A SUSY RG-Flow
Using Hamiltonian Methods

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True for: Yukawa in 3D or fermion mass in 4D

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There's no dependence of observables on the volume

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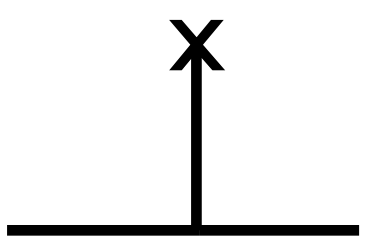

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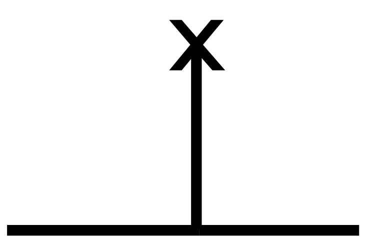

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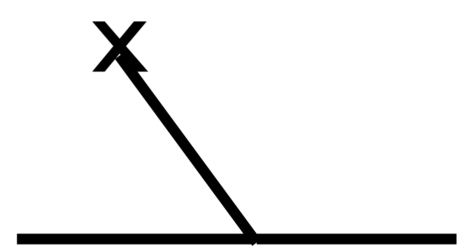

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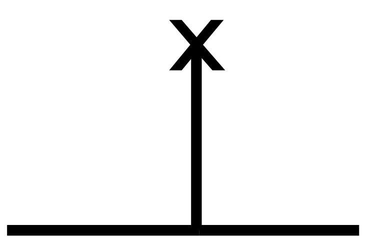

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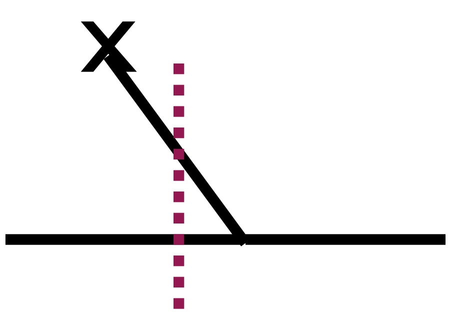

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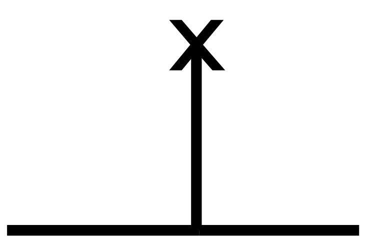

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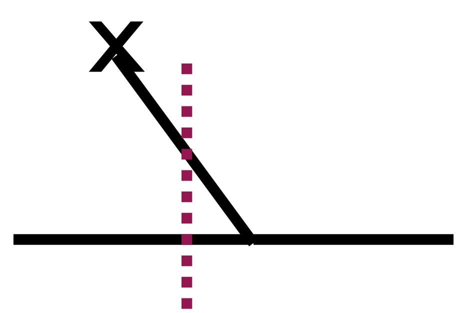

LC lacks appropriate intermediate states!

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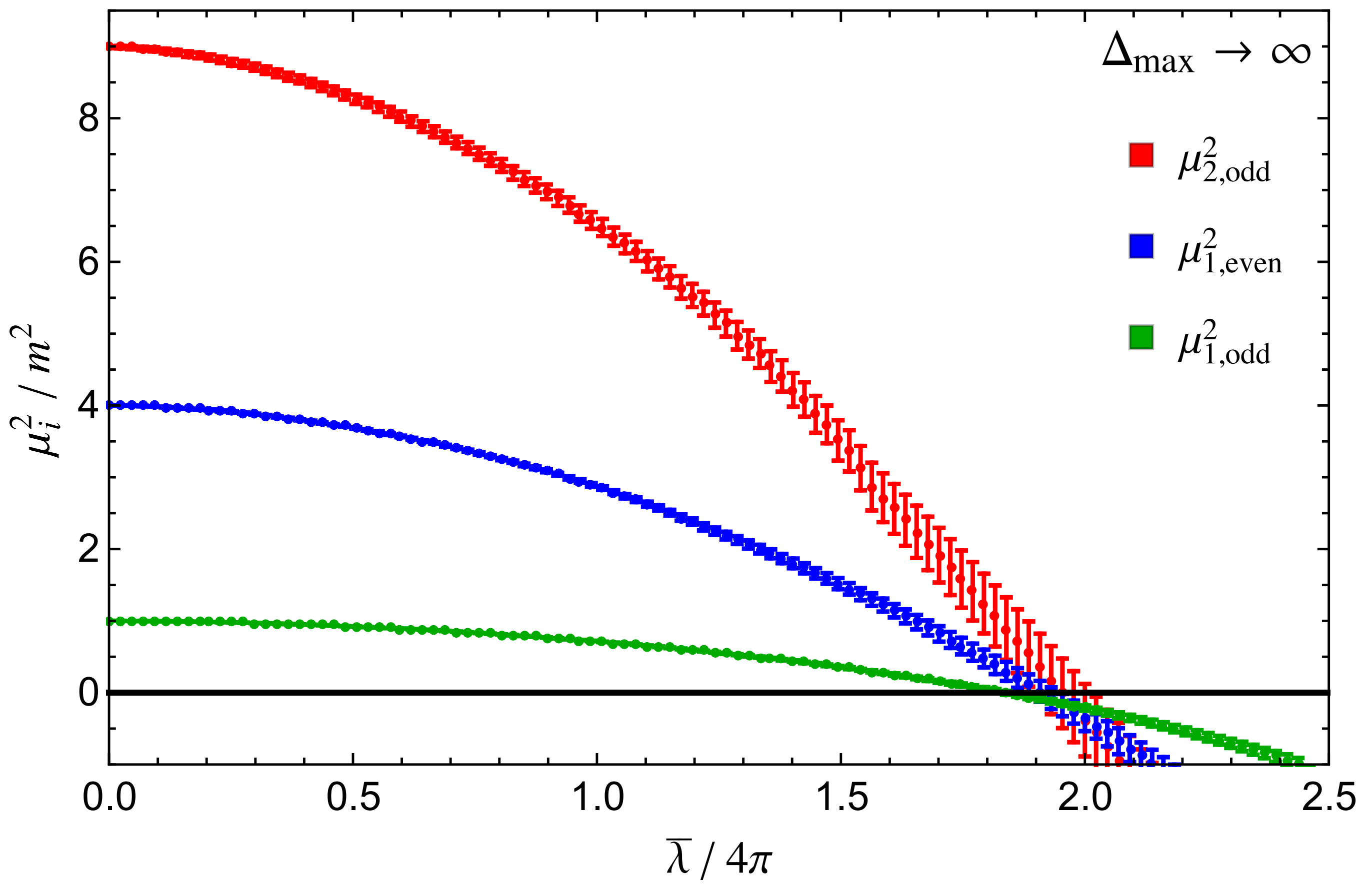

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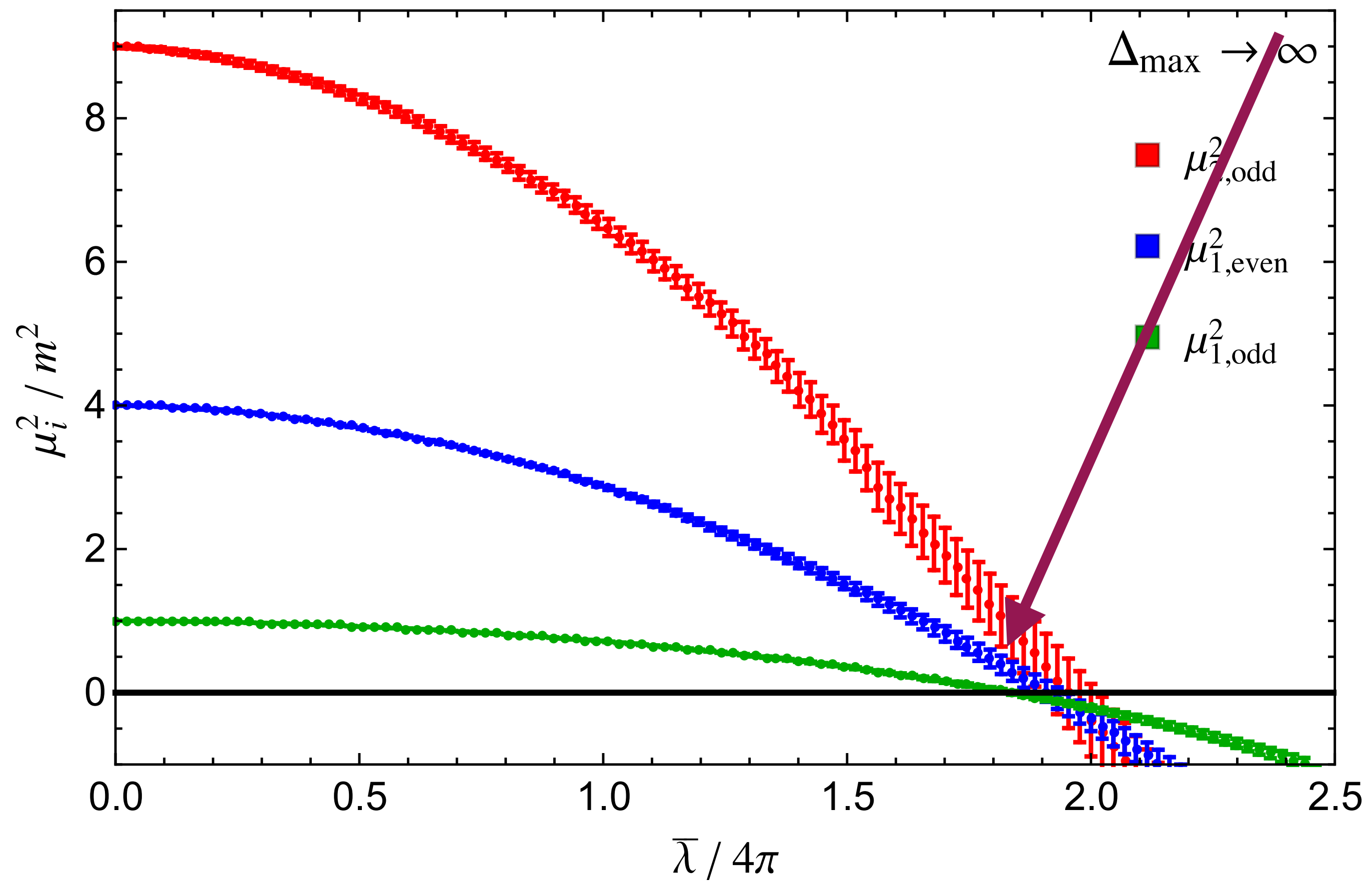

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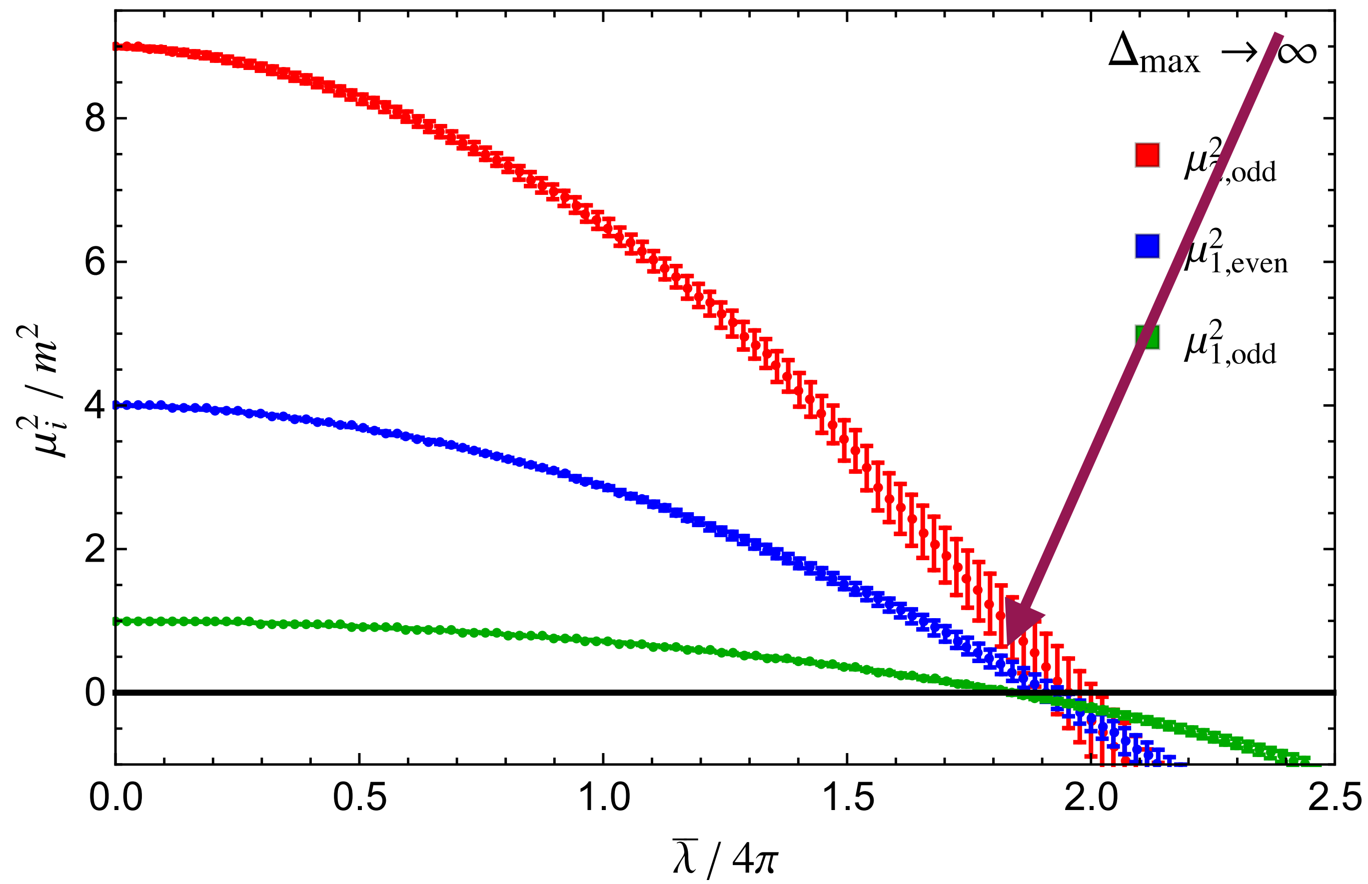

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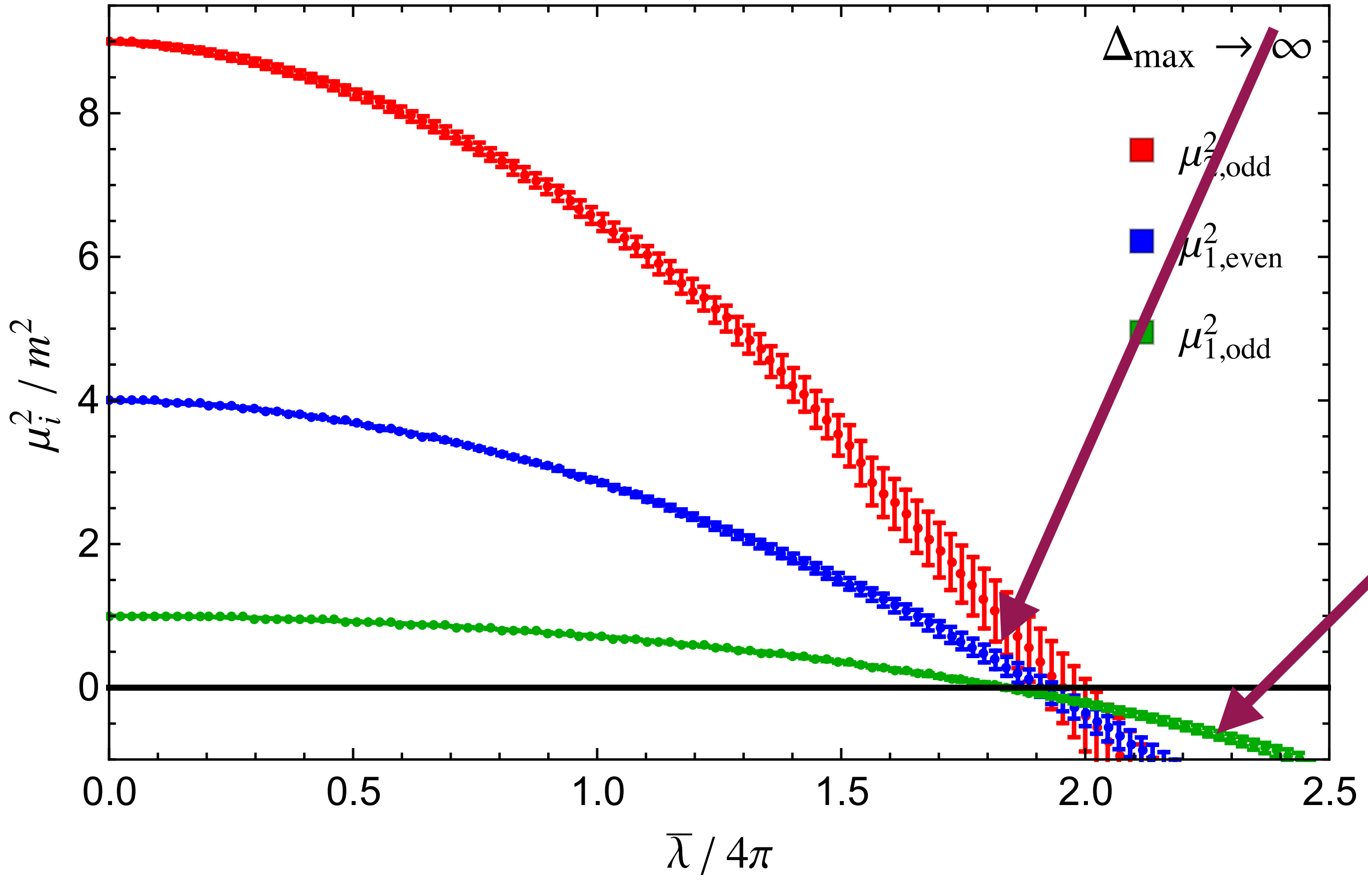
NEC violation!



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I.e. "Zero-Modes" have large energies and need to be integrated out properly!

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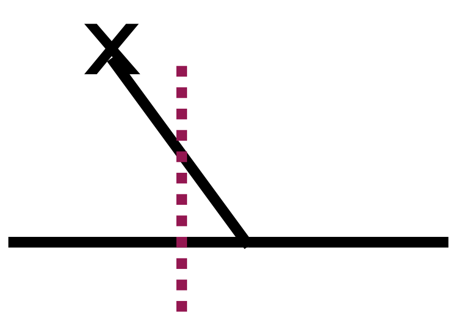

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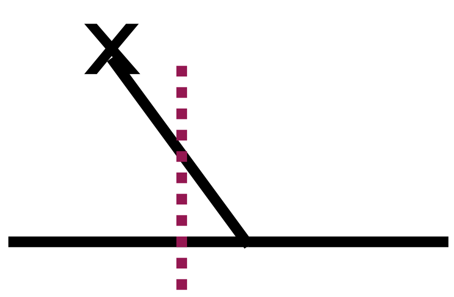

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Non-Analytic in q0

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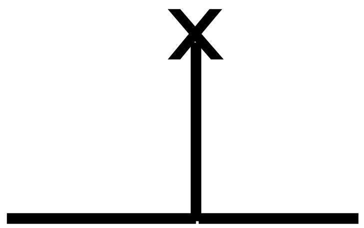
$$H_{eff} \supset$$ 

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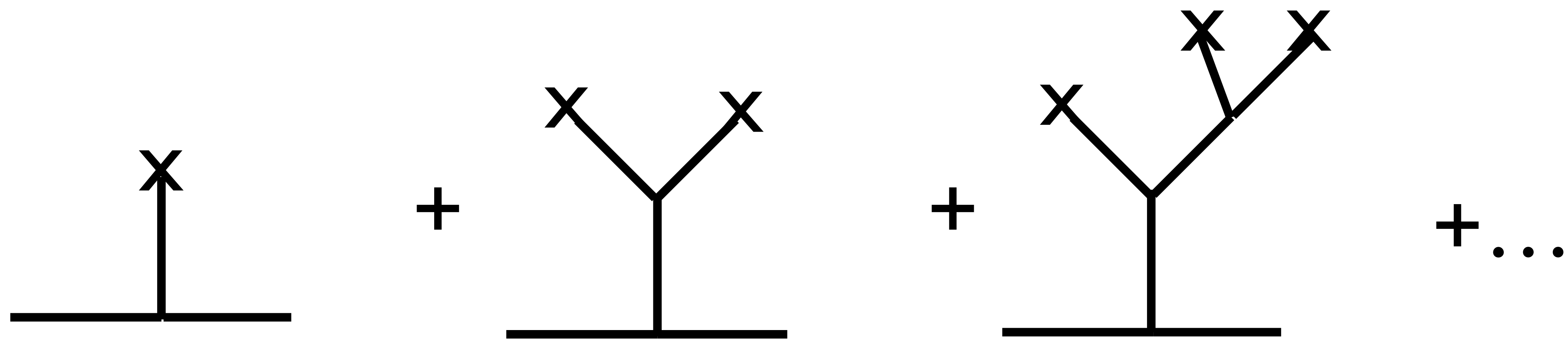

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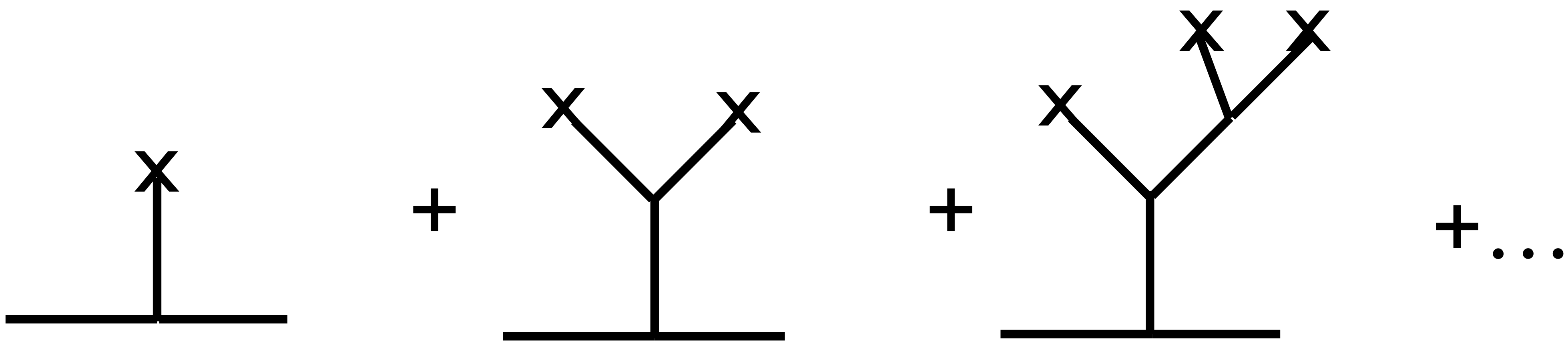

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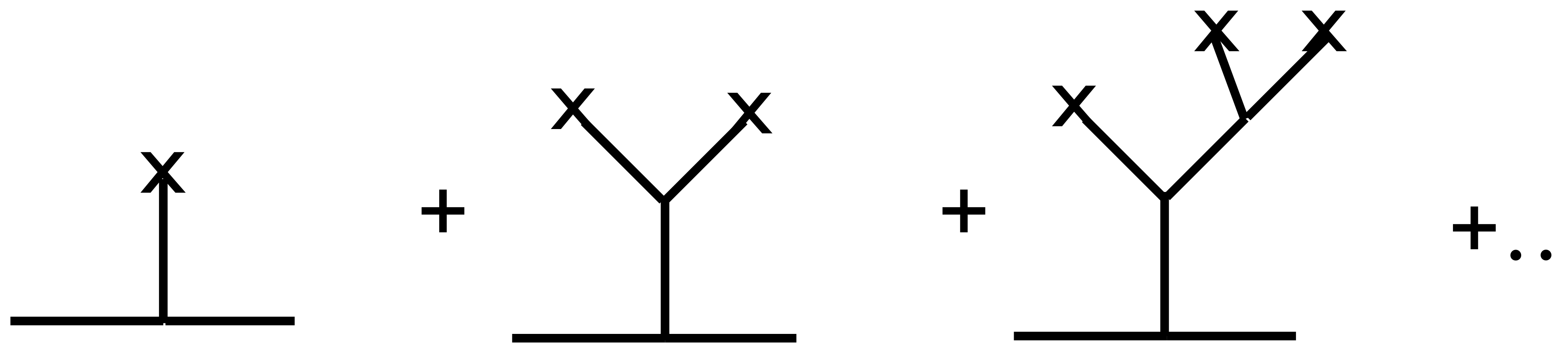

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Prescription reproduces missing vevs!

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Only possible if some 3pt-functions vanish on the LC:

$$\Delta' = \Delta + \Delta_R + 2n$$

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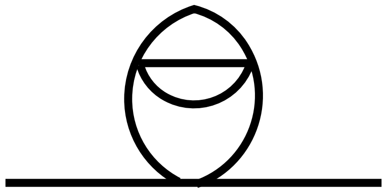
$H_{eff} \supset$  + ...

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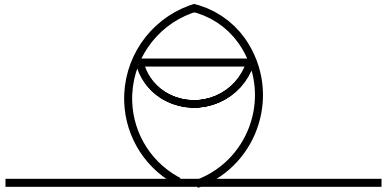
$H_{eff} \supset$  + …

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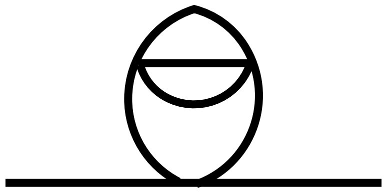
$H_{eff} \supset$  **+ ...**

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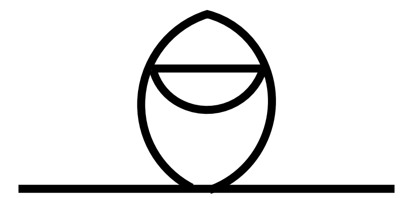
$H_{eff} \supset$  + ...

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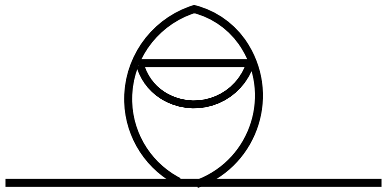
$H_{eff} \supset$  + ...

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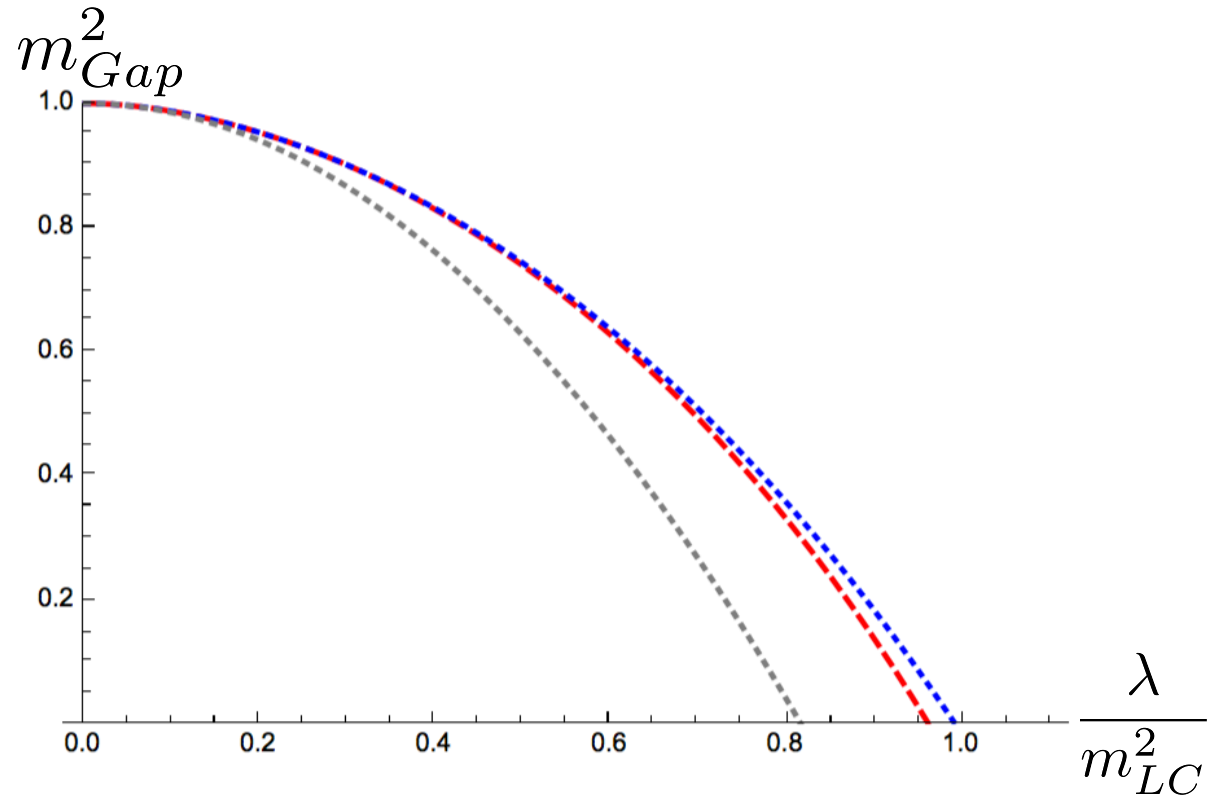

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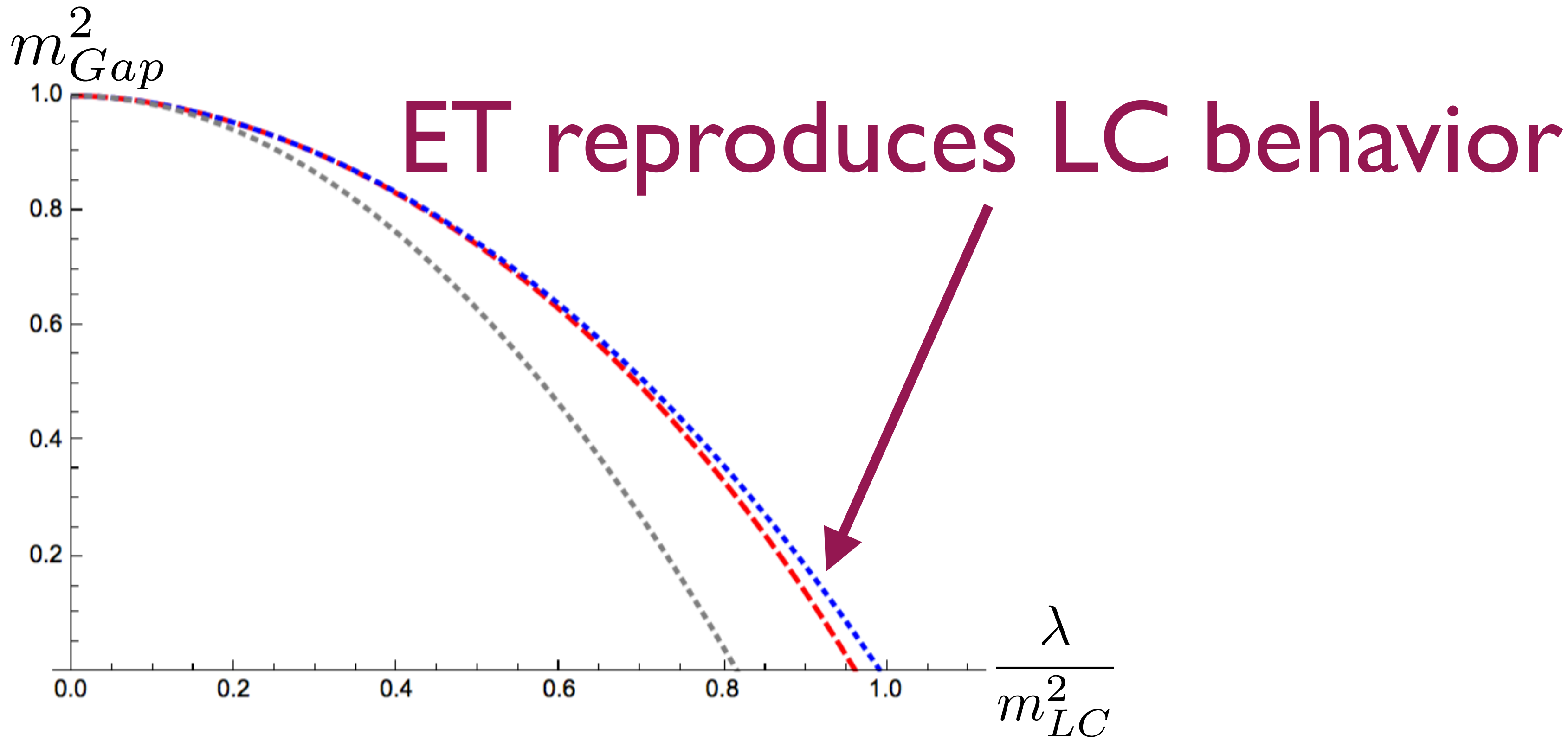

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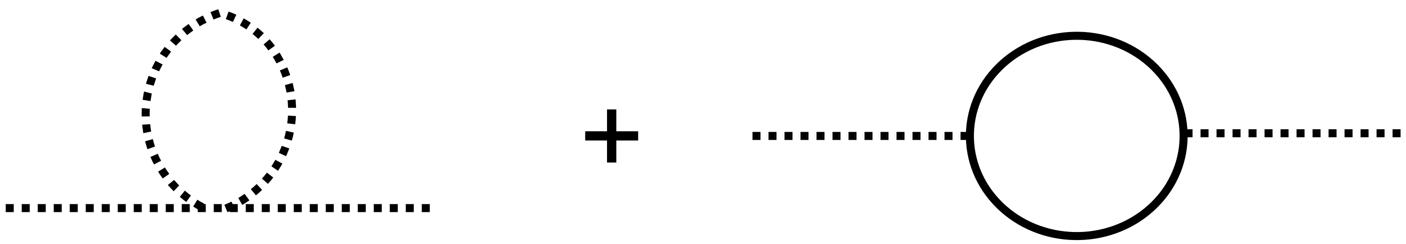

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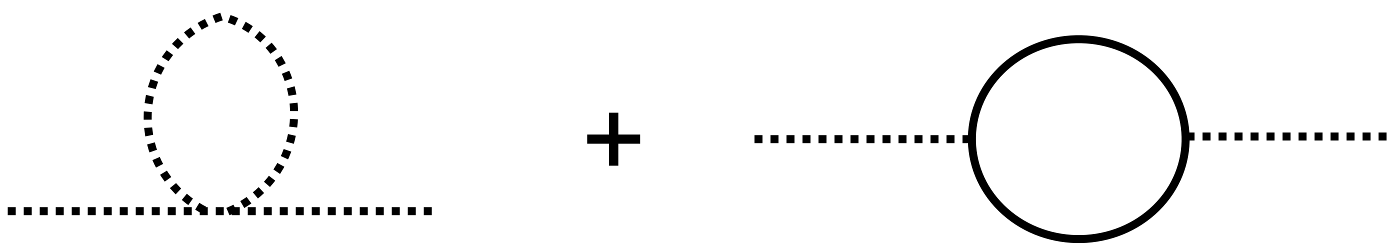

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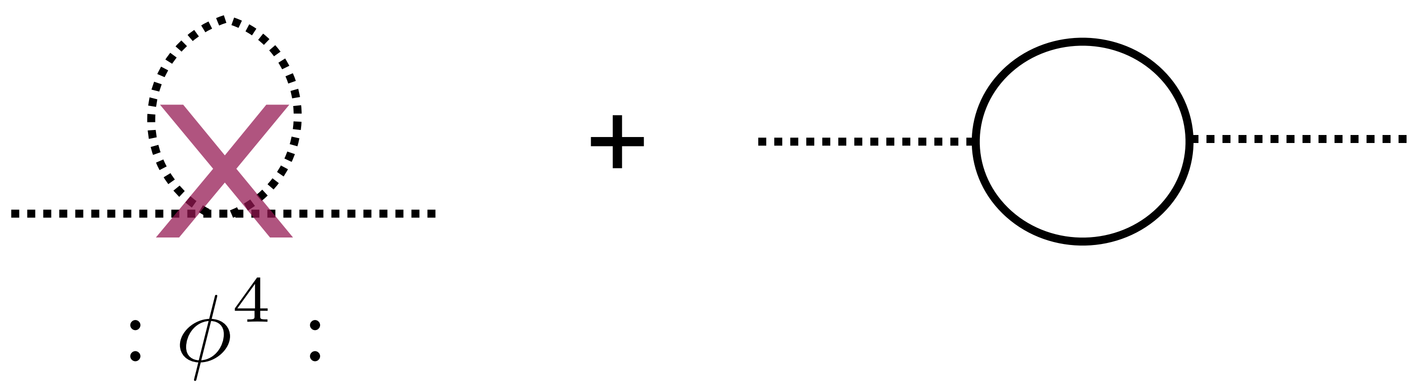

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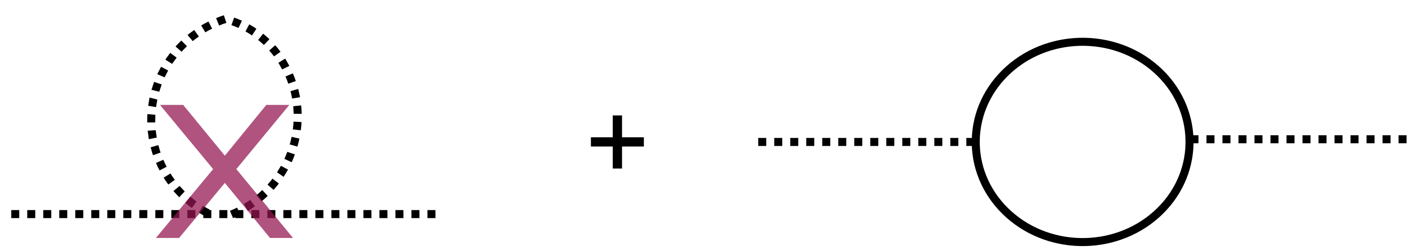

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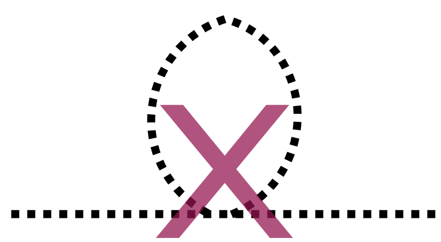

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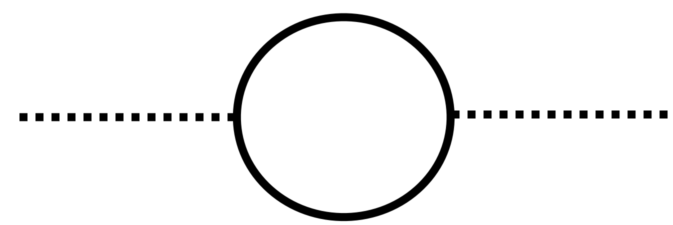

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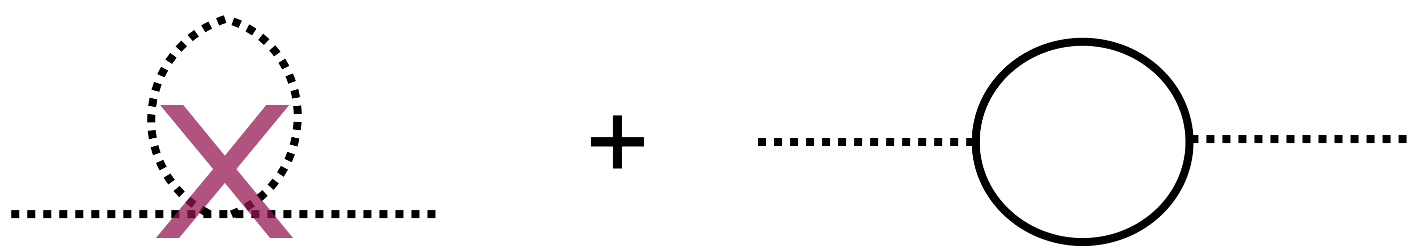

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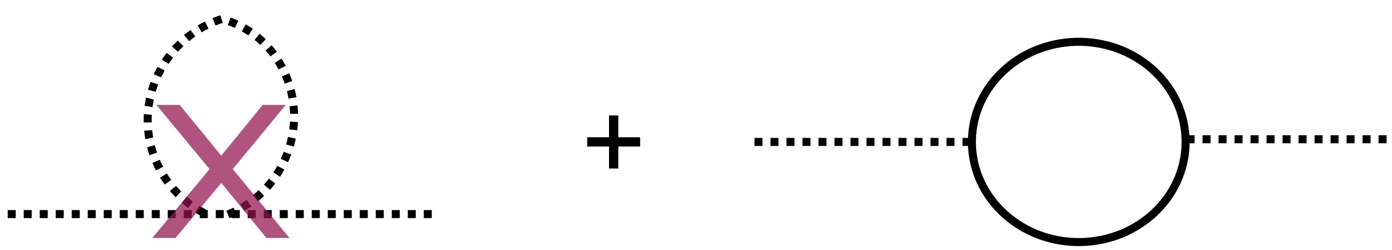

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$P_+ = Q_+^2 > 0$ (NEC)

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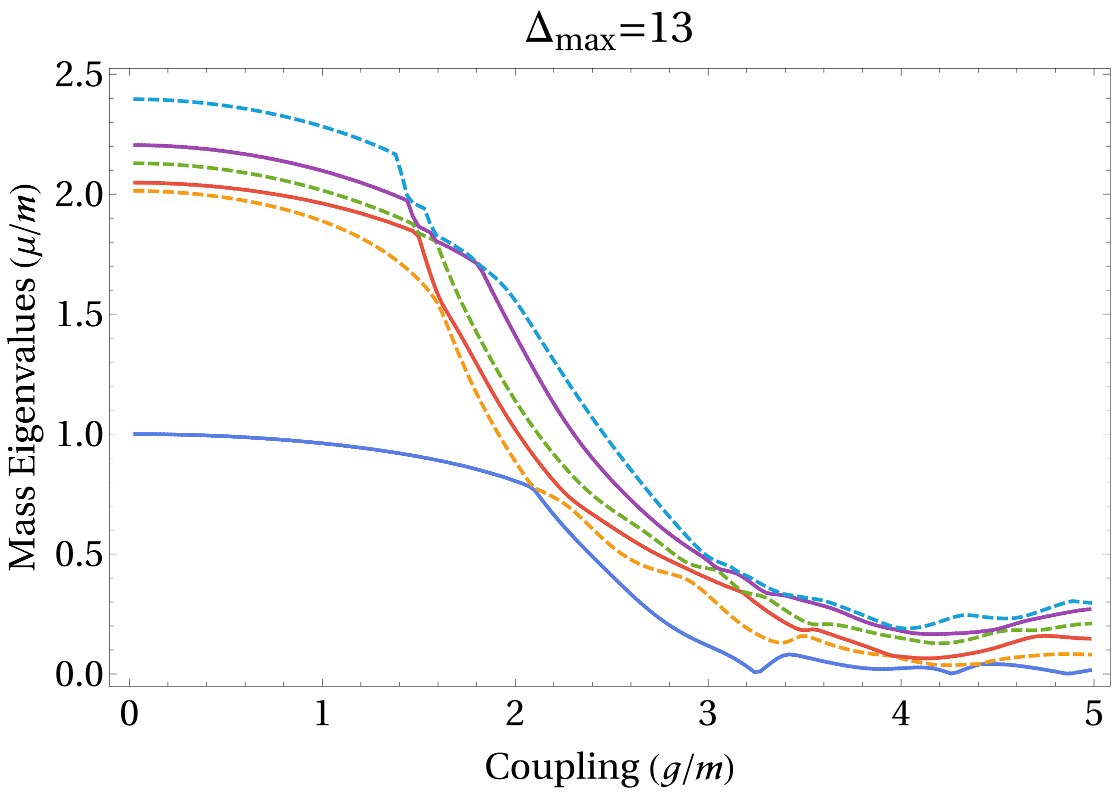

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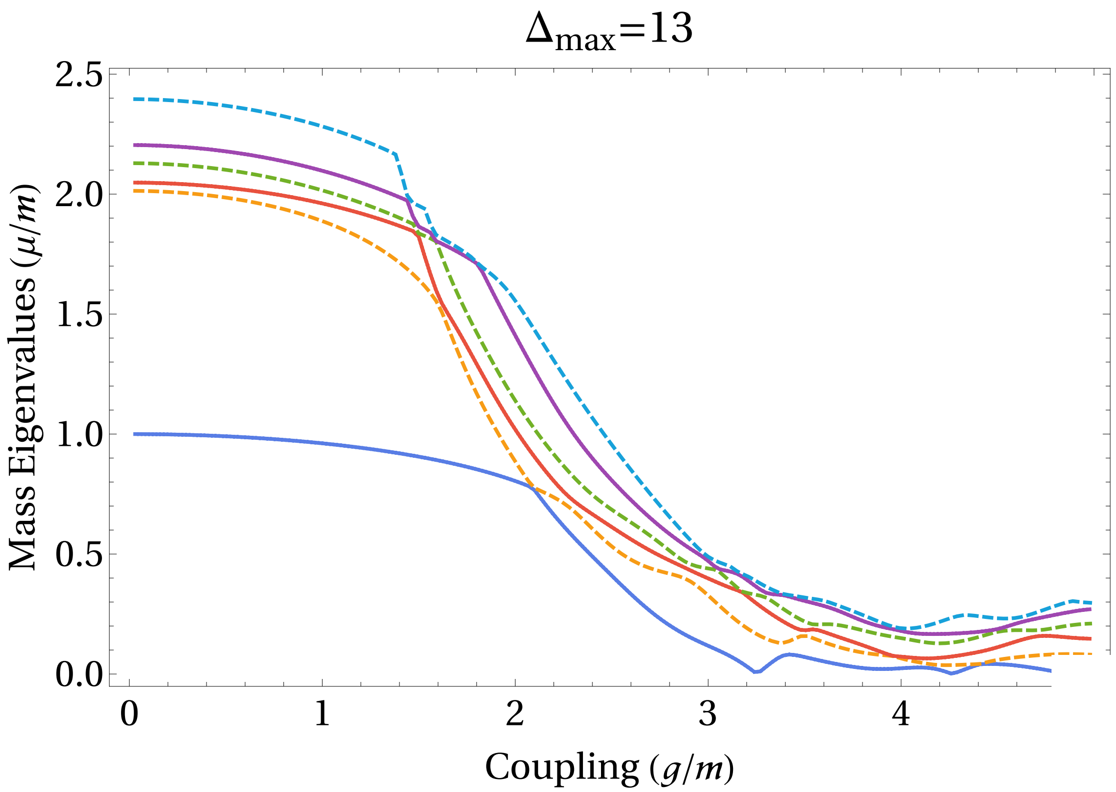

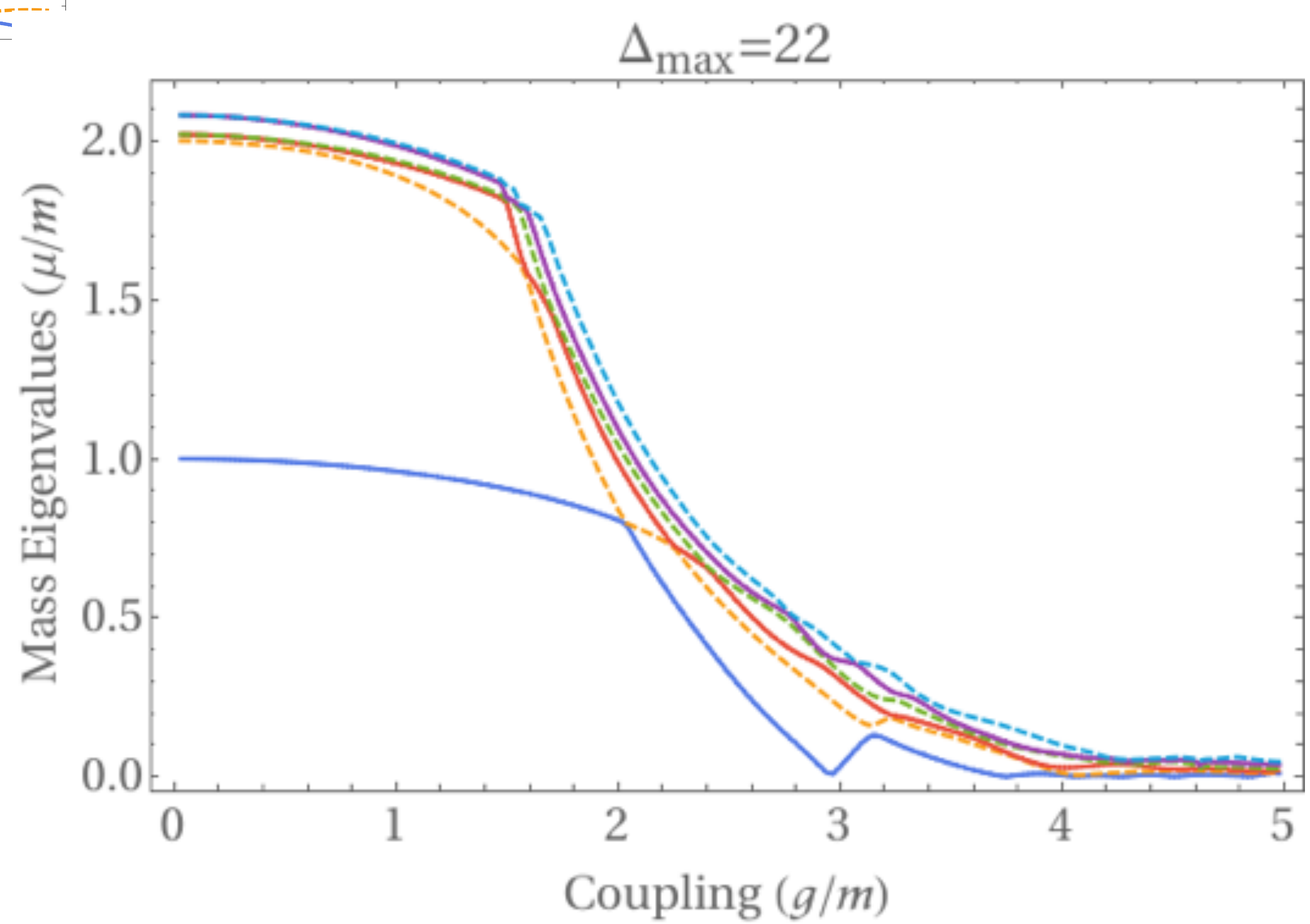

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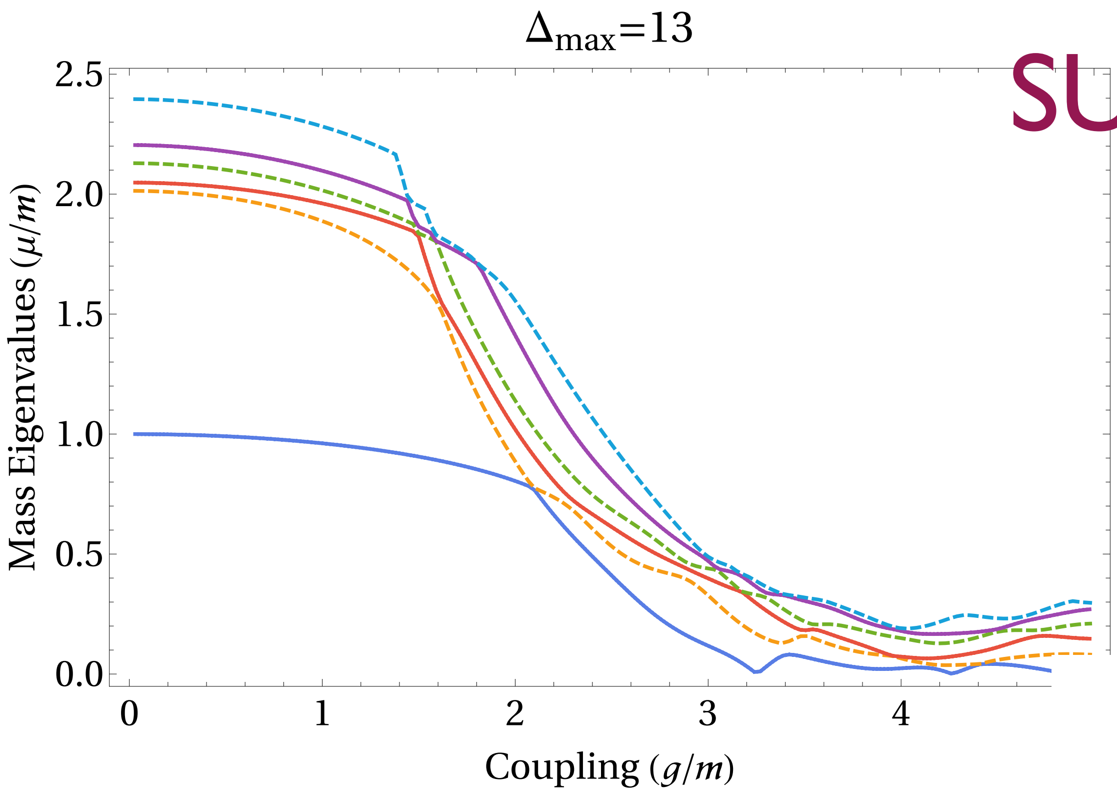

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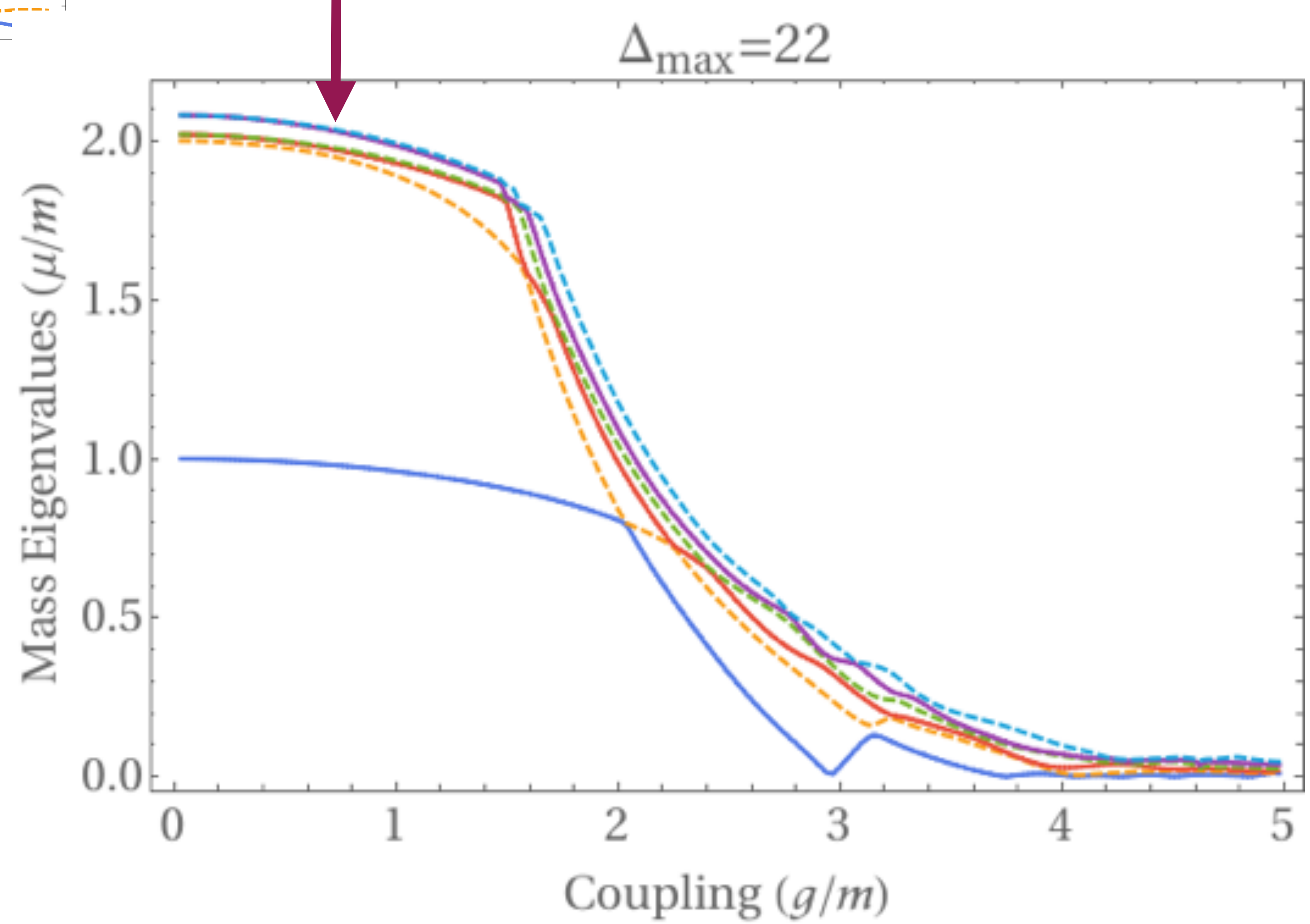

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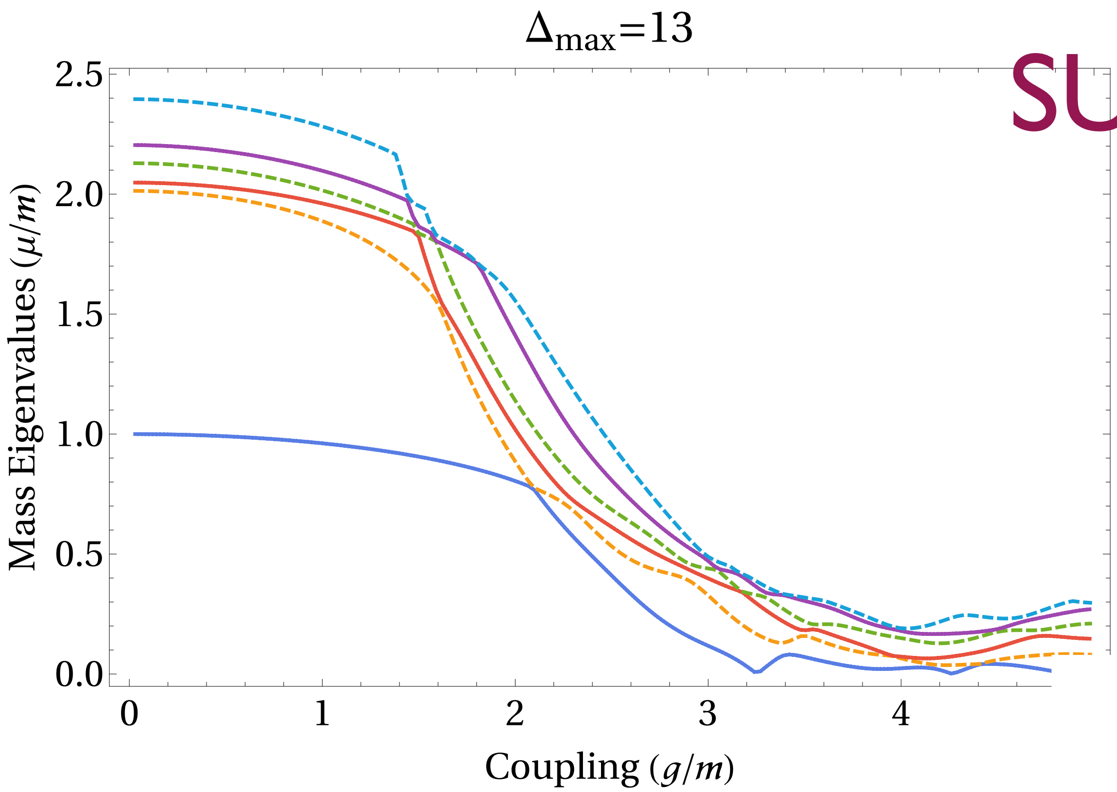

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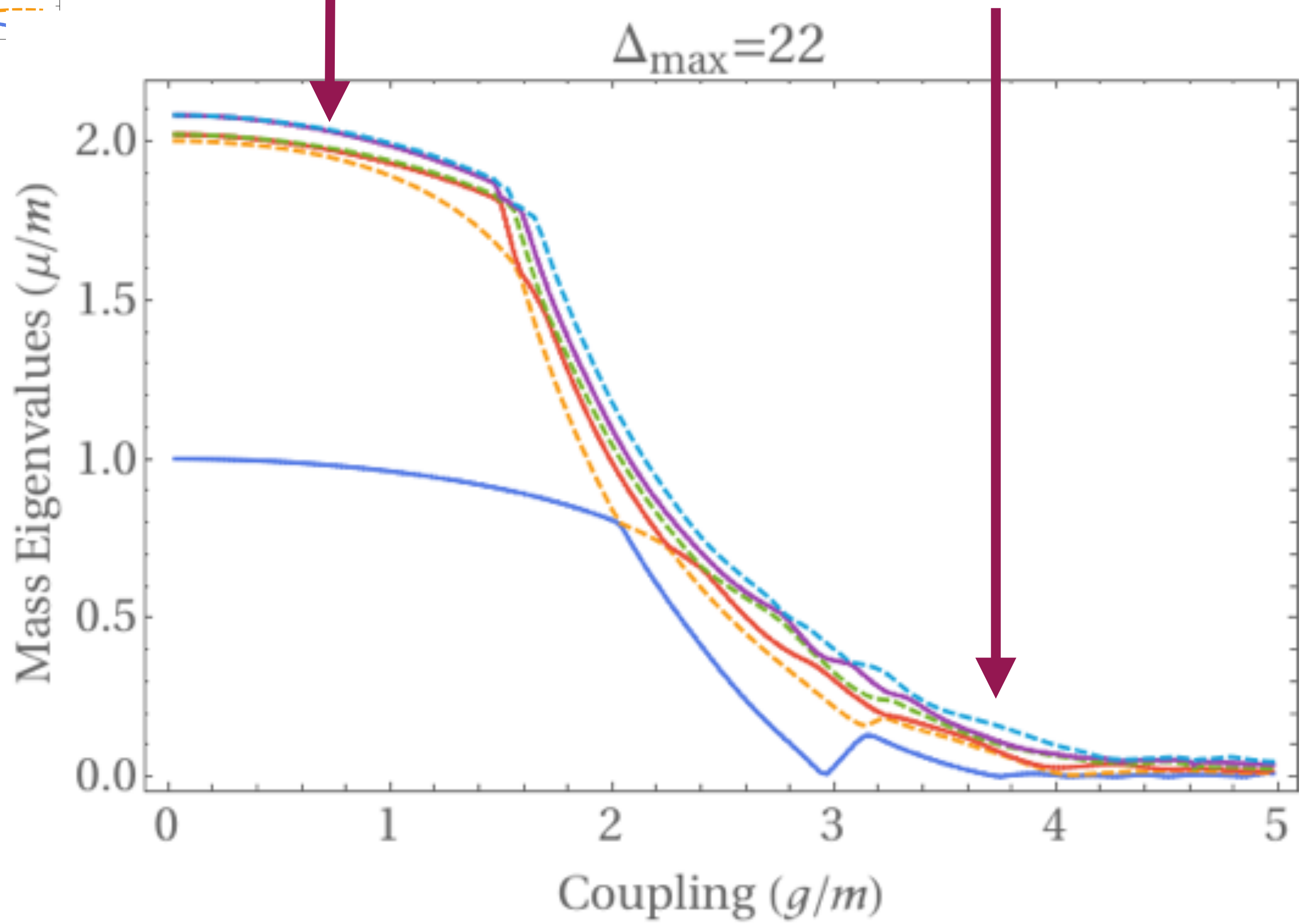
Evidence for Ising gapless phase



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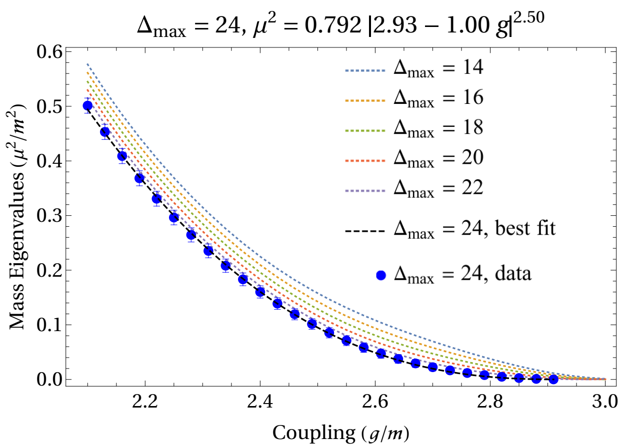

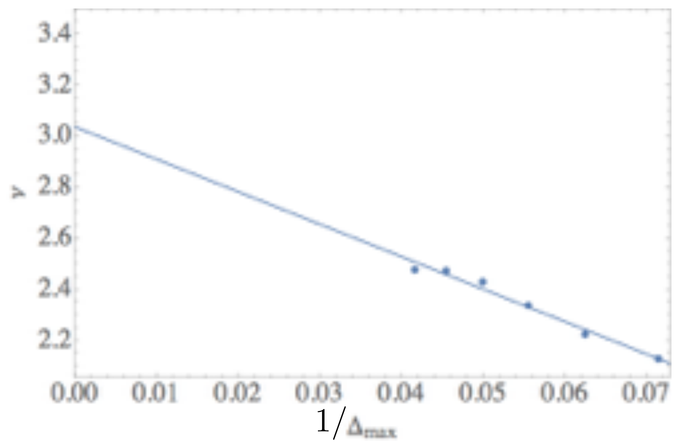

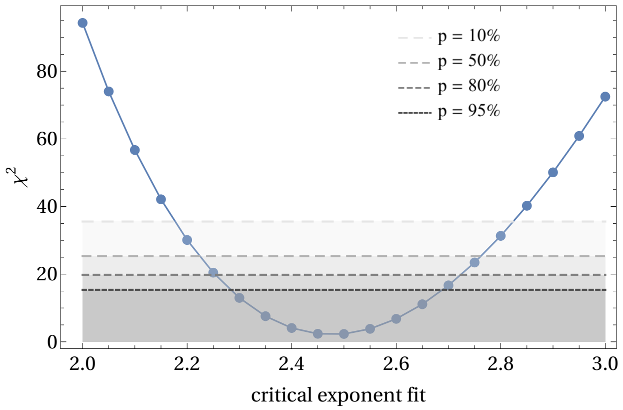

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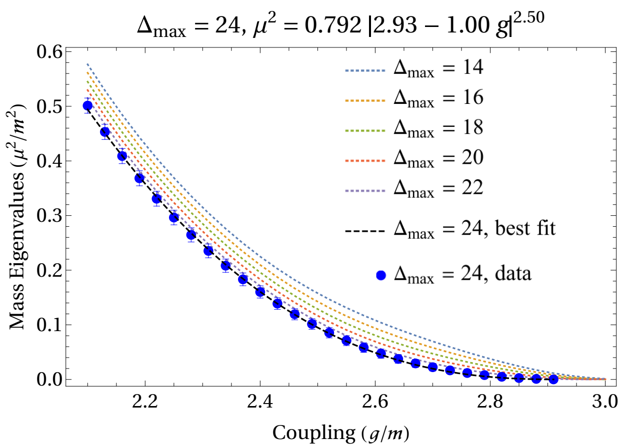

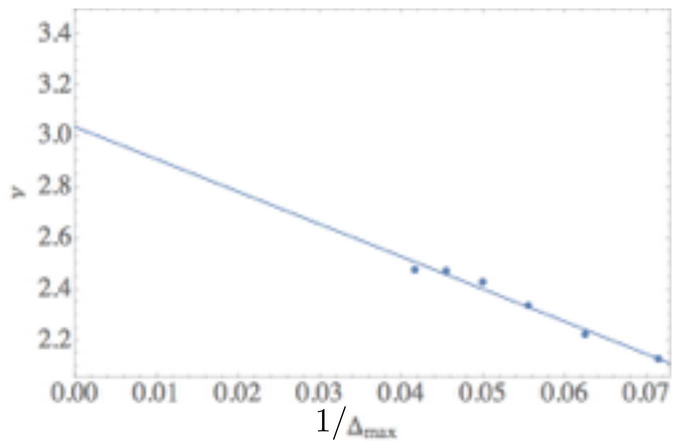

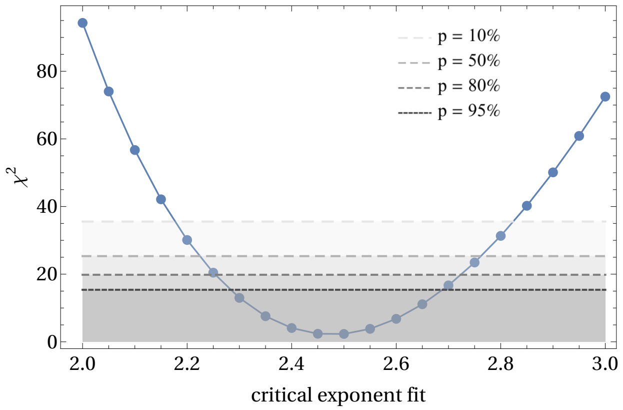

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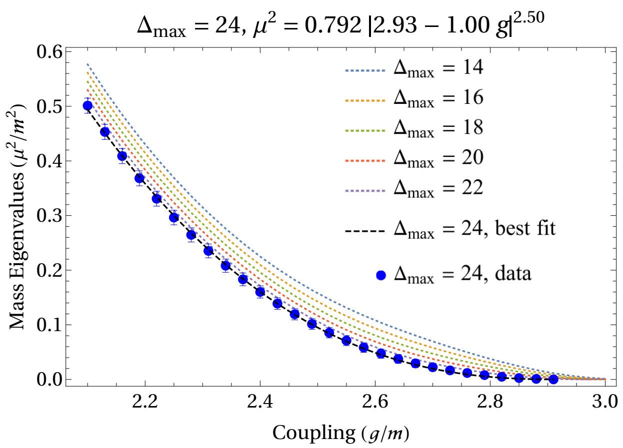

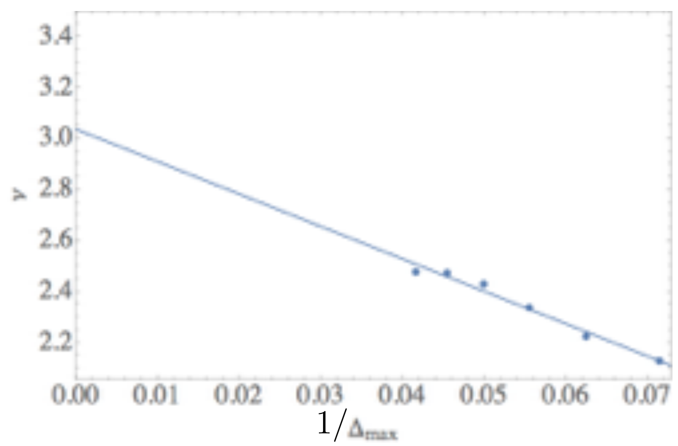

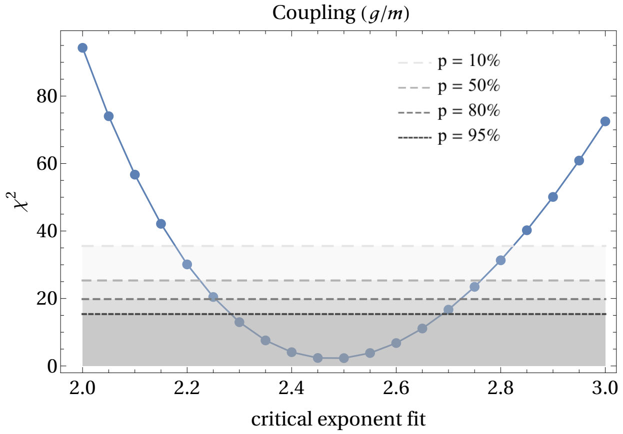

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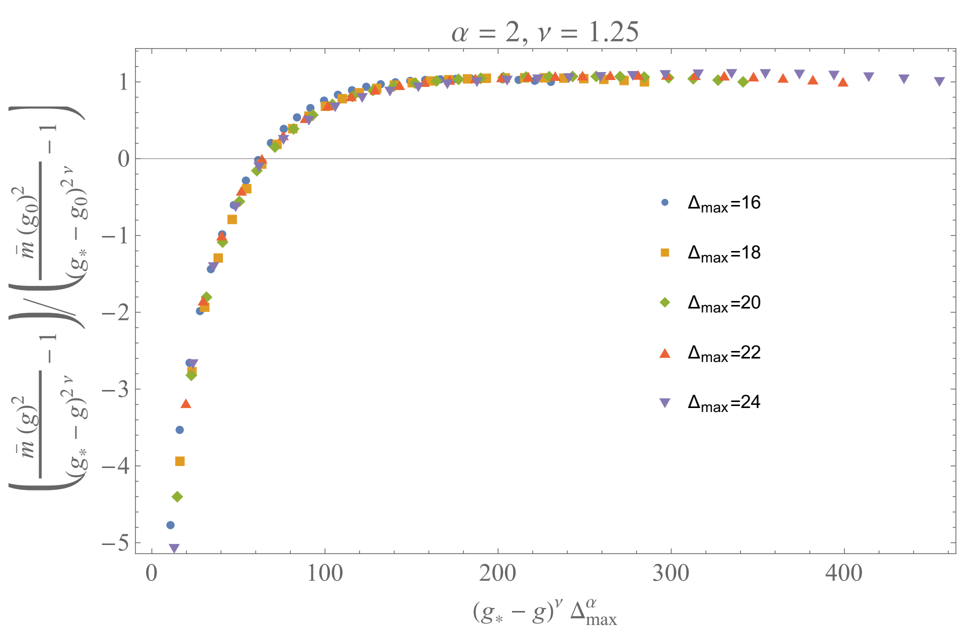

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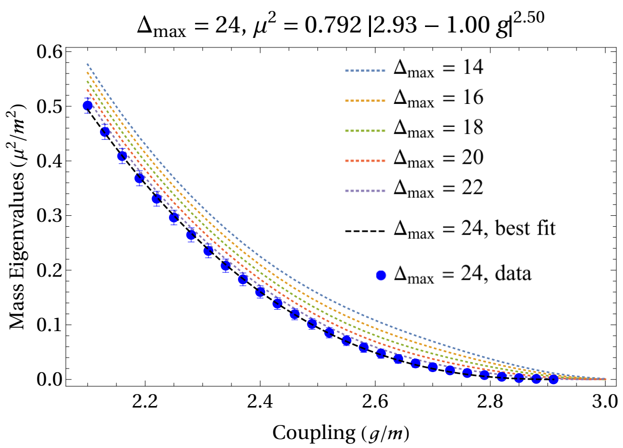

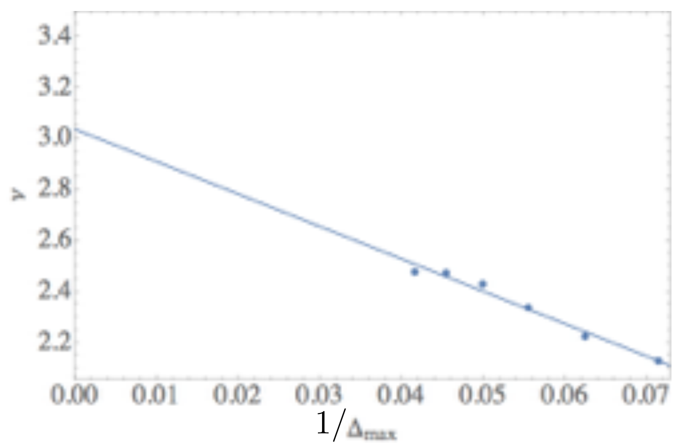

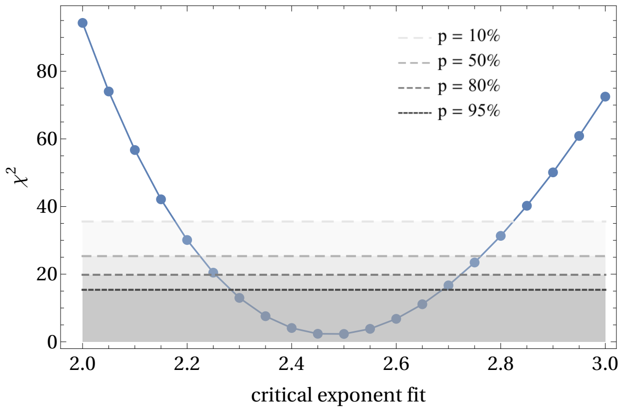

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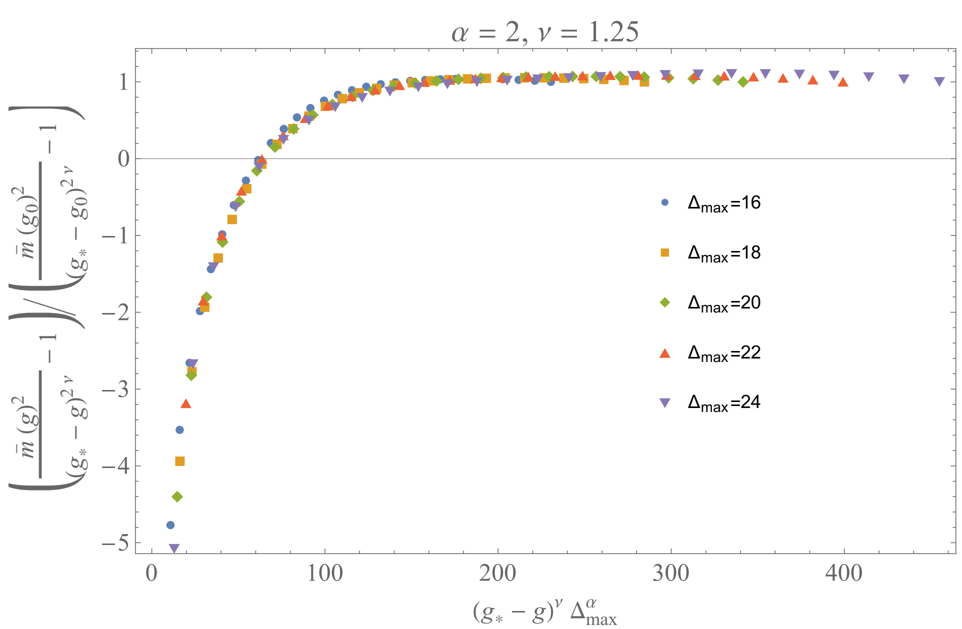

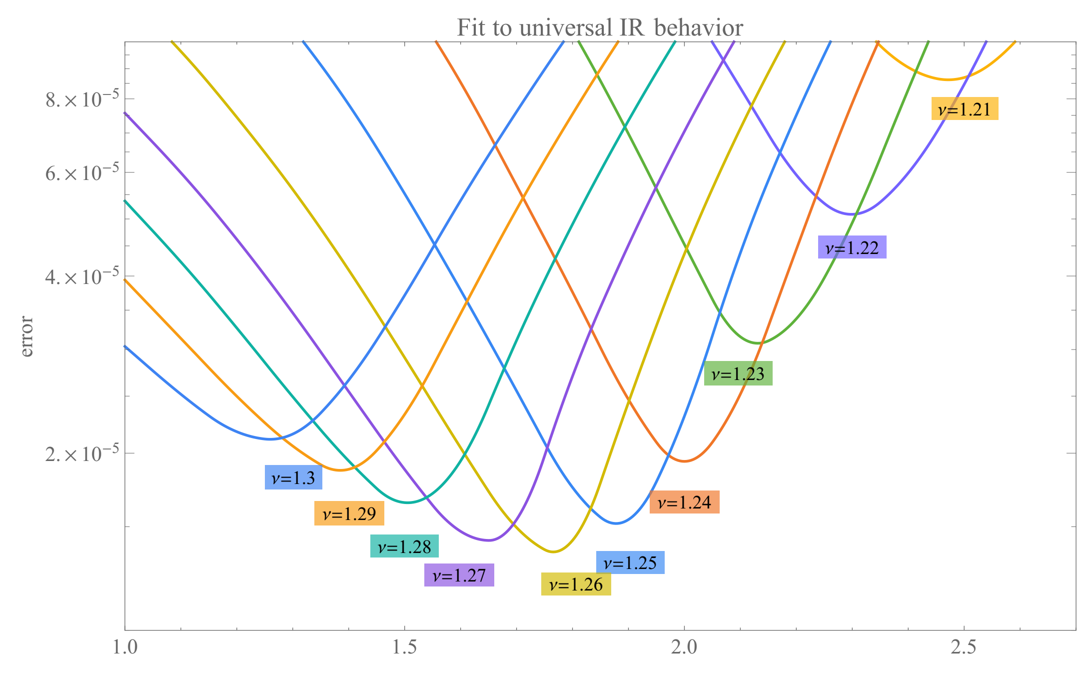

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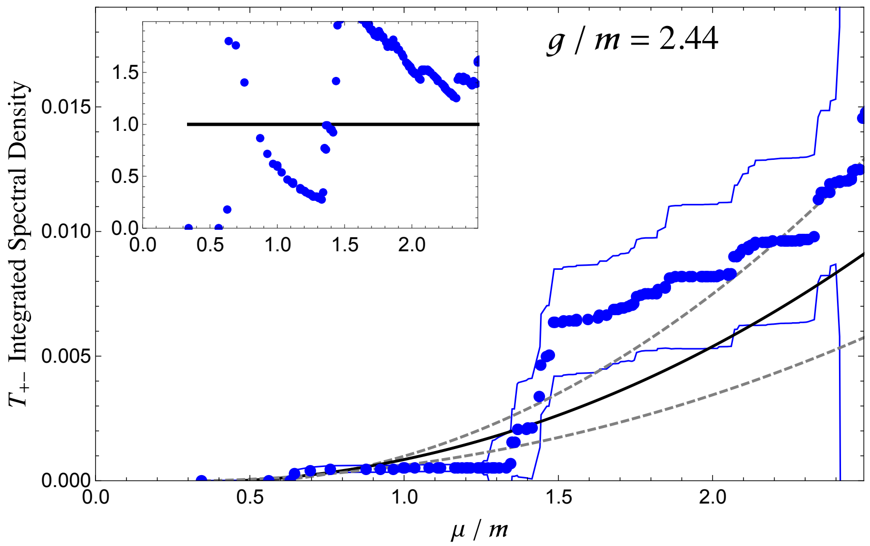

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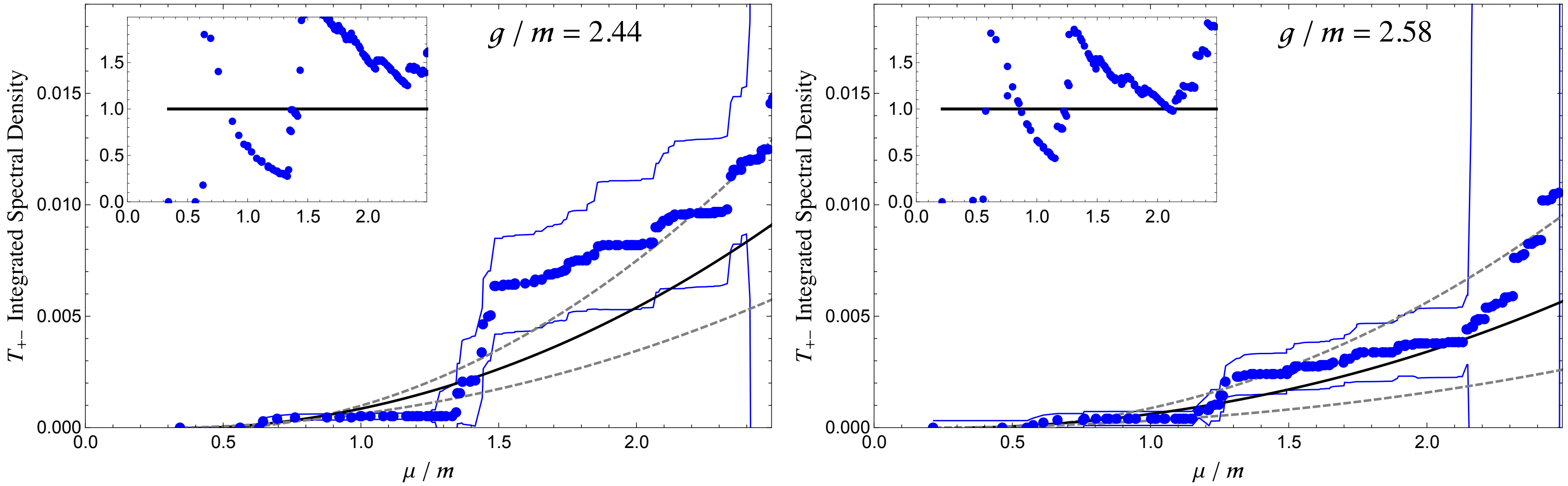

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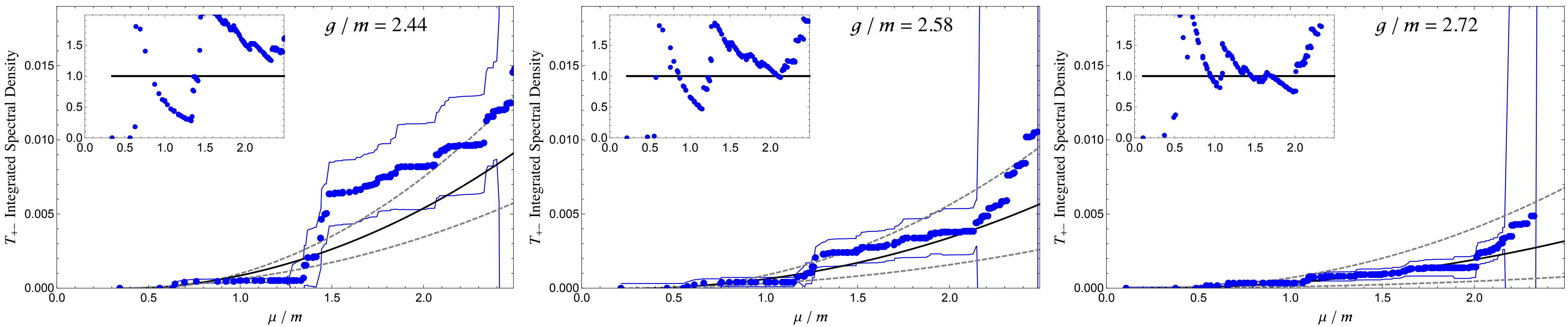

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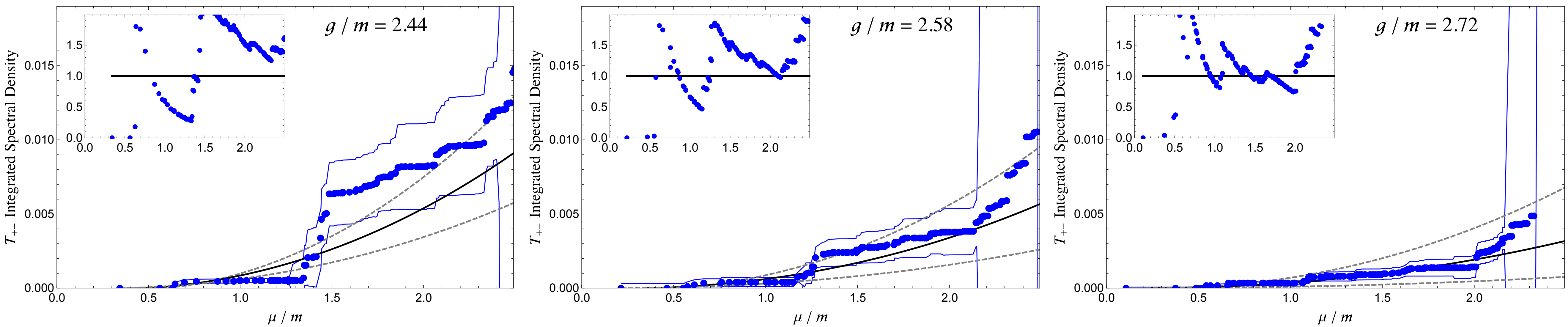

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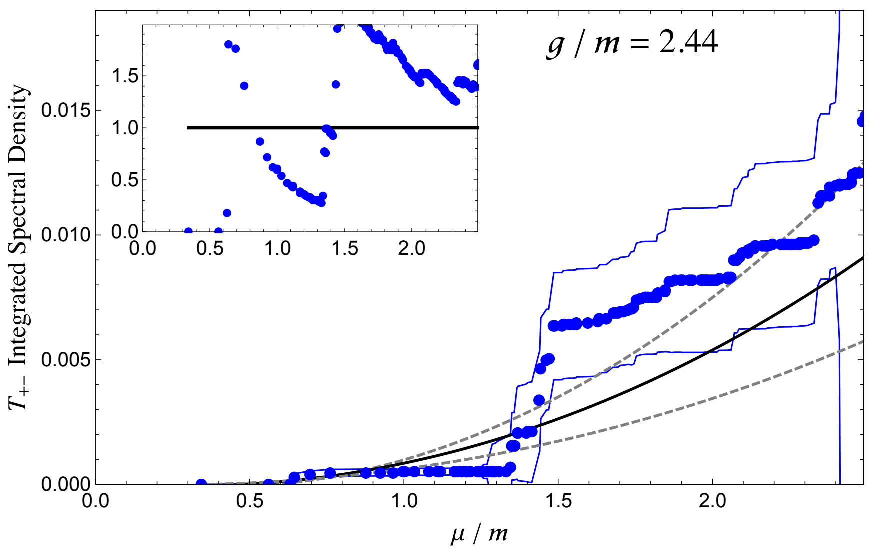

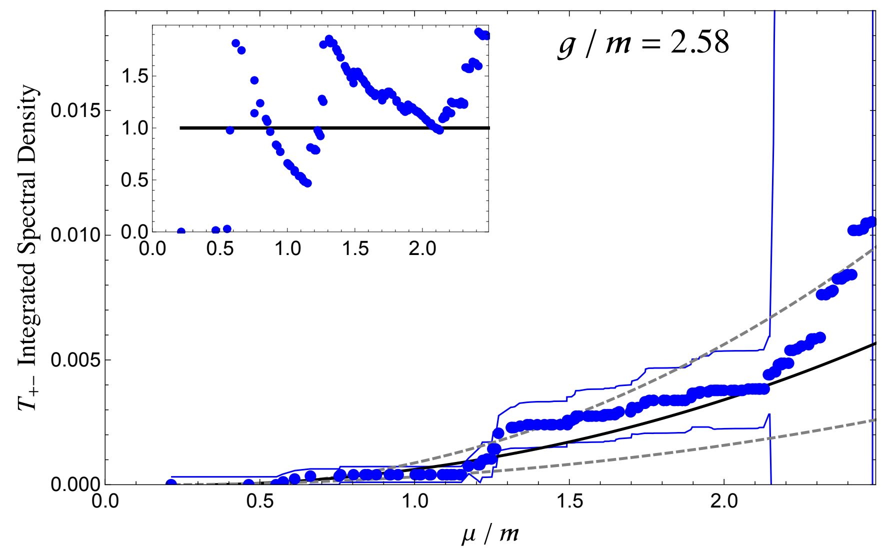

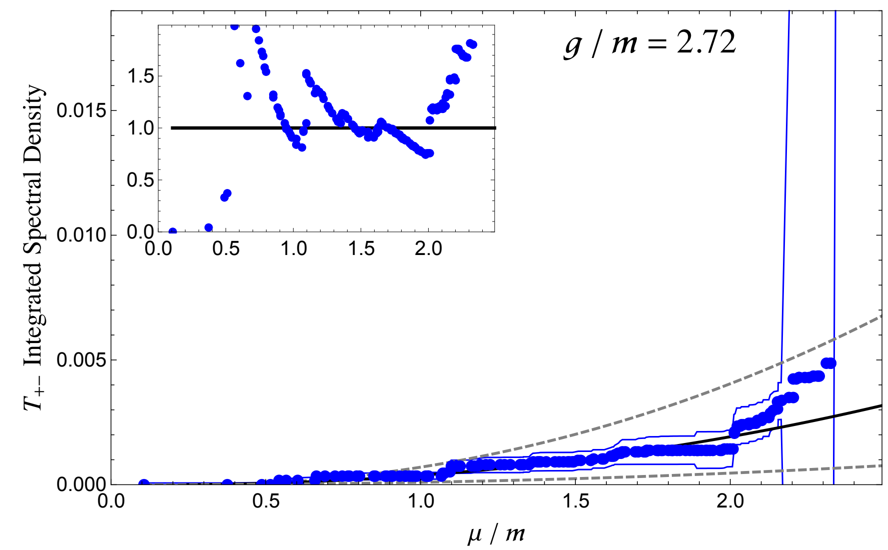

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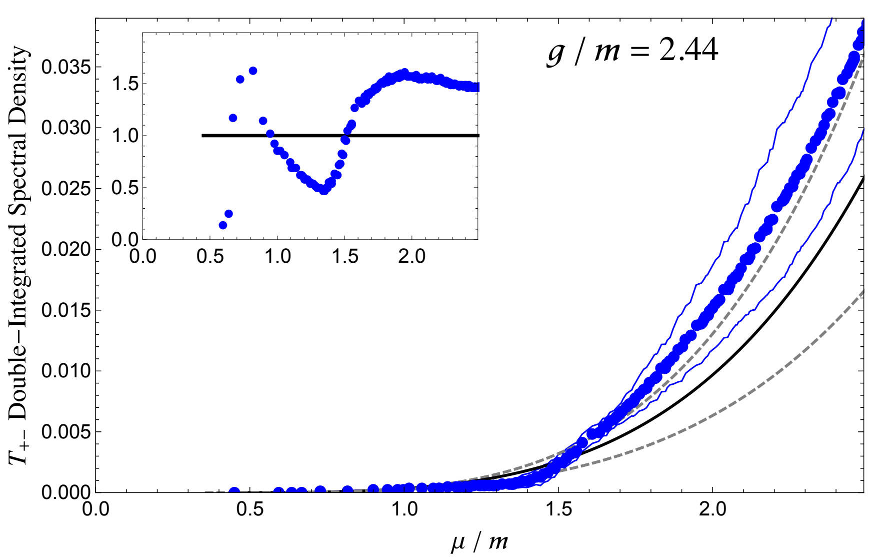

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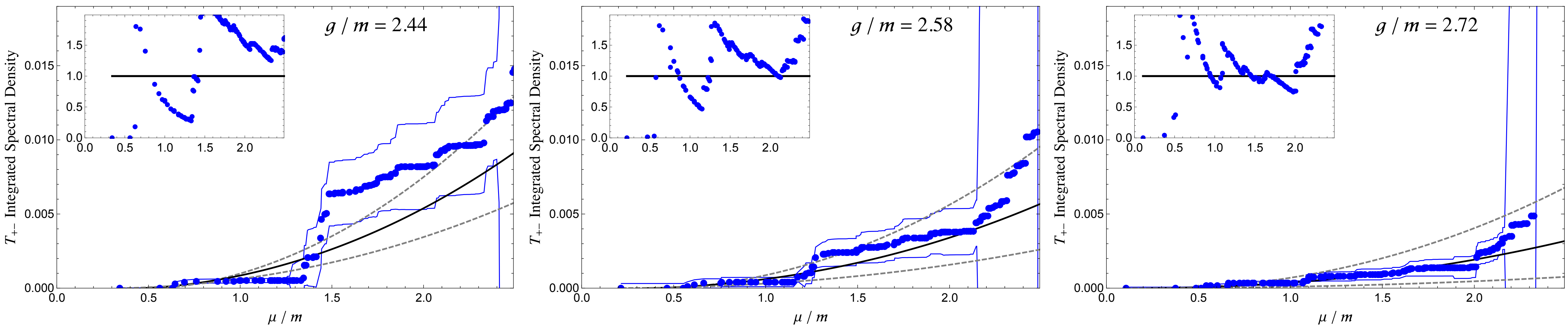

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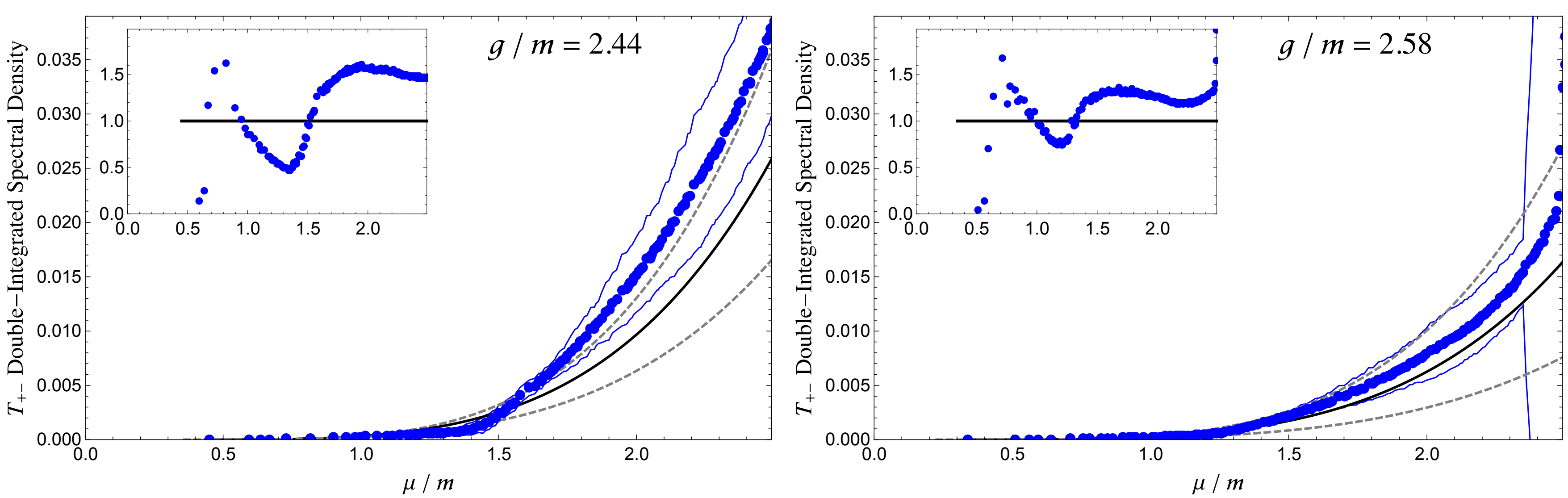
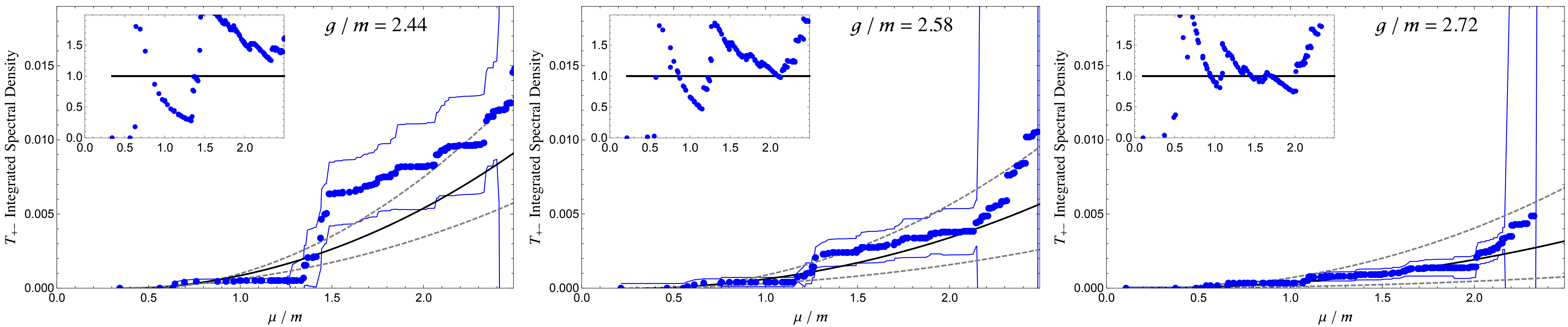
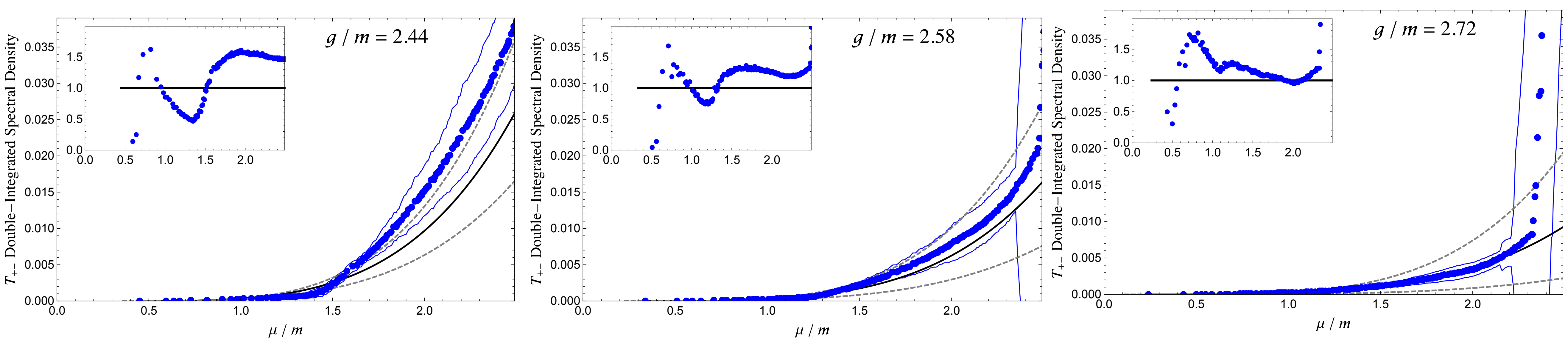


The spectral density of: $T^{\mu}_{\mu} = 2T_{+-}$

Known TIM behavior in the IR: $T_{+-} = m_{kink}^{\frac{4}{5}} \epsilon' + \cdots$



For a smoother taste, let's integrate again:



A Dream

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Large-N RG-flow:

$$\mathcal{N} = 4 \quad SYM \rightarrow \mathcal{N} = 2 \quad SYM$$

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$$\delta Q_+ = \int d^{d-1}x \ m \ tr(\phi\psi)$$

A Dream

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$$\delta Q_+ = \int d^{d-1}x \; m \; tr(\phi\psi)$$

SUSY Non-Renorm Thms protect
naive LC from “zero-modes”!