Domain Walls in Super-QCD

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Non-Perturbative Methods in Quantum Field Theory
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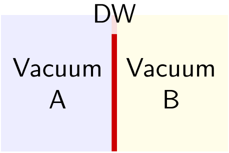
with Vladimir Bashmakov, Sergio Benvenuti, Matteo Bertolini, Paolo Spezzati

arXiv: 1812.04645 and in progress

3D/4D connection:



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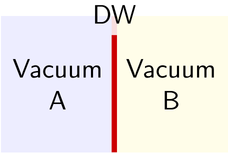

4D theory with multiple gapped vacua $\Rightarrow$ Domain Walls separating them with 3D worldvolume theory

- Relation between 4D and 3D dynamics, their symmetries, anomalies, ...
- Changing "bulk" parameters $\rightarrow$ transitions in the worldvolume theory
  - ⋆ 3D transition with no bulk transition
- Different descriptions in the bulk $\longleftrightarrow$ Different phases or dualities in 3D
- Beautifully applied to YM and QCD [Gaiotto, Kapustin, Komargodski, Seiberg 17]

[Gaiotto, Komargodski, Seiberg 17; di Vecchia, Rossi, Veneziano, Yankielowicz 17]

Multiple gapped vacua:

- spontaneously broken discrete symmetry
- tuning to 1st order phase transition
- supersymmetry

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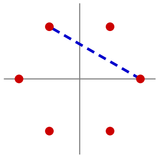
We look at 4D $\mathcal{N} = 1$ (massive) SQCD:



- Multiple vacua for generic parameters
- BPS domain walls host a 3D $\mathcal{N} = 1$ worldvolume theory

  SUSY $\Rightarrow$ 2^{nd} order phase transitions on DWs

A lot was already done, but not complete classification

[Acharya, Armoni, de Carlos, Dvali, Giveon, Hindmarsh, Hollowood, Israel, Kaplunovsky, Kovner, McNair, Moreno, Niarchos, Poppitz, Ritz, Shifman, Smilga, Sonnenshein, Vafa, Vainshtein, Vaselov, Witten, Yankielowicz, . . .]

4D $\mathcal{N} = 1$ (massive) Super-QCD

$SU(N)$ SQCD with F flavors $Q, \widetilde{Q}$

- Restrict to $F < N$: only mesons (not baryons) are relevant

  $\rightarrow$ gauge-invariant meson superfields $M = \widetilde{Q}Q$

- Parameters: $\Lambda^{3N-F} = \mu^{3N-F} \, e^{-\frac{8\pi^2}{g(\mu)^2} + i\theta}$, mass matrix m_{4d} $(F \times F)$

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Without mass m_{4d}: *classical* mesonic moduli space [Affleck, Dine, Seiberg 84]

Dynamically generated superpotential $W_{\mathsf{ADS}} = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$

$\rightarrow$ runaway behavior

Add flavor-invariant diagonal mass $m_{\mathsf{4d}} \propto \mathbb{1}_F$ $\rightarrow$ N massive vacua

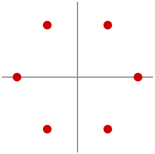
4D $\mathcal{N} = 1$ (massive) Super-QCD

$$W_{\text{eff}} = m_{4d}\,\mathrm{Tr}\,M + (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

- Symmetries: $\mathbb{Z}_{2N} \times SU(F) \times U(1)_B$

N gapped vacua from spontaneous R-symmetry breaking $\mathbb{Z}_{2N} \to \mathbb{Z}_2$

$$\langle M \rangle = \widetilde{M}\,\mathbb{1}_F \qquad \langle \lambda\lambda \rangle = m_{4d}\,\widetilde{M} \qquad \widetilde{M} = \left(\frac{\Lambda^{3N-F}}{m_{4d}^{N-F}}\right)^{1/N}$$



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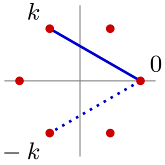
- Domain walls from j^{th} vacuum to $(j+k)^{\text{th}}$ vacuum

Broken R-symmetry: k-walls are all equivalent

- An $(N-k)$-wall is parity reversal of a k-wall



- 4D $\mathcal{N} = 1$ superalgebra admits a two-brane charge:

[Azcarraga, Gauntlett, Izquierdo, Townsend 89; Dvali, Shifman 96]

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\sigma^{\mu}_{\alpha\dot\alpha}\, P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \sigma^{\mu\nu}_{\alpha\beta}\, Z_{\mu\nu} \qquad \text{and c.c.}$$

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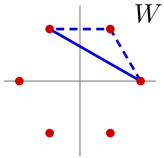
$\Rightarrow$ BPS domain walls

- Protected tension in terms of "central charge" [Abraham, Townsend 91; Cecotti, Vafa 92]

$$T = |Z| \,, \qquad \boxed{Z = 2\,\Delta W}$$



Tension is controlled by the shift of superpotential

$\Rightarrow$ ⋆ Parallel multiple walls decay to a bound state

⋆ There can be degenerate walls connecting the same two vacua

3D phase transitions

- Large mass $m_{4d} \gg \Lambda$: $SU(N)$ Super-Yang-Mills

  Domain walls support TQFT (gapped) [Acharya, Vafa 01]

- Small mass $m_{4d} \ll \Lambda$:

  almost-weakly-coupled Wess-Zumino description on *mesonic space*

  Inequivalent degenerate walls, with 3d Goldstones + TQFT

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- In the two regimes we see very different walls:

  phase transition at $m_{4d} = m_*$

We seek a 3D $\mathcal{N} = 1$ worldvolume description

Domain walls in Super-Yang-Mills

$SU(N)$ SYM

- N gapped vacua

Gaugino condensate $\langle\lambda\lambda\rangle = \Lambda^3\, e^{\frac{2\pi i}{N}k}$ breaks R-symmetry $\mathbb{Z}_{2N} \to \mathbb{Z}_2$

- 3D $\mathcal{N}=1$ low-energy theory on k-wall: [Acharya, Vafa 01]

[also Armoni, Hollowood 05; *ibid.* 06; Bashmakov, Gomis, Komargodski, Sharon 18]

3D $\mathcal{N}=1$ $U(k)_{N-\frac{k}{2},N}$ + singlet Φ_0

$\downarrow$

$U(k)_{N-k,N}$ Chern-Simons TQFT

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$$\text{3D} \quad \mathcal{N} = 1 \quad U(k)_{N-\frac{k}{2},N} \qquad + \quad \text{singlet } \Phi_0$$

$$\downarrow$$

$$U(k)_{N-k,N} \qquad \text{Chern-Simons TQFT}$$

- k-wall $\xleftrightarrow{\text{parity reversal}}$ $(N-k)$-wall from level-rank duality

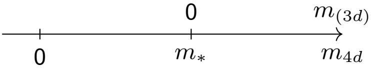
Domain walls in Super-QCD

$SU(N)$ massive SQCD with F flavors

Our proposal for 3D worldvolume theory:

$$\text{3D} \quad \mathcal{N}=1 \quad U(k)_{N-\frac{k}{2}-\frac{F}{2},N-\frac{F}{2}} \quad \text{with } F \text{ flavors of } X\text{'s}$$

$$\mathcal{W} = \mathrm{Tr}\, X^\dagger X X^\dagger X + \alpha \big(\mathrm{Tr}\, X^\dagger X\big)^2 + m_{(3d)}\, \mathrm{Tr}\, X^\dagger X$$



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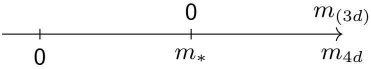

⋆ k-walls and $(N-k)$-walls related by parity reversal: [Choi, Roček, Sharon 18]

$$U(k)_{N-\frac{k+F}{2},N-\frac{F}{2}} \text{ with } F \text{ flavors} \quad \leftrightarrow \quad U(N-k)_{-\frac{N+k-F}{2},-N+\frac{F}{2}} \text{ with } F \text{ flavors}$$

Analysis of vacua

- $m_{(3d)} > 0$: unique vacuum at $X^\dagger X = 0$ $\quad\rightarrow\quad$ $\mathcal{N} = 1$ $\;U(k)_{N-\frac{k}{2},N}$ (AV)

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- $m_{(3d)} > 0$: unique vacuum at $X^\dagger X = 0 \quad \rightarrow \quad \mathcal{N}=1 \;\; U(k)_{N-\frac{k}{2},N}$ (AV)
- $m_{(3d)} < 0$: multiple vacua $\quad X^\dagger X \propto \begin{pmatrix} \mathbb{1}_J & \\ & 0 \end{pmatrix}$ flavor symmety breaking

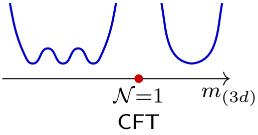
$$\mathcal{N}=1 \qquad U(k-J)_{N-\frac{k}{2}-F+\frac{J}{2},\,N-F} \quad \times \quad \text{NLSM} \quad \frac{U(F)}{U(J)\times U(F-J)}$$

with $\quad \max(0, F+k-N) \leq J \leq \min(k,F)$

- J-vacua: inequivalent degenerate BPS k-walls between same two 4D vacua

- SUSY $\Rightarrow$ 2^{nd} order phase transition (CFT)

  Multiple vacua coalesce into one at a single point



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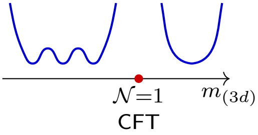

- Enhanced 3D $\mathcal{N} = 2$ SUSY (SCFT) for special values

  $F = 1$ or $k = 1$: only one quartic superpotential term

  At large CS level can be seen perturbatively

  [Avdeev, Grigorev, Kazakov 92]

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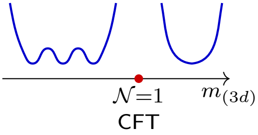

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- Enhanced 3D $\mathcal{N}=4$ SUSY for $SU(2)$, $F=1$, $k=1$ [Gang, Yamazaki 18]

$$\mathcal{N}=1 \quad U(1)_{\pm\frac{3}{2}} \text{ with 1 flavor} \quad \longleftrightarrow \quad \mathcal{N}=1 \quad SU(2)_{\pm\frac{3}{2}} \text{ with 1 flavor}$$

4D construction of domain walls

- For $m_{4d} \ll \Lambda$: 4D vacua lie in the weakly-coupled Higgsed phase

Low energy effective theory: Wess-Zumino (WZ) model on mesonic space

$$\mathcal{K} = 2\,\mathrm{Tr}\,\sqrt{\overline{M}M} \qquad\qquad W = m_{4d}\,\mathrm{Tr}\,M + W_{\mathsf{ADS}}(M)$$

$\Rightarrow$ Construct WZ-type walls

Valid as long as we remain in the Higgsed phase throughout the wall

⋆ *Caveat*: unbroken $SU(N-F)$ SYM theory on mesonic space

Domain walls in WZ theory

[Fendley, Mathur, Vafa, Warner 90; Cecotti, Vafa 92]

BPS domain wall equation (central charge $Z = 2\,\Delta W = e^{i\gamma}|Z|$):

$$\mathcal{K}_{a\bar{b}}\;\partial_x \Phi^a = e^{i\gamma}\;\frac{\partial \overline{W}}{\partial \overline{\Phi}^{\bar{b}}}$$

where Φ^a are the meson components $M^i{}_j$

$$W = m\,\mathrm{Tr}\,M + (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

- $F = N - 1$: WZ model on mesonic space (no SYM sector)

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- $F < N-1$: unbroken $SU(N-F)$ SYM on mesonic space

$$\Lambda^{3(N-F)}_{\text{unbroken}} = \frac{\Lambda^{3N-F}}{\det M} \qquad W_{\text{unbroken}} = (N-F)\big(\Lambda^{3(N-F)}_{\text{unbroken}}\big)^{1/(N-F)}$$

Multivaluedness of W_{ADS} from $N-F$ vacua of unbroken gauge theory

$\Rightarrow \quad N-F$ sheets over each point on mesonic space

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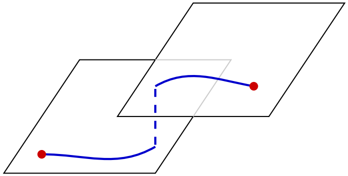
Size of walls:

$$\ell_{\text{WZ}} \sim \frac{M}{\partial_x M} \sim \frac{1}{m_{4d}} \gg \ell_{SYM} \sim \frac{1}{\Lambda_{\text{unbroken}}} \sim \frac{1}{m_{4d}^{F/3N}\Lambda^{1-F/3N}}$$

“Hybrid” SQCD walls

Hybrid walls: WZ type + “instantaneous” jumps from one sheet to another



⋆ 3D theory:

AV topological sector associated to jump Δ in unbroken SYM

$$\mathcal{N} = 1 \quad U(\Delta)_{N-F-\frac{\Delta}{2}, N-F}$$

$\times$

Goldstone bosons for broken flavor symmetry

NLSM

Results

The various walls can be studied:

- algebraically — for $M(x) \propto \mathbb{1}_F$
- numerically

⋆ E.g. at small N:

$SU(2)$	$F=1$	$k=1:$ gap, gap

$SU(3)$	$F=2$	$k=1:$ gap, $\mathbb{P}^1$
	$F=1$	$k=1:$ $U(1)_2$, gap

$SU(4)$	$F=3$	$k=1:$ gap, $\mathbb{P}^2$	$k=2:$ $\mathbb{P}^2$, $\mathbb{P}^2$
	$F=2$	$k=1:$ $U(1)_2$, $\mathbb{P}^1$	$k=2:$ gap, $U(1)_2 \times \mathbb{P}^1$, gap
	$F=1$	$k=1:$ $U(1)_3$, gap	$k=2:$ $U(1)_{-3}$, $U(1)_3$

⋆ All cases analyzed match with 3D prediction

Some generalizations

⋆ 4D $SU(N)$ SQCD with $F = N$ flavors [in progress]

- Baryons $B = Q^N \quad \widetilde{B} = \widetilde{Q}^N$ enter into play
- Deformed moduli space $\det M - B\widetilde{B} = \Lambda^{2N}$ [Seiberg 94]

3D theory at $m_{(3d)} < 0 \quad \Rightarrow \quad U(1)_0 \times$ NLSM $\frac{U(N)}{U(J)\times U(N-J)}$

Some DWs spontaneously break baryonic symmetry (3D top. symm.)

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⋆ 4D $Sp(N)$ SQCD with $F < N+1$ or $F = N+1$ flavors

Similar results:

- $F = 0$: TQFT $Sp(k)_{N+1-k}$
- $F > 0$: $\mathcal{N} = 1$ $Sp(k)_{N-\frac{F+k-3}{2}}$ with F fund. and $\mathcal{W}$

Summary

- Proposed 3D description of DWs in 4D $SU(N)$ SQCD with $F \leq N$ flavors
- Predicts 2^{nd} order phase transition for $m_{4d} = m_*$ and intricate zoo of walls
- Predicts supersymmetry enhancement
- Large and small m_{4d} limits reproduced by 4D QFT arguments

Some open questions

- More flavors: $F > N$ (baryons, Seiberg duality, ...)

  [FB, Benvenuti, Bertolini, Spezzati: in progress]
- Other gauge groups ($Spin(N)$, quivers, ...) and representations
- Domain wall junctions: 2D $\mathcal{N} = (0,1)$ [*cfr.* Gaiotto 13]