Hot and cold domain walls and anomaly matching

Erich Poppitz UofToronto

w/ **Mohamed Anber** (Lewis & Clark College)

1807.00093, 1811.10642

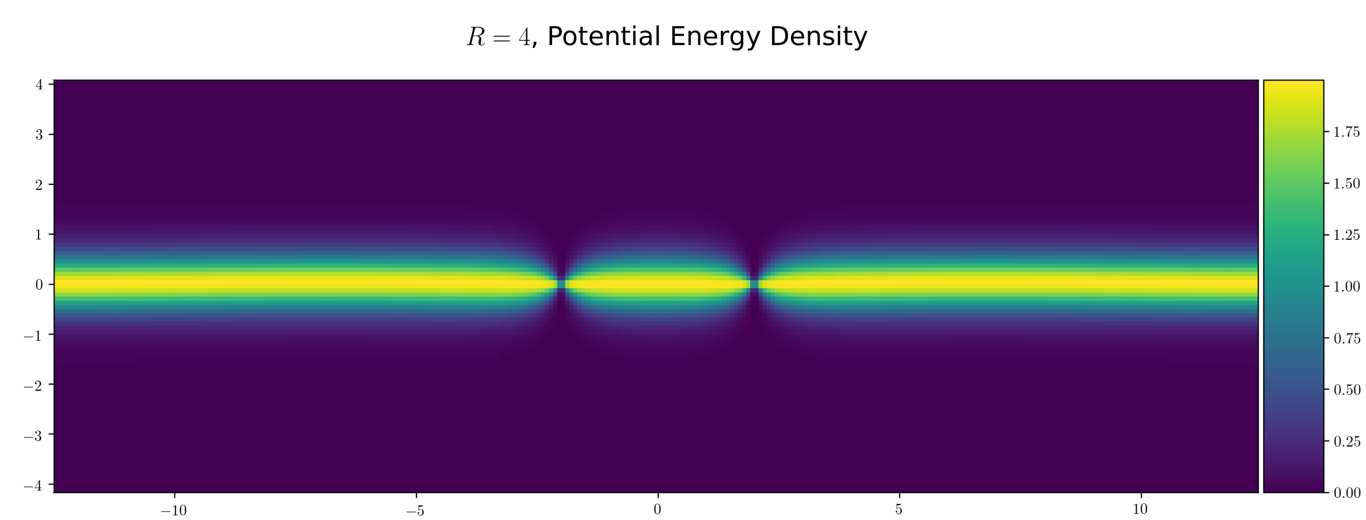
w/ **Andrew Cox & Samuel Wong**

in progress, more domain walls

w/ **Anber, Tin Sulejmanpasic**

1501.06773 on DWs,

pre-0-form/1-form anomaly



(discrete) anomaly inflow: SYM (& dYM at $\theta = \pi$) $(R^3 \times S^1)$

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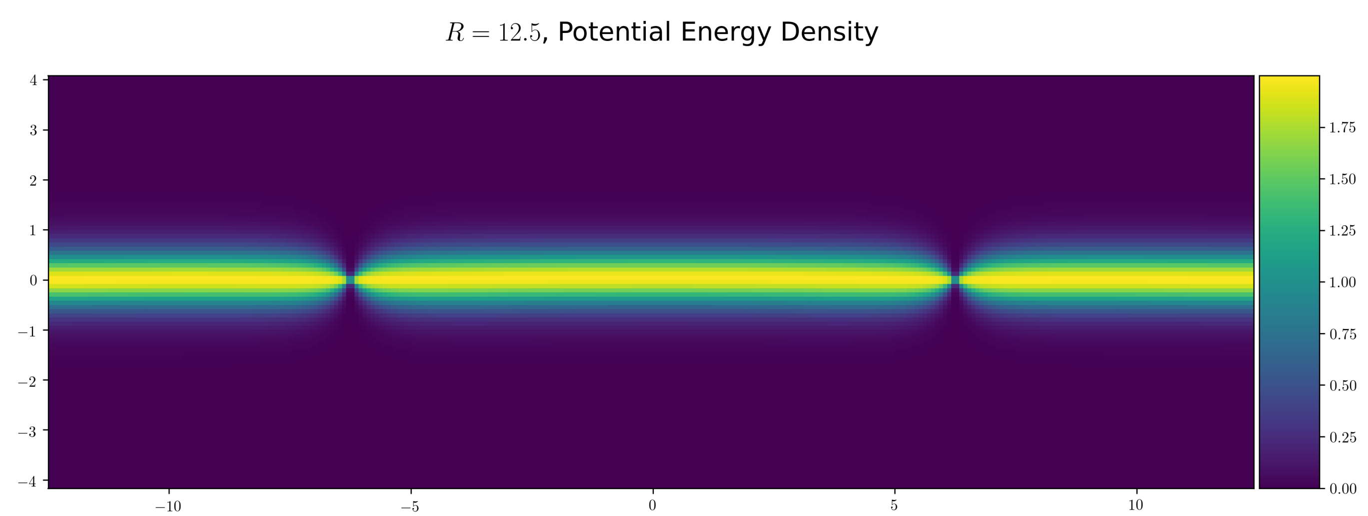

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1501.06773 on DWs,

pre-0-form/1-form anomaly

(discrete) anomaly inflow: SYM (& dYM at $\theta = \pi$) $(R^3 \times S^1)$

QCD
quarks, gluons

UV

other gauge
theories

complicated
RG flow

hadrons

IR

what is the IR?

few general constraints:

- inequalities (“a theorem”)
- a rare equality: ’t Hooft anomalies, UV = IR!
- ca. 1980, all done?
- NOT!

missed anomalies involving higher-form symmetries

Gaiotto, Kapustin, Komargodski, Seiberg, Willett… 2014-2017

hence, new anomaly matching conditions!

new anomaly matching conditions!

e.g. implications for phases of 4D adjoint QCD

Anber-EP; ***Cordova-Dumitrescu***; Bi-Senthil; ***Wan-Wang,*** Ryttov-EP (2018-2019)

*crucial subtleties clarified; ultimately, need lattice to figure out IR phases… **won't discuss here**.*

0-form/1-form 't Hooft anomalies are shown/believed to imply:

- IR phases can't be "trivial"
- domain walls "nontrivial" due to 'discrete anomaly inflow'

this talk:

- *examples of nontrivial DWs, where mechanism of anomaly inflow can be described semiclassically*
- *walls in high-T phase exhibit features of low-T phase and v.v.*
- *related [for sure or perhaps…] to confinement mechanism*

Higher form symmetry $\supset$ 1-form symmetry $\supset$ $\mathbb{Z}_N^{(1)}$ center symmetry

2D compact U(1) with (integer) charge-N massless Dirac

"charge N Schwinger model"

4D SU(N) with n_f massless Weyl adjoints

$n_f = 1$ = SYM

" n_f *QCD(adj)*"

Higher form symmetry $\supset$ 1-form symmetry $\supset$ $\mathbb{Z}_N^{(1)}$ center symmetry

2D compact U(1) with (integer) charge-N massless Dirac

"charge N Schwinger model"

4D SU(N) with n_f massless Weyl adjoints

$n_f = 1$ = SYM

" n_f QCD(adj)"

1 remarkably alike

both have similar mixed 0-form/1-form anomalies

2 high-T domain walls in SU(2) SYM (high-T "center vortices")

world-volume theory "=" charge-2 Schwinger model

(realization of anomaly inflow)

3 simplest interacting QFT (solvable) with new anomaly

interesting generalizations/applications: Armoni, Sugimoto '18; Misumi, Tanizaki, Unsal '19

Higher form symmetry $\supset$ 1-form symmetry $\supset$ $\mathbb{Z}_N^{(1)}$ center symmetry

2D compact U(1) with (integer) charge-N massless Dirac

"charge N Schwinger model"

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}_+ (\partial_- - iNA_-)\Psi_+ + i\bar{\Psi}_- (\partial_+ - iNA_+)\Psi_-$$

$$U(1)_A : \quad \Psi_\pm \to e^{\pm i\chi} \Psi_\pm$$

axial anomaly $$[\mathcal{D}\Psi] \to [\mathcal{D}\Psi]\, e^{i 2N\chi \cdot \overbrace{\int \frac{d^2x\, F_{12}}{2\pi}}^{\mathbf{Q_{top.}}}}$$

$$e^{i 2N\chi Q_{top.}}$$

$\hookrightarrow$ quantized $\in \mathbb{Z}$ ("1st Chern class")

phase is unity when $\chi = \frac{2\pi}{2N}$ $\quad \mathbb{Z}_{2N}^{d\chi} = \mathbb{Z}_{2N}^{(0)}$

discrete chiral

(likewise, 4D QCD(adj) has $SU(n_f) \times \mathbb{Z}_{2Nn_f}^{d\chi}$ *global chiral symmetry)*

We want to know what

charge-N Schwinger model or QCD(adj) “do” in the IR?

assisted by **claim** that: there is a mixed anomaly between

$\mathbb{Z}^{d\chi}_{2Nn_f}$ discrete “0-form” chiral, present in both models

$(n_f \to 1 \text{ in } 2D)$

$\mathbb{Z}^{(1)}_N$ discrete “1-form” center, present in both models

$$e^{i\oint dx^1 A_1} \longrightarrow e^{i\frac{2\pi}{N}k_1}\, e^{i\oint dx^1 A_1}$$

mixed chiral/center ’t Hooft anomaly in three lines:

gauging the center (turning on nondynamical background) explicitly breaks the chiral!

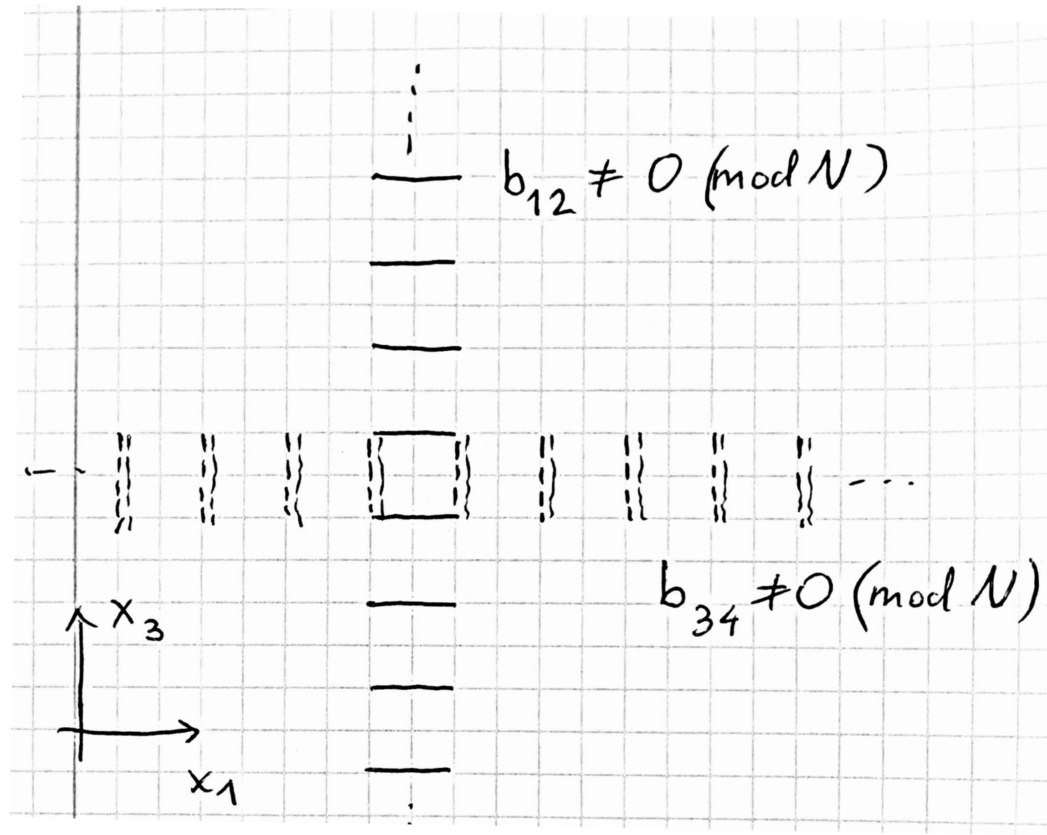
- “’t Hooft flux” (twisted b.c.) or “thin center vortex,”

  results in topological charge ~ 1/N, not integer

't Hooft fluxes in 1-2 and 3-4 planes **=** **intersecting center vortices = 2 codimension two objects** (here: two 2-planes of plaquettes w/ empty coboundary)

II

gauge background $\in SU(N)/Z_N$ **bundle**



II

topological (no flux thru cubes) **background for** $B^{(2)}_{\mu\nu} dx^\mu \wedge dx^\nu$ **- 2-form** Z_N **gauge field, introduced to gauge 1-form** Z_N **center symmetry**

mixed chiral/center 't Hooft anomaly in three lines:

gauging the center (turning on nondynamical background) explicitly breaks the chiral!

- "'t Hooft flux" (twisted b.c.) or "thin center vortex,"

  results in topological charge ~ 1/N, not integer given that, simply recall measure transform under anomaly-free chiral:

gauging $\mathbb{Z}_N^{(1)}$ explicitly breaks $\mathbb{Z}_{2N}^{d\chi}$

$$[\mathcal{D}\psi] \xrightarrow[\mathbb{Z}_{2N}^{d\chi}]{} [\mathcal{D}\psi]\, \underbrace{e^{i 2\pi Q_{top}}}_{} = e^{i\frac{2\pi}{N}}, \text{ since } Q_{top} = \frac{1}{N}$$

in theory with gauged center

- **phase IS the mixed ’t Hooft anomaly!**
- **RG invariant, same at all scales (eg torus size-independent)**

---

likewise, in a theory without fermions but with θ term, fractionalization of topological charge breaks the 2π periodicity

“anomaly in the space of couplings” [Cordova, Freed, Lam, Seiberg ’19]

(or, at $\theta = \pi$ there is a mixed anomaly with CP) now, to **mixed ’t Hooft anomaly in charge-N Schwinger model:**

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}_+ (\partial_- - iNA_-)\Psi_+ + i\bar{\Psi}_- (\partial_+ - iNA_+)\Psi_-$$

operator language - Hamiltonian, $A_0 = 0$ gauge, on S^1 space:

$$\hat{U}_{Z_{2N}^{(0)}} = e^{i\frac{2\pi}{2N}\left(\int_0^L dx\, \hat{J}_0^A(x) - \frac{2N}{2\pi}\int_0^L dx\, \hat{A}_1(x)\right)} = e^{i\frac{2\pi}{2N}\int_0^L dx\, \hat{J}_0^A(x)} \; e^{-i\int_0^L dx\, \hat{A}_1(x)}$$

discrete chiral generator

conserved charge involves 1D CS term $\quad \partial_\mu J^{\mu A} = \frac{2N}{2\pi} F_{01}$

now, to **mixed 't Hooft anomaly in charge-N Schwinger model:**

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_+(\partial_- - iNA_-)\psi_+ + i\bar{\psi}_-(\partial_+ - iNA_+)\psi_-$$

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nonperiodic "gauge transformation" $e^{i\omega(x)} = e^{i\frac{2\pi x}{LN}}$, $e^{i\omega(L)} = e^{i\frac{2\pi}{N}}\, e^{i\omega(0)}$

center symmetry generator:

$$\hat{U}_{\mathbb{Z}_N^{(1)}} = e^{i\int_0^L dx\left(\partial_x\left(\frac{2\pi x}{LN}\right)\hat{\Pi}_1(x) + \frac{2\pi x}{LN}\hat{J}_0^V(x)\right)} = e^{i\frac{2\pi}{N}\hat{\Pi}_1(L)}\; e^{i\int_0^L dx\, \frac{2\pi x}{LN}\underline{\left(-\partial_x\hat{\Pi}_1(x) + \hat{J}_0^V(x)\right)}}$$

codimension-2 operator; links w/lines

=0 on physical states (needed to commute with H) now, to **mixed 't Hooft anomaly in charge-N Schwinger model:**

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$$\hat{U}_{Z_{2N}^{(0)}} = e^{i\frac{2\pi}{2N}\left(\int_0^L dx\, \hat{J}_0^A(x) - \frac{2N}{2\pi}\int_0^L dx\, \hat{A}_1(x)\right)} = e^{i\frac{2\pi}{2N}\int_0^L dx\, \hat{J}_0^A(x)}\; e^{-i\int_0^L dx\, \hat{A}_1(x)}$$

commute

$$\hat{U}_{Z_N^{(1)}} = e^{i\int_0^L dx\left(\partial_x\left(\frac{2\pi x}{LN}\right)\hat{\Pi}_1(x) + \frac{2\pi x}{LN}\hat{J}_0^V(x)\right)} = e^{i\frac{2\pi}{N}\hat{\Pi}_1(L)}\; e^{i\int_0^L dx\, \frac{2\pi x}{LN}\left(-\partial_x\hat{\Pi}_1(x) + \hat{J}_0^V(x)\right)}$$

now, to **mixed 't Hooft anomaly in charge-N Schwinger model:**

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$$\hat{U}_{\mathbb{Z}^{(0)}_{2N}} = e^{i\frac{2\pi}{2N}\left(\int_0^L dx\, \hat{J}^A_0(x) - \frac{2N}{2\pi}\int_0^L dx\, \hat{A}_1(x)\right)} = e^{i\frac{2\pi}{2N}\int_0^L dx\, \hat{J}^A_0(x)}\; e^{-i\int_0^L dx\, \hat{A}_1(x)}$$

do not commute

$$\hat{U}_{\mathbb{Z}^{(1)}_{N}} = e^{i\int_0^L dx\left(\partial_x\left(\frac{2\pi x}{LN}\right)\hat{\Pi}_1(x) + \frac{2\pi x}{LN}\hat{J}^V_0(x)\right)} = e^{i\frac{2\pi}{N}\hat{\Pi}_1(L)}\; e^{i\int_0^L dx\, \frac{2\pi x}{LN}\left(-\partial_x\hat{\Pi}_1(x) + \hat{J}^V_0(x)\right)}$$

't Hooft anomaly

$$\hat{U}_{\mathbb{Z}^{(1)}_N}\,\hat{U}_{\mathbb{Z}^{(0)}_{2N}}\,\hat{U}^{-1}_{\mathbb{Z}^{(1)}_N} = e^{-i\frac{2\pi}{N}}\,\hat{U}_{\mathbb{Z}^{(0)}_{2N}}$$

(recall $A\,B = e^{-i\frac{2\pi}{N}}\,B\,A$ 't Hooft loop/Wilson loop algebra)

now, to **mixed 't Hooft anomaly in charge-N Schwinger model:**

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't Hooft anomaly

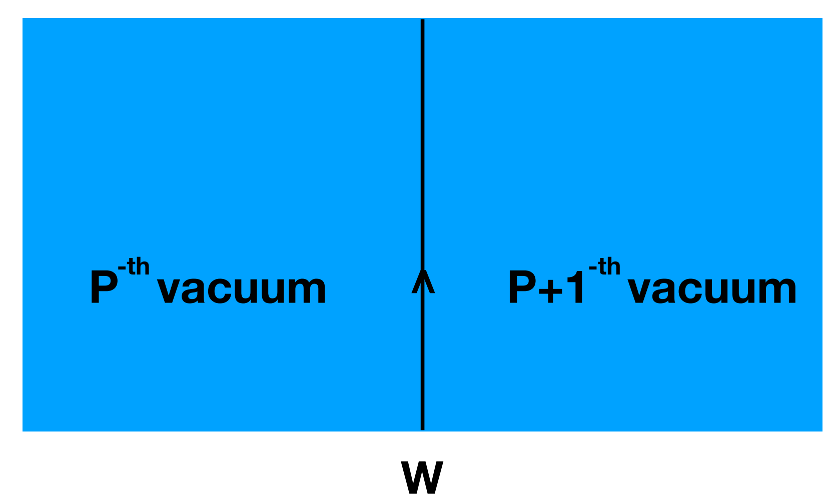
$$\hat{U}_{Z_N^{(1)}}\,\hat{U}_{Z_{2N}^{(0)}}\,\hat{U}^{-1}_{Z_N^{(1)}} = e^{-i\frac{2\pi}{N}}\,\hat{U}_{Z_{2N}^{(0)}}$$

N vacua; discrete chiral broken by fermion bilinear; massive boson in each vacuum

$$\hat{U}_{Z_{2N}^{(0)}}|P\rangle = |P+1\rangle \pmod N$$

$$\hat{U}_{Z_N^{(1)}}|P\rangle = |P\rangle e^{i\frac{2\pi}{N}P}$$

$$\langle \hat{E}_1\rangle_P = g^2 P$$ - discrete E-field



$$\langle \hat{E}_1\rangle_P - \langle \hat{E}_1\rangle_{P-1} = g^2$$ - "DW" = 'fundamental' unit charge Wilson loop

now, to **mixed ’t Hooft anomaly in charge-N Schwinger model:**

as spectrum is gapped, what matches the anomaly below mass gap?
- **an IR TQFT, a “chiral lagrangian” describing the N vacua.**

this is usually not trivial to derive from the UV theory, but here it is

TQFT: N-dim Hilbert space (the N vacua) - compact scalar and compact U(1)

$$S_{2-D} = i\,\frac{N}{2\pi}\int\limits_{M_2} \varphi^{(0)} da^{(1)}$$

chiral $\phi^{(0)} \to \phi^{(0)} + \frac{2\pi}{N}$ center $a^{(1)} \to a^{(1)} + \frac{1}{N}\epsilon^{(1)}$

…upon gauging center in TQFT, the phase of partition function under chiral transform matches anomaly, so all is consistent and as explicit as can be!

The charge-N Schwinger model is the simplest solvable *interacting* QFT with a mixed 0-form/1-form anomaly, so has at least pedagogical value…

…now, to promised relation to 4D SYM:

… promised relation to 4D SYM (in words/pictures):

high-T domain walls in **SU(2) SYM,** or high-T “center vortices”

worldvolume theory “=“ charge-2 Schwinger model

(realization of anomaly inflow)

(for SU(N), see 1811.10642 Anber, EP)

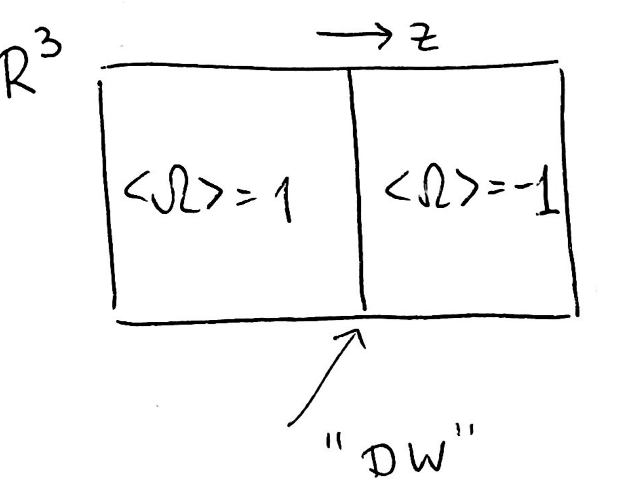
… promised relation to 4D SYM (in words/pictures):

First, what are high-T "center vortices"? $R^3 \times S^1_\beta$

Polyakov loop $\Omega = \mathrm{tr}\, e^{i \oint_{S^1_\beta} A}$ $\langle \Omega \rangle = 0$ confinement, low-T

$T \gg \Lambda$ deep in deconfined high-T phase $\langle \Omega \rangle = \pm 1$ $Z_2^{(1), S^1_\beta} \to \emptyset$ SU(2)!

high-T phase breaks "0-form" center (in modern parlance; preserves 1-form, or R^3 center)



twisted boundary conditions $B^{(2)}_{z\beta} \neq 0$ (say, unit 't Hooft flux): k=1 wall

these "DW"s are the high-T "center vortices" (semiclassical!)

- codimension-2 objects, link with Wilson loops

width $\sim \frac{1}{gT}$ (Debye)

DWs:

Bhattacharya, Gocksch, Korthals-Altes, Pisarski,…~'92

lattice, down to Tc: Bursa, Teper '05;…

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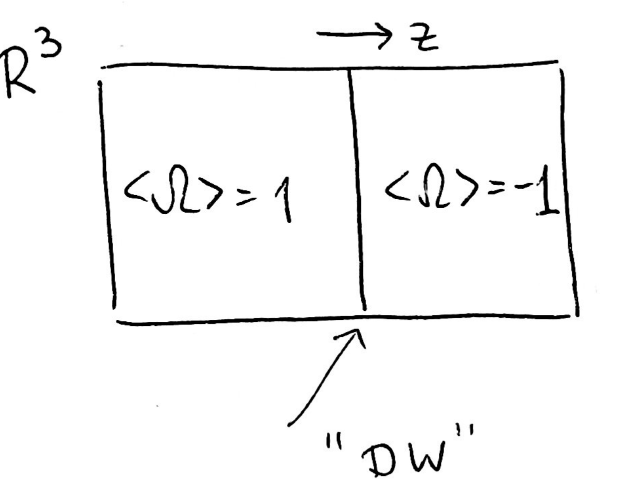

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these "DW"s are the high-T "center vortices" (semiclassical!)

- codimension-2 objects, link with Wilson loops
- "heavy" at high-T: semiclassical and unlikely to appear; **pure YM:**
- "light" at low-T: condense, disorder nonzero N-ality Wilson loops: area law, confinement, N-ality dependence of string tensions…

of course, not theoretically controlled confinement but

lattice evidence: Greensite et al, '97; D' Elia, de Forcrand '99,…

DWs:

Bhattacharya, Gocksch, Korthals-Altes, Pisarski…~'91

lattice, down to Tc: Bursa, Teper '05;…

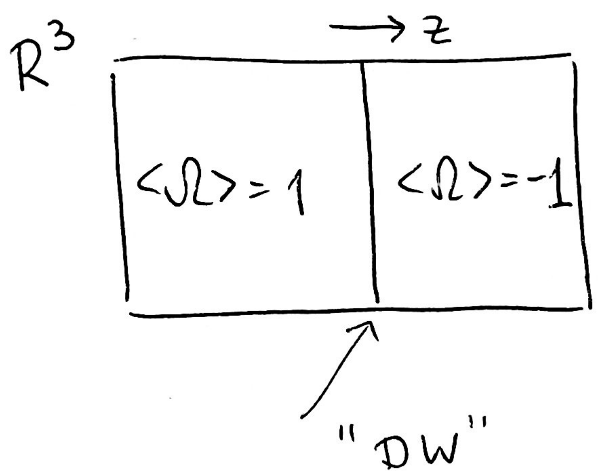
… promised relation to 4D SYM (in words/pictures):

high-T domain walls in SU(2) SYM, or high-T "center vortices" (for SU(N), see 1811.10642 Anber, EP)

worldvolume theory "=" charge-2 Schwinger model

(realization of anomaly inflow)

Next, what about high-T "center vortices" in SYM? $T \gg \Lambda$ $\quad \mathbb{R}^3 \times S^1_\beta$



- high-T "center vortices" also exist in SYM
- SYM has a $Z_4^{(0)} - Z_2^{(1)}$ chiral center anomaly

- has to be matched at any size/shape of torus, hence at any T

- recall that anomaly requires turning on perpendicular 't Hooft fluxes

- turning on $\langle B^{(2)}_{z\beta} \rangle \neq 0$ produces a k=1 wall (SU(2)) in high-T phase

$$\Omega(z) = e^{i \frac{A_0(z)}{T} \frac{\tau_3}{2}} \qquad \begin{array}{c} A_0(z \to -\infty) = 0 \\ \Omega(-\infty) = 1 \end{array} \qquad \begin{array}{c} A_0(z \to +\infty) = 2\pi T \\ \Omega(+\infty) = -1 \end{array}$$

- at center of wall $\Omega(0) = \mathrm{diag}(i, -i)$ $\quad SU(2) \to U(1)$ *, massless photon, W-boson mass ~ T*

- localized fermion zero modes: ψ_+ *charge 2,* ψ_- *-2: "axial charge-2 Schwinger model"*

we saw it has $Z_4^{(0)}$ *vector and* $Z_2^{(1)}$ *center with mixed anomaly, turning on* $\langle B^{(2)}_{12} \rangle \neq 0$ *on worldvolume:*

matches the bulk SYM anomaly (= "anomaly inflow")

… promised relation to 4D SYM (in words/pictures):

formally, anomaly (bulk) from 5D CS: $S_{5-D} = i\,\frac{2\pi}{N} \int\limits_{M_5\ (\partial M_5 = M_4)} \frac{2NA^{(1)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi}$

$\langle B^{(2)}_{z\beta} \rangle \neq 0$ anomaly inflow (wall) from 3D CS : $S_{3-D} = i\,\frac{2\pi k}{N} \int\limits_{M_3\ (\partial M_3 = M_2)} \frac{2NA^{(1)}}{2\pi} \wedge \frac{NB^{(2)}}{2\pi}$ (*)

we just argued wall theory matches (*) in the regime of perturbative wall theory

What does the wall worldvolume do at large distances?

-anomaly has to be matched at any scale

-bulk is gapped (confinement in 3D pure YM)

-either fermions on wall remain massless (unlikely, as flow to strong coupling), or as in charge-2 Schwinger $\langle \psi_+ \psi_- \rangle_P \neq 0$ breaking $Z_4^{(0)} \to Z_2^{(0)}$

-above is more likely, but not proven, as bulk and DW expected to become strongly coupled at about the same scale, the bulk confinement scale ~ g^2T

we take this to predict that, at $T \gg \Lambda$ $Z_{2N}^{(0)}\ Z_N^{(1)}$ 't Hooft anomaly matched by

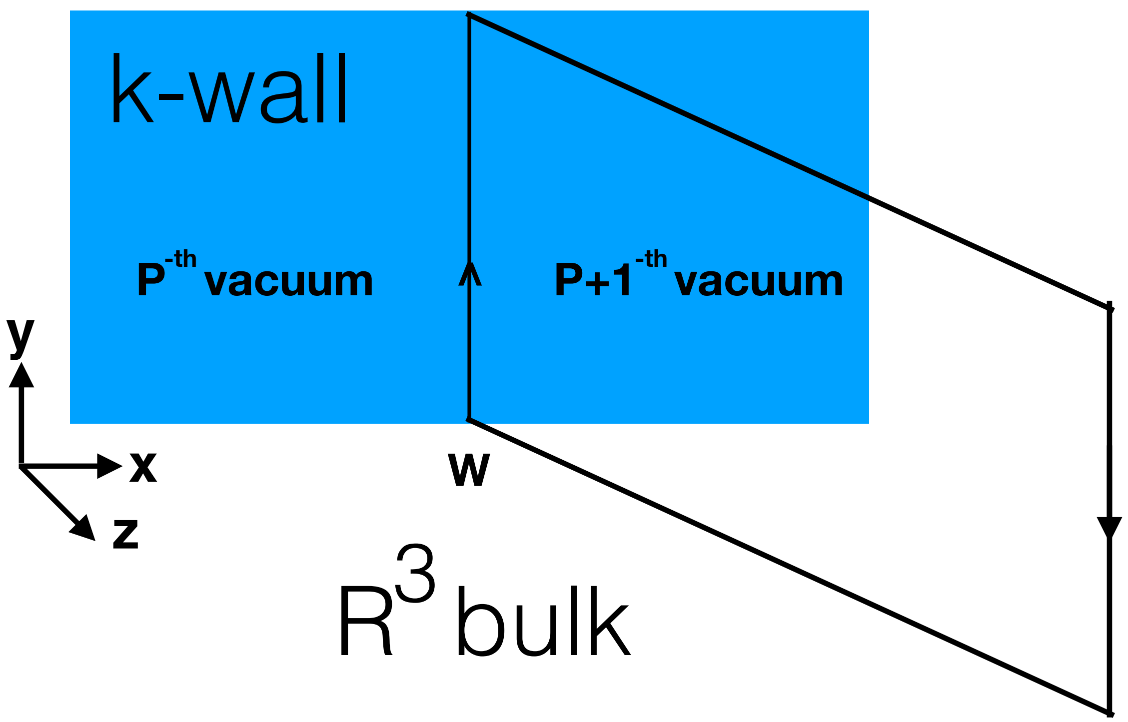
- nonvanishing fermion condensate on k-wall: **at high-T, in chirally restored and deconfined phase wall shows features of low-T phase -** perhaps testable on lattice?

-quarks "deconfined" on k-wall, $Z_N^{(1)}$ also broken, as per the Z_N IR TQFT…

… promised relation to 4D SYM (in words/pictures):

$T \gg \Lambda$

$Z_{2N}^{(0)}\, Z_N^{(1)}$ 't Hooft anomaly:



1 fermion condensate on k-wall

(in high-T phase! "testable" - lattice?)

$$\langle \psi_+ \psi_- \rangle_P \sim e^{i\frac{2\pi P}{N}}$$

2 quarks "deconfined" on k-wall, so bulk confining strings end

first via holography: *F1* on *D1*

[Aharony, Witten 1999;…]

(one can't help but wonder whether different worldvolume of high-T center vortex reflected in different confinement mechanism in SYM/YM?)

… finally, some pictures about cold DWs in 4D SYM on small $R^3 \times S^1$:

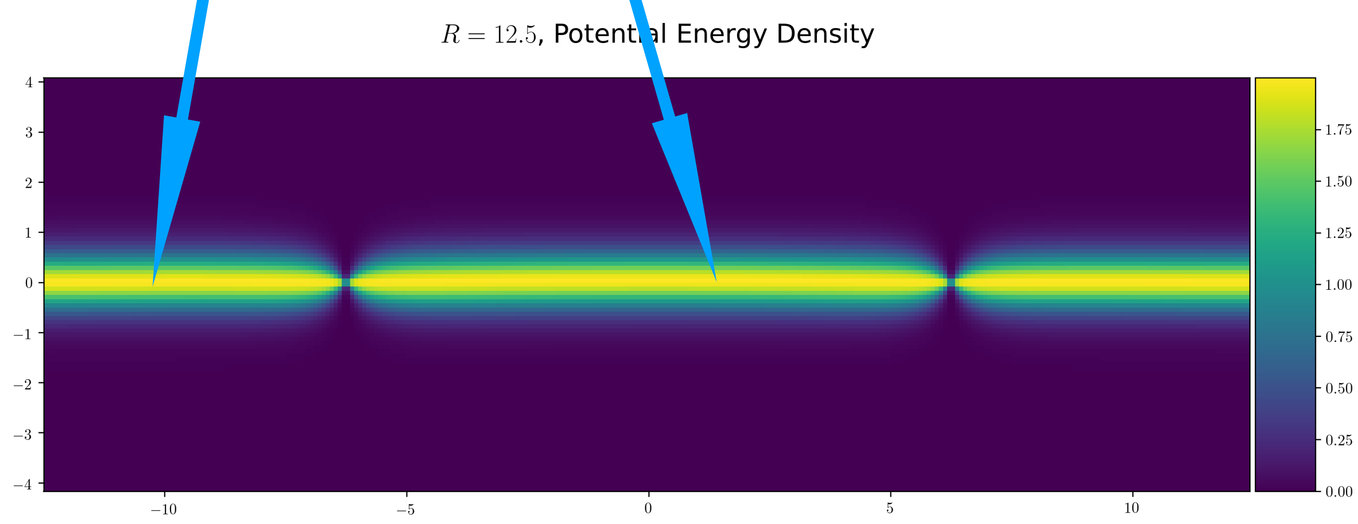
cold DWs [lines!] between $Z_{2N}^{(0)} \to Z_N^{(0)}$ chirally broken vacua

- consider k=1 DWs between neighbouring chirally broken vacua

- 0-form center $Z_N^{(1),S^1}$ and 1-form center $Z_N^{(1),R^3}$ broken on the DW

k=1 DWs have N "vacua"

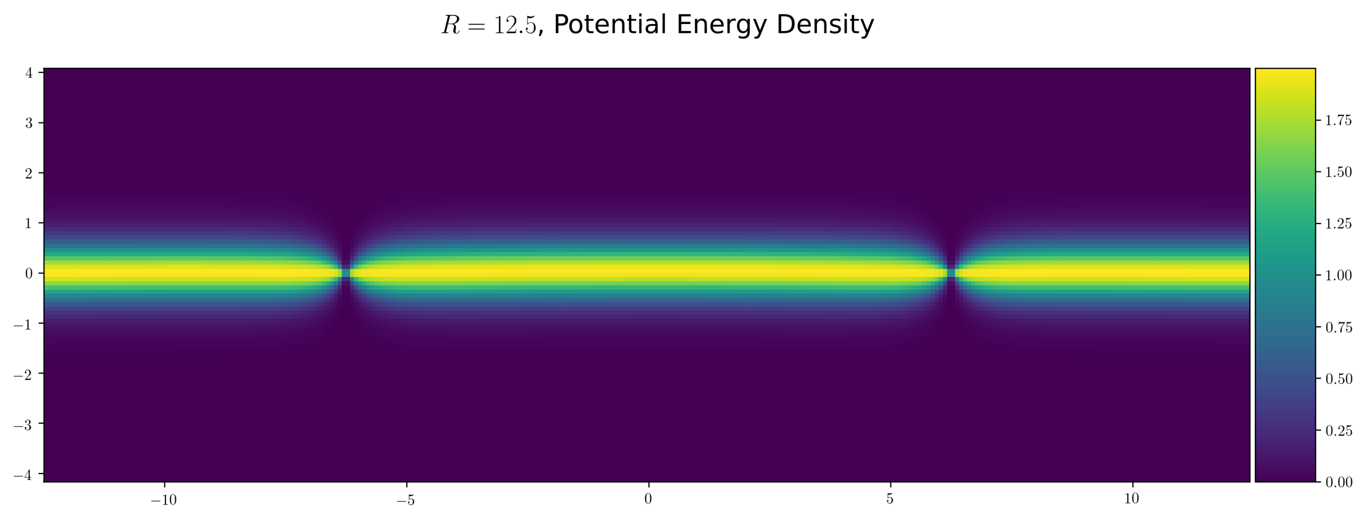
quarks are deconfined on k=1 DWs



(there are $\binom{N}{k}$ BPS k walls)

- there are N different BPS walls between neighbouring vacua
- these walls each carry a fraction of a flux of a quark
- each quark has its flux split between two walls of equal tension
- hence, quarks deconfined on walls

… finally, some pictures about cold DWs in 4D SYM on small $R^3 \times S^1$:



for the above k=1 'cold' walls the 2D TQFT is the same as
for the 'hot' k=1 walls described above (replace 0-form center with 0-form chiral)

'cold' wall story under complete control $R^3 \times S^1$

= magnetic bion confinement

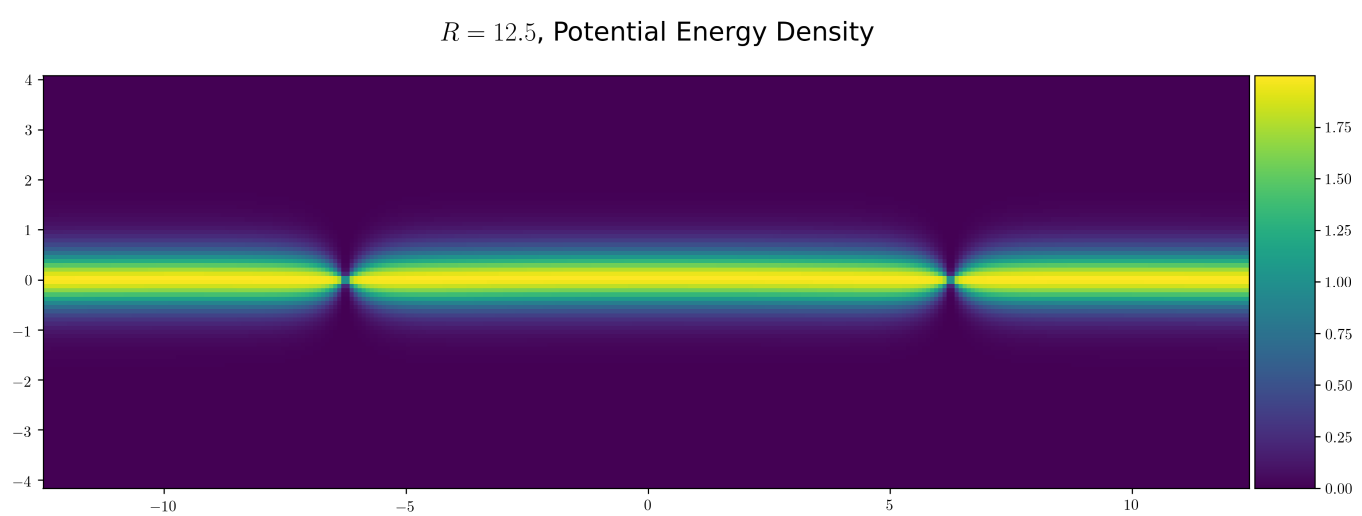
'hot' wall story needs further (lattice) studies, as strong coupling…

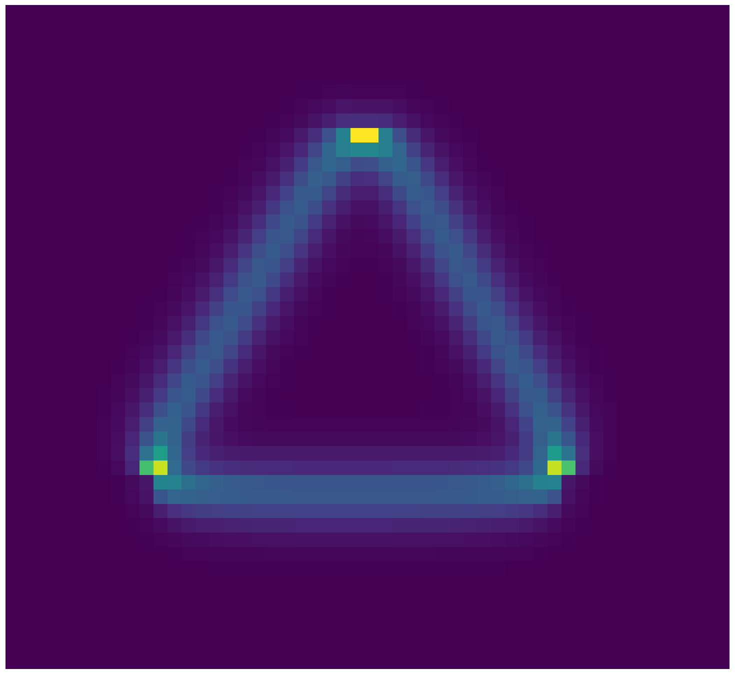
… finally, some pictures about cold DWs in 4D SYM on small $R^3 \times S^1$:

anomaly matching implies deconfinement of quarks
on walls between chirally broken vacua

magnetic bion mechanism realizes deconfinement using the DW'
properties, namely the electric flux carried by them

it also implies that heavy baryons in SYM shaped like Δ (lattice anyone?)



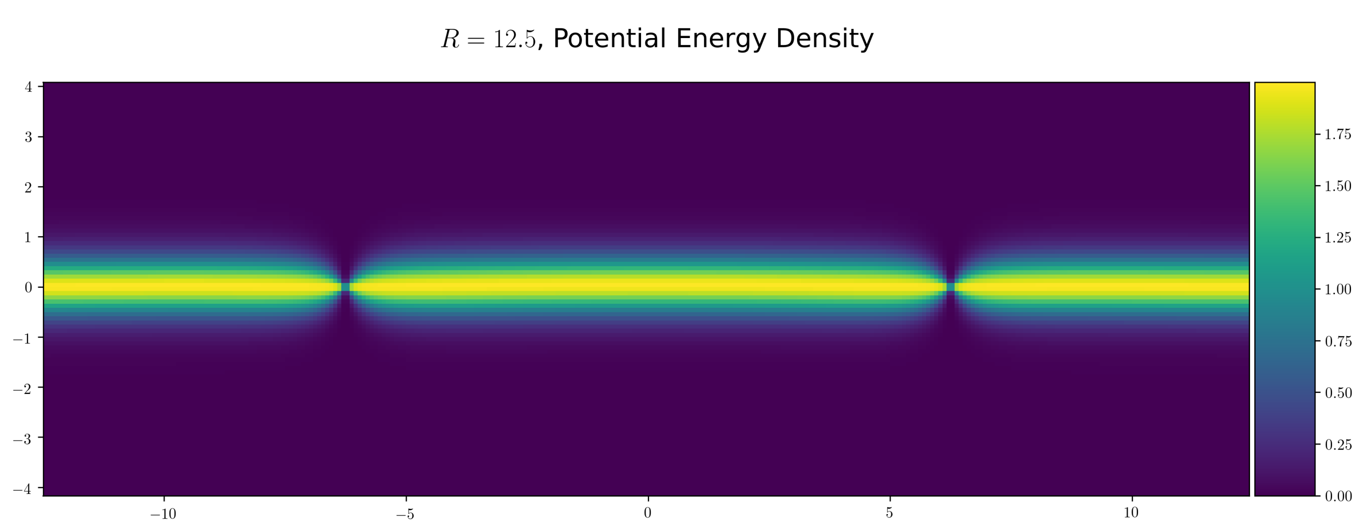


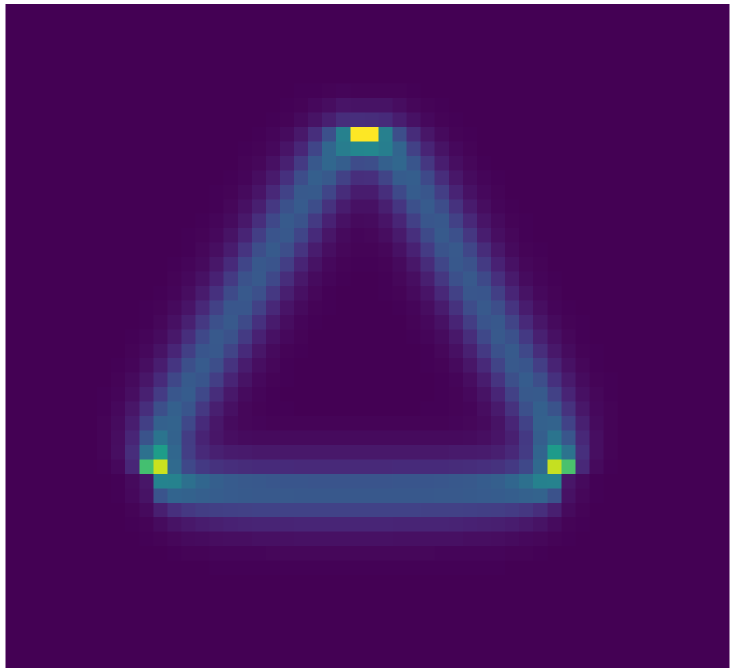
this talk:

- *examples of nontrivial DWs, where mechanism of anomaly inflow can be described semiclassically*

- *walls in high-T phase exhibit features of low-T phase and v.v.*

- *related [for sure or perhaps…] to confinement mechanism*





conclusion:

there is more to these anomalies than we have found out so far