QFT Dynamics from CFT Data

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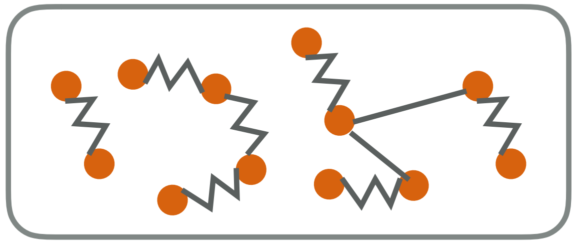
Preface

This talk: A new numerical method ("conformal truncation") to study real-time, infinite-volume dynamics of strongly-coupled QFTs

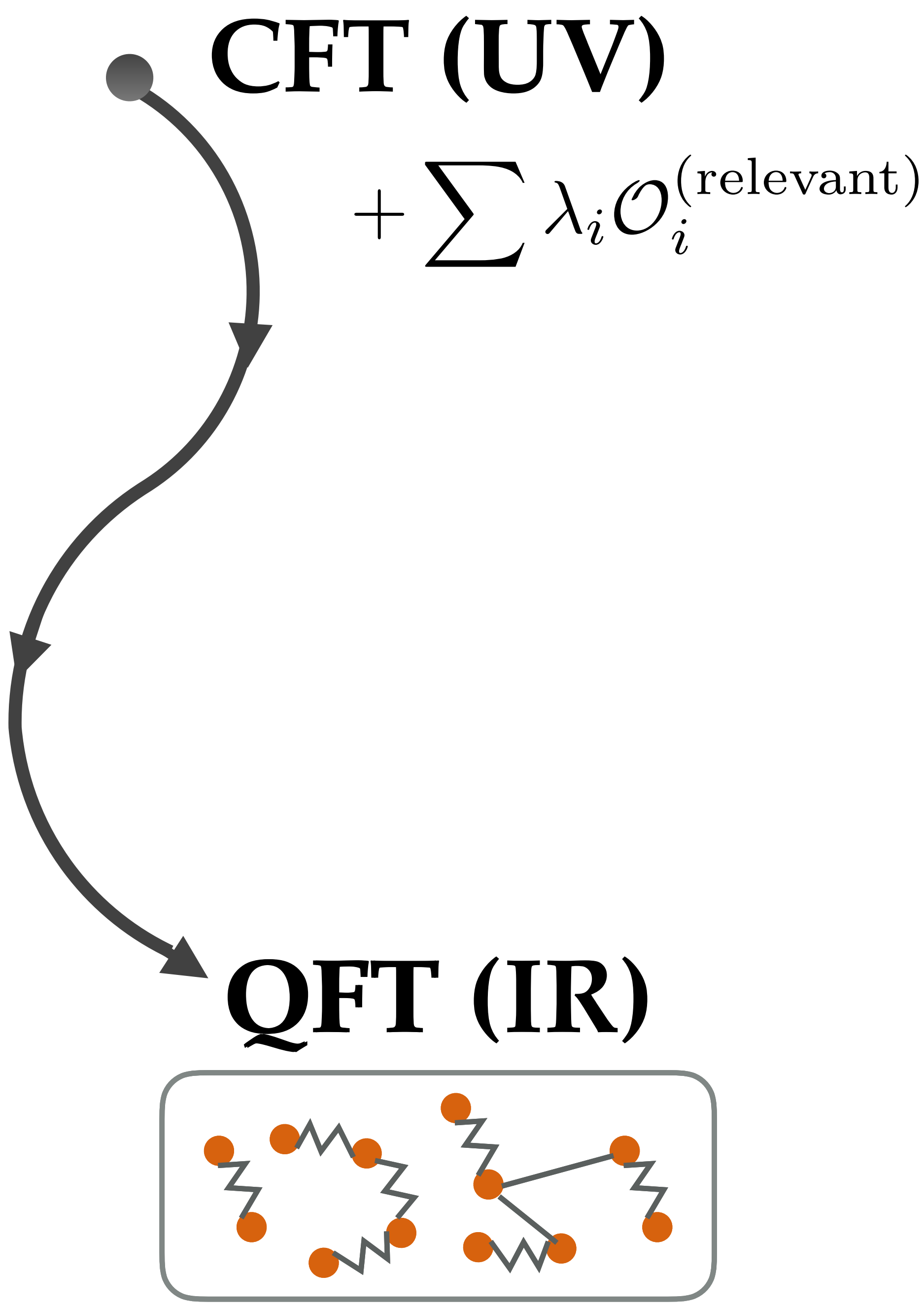
Basic Strategy

QFT



Basic Strategy

Write QFT as deformation of UV CFT.
Use CFT data to organize QFT calculation.



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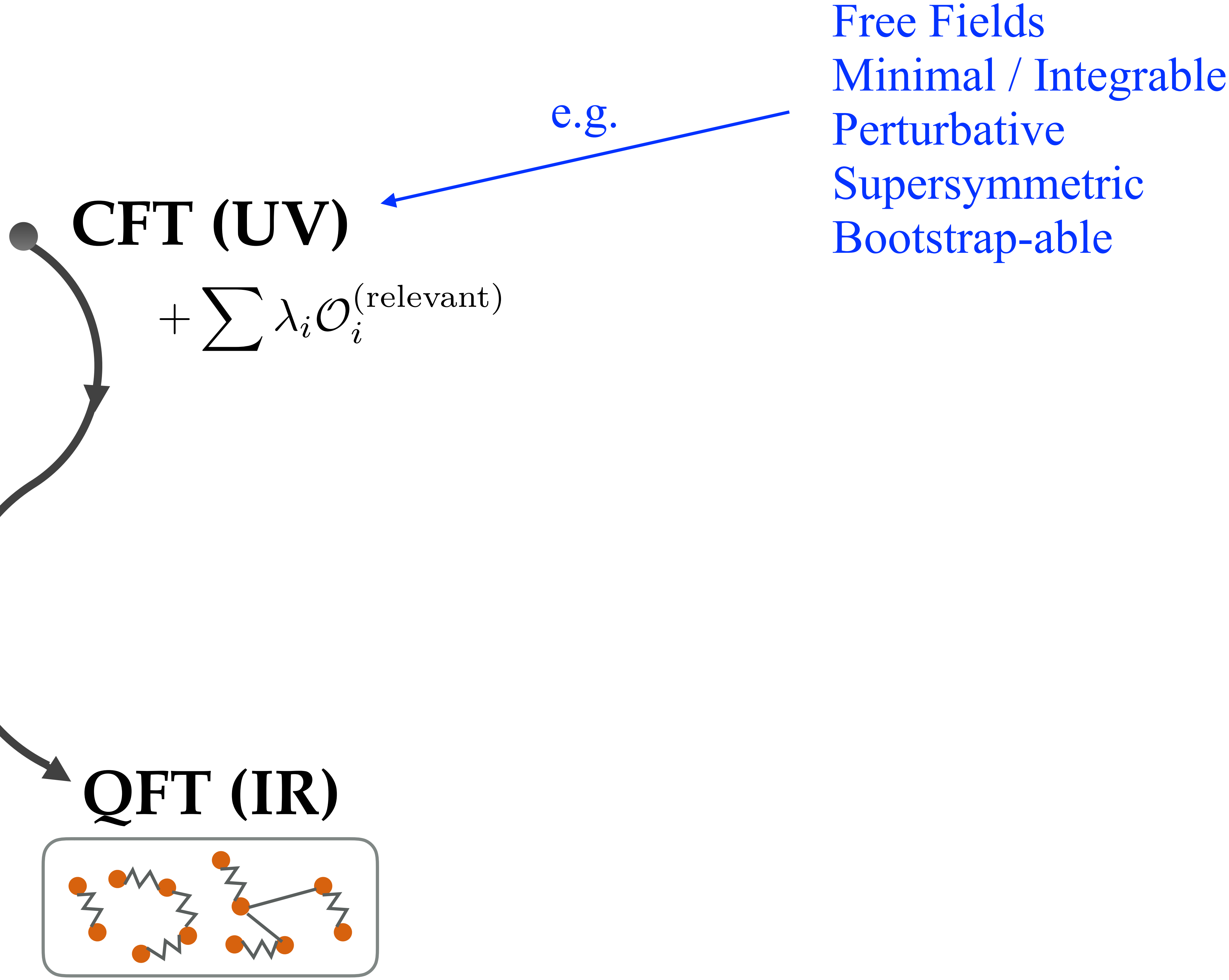
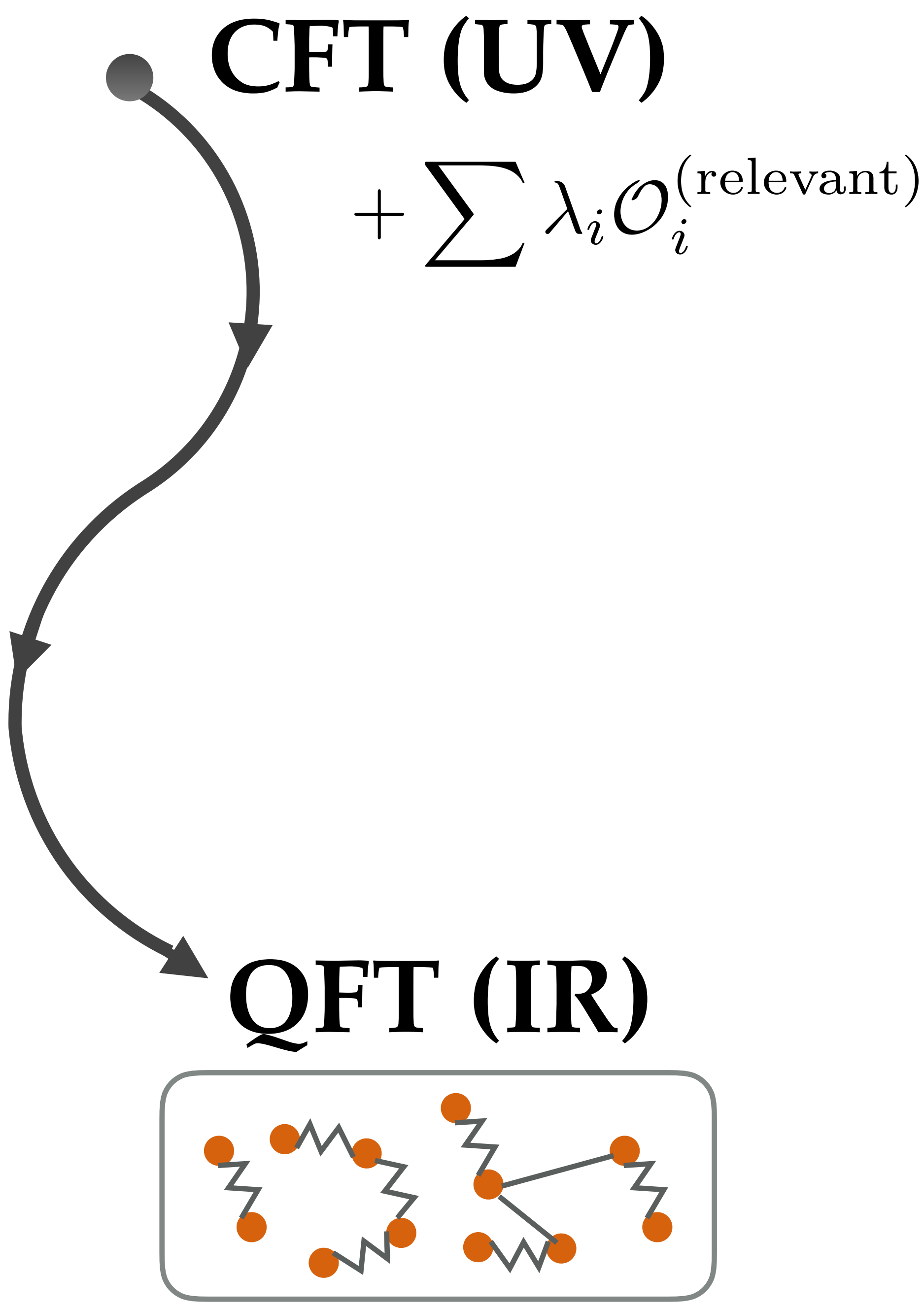


Basic Strategy

Goal: Extract QFT dynamics from CFT data



Input

UV CFT Data:
Δ's + OPE coefficients

Output

IR QFT Observables:

- Spectrum
- Correlation Functions
  (real-time, infinite-volume)

Novel Feature of Conformal Truncation

Formulated so that entire computation takes place in real time and infinite volume, allowing access to dynamics

No Wick rotation, no lattice, no compactification

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Conformal truncation is a specific implementation of Hamiltonian truncation.

Hamiltonian Truncation

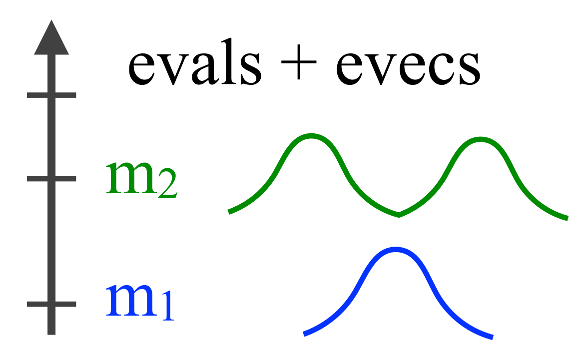
1. Identify a basis of QFT states $|b_1\rangle,\ |b_2\rangle,\ |b_3\rangle, \ldots$ (infinite)

2. Write Hamiltonian in chosen basis

$$H = \begin{pmatrix} H_{11} & H_{12} & \cdots \\ H_{21} & H_{22} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}$$

3. Truncate in some way

4. Diagonalize numerically



5. Look for convergence w/ truncation level

Hamiltonian Truncation

Heart of any truncation scheme.
How to discretize QFT???

1. Identify a basis of QFT states

$|b_1\rangle, |b_2\rangle, |b_3\rangle, \ldots$ (infinite)

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$$H = \begin{pmatrix} H_{11} & H_{12} & \cdots \\ H_{21} & H_{22} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}$$

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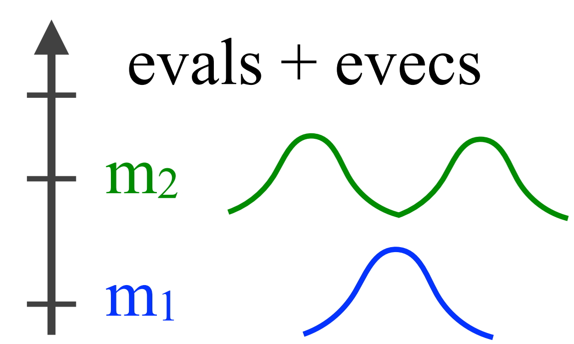

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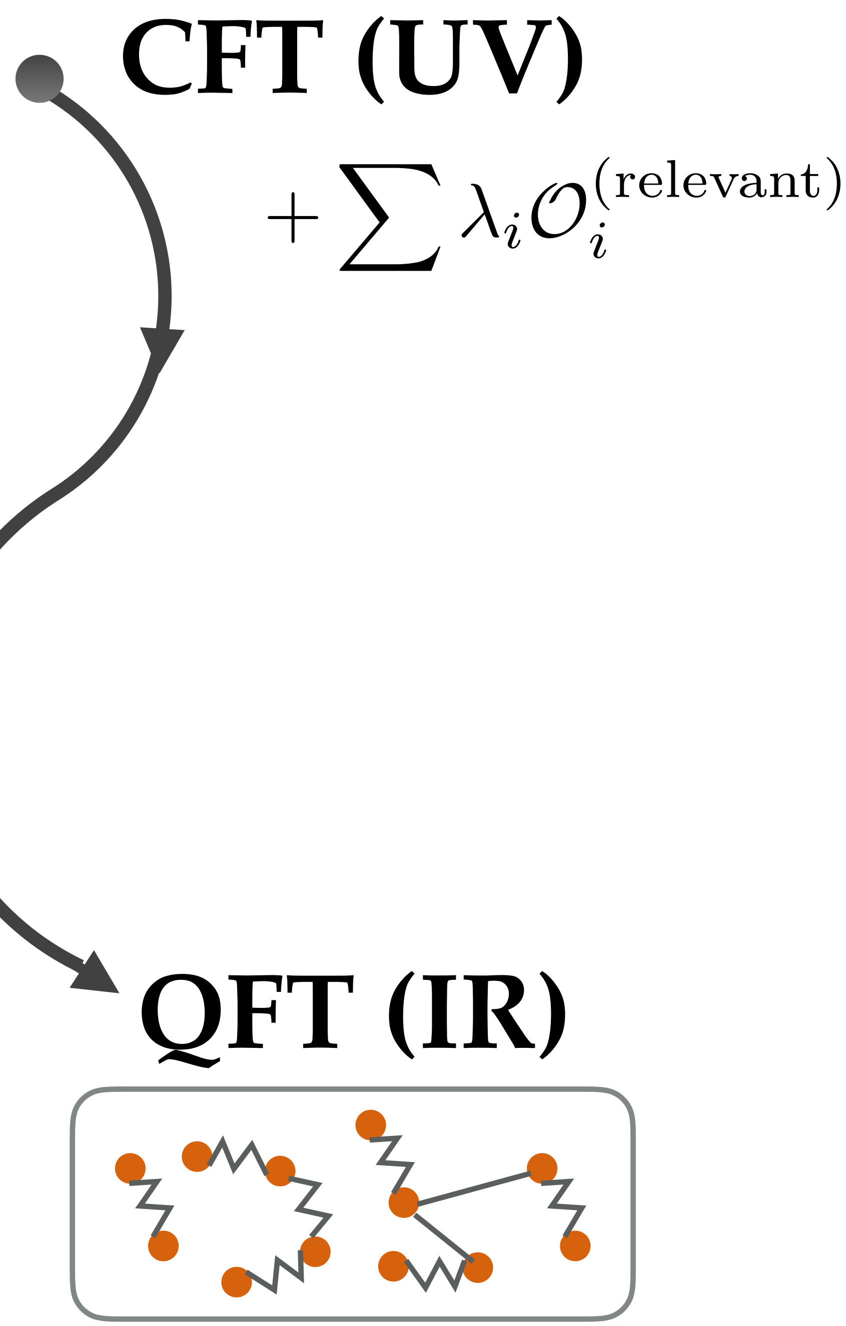
Conformal Truncation Basis

Use UV CFT operators $\mathcal{O}_\Delta(x^\mu)$ to construct basis $|b_1\rangle, |b_2\rangle, |b_3\rangle, \ldots$

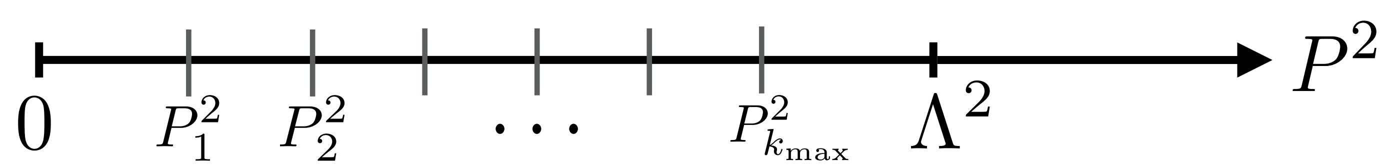


Conformal Truncation Basis

Use UV CFT operators $\mathcal{O}_\Delta(x^\mu)$ to construct basis $|b_1\rangle, |b_2\rangle, |b_3\rangle, \ldots$

Think: $[H, \vec{P}] = 0.$

$$\mathcal{O}_\Delta(x) \quad \longrightarrow \quad |\Delta, \vec{P}, P^2\rangle = \int d^dx \, e^{-iP\cdot x} \, \mathcal{O}_\Delta(x)|0\rangle$$



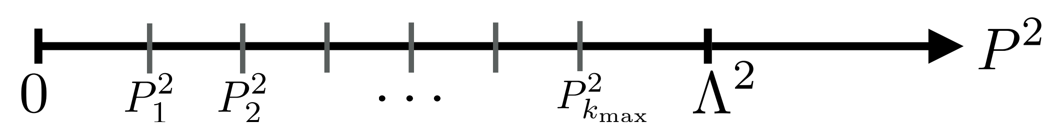
$$\longrightarrow \quad |\Delta, \vec{P}, P_k^2\rangle \qquad (k = 1, \ldots, k_{\max})$$

Final basis states

Note: Still real time and infinite volume

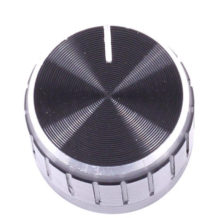
Truncation Parameters: $\Delta_{\max}$, $k_{\max}$

$$\mathcal{O}_\Delta(x) \longrightarrow |\Delta, \vec{P}, P^2\rangle = \int d^d x\, e^{-iP\cdot x}\, \mathcal{O}_\Delta(x)|0\rangle$$



$$\longrightarrow |\Delta, \vec{P}, P_k^2\rangle \qquad (k = 1, \ldots, k_{\max})$$

$\Delta_{\max}$



$k_{\max}$

Why Truncate in $\Delta_{\max}$?

Holographic Intuition:

$$\text{CFT}_d \qquad\qquad \text{AdS}_{d+1}$$

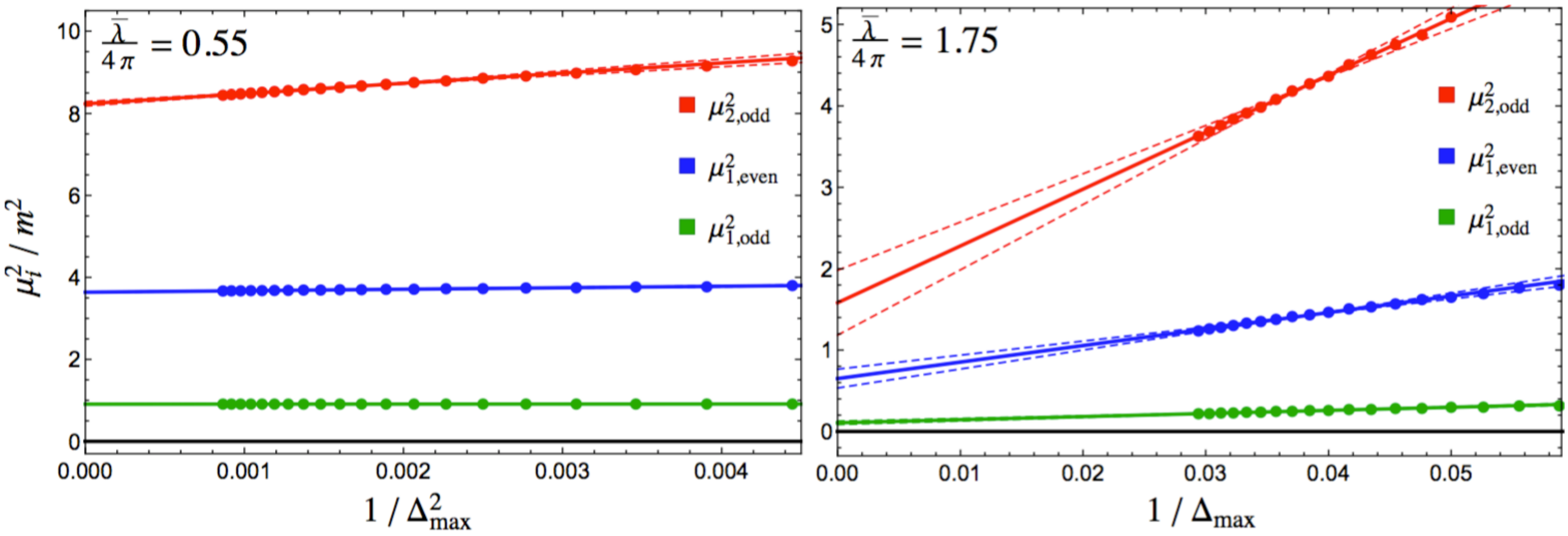
$$\mathcal{O}_\Delta(x) \quad \longleftrightarrow \quad \Phi(x,z) \qquad M^2_{\text{AdS}} \sim \Delta^2$$

Large Δ operators = **heavy** objects in AdS

(expect to decouple)

Why Truncate in $\Delta_{\max}$?

<u>Experimental Evidence:</u> (1+1)d $\lambda\phi^4$-theory



$$\mu_i^2(\Delta_{\max}) = A + \frac{B}{(\Delta_{\max})^{\#}}$$

small parameter: $\frac{1}{(\Delta_{\max})^{\#}}$!

Hamiltonian Matrix Elements

$$\text{CFT Spectrum} \longrightarrow \text{basis}$$

$$\text{OPE Coefficients} \longrightarrow H \text{ matrix elements}$$

$$H_{QFT} = H_{CFT} + \lambda \int d\vec{x}\, \mathcal{O}_{\text{rel}}(\vec{x})$$

$$\langle \Delta, P | \delta H | \Delta', P' \rangle = \delta(\vec{P} - \vec{P}') \int d^d x\, d^d x'\, e^{i(P\cdot x - P'\cdot x')} \langle \mathcal{O}(x) \mathcal{O}_{\text{rel}}(0) \mathcal{O}'(x') \rangle$$

H matrix element

Fourier transform of CFT 3PF

Quantization scheme: Lightcone

Technology

$$\text{CFT Spectrum} \longrightarrow \text{basis}$$
$$\text{OPE Coefficients} \longrightarrow H \text{ matrix elements}$$

1. How to enumerate all primary operators in a CFT (even just free CFT)?

2. How to efficiently compute OPE coefficients (even just free CFT)?

3. How to Fourier transform general-spin CFT 3PFs?

   specifically, Wightman functions

Conformal Truncation Deliverables

- Spectrum: bound states, onset of critical behavior, etc.

- Real-time, infinite-volume correlation functions:

e.g., Källén-Lehmann spectral density $\rho_{\mathcal{O}}(\mu)$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \int d\mu^2 \boxed{\rho_{\mathcal{O}}(\mu)} \int \frac{d^d p}{(2\pi)^d}\, e^{-ip\cdot x}\theta(p_0)(2\pi)\delta(p^2-\mu^2)$$

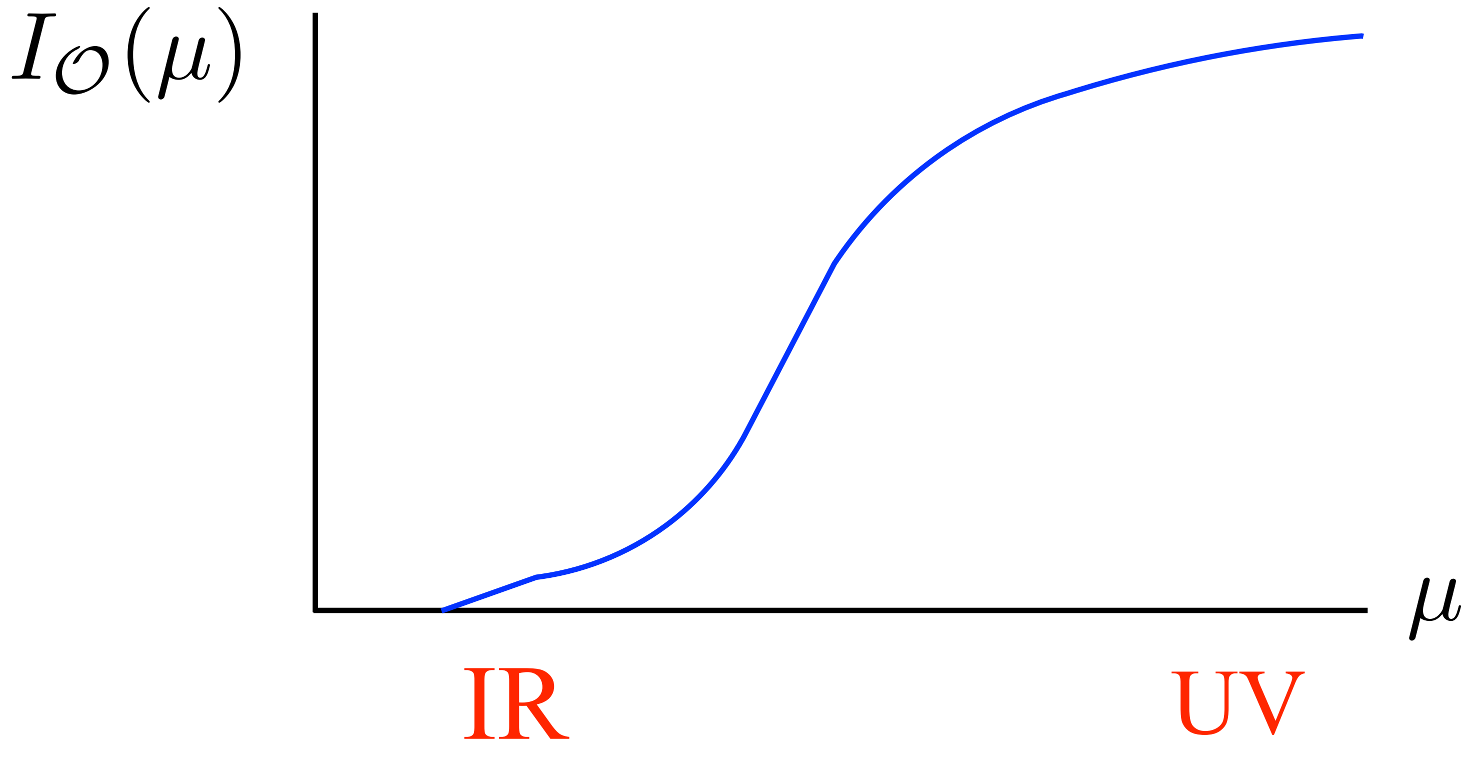
$$I_{\mathcal{O}}(\mu) \equiv \int_0^{\mu^2} d\mu'^2 \boxed{\rho_{\mathcal{O}}(\mu')}$$

Conformal Truncation Deliverables

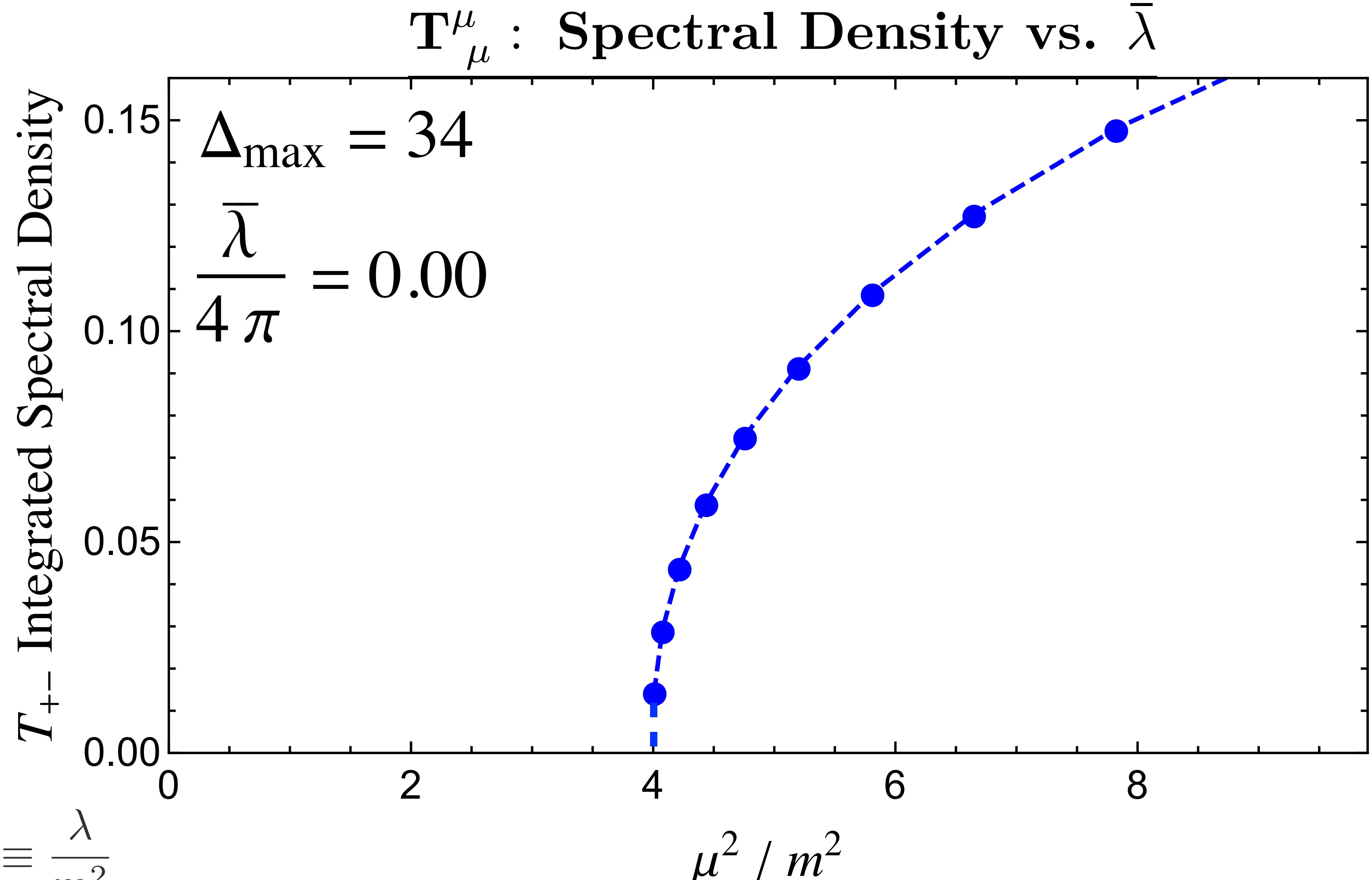
$$I_{\mathcal{O}}(\mu) \equiv \int_0^{\mu^2} d\mu'^2 \, \rho_{\mathcal{O}}(\mu')$$

Källén-Lehmann spectral density $\rho_{\mathcal{O}}(\mu)$

Encodes RG



Example: (1+1)d $\lambda\phi^4$-theory



$\bar{\lambda} \equiv \frac{\lambda}{m^2}$

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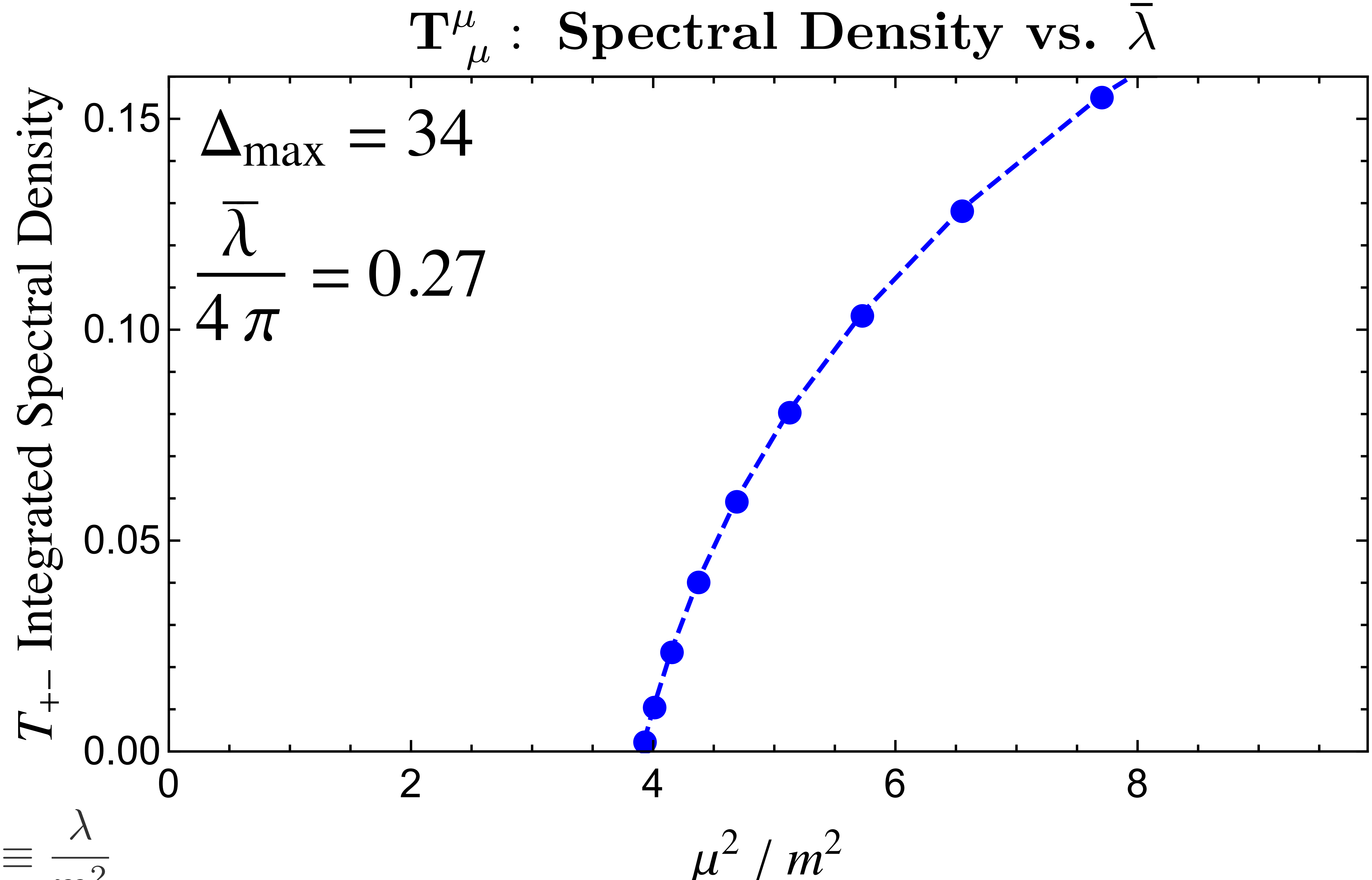

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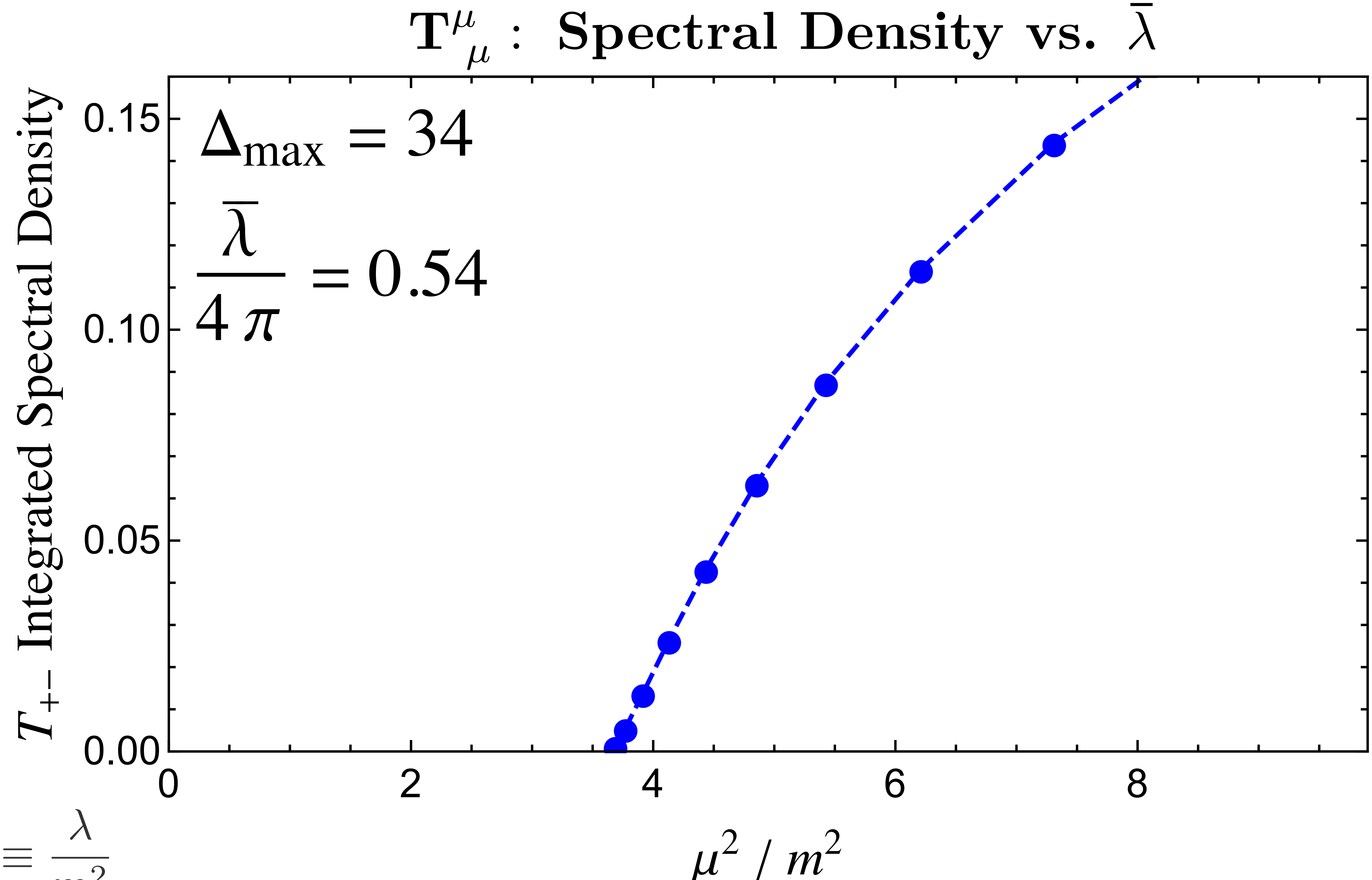

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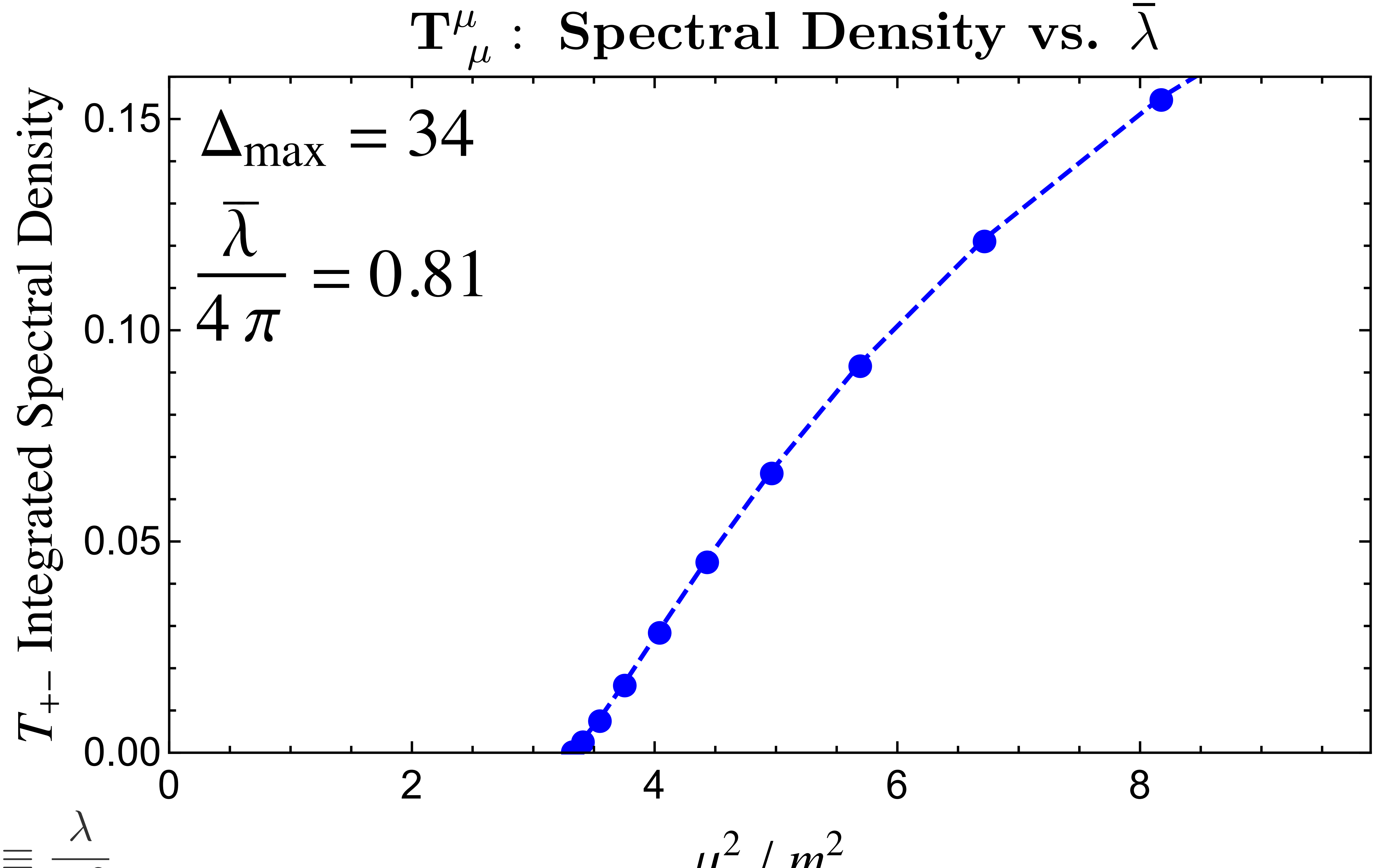

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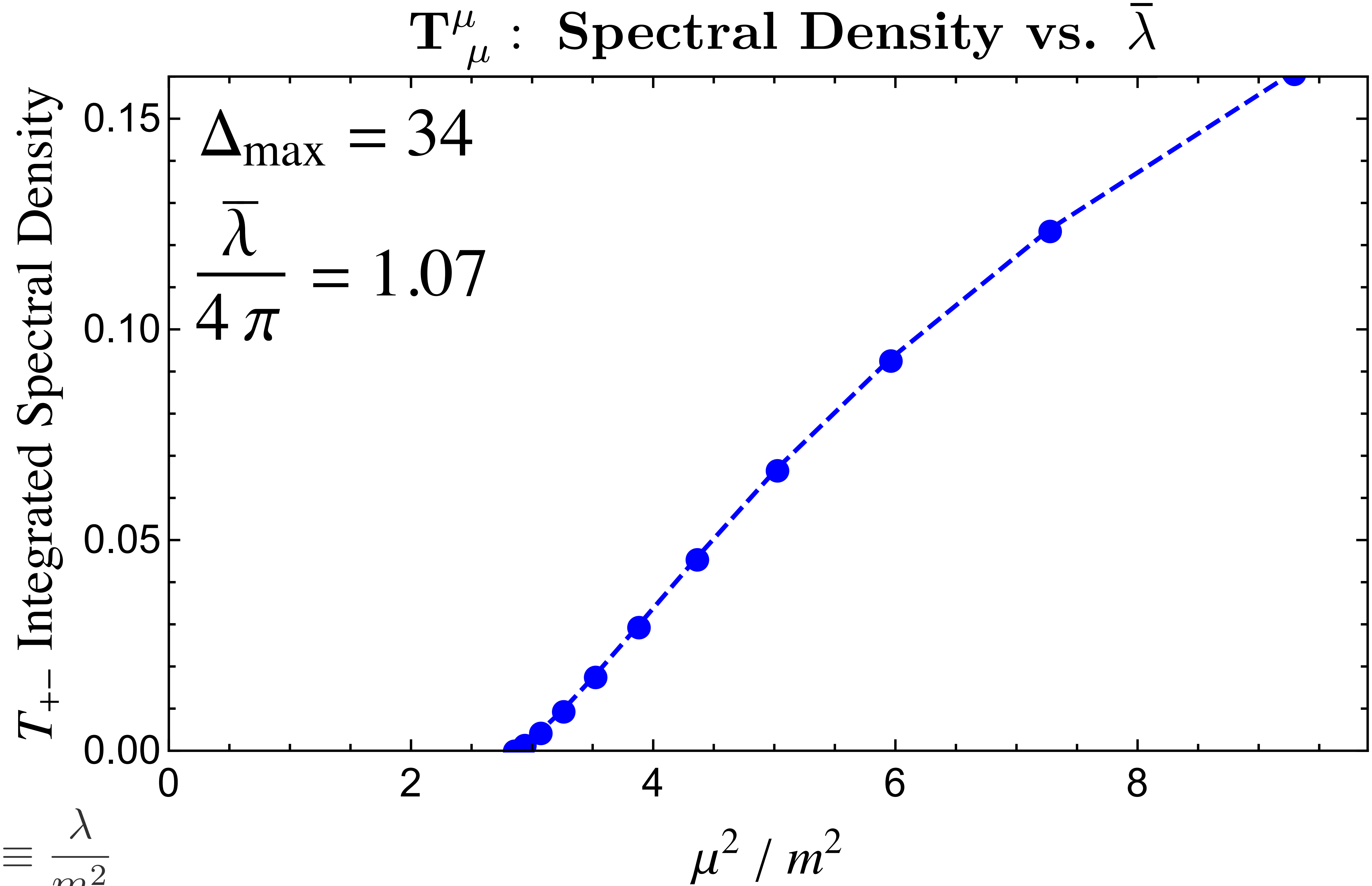

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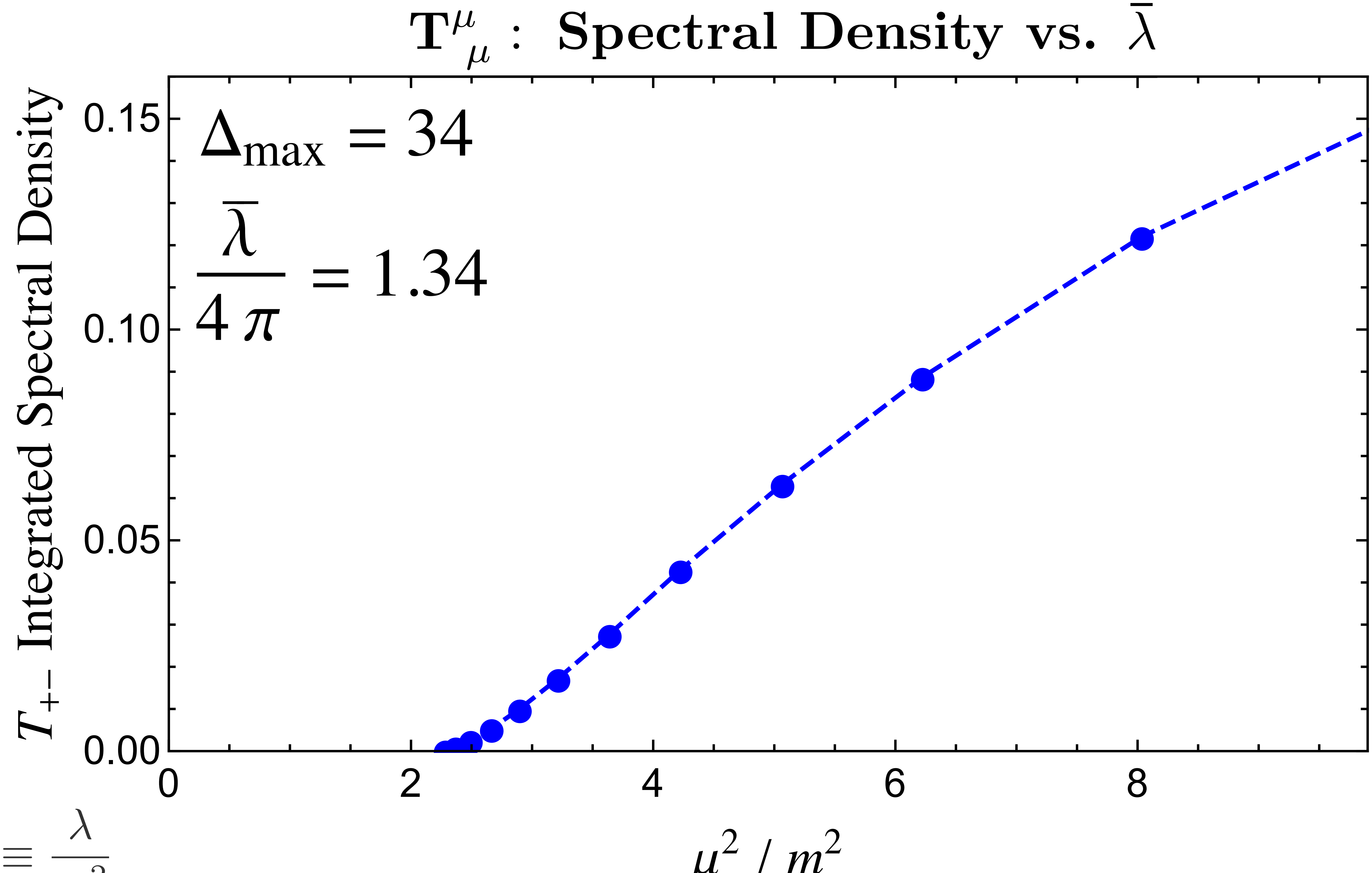

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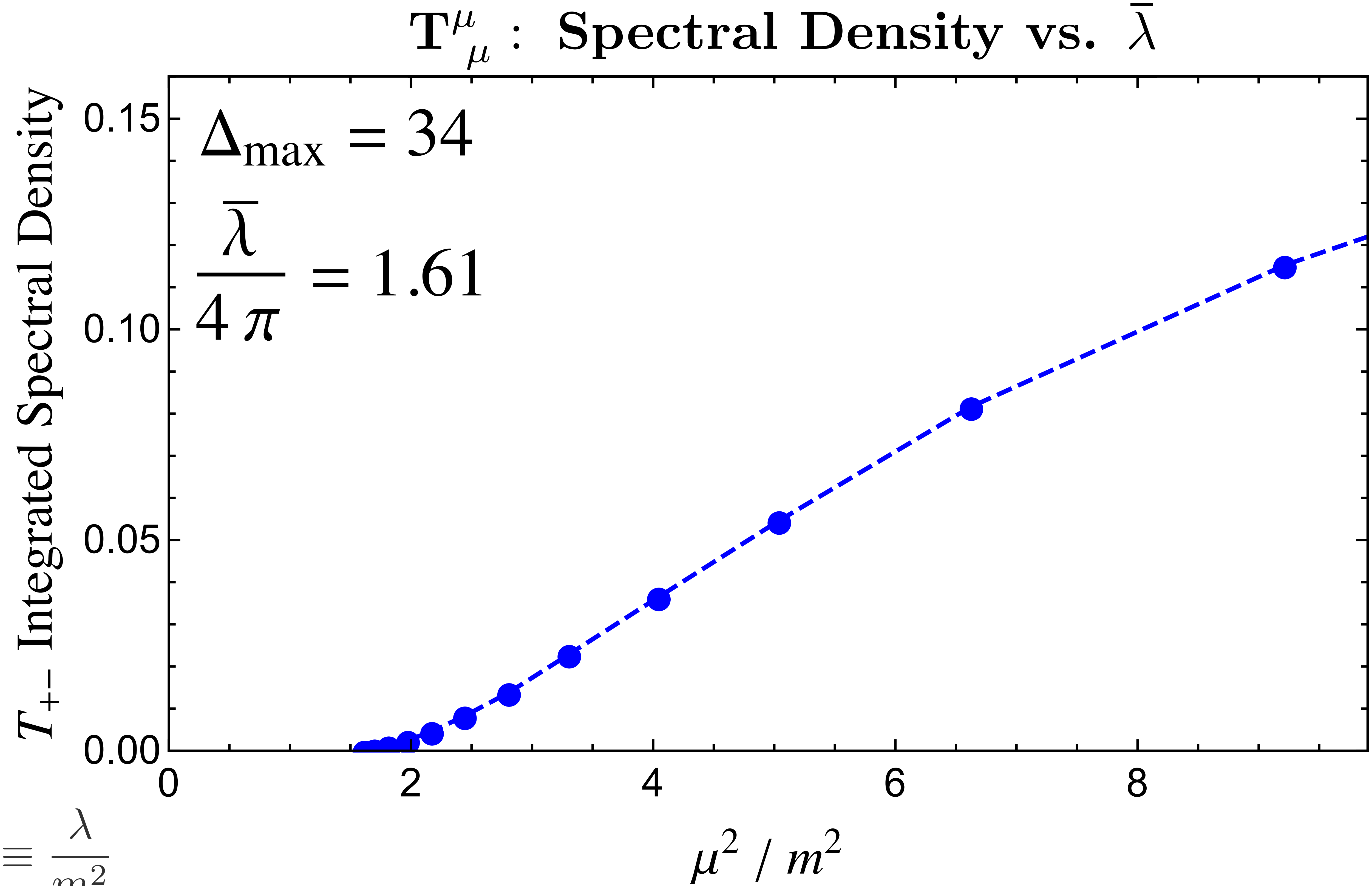

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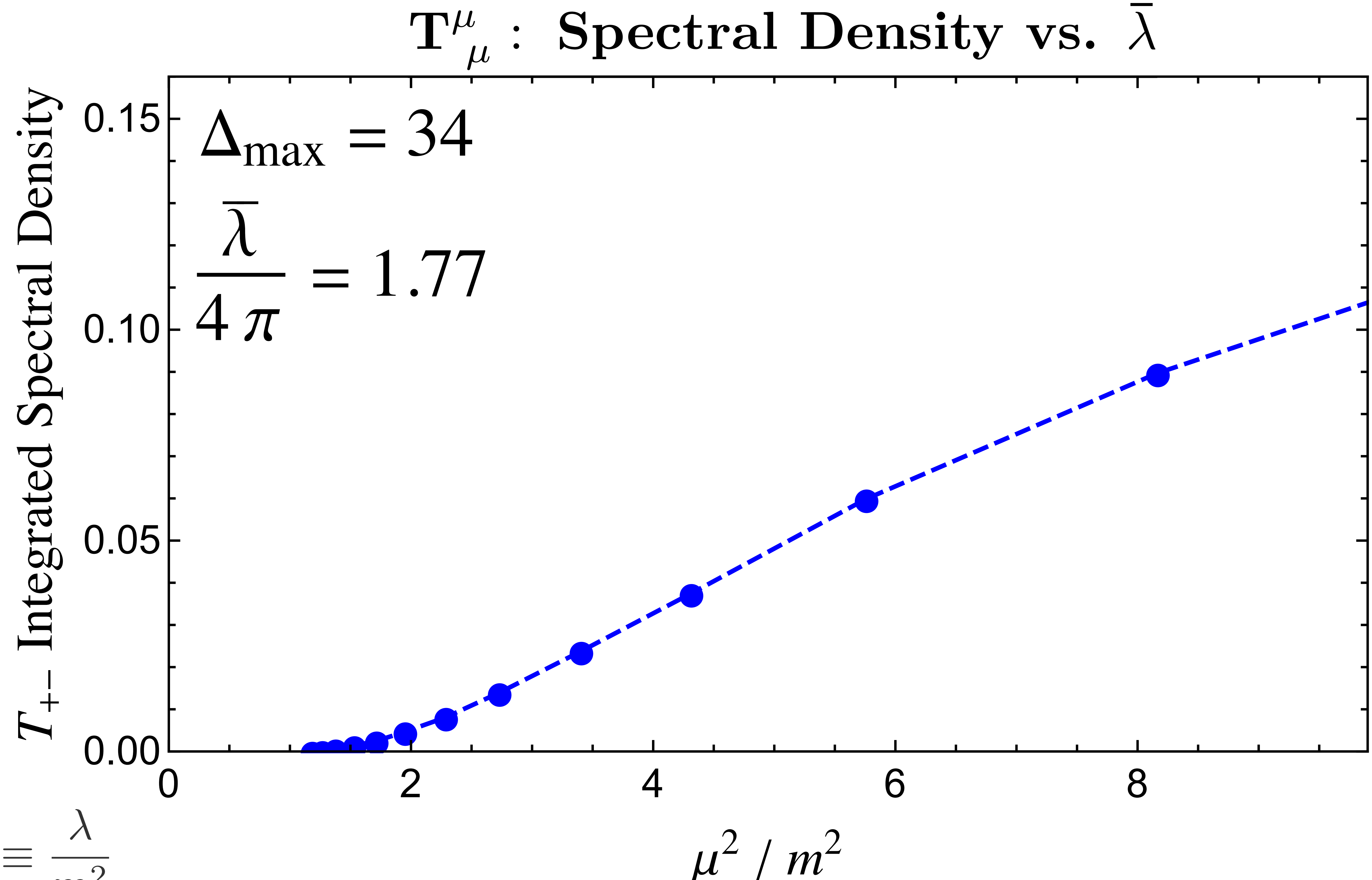

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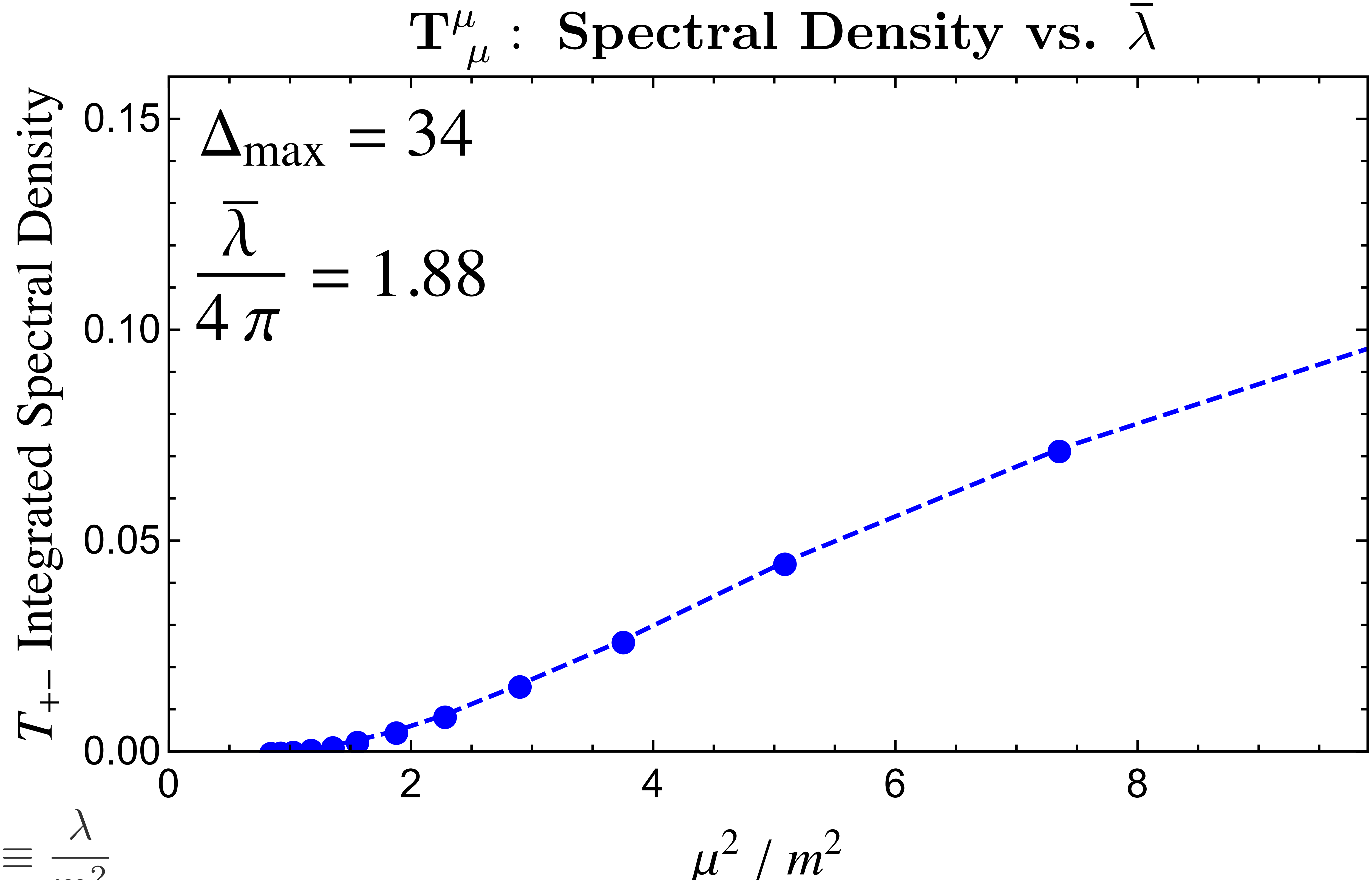

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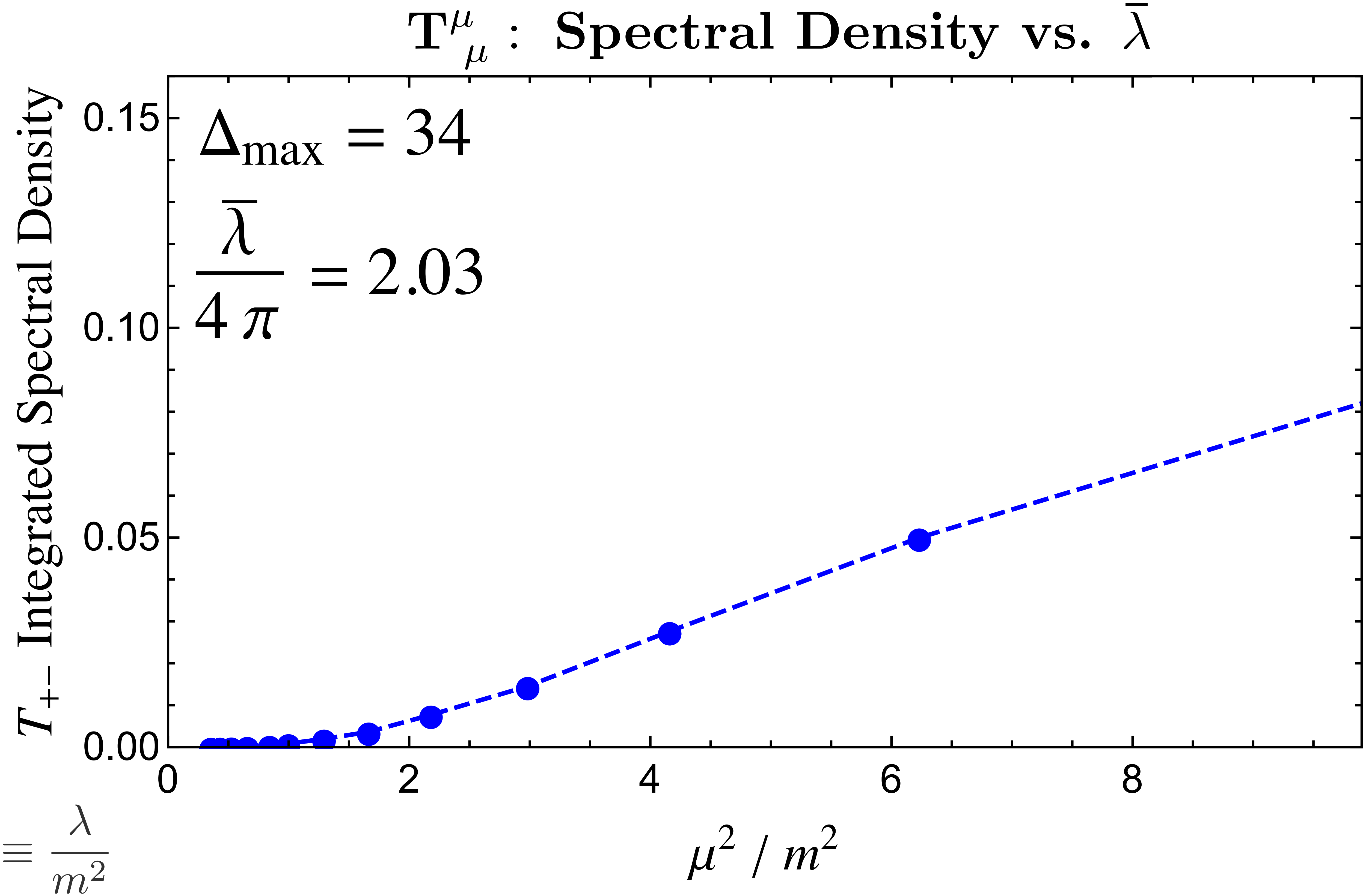

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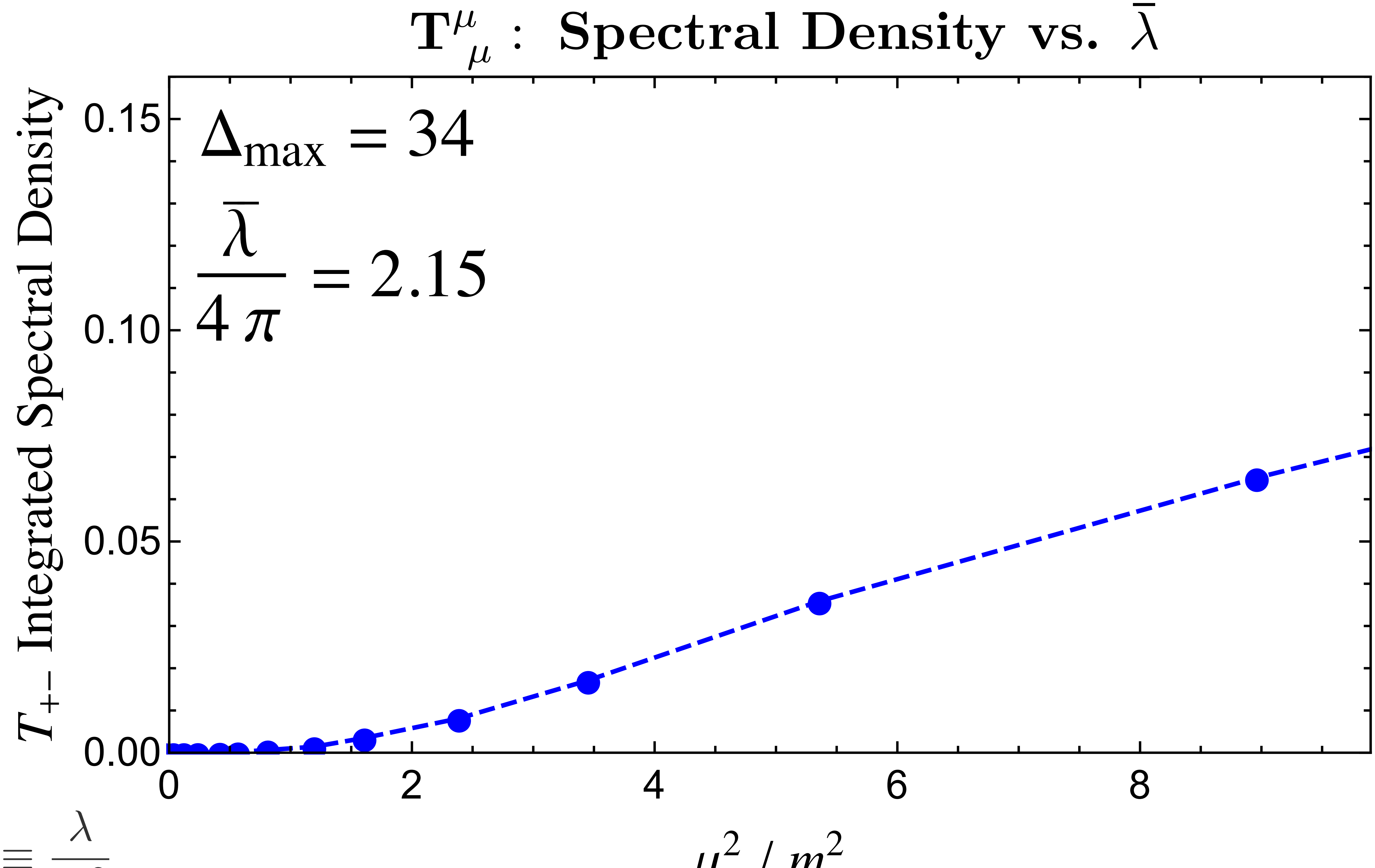

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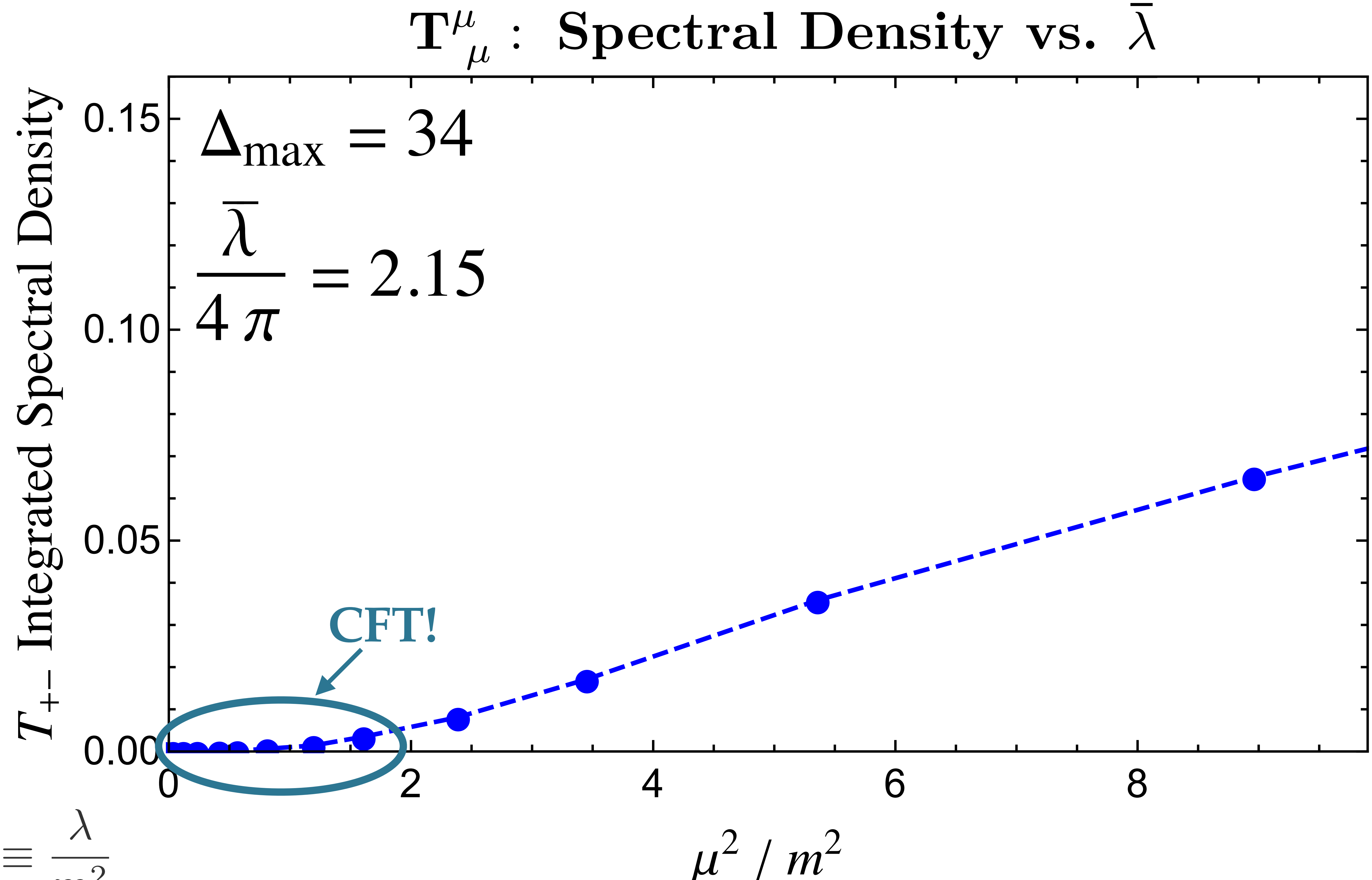

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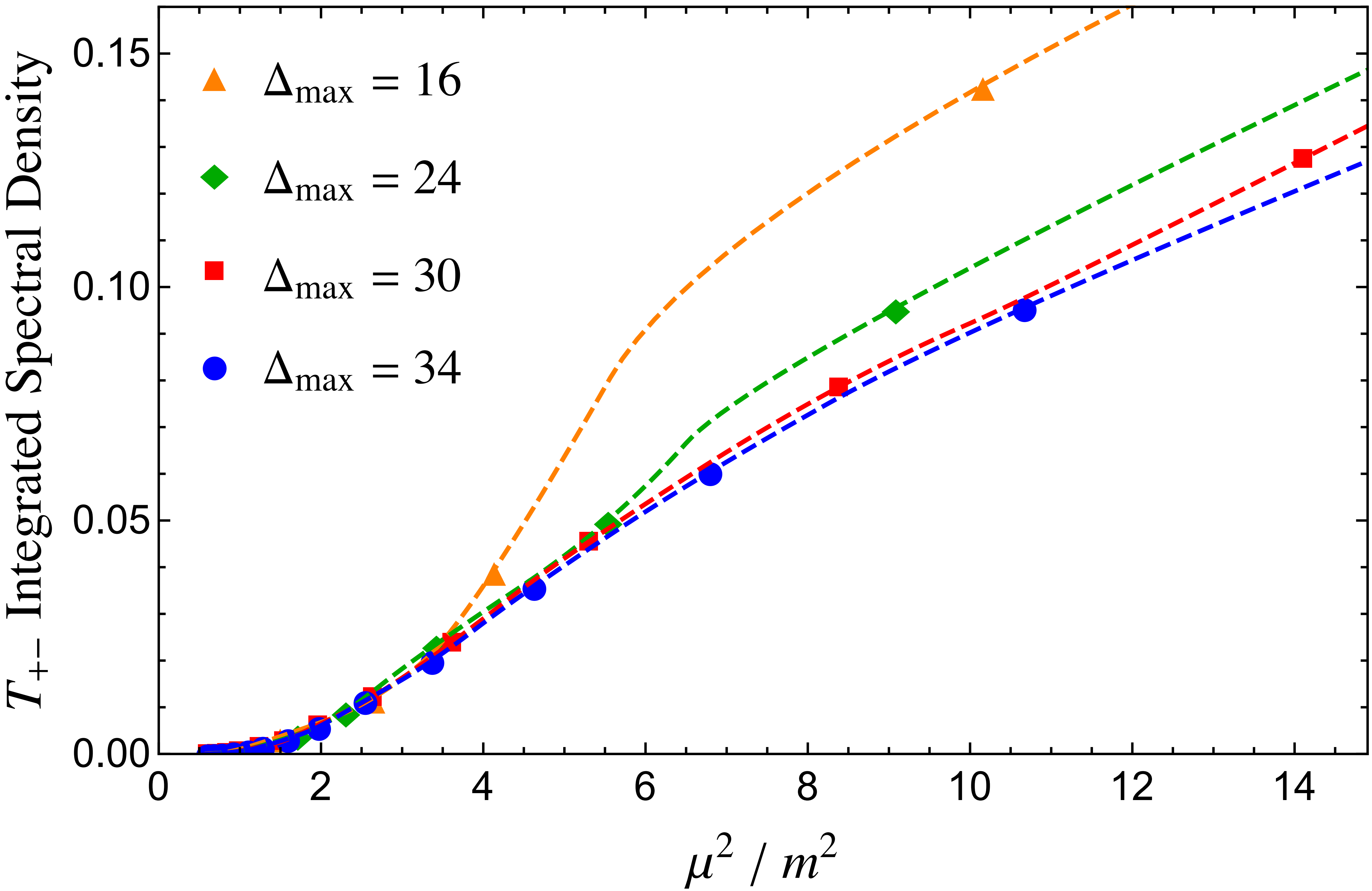
$\Delta_{\max}$ Convergence (@ fixed λ)



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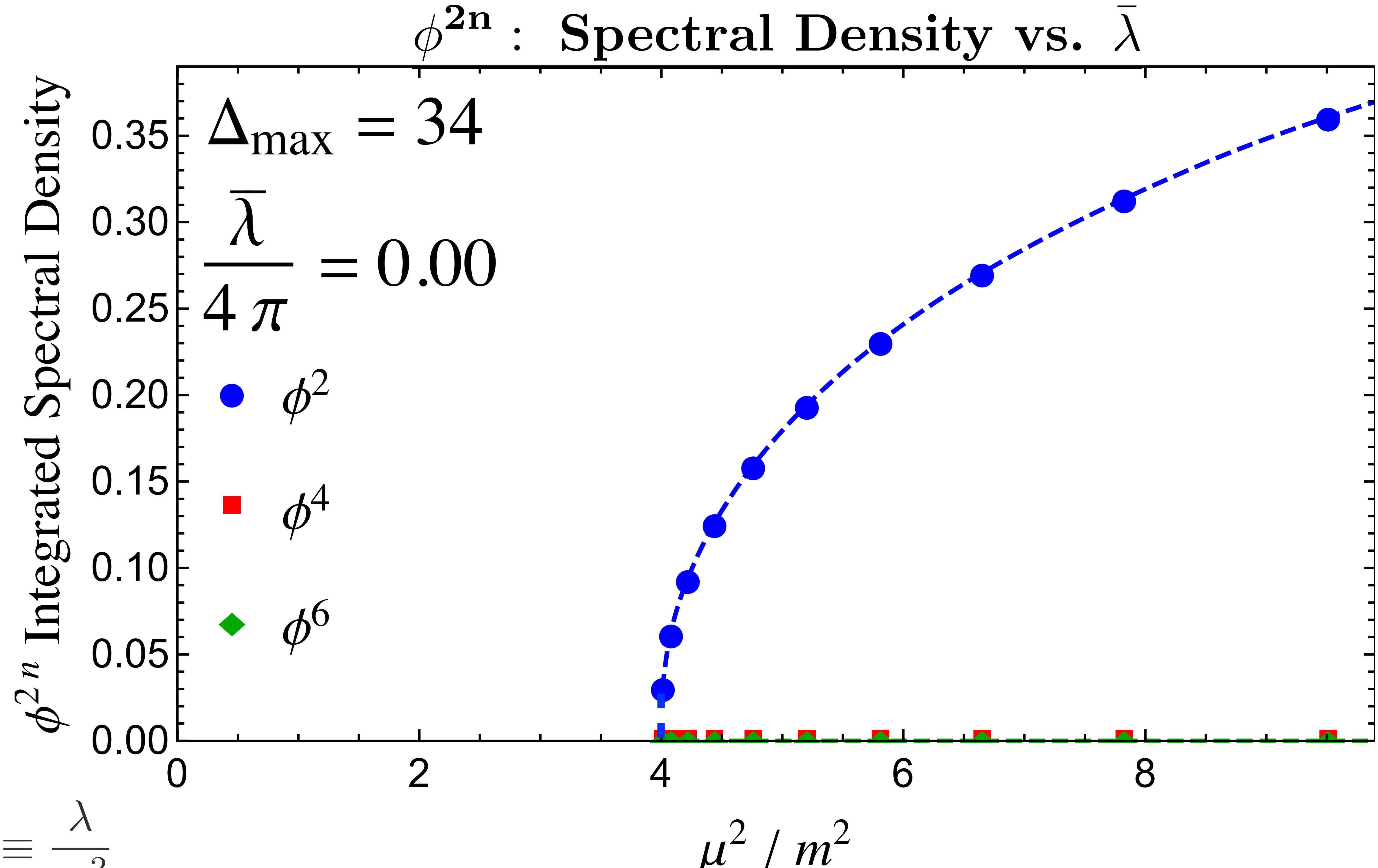

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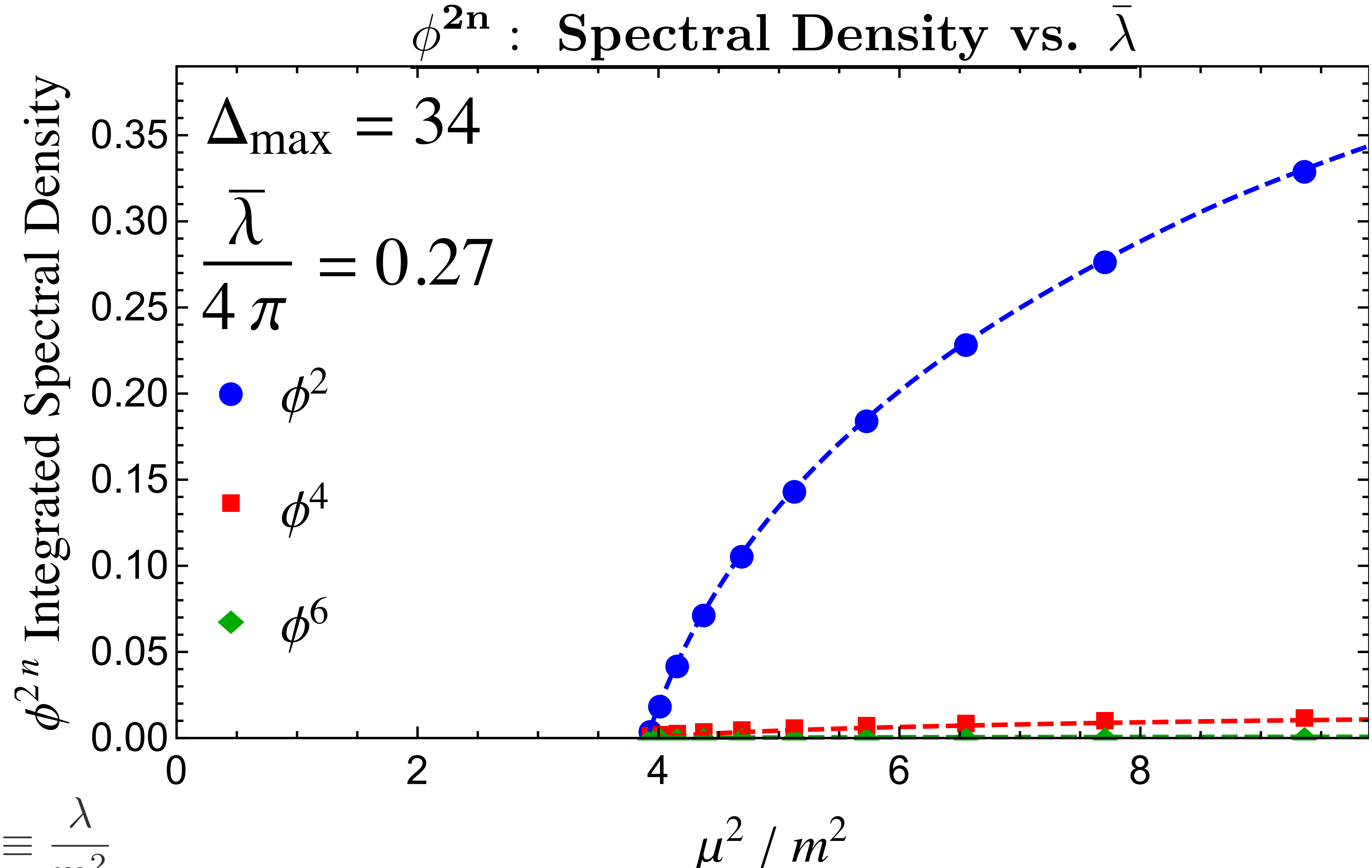

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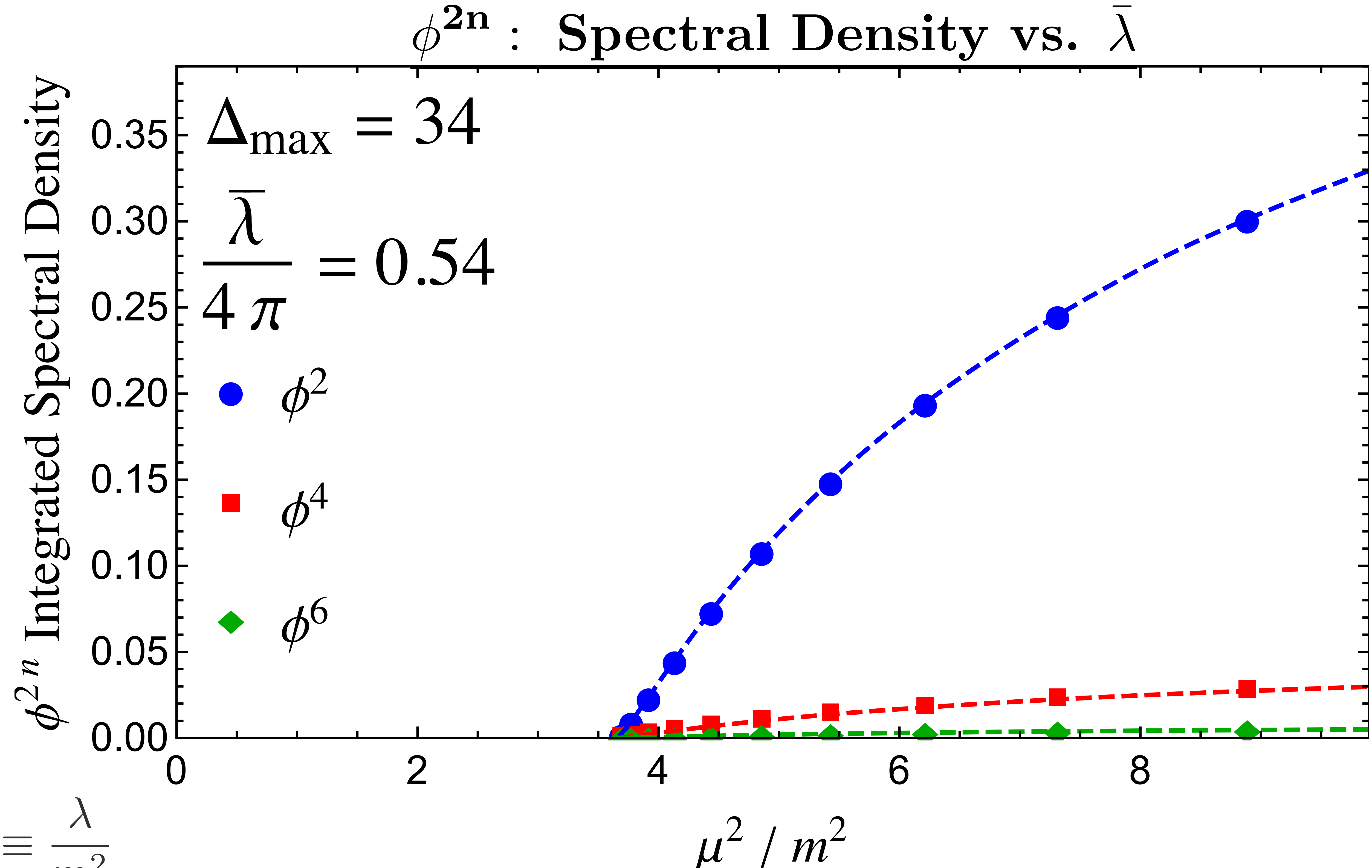

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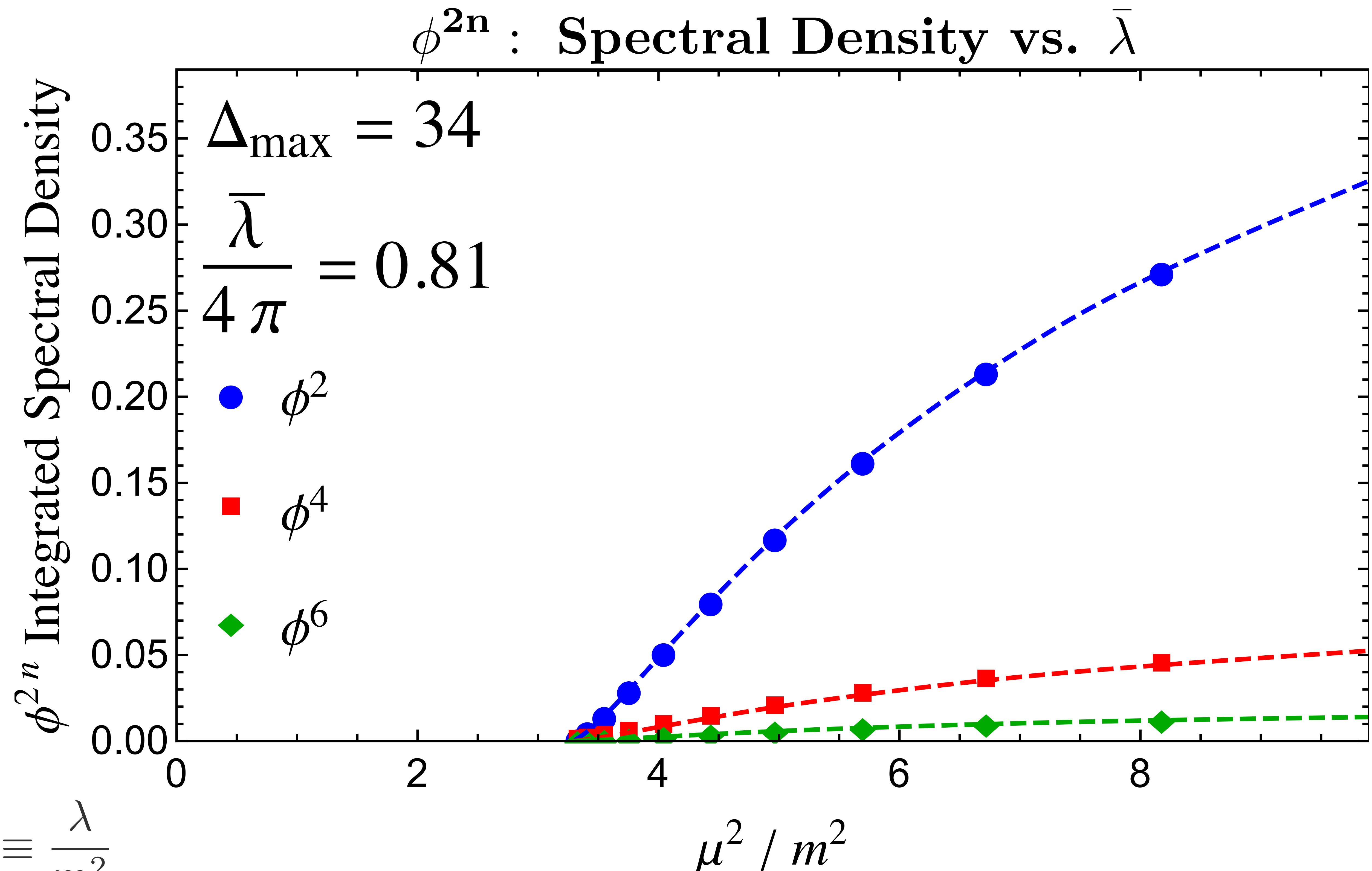

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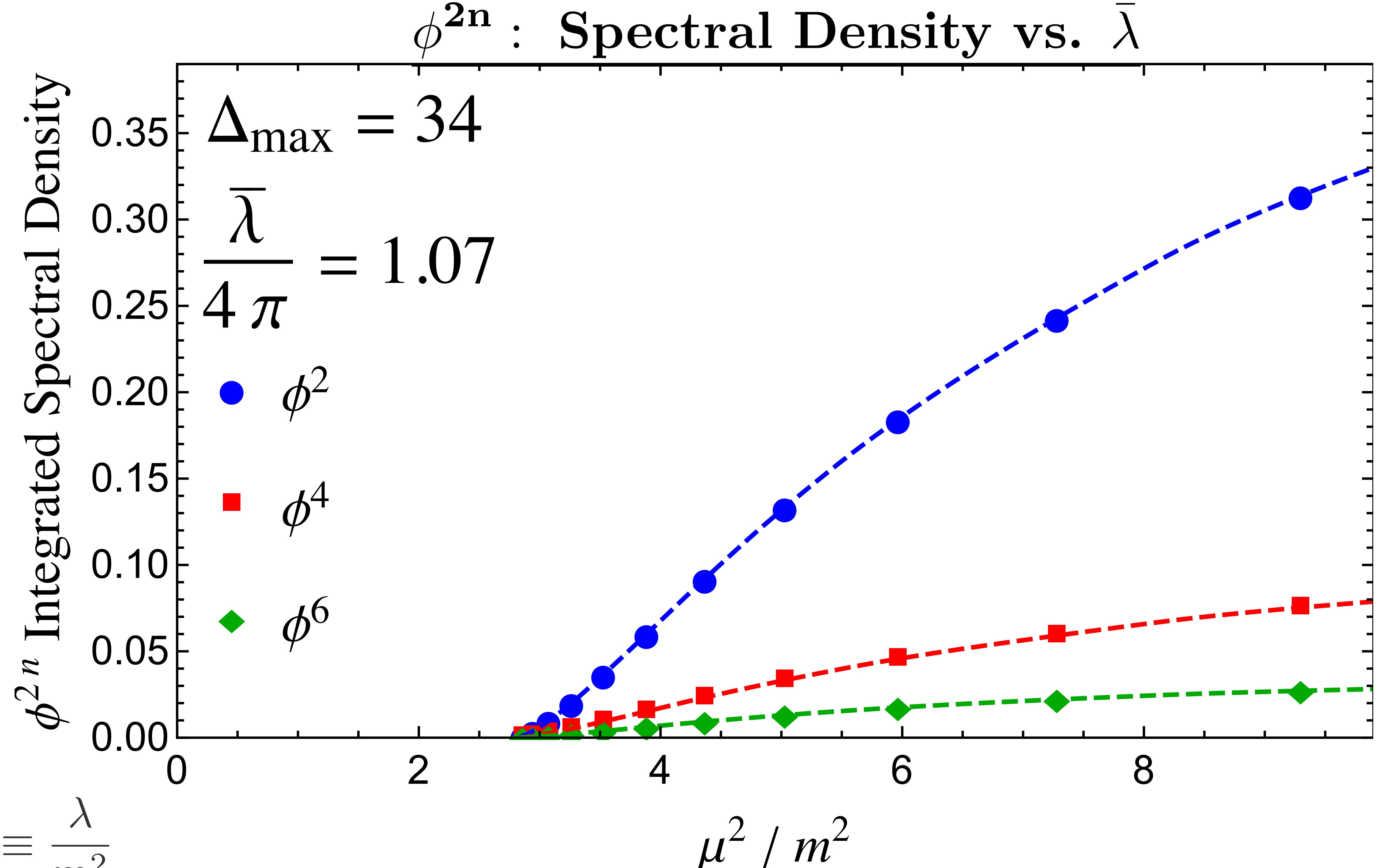

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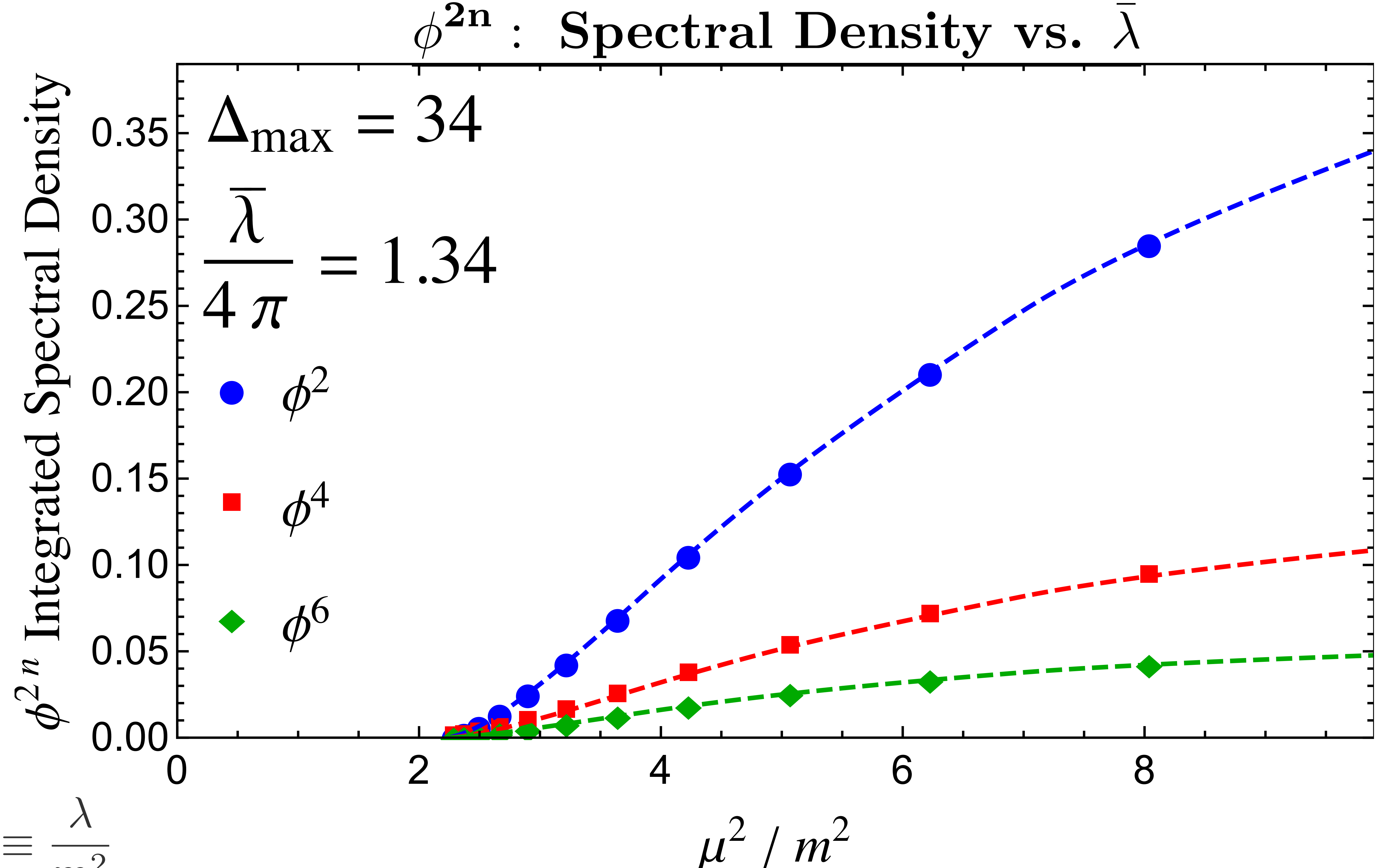

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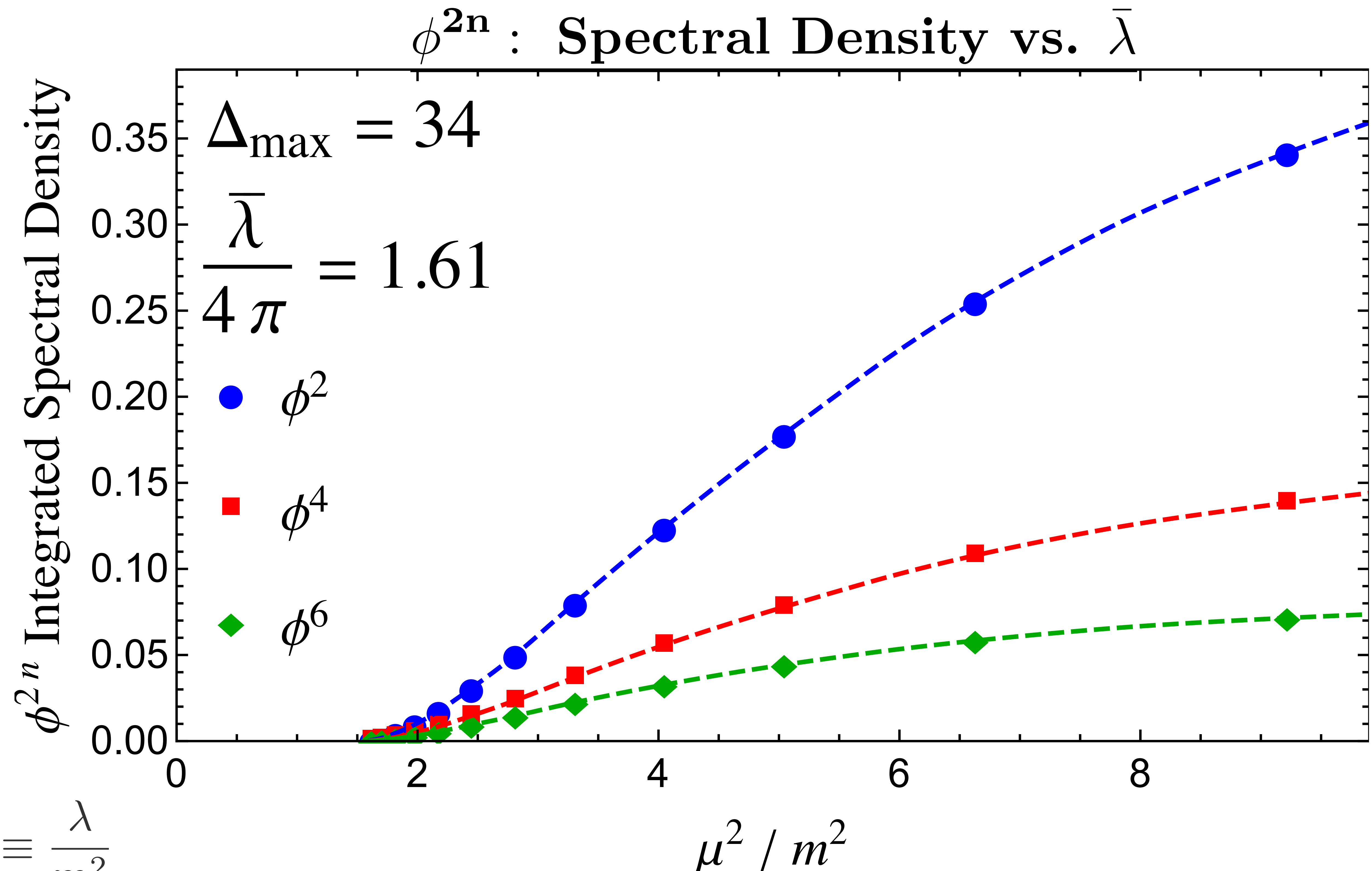

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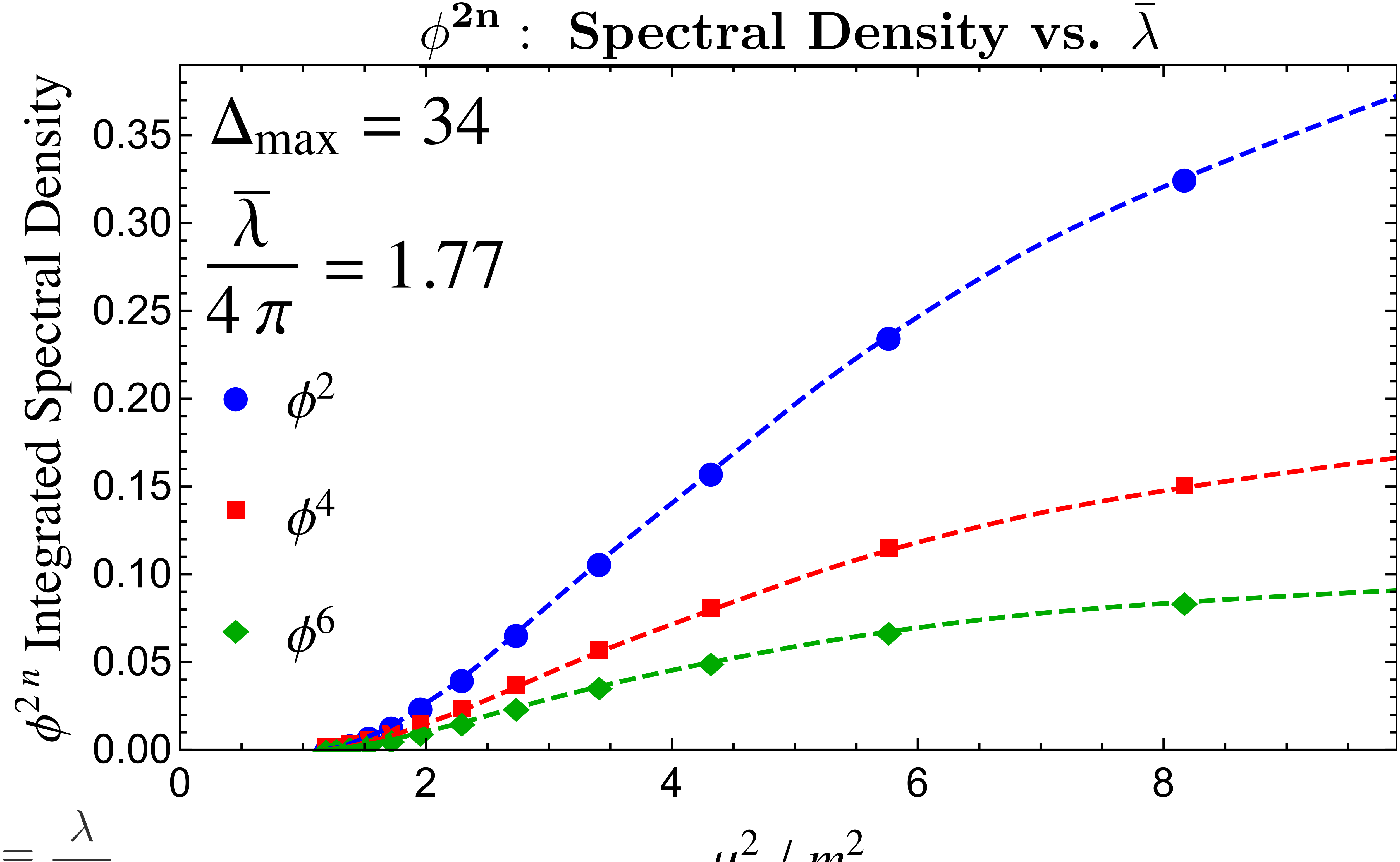

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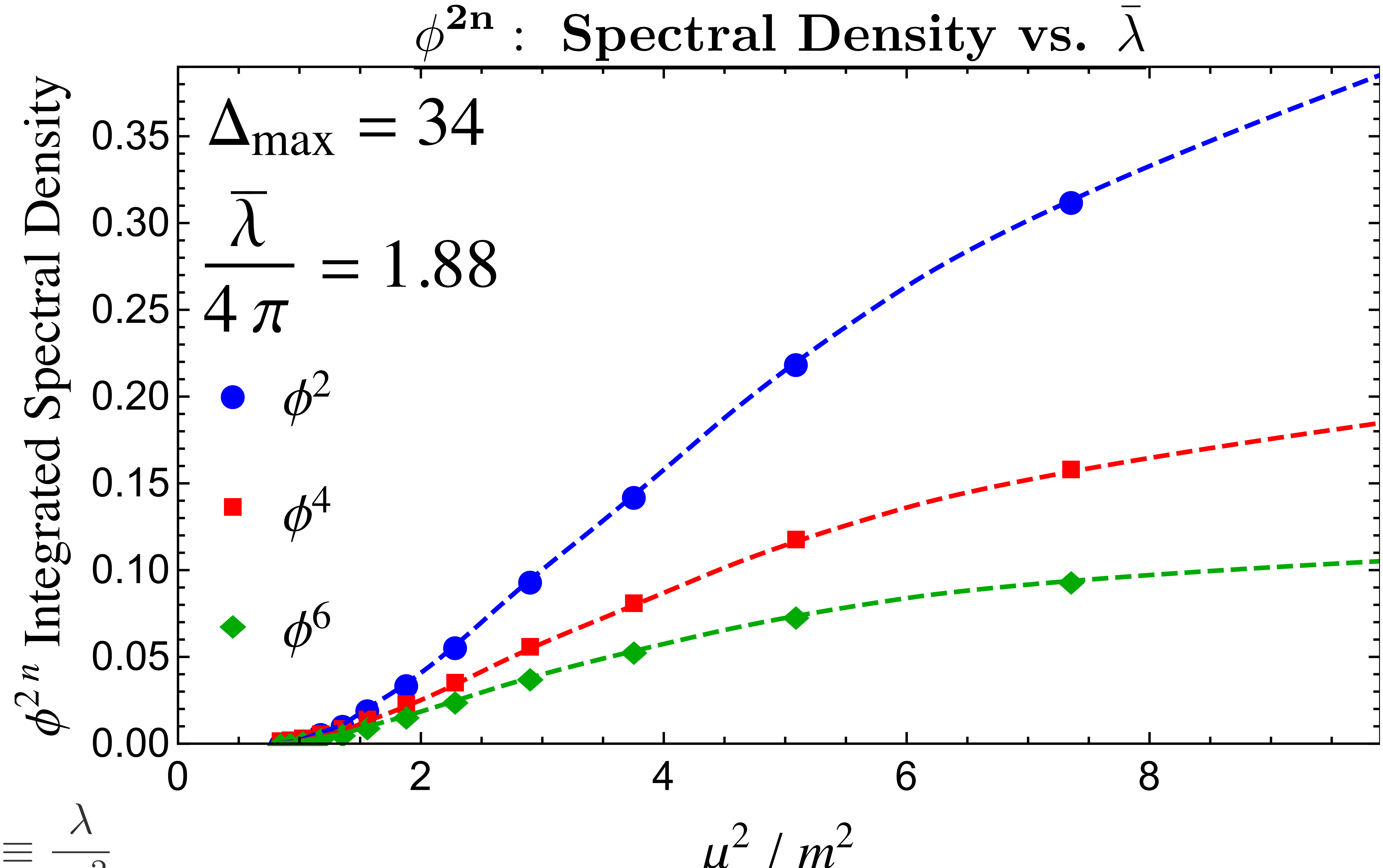

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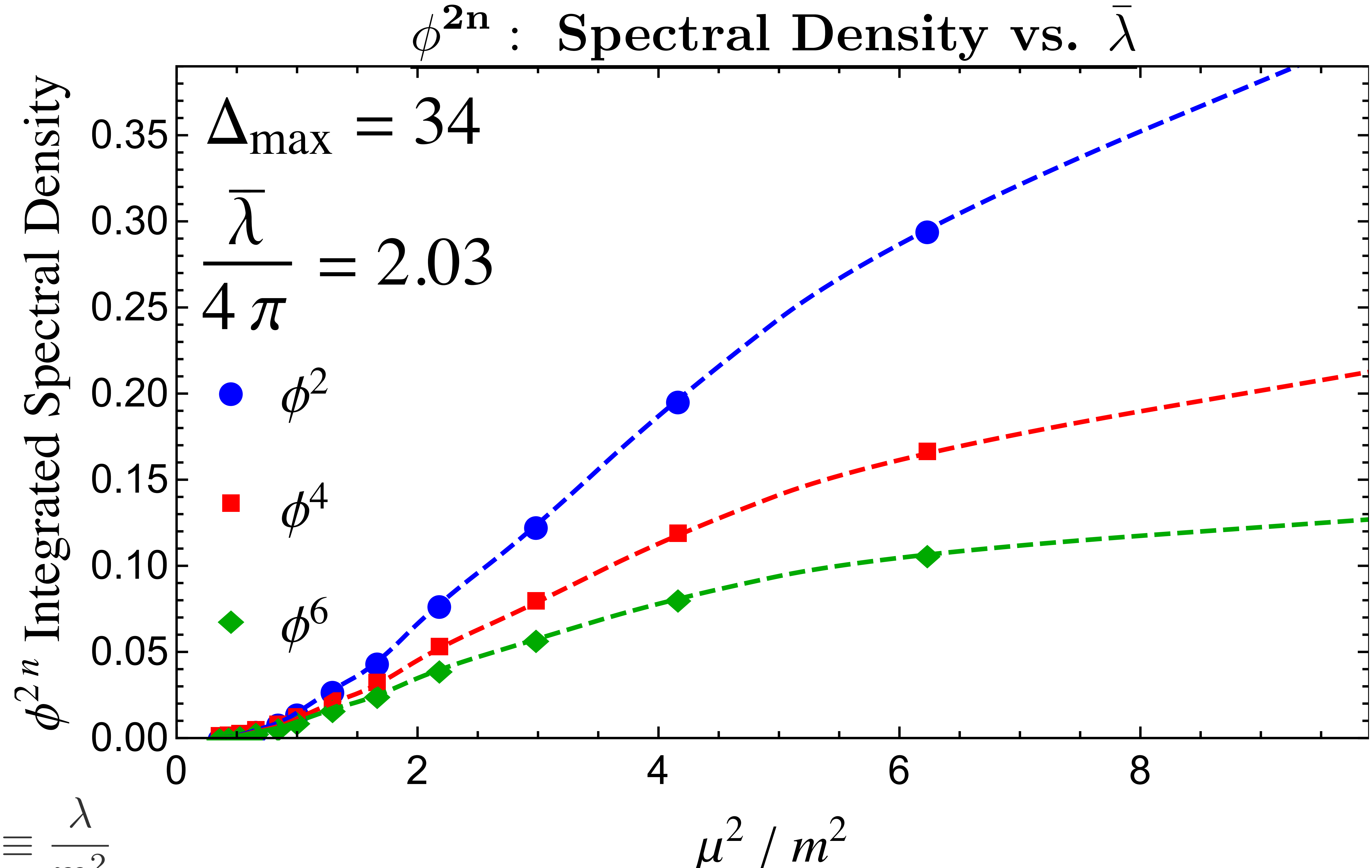
Example: $(1{+}1)\mathrm{d}\ \lambda\phi^4$-theory



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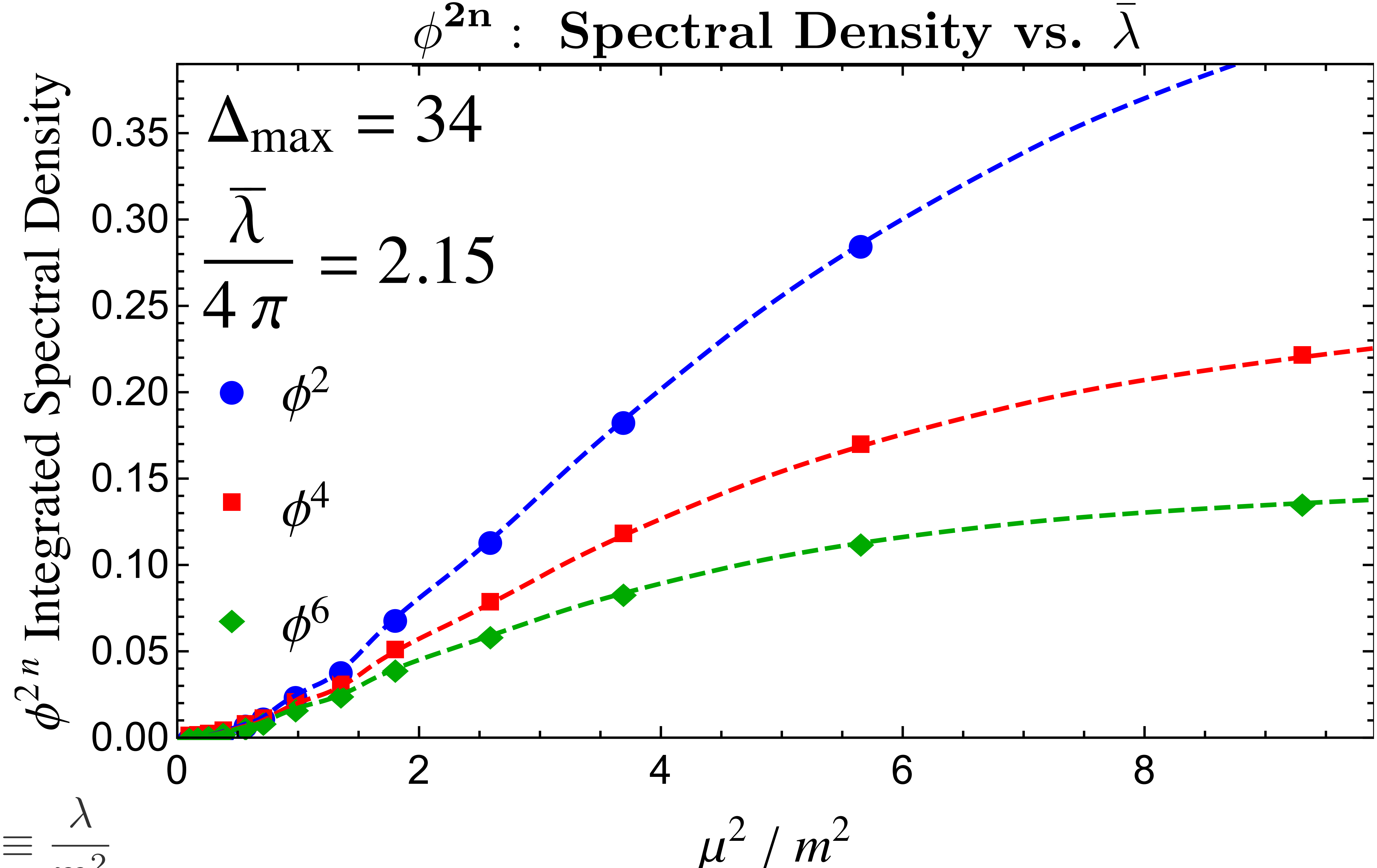
Example: (1+1)d $\lambda\phi^4$-theory



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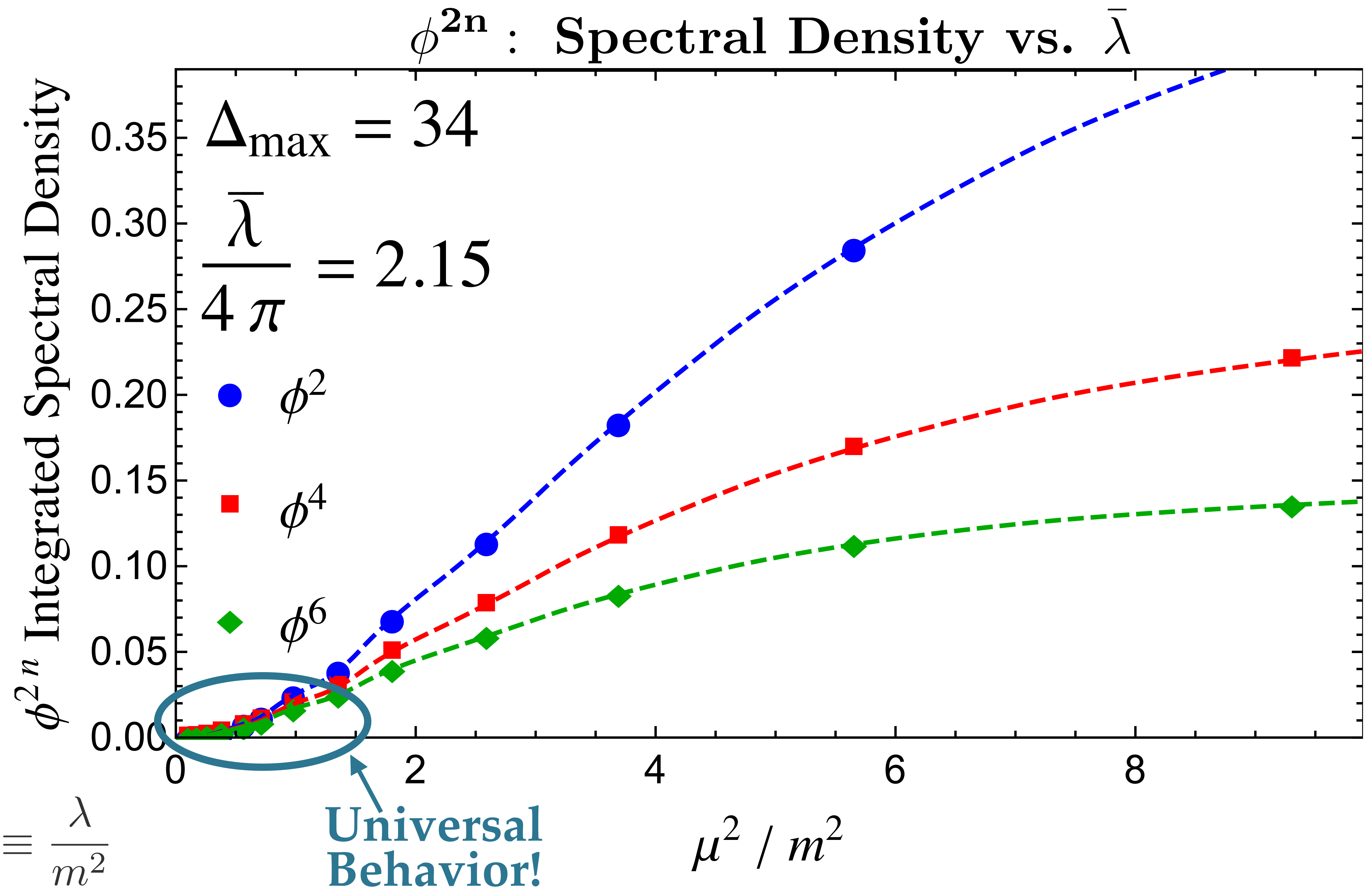

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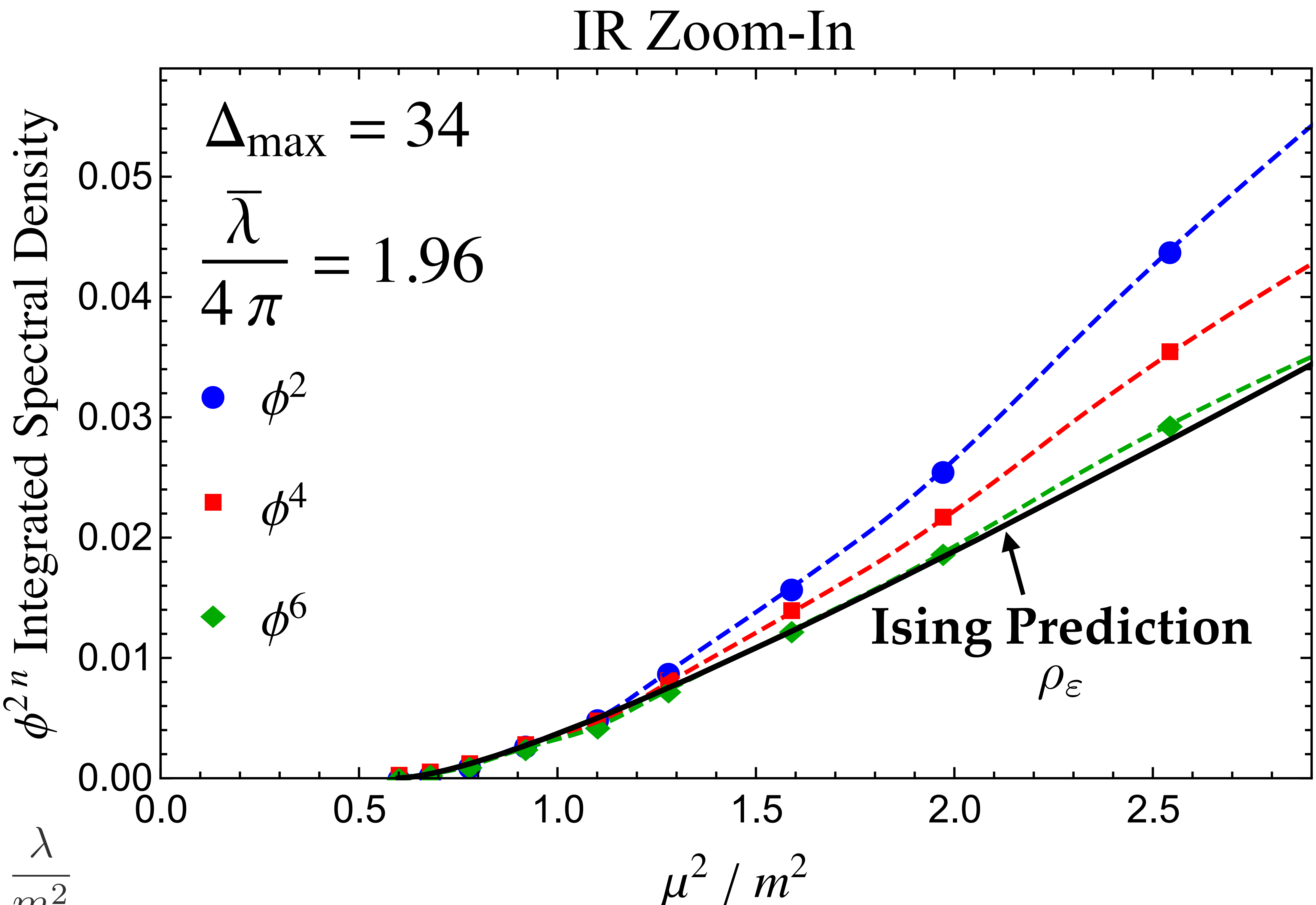

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Summary of Conformal Truncation

It's a Hamiltonian truncation method formulated directly in real time and infinite volume, allowing access to nonperturbative dynamics.

Input is CFT data. Output is QFT dynamics.

Tries to harness small parameter: $\frac{1}{(\Delta_{\rm max})^{\#}}$