$T\bar{T}$ and the mirage of a bulk cutoff

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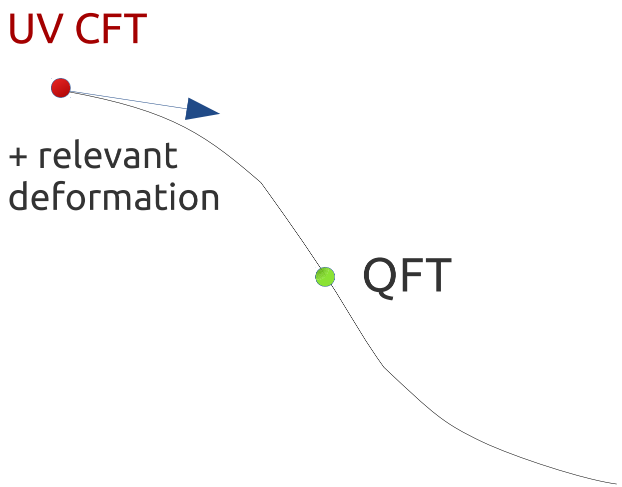
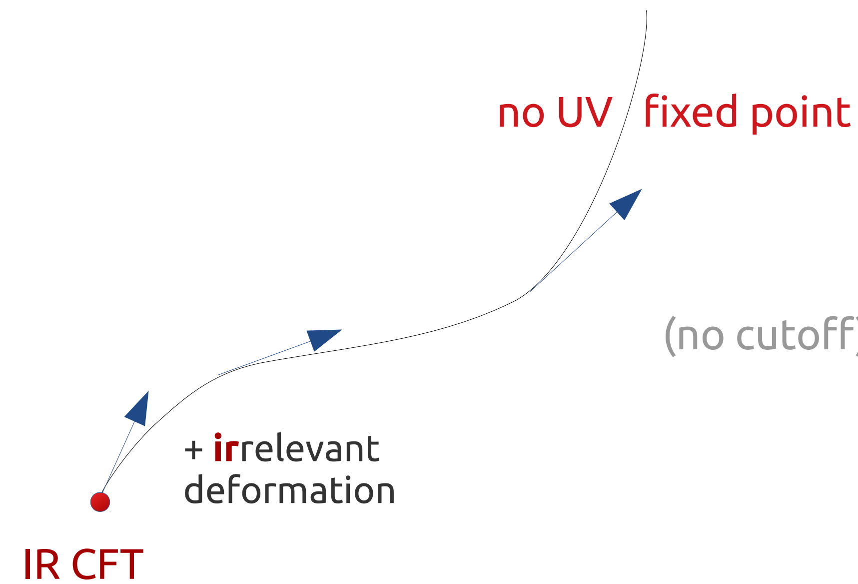
based on 1906.11251: with Ruben Monten

Motivation

Usual framework: local, UV complete QFTs



Examples of non-local, UV complete QFTs



- Quantum gravity ?
- Holography in non-asymptotically AdS spacetimes

$T\bar{T}$ – deformed CFTs

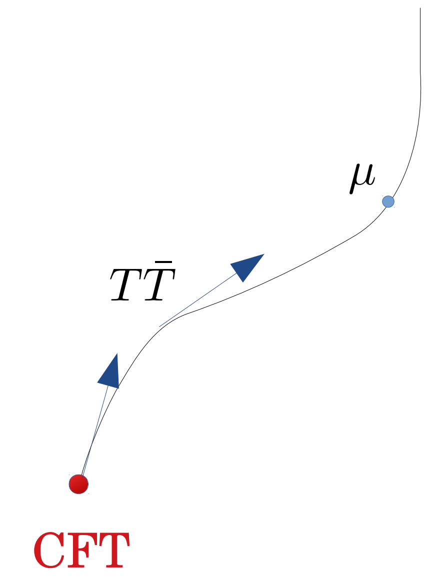
- universal deformation of 2d CFTs/QFTs

$$\frac{\partial S}{\partial \mu} = \int d^2 z \, \underbrace{(T_{zz} T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2)_\mu}_{\text{“}T\bar{T}\text{”}}$$

- deformation irrelevant (dim = (2,2)) but integrable

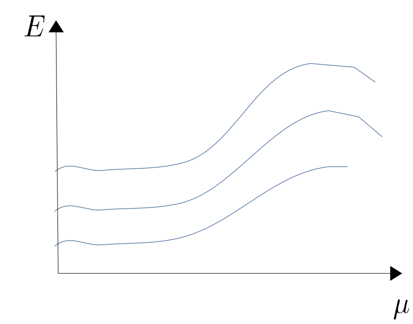
  finite size spectrum, partition function, thermodynamics

  Smirnov & Zamolodchikov, Cavaglia et al, Cardy



- energy levels smoothly deformed

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E_0}{R} + \frac{4\mu^2 P_0^2}{R^2}}\right)$$



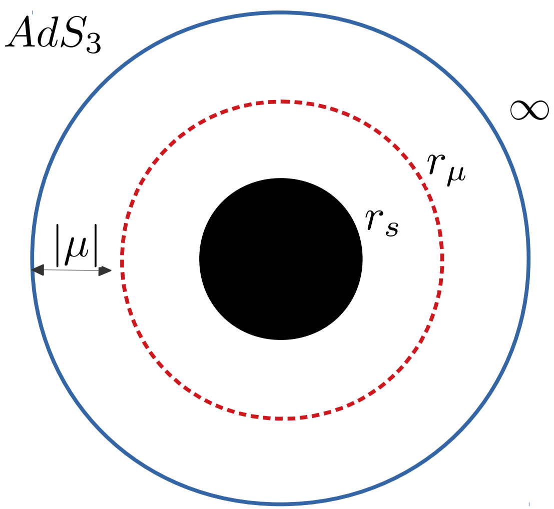
- deformed theory non-local (scale μ) but argued UV complete

  S-matrix (2→ 2): $\mathcal{S}_\mu = e^{\frac{i\mu s}{4}} \mathcal{S}_0$ Dubovsky et al.

$T\bar{T}$ and the finite bulk cutoff

- energy spectrum of $T\bar{T}$-deformed CFTs with $\mu < 0$ exactly matches energy of a ``black hole in a box''

McGough, Mezei, Verlinde '16



$$E(\mu) = \frac{R}{2|\mu|}\left(1 - \sqrt{1 - \frac{4|\mu|M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$$

$=$ energy measured by an observer on a fixed radial slice r_μ

- imaginary energies for large M ⟺ $r_s > r_\mu$

  at μ, R fixed

- matter fields ? $\mu > 0$?

Finite bulk cutoff usually associated with integrating out degrees of freedom in bulk/boundary

(holographic Wilsonian RG) ⟶ ⟵ Integrability & UV completeness of $T\bar{T}$?

This talk

- first principles derivation of the holographic dictionary for T$\bar{\text{T}}$ - deformed CFTs for both signs of μ

- as expected for double trace: AdS_3 with
  - mixed boundary conditions at ∞ for the metric
  - unchanged (Dirichlet) for the matter fields

- for $\mu < 0$ and pure gravity and on-shell $\approx$ Dirichlet at finite radius r_μ independent of the mass

  ? pure coincidence

- when matter field profiles (vevs) are present, no special reinterpretation in terms of Dirichlet at finite radius

Double-trace deformations in AdS/CFT

- $T\bar{T}$ is a double-trace deformation → mixed boundary conditions for dual bulk fields

- e.g. scalar

$$\underbrace{\Phi = \mathcal{J}\, z^{d-\Delta}}_{\text{source (fixed)}} + \ldots + \underbrace{\langle \mathcal{O} \rangle\, z^{\Delta}}_{\text{vev (fluctuates)}} + \ldots$$

- $S_\mu = S_{CFT} + \mu \int \mathcal{O}^2$

- 1) variational principle (equivalent to Hubbard-Stratonovich, only uses large N field theory)

$$\delta S_\mu = \delta S_{CFT} - \delta\left(\mu \int \mathcal{O}^2\right) = \int \mathcal{O}\delta\mathcal{J} - \mu \int \delta\mathcal{O}^2 = \int \mathcal{O}\,\delta(\underbrace{\mathcal{J} - 2\mu\mathcal{O}}_{\text{new source } \tilde{\mathcal{J}}})$$

new vev

- 2) translate into boundary conditions on the bulk field

Sources and vevs in $T\bar{T}$ - deformed CFTs

- variational principle approach:

$$\delta S_{CFT} - \Delta\mu\,\delta S_{T\bar{T}} = \int d^2x \underbrace{\sqrt{\gamma}\,T_{\alpha\beta}\,\delta\gamma^{\alpha\beta}}_{\text{CFT}} - \Delta\mu \int d^2x\,\delta\underbrace{(\sqrt{\gamma}\,\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}T_{\alpha\beta}T_{\gamma\delta})}_{\text{deformation}} = \int d^2x \underbrace{\sqrt{\tilde{\gamma}}\,\tilde{T}^{\alpha\beta}\delta\tilde{\gamma}^{\alpha\beta}}_{\text{new sources \& vevs}}$$

- flow equations

$$\partial_\mu\gamma_{\alpha\beta} = -2(T_{\alpha\beta} - \gamma_{\alpha\beta}\,T) \equiv -2\,\hat{T}_{\alpha\beta} \qquad \partial_\mu\hat{T}_{\alpha\beta} = -\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta} \qquad \partial_\mu(\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}) = 0$$

- exact solution

$$\boxed{\begin{aligned}\gamma_{\alpha\beta}(\mu) &= \gamma_{\alpha\beta}(0) - 2\mu\hat{T}_{\alpha\beta}(0) + \mu^2\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)\\ \hat{T}_{\alpha\beta}(\mu) &= \hat{T}_{\alpha\beta}(0) - \mu\,\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)\end{aligned}}$$

  - both signs of μ
  - other (matter) vevs can be on
  - large N field theory

- sources for matter operators unaffected at linear level $\boxed{\sqrt{\gamma}\,\mathcal{O}\,\delta\mathcal{J} = \sqrt{\tilde{\gamma}}\,\tilde{\mathcal{O}}\,\delta\tilde{\mathcal{J}}}$

- flow equations $\Rightarrow$ $\partial_\mu(R\sqrt{\gamma}) = 0\,, \qquad \partial_\mu(\sqrt{\gamma}\,\mathcal{O}_{T\bar{T}}) = 0\,, \qquad T(\mu) = \dfrac{c}{24\pi}R(\mu) - \mu\,\mathcal{O}_{T\bar{T}}(\mu)$

The $T\bar{T}$ holographic dictionary

- new sources $\gamma_{\alpha\beta}(\mu) = \gamma_{\alpha\beta}(0) - 2\mu\hat{T}_{\alpha\beta}(0) + \mu^2\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)$
- new vevs $\hat{T}_{\alpha\beta}(\mu) = \hat{T}_{\alpha\beta}(0) - \mu\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)$

large N field theory

Holography → Fefferman Graham expansion

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \left(\frac{g^{(0)}_{\alpha\beta}}{\rho} + g^{(2)}_{\alpha\beta} + \ldots\right) dx^\alpha dx^\beta$$

$$\underbrace{g^{(0)}_{\alpha\beta} \leftrightarrow \gamma_{\alpha\beta}(0)\,, \qquad g^{(2)}_{\alpha\beta} \leftrightarrow 8\pi G\ell\,\hat{T}_{\alpha\beta}(0)}^{\text{in original CFT}}$$

- $\gamma_{\alpha\beta}(\mu)$ fixed ⟺ mixed non-linear boundary conditions for the AdS_3 metric

$$\boxed{g^{(0)}_{\alpha\beta} - \frac{\mu}{4\pi G\ell}g^{(2)}_{\alpha\beta} + \frac{\mu^2}{(8\pi G\ell)^2}g^{(2)}_{\alpha\gamma}g^{(0)\gamma\delta}g^{(2)}_{\delta\beta} = fixed}$$

- stress tensor expectation value non-linearly related to $g^{(2)}_{\alpha\beta}$
- matter field boundary conditions unchanged, since $\mathcal{J}(\mu) = \mathcal{J}(0)$

Pure gravity

- pure 3d gravity → Fefferman-Graham expansion truncates

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{g^{(0)}_{\alpha\beta} + \rho\, g^{(2)}_{\alpha\beta} + \rho^2\, g^{(4)}_{\alpha\beta}}{\rho}\, dx^\alpha dx^\beta \qquad\qquad g^{(4)}_{\alpha\beta} = \frac{1}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta}$$

- mixed boundary conditions at ∞ → coincide precisely with Dirichlet at $\boxed{\rho_c = -\frac{\mu}{4\pi G \ell}}$

$\mu < 0$ coincides with McGough, Mezei, Verlinde

- deformed stress tensor → coincides precisely with Brown-York + counterterm at ρ_c

$$T_{\alpha\beta}(\mu) = \hat{T}_{\alpha\beta}(\mu) - \gamma_{\alpha\beta}(\mu)\hat{T}(\mu) = -\frac{1}{8\pi G}(K_{\alpha\beta} - K g_{\alpha\beta} - g_{\alpha\beta}/\ell)(\rho_c)$$

- fixed by variational principle → no ambiguity!

The "asymptotically mixed" phase space

- most general pure gravity solution with $\gamma_{\alpha\beta}(\mu) = \eta_{\alpha\beta}$ (T$\bar{\text{T}}$ on flat space with coordinates U, V)

- $R(\mu) = 0 \;\Rightarrow\; R(0) = 0 \;\Rightarrow\; \gamma_{\alpha\beta}(0) dX^\alpha dX^\beta = du dv$ for some auxiliary coordinates u, v

- in these coordinates, the most general bulk solution is

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{du dv}{\rho} + \underline{\mathcal{L}(u)} du^2 + \underline{\bar{\mathcal{L}}(v)} dv^2 + \rho \mathcal{L}(u) \bar{\mathcal{L}}(v)$$

- boundary condition: $ds_c^2 = \rho_c ds^2(\rho = \rho_c) = dU dV$ → relation between u, v and T$\bar{\text{T}}$ coordinates U, V

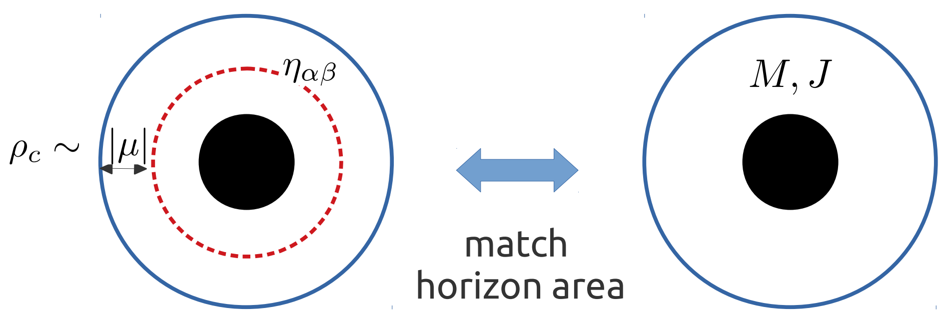
$$U = u + \rho_c \int^v dv' \bar{\mathcal{L}}(v') \,, \qquad V = v + \rho_c \int^u du' \mathcal{L}(u')$$

- $g^{(0)}, g^{(2)}$ metric above in the U, V coordinate system (asymptotically mixed)

- most general solution parametrized by two arbitrary functions of the state-dependent coordinates u, v

Energy match

- high energy eigenstates → black holes : can we reproduce $E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$?
- deformed black hole: constant $\mathcal{L}_\mu\ ,\bar{\mathcal{L}}_\mu$; energy $E(\mu) = \int d\phi\, T_{TT}(\mu)$
- relation to undeformed $M \sim \mathcal{L}_0 + \bar{\mathcal{L}}_0\ ,\quad J \sim \mathcal{L}_0 - \bar{\mathcal{L}}_0$?



deformed state

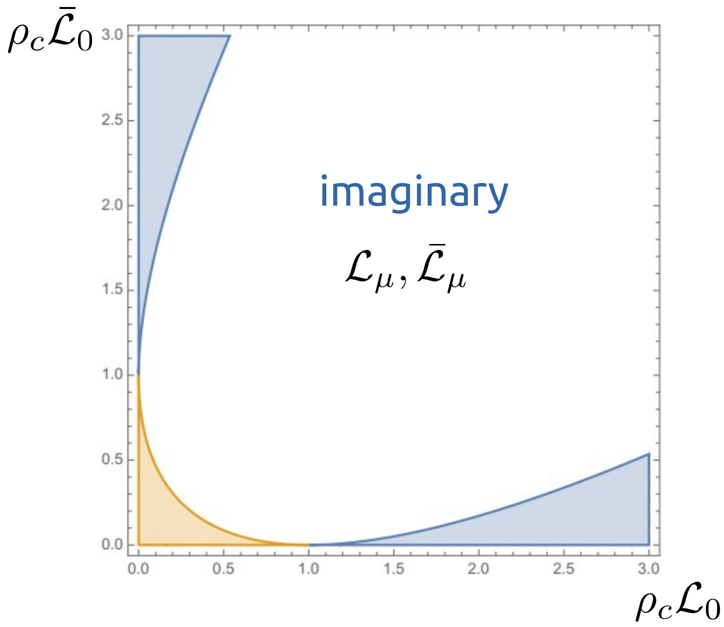
undeformed state

- energy eigenstates smoothly deformed
  → unchanged degeneracy
- angular mometum quantized → unchanged
- perfect match for both signs of μ ✓

- McGough et al computed energy on undeformed BTZ at Schwarzschild coordinate $r_c \sim |\mu|^{-\frac{1}{2}}$
- map: $r^2(\rho) = \frac{(1 + \mathcal{L}_\mu(\rho - \rho_c) - \mathcal{L}_\mu\bar{\mathcal{L}}_\mu\rho\rho_c)(1 + \bar{\mathcal{L}}_\mu(\rho - \rho_c) - \mathcal{L}_\mu\bar{\mathcal{L}}_\mu\rho\rho_c)}{\rho(1 - \rho_c^2\mathcal{L}_\mu\bar{\mathcal{L}}_\mu)^2}$ $\qquad r^2(\rho_c) = \frac{1}{\rho_c}$

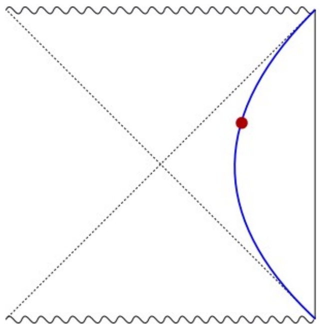
Imaginary energies

- for $\mu < 0$ the energy $E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$ can become imaginary



- orange region ~ energies measured by observer outside outer horizon

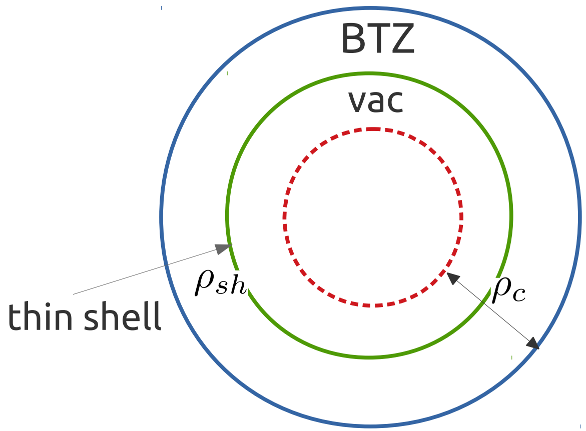
- blue region ~ energies measured by observer inside inner horizon ($g^{(0)}$ has CTCs)



- McGough et al picture still valid in typical states

Adding matter

- difference between mixed at infinity and Dirichlet at finite radial distance for $\mu < 0$



- shell outside ρ_c

  → mixed b.c. picture only depends on the asymptotic behaviour of the metric = BTZ ⇒ energy matches field theory ✓

  → Dirichlet b.c. yield vacuum answer ✗

- configurations outside this surface ✓ → 2d $T\bar{T}$ describes entire spacetime : UV completeness & integrability

- imaginary energies ? → breakdown of coordinate transformation used to make $\gamma_{\alpha\beta}(\mu) = \eta_{\alpha\beta}$ which only depends on the asymptotic value of the metric (no details of the interior matter)

Take-home: universal formula for energy ↔ universal asymptotic behaviour

Asymptotic symmetries

- diffeomorphisms that preserve asymptotically mixed boundary conditions
- parametrized by two arbitrary functions $f(u)\,,\; g(v)$ & strongly background dependent ($\mathcal{L}(u)\,,\; \bar{\mathcal{L}}(v)$)

$$U = u + \rho_c \int^v dv' \bar{\mathcal{L}}(v')\,, \qquad V = v + \rho_c \int^u du' \mathcal{L}(u')$$

state-dependent coordinates

- **NB:** on a purely gravitational background and for $\mu < 0$: ≈ asymptotic symmetries of a finite box
- asymptotic symmetry group: $Virasoro(u) \times Virasoro(v)$ with same **c** as in CFT

non-trivial

→ compare with $U(1)_L \times U(1)_R$ naively preserved by T$\overline{\text{T}}$

- non-local, "state-dependent" deformation of original Virasoro
- ASG ↔ symmetries of field theory: field theoretical interpretation ??

Conclusions

Summary and future directions

- large N holographic dictionary for $T\bar{T}$ – deformed CFTs

  → derivation from variational principle: precision holography

  → both signs of μ and in presence of matter

  → mixed boundary conditions at infinity for the metric (no finite bulk cutoff)

  → ASG: non-local & state-dependent generalization of Virasoro

Future directions:

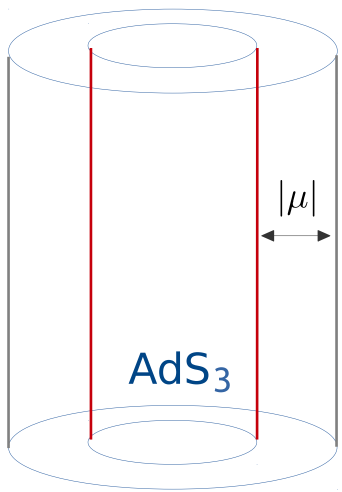
- precision match between all observables (e.g. correlation functions)? can holography help?
- 1/N corrections?
- field theory interpretation of the Virasoro symmetries → constraints on the theory/ non-locality?
- generic $\underbrace{\text{single trace}}_{\text{non- aAdS spacetimes}}$ generalisations of these UV-complete irrelevant deformations?

Thank you !

Holography: why interesting

Double-trace $T\bar{T}$ deformation

- universal , $\forall$ large c CFT



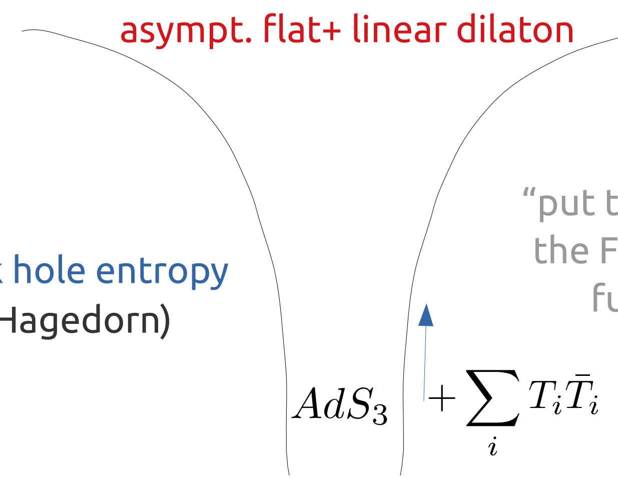
AdS_3 with mixed bnd. conditions at ∞

$\mu < 0 \quad \approx$ Dirichlet at finite radius

McGough, Mezei, Verlinde

Single-trace $T\bar{T}$ deformation $\sum_{i=1}^{p} T_i \bar{T}_i$

- near horizon NS5-F1 $\rightarrow$ $\mathcal{M}^p/S_p$



black hole entropy
(Hagedorn)

"put the 1 back in the F1 harmonic function"

Giveon, Itzhaki, Kutasov

Generalisations?

- tractable single-trace irrelevant flows with no UV fixed point?