Vacuum structure of 2d adjoint QCD
– anomaly, mod 2 index, and semiclassics –

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References: 1908.09858[hep-th]

2d adjoint QCD

We consider 2d $SU(N)$ YM + one adjoint Majorana fermion:

$$S = \frac{1}{2g^2}\int_{M_2} \mathrm{tr}[G \wedge \star G] + \int_{M_2} \mathrm{tr}[\psi_+ D_+ \psi_+ + \psi_- D_- \psi_- + m\psi_+\psi_-]$$

In this talk, we shall elucidate its ground-state properties based on

- careful analysis of symmetries and 't Hooft anomalies, and
- explicit exploration of dynamics with semiclassical analysis on small $\mathbb{R} \times S^1$.

Motivations

There are some similarities with 4d confining gauge theories:

- Theory is not solvable.
- Theory has a $\mathbb{Z}_N$ center symmetry. Confined or deconfined?
  - It's an interesting question if 1-form symmetry in 2d can be spontaneously broken.
  - Indeed, most likely, 1-form symmetry in 2d is unbroken, unless anomaly requires. (cf. Gaiotto, Kapustin, Seiberg, Willet, '14)
- 2d pure YM is also good, but it does not have any propagating modes by gauge invariance.
  2d adjoint QCD has $O(N^2)$ microscopic DOF thanks to adjoint Majorana fermions ψ.
- In large-N, there are infinitely many Regge-like trajectories.
  ('t Hooft model (i.e. 2d YM + fundamental) only has one.)

Main result

We find various $\mathbb{Z}_2$ anomaly for the symmetry at $m=0$:

$$G = \underbrace{\mathbb{Z}_N^{[1]}}_{\text{center sym.}} \rtimes \underbrace{(\mathbb{Z}_2)_C}_{\text{charge conj.}} \times \underbrace{(\mathbb{Z}_2)_F}_{(-1)^F} \times \underbrace{(\mathbb{Z}_2)_\chi}_{\text{discrete chiral}} .$$

Minimal requirement of anomaly matching shows

- Spontaneous chiral symmetry breaking,

$$(\mathbb{Z}_2)_\chi \to 1,$$

  for $N=4n, 4n+2, 4n+3$ but not for $N=4n+1$.
- For odd N, center symmetry is unbroken.
  For even N, partial deconfinement is required,

$$\mathbb{Z}_N^{[1]} \to \mathbb{Z}_{N/2}^{[1]}$$

Some remarks

- 2d adjoint QCD has the marginally relevant four-fermion interactions,
$$(\mathrm{tr}[\psi_+\psi_-])^2,\ \mathrm{tr}[\psi_+\psi_+\psi_-\psi_-].$$
Adding these does not break any symmetry of the Lagrangian.

- Since our analysis is based only on symmetry, our result generalizes to any local deformations like this!
(assuming mass gap)

- Moreover, the semiclassical analysis on small $\mathbb{R}\times S^1$ prefers minimal scenario of anomaly matching.

Symmetry and Anomaly

Symmetry of adjoint QCD

(Internal) Symmetry of 2d adjoint QCD with $m=0$:

$$G = \underbrace{\mathbb{Z}_N^{[1]}}_{\text{center sym.}} \rtimes \underbrace{(\mathbb{Z}_2)_C}_{\text{charge conj.}} \times \underbrace{(\mathbb{Z}_2)_F}_{(-1)^F} \times \underbrace{(\mathbb{Z}_2)_\chi}_{\text{discrete chiral}} .$$

The first three factors are the vector-like symmetry:

- $\mathbb{Z}_N^{[1]}$: $W(C) \mapsto \mathrm{e}^{2\pi \mathrm{i}/N} W(C)$.
- $(\mathbb{Z}_2)_C$: $a_{ij,\mu} \mapsto -a_{ji,\mu}$, $\psi_{ij} \mapsto \psi_{ji}$.
- $(\mathbb{Z}_2)_F$: $\psi \mapsto -\psi$.

The last one is the chiral symmetry:

- $(\mathbb{Z}_2)_\chi$: $\psi_+ \mapsto \psi_+$ and $\psi_- \mapsto -\psi_-$.

Mixed Anomaly

We will see that the partition function $\mathcal{Z}$ transforms as

$$(\mathbb{Z}_2)_\chi : \mathcal{Z} \to (-1)^\zeta \mathcal{Z},$$

under the background gauge field (or twisted b.c.) of vector-like symmetry.

List of mixed 't Hooft anomalies: ✓ if $(-1)^\zeta = -1$.

Anomalous symmetry	$N = 4n$	$4n+1$	$4n+2$	$4n+3$
$\mathbb{Z}_N^{[1]} \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_C \times (\mathbb{Z}_2)_\chi$			✓	✓

(Note: $\mathbb{Z}_N^{[1]} \times (\mathbb{Z}_2)_\chi$-anomaly was partly discovered in Lenz, Shifman, Thies (hep-th/9412113))

Absence of familiar Dirac index

In 4d gauge theory (also in 2d U(1) gauge theory), we are familiar with the index theorem, stating that

$$\text{Imbalance of chirality} = \text{Topological charge}$$

BUT $SU(N)$ gauge field is traceless, and this index theorem is not useful:

$$\#(\text{Zero modes with + chirality}) - \#(\text{Zero modes with - chirality})$$
$$= \frac{1}{2\pi}\int \mathrm{tr}(G) = 0$$

$\Rightarrow$ No imbalance between ψ_+ and ψ_-

Mod 2 index & $\mathbb{Z}_2$ mixed anomaly

Theorem (Mod 2 index theorem)

Let us set

$$\begin{aligned}\zeta &= \#(\textit{Zero modes with } + \textit{ chirality}) \\ &= \#(\textit{Zero modes with - chirality}).\end{aligned}$$

Then, ζ is a topological invariant mod 2.

The $\mathbb{Z}_2$ topological invariant $(-1)^\zeta$ determines the mixed anomaly!

$$\mathcal{D}\psi \sim (\mathrm{d}\psi_{+(0)})^\zeta (\mathrm{d}\psi_{-(0)})^\zeta \prod_{i:\lambda_i\neq 0} \mathrm{d}\psi_{+i}\mathrm{d}\psi_{-i},$$

$$(\mathbb{Z}_2)_\chi : \mathcal{D}\psi \mapsto (-1)^\zeta \mathcal{D}\psi.$$

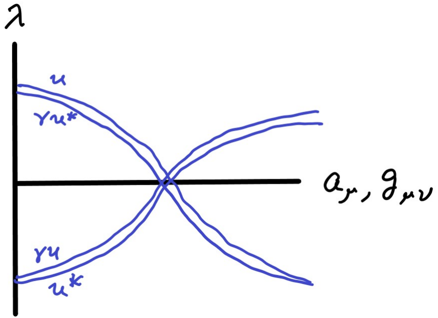
Proof of mod 2 index theorem

The Dirac operator $\not{D}$ in 2d adjoint fermions is real anti-symmetric.

$$\not{D} u = \mathrm{i}\lambda u.$$

For $\lambda \neq 0$, we get (cf. Witten 1508.04715):

$+\mathrm{i}\lambda$	u	$(\mathrm{i}\gamma_1\gamma_2)u^*$
$-\mathrm{i}\lambda$	u^*	$(\mathrm{i}\gamma_1\gamma_2)u$



$\mathbb{Z}_2$ anomalies

We can find mixed anomalies with chiral symmetry.

- With anti-periodic (AP) B.C. on T^2: $\mathcal{Z}_{\mathrm{AP/AP}} \xrightarrow{(\mathbb{Z}_2)_\chi} \mathcal{Z}_{\mathrm{AP/AP}}$.
- With periodic (P) B.C. on T^2 = Background flux on $(\mathbb{Z}_2)_F$:

$$\mathcal{Z}_{\mathrm{P/P}} \xrightarrow{(\mathbb{Z}_2)_\chi} (-1)^{N-1}\mathcal{Z}_{\mathrm{P/P}}.$$

- With AP B.C. with the minimal 't Hooft flux $\int B = \frac{2\pi}{N}$:

$$\mathcal{Z}_{\mathrm{AP/AP}}[B] \xrightarrow{(\mathbb{Z}_2)_\chi} (-1)^{N-1}\mathcal{Z}_{\mathrm{AP/AP}}[B].$$

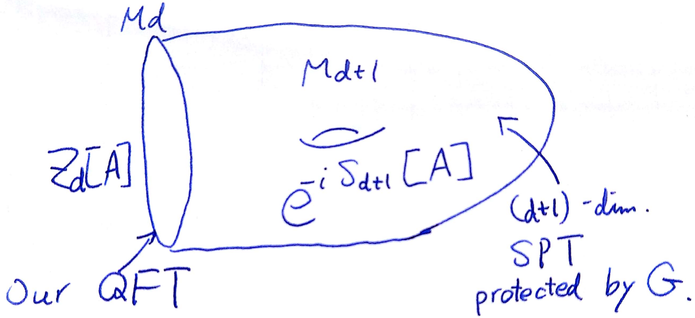
- With P/C-twisted B.C.:

$$\mathcal{Z}_{\mathrm{P}/C} \xrightarrow{(\mathbb{Z}_2)_\chi} (-1)^{N(N-1)/2}\mathcal{Z}_{\mathrm{P}/C}.$$

Anomaly matching and low-energy behaviors

Anomaly matching

We cannot gauge G with anomaly.
We regard our theory as a boundary of $(d+1)$-dim. SPT phase protected by G (Wen, '13, Kapustin, Thorngren, '14, Cho, Teo, Ryu, '14, ...) :



$\Rightarrow$ Low-energy DOF must also cancel the anomaly inflow from bulk.

Anomaly matching: Chiral symmetry breaking

Anomaly matching Low-energy DOF reproduce the same anomaly.
Possible options of low-energy physics:

- massless excitations,
- spontaneous symmetry breaking, or
- topological order.

The list of our 't Hooft anomaly is

Anomalous symmetry	$N=4n$	$4n+1$	$4n+2$	$4n+3$
$\mathbb{Z}_N^{[1]} \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_C \times (\mathbb{Z}_2)_\chi$			✓	✓

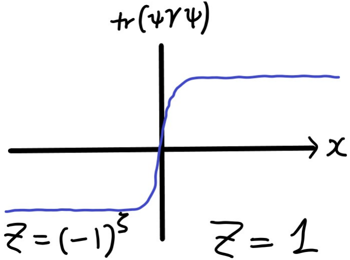
$\Rightarrow$ Chiral symmetry breaking seems to be a natural option.

(For even N, this is indeed the unique option. For $N=4n+3$, C-breaking is also a possibility)

What about deconfinement?

In the following, let's assume chiral symmetry breaking.
Can we say anything useful about confinement/deconfinement?

Yes, domain wall physics tells us that $\mathbb{Z}_N^{[1]} \to \mathbb{Z}_{N/2}^{[1]}$ is required for even N.



Massive adjoint QCD as nontrivial SPT

To see the nontrivial property of the wall, we first consider the massive deformation $m \neq 0$.

$$\mathrm{Pf}(\mathrm{i}\not{D} - m\gamma) = m^\zeta \prod_i{}' (\lambda_i^2 + m^2).$$

This means that $m < 0$ is a nontrivial SPT compared with $m > 0$ if $(-1)^\zeta = -1$:

$$\frac{\mathcal{Z}_{m=-M}}{\mathcal{Z}_{m=M}} = (-1)^\zeta.$$

Since

Domain wall $\simeq$ Boundary of nontrivial SPT,

there must be gappless excitations on the domain wall with appropriate charge.

Partial deconfinement for even N

Recall that, for even N,

$$\pi\zeta = \underbrace{\pi\zeta_{\text{free}}}_{(-1)^F} + \underbrace{\frac{N}{2}\int B}_{\mathbb{Z}_N^{[1]}}.$$

Thus, boundary excitation is fermionic, and has N-ality $N/2$.
$\Rightarrow$ In order for two vacua having the same energy density,
$N/2$-string tension must vanish:

$$\sigma_{N/2} = 0.$$

We have no symmetry reasonings for deconfinement of other strings, so that we propose

$$\sigma_k \sim N g^2 \left(1 - \cos\left(\frac{4\pi k}{N}\right)\right).$$

Semiclassical analysis

Analysis on small $\mathbb{R}\times S^1$

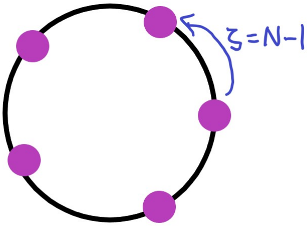
With $gL \ll 1$, the semiclassical treatment becomes reliable

(Smilga hep-th/9402066, Lenz, Shifman, Thies, hep-th/9412113)

With AP B.C., the Polyakov-loop potential has N minima,

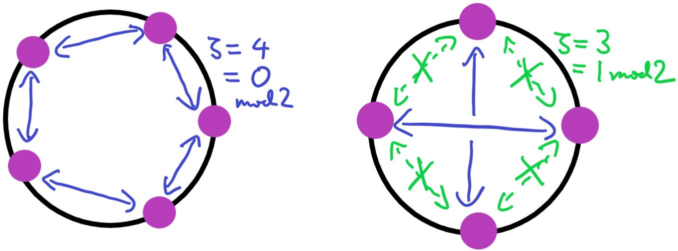
$$P = \mathrm{e}^{2\pi \mathrm{i} k/N} \quad (k = 0, 1, \dots, N-1).$$

Tunneling between them is associated with fermionic zero modes with $\zeta = N - 1$.



Tunneling and Mod 2 index

Note that the fermionic zero modes ζ is protected only mod 2. Tunneling is possible if $\zeta = 0$ mod 2.



- Odd $N \Rightarrow$ Unique ground state. No SSB.
- Even $N \Rightarrow$ Two vacua: Chiral SSB. Also,

$$\sigma_k = \frac{\Delta E}{L}\left(1 - \cos\frac{4\pi k}{N}\right).$$

Brief comments on previous results

Almost all studies before us claim that
(Gross, Klebanov, Matytsin, Smilga hep-th/9511104, ...)

- Chiral symmetry must be always broken,
- Complete deconfinement $\mathbb{Z}_N \to 1$ must always happen.

We find no symmetry reasonings to claim these results (especially the second one).
The reason why the results disagree is that the following point was missed:

- fermionic zero modes are protected only mod 2 not by integers.

Summary

- We revisit the vacuum structures of 2d adjoint QCD in view of recent developments of 't Hooft anomaly matching.
- Minimal requirement of anomaly matching is
  - Chiral SSB for $N = 4n, 4n+2, 4n+3$.
  - Partial deconfinement $\mathbb{Z}_N^{[1]} \to \mathbb{Z}_{N/2}^{[1]}$ for even N.
- Previous studies, starting from GKMS, have claimed too strong results in view of symmetry.

Backups

$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_\chi$ anomaly

Put our theory on T^2, then we can choose the fermion BC as periodic (P) or anti-periodic (AP) on each direction.

Since $(-1)^\zeta$ is topological, it is sufficient to compute ζ for free Dirac fermions, $a_\mu = 0$:

- For AP/AP, AP/P b.c.
$$\zeta = 0.$$
- For P/P b.c.,
$$\zeta = N^2 - 1.$$

Thus, $(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_\chi$ anomaly $(-1)^\zeta = -1$ is present for even N.

$\mathbb{Z}_N^{[1]} \times (\mathbb{Z}_2)_\chi$ anomaly

Start from AP/AP b.c. on T^2. Adding minimal 't Hooft flux,

$$\psi(x_1+1,x_2) = -\Omega_1(x_2)^\dagger \psi(x_1,x_2)\Omega_1(x_2),$$
$$\psi(x_1,x_2+1) = -\Omega_2(x_1)^\dagger \psi(x_1,x_2)\Omega_2(x_1),$$

with

$$\Omega_1(x_2+1)\Omega_2(x_1) = \mathrm{e}^{-2\pi\mathrm{i}/N}\Omega_2(x_1+1)\Omega_1(x_2).$$

Solving Dirac equation with this b.c. in a good setup, we find

$$\zeta = \begin{cases} 1 & \text{even} N, \\ 0 & \text{odd} N. \end{cases}$$

For odd N, we find no anomaly so far. For even N,

$$\pi\zeta = \underbrace{\pi\zeta_{\mathrm{free}}}_{(-1)^F} + \underbrace{\frac{N}{2}\int B}_{\mathbb{Z}_N^{[1]}}.$$

Anomaly for odd N

So far, no anomaly is found for odd N.
Using the P/C-twisted b.c., only off-diagonal fermions can be gapless, so that

$$\zeta = \frac{N(N-1)}{2}.$$

$\zeta = 1$ mod 2 for $N = 4n+3$, so this is $(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_C \times (\mathbb{Z}_2)_\chi$ anomaly.

Anomaly \ N	$4n$	$4n+1$	$4n+2$	$4n+3$
$\mathbb{Z}_N^{[1]} \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_\chi$	✓		✓	
$(\mathbb{Z}_2)_F \times (\mathbb{Z}_2)_C \times (\mathbb{Z}_2)_\chi$			✓	✓

Objections to previous studies

Our result disagrees with previous studies. They claim

$$\sigma \sim mg,$$

and the complete deconfinement happens with massless adjoint fermions.

We here argue that this claim by previous studies cannot be justified.

Summary of previous studies and objections

Arguments	Our objections
Kutasov-Schwimmer universality maps adjoint QCD to N-flavor fundamental QCD. String breaking thus should happen for any reps.	Universality applies only for massive flavor-singlet mesons. One cannot use it to identify ground states.
$SU(N)/\mathbb{Z}_N$ gauge fields has N topological sectors. They may be disconnected by fermionic zero modes.	Number of zero modes are protected only mod 2. Having 2 disconnected sectors is natural.
Chiral rotation can eliminate the fractional charges at infinities	Possible chiral rotation is $\mathbb{Z}_2$ for Majorana fermion.

$\Rightarrow$ Previous study shows no evidence that deconfinement happens.