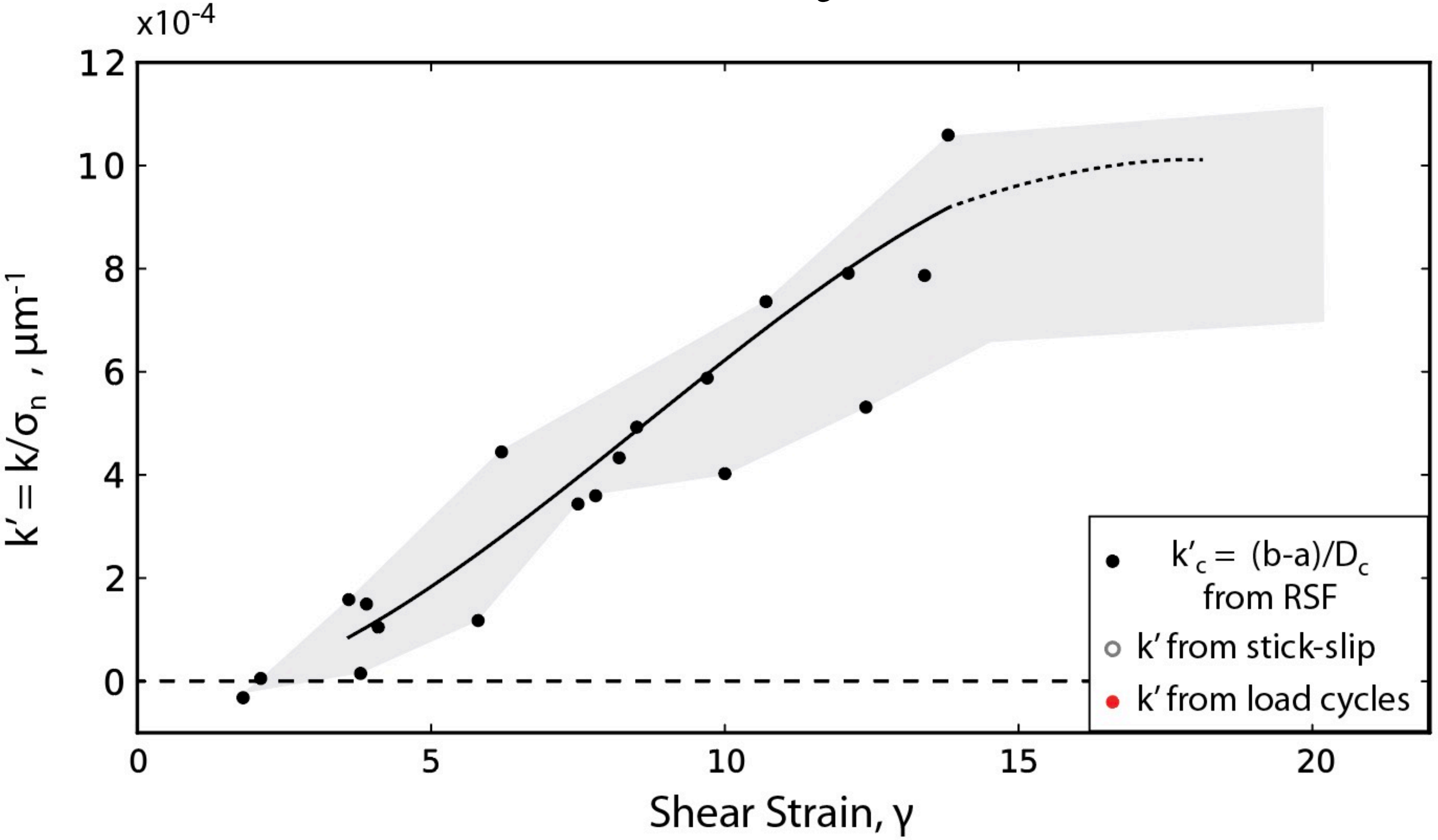
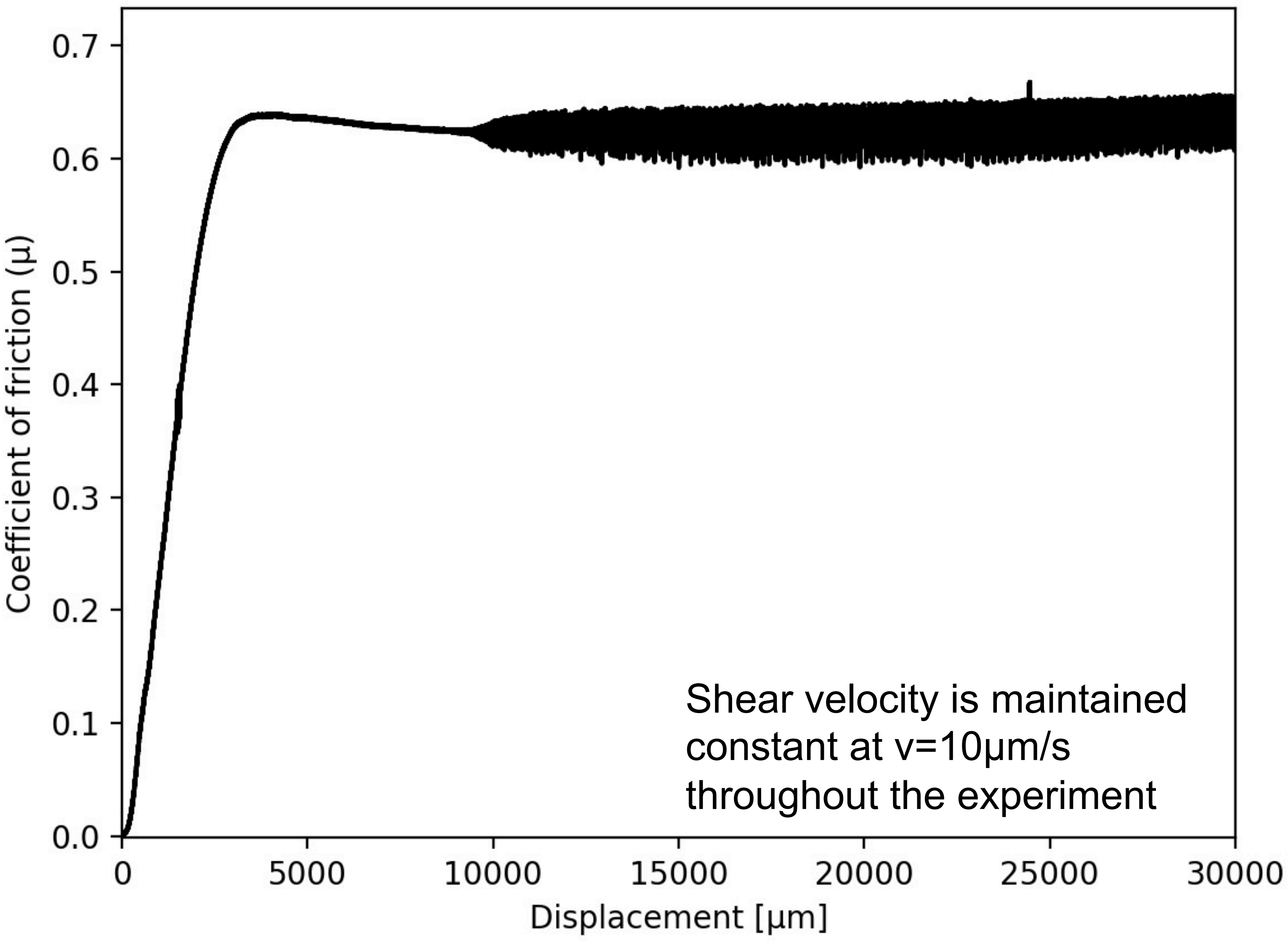


From aseismic creep to dynamic stick-slip passing through slow earthquakes

Typical experimental curve



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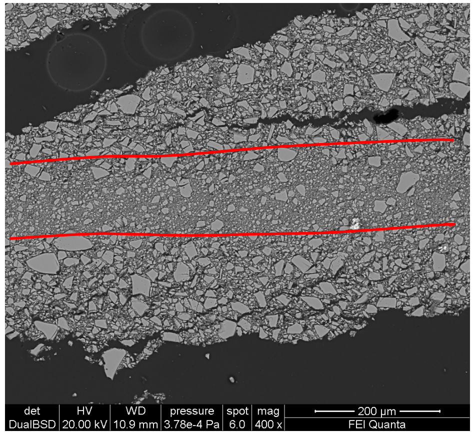
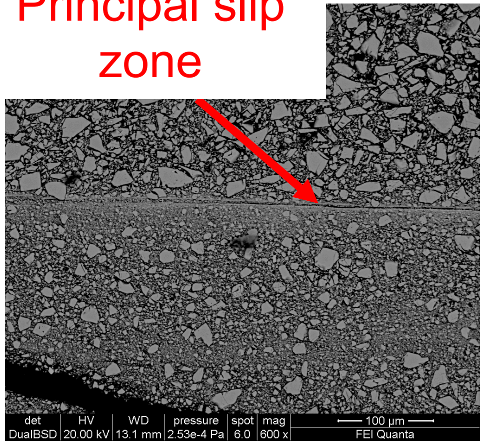
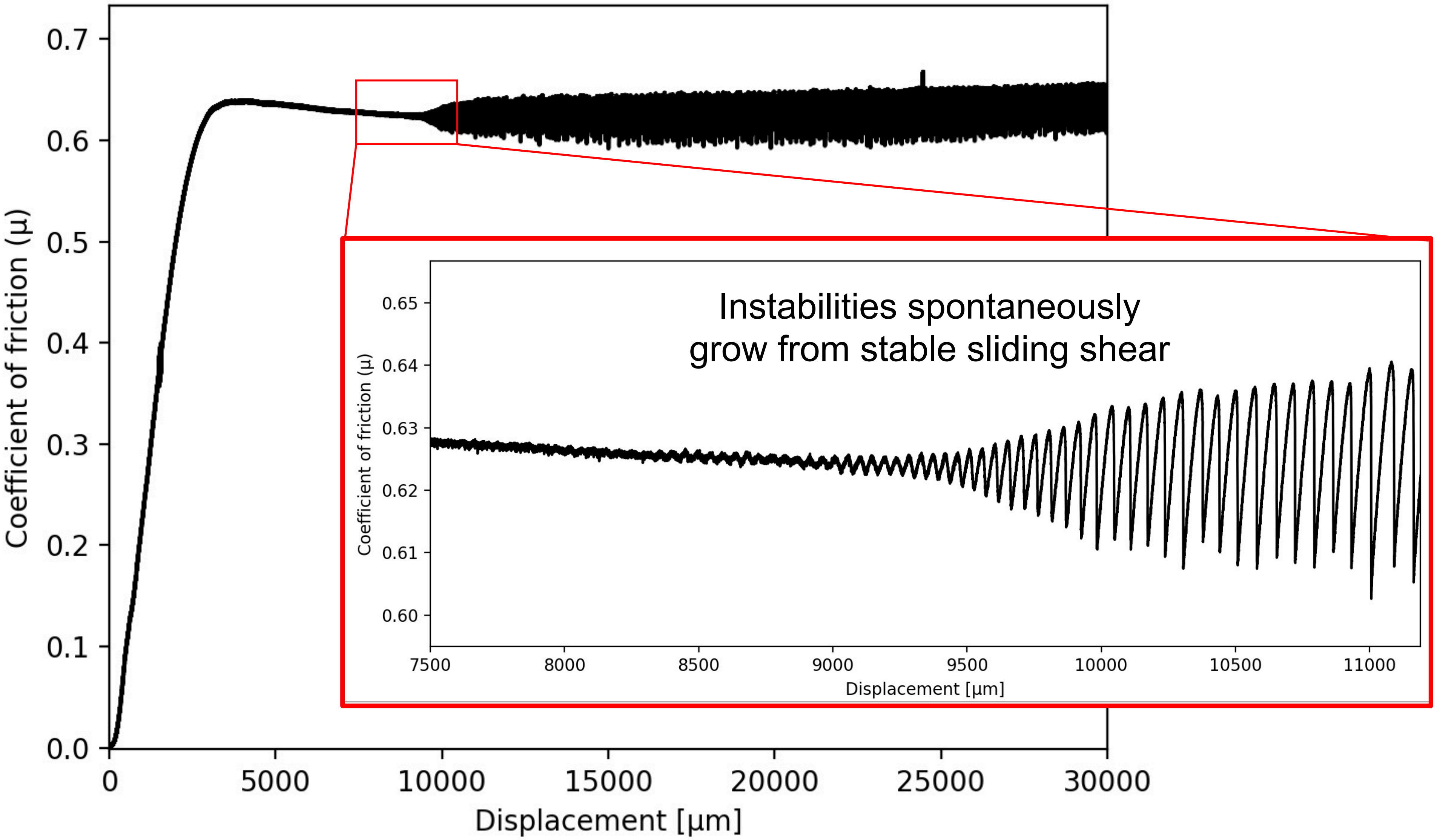


Principal slip zone





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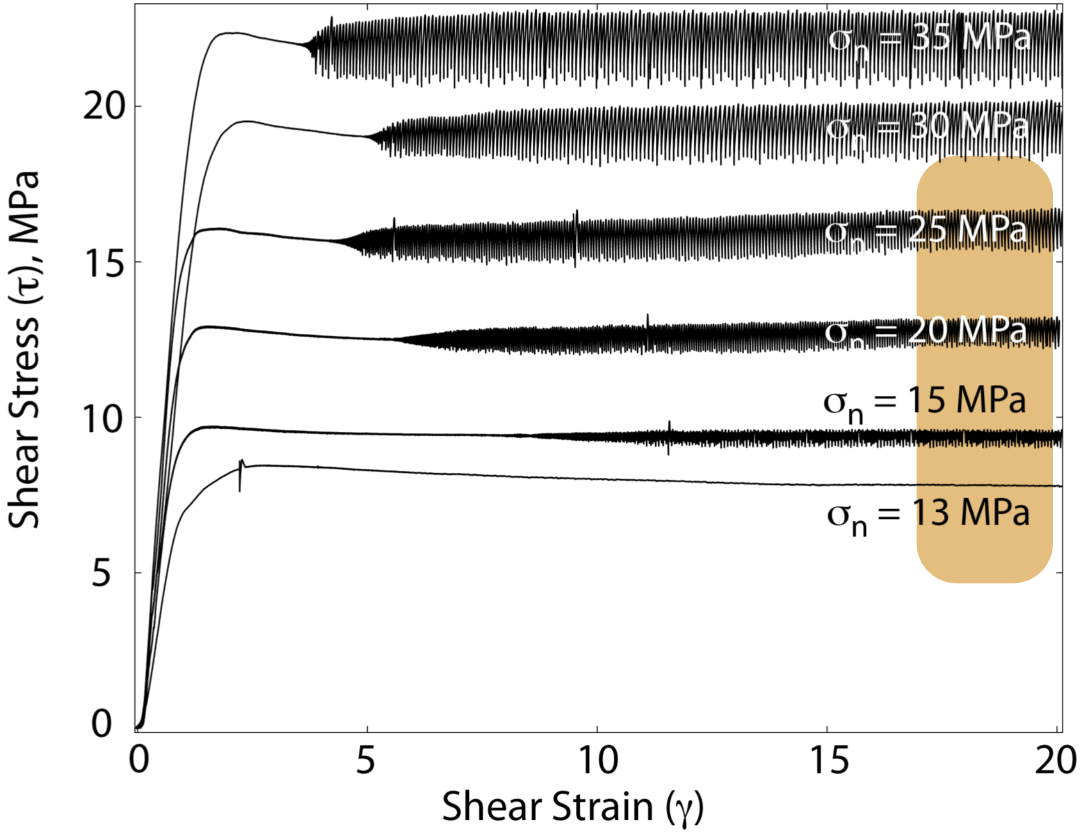
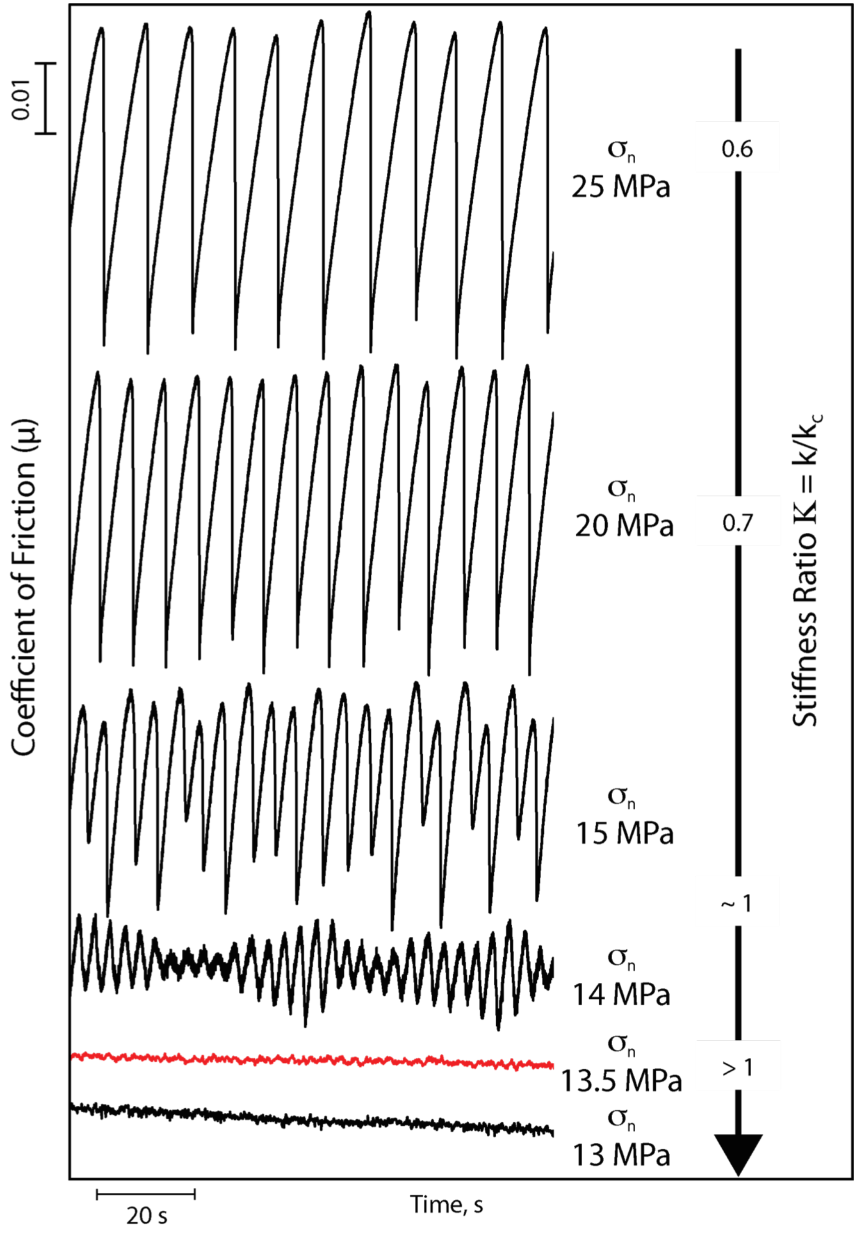
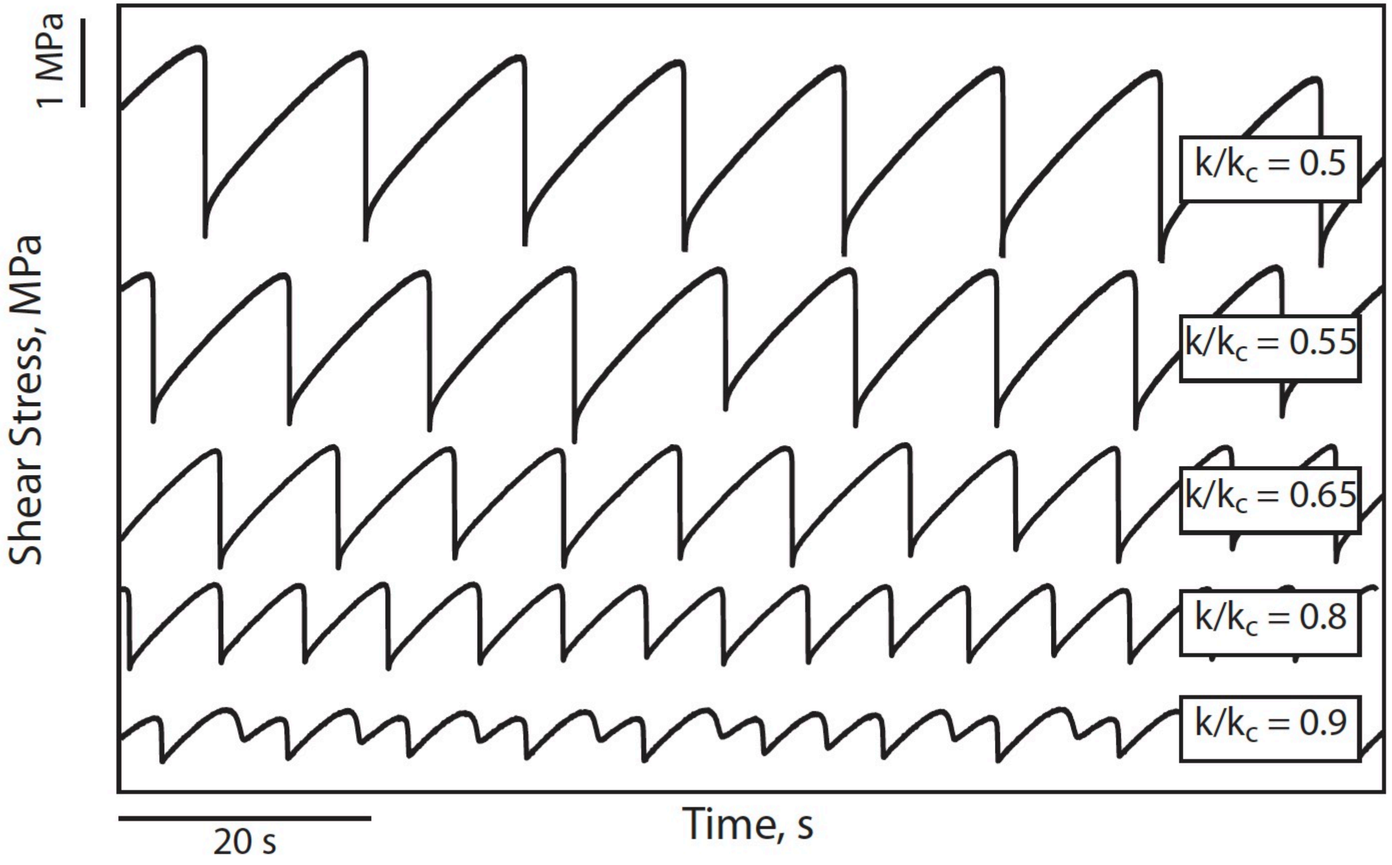




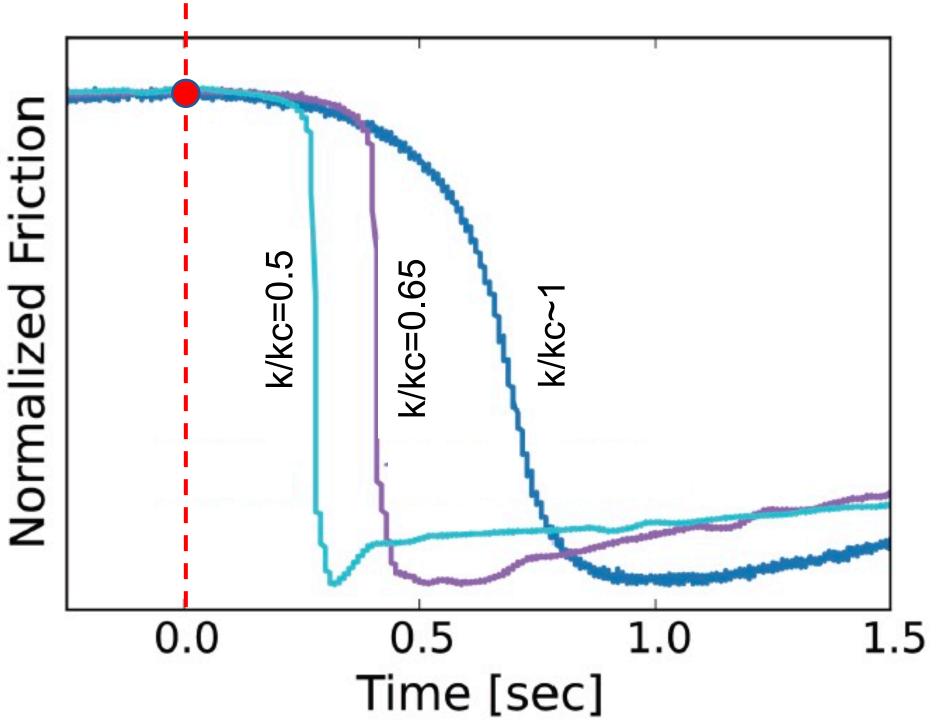
Spectrum of fault slip behavior:

- Stable sliding k/kc>1 (i.e. $\sigma_n \leq 13.5$MPa)
- Slow stick-slip k/kc~1 (i.e. σ_n~15MPa)
- Fast dynamic stick-slip k/kc<1 (i.e. σ_n>20MPa)

From aseismic creep to dynamic stick-slip passing through slow earthquakes

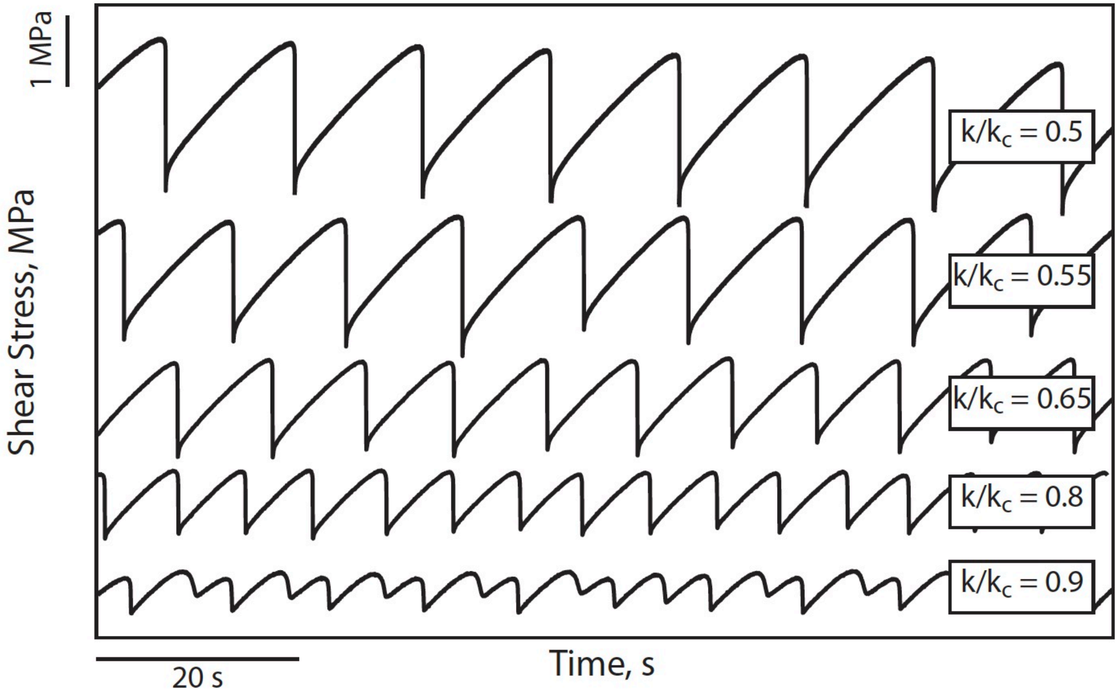


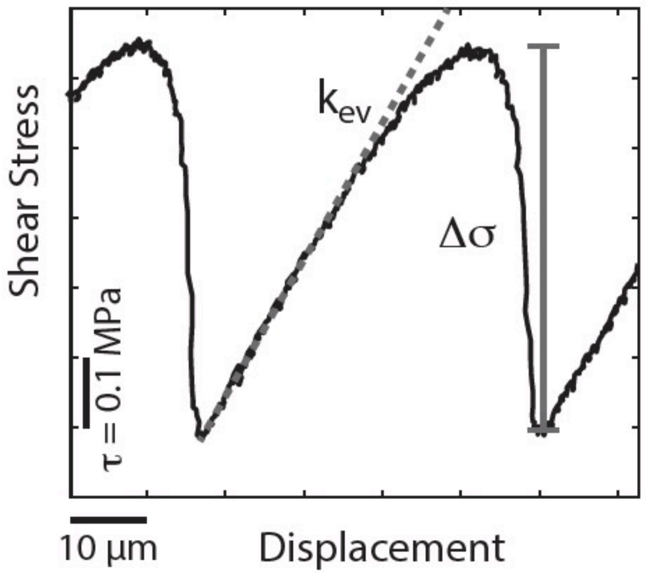
Maximum friction before weakening and stress drop (μ_s)

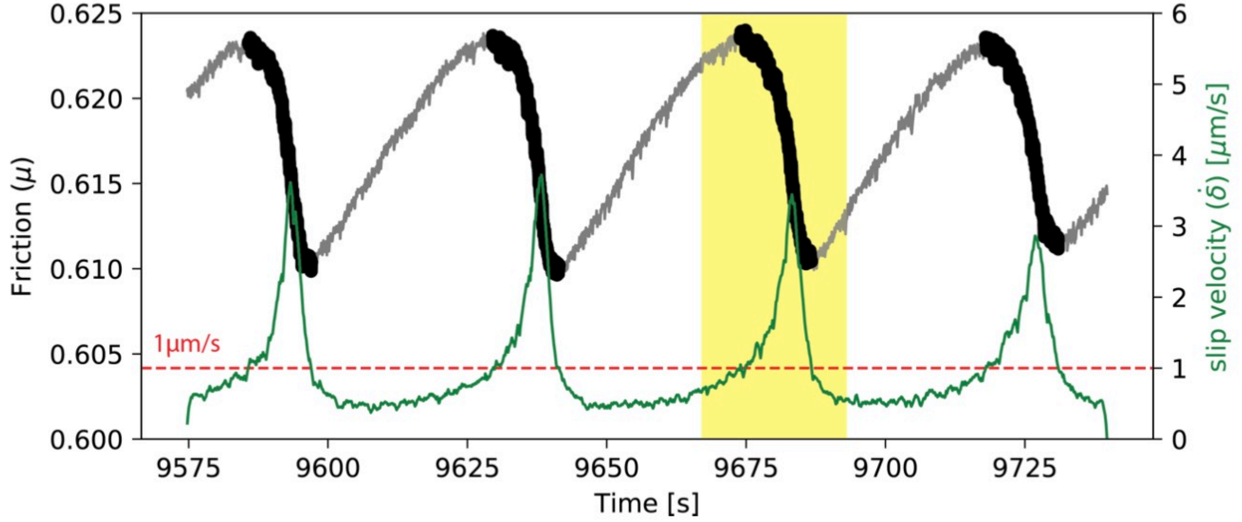


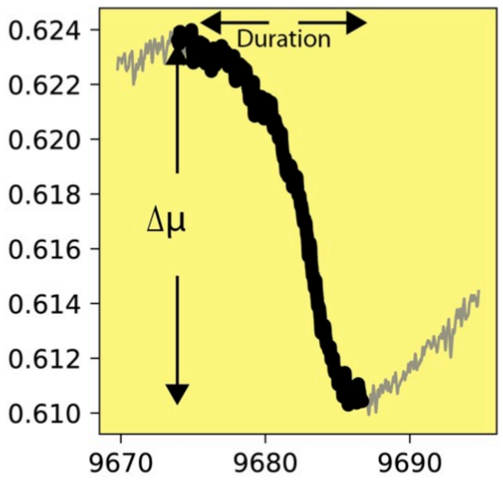
Scuderi et al., 2016 *NatGeosc;* Leeman et al., 2016 *NatComml;* Tinti et al., 2016 JGR

From aseismic creep to dynamic stick-slip passing through slow earthquakes



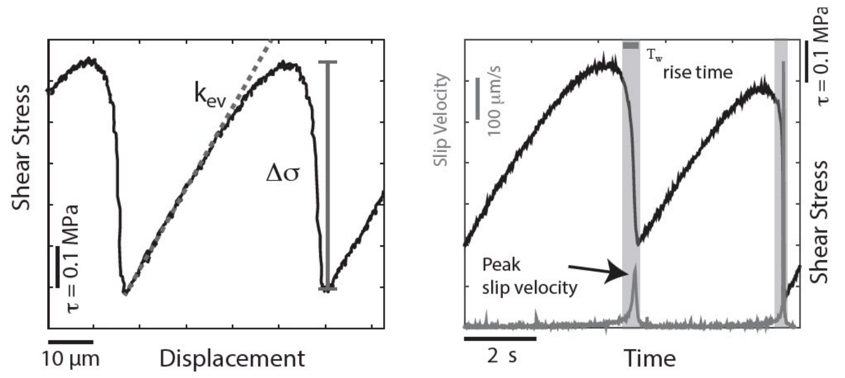


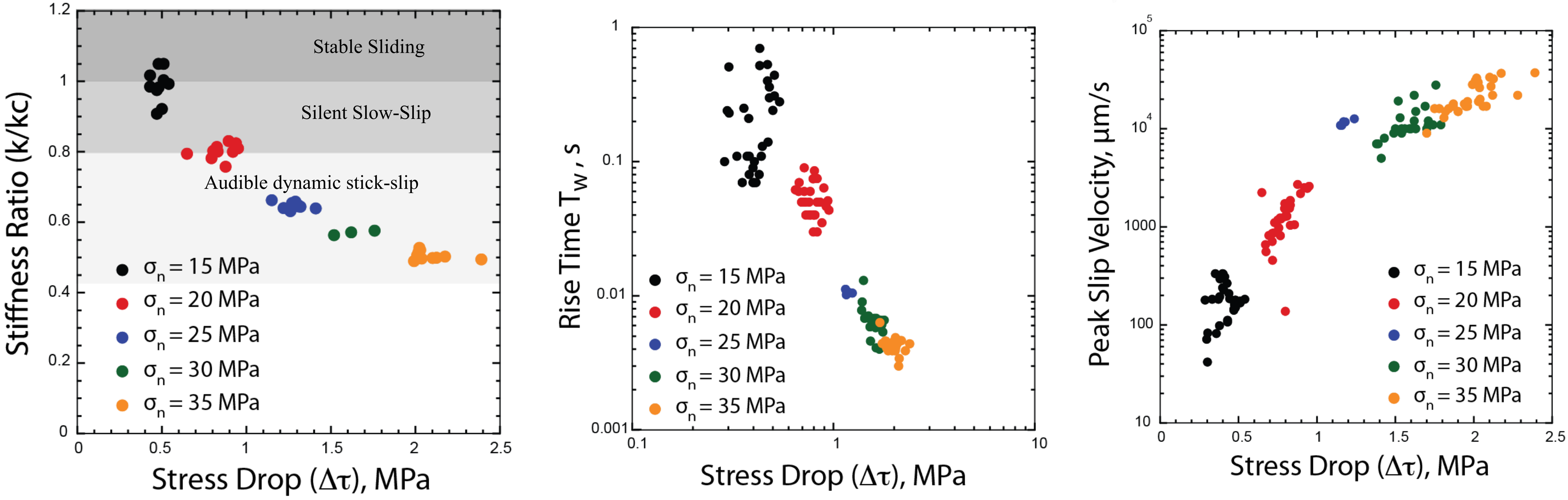




From aseismic creep to dynamic stick-slip passing through slow earthquakes

Stress drop, Rise Time and Peak slip velocity all scales with the stiffness ratio K/K_c showing a continuum spectrum of fault slip behaviors





Scuderi et al., 2016 *Nature Geoscience;* Tinti et al., 2016 *JGR*

From aseismic creep to dynamic stick-slip passing through slow earthquakes

How a frictional instability begins

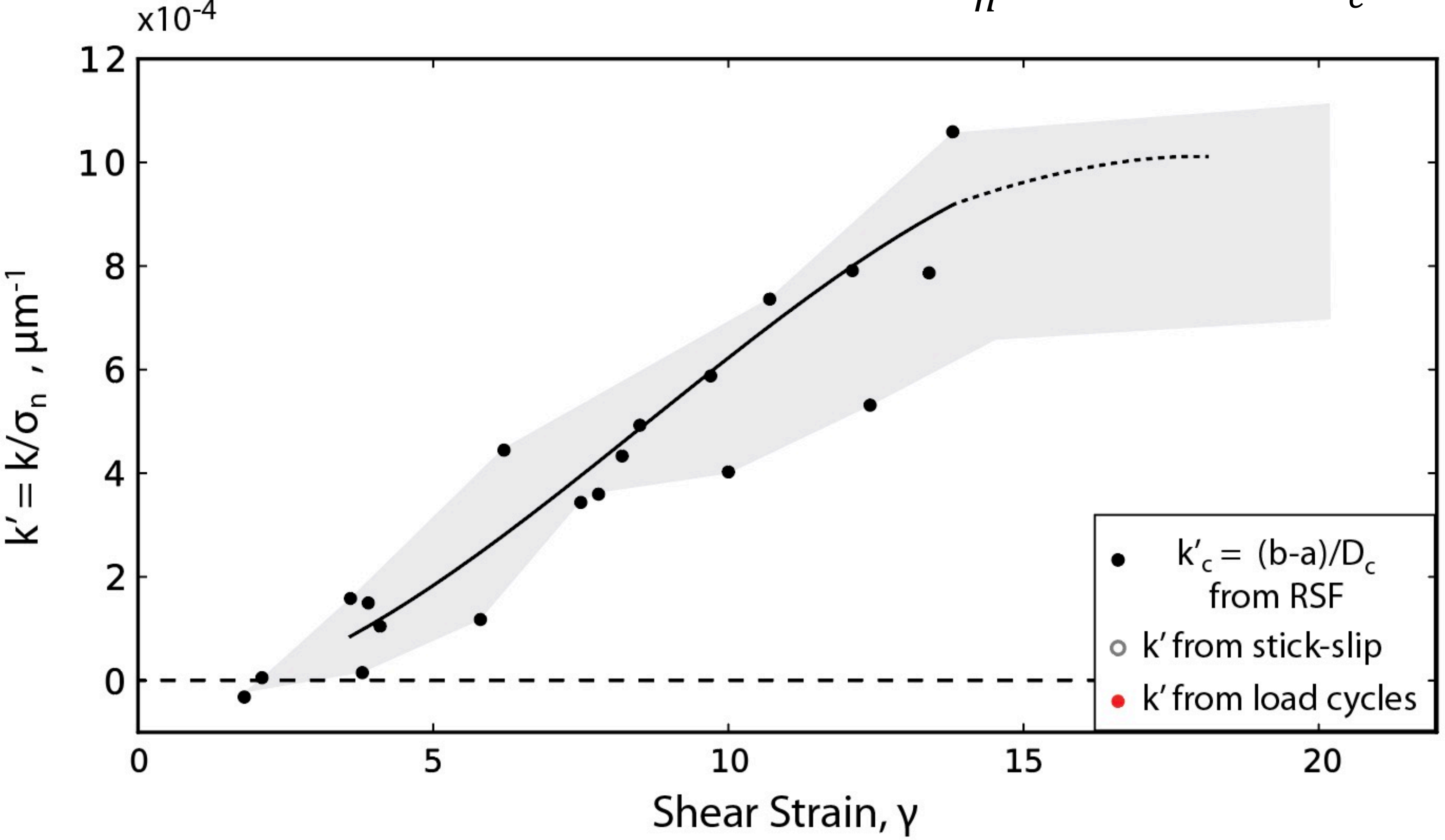
Unstable slip requires $k<k_c$

Surrounding stiffness

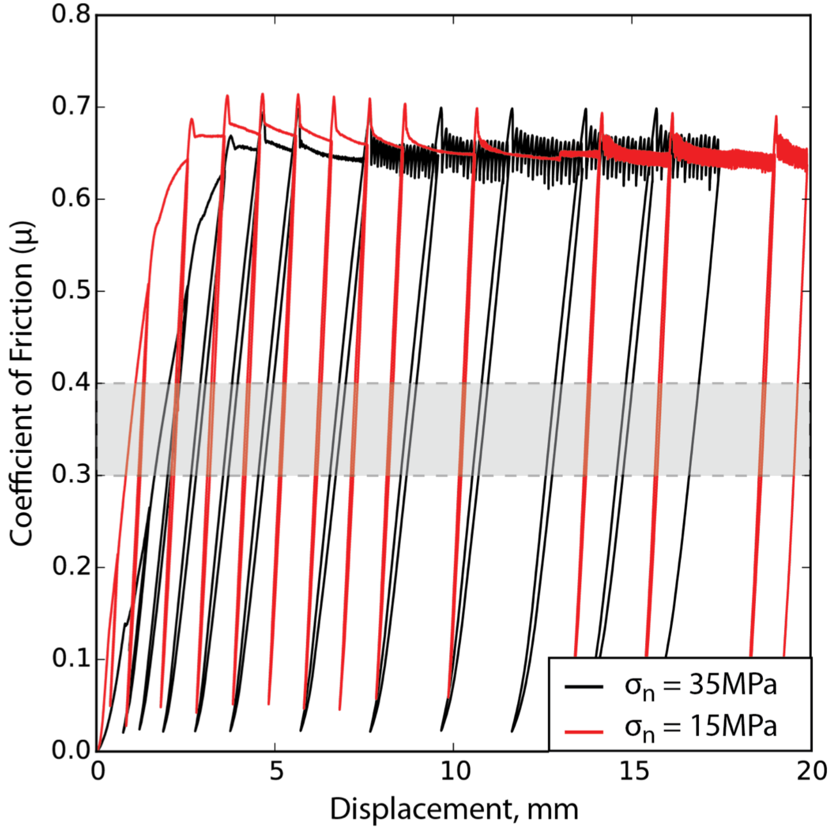
$$k' = \frac{k}{\sigma_n}$$

Critical rheologic stiffness

$$k_c = \frac{(b-a)}{D_c}$$



From aseismic creep to dynamic stick-slip passing through slow earthquakes



Measure the evolution of fault zone stiffness during deformation

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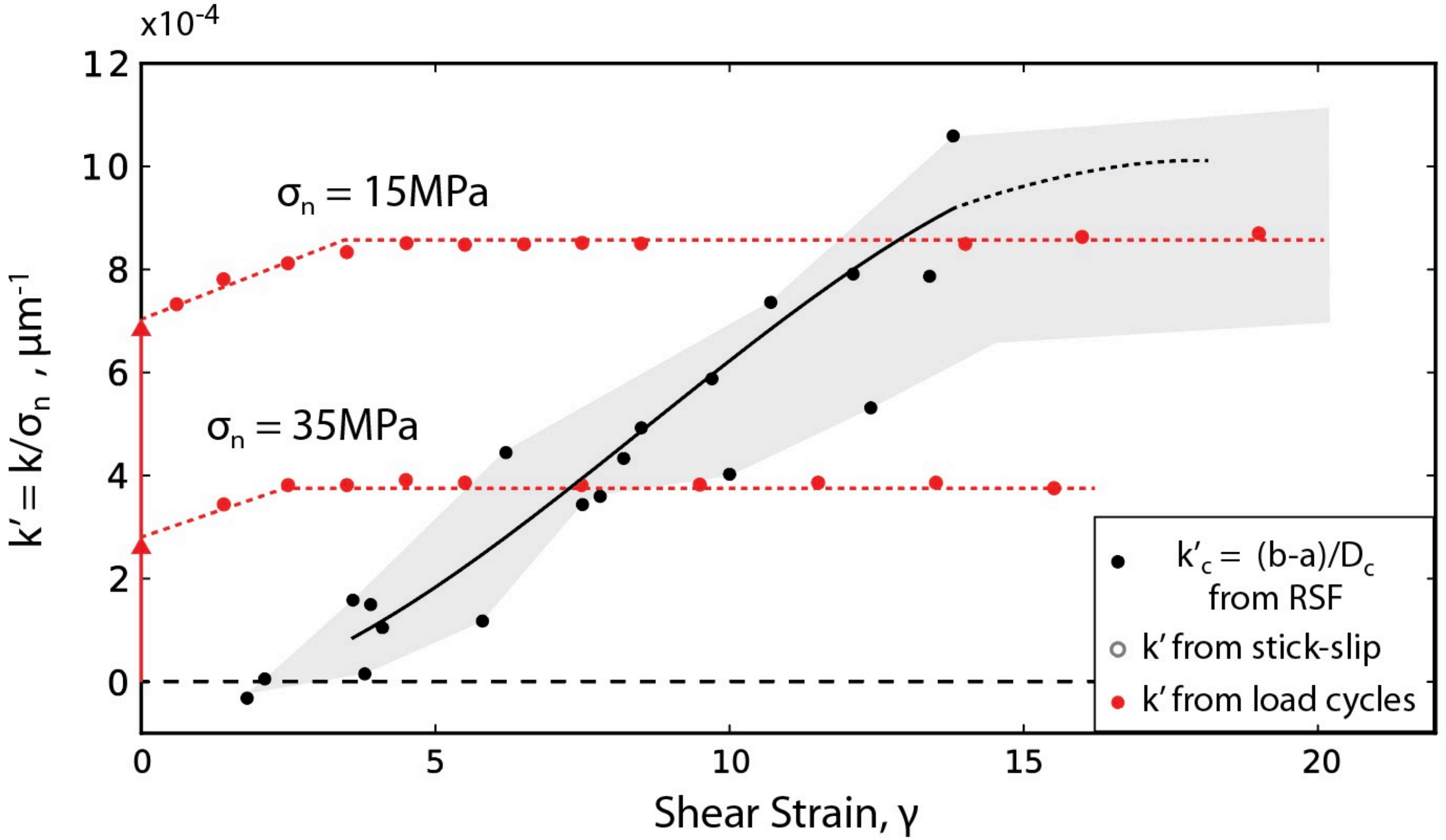
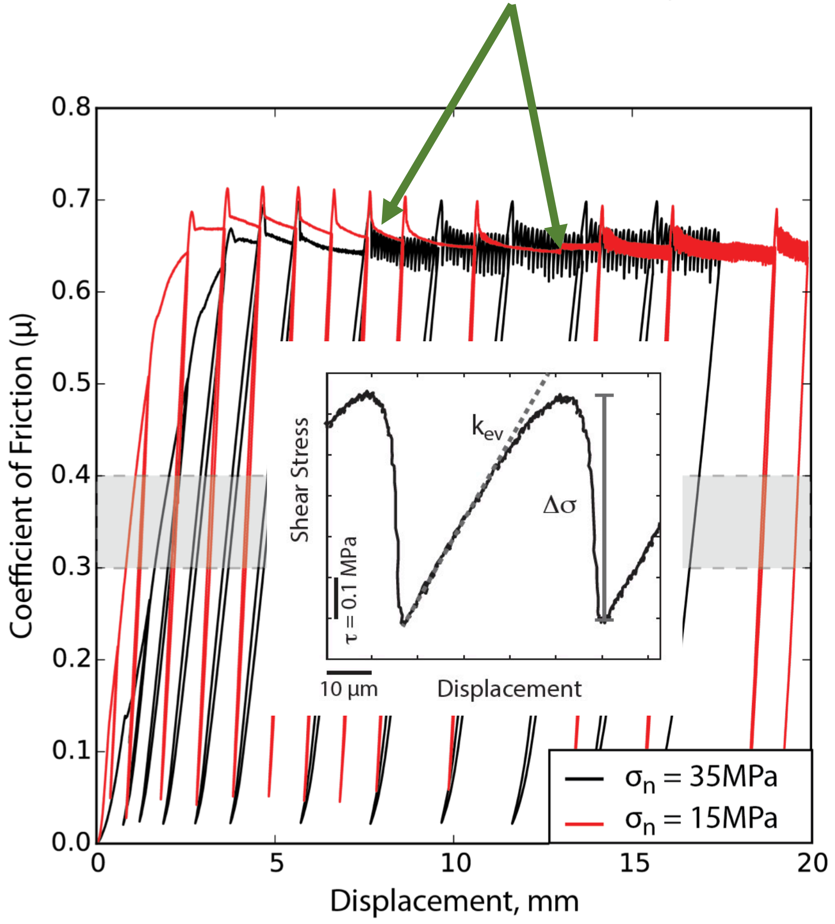


From aseismic creep to dynamic stick-slip passing through slow earthquakes

Instability arise spontaneously from stable sliding



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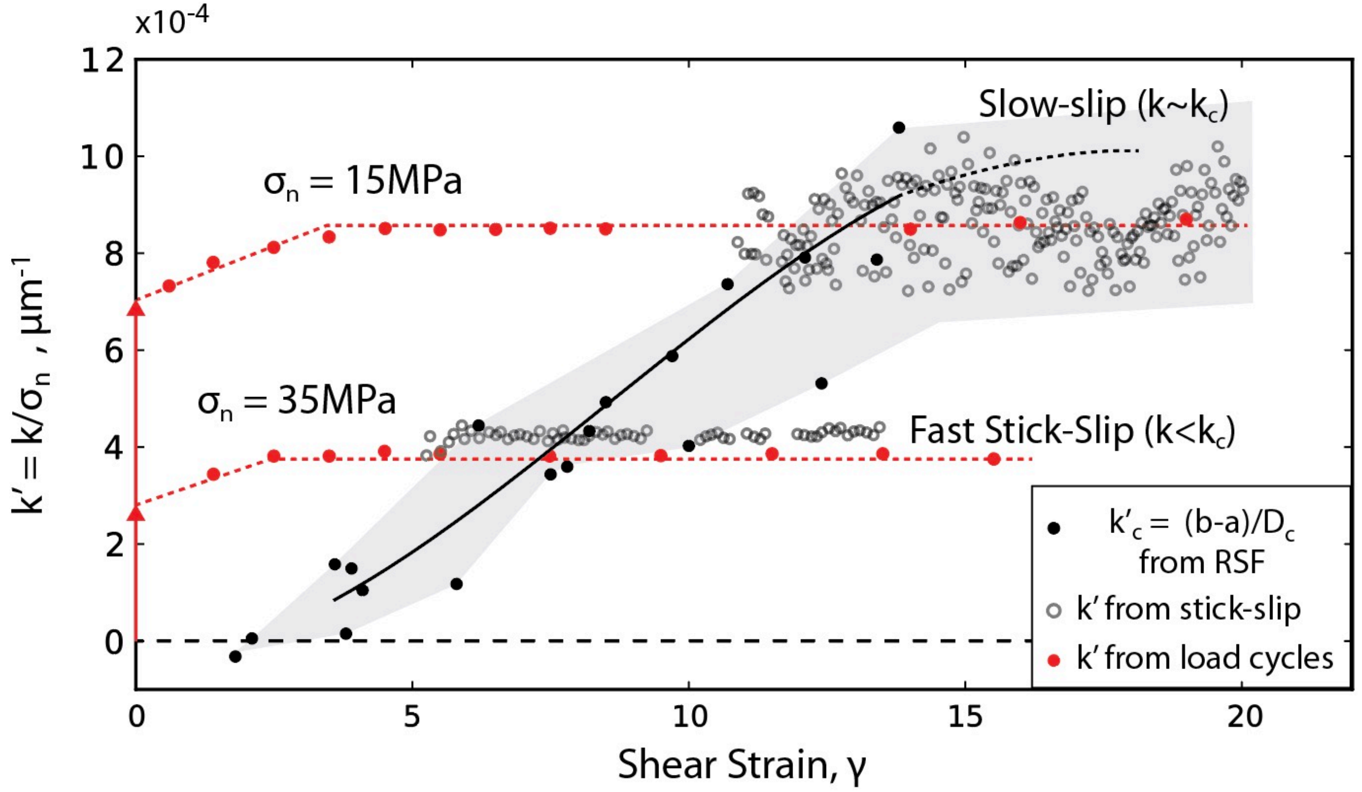
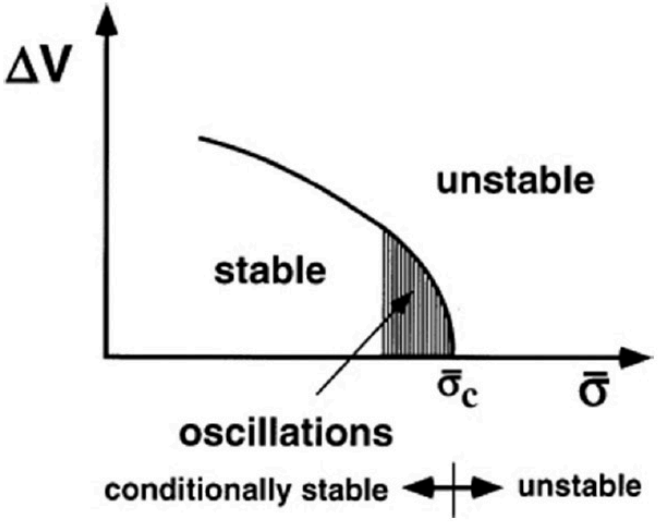
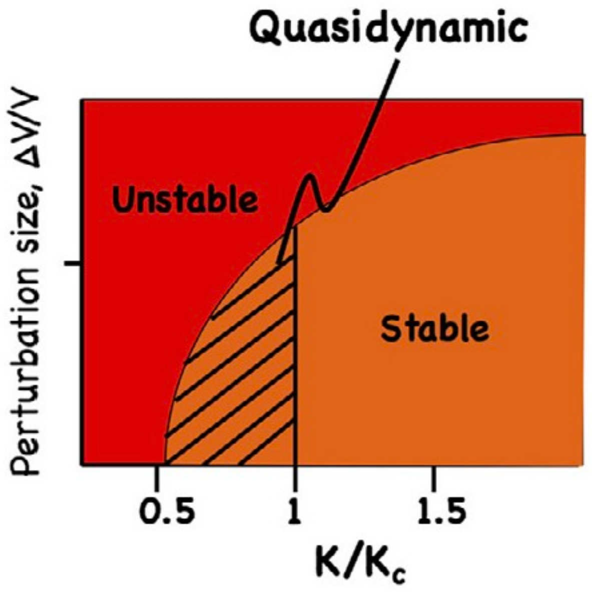
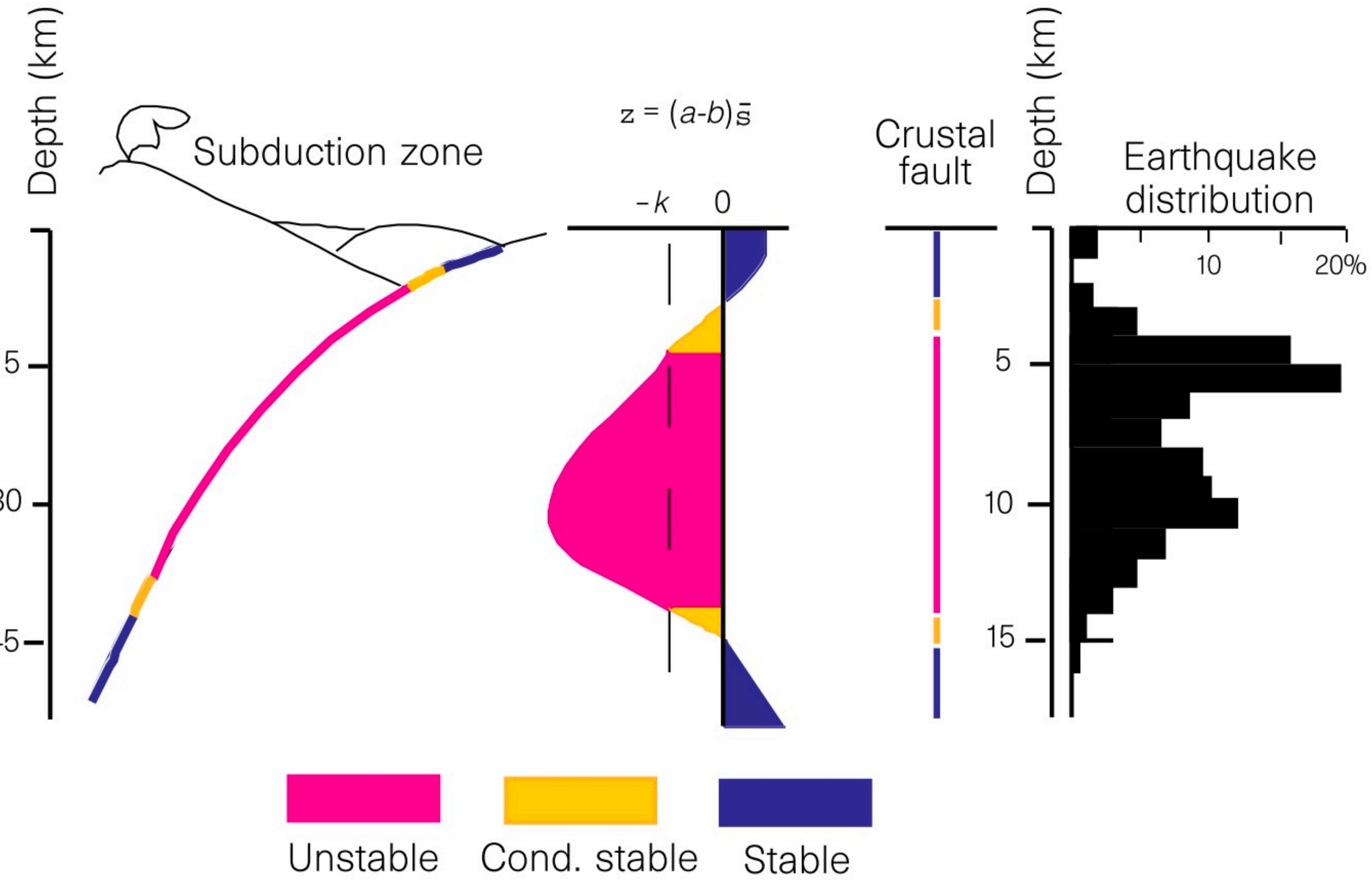


From aseismic creep to dynamic stick-slip passing through slow earthquakes





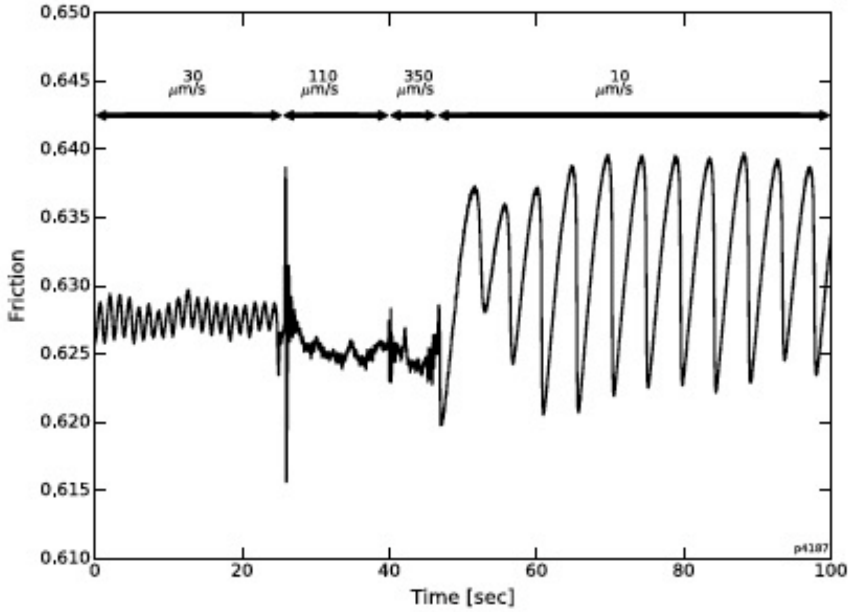


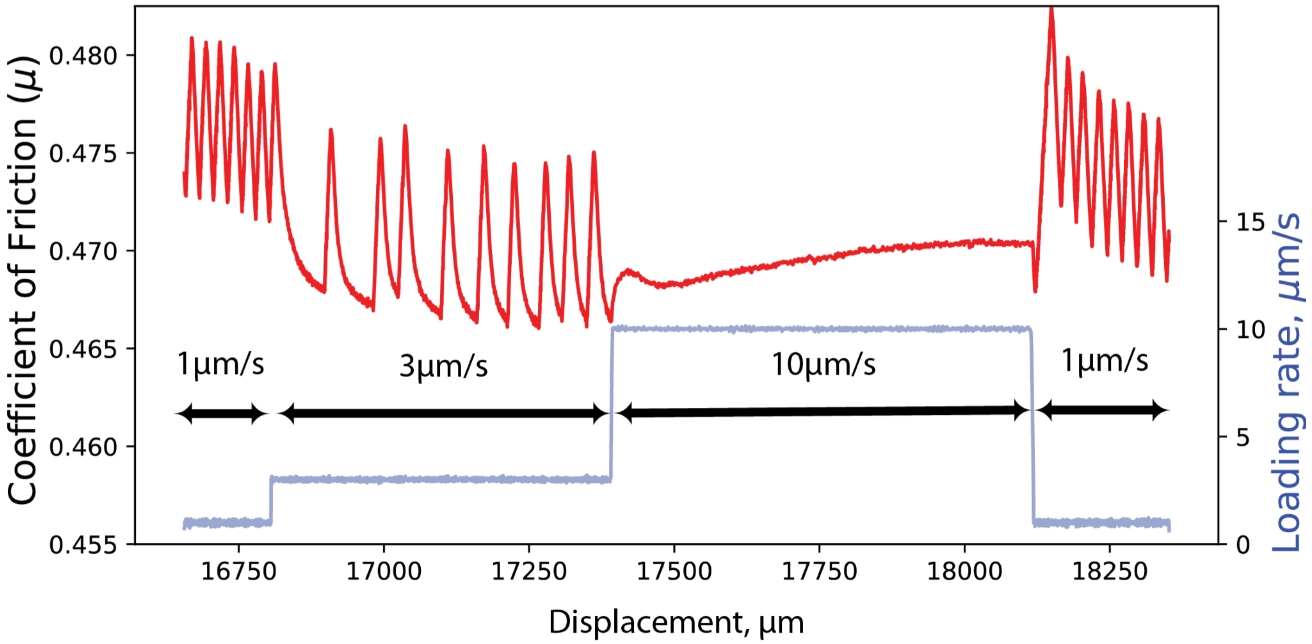
Gradual stability transition with normal stress and not a Hopf bifurcation between stable and unstable.

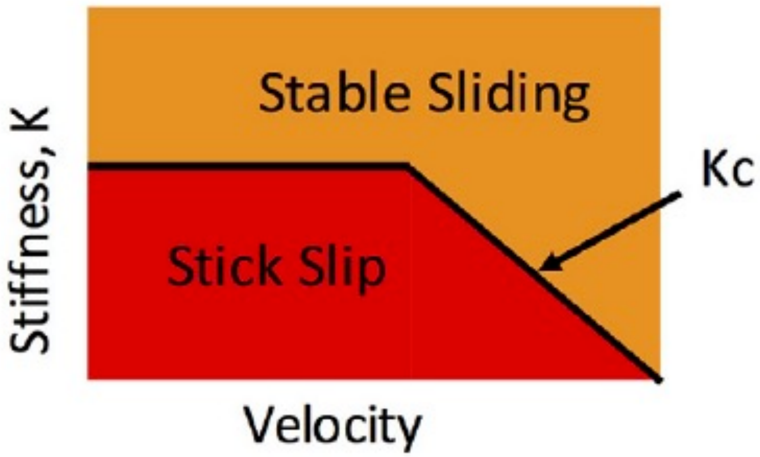
Slip acceleration and maximum slip velocity vary systematically across the transition.

From aseismic creep to dynamic stick-slip passing through slow earthquakes

Velocity dependence of stability





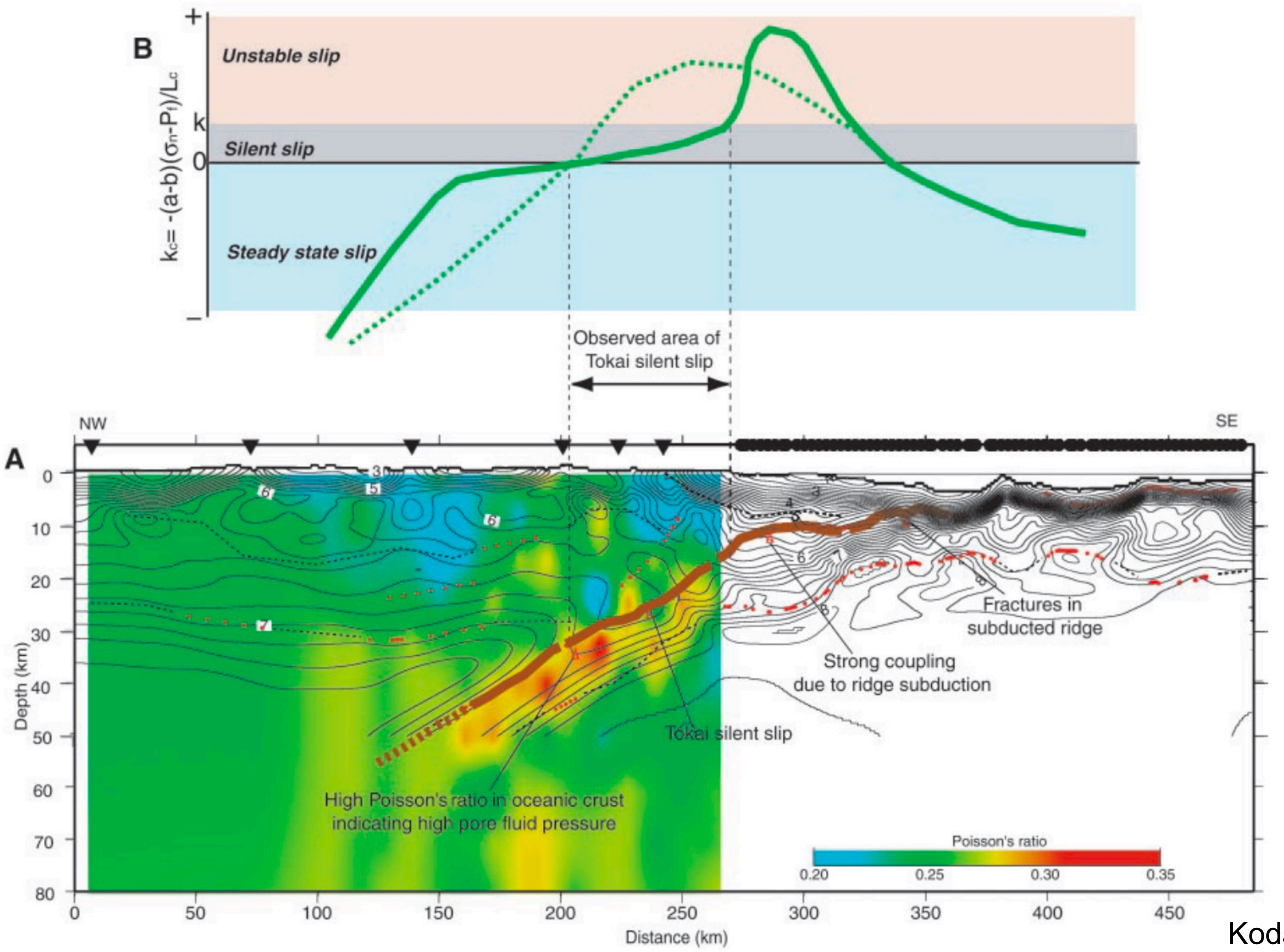


The fact that Kc decreases with slip velocity means that an unstable slip event may be quenched as it accelerates above a critical slip velocity

Leeman et al., 2018; Rubin, 2008

From aseismic creep to dynamic stick-slip passing through slow earthquakes

Elevated fluid pressure



Kodaira et al., 2004 Science

Lab is fun !!

Thank you